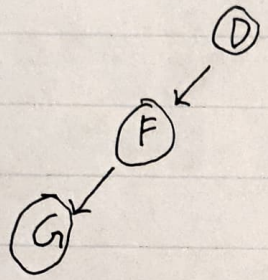
1.

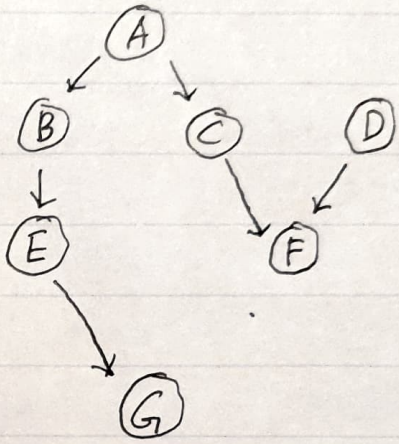
(a) $I(D, G \mid \{3\})$

i)



Not blocked by F

ii)

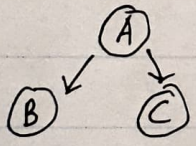


"Blocked by F" (Case 3)

$\therefore I(D, G \mid \{3\}) = \text{False}.$

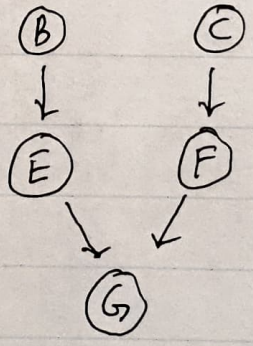
(b) $I(B, C \mid \{3\})$

i)



"Not blocked by A"

ii)

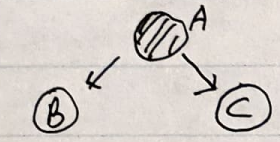


"Blocked by G" (Case 3)

~~$I(A, D \mid \{3\})$~~ $I(B, C \mid \{3\}) = \text{False}.$

(c) $I(B, C \mid \{A\})$

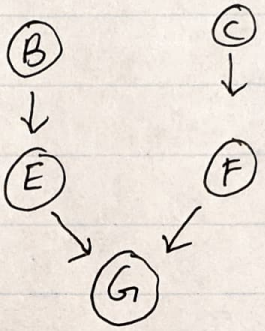
i)



"Blocked by A"

(Case 1)

ii)



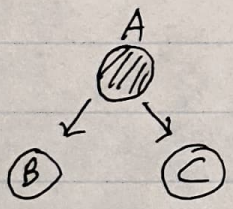
"Blocked by G"

(Case 3)

$\therefore I(B, C \mid \{A\}) = \text{True}.$

(d) $I(B, C \mid \{A, G\})$

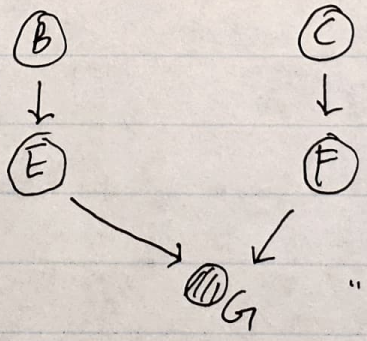
i)



"Blocked by A"

(Case 1)

ii)



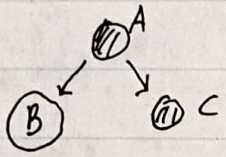
"Not blocked by G"

(No Case 1, Case 2, or Case 3)

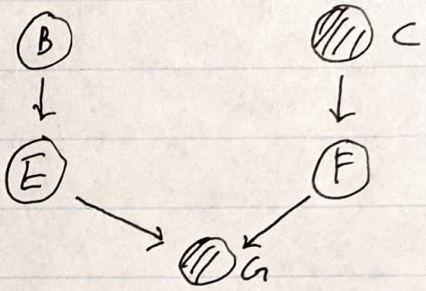
$\therefore I(B, C \mid \{A, G\}) = \text{False}$

(e) $I(B, C \mid \{A, C, G\})$

i)



ii)



Since $B \in V - E$, $C \in E$ and $C \notin V - E$,
B and C are not d-separated.

$\therefore I(B, C \mid \{A, C, G\}) = \text{False}$

2.

(a) $P(A=T, B=F, C=T, D=F, E=F)$

$= P(A=T)\,P(B=F|A=T)\,P(C=T|A=T)\,P(D=F|C=T)\,P(E=F|B=F, D=F)$

$= 0.4 \times 0.1 \times 0.25 \times 0.25 \times 0.6 = 0.0015$

$\therefore P(A=T, B=F, C=T, D=F, E=F) = 0.0015.$

(b) $P(B=F, C=F)$

$= \sum_a P(A=a)P(B=F|A=a)P(C=F|A=a) \sum_d P(D=d|C=F) \sum_e P(E=e|B=F, D=d)$

i) $\sum_e P(E=e|B=F, D=d) = 1$

$\Rightarrow \sum_a P(A=a)\,P(B=F|A=a)\,P(C=F|A=a) \sum_d P(D=d|C=F)$

ii) $\sum_d P(D=d|C=F) = 1$

$\Rightarrow \sum_a P(A=a)P(B=F|A=a)P(C=F|A=a) = P(B=F, C=F)$

① $A=T$

$P(A=T)\,P(B=F|A=T)\,P(C=F|A=T) = 0.4 \times 0.1 \times 0.75 = 0.03$

② $A=F$

$P(A=F)\,P(B=F|A=F)\,P(C=F|A=F) = 0.6 \times 0.75 \times 0.2 = 0.09$

$\therefore P(B=F, C=F) = 0.12$

c. $P(A=T \mid B=F, C=F)$

i) $P(A=T \mid B=F, C=F)$

$$= \frac{P(A=T, B=F, C=F)}{P(B=F, C=F)}$$

$P(A=T, B=F, C=F)$

$= P(A=T)P(B=F \mid A=T)P(C=F \mid A=T) \sum_d P(D=d \mid C=F) \sum_e P(E=e \mid B=F, D=d)$

$= P(A=T)P(B=F \mid A=T)P(C=F \mid A=T)$

$= 0.4 \times 0.1 \times 0.75 = 0.03$

$$\therefore \frac{P(A=T, B=F, C=F)}{P(B=F, C=F)} = \frac{0.03}{0.12} = \frac{1}{4} = 0.25$$

$$\therefore P(A=T \mid B=F, C=F) = 0.25$$