

$$1. \begin{aligned} S &\rightarrow aS | bBb | aaA | A \\ A &\rightarrow aaA | Aa \\ B &\rightarrow Bb | bba | \lambda \end{aligned}$$

$S \rightarrow A$ has to be removed.

$B \rightarrow \lambda$ has to be removed.

$$\Downarrow$$

$$\begin{aligned} S &\rightarrow aS | bBb | aaA | Aa | bB \\ A &\rightarrow aaA | Aa \\ B &\rightarrow Bb | bba | b \end{aligned}$$

A is useless

because A never lead to a sentence.

$$\Downarrow$$

$$\begin{aligned} S &\rightarrow aS | bBb | bb \\ B &\rightarrow Bb | bba | b \end{aligned}$$

$$\begin{aligned} S &\rightarrow aS | bBb | bb \\ B &\rightarrow Bb | bba | b \end{aligned}$$

$$2. \begin{aligned} S &\rightarrow aAbC | aaA | bbS \\ A &\rightarrow bbC | bb \\ B &\rightarrow Ab | ba | \lambda \\ C &\rightarrow abC | C | \lambda \end{aligned}$$

B is useless

$B \rightarrow \lambda, C \rightarrow \lambda$ has to be removed.

$C \rightarrow C$ is useless.

$$\Downarrow$$

$$\begin{aligned} S &\rightarrow aAbC | aaA | bbS | aAb \\ A &\rightarrow bbC | bb \\ C &\rightarrow abC | ab \end{aligned}$$

$$\begin{aligned} S &\rightarrow aAbC | aaA | bbS | aAb \\ A &\rightarrow bbC | bb \\ C &\rightarrow abC | ab \end{aligned}$$

3. We find the grammar is

$$S \rightarrow aS \mid bBb \mid bb$$

$$B \rightarrow Bb \mid bba \mid b.$$

So, the grammar in Chomsky normal form is...

$$S \rightarrow B_a S \mid D_1 B_b \mid B_b B_b$$

$$D_1 \rightarrow bB$$

$$B \rightarrow B B_b \mid D_2 B_a \mid b$$

$$D_2 \rightarrow B_b B_b$$

$$B_a \rightarrow a \quad B_b \rightarrow b$$

$$S \rightarrow B_a S \mid D_1 B_b \mid B_b B_b$$

$$D_1 \rightarrow bB$$

$$B \rightarrow B B_b \mid D_2 B_a \mid b$$

$$D_2 \rightarrow B_b B_b$$

$$B_a \rightarrow a \quad B_b \rightarrow b$$

4. We find the grammar is...

$$S \rightarrow aAbC \mid aaA \mid bbS \mid aAb$$

$$A \rightarrow bbC \mid bb$$

$$C \rightarrow abC \mid ab$$

So the grammar in Greibach normal form is...

$$S \rightarrow aAB_bC \mid aBaA \mid bB_bS \mid aAB_b$$

$$A \rightarrow bB_bC \mid bB_b$$

$$C \rightarrow aB_bC \mid aB_b$$

$$Ba \rightarrow a \quad B_b \rightarrow b$$

$$S \rightarrow aAB_bC \mid aBaA \mid bB_bS \mid aAB_b$$

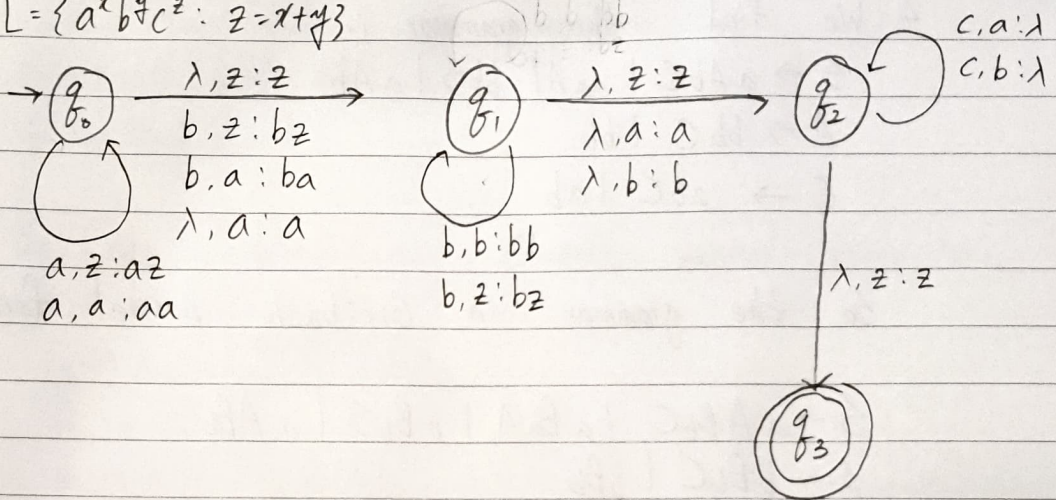
$$A \rightarrow bB_bC \mid bB_b$$

$$C \rightarrow aB_bC \mid aB_b$$

$$Ba \rightarrow a$$

$$B_b \rightarrow b$$

5. $L = \{a^x b^y c^z : z = x + y\}$



6. $L = \{a^n b^n : n \text{ is not a multiple of } 3\}$

