

# CS331: Introduction to Artificial Intelligence

## Written Assignment #3

Date handed out: May 9, 2022

Date due: May 16, 2022 at 10:00

Total: 25 points

This assignment is to be done individually. Please hand in a pdf on Canvas.

1. Using the full joint probability distribution below, write out what the following probability distributions look like. Notice that the P is in boldface to emphasize that these are distributions (i.e., probability tables). **This means you have to write out the probability distributions for all uninstantiated random variables e.g., for (a), write out  $P(\text{Catch}=\text{true})$  and  $P(\text{Catch}=\text{false})$ .**

Toothache	Cavity	Catch	<b>P(Toothache, Cavity, Catch)</b>
false	false	false	0.582
false	false	true	0.128
false	true	false	0.009
false	true	true	0.071
True	false	false	0.066
True	false	true	0.014
True	True	false	0.013
True	True	true	0.117

a) **P(Catch) [2 points]**

b) **P(Toothache, Catch) [4 points]**

c) **P(Toothache | Catch) [4 points]**

2. Consider a different variant of the Monty Hall problem we did in class. In this variant, there are 4 doors; one door reveals a car, and the other four doors reveal goats. After you select a door, the host will show you 2 goats. The host will not open the door you chose, nor will they reveal the car. Let  $C \in \{1,2,3,4\}$  be the location of the car,  $D \in \{1,2,3,4\}$  be your initial choice of door,  $H_1 \in \{1,2,3,4\}$  be the first door opened by the host, and  $H_2 \in \{1,2,3,4\}$  be the second door opened by the host. Suppose  $P(C) = P(D) = \{\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\}$ .

a) What is  $P(H_1|C, D)$ ? [2 points]

b) What is  $P(H_2|H_1 = 1, C, D = 3)$ ? [2 points]

c) Suppose you select door 1. The host opens doors 2 and 3. What are  $P(C = 1|D = 1, H_1 = 2, H_2 = 3)$  and  $P(C = 4|D = 4, H_1 = 2, H_2 = 3)$ ? (At this point, the car cannot be behind doors 2 or 3 by the rules of the game.) Should you stick with door 1 or switch to door 4? [4 points]

3. A breast cancer test has a sensitivity of 92% and a specificity of 97.7%. Sensitivity means the probability of a positive result, given that you have the disease. Specificity means the probability of a negative result, given that you do NOT have the disease. The American breast cancer rate is 13%.

a) Based on these numbers, compute the probability that a patient has breast cancer, given that they get a positive test. [5 points]

b) What if the breast cancer rate is actually 8%? How does your answer to part (a) change? [2 points]