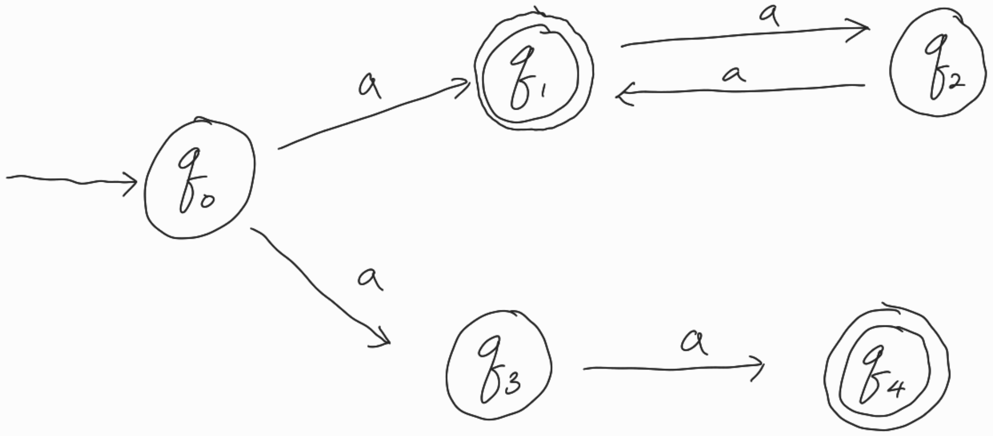
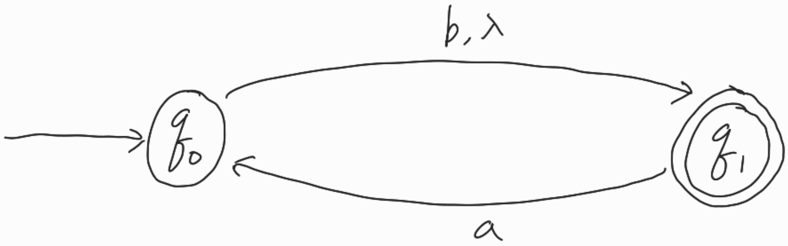
1.



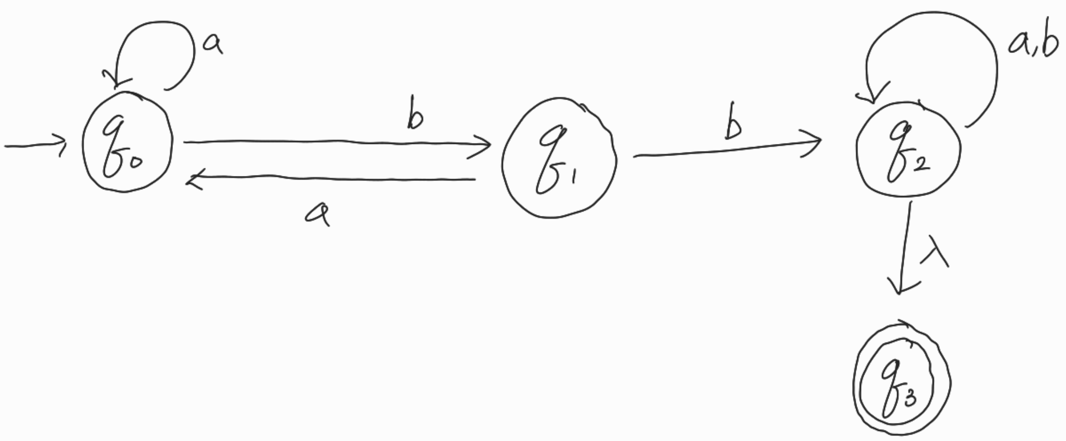
2.

NFA for L

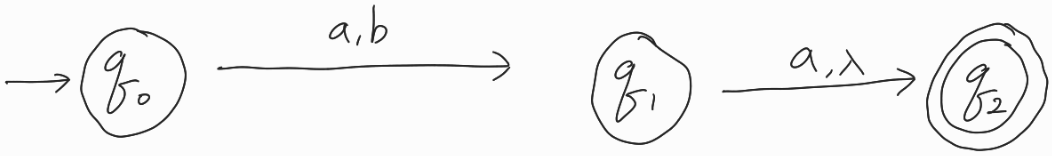


The element of $\bar{L}$ will not be accepted by the NFA.
The string in which 'b' is occuring more than one in a row is not accepted by the NFA. So $\bar{L}$ is a set of string in which 'b' occurs more than one in a row.

NFA for $\bar{L}$



3. NFA for L



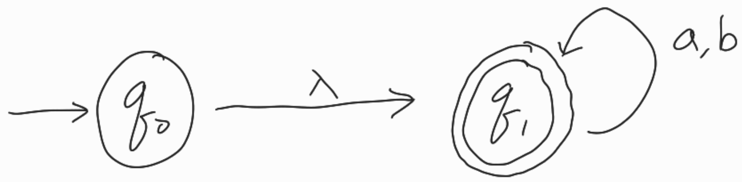
$\therefore L = \{a, b, aa, ba\}$

$L^* = \{\lambda, a, b, aa, ba, bb, \cdots, \}$

Let $A = \{a,b\}$. Then $A^* \supseteq L^*$.

