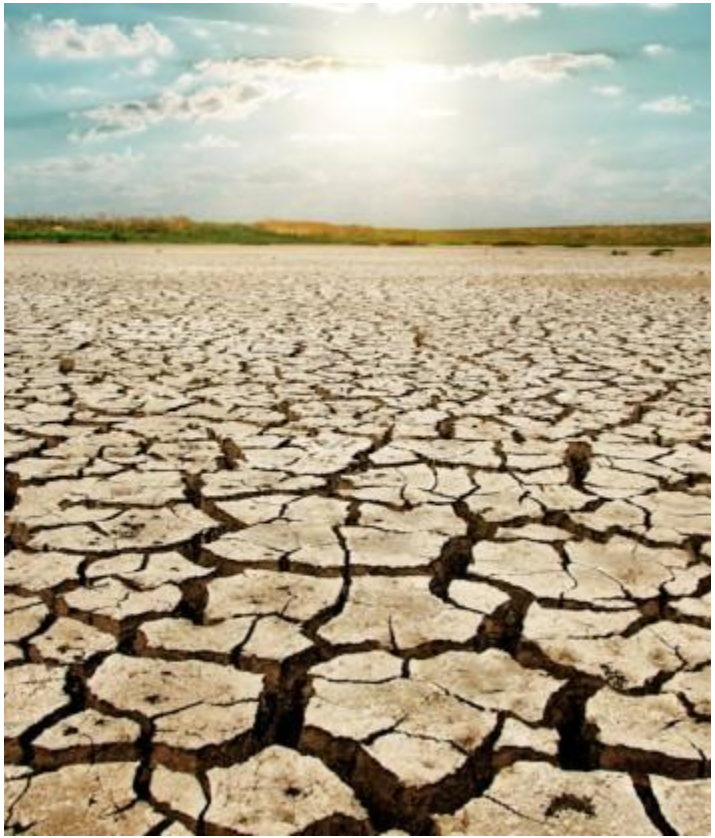
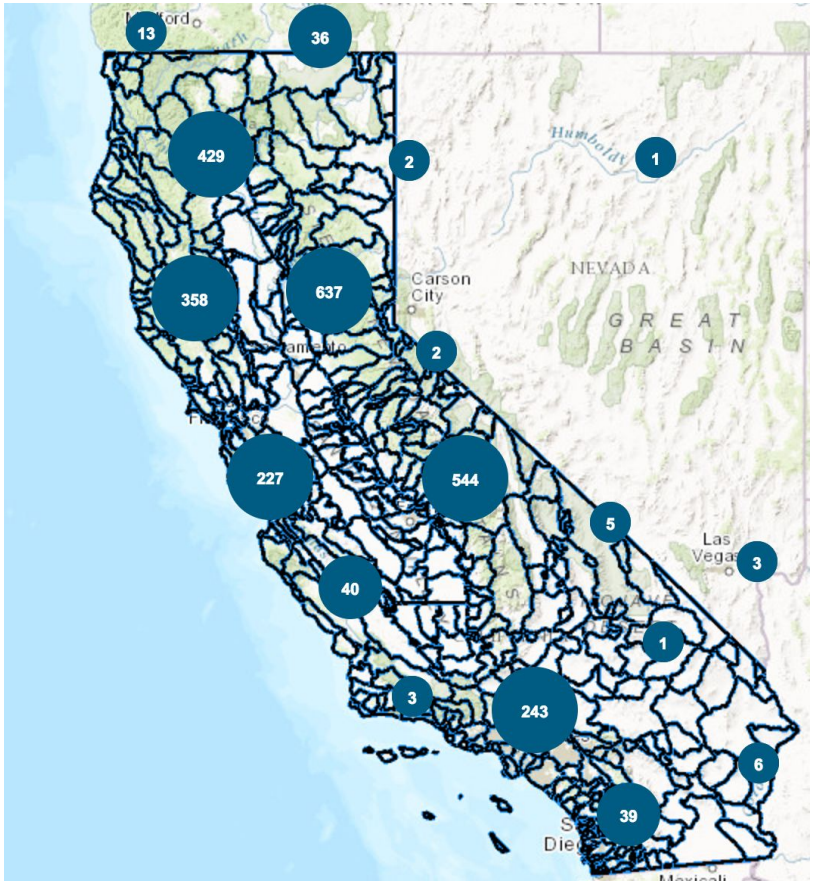
How to fit a nonlinear mixed model?

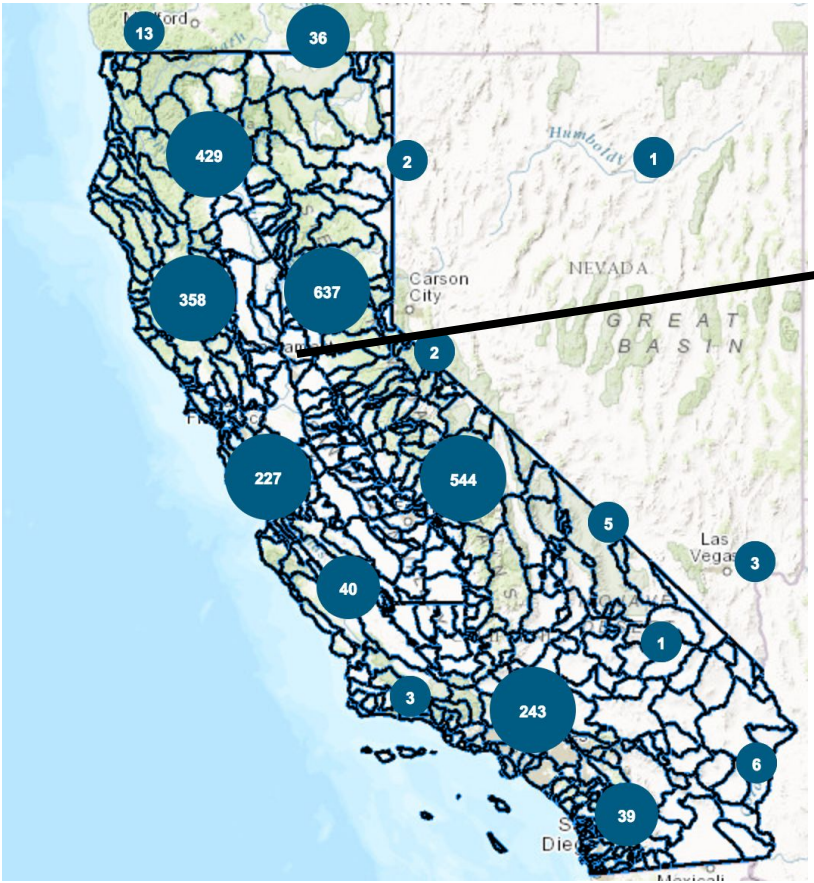
Page Piccinini & Eric Kramer

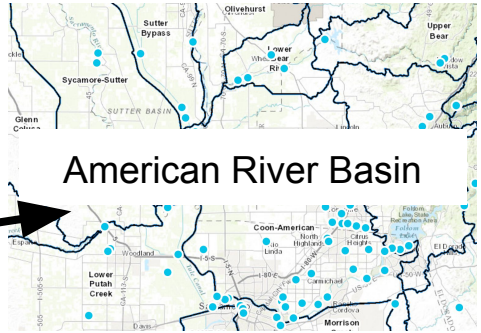


?

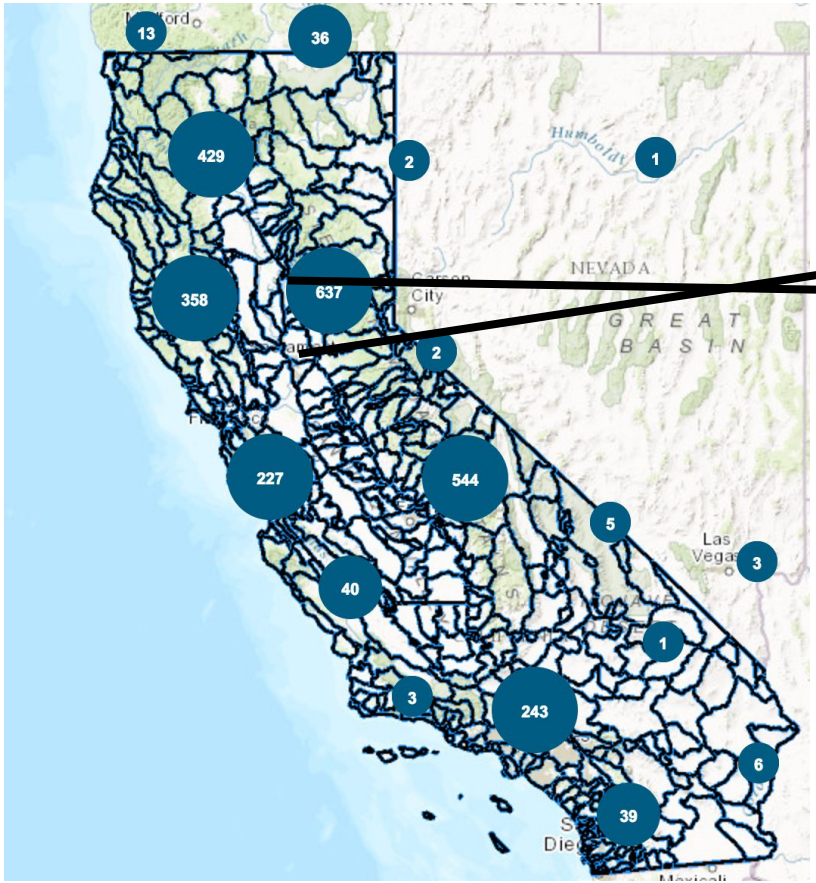


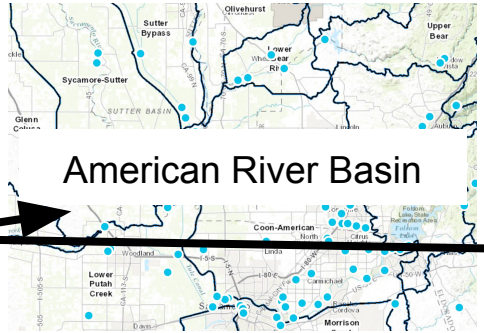
Data from California Data Exchange Center, Department of Water Resources (http://cdec.water.ca.gov)





Data from California Data Exchange Center, Department of Water Resources (http://cdec.water.ca.gov)

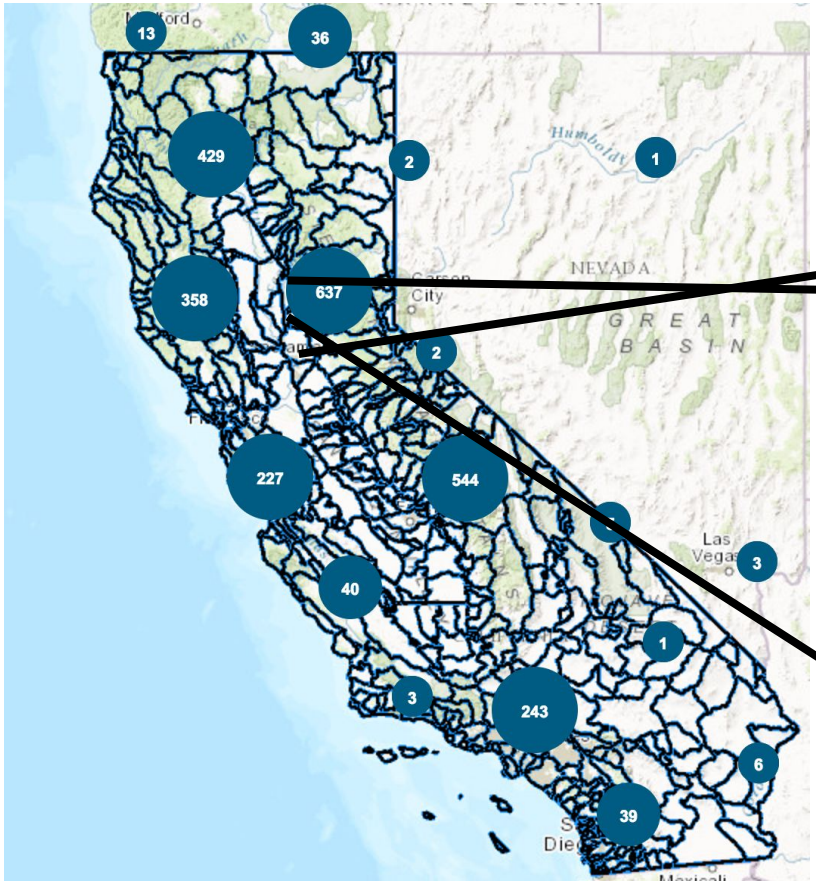


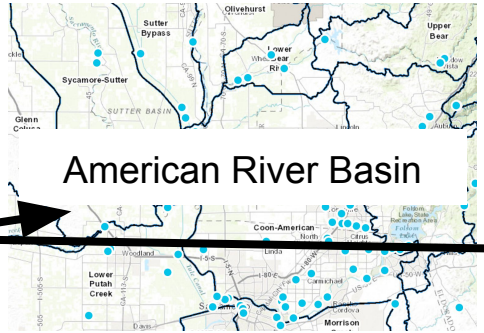


American River Basin

Sacramento River Basin

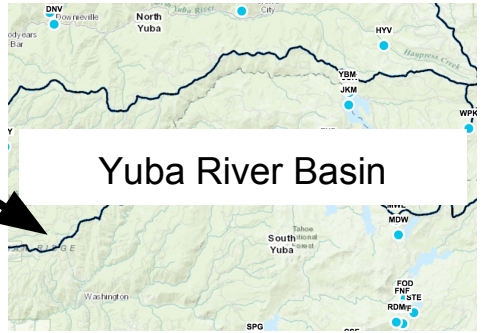
Data from California Data Exchange Center, Department of Water Resources (http://cdec.water.ca.gov)





