MATH-131 (Numerical Methods for Scientists and Engineers) — Worksheet 8

Semester: Spring 2019, Instructor: Nicholas Knight

Due Apr. 2 at 2359. *Please remember to cite your sources, including collaborators.*

Deliverable: Submit a Live script titled `worksheet8.mlx` via CatCourses (under Assignments). Divide this file into sections, one for each of the following questions, plus an extra (final) section containing all the function definitions. Document each function definition to explain the input and output arguments. Also document key portions of the algorithm to make it clear you understand how your code works.

1. Consider the Runge function, $f : x \mapsto 1/(1 + 25x^2)$.

   (a) Use your Lagrange interpolation code (from the previous worksheets) to approximate f using 10, 20, 30, and 40 equispaced points from -1 and 1 (inclusive). Make a (single) plot comparing these four approximations with the (exact) function f. Use a legend to help distinguish the five curves. Intuitively, increasing the number of sample points should give a 'better' approximation. Does it? (A qualitative answer is sufficient.)

   (b) Repeat Part (a) using piecewise linear interpolation ("connect-the-dots"), by applying your Lagrange interpolation code to each pair of successive points. Are these approximations 'better' than those from Part (a)?

   (c) Repeat Part (b) using Matlab's `interp1` or `griddedInterpolant` routines. (You can use either: `interp1` evaluates the interpolant at given points, whereas `griddedInterpolant` returns the interpolant as a function that you can evaluate later.)

   (d) Repeat Part (c) but using piecewise cubic Hermite interpolation. (Pass `'pchip'` as the `method` argument, rather than `'linear'`, the default.) Are these approximations 'better' than those from Parts (b) and (c)?