

$$1. (111.111)_2$$

$$= 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 + 1 \cdot 2^{-1} + 1 \cdot 2^{-2} + 1 \cdot 2^{-3}$$

$$= 4 + 2 + 1 + 0.5 + 0.25 + 0.125 = 7 + 0.875 = 7.875$$

$$\therefore (111.111)_2 = (7.875)_{10}$$

$$2. (14.6)_{16} = 1 \cdot 16^1 + 4 \cdot 16^0 + 6 \cdot 16^{-1}$$

$$= 16 + 4 + \frac{6}{16} = 16 + 4 + \frac{3}{8} = 20 + 3 \cdot 0.125 = 20 + 0.375$$

$$= 20.375$$

$$\therefore (14.6)_{16} = (20.375)_{10}$$

$$3. (11)_{16} = 1 \cdot 16^1 + 1 \cdot 16^0 = 1 \cdot 2^4 + 0 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0$$

$$= (10001)_2$$

$$\therefore (11)_{16} = (10001)_2$$

$$4. (101110101101)_2 = \underbrace{(1011)}_{(i)} \underbrace{(1010)}_{(ii)} \underbrace{(1101)}_{(iii)}$$

$$(i) 1011_2 = 11_{10} = B_{16}$$

$$\therefore (101110101101)_2 = (BAD)_{16}$$

$$(ii) 1010_2 = 10_{10} = A_{16}$$

$$(iii) 1101_2 = 13_{10} = D_{16}$$

$$5. (19.25)_{10} = 1 \cdot 2^4 + 0 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 + 0 \cdot 2^{-1} + 1 \cdot 2^{-2}$$

$$\Rightarrow 1.0011.01_2 \times 2^0 = 1.001101 \times 2^4$$

$$\cdot \text{Sign: } 0$$

$$\cdot \text{Biased Exponent: } 4 + 127 = 131 = 128 + 2 + 1 = 10000011_2$$

$$\cdot \text{Fraction: } 00110100000000000000$$

$$\text{Sign: } 0$$

$$\cdot \text{Biased Exponent: } 10000011$$

$$\text{Fraction: } 00110100000000000000$$

6.	Decimal	Octal	Hexadecimal
	$\begin{array}{r} 7322 \\ + 1933 \\ \hline 9255 \end{array}$	$\begin{array}{r} 333 \\ + 257 \\ \hline 612 \end{array}$	$\begin{array}{r} \text{A} \text{B} \text{A} \text{D} \\ - \text{A} \text{B} \text{A} \\ \hline 0 \text{F} 3 \end{array}$

7.  $(11111100)_2$

$$\begin{array}{r} 00000011 \\ + 00000001 \\ \hline 00000100 \end{array}$$

$\therefore$  2's complement operation on

$$(11111100)_2 = 00000100$$

8. 1's complement : Swap 0s and 1s (i.e.  $0 \Rightarrow 1$  or  $1 \Rightarrow 0$ )

$$(11111100)_2 \xrightarrow{\text{1's complement}} (00000011)_2$$

$$\therefore (00000011)_2$$

$$Z(A, B, C, D) = \sum 0, 2, 4, 8, 10$$

A	B	C	D	Z
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

<K-map>

CD \ AB	00	01	11	10
00	1	1		1
01				
11				
10	1			1

$$i) \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}B\bar{C}\bar{D} = \bar{A}\bar{C}\bar{D}$$

$$ii) \bar{A}\bar{B}\bar{C}D + \bar{A}B\bar{C}D = \bar{B}\bar{C}D$$

$$iii) \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}B\bar{C}\bar{D} = \bar{A}\bar{B}\bar{D}$$