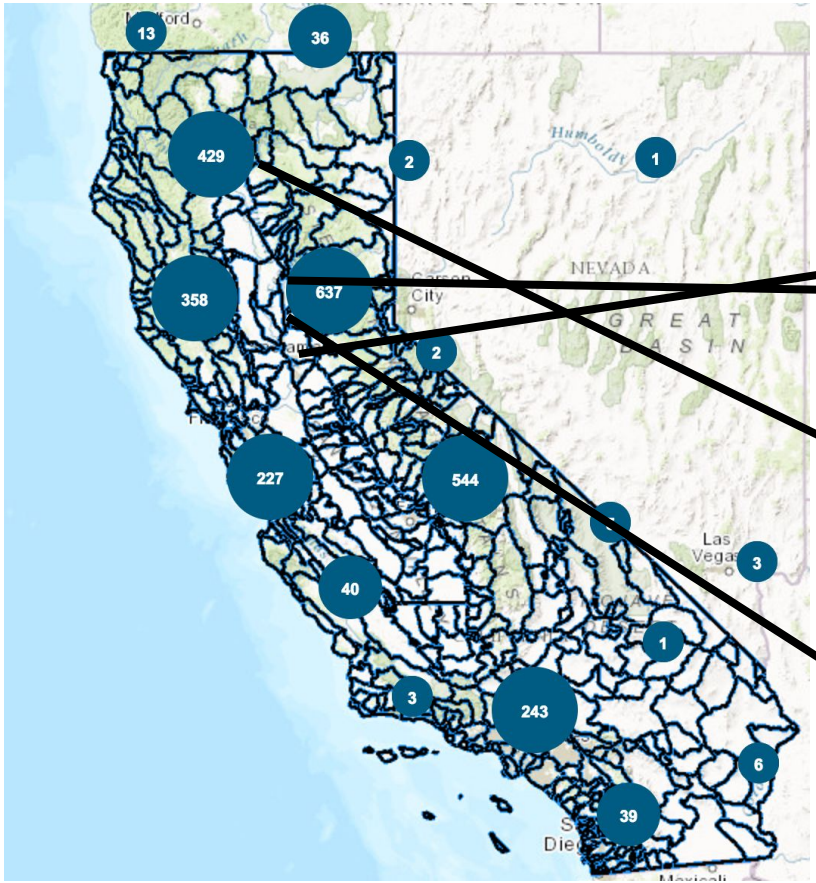
American River Basin

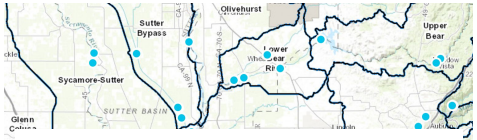
Sacramento River Basin



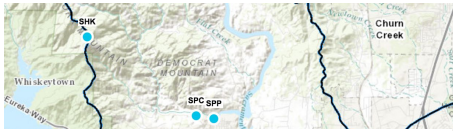
Yuba River Basin

Data from California Data Exchange Center, Department of Water Resources (http://cdec.water.ca.gov)





American River Basin

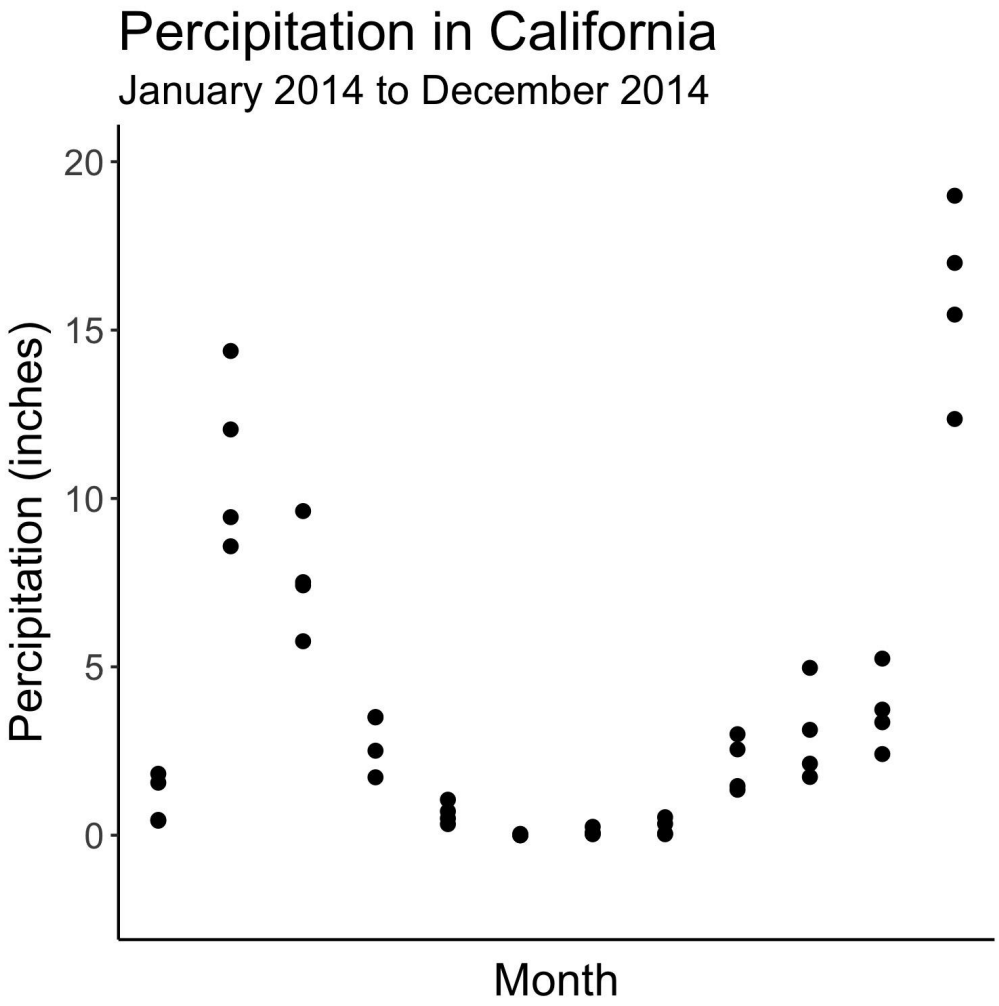


Sacramento River Basin

Yuba River Basin

McCloud River Basin

Data from California Data Exchange Center, Department of Water Resources (http://cdec.water.ca.gov)


Percipitation in California
January 2014 to December 2014
Percipitation (inches)
20
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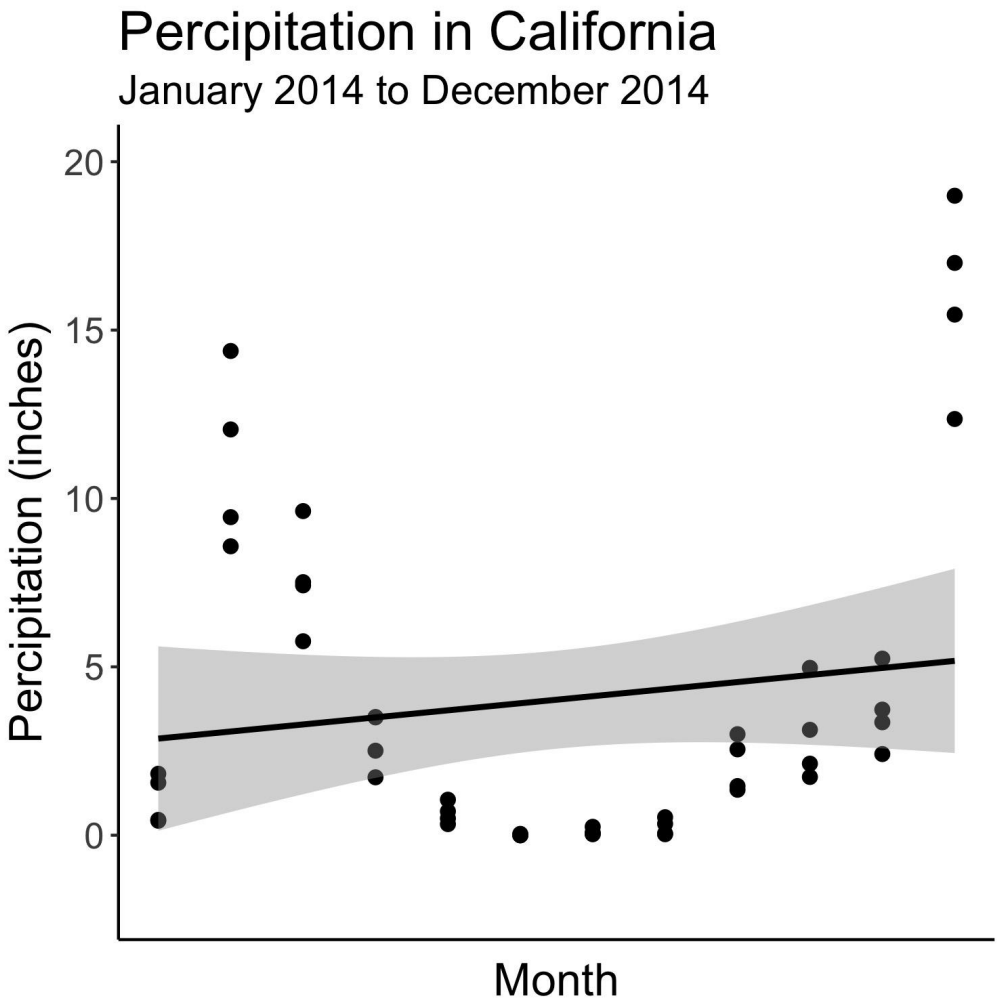
Month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January 2014 to December 2014
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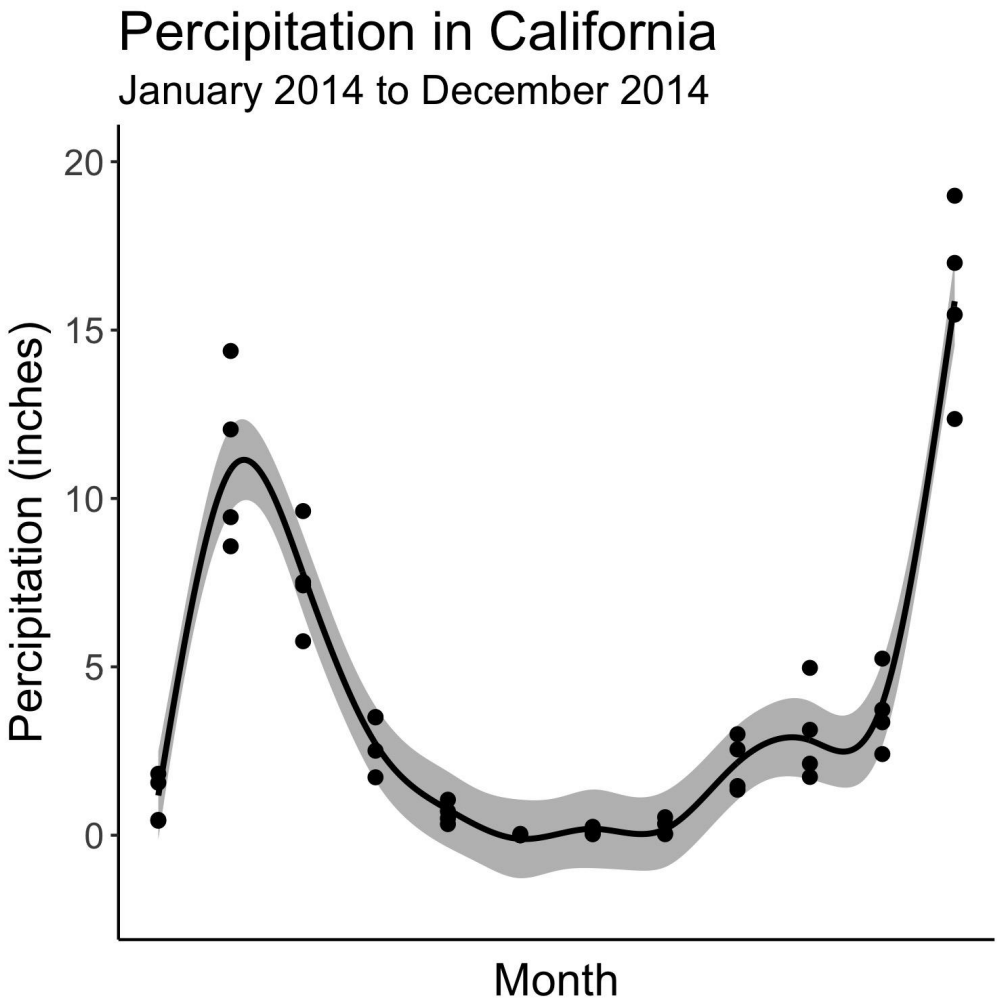
Percipitation in California
January 2014 to December 2014
Percipitation (inches)
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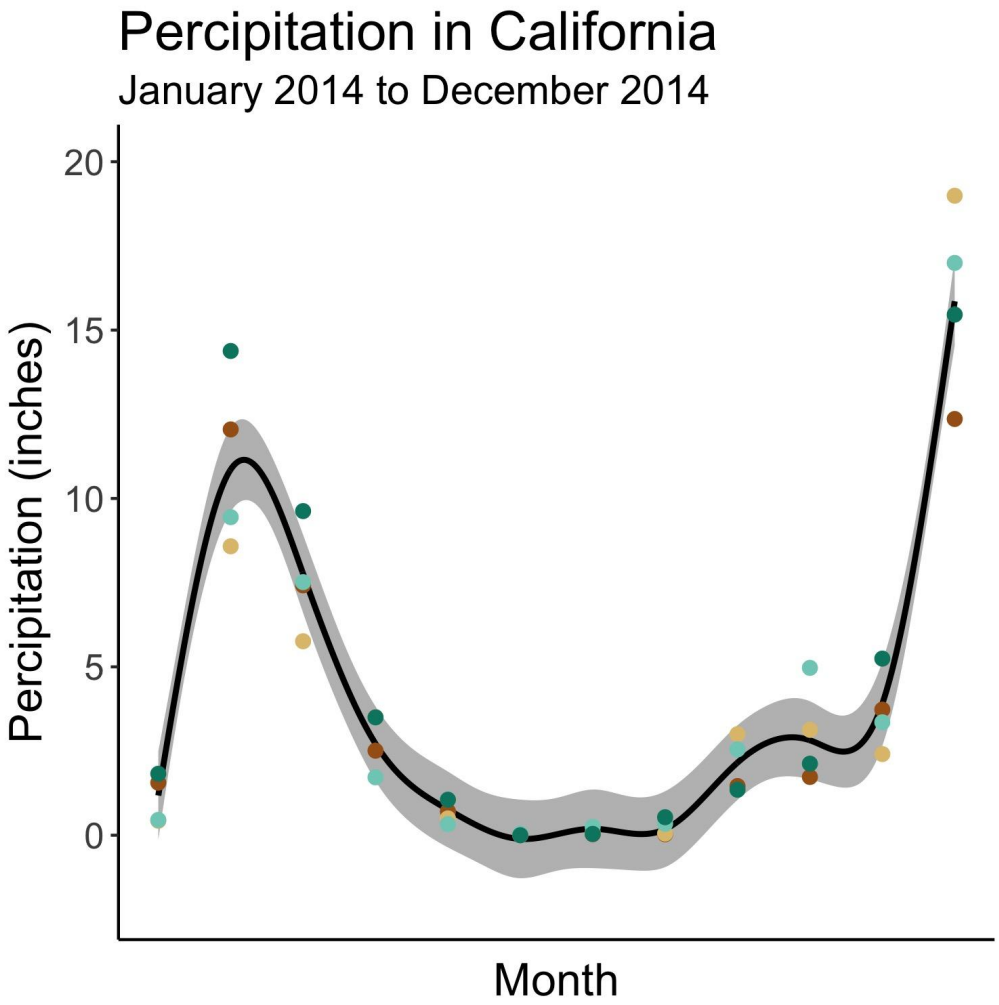
Month


