Date . . No.

1. A turing machine M can either change the tape symbol or move the read/write head, but not both on each move.

$M = (Q, \Sigma, \Gamma, \delta, q_0, \square, F\}$.

$\delta: Q \times \Gamma \longrightarrow \{Q \times \Gamma\} \cup \{Q \times \{L, R\}\}$.

We can simulate M by using a standard Turing machine $\hat{M}$.

$\hat{M} = (\hat{Q}, \Sigma, \Gamma, \hat{\delta}, \hat{q}_0, \square, \hat{F})$.

First in case of changing the tape symbol, we can stop $\hat{M}$ after changing the tape symbol.

$\delta(q_i, a) = \delta(q_j, b)$

$\Downarrow$

$\hat{\delta}(\hat{q}_i, a) = \hat{\delta}(\hat{q}_{js}, b, R)$ and $\hat{\delta}(\hat{q}_{js}, c) = \hat{\delta}(\hat{q}_j, c, L)$.

$\forall c \in \Gamma$

In case of moving the read/write head, we can just move $\hat{M}$ without changing the tape symbol.

$\delta(q_k, d) = \delta(q_l, L \text{ or } R)$

$\Downarrow$

$\hat{\delta}(\hat{q}_k, d) = \hat{\delta}(\hat{q}_l, d, L \text{ or } R)$.

Thus, M is simulated by a standard Turing Machine.

Date . . No.

2. Let two independent stacks of two-stack npda of NTM are S_L and S_R.

NTM : Nondeterministic Turing Machine

Assume two stacks are starting at z, and the tape is empty. A move depends on the tops of S_L and S_R, and results in new values being pushed on S_L and S_R.

We will write the top of S_L to the left of the head and the top of S_R to the right of the head of nondeterminstic Turing machine.

i) Initiation

$\delta(q_0, \square) = (q_{0s}, \#, R)$, $\delta(q_0, \#) = \{(q_1, SL_1, L), (q_2, SR_1, R)\}$

$\delta(q_{0s}, \square) = (q_0, \square, L)$

SL_i is top of S_L

SR_i is top of S_R $(i = 1, 2, 3, \cdots)$

ii) For pushing new values.

$\delta(q_i, \square_k) = (q_j, SL_{k+1}, L)$

$\delta(q_j, \square) = (q_k, \square, R)$ for S_L

$\delta(q_l, \square) = (q_m, SR_{m+1}, R)$.

$\delta(q_m, \square) = (q_n, \square, L)$. for S_R

iii) For a movement depending on the tops.

$\delta(q_a, SL_i) = (q_{as}, SL_i, L)$

$\delta(q_{as}, \square_i) = (q_a, \square, R)$ (for S_L)

$\delta(q_d, SR_k) = (q_{ds}, SR_k, R)$

$\delta(q_{ds}, \square) = (q_d, \square, L)$ (for S_R)

iv) For poping tops of S_L and S_R.

TM: Turing Machine

$f(q_i, SL_j) = (q_{i+1}, \square, L)$

$f(q_{i+1}, \square) = (q_{i+1}, \square, R)$ for S_L

$f(q_k, SR_l) = (q_{k+1}, \square, R)$

$f(q_{k+1}, \square) = (q_{k+1}, \square, L)$ for S_R

v) Halt: $f(q_x, \square) = (q_x, \square, R)$, $f(q_x, \#) = (q_{fL}, \#, L)$, for S_L

$f(q_y, \square) = (q_y, \square, L)$, $f(q_y, \#) = (q_{fR}, \#, R)$ for S_R

Two-stack npda is equivalent to nondeterministic TM because every automation of NTM is in an automation of two-stack npda.
By Theorem 10.2 in the textbook, nondeterministic Turing machine is equivalent to standard Turing machine.

Therefore, two-stack npdas are equivalent to standard Turing machines.

Date . . No.

3. S_1 and S_2 are countable sets.
Countable sets can be written by enumeration procedure.
Turing machine can implement the enumeration procedure used in countable sets.
Thus, Countable sets are accepted by Turing machines.

$S_1 \cup S_2$ can be accepted by two-tape Turing machine or a standard Turing machine with four tracks.

$S_1 \times S_2$ can be accepted by two-dimensional Turing machine,

By Theorem 10.3 in the textbook, Turing machinces which accept $S_1 \cup S_2$ or $S_1 \times S_2$ (but not both) are countable.

Therefore, $S_1 \cup S_2$ and $S_1 \times S_2$ are countable.