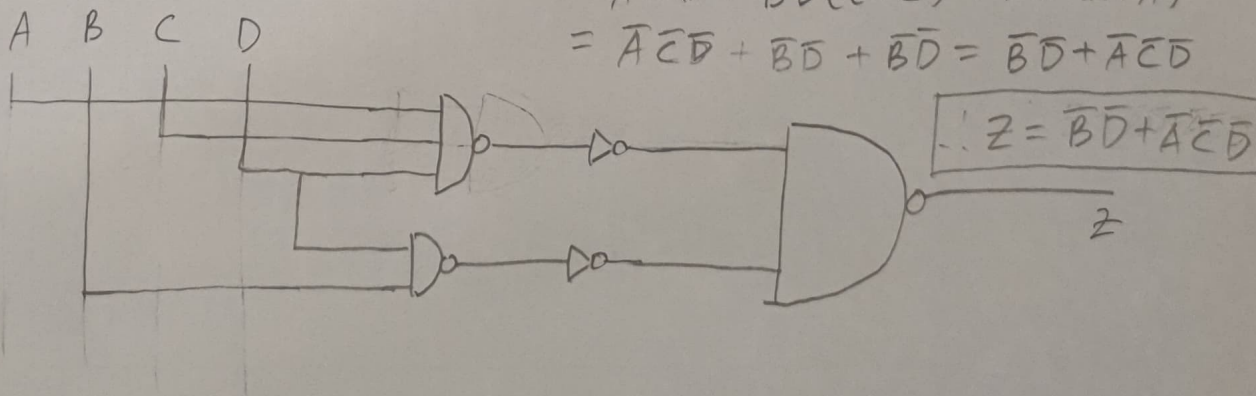
$$iv) \bar{A}\bar{B}C\bar{D} + \bar{A}BC\bar{D} = \bar{B}C\bar{D}$$

$$v) A\bar{B}\bar{C}\bar{D} + A\bar{B}C\bar{D} = A\bar{B}\bar{D}$$

$$\therefore Z = \bar{A}\bar{C}\bar{D} + \bar{B}\bar{C}D + \bar{A}\bar{B}\bar{D} + \bar{B}C\bar{D} + A\bar{B}\bar{D}$$

$$= \bar{A}\bar{C}\bar{D} + \bar{B}\bar{D}(C + \bar{C}) + \bar{B}\bar{D}(A + \bar{A})$$

$$= \bar{A}\bar{C}\bar{D} + \bar{B}\bar{D} + \bar{B}\bar{D} = \bar{B}\bar{D} + \bar{A}\bar{C}\bar{D}$$



2 x 2-input NAND + 2 x NOT Gate + 1 x 3-input - NAND Gate

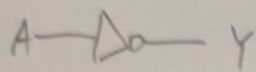
$$= 2 \times 2 + 2 \times 1 + 1 \times 3 = 4 + 2 + 3 = 9$$

$\therefore$  9 transistors

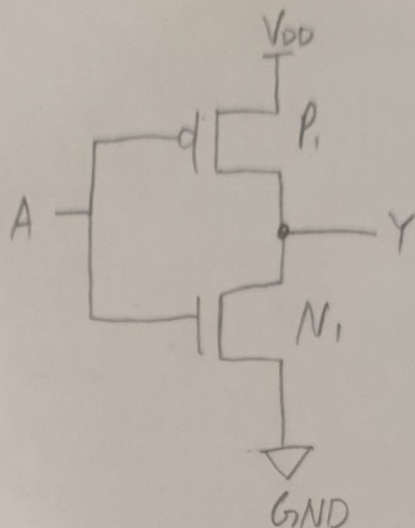


[CMOS]