Percipitation in California
January 2014 to December 2014
Percipitation (inches)
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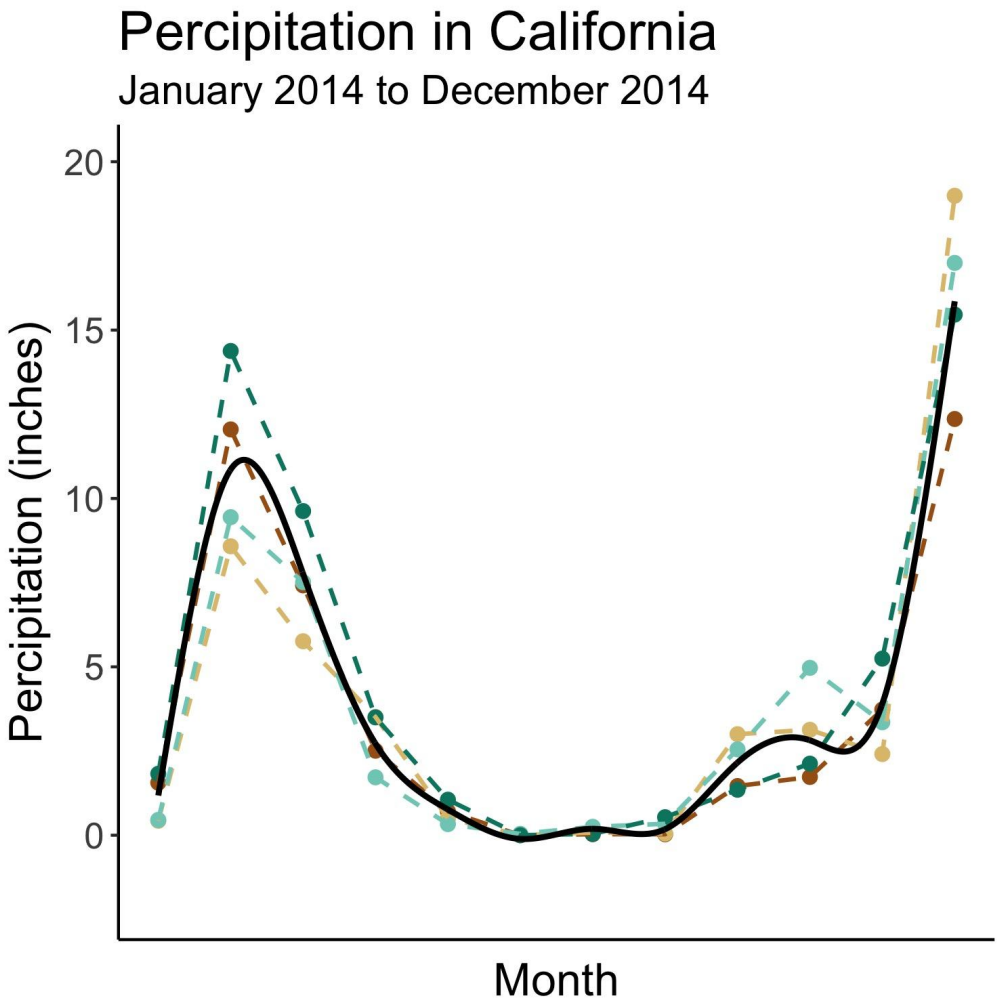
Month


Percipitation in California
January 2014 to December 2014
Percipitation (inches)
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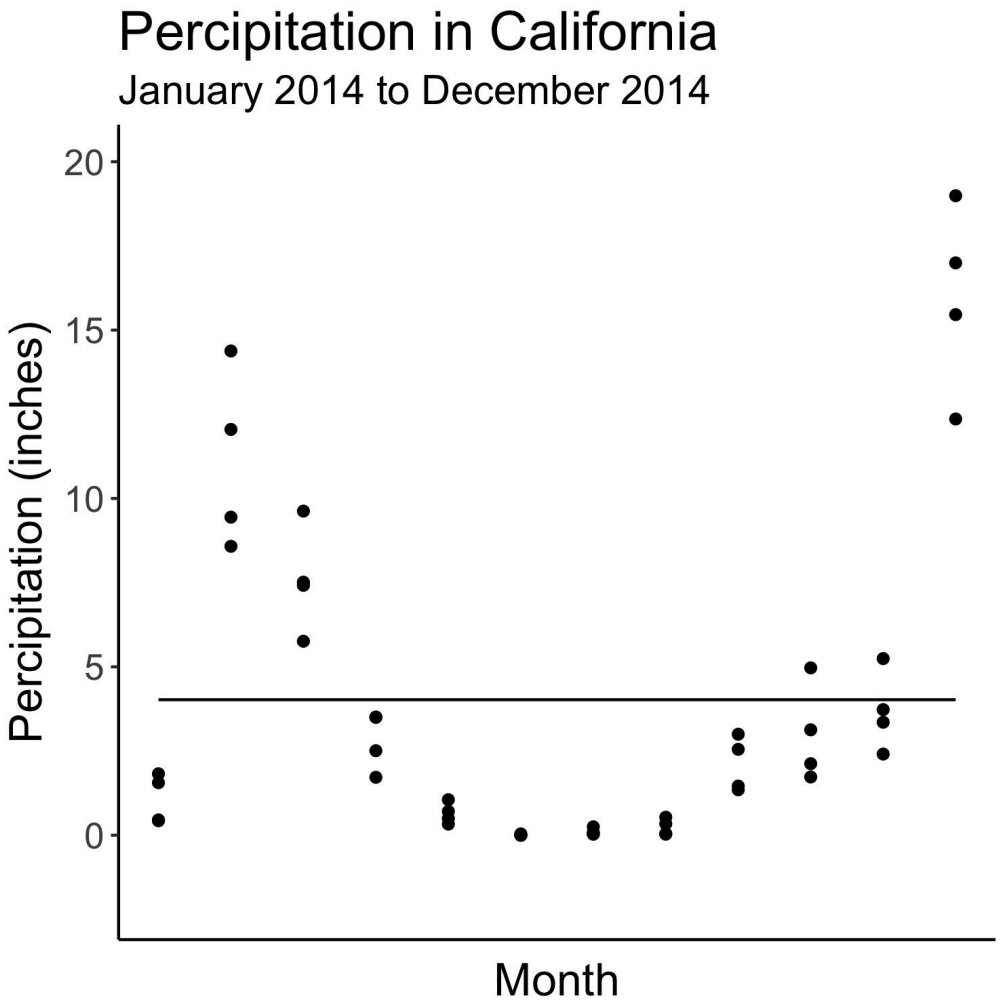
Month

Nonlinear Models


Percipitation in California
January 2014 to December 2014
Percipitation (inches)
20
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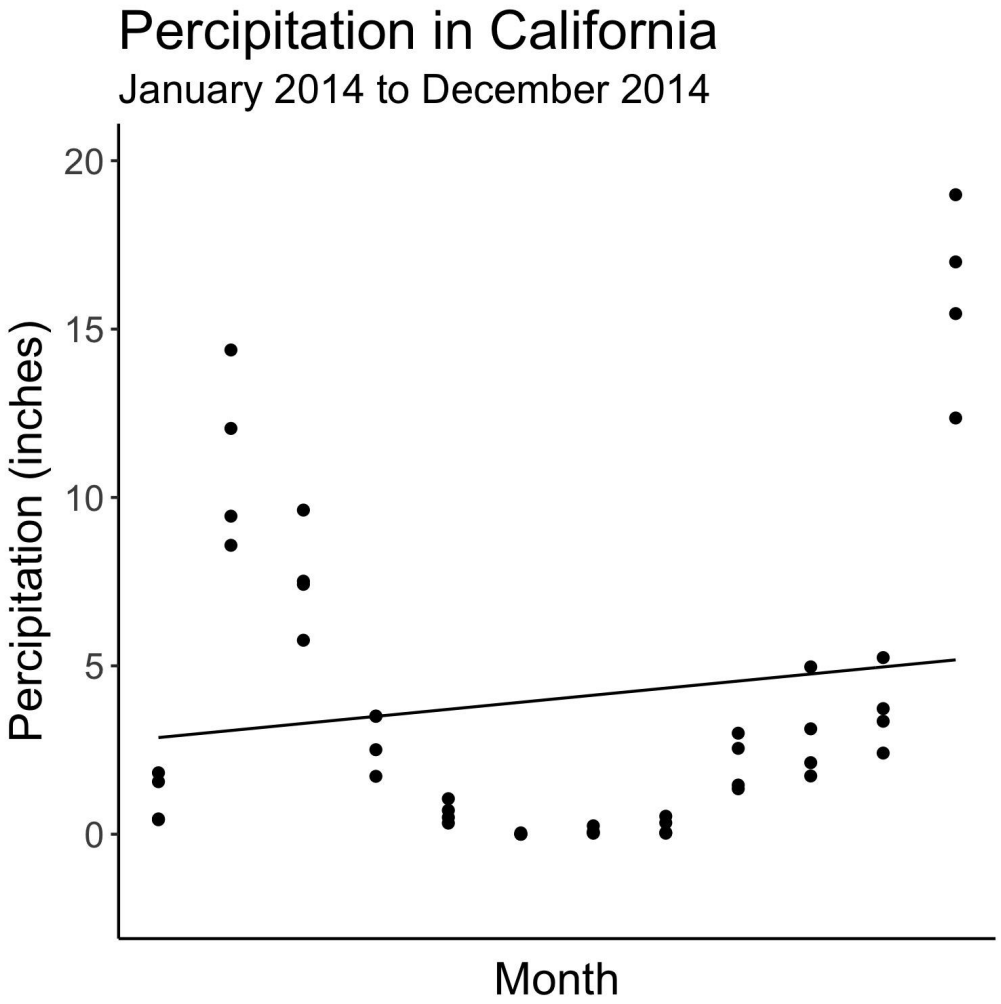
Month

