CSE 180 – Introduction to Robotics
Homework N.3 – Spring 2018

Important Information

1. This homework is due on March 22, 2018 at 4:15pm. The deadline is strict and will be enforced by CatCourses. **No exceptions.**
2. Your solution must be submitted in electronic form through CatCourses. Paper submissions will not be accepted. **No exceptions.**
3. If you write your solution by hand, write clearly. Unreadable documents will not be graded.
4. The method you follow to determine the solution is more important than the final results. Clearly illustrate and explain the intermediate steps. If you just write the final result you will get not credit.
5. Before solving this homework, make sure you download the latest version of the lecture notes from CatCourses and carefully study appendix A (up to section A.9 included).
6. **THIS HOMEWORK MUST BE SOLVED INDIVIDUALLY. NO EXCEPTIONS**.

1 Random Variables

Consider the probability space $(\Omega, \mathcal{A}, P)$ defined as follows:

- $\Omega = \{R, G, B, Y\}$;
- $\mathcal{A} = 2^{\Omega}$, i.e., the event space is the power set of Ω;
- $P(R) = 1/3$, $P(G) = P(B) = P(Y) = 2/9$, where we define the probability only for the elementary outcomes, and the probability of every event in $\mathcal{A}$ can be deduced from these values (as per discussion in section A.5).

This probability space can model, for example, the outcome of rolling a four-sided die whose faces are red, green, blue, and yellow. The outcome of the experiment is the color of the downward face. Next, let us consider the function $X : \Omega \to \mathbb{R}$ defined as follows: $X(R) = 2; X(G) = -2; X(B) = X(Y) = 0$.

1. Prove that X defines a random variable over the given probability space.
2. Compute the cumulative function of X

2 Bayes Rule

Let A be the event *Person X has disease Z*. Let moreover assume that Z is a rare disease, i.e., the probability that a X has disease Z is 0.01. Let us assume a certain diagnostic exam is performed on X and it indicates that X indeed has disease Z. However, the exam is not perfect, i.e., it has a false positive rate of 0.1, i.e., there is 0.1 probability that the exam determines X has the disease, even though this is not the case. In the light of the exam result, what is the posterior probability that X has disease Z? If the exam is performed twice, and both times it indicates that person X has the disease, would the probability change? If your answer is "yes" explain why and how it will change. If your answer is "no" explain why it will not change.

3 Sum of Random Variables

Let $X \sim \mathcal{N}(0,1)$ and $Y \sim \mathcal{N}(2,1.5)$ and let X and Y be independent. Let $Z = 2X + Y$ and let f_Z be its PDF. Determine the interval $[a,b]$ such that

$$\int_a^b f_Z(\zeta) d\zeta = 0.95.$$

4 Confidence region

Consider a bivariate Gaussian variable with $\mu_x = [2 \quad 3]^T$ and covariance matrix

$$\mathbf{\Sigma} = \begin{bmatrix} 1 & 0.2 \\ 0.2 & 2.5 \end{bmatrix}$$

Let f_x its probability density. Determine the region $\mathcal{D}$ of $\mathbb{R}^2$ such that

$$\int_{\mathcal{D}} f_x(\zeta) d\zeta = 0.975$$