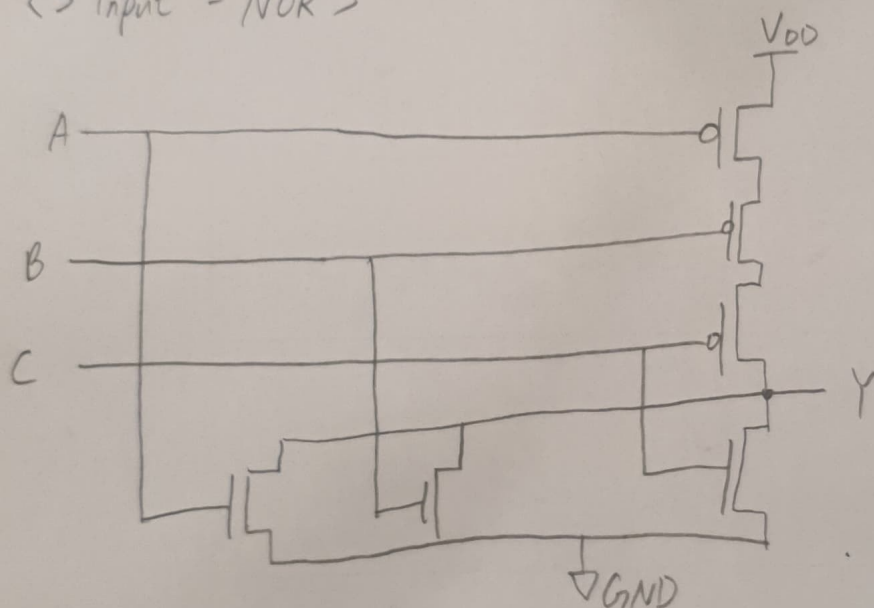
<NOT>



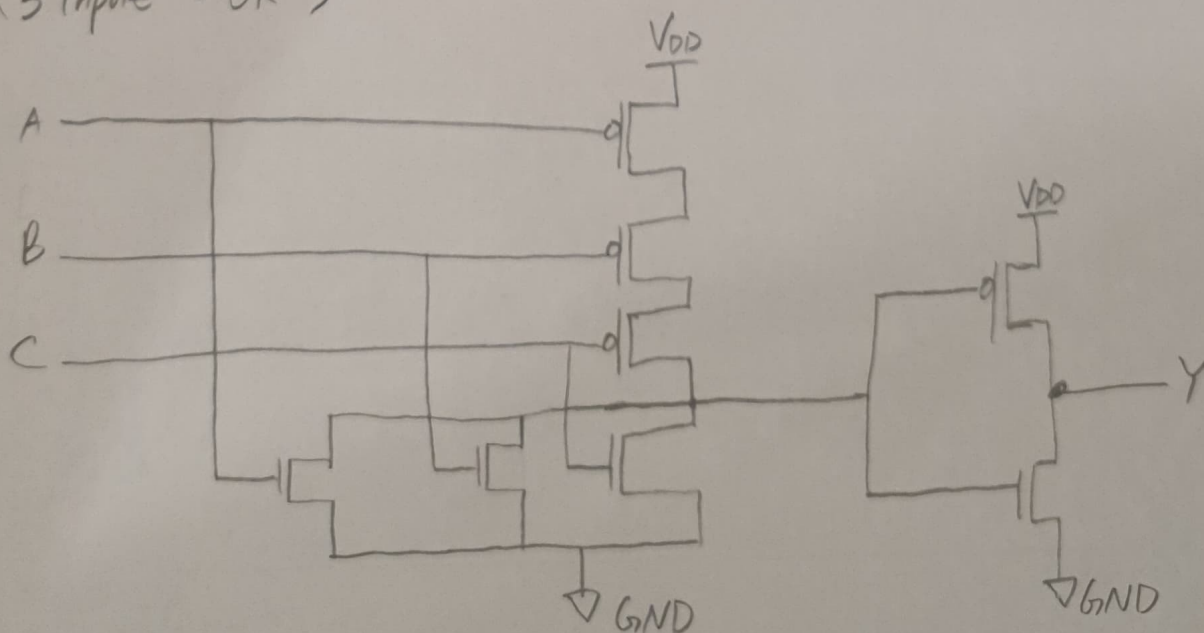
$$Y = \bar{A}$$



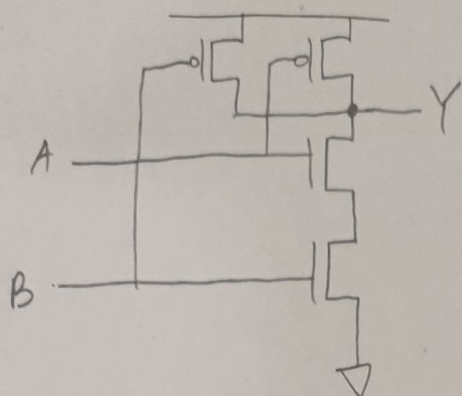
<3 input - NOR>



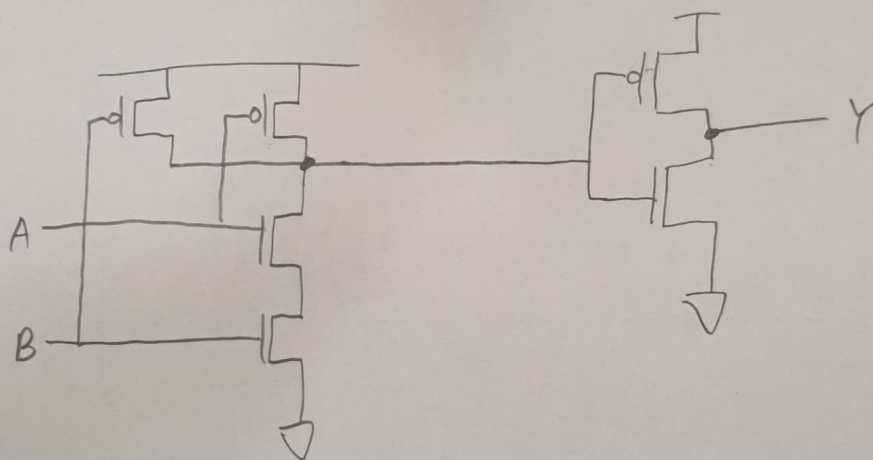
<3 input - OR>



<2-input NAND>



<2-input AND>



## < Sequential Logic Timing >

1.

Setup Time Constraint:  $T_c \geq t_{pcq} + t_{pd} + t_{setup}$

$$T_c = 200ps, \quad t_{setup} = 90ps, \quad t_{pcq} = 75ps.$$

In the pipeline, there are four of Routing delay and delays by three of LUT.

$$\text{So, entire } t_{pd} \text{ is } 4 \times 10ps + 3 \times 15ps = 85ps.$$

$$200ps \geq 75ps + 85ps + 90ps \Rightarrow 200ps \geq 250ps.$$

This doesn't make sense!

Thus, through these reasons, if the clock period is 200ps, there is a setup time violation.

2.

Hold Time Constraint:  $t_{hold} < t_{ccq} + t_{cd}$

$$t_{hold} = 60ps, \quad t_{ccq} = 15ps.$$

The short path is the entire pipeline of digital logic.

The entire  $t_{cd}$  is  $3 \times 5ps + 4 \times 5ps = 35ps$  because there are 3 LUTs and 4 Routing delays in pipeline.

$$60ps < 15ps + 35ps \Rightarrow 60ps < 50ps. \text{ This doesn't make sense!}$$

Thus, through these reasons, if the clock period is 400ps, there is a hold time violation.



3. To fix setup violation, there are three ways.

Setup violation can be fixed by increasing the clock period, by redesigning the combinational logic to have a shorter propagation delay, or using synchronizer.