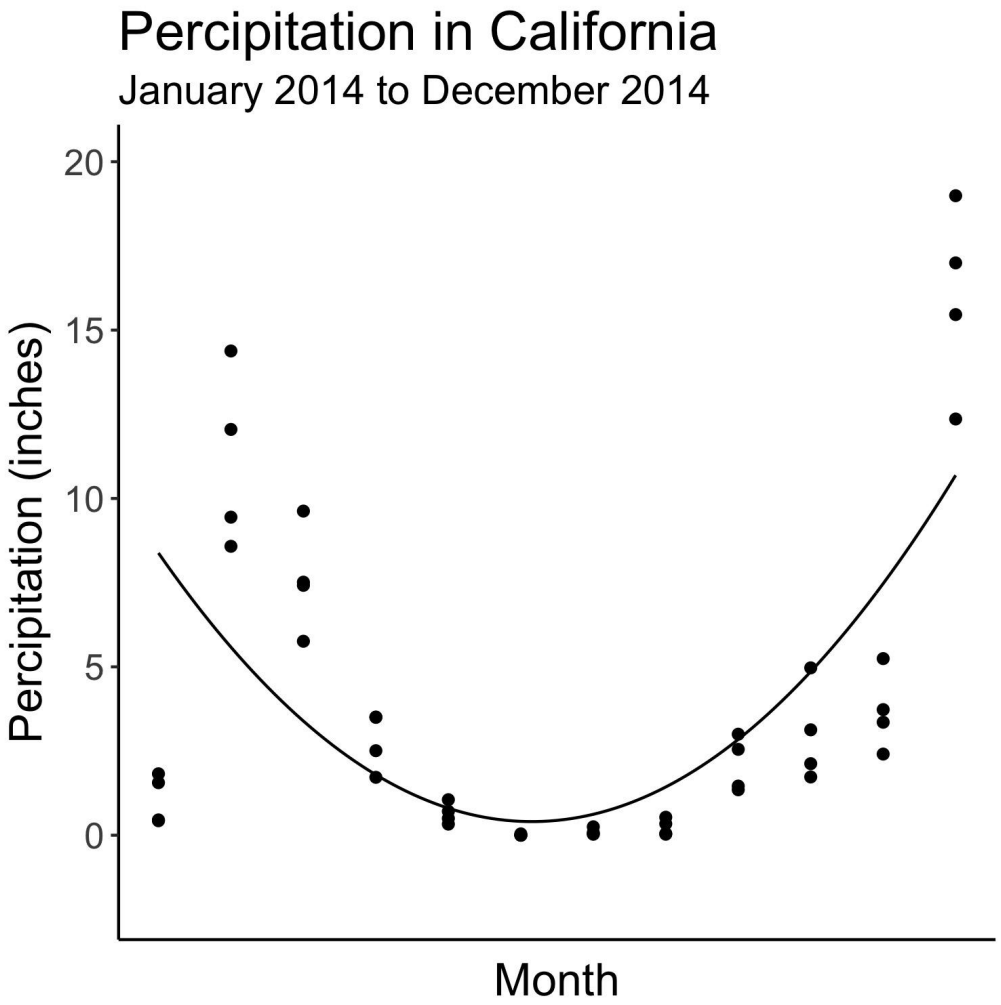
$$y = \beta_0$$

```
lm(y ~ 1)
```



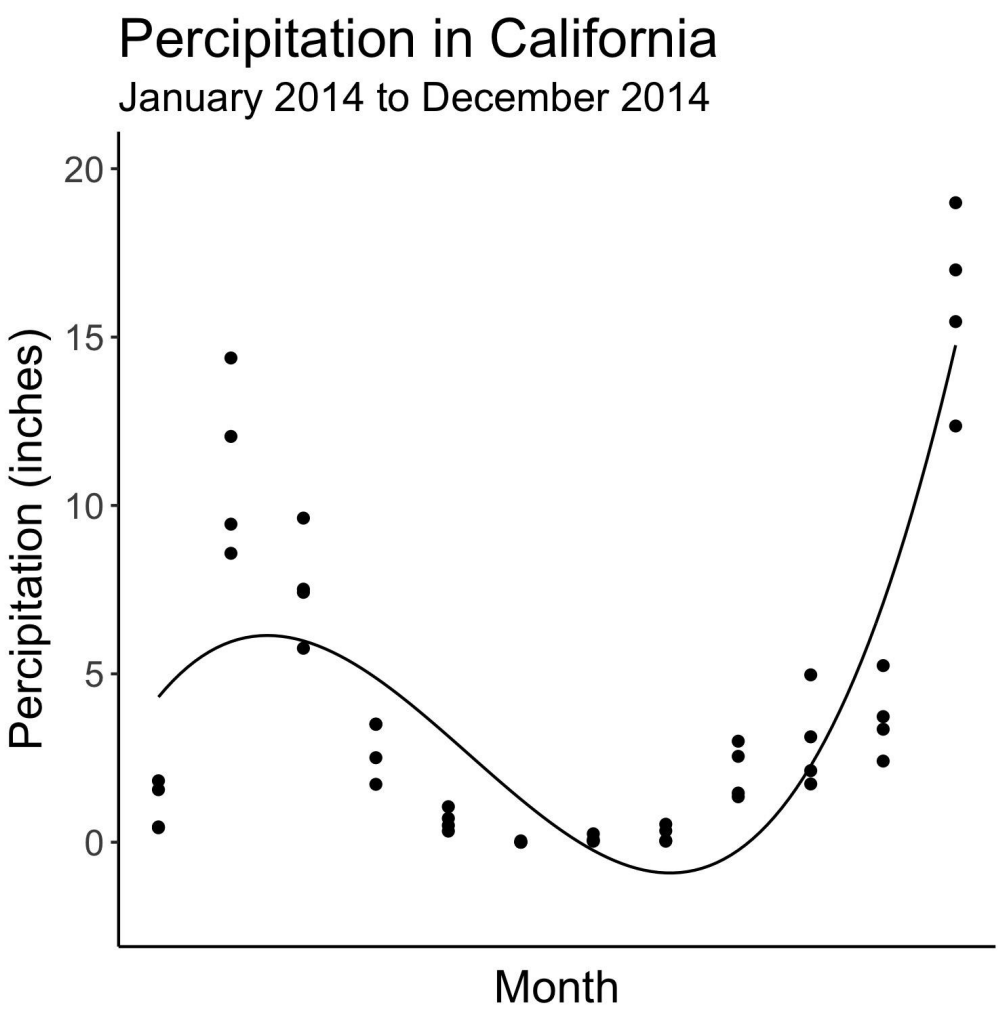
$$y = \beta_0 + \beta_1 x$$

```
lm(y ~ x)
```



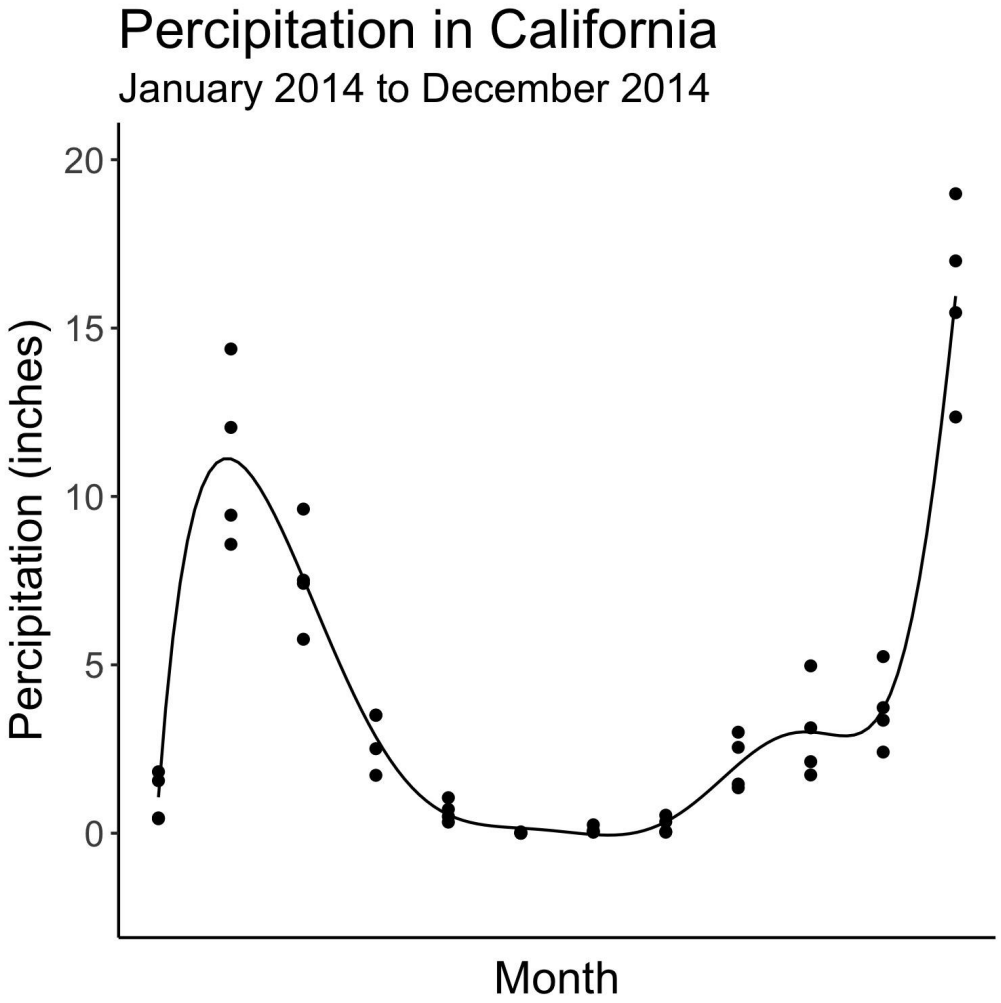
$$y = \beta_0 + \beta_1 x + \beta_2 x^2$$

```
lm(y ~ x +
       I(x^2))
```



$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3$$

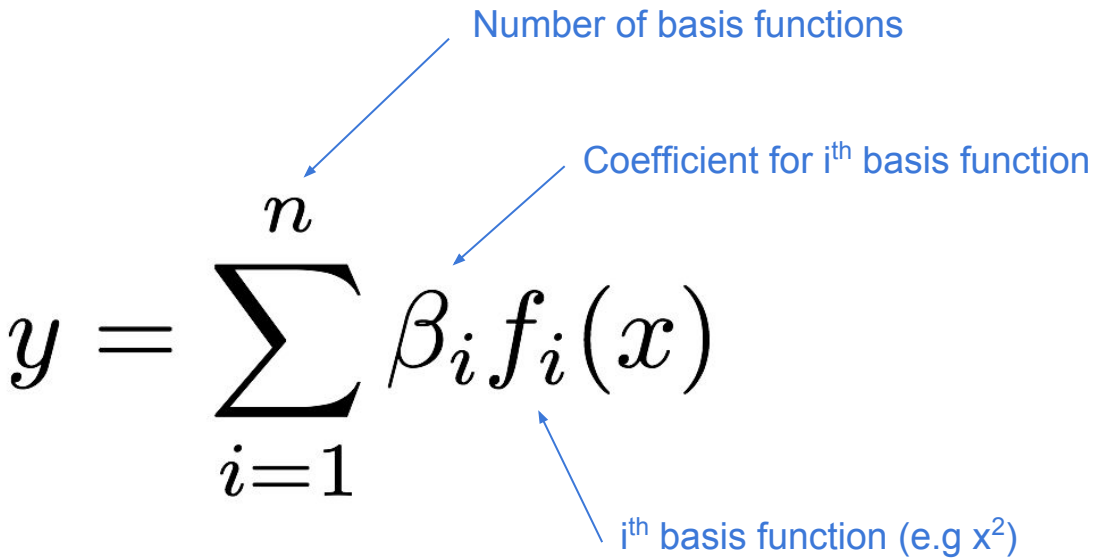
```
lm(y ~ x +
       I(x^2) +
       I(x^3))
```



$$y = \sum_{i=0}^{10} \beta_i x^i$$

```
lm(y ~ x +
       I(x^2) +
       I(x^3) +
       I(x^4) +
          ... )
```

Generalization



Questions

How do I choose n ?

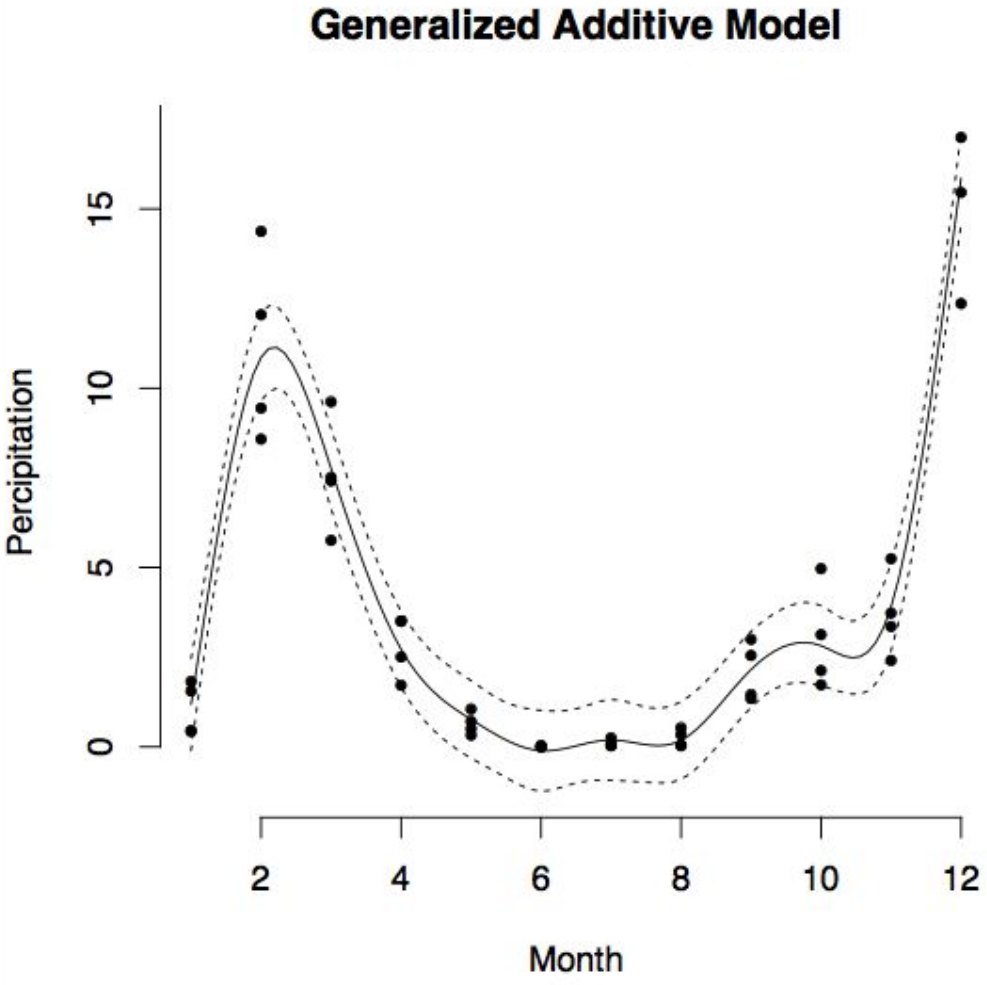
Try several values. Balance goodness-of-fit with generalizability

How do I choose $f_i(x)$?

Use thin plate splines or cubic splines -- don't use polynomials!

Should I do this manually with `lm`?

No! Use `gam` from the `mgcv` package



No longer using `lm`

Use “thin plate” bases

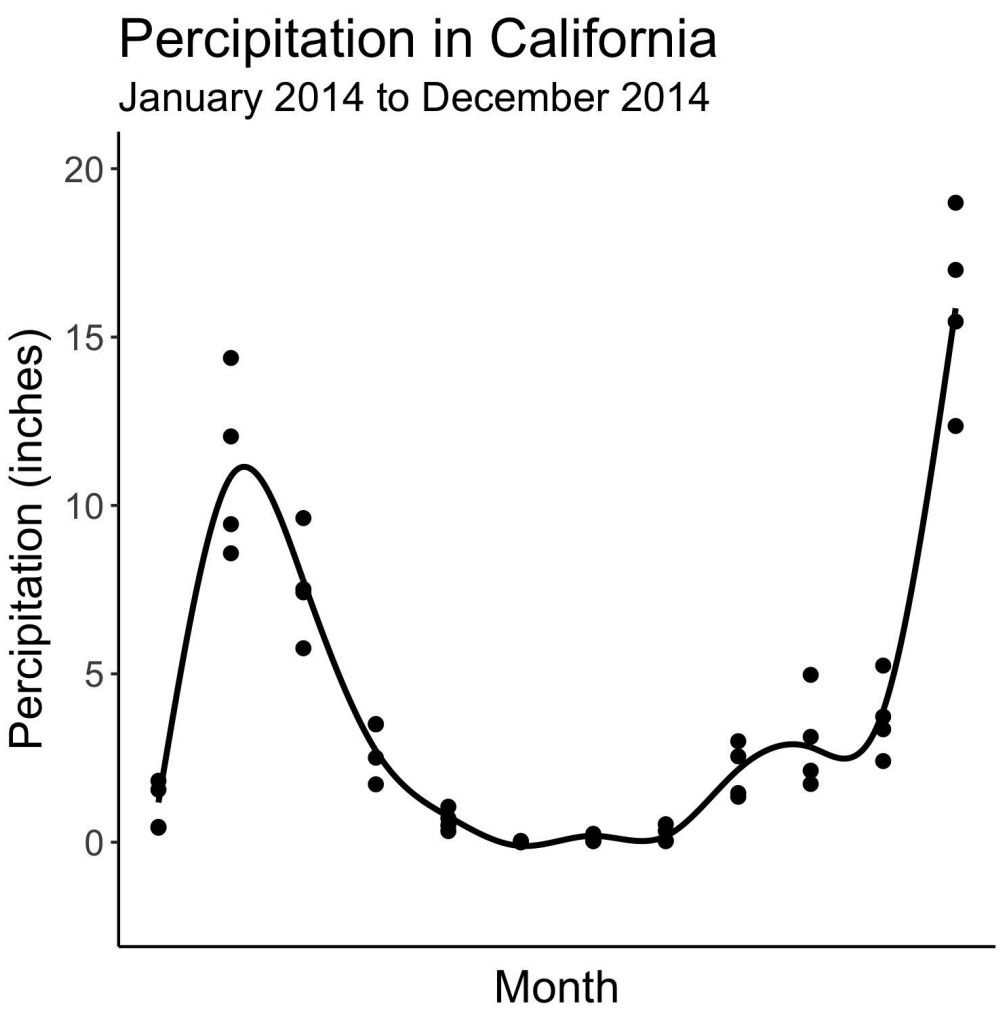
```
m = gam(y ~ s(x, k=10, bs="tp"))
```

