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1. $S \to aSbA \mid b$
$A \to abA \mid \lambda$

We have to remove λ-production.

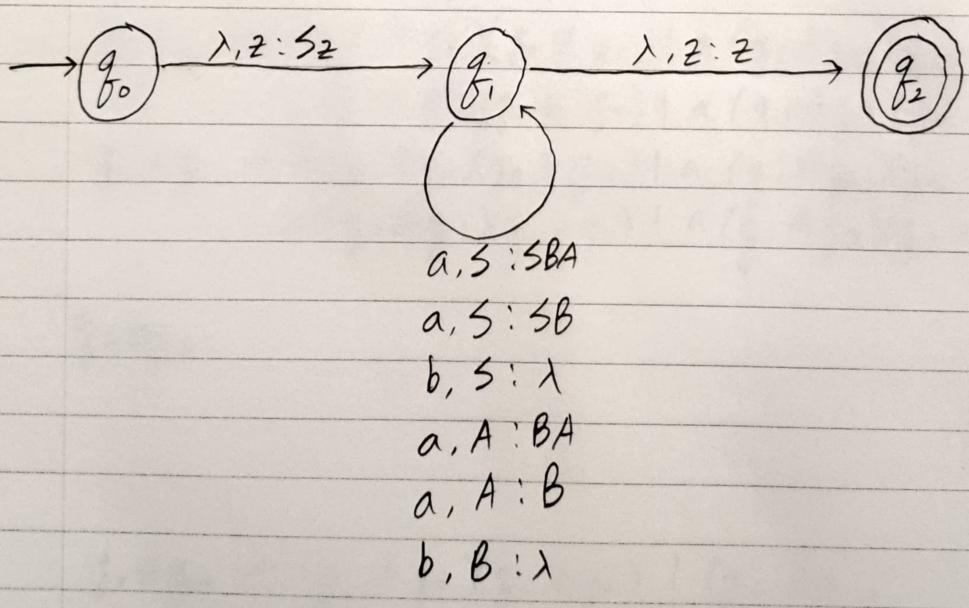
$\Downarrow$

$S \to aSbA \mid aSb \mid b$
$A \to abA \mid ab$.

We have to change in Greibach Normal Form.

$\Downarrow$

$S \to aSBA \mid aSB \mid b$
$A \to aBA \mid aB$
$B \to b$.



2.

$\delta(q_2, \lambda, z) = \{(q_3, \lambda)\}$ $\Rightarrow$ $q_2 z q_3 \to \lambda$
$\delta(q_0, \lambda, z) = \{(q_1, Az)\}$
$\delta(q_2, a, z) = \{(q_1, Az)\}$
$\delta(q_1, b, A) = \{(q_2, \lambda)\}$ $\Rightarrow$ $q_1 A q_2 \to b$

The npda satisfies condition 1 and 2.

$\delta(q_2, \lambda, Z) = \{(q_3, \lambda)\} \Rightarrow q_2 Z q_3 \to \lambda$
$\delta(q_1, b, A) = \{(q_2, \lambda)\} \Rightarrow q_1 A q_2 \to b.$

By $\delta(q_2, a, z)$, $\delta(q_0, \lambda, z)$ we get ...

$q_2 z q_0 \to a(q_1 A q_0)(q_0 z q_0) \mid a(q_1 A q_2)(q_2 z q_0) \mid$
$a(q_1 A q_1)(q_1 z q_0) \mid a(q_1 A q_3)(q_3 z q_0)$
$q_2 z q_1 \to a(q_1 A q_0)(q_0 z q_1) \mid a(q_1 A q_2)(q_2 z q_1) \mid$
$a(q_1 A q_1)(q_1 z q_1) \mid a(q_1 A q_3)(q_3 z q_1)$

$q_2 z q_3$

⋮

$q_0 z q_0 \to (q_1 A q_0)(q_0 z q_0) \mid (q_1 A q_1)(q_1 z q_0) \mid$
$(q_1 A q_2)(q_2 z q_0) \mid (q_1 A q_3)(q_3 z q_0)$

⋮

$q_0 z q_3 \to (q_1 A q_0)(q_0 z q_3) \mid (q_1 A q_1)(q_1 z q_3) \mid$
$(q_1 A q_2)(q_2 z q_3) \mid (q_1 A q_3)(q_3 z q_1)$

Since there are only paths from q_2 to q_3, q_0 to q_1, q_2 to q_1 and q_1 to q_2.

The context-free grammar is....

$(q_2 Z q_3) \rightarrow \lambda$
$(q_1 A q_2) \rightarrow b$
$(q_0 Z q_0) \rightarrow (q_1 A q_2)(q_2 Z q_0)$
$(q_0 Z q_1) \rightarrow (q_1 A q_2)(q_2 Z q_1)$
$(q_0 Z q_2) \rightarrow (q_1 A q_2)(q_2 Z q_2)$
$(q_0 Z q_3) \rightarrow (q_1 A q_2)(q_2 Z q_3)$

$(q_2 Z q_0) \rightarrow a(q_1 A q_2)(q_2 Z q_0)$
$(q_2 Z q_1) \rightarrow a(q_1 A q_2)(q_2 Z q_1)$
$(q_2 Z q_2) \rightarrow a(q_1 A q_2)(q_2 Z q_2)$
$(q_2 Z q_3) \rightarrow a(q_1 A q_2)(q_2 Z q_3)$

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3.

Given the opponent's choice for m; we pick a^p where $p = 2^m$ so that $p \geq m$.
The opponent chooses a decomposition where $v = a^k$ and $y = a^l$ with $k+l \geq 1$.
If we pump i times, then $w_i = a^{p+(k+l)(i-1)}$.
So, if we pump 0 times, then $w_0 = a^{p-(k+l)}$.
Since $1 \leq (k+l) \leq m$ and $p = 2^m$, $2^m - (k+l) < 2^m$ and $2^m - (k+l) > 2^{m-1}$.
$2^m - (k+l)$ is not a power of 2.
Since $w_0 = a^{2^m-(k+l)}$ and $w_0 \notin L$, L is not context-free

4.

Given the opponent's choice for m, we pick $a^m b^m c^{m+1}$.
The opponent chooses a decomposition where $v = c^k$ and $y = c^l$ with $k+l \geq 1$.
If we pump i times, then $w_i = a^m b^m c^{m+1+(k+l)(i-1)}$
So, if we pump 0 times, then $w_0 = a^m b^m c^{m+1-(k+l)}$.

Since $1 \leq (k+l) \leq m$, $1 \leq m+1-(k+l) \leq m$.
So, $m+1-(k+l) > m$ is false so that $w_0 \notin L$.
Therefore, L is not context-free.

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5. $L = \{ \{a,b,c\}^* : n_a(w) = n_b(w) = n_c(w) \}$.

Let's use closure under regular intersection to prove that the complement of L is context-free.

$\bar{L} = \{ \{a,b,c\}^* : n_a(w) = n_b(w)$ or $n_b(w) = n_c(w)$ or $n_a(w) = n_c(w)$, or $n_a(w) \neq n_b(w) \neq n_c(w) \}$

Let $L_1 = \{ab^3c^5\}$.
Since L_1 is finite, L_1 is regular.

Then, $\bar{L} \cap L_1 = L_1 = \{ab^3c^5\}$.
Since all regular language is context-free, $\bar{L} \cap L_1$ is context-free.

By the closure under regular intersection, the complement of L is context-free.