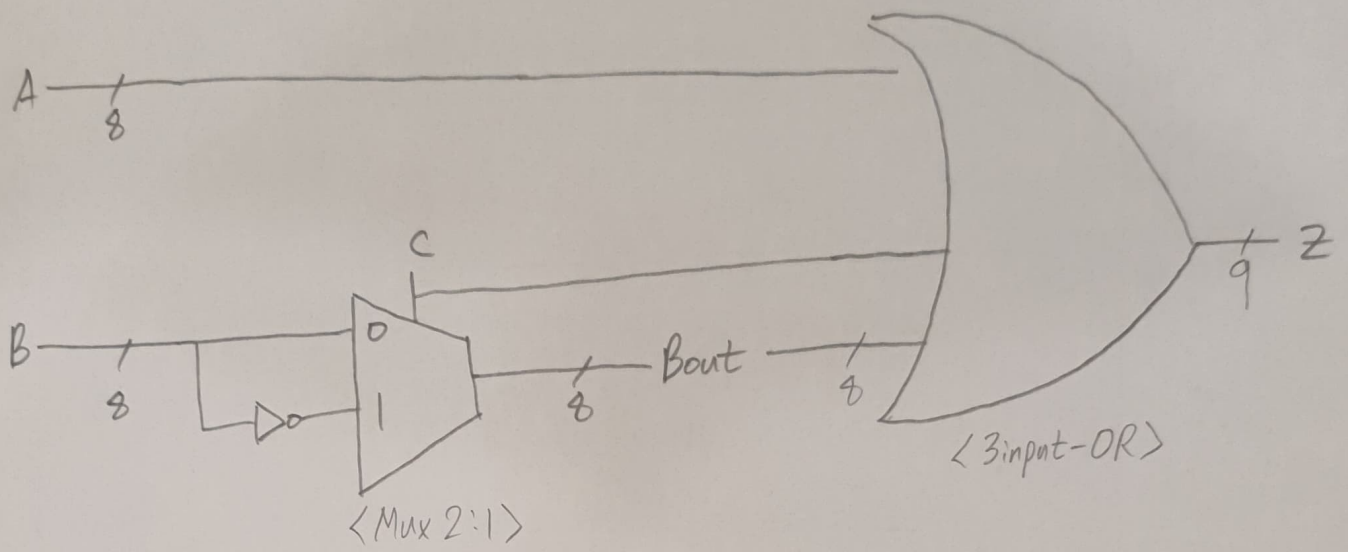
4.

Hold violation can be fixed by adding more logic gates the short path of the combinational logic to increase entire contamination delay.

5.

The acronym LUT represents lookup table which enables to perform combinational logic in FPGA.

# <Mystery Block>





<PLA>

$$X(A,B,C) = \sum 0, 4, 6, 7 = \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + AB\bar{C} + ABC$$

$$Y(A,B,C) = \sum 0, 2, 3, 4 = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}\bar{C}$$

