6.s02: EECS II - From A Medical Perspective

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1.
   - Area of probability distribution sums up to one, so area of triangle is one.

     $1 = \frac{1}{2}(b \cdot h) = \frac{1}{2}(4 \cdot a)$

     $a = \frac{1}{2}$

   - Mean of distribution

     $$\mu_x = \int_{-\infty}^{\infty} x f_x(x) dx$$
     $$\mu_x = \int_0^2 x\left(\frac{x}{4}\right) dx + \int_2^4 x\left(1 - \frac{x}{4}\right) dx$$
     $$\mu_x = \tfrac{2}{3} + \tfrac{4}{3} = 2$$

   - Variance of distribution

     $$\sigma_x^2 = \int_{-\infty}^{\infty} (x - \mu_x)^2 f_x(x) dx$$
     $$\sigma_x^2 = \int_0^2 (x-2)^2 \left(\frac{x}{4}\right) dx + \int_2^4 (x-2)^2 \left(1 - \frac{x}{4}\right) dx$$
     $$\sigma_x^2 = \tfrac{1}{3} + \tfrac{1}{3} = \tfrac{2}{3}$$

2. (a) 4
   (b) 3
   (c) 5
   (d) 2
   (e) 6
   (f) 1
   (g) 7
   (h) The patient moves toward the E region. For the top C the patient is measured at a higher level than they have and will move to an even lower level. For the bottom C region the patient is measured at a lower level than they have and will move to an even higher level.

3. (a) $\sin\left(\frac{6\pi}{7}(n+N)\right) = \sin\left(\frac{6\pi}{7}n + \frac{6\pi}{7}N\right)$

$\sin\left(\frac{6\pi}{7}n + \frac{6\pi}{7}N\right) = \sin\left(\frac{6\pi}{7}n + 2\pi k\right)$

$\therefore \frac{6\pi}{7}N = 2\pi k$

$N = 7$ (satisfied by $k = 3$)

(b) $\sin\left(\frac{6\pi}{7}(n+N) + 1\right) = \sin\left(\left(\frac{6\pi}{7}n + \frac{6\pi}{7}N\right) + 1\right)$

The one is just a shift and does not affect the period, so we can ignore it. Then the equation becomes the same as the above.

$\sin\left(\frac{6\pi}{7}n + \frac{6\pi}{7}N\right) = \sin\left(\frac{6\pi}{7}n + 2\pi k\right)$

$\therefore \frac{6\pi}{7}N = 2\pi k$

$N = 7$ (satisfied by $k = 3$)

(c) $\cos\left(\frac{6(n+N)}{7} - \pi\right) = \cos\left(\left(\frac{6n}{7} + \frac{6N}{7}\right) - \pi\right)$

As before, we can ignore the shift term by dropping π from our concerns.

$\cos\left(\frac{6n}{7} + \frac{6N}{7}\right) = \cos\left(\frac{6n}{7} + 2\pi k\right)$

$\therefore \frac{6N}{7} = 2\pi k$

$N = \frac{7}{3}\pi k$

No integer value of k satisfies, so it is not a DT periodic signal.

(d) $\cos\left(\frac{\pi}{2}(n+N)\right)\cos\left(\frac{\pi}{4}(n+N)\right) = \cos\left(\frac{\pi}{2}n + \frac{\pi}{2}N\right)\cos\left(\frac{\pi}{4}n + \frac{\pi}{4}N\right)$

$\cos\left(\frac{\pi}{2}n + \frac{\pi}{2}N\right)\cos\left(\frac{\pi}{4}n + \frac{\pi}{4}N\right) = \cos\left(\frac{\pi}{2}n + 2\pi k_1\right)\cos\left(\frac{\pi}{4}n + 2\pi k_2\right)$

$\therefore \frac{\pi}{2}N = 2\pi k_1$ and $\frac{\pi}{4}N = 2\pi k_2$

$N = 8$ (satisfied by $k_1 = 2$ and $k_2 = 1$)

4. (a) $X[k] = \frac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi}{N}kn}$

$$X[k] = \frac{1}{6}\sum_{n=0}^{5}\left(1+\cos\left(\frac{2\pi}{6}n\right)\right)e^{-j\frac{2\pi}{6}kn}$$

Solved with WolframAlpha

$$X[k] = \tfrac{1}{4}e^{\left(-5j\frac{\pi}{3}k\right)} + \tfrac{1}{12}e^{\left(-4j\frac{\pi}{3}k\right)} + \tfrac{1}{12}e^{\left(-2j\frac{\pi}{3}k\right)} + \tfrac{1}{4}e^{\left(\frac{\pi}{3}(-k)\right)} + \tfrac{1}{3}$$

(b) $X[k] = \frac{1}{8}\sum_{n=0}^{3} e^{-j\frac{2\pi}{8}kn} - \frac{1}{8}\sum_{n=4}^{7} e^{-j\frac{2\pi}{8}kn}$

5. $$x[n] = \sum_{k=-\frac{N}{2}}^{\frac{N}{2}-1} X[k]\, e^{j\frac{2\pi}{N}kn}$$

$$x[n] = \sum_{k=-4}^{3} X[k]\, e^{j\frac{2\pi}{7}kn}$$

$$x[n] = -\frac{1}{2j}e^{j\frac{-4\pi}{7}n} + e^{j\frac{-2\pi}{7}n} + e^{j\frac{2\pi}{7}n} + \frac{1}{2j}e^{j\frac{4\pi}{7}n}$$

$$x[n] = \frac{1}{2j}\left(-e^{j\frac{-4\pi}{7}n} + e^{j\frac{4\pi}{7}n}\right) + 1\left(e^{j\frac{-2\pi}{7}n} + e^{j\frac{2\pi}{7}n}\right)$$

$$x[n] = \sin\left(\frac{4\pi}{7}n\right) + 2\cos\left(\frac{2\pi}{7}n\right)$$