1)

$\log_2 2048 = \log_{10} 2048 / \log_{10} 2$

$= 3.311 / 0.301 = 11$

2)

$3+5+7+9+\dots+2k+1 \quad \sum_{i=1}^{k} i = \frac{k(k+1)}{2}.$

$2(1+2+3+\dots+k) + (1+1+\dots+1)$

$= \cancel{2} \times \frac{k(k+1)}{\cancel{2}} \quad + k$

$= k^2+k+k = k^2+2k = k(k+2)$

3)

Prove that the following statement is false:

$n^3 > 2^n$ for any $n \geq 1$

when $n = 1$

$1^3 < 2^1 \rightarrow 1 < 2 \Rightarrow$ be not established $n^3 > 2^n$ when $n = 1$

Hence, '$n^3 > 2^n$ for any $n \geq 1$' is false because '$n^3 > 2^n$ for any $n \geq 1$' is not established when $n = 1$

4)

Prove that the following statement is true:

the square of an even number is also even.

① Assume it is false

: the square of an even number is odd

if x^2 is odd, then $x^2 = 2c+1$

But $x = 2a$ means $4a^2 = 2c+1$

So, $4(a^2) = 2c+1$, but this says an even number equals an odd number, which is impossible

Therefore, the square of an even number is also even.

5) (a) prove by induction

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

① Basic step n=1

$$\sum_{i=1}^{1} i^3 = \frac{1^2(\cancel{2^2})}{\cancel{4}} = 1 \qquad \sum_{i=1}^{1} i^3 = 1^3 = 1$$

true

② Inductive step:

Assume true for k:

$$\sum_{i=1}^{k} i^3 = \frac{k^2(k+1)^2}{4}$$

Show true for k+1:

$$\sum_{i=1}^{k+1} i^3 = \boxed{\frac{(k+1)^2(k+2)^2}{4}}$$

$$\sum_{i=1}^{k+1} i^3 = \frac{k^2(k+1)^2}{4} + (k+1)^3$$

$$= \frac{k^2(k+1)^2}{4} + \frac{4(k+1)^3}{4} = \frac{(k+1)^2(k^2+4k+4)}{4}$$

$$= \boxed{\frac{(k+1)^2(k+2)^2}{k.}}$$

Conclusion: Since the base case was true and the inductive step was shown to be true, by induction it is true.

(b) Prove by induction

$n^2 - n$ is even for any $n \geq 1$

① Basic step $n = 1$

$1 - 1 = 0 \Rightarrow 0$ is even.

② Inductive step

Assume true for k.

$k^2 - k \Rightarrow$ even.

Show true for $k+1$.

$(k+1)^2 - (k+1) = k^2 + 2k + \cancel{1} - \cancel{k} - \cancel{1}$

$= k^2 + k. = k^2 - k + 2k.$

$k^2 - k$ is even and $2k$ is even

so, $k^2 - k + 2k$ is also even.

Conclusion : Since the base case was true and the inductive step was shown to be true, by induction it is true.