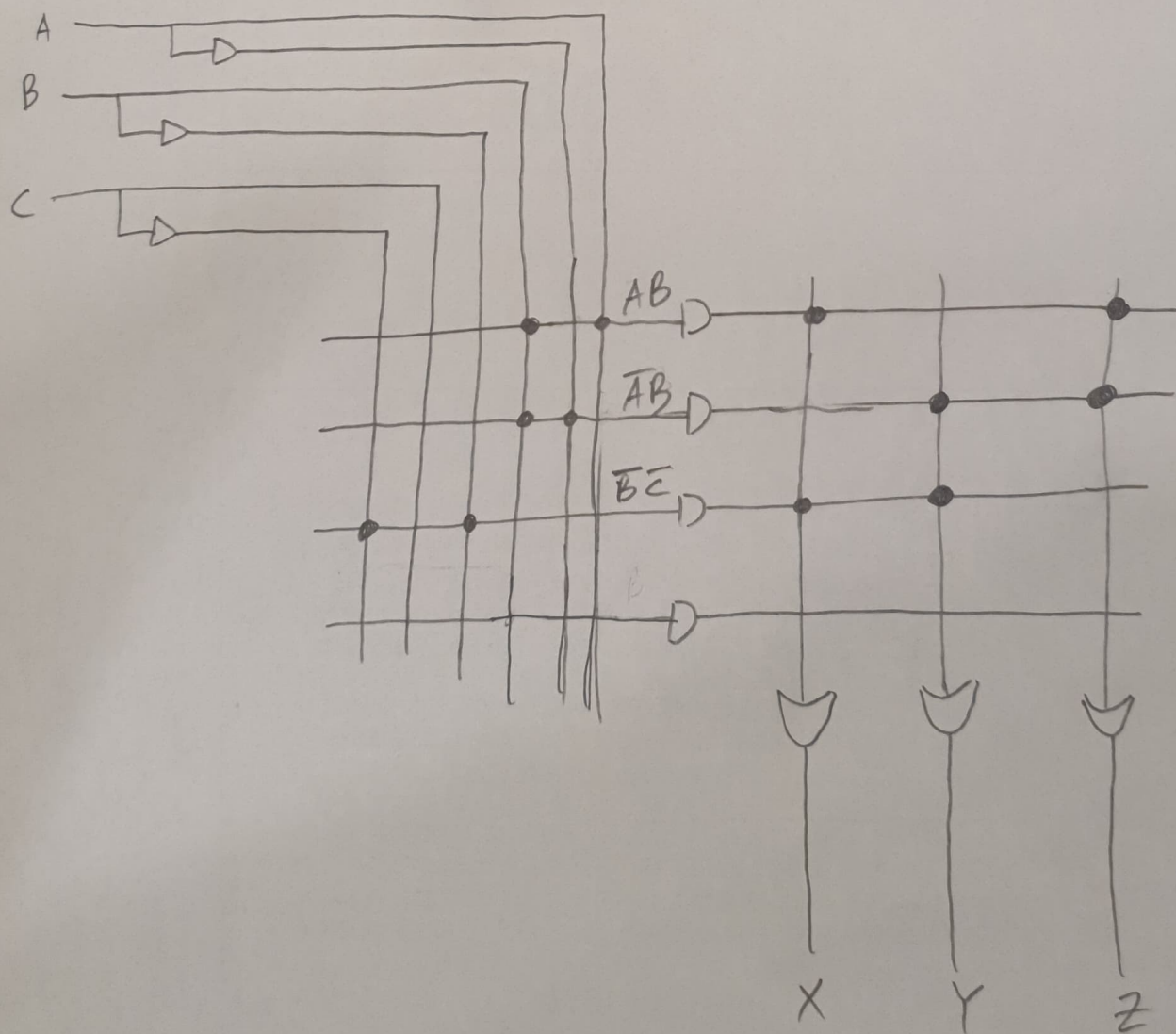
$$Z(A,B,C) = \sum 2, 3, 6, 7 = \bar{A}B\bar{C} + \bar{A}BC + AB\bar{C} + ABC$$

$$\Rightarrow X(A,B,C) = \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + AB\bar{C} + ABC = \bar{B}\bar{C} + AB$$

$$Y(A,B,C) = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}\bar{C}$$

$$= \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC = \bar{B}\bar{C} + \bar{A}B$$

$$Z(A,B,C) = \bar{A}B\bar{C} + \bar{A}BC + AB\bar{C} + ABC = \bar{A}B + AB = B \quad (\text{since } \bar{A} + A = 1)$$



# < State Machines >

1. 2 bits of memory are used in this state machine.
2. Let current state and next state of upper D Flip-Flop in the schematic be  $s_0, s_0'$

Let current state and next state of lower D Flip-Flop in the schematic be  $s_1, s_1'$ .

$\therefore$  current state:  $s_0, s_1$ , next state:  $s_0', s_1'$ ,

Input:  $D_{in}$

$$s_0' = D_{in} \overline{s_0} \overline{s_1} + \overline{D_{in}} \overline{s_0} s_1 + D_{in} s_0 s_1 + \overline{D_{in}} s_0 \overline{s_1}$$

$$s_1' = \overline{s_1}$$

3.

$$1) Q_0 = s_1 \quad 3) A = \overline{s_0} \overline{s_1} \quad 5) C = s_0 \overline{s_1}$$