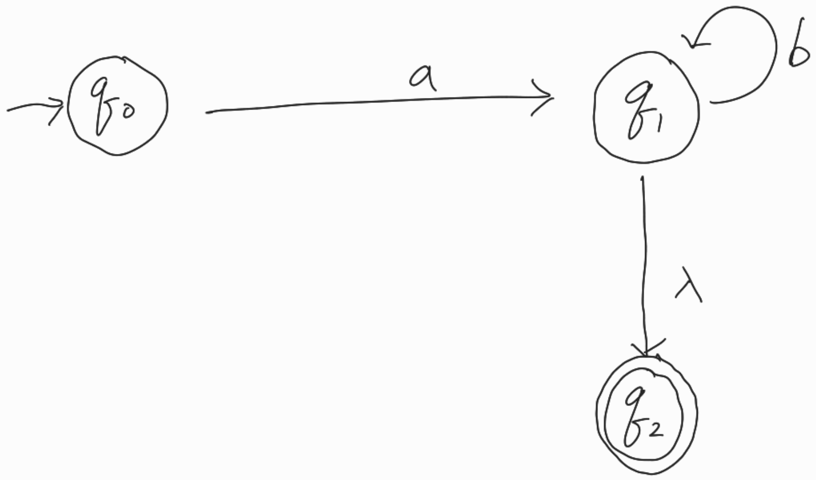
So, NFA for L^* accepts all the string which is combination of 'a' and 'b'.

$\therefore$ NFA for L^*

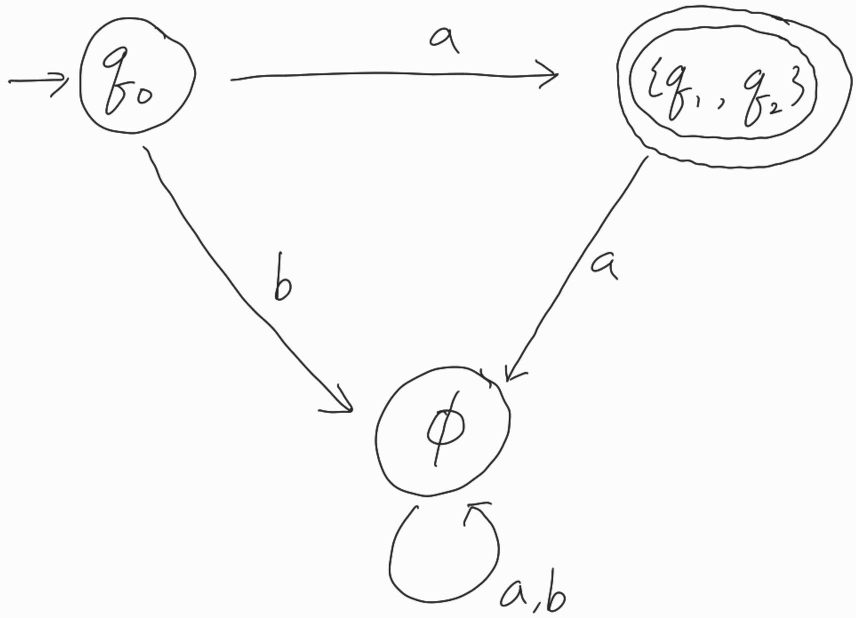


4.

<NFA>



∴ <DFA>



5. Prove all finite languages are regular.

Base Case)

- Let finite language $L = \phi$. Then L is regular
- Let finite language $L = \{\lambda\}$. Then L is regular.

$\therefore$ Base case holds.

Inductive step)

Let $L = \{a_1, a_2, a_3, \cdots, a_n\}$, $n \geq 0$ be a finite number, and $a_{1,2,\cdots n} \in \Sigma^*$.

Assume L is a regular.

($\because a \in \Sigma \rightarrow$ regular, concatenation of regular $(r_1 \cdot r_2) \longrightarrow$ regular)

Then $L \cup \{w\}$. ($w \notin L$, $w \in \Sigma^*$ and $w \neq \lambda$)

As $\{w\}$ is regular, the union between two regular languages is regular.

$\therefore L \cup \{w\} = \{a_1, a_2, a_3, \cdots, a_n, a_{n+1}\} \ (a_{n+1} = w)$, is regular.

Thus, if our claim is true for n, it must be true for $n+1$. Since n can be any finite number, the statement is true for any finite languages. ■

6.

If language L is regular, there are some DFA which accepts L.

If there are multiple final states in the DFA, let's connect them to a new final state with λ transection. This is NFA which is equivalent to DFA.

So, NFA has one initial state and one final state.

When we set the initial state as final state and the final state as initial state, and reverse the direction of transection lines, the renewed NFA or DFA will accept strings which are reversed form of strings in L. (If DFA has one initial state and one final state, we don't have to change it into NFA)

Since we found NFA or DFA which accepts the reversed strings and NFA is equivalent to DFA, L^R is regular if L is regular. ■