Create “smooth” for x

Number of basis functions

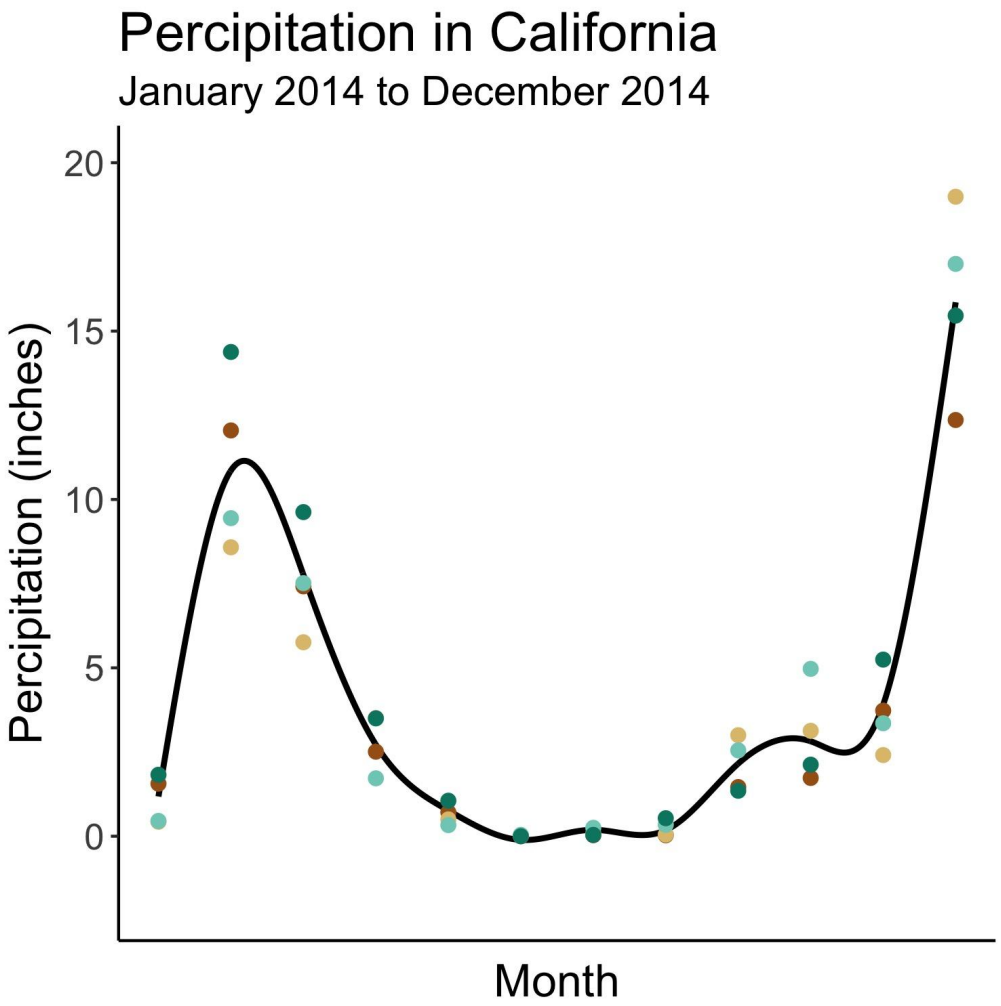
```
library(mgcv)
```

Linear Mixed Effects Models




```
m = gam(y ~
        s(x, k=10, bs="tp"))
```

```
library(mgcv)
```


Percipitation in California
January 2014 to December 2014
Percipitation (inches)
20
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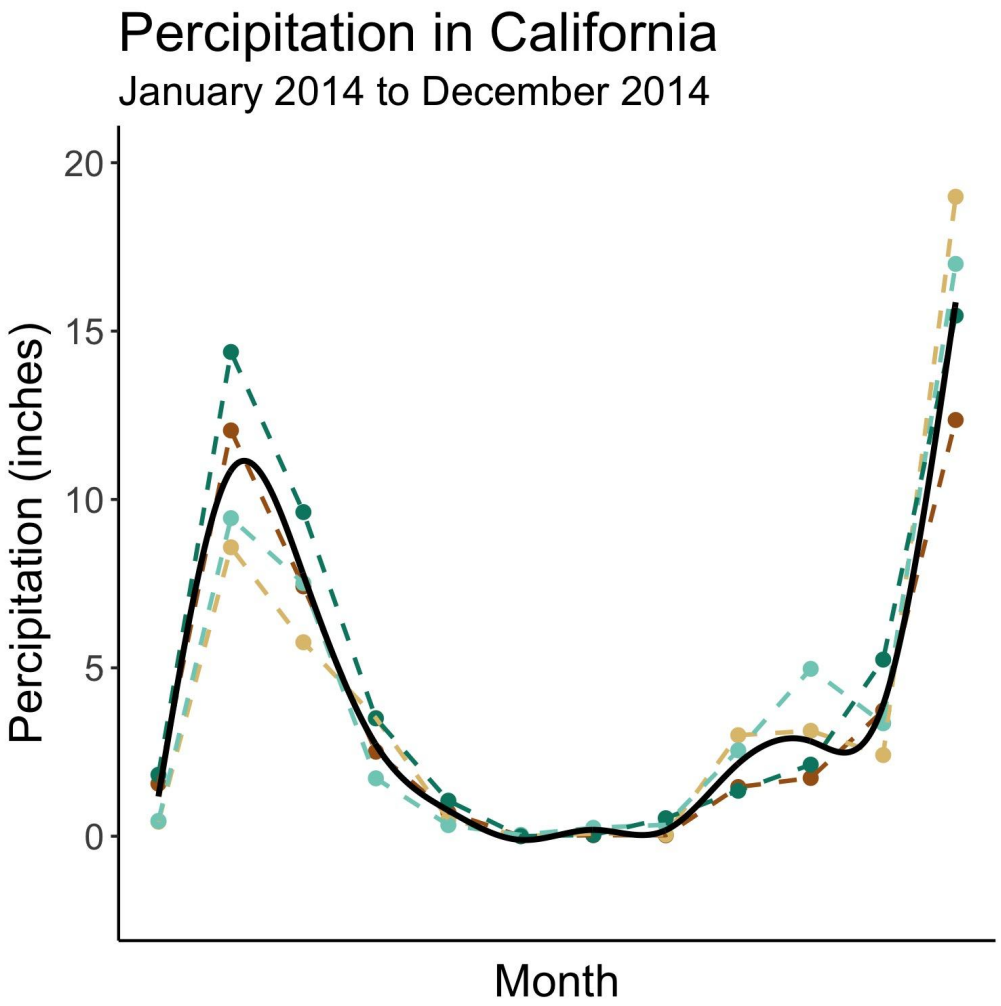
Month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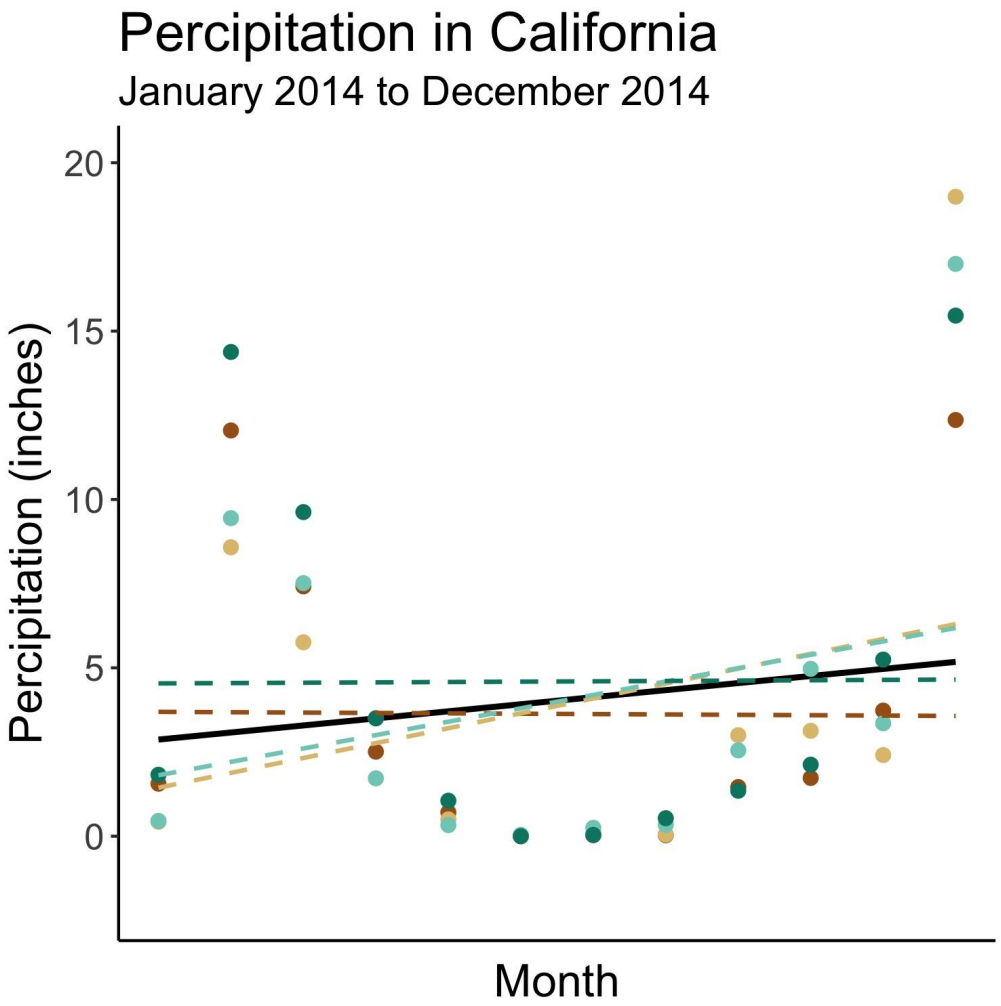
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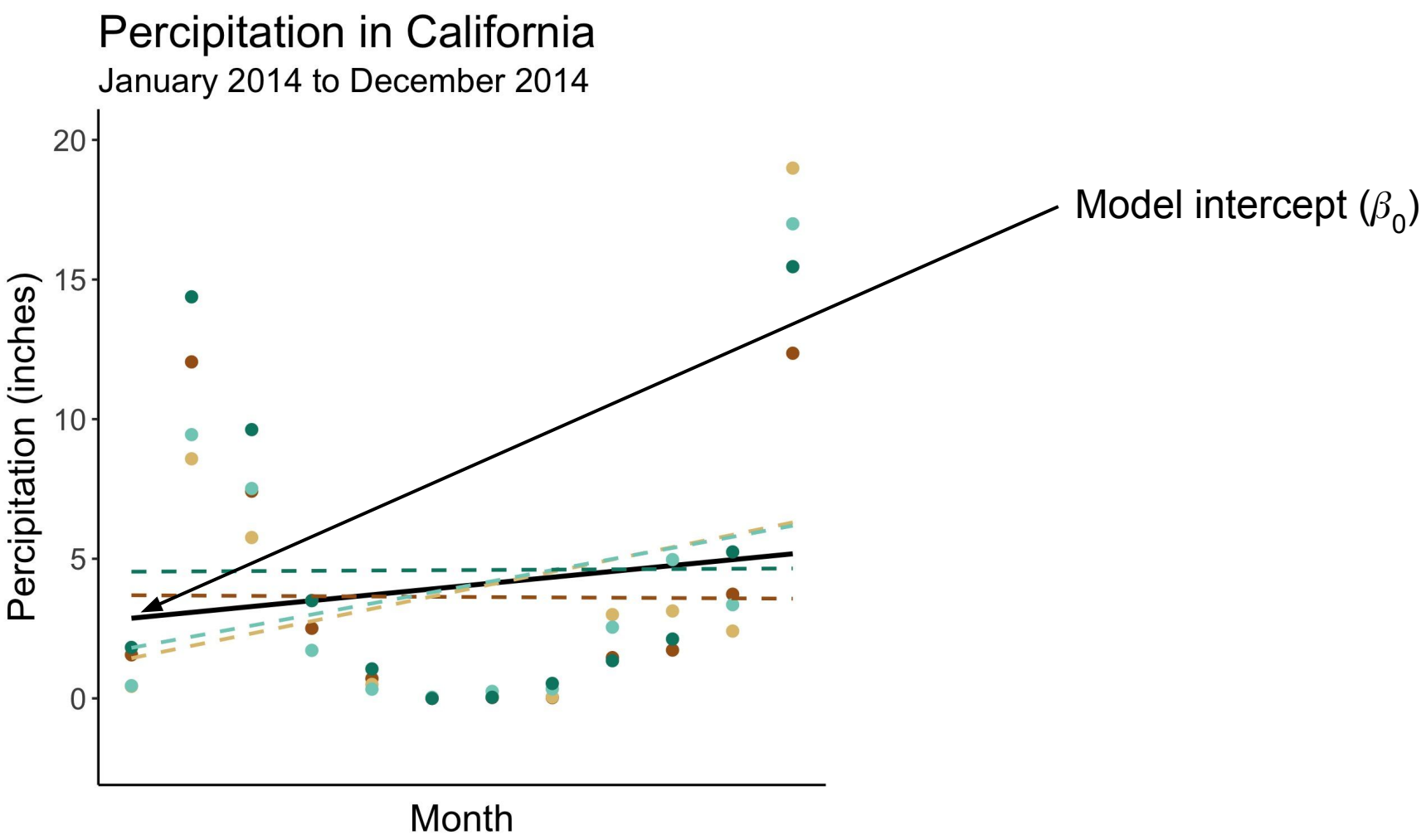
Percipitation in California
January 2014 to December 2014
Percipitation (inches)
20
15
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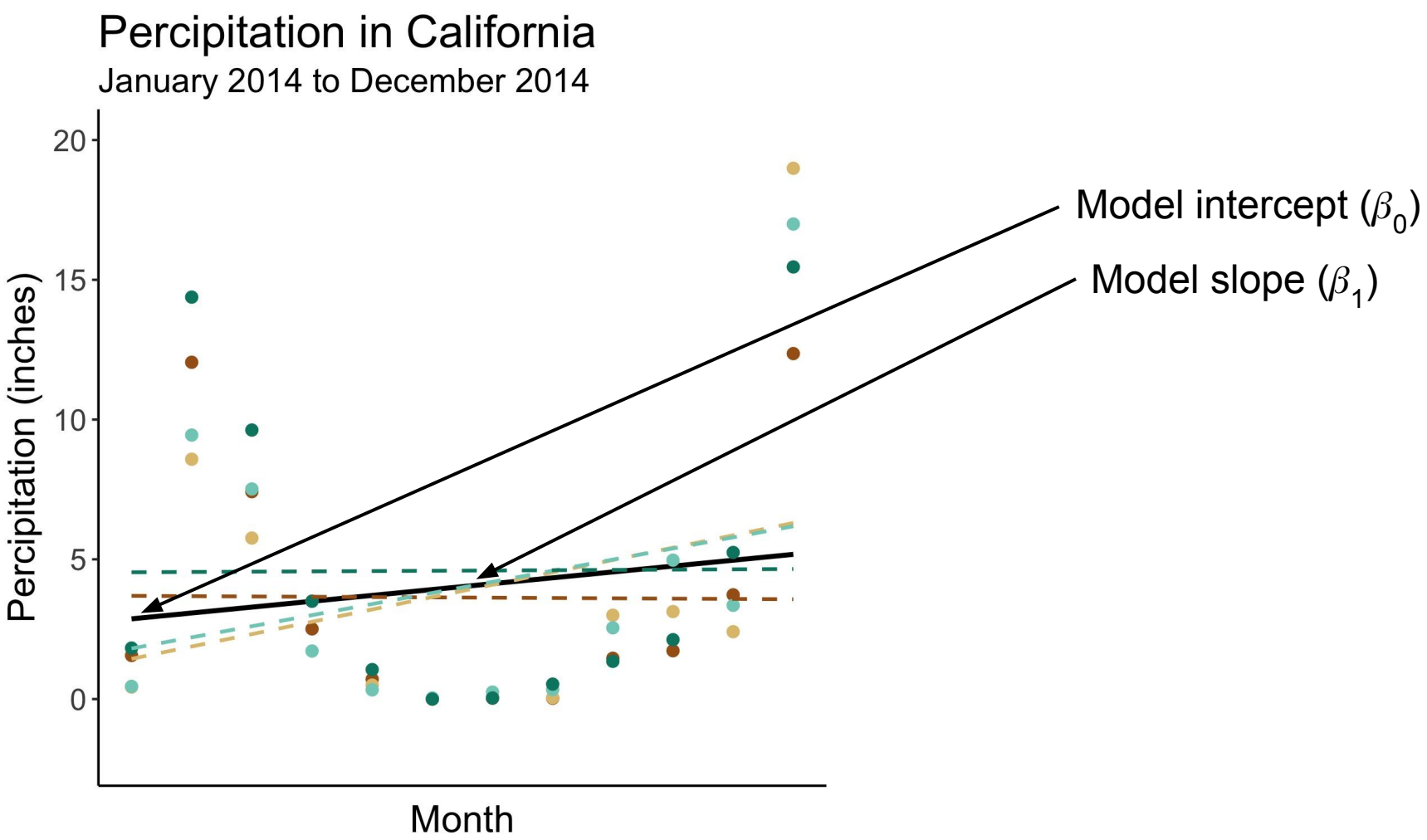
5
0
Month
Model intercept (β_0)