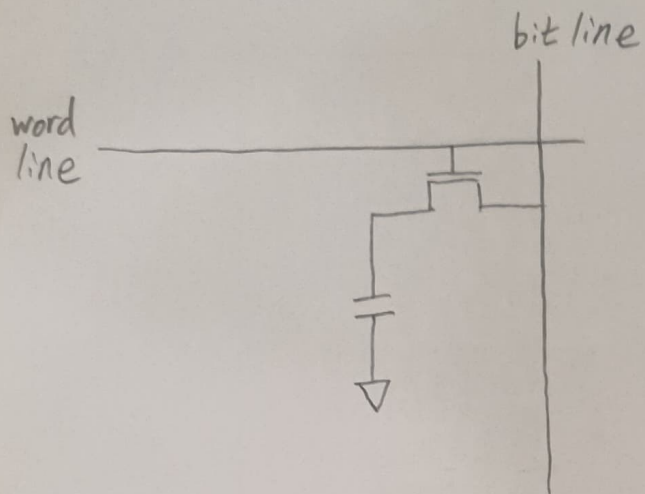
$$2) Q_1 = s_0 \quad 4) B = \overline{s_0} s_1 \quad 6) D = s_0 s_1$$

4. The output of this state machine is only depending on the current state;  $s_0$  and  $s_1$ .

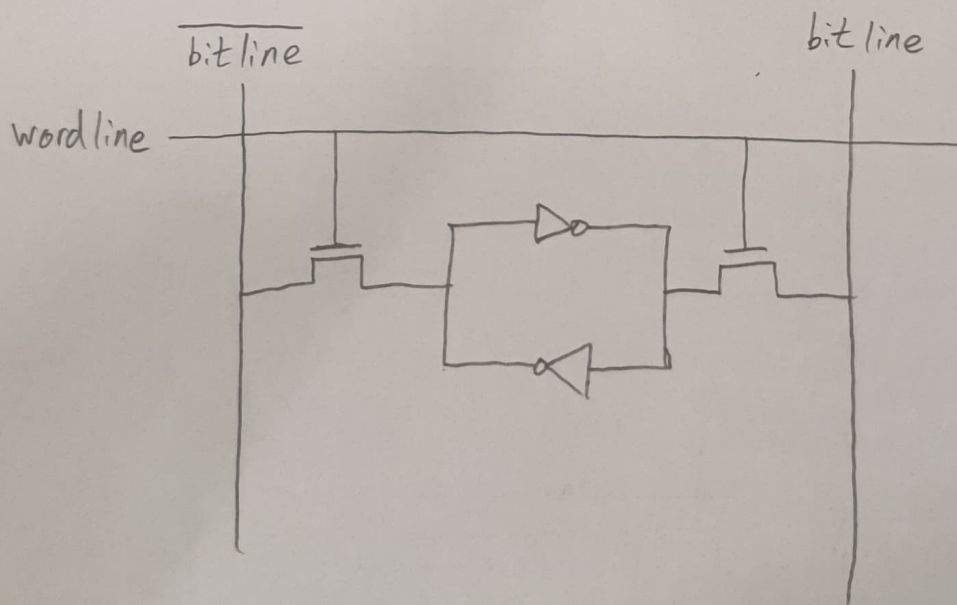
Therefore, this state machine is a Moore State Machine.

# <Memory>

## 1. DRAM



## 2. SRAM





# < Adders >

## 1. Size Table

	8 bits	16 bits	32 bits	64 bits
Ripple Carry	40	80	160	320
Prefix Adder	68	160	304	704

## 2. Time Table

$$t_{\text{ripple}} = N t_{\text{FA}}$$

$$t_{\text{PA}} = t_{\text{pg}} + \log_2 N (t_{\text{pg-prefix}}) + t_{\text{xor}}$$

$$\left( \begin{array}{l} t_{\text{FA}} = 500\text{ps}, t_{\text{pg}} = 100\text{ps} \\ t_{\text{xor}} = 100\text{ps}, t_{\text{pg-prefix}} = 200\text{ps} \end{array} \right)$$

	8 bits	16 bits	32 bits	64 bits
Ripple Carry	4000	8000	16000	32000
Prefix Adder	800	1000	1200	1400

# <Validation>

