6.s02: EECS II - From A Medical Perspective

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1. (a) $M_{xy}(t) = M_{xy}(0)e^{-t/T_2}$
   $\therefore M_{xyA}(t) = M_{xy}(0)e^{-t/T_2A}$

   $M_z(t) = M_0 + (M_z(0) - M_0)e^{-t/T_1}$
   No component of field in longitudinal direction after excitation.
   $\therefore M_{zA}(t) = M_0 - M_0e^{-t/T_{1A}}$

   (b)

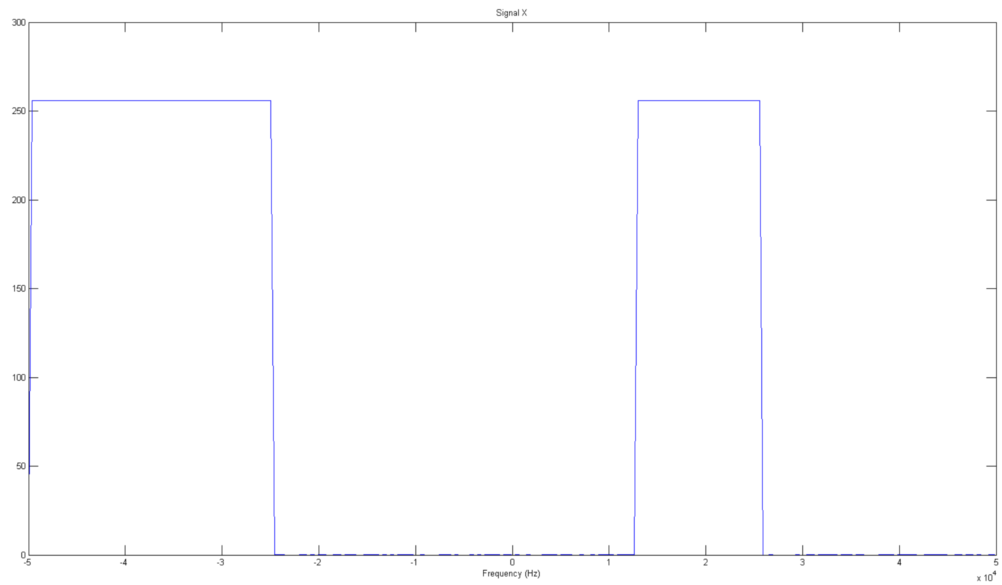
$$\begin{aligned} |\Delta S_{xy}(t)| &= |M_{xyA}(t) - M_{xyB}(t)| \\ &= M_0e^{-t/T_{2A}} - M_0e^{-t/T_{2B}} \\ \text{Maximize} \\ 0 &= \frac{d}{dt}M_0\left(e^{-t/T_{2A}} - e^{-t/T_{2B}}\right) \\ t &= \left(\frac{1}{T_{2A}} - \frac{1}{T_{2B}}\right)^{-1} \ln\left(\frac{T_{2B}}{T_{2A}}\right) \end{aligned}$$

   (c)

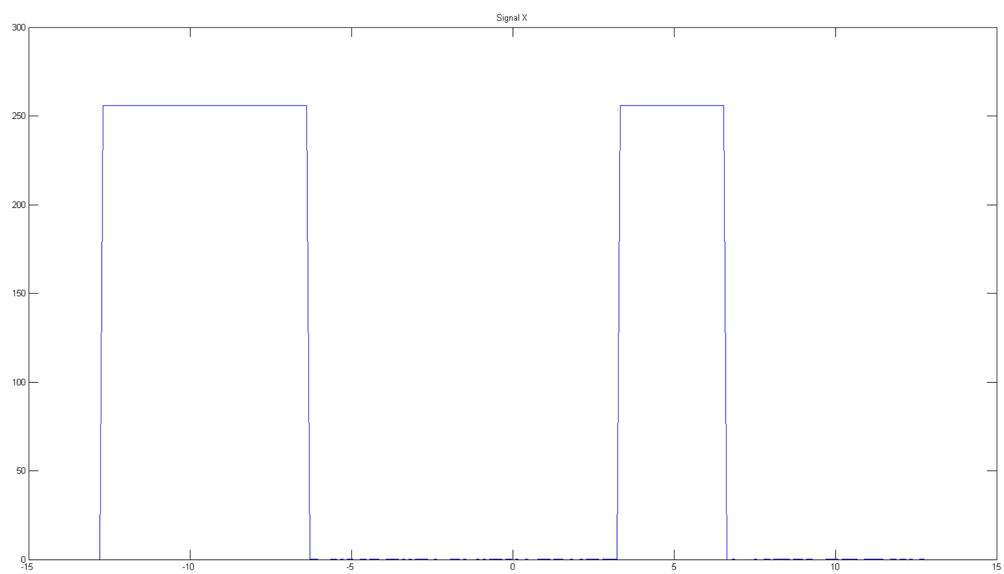
$$\begin{aligned} |\Delta S_z(t)| &= |M_{zA}(t) - M_{zB}(t)| \\ &= \left(M_0 - M_0e^{-t/T_{1A}}\right) - \left(M_0 - M_0e^{-t/T_{1B}}\right) \\ \text{Maximize} \\ t &= \left(\frac{1}{T_{1A}} - \frac{1}{T_{1B}}\right)^{-1} \ln\left(\frac{T_{1B}}{T_{1A}}\right) \end{aligned}$$

