1. Notations: True = "T", False = "F"

(a) P(Catch)

i) P(Catch = T)

= P(Toothache = F, Cavity = F, Catch = T) + P(Toothache = F, Cavity = T, Catch = T) + P(Toothache = T, Cavity = F, Catch = T) + P(Toothache = T, Cavity = T, Catch = T)

= 0.128 + 0.071 + 0.014 + 0.117

= 0.33

∴ P(Catch = T) = 0.33

ii) P(Catch = F) = 1 - P(Catch = T) = 1 - 0.33 = 0.67

∴ P(Catch = F) = 0.67

∴ P(Catch = T) = 0.33, P(Catch = F) = 0.67

(b) P(Toothache, Catch)

i) P(Toothache = T, Catch = T)

= P(Toothache = T, Cavity = T, Catch = T) + P(Toothache = T, Cavity = F, Catch = T)

= 0.117 + 0.014 = 0.131

∴ P(Toothache = T, Catch = T) = 0.131

ii) P(Toothache = F, Catch = T)

= P(Toothache = F, Cavity = F, Catch = T) + P(Toothache = F, Cavity = T, Catch = T)

= 0.128 + 0.071 = 0.199

iii) P(Toothache = F, Catch = F)

= P(Toothache = F, Cavity = F, Catch = F) + P(Toothache = F, Cavity = T, Catch = F)

= 0.582 + 0.009 = 0.591

iv) P(Toothache = T, Catch = F) = P(Catch = F) - P(Toothache = F, Catch = F)

= 0.67 - 0.591 = 0.079

∴ ~~P(Toothache = T, Catch = T) = 0.131, P(Too~~

(c) $P(\text{Toothache} \mid \text{Catch}) = \dfrac{P(\text{Toothache}, \text{Catch})}{P(\text{Catch})}$

i) $P(\text{Toothache}=T \mid \text{Catch}=T) = \dfrac{0.131}{0.33} = \dfrac{131}{330} \simeq 0.4$

ii) $P(\text{Toothache}=F \mid \text{Catch}=T) = \dfrac{0.191}{0.33} = \dfrac{191}{330} \simeq 0.6$

iii) $P(\text{Toothache}=T \mid \text{Catch}=F) = \dfrac{0.079}{0.67} = \dfrac{79}{670} \simeq 0.12$

iv) $P(\text{Toothache}=F \mid \text{Catch}=F) = \dfrac{0.591}{0.67} = \dfrac{591}{670} \simeq 0.88$

$\therefore$ i) $P(\text{Toothache}=T \mid \text{Catch}=T) = \dfrac{131}{330}$

ii) $P(\text{Toothache}=F \mid \text{Catch}=T) = \dfrac{191}{330}$

iii) $P(\text{Toothache}=T \mid \text{Catch}=F) = \dfrac{79}{670}$

iv) $P(\text{Toothache}=F \mid \text{Catch}=F) = \dfrac{591}{670}$

2.

(a) $P(H_1 \mid C, D)$

i) $D=3$

C	H_1	$P(H_1 \mid C, D)$
1	1	0
1	2	$\frac{1}{2}$
1	3	0
1	4	$\frac{1}{2}$
2	1	$\frac{1}{2}$
2	2	0
2	3	0
2	4	$\frac{1}{2}$
3	1	$\frac{1}{3}$
3	2	$\frac{1}{3}$
3	3	0
3	4	$\frac{1}{3}$
4	1	$\frac{1}{2}$
4	2	$\frac{1}{2}$
4	3	0
4	4	0

From this case $\Rightarrow$ $P(H_1 \mid C, D) \in \{0, \frac{1}{3}, \frac{1}{2}\}$

$\therefore P(H_1 \mid C, D) \in \{0, \frac{1}{3}, \frac{1}{2}\}$

(b) $P(H_2 \mid H_1 = 1, C, D = 3)$

$D=3$	C	$H_1=1$	H_2	$P(H_2 \mid H_1=1, C, D=3)$
	1	1	1	0
	1	1	2	0
	1	1	3	0
	1	1	4	0
	2	1	1	0
	2	1	2	0
	2	1	3	0
	2	1	4	1
	3	1	1	0
	3	1	2	$\frac{1}{2}$
	3	1	3	0
	3	1	4	$\frac{1}{2}$
	4	1	1	0
	4	1	2	1
	4	1	3	0
	4	1	4	0

