# MATH-131 (Numerical Methods for Scientists and Engineers) — Worksheet 11*

Semester: Spring 2019, Instructor: Nicholas Knight

***Due Apr. 30 at 2359.*** *Please remember to cite your sources, including collaborators.*

*Deliverable:* Submit a Live script titled `worksheet11.mlx` via CatCourses (under Assignments). Divide this file into sections, one for each of the following questions, plus an extra (final) section containing all the function definitions. Document each function definition to explain the input and output arguments. Also document key portions of the algorithm to make it clear you understand how your code works.

1. Use the forward Euler method,

   $$y_{i+1} = y_i + (t_{i+1} - t_i) f(t_i, y_i) \text{ for } i = 0, 1, 2, \ldots,$$

   taking $y_0 = y(t_0)$ to be the initial condition, to approximate the solution at $t = 2$ of the IVP

   $$y' = y - t^2 + 1, \qquad 0 \le t \le 2, \qquad y(0) = 0.5.$$

   Use $N = 2^k$, $k = 1, 2, \ldots, 20$ equispaced timesteps (so $t_0 = 0$ and $t_{N-1} = 2$). Make a convergence plot, computing the error by comparing with the exact solution, $y \colon t \mapsto (t+1)^2 - \exp(t)/2$, and plotting the error as a function of the step-size.

   *Challenge (ungraded):* This ODE is of a very special type that can be solved analytically. Find a procedure to analytically solve ODEs of the form $y' + Py = Q$, where $y, P, Q$ are univariate functions.

2. Use Matlab's ODE solver, `ode45`, to approximate $y(2)$. How many timesteps did it take? How accurate was its approximation?

   *Challenge (ungraded):* Experiment with `ode45`'s tolerance options (`RelTol` and `AbsTol`) to improve the approximation quality.

*Revision History:

Version 0425: fixed error in Q1 (added missing "$y_i+$")

Version 0423: initial version