   (d) Transverse: $\left(\frac{1}{92} - \frac{1}{100}\right)^{-1} \ln\left(\frac{100}{92}\right) = 95.88s$
   Longitudinal: $\left(\frac{1}{680} - \frac{1}{810}\right)^{-1} \ln\left(\frac{810}{680}\right) = 741.21s$

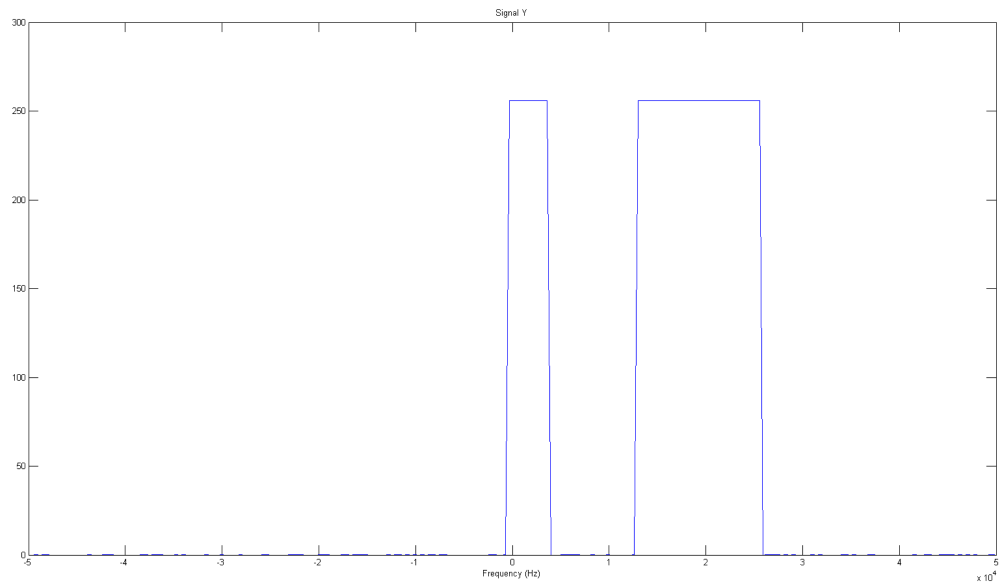
2. (a)

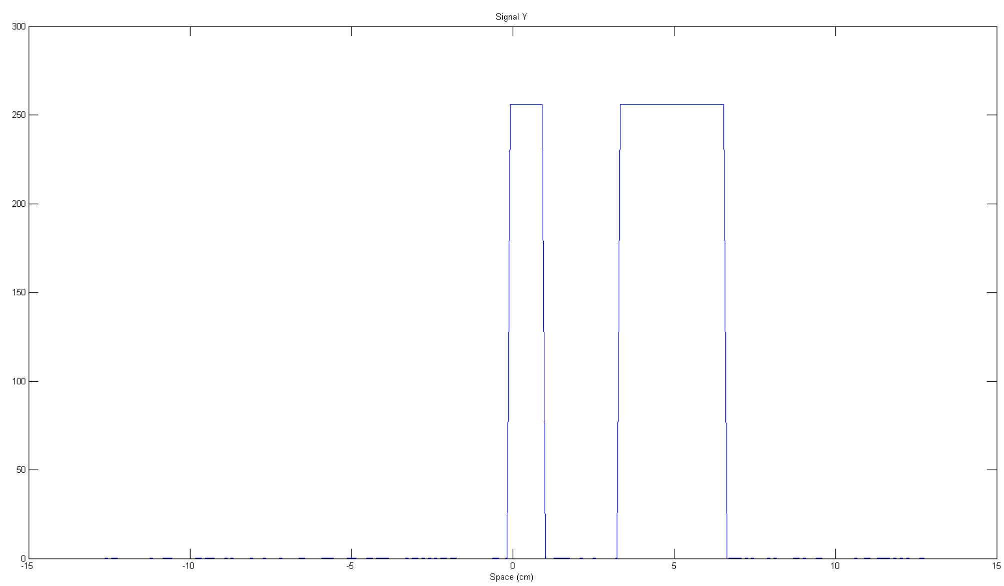

Signal X
300
250
200
150
100
50
0
-5
-4
-3
-2
-1
0
1
2
3
4
5
Frequency (Hz)
x 10^4

