MATH-131 (Numerical Methods for Scientists and Engineers) — Worksheet 5

Semester: Spring 2019, Instructor: Nicholas Knight

Due Feb. 26 at 2359. *Please remember to cite your sources, including collaborators.*

Deliverable: Submit a Live script titled `worksheet5.mlx` via CatCourses (under Assignments). Divide this file into sections, one for each of the following questions, plus an extra (final) section containing all the function definitions. Document each function definition to explain the input and output arguments. Also document key portions of the algorithm to make it clear you understand how your code works.

1. Implement Newton's method (Alg. 2.3). Your signature should be

   ```
   function p = newtons(f, fp, p0, tol, maxits)
   ```

   where `f` and `fp` are function handles and `p0`, `tol`, `maxits`, and `p` are numbers. (`fp` is the derivative of `f`)

2. Implement the secant method (Alg. 2.4). Your signature should be

   ```
   function p = secant(f, p0, p1, tol, maxits)
   ```

   where `f` is a function handle and `p0`, `p1`, `tol`, `maxits`, and `p` are numbers.

3. Find the root(s) of the following functions using both Newton's method and the secant method, using `tol` = `eps`.
   - You will need to experiment with the parameters `p0`, `p1`, and `maxits`.
   - For each root, visualize the iteration history of both methods by plotting the absolute errors, as a function of iteration number, on a (single) `semilogy` plot.
   - Label the two curves (Newton's method and secant method) using a legend, describing the `p0` and `p1` values you used, e.g., "Newton's (`p0` = 7)" and "Secant (`p0` = 0, `p1` = 1)".

   (a) $f\colon x \mapsto x^2 - 2$, which has two roots, $x = \pm\sqrt{2}$.
   Rewrite the iteration formula to verify it is the same as the Babylonian method (see Worksheet 4 Q1(a)). Explain the problem with the starting guess `p0` $= 0$ in terms of the derivative of f.

   (b) $f\colon x \mapsto \tanh(x)$, which has one root, $x = 0$.
   Characterize (roughly) when the methods converge or diverge in terms of the starting guesses. (E.g., try Newton's method with `p0` $= 1.08$ and 1.09.)

   (c) $f\colon x \mapsto x^3 - 2x + 2$, which has one root, $x = -1.769\cdots$.
   What happens with Newton's method when `p0` $= 0$ or 1?

   (d) $f\colon x \mapsto \sqrt[3]{x}$, which has one root, $x = 0$. (In Matlab, use `nthroot(x,3)` instead of `x^(1/3)`.)

   (e) (*Challenge; ungraded* $f\colon x \mapsto \begin{cases} 0 & x = 0 \\ x + x^2\sin(2/x) & x \neq 0 \end{cases}$, which has a root at $x = 0$.

   Can you find the root? (Explain what you observe.) Note that $f'\colon x \mapsto \begin{cases} 1 & x = 0 \\ 1 + 2x\sin(2/x) - 2\cos(2/x) & x \neq 0 \end{cases}$.