CSE 180 – Introduction to Robotics
Homework N.4 – Spring 2018

Important Information

1. This homework is due on April 24, 2018 at 4:15pm. The deadline is strict and will be enforced by CatCourses. **No exceptions.**
2. Your solution must be submitted in electronic form through CatCourses. Paper submissions will not be accepted. **No exceptions.**
3. If you write your solution by hand, write clearly. Unreadable documents will not be graded.
4. The method you follow to determine the solution is more important than the final results. Clearly illustrate and explain the intermediate steps. If you just write the final result you will get not credit.
5. Before solving this homework, make sure you download the latest version of the lecture notes from CatCourses.
6. **THIS HOMEWORK MUST BE SOLVED INDIVIDUALLY. NO EXCEPTIONS**.

1 Extended Kalman Filter – Linearization

The motion model for a differential drive considered in the example discussed in section 6.7.3 in the lecture notes is simplified, i.e., it assumes that the robot either translates but does not rotate ($v(t) \neq 0$ and $r(t) = 0$) or it rotates but does not translate ($v(t) = 0$ and $r(t) \neq 0$). The following model instead considered the case where the robot can move along a circular arc (we assume $\Delta t = 1$ for simplicity):

$$\begin{cases} x(t+1) = x(t) + \left[-\frac{v(t)}{r(t)} \sin\theta(t) + \frac{v(t)}{r(t)} \sin(\theta(t) + r(t)) \right] \\ y(t+1) = y(t) + \left[\frac{v(t)}{r(t)} \cos\theta(t) - \frac{v(t)}{r(t)} \cos(\theta(t) + r(t)) \right] \\ \theta(t+1) = \theta(t) + r(t) \end{cases} \tag{1}$$

1. Write the linearized motion model for this state transition equation.
2. The given model is not defined when $r(t) = 0$, i.e., when the robot translates only. Write the model that should be used when the robot translates only, and write its linearization.
3. Is the above model accurate for the case where the robot rotates but does not translate? Explain and your answer. If it is not, write an updated model and its linearization.

2 Extended Kalman Filter – Uncertainty Propagation

Consider now a robot governed by the same motion model given in the previous exercise. Its initial pose is modeled by a multivariate Gaussian distribution with mean $(0, 0, 0)$ and covariance

$$\mathbf{\Sigma}_0 = \begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 0.01 \end{bmatrix}$$

The covariance of the state evolution noise is

$$\mathbf{R} = \begin{bmatrix} 0.05 & 0 & 0 \\ 0 & 0.05 & 0 \\ 0 & 0 & 0.08 \end{bmatrix}$$

The robot is equipped with a sensor returning the distance from two landmarks at known locations. The first landmark is at position $(0, 4)$ while the second is at $(5, 0)$. The covariance of the sensor noise is

$$\mathbf{Q} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}$$

The robot first receives the input $(0.2, 0.2)$ and then the sensor measurement $(3.9, 4.7)$.

1. determine the uncertainty ellipsoid (95% confidence level) after the prediction step, and compute its volume;
2. determine the uncertainty ellipsoid (95% confidence level) after the correction step and compute its volume.

Hint: to solve this exercise write a script in Matlab or a language of your choice; manual calculations are discouraged. Recall that the volume of tridimensional ellipsoid is $\frac{4}{3}\pi abc$, where a, b, c are the lengths of its semi-axes.