(b)


Signal X
300
250
200
150
100
50
0
-15
-10
-5
0
5
10
15
Space (cm)

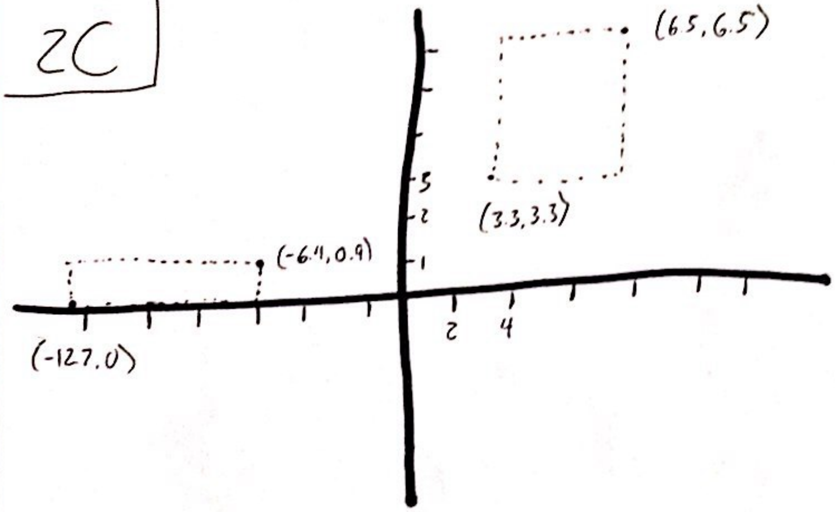
(c)


Signal Y
300
250
200
150
100
50
0
-5
-4
-3
-2
-1
0
1
2
3
4
5
Frequency (Hz)
x 10^4

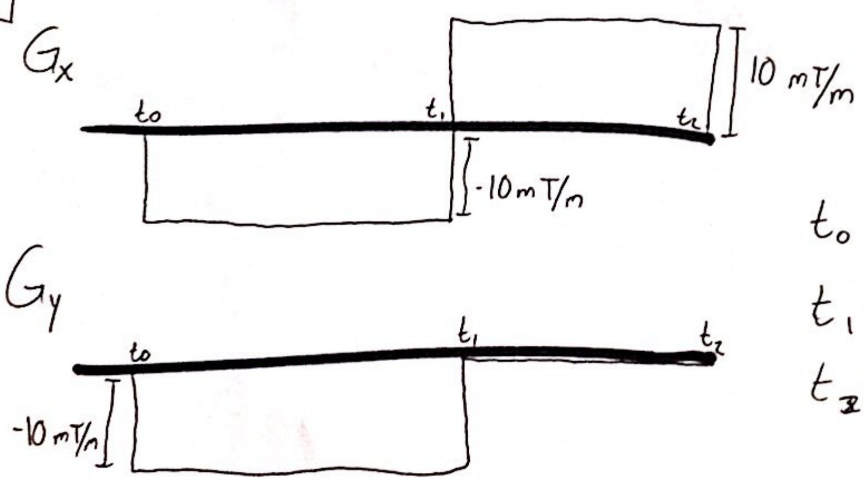

Signal Y
300
250
200
150
100
50
0
-15
-10
-5
0
5
10
15
Space (cm)

```
f = linspace(-50000, 100000*(1-1/256)/2, 256);
xfft = fftshift(fft(signal_x));
yfft = fftshift(fft(signal_y));
gamma = 42577;
G = 9.176;
dist = f*100 / (G*gamma);
% Frequency X
plot(f, abs(xfft))
xlabel('Frequency (Hz)')
title('Signal X')
% Frequency Y
plot(f, abs(yfft))
xlabel('Frequency (Hz)')
title('Signal Y')
% Space X
plot(dist, abs(xfft))
xlabel('Space (cm)')
title('Signal X')
% Space Y
plot(dist, abs(yfft))
xlabel('Space (cm)')
title('Signal Y')
```

2C



3



$t_0 = 0$

$t_1 = 0.001\ s = 1\ ms$

$t_2 = 0.003\ s = 3\ ms$

$$\frac{\gamma}{2\pi} \cdot G \cdot t = \text{displacement} \Rightarrow t = \frac{\text{displacement}}{\frac{\gamma}{2\pi} \quad G}$$

$$t = \frac{(4.257\ cm^{-1})}{(42.57 \times 10^4\ Hz/mT)\ (10\ mT/m)\left(\frac{1m}{100\ cm}\right)} = 1\ ms$$