From this case, $P(H_2 \mid H_1 = 1, C, D = 3) \in \{0, \frac{1}{2}, 1\}$

$$\therefore P(H_2 \mid H_1 = 1, C, D = 3) \in \{0, \tfrac{1}{2}, 1\}$$

(c)

i) $D=1$

C	H_1	$P(H_1 \mid C, D)$
1	1	0
1	2	$\frac{1}{3}$
1	3	$\frac{1}{3}$
1	4	$\frac{1}{3}$
2	1	0
2	2	0
2	3	$\frac{1}{2}$
2	4	$\frac{1}{2}$
3	1	0
3	2	$\frac{1}{2}$
3	3	0
3	4	$\frac{1}{2}$
4	1	0
4	2	$\frac{1}{2}$
4	3	$\frac{1}{2}$
4	4	0

ii) $D=1$, C, $H_1=2$ $H_2=$ $P(H_2 \mid H_1=2, C, D=1)$

C	$H_1=2$	$H_2=$	$P(H_2 \mid H_1=2, C, D=1)$
1	2	1	0
1	2	2	0
1	2	3	$\frac{1}{2}$
1	2	4	$\frac{1}{2}$
2	2	1	0
2	2	2	0
2	2	3	0
2	2	4	0
3	2	1	0
3	2	2	0
3	2	3	0
3	2	4	1
4	2	1	0
4	2	2	0
4	2	3	1
4	2	4	0

iii) $P(C \mid D=1, H_1=2, H_2=3)$

$$= \frac{P(C, D=1, H_1=2, H_2=3)}{P(D=1, H_1=2, H_2=3)} = \frac{P(C, D=1, H_1=2, H_2=3)}{\sum_c P(C=c, D=1, H_1=2, H_2=3)}$$

$$= \frac{P(H_2=3 \mid C, D=1, H_1=2) P(C, D=1, H_1=2)}{\sum_c \{P(H_2=3 \mid C=c, D=1, H_1=2) P(C=c, D=1, H_1=2)\}}$$

$$= \frac{P(H_2=3 \mid C, D=1, H_1=2) P(H_1=2 \mid C, D=1) P(C) P(D=1)}{\sum_c \{P(H_2=3 \mid C=c, D=1, H_1=2) P(H_1=2 \mid C=c, D=1) P(C=c) P(D=1)\}}$$

$$= \frac{P(H_2=3 \mid C, D=1, H_1=2) P(H_1=2 \mid C, D=1) P(C) \cancel{P(D=1)}}{\sum_c \{P(H_2=3 \mid C=c, D=1, H_1=2) P(H_1=2 \mid C=c, D=1) P(C=c) \cancel{P(D=1)}\}}$$

$$= \propto \langle \overset{C=1}{\frac{1}{4} \cdot \frac{1}{3} \cdot \frac{1}{2}} \quad \overset{C=2}{\frac{1}{4} \cdot 0 \cdot 0} \quad \overset{C=3}{\frac{1}{4} \cdot \frac{1}{2} \cdot 0} \quad \overset{C=4}{\frac{1}{4} \cdot \frac{1}{2} \cdot 1} \rangle$$

$$= \propto \langle \frac{1}{4} \cdot \frac{1}{6}, 0, 0, \frac{1}{4} \cdot \frac{1}{2} \rangle \quad = \langle \frac{1}{4}, 0, 0, \frac{3}{4} \rangle$$

$\therefore P(C=1 \mid D=1, H_1=2, H_2=3) = \frac{1}{4}$

$P(C=4 \mid D=1, H_1=2, H_2=3) = \frac{3}{4}$

Thus, you have to switch to door 4 because there is 75% of probability that the car is behind the door.

3.

Notations: Breast Cancer = "BC", Not Breast Cancer = "¬BC",
Positive = "P", Negative = "N"

a. $P(BC) = 0.13$, $P(P|BC) = 0.92$, $P(N|\neg BC) = 0.907$

	P	N	
BC	$0.92 \cdot 0.13$	$0.08 \cdot 0.13$	0.13
¬BC	$0.87 \cdot 0.023$	$0.87 \cdot 0.977$	0.87

⇓

	P	N	
BC	0.1196	0.0104	0.13
¬BC	0.02001	0.84999	0.87
	0.13961	0.86039	1

$$\therefore P(BC|P) = \frac{P(BC, P)}{P(P)} = \frac{0.1196}{0.13961} \doteq 0.857$$

$$\therefore P(BC|P) \simeq 85.7\%$$

(b)

	P	N	
BC	$0.92 \cdot 0.08$	$0.08 \cdot 0.08$	0.08
¬BC	$0.023 \cdot 0.92$	$0.907 \cdot 0.92$	0.92
	0.09476	0.90524	1

$$\therefore P(BC|P) = \frac{0.92 \cdot 0.08}{0.09476} = \frac{0.0736}{0.09476} \simeq 0.777$$

$$\therefore P(BC|P) \simeq 77.7\%$$