1.

Assume L_1 and L_2 are regular languages.

Then, L_1 and L_2 are closed under union, intersection, complement, concatenation, and star-closure by Theorem 4.1 in textbook.

As the symmetric difference of two sets is a new set that contains every elements in either set except the elements in both sets, the symmetric difference of L_1 and L_2 can be denoted as $(L_1 - L_2) \cup (L_2 - L_1)$.

$L_1 - L_2 = L_1 \cap \overline{L_2}$. Since L_1 and L_2 are closure under intersection and complement, $\overline{L_2}$ is regular so that $L_1 \cap \overline{L_2}$ is regular.

$L_2 - L_1 = L_2 \cap \overline{L_1}$. Since L_1 and L_2 are closure under intersection and complement, $\overline{L_1}$ is regular so that $L_2 \cap \overline{L_1}$ is regular.

As $(L_1 \cap \overline{L_2})$ and $(L_2 \cap \overline{L_1})$ are regular, $(L_1 \cap \overline{L_2})$ and $(L_2 \cap \overline{L_1})$ are closed under union, intersection, complement, concatenation, and star-closure.

Thus, $(L_1 \cap \overline{L_2}) \cup (L_2 \cap \overline{L_1})$ is regular.

We proved that symmetric difference of regular languages is regular with regular languages L_1 and L_2.

Therefore, the family of regular languages is closed under symmetric difference. ∎

2.

Assume L is regular language. We're given m.

Let's pick $w = a^m b^m$. Suppose $|xy| \le m$, $|y| \ge 1$, and $|y| = k$ $(1 \le k \le m)$.

Let $xy = a^m$, $y = a^k$. Then $w = xyz = a^{m-k} a^k b^m$ so that $w_i = a^{m-k} (a^k)^i b^m$ by Theorem 4.8.

If we choose $i = 0$, $w_0 = a^{m-k} b^m$. Since $1 \le k \le m$, $m-k \ne m$ so that $w_0 \notin L$.

Thus, we have successfully pumped the string out of the language. ∎

3.

Assume L is regular language and we're given m.

Let's pick $w = a^{2^m}$. Suppose $|xy| \le m$, $|y| \ge 1$, and $|y| = k$ $(1 \le k \le m)$.

Let $xy = a^m$ and $y = a^k$. Then $w = xyz = a^{m-k} a^k a^{2^m - m}$ so that

$w_i = a^{m-k} (a^k)^i a^{2^m - m}$ by Theorem 4.8 in textbook.

If we choose $i = 0$, $w_0 = a^{2^m - k}$.

Since $1 \le k \le m$, $2^m - k > 2^{(m-1)}$ so that $w_0 \notin L$.

Thus, we have successfully pumped the string out of the language. ∎