Percipitation in California
January 2014 to December 2014
Percipitation (inches)
20
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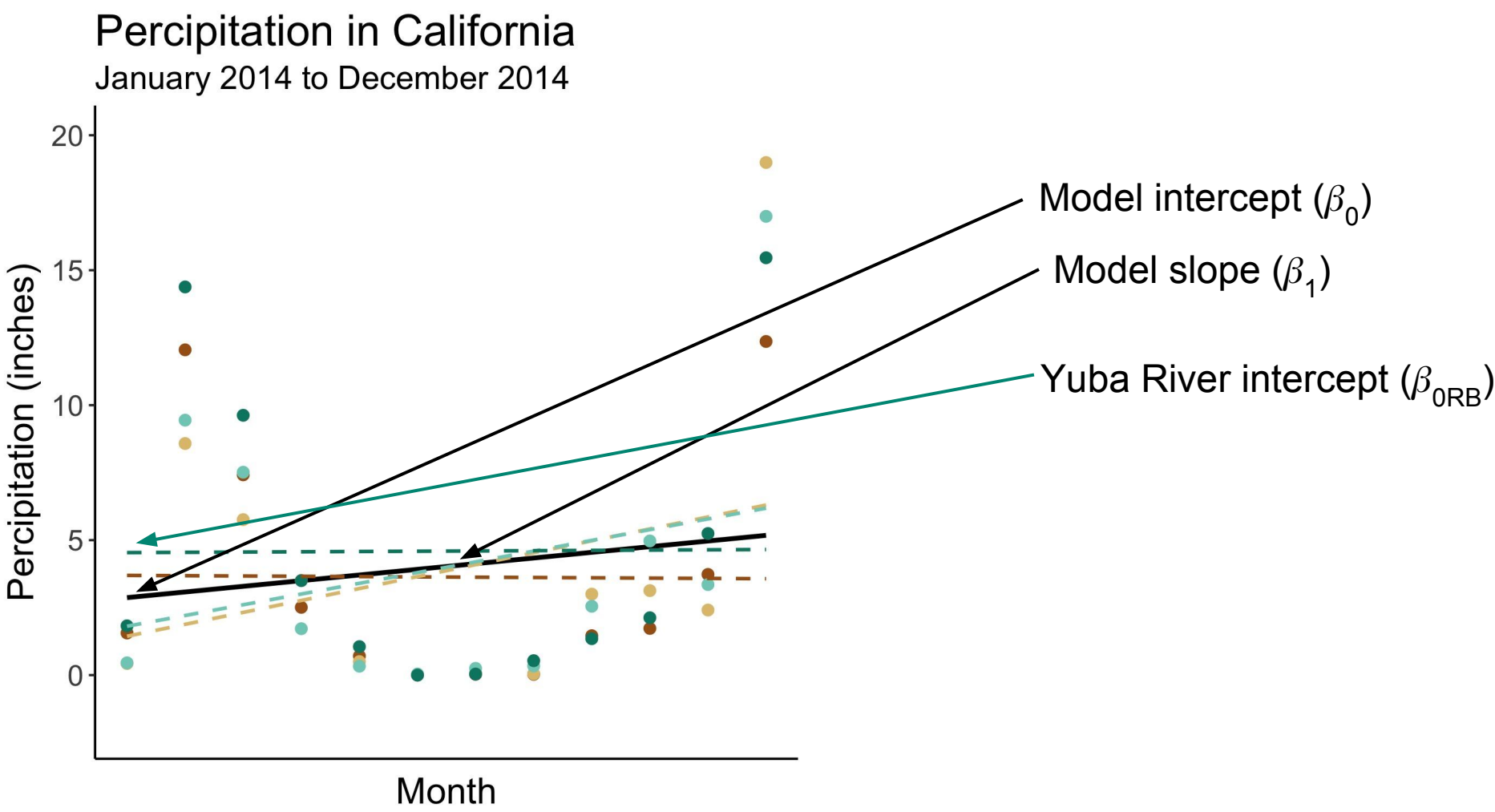
Month
Model intercept (β_0)
Model slope (β_1)


Percipitation in California
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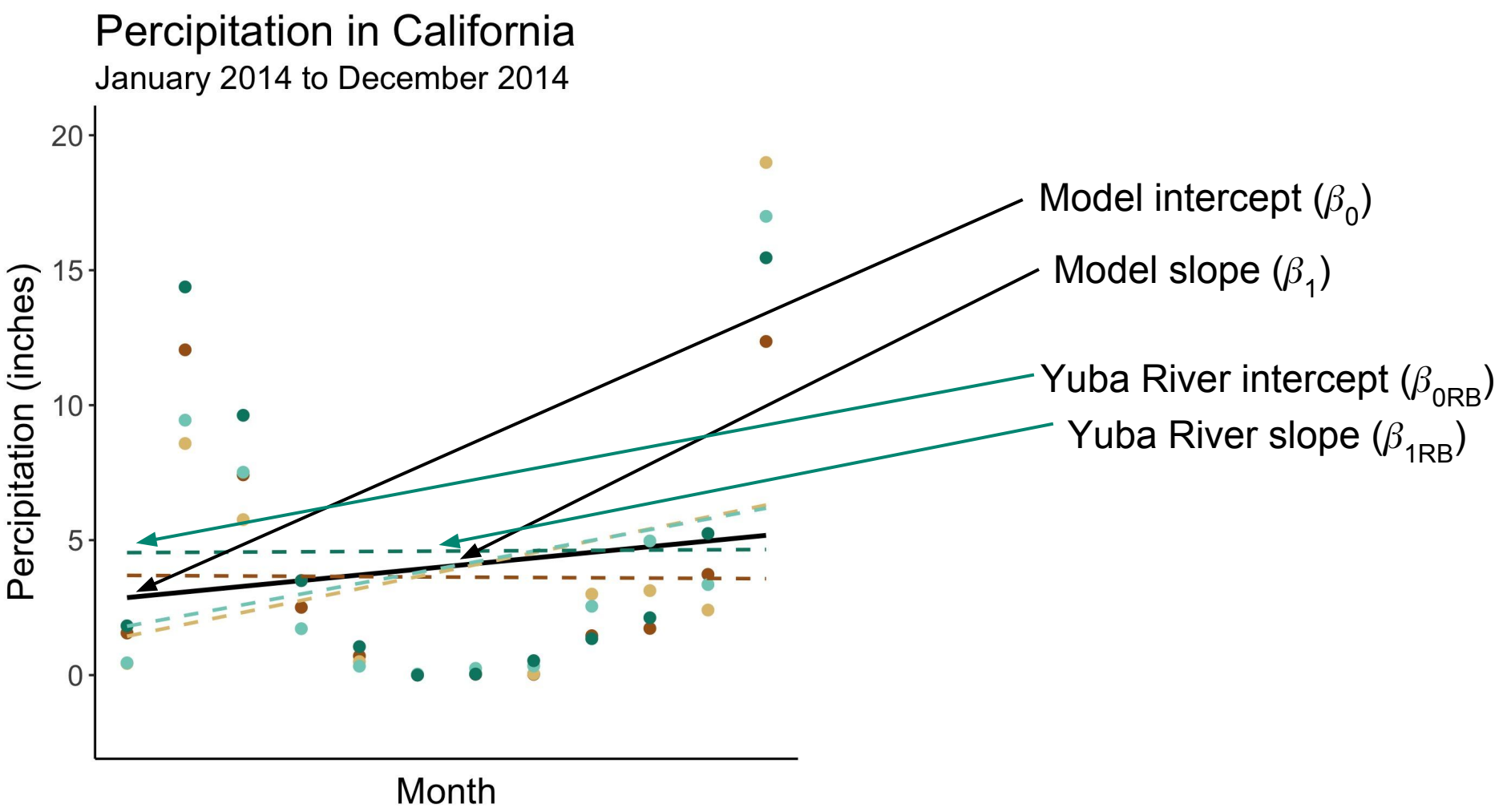
Month
Model intercept (β_0)
Model slope (β_1)
Yuba River intercept (β_{0RB})


