MATH-131 (Numerical Methods for Scientists and Engineers) — Worksheet 12

Semester: Spring 2019, Instructor: Nicholas Knight

Due May 7 at 2359. *Please remember to cite your sources, including collaborators.*

Deliverable: Submit a Live script titled `worksheet12.mlx` via CatCourses (under Assignments). Divide this file into sections, one for each of the following questions, plus an extra (final) section containing all the function definitions. Document each function definition to explain the input and output arguments. Also document key portions of the algorithm to make it clear you understand how your code works.

1. Write a function that solves a linear system $Ax = b$ via elimination and substitution.

   ```
   function x = my_linsolve(A, b)
   ```

   where `A` is an n-by-n array and `b` and `x` are n-by-1 arrays. You may only use elementary row operations:

   - Exchange/swap two rows
   - Add a nonzero multiple of one row to another row
   - Scale a row by a nonzero

   Here, 'row' means a row of the augmented matrix $[A \,|\, b]$ (i.e., a linear equation).

   Test your code on random systems of dimension $n = 2^k$, $k = 1, 2, \ldots, 8$. To construct these systems, start by forming `A` and the solution `x`, then determine the right-hand side by `b = A * x;`. Use `rng` to seed your random number generator so that it produces the same sequence of linear systems every time your code is run.

   Compare your answers and the ones obtained by Matlab's `linsolve`: plot the error (in the infinity norm) as a function of n.

2. Repeat Question 1 but extend your code to support partial pivoting.