

1.

(a) $[A:s1, B:s1]$

(b) $s1$ is a strictly dominant strategy for A.

$\therefore A:s1$

(c) No.

There is no always better strategy for B regardless of what A plays.

For instance, if B plays $s1$, then $[A:s1, B:s1]$ is the best.

However, if A switches to $s2$, then $[A:s2, B:s3]$ is better than $[A:s2, B:s1]$ for B.

\therefore B has no strictly dominant strategy.

(d) $[A:s1, B:s1] \Rightarrow [A=5, B=9]$

There is no other outcome that A and B would prefer.

\therefore Pareto optimal outcome = $(A=5, B=9)$

(e) Since there is a pure Nash equilibrium, this game is not a zero-sum game.

2.

(a) i) Mixed Strategy for A.

$$[S1: p, S2: 1-p]$$

B's ~~expe~~ payoff for S1

$$\therefore -2p + 3(1-p) = 3-5p$$

B's payoff for S2

$$\therefore 5p - 4(1-p) = 9p - 4$$

$$3-5p = 9p-4 \Rightarrow 7 = 14p \Rightarrow p = \frac{1}{2}$$

$$\therefore [S1: \frac{1}{2}, S2: \frac{1}{2}]$$

ii) Mixed strategy for B.

$$[S1: q, S2: 1-q]$$

A's payoff for S1

$$\therefore 2q - 5(1-q) = 7q - 5$$

A's payoff for S2

$$-3q + 4(1-q) = 4-7q$$

$$4-7q = 7q-5 \Rightarrow 9 = 14q \Rightarrow q = \frac{9}{14}$$

~~$$[S1: \frac{9}{14}, S2: \frac{5}{14}]$$~~

$$\therefore [S1: \frac{9}{14}, S2: \frac{5}{14}]$$

(b)

$$i) A's \text{ expected payoff} = 4-7q = 4-7 \cdot \frac{9}{14} = 4-\frac{9}{2} = -\frac{1}{2}$$

$$\therefore -\frac{1}{2}$$

$$ii) B's \text{ expected payoff} = 9p-4 = 9 \cdot \frac{1}{2} - 4 = \frac{1}{2}$$

$$\therefore \frac{1}{2}$$