Percipitation in California
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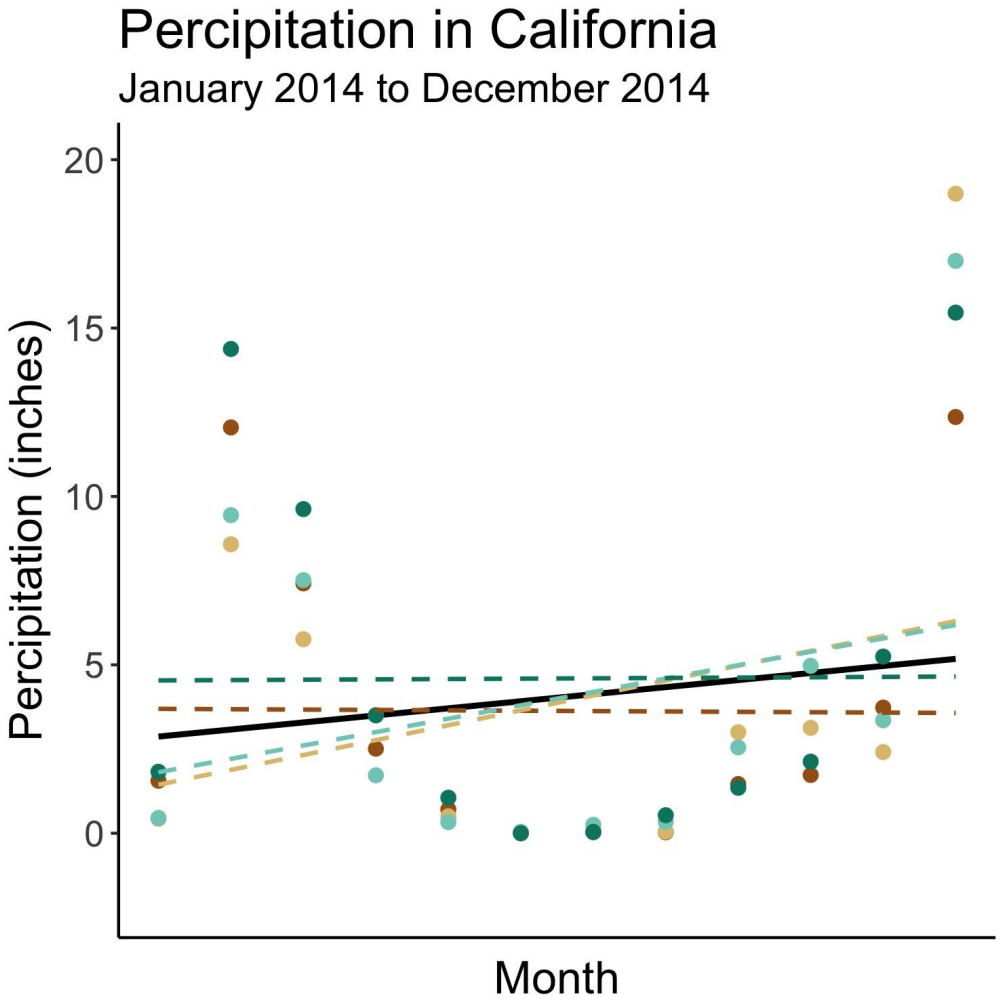
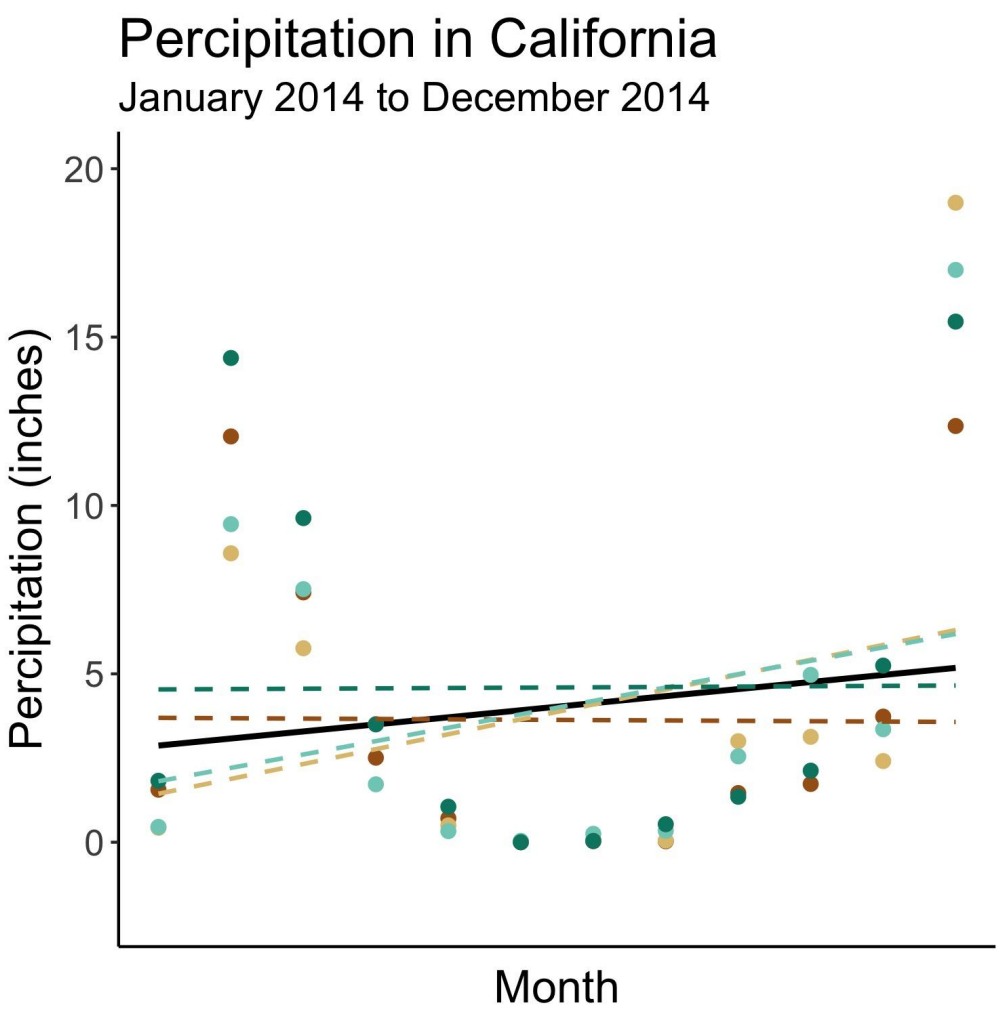
Model intercept (β_0)
Model slope (β_1)
Yuba River intercept (β_{0RB})
Yuba River slope (β_{1RB})



$$y = \beta_0 + \beta_{r_0} + (\beta_1 + \beta_{r_1})x$$

```
lmer(y ~ x +
       (1|r))
```

```
library(lme4)
```

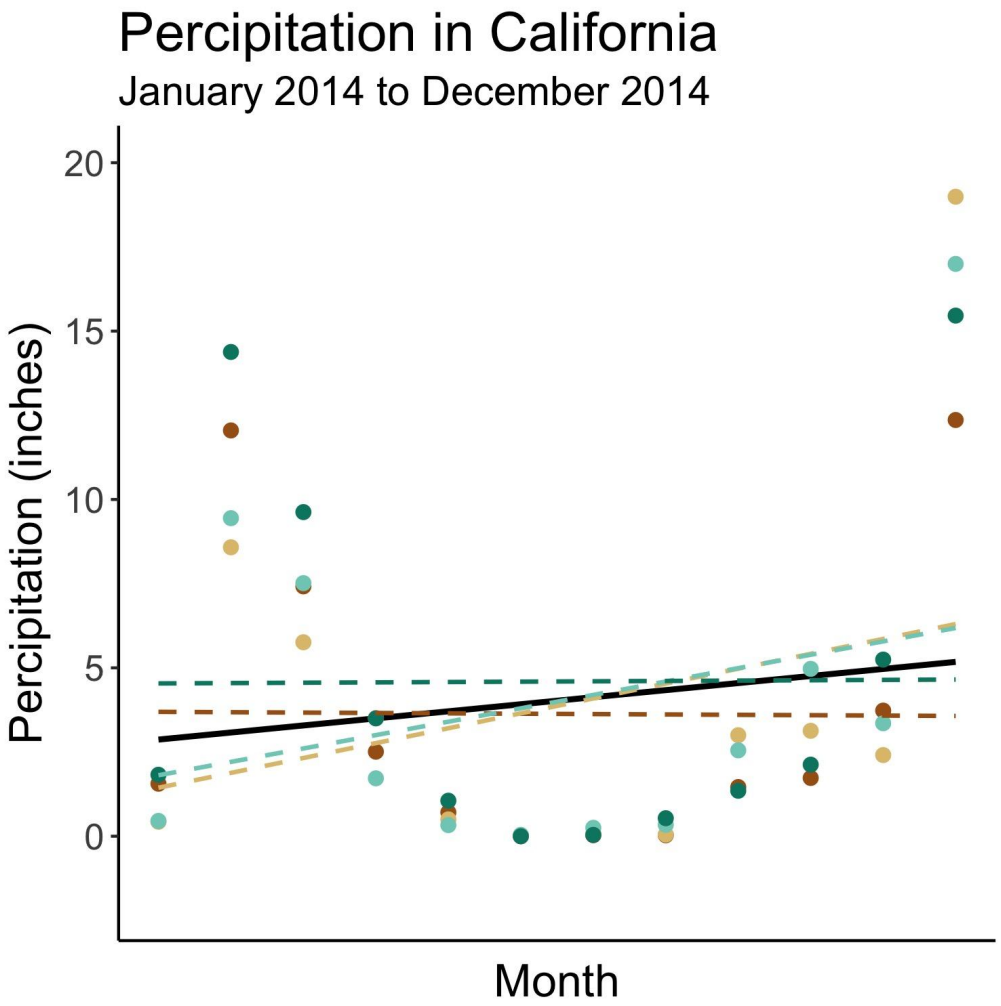


$$y = \beta_0 + \beta_{r_0} + (\beta_1 + \beta_{r_1})x$$

```
lmer(y ~ x +
       (1+x|r))
```

```
library(lme4)
```

Non-Linear Mixed Effects Models


Percipitation in California
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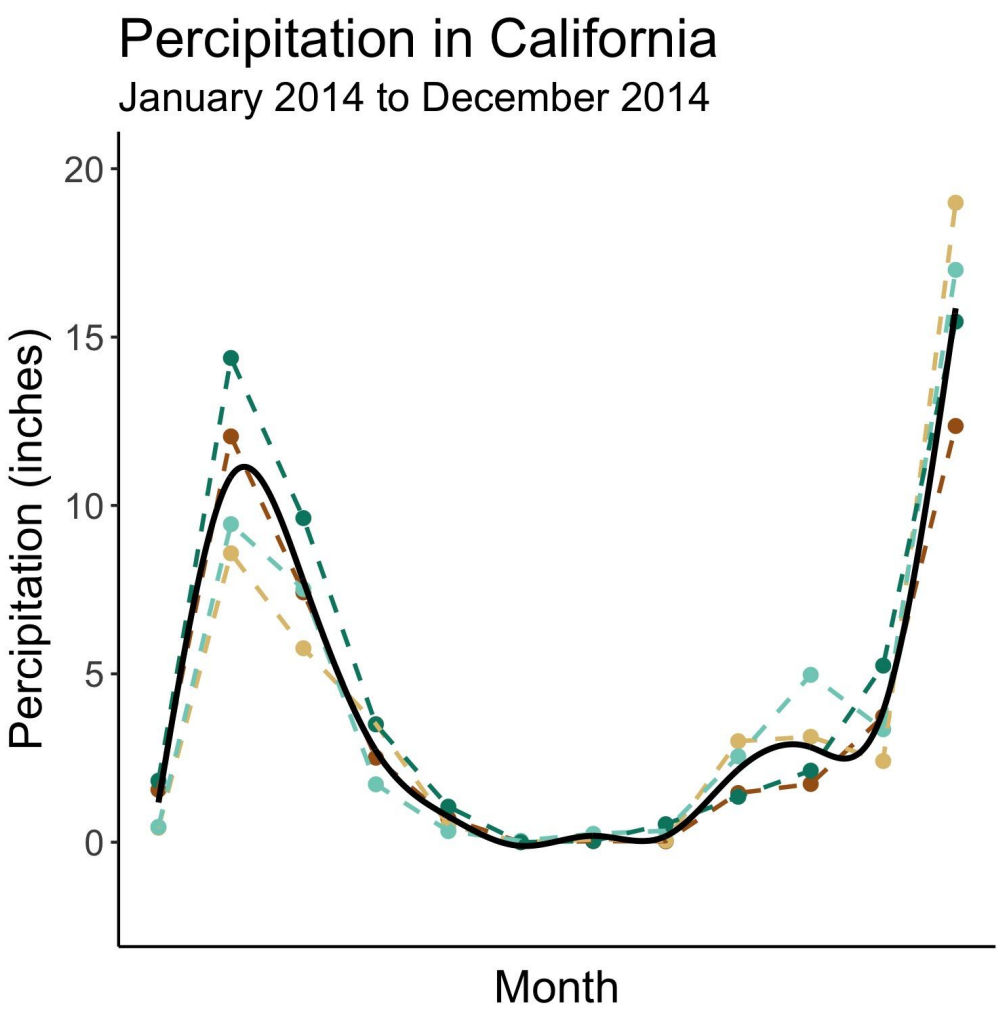
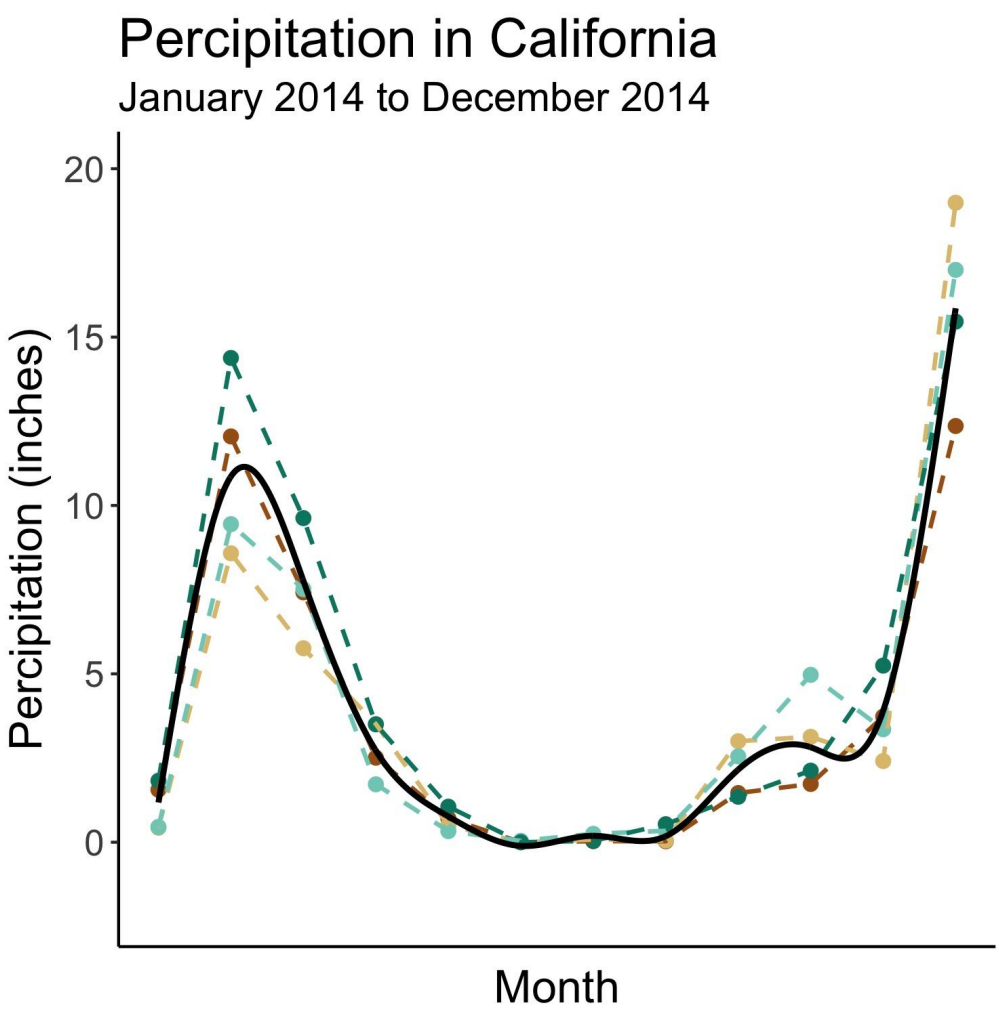


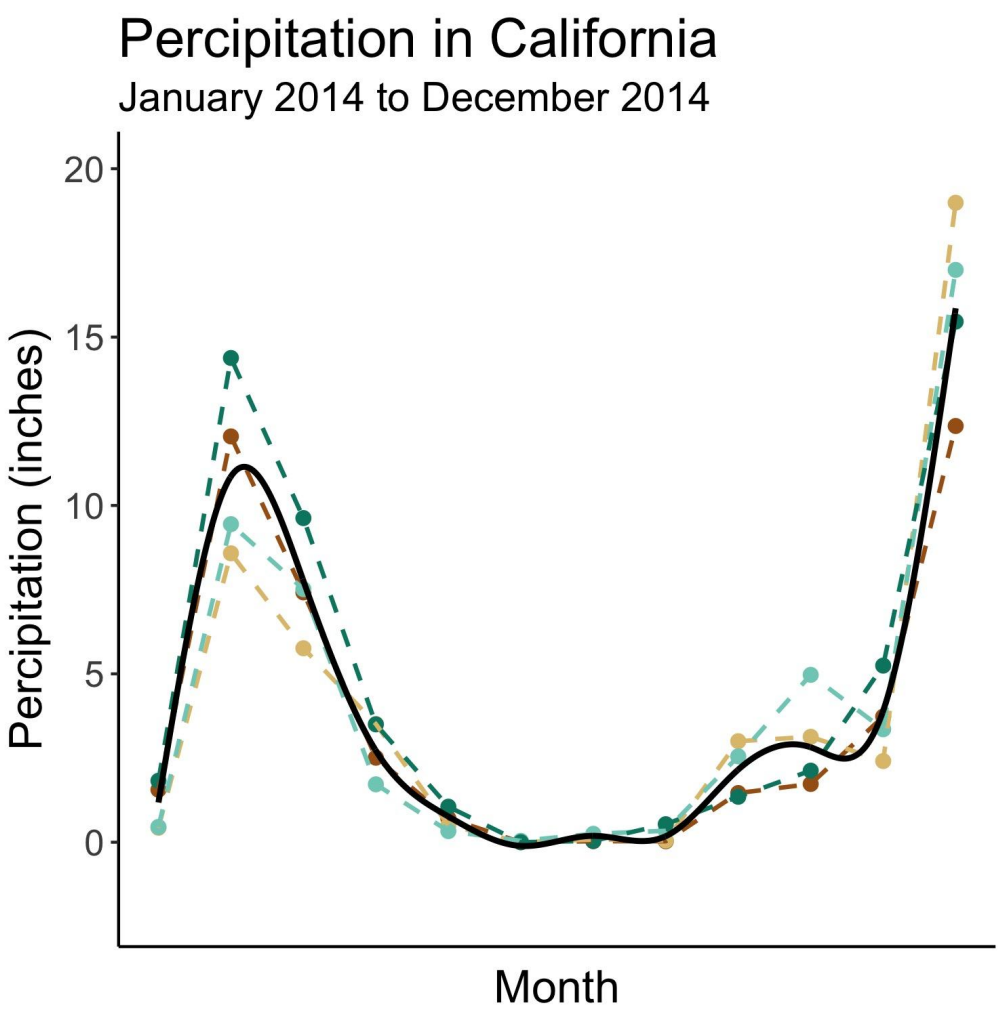
Random **slope** → Random **smooth**




```
m = gam(y ~ s(x, k=10, bs="tp") +

                                )
```

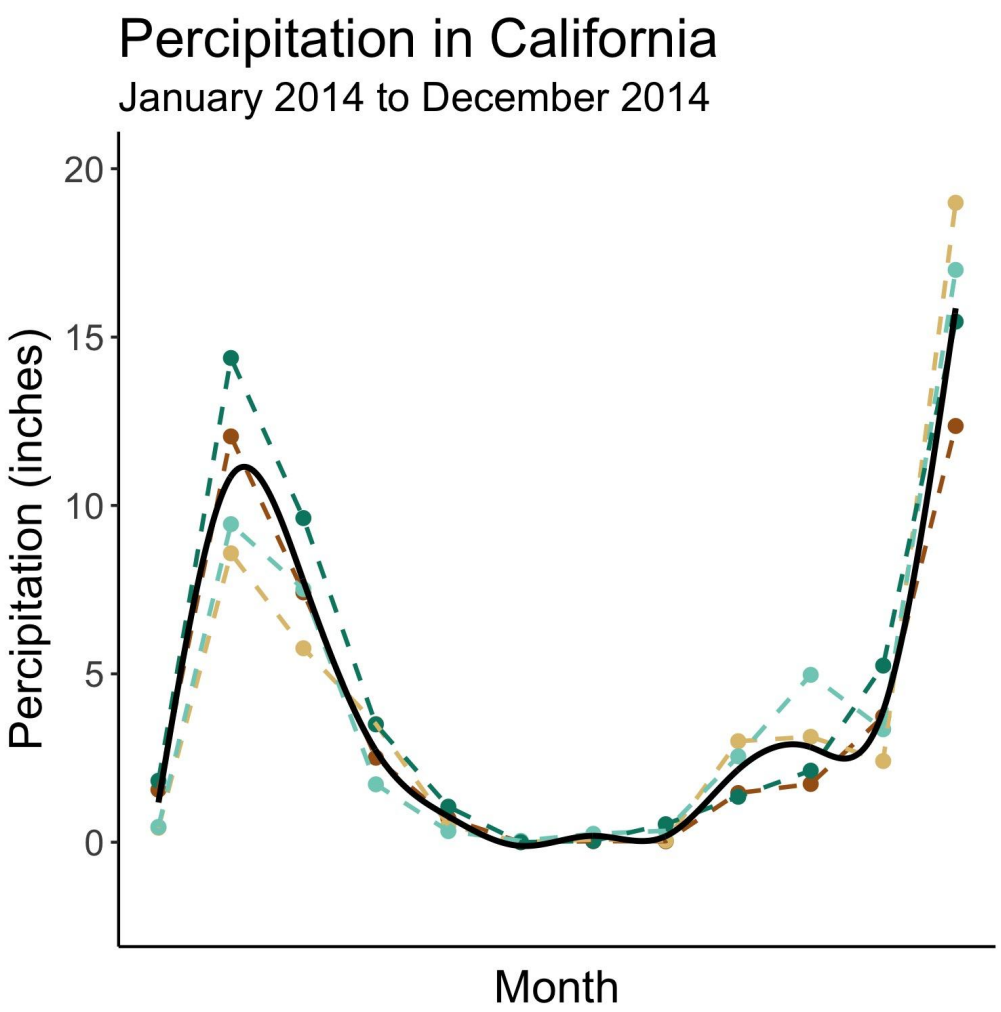
```
library(mgcv)
```




```
m = gam(y ~ s(x, k=10, bs="tp") +
            s(r, bs="re") +
                          )
```

Random intercept for r

```
library(mgcv)
```

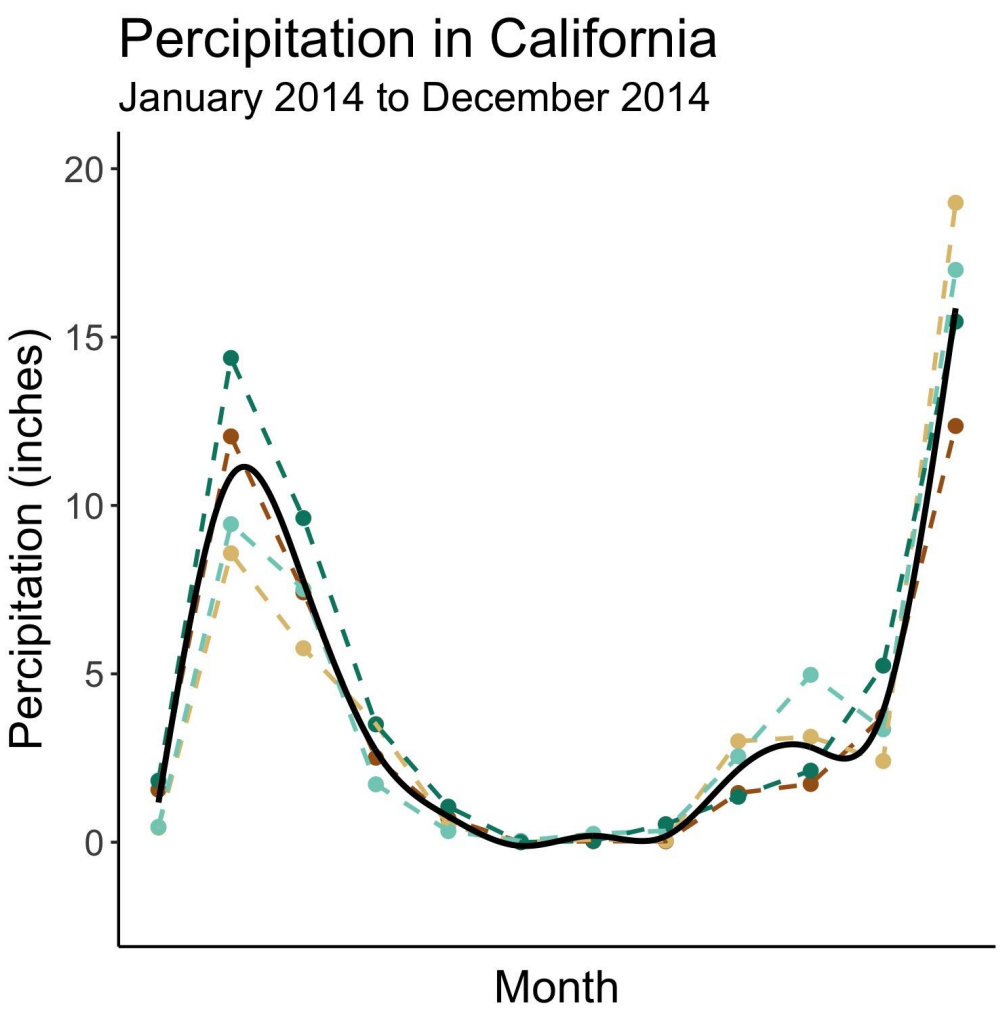



```
m = gam(y ~ s(x, k=10, bs="tp") +
            s(r, bs="re") +
            s(r, x, bs="re"))
```

Random intercept for r

Random slope for r by x

```
library(mgcv)
```



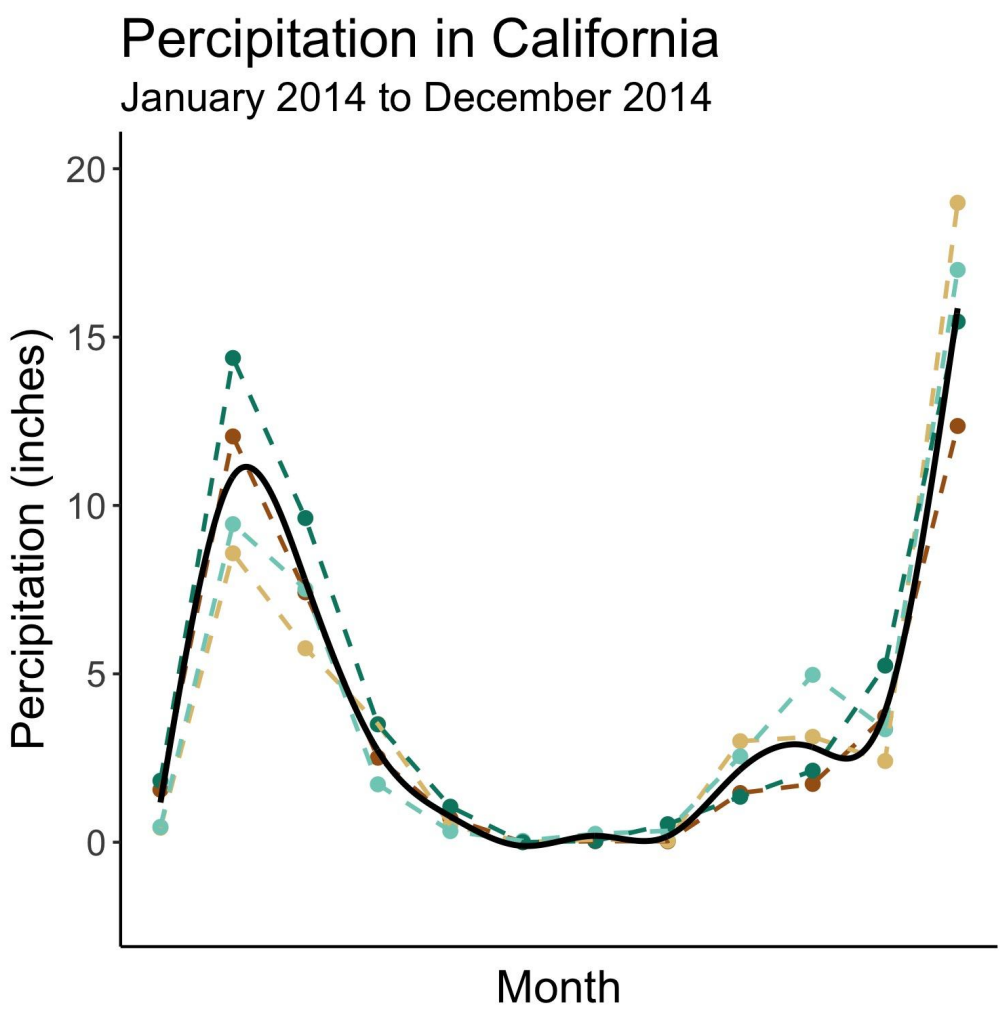

```
m = gam(y ~ s(x, k=10, bs="tp") +
            s(r, bs="re") +
            s(r, x, bs="re"))
```

lmer()
`(1+x|r)`

=

gam()
```
s(r, bs="re") +
s(r, x, bs="re"))
```

```
library(mgcv)
```




```
m = gam(y ~ s(x, k=10, bs="tp") +
          s(x, r, bs="fs", m=1))
```

Random smooth for r by x
NOTE! Order of vars switched

```
library(mgcv)
```

Detailed Resources

- GENERALISED ADDITIVE MIXED MODELS FOR DYNAMIC ANALYSIS IN LINGUISTICS: A PRACTICAL INTRODUCTION by Márton Sóskuthy
- Wood, S. (2006). Generalized additive models: an introduction with R. Boca Raton: CRC Press.
- Baayen, R. H., van Rij, J., de Cat, C., & Wood, S. N. (2016). Autocorrelated errors in experimental data in the language sciences: Some solutions offered by generalized additive mixed models. arXiv preprint arXiv:1601.02043.
- Kelly, R. (2014). Extending linear models: Non-linearity. https://rstudio-pubs-static.s3.amazonaws.com/24589_7552e489485b4c2790ea6634e1afd68d.html.
- Simpson, G. (2014). Modelling seasonal data with GAMs. http://www.fromthebottomoftheheap.net/2014/05/09/modelling-seasonal-data-with-gam/.