





A *query language* is used to update and retrieve data that is stored in a data model.

Relational algebra is a set of relational operations for retrieving data.

• Just like algebra with numbers, relational algebra consists of operands (which are relations) and a set of operators.

Every relational operator takes as input one or more relations and produces a relation as output.

A sequence of relational algebra operators is called a *relational* algebra expression.

Relational algebra is the foundation of all relational database systems. SQL gets translated into relational algebra.

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# **Relational Algebra Operators**

| Operator          | Symbol    | Purpose   |
|-------------------|-----------|---|
| Selection         | σ         | Filter rows                                     |
| Projection        | П         | Keep only certain columns                       |
| Cartesian Product | ×         | Combine two tables in all possible ways         |
| Join              | $\bowtie$ | Combine two tables based on a condition         |
| Union             | $\cup$    | Keep rows in either of two tables               |
| Difference        | -         | Keep rows in first table that are not in second |
| Intersection      | $\cap$    | Keep rows that are in both tables               |

Relational algebra operators are fundamental to data processing and occur in other data systems (even non-relational).





The *selection operation* takes a relation as input and returns a new relation as output that contains tuples that satisfy a condition, called a *predicate*.

- The predicate is similar to a condition in an if statement.
- The output relation has the same number of columns as the input relation, but may have less rows.

Selection operation on relation R with predicate F is denoted by  $\sigma_{F}(R)$ .

• The predicate uses operands that are attributes or constants/expressions, comparison operators (<, >, =,  $\neq$ ,  $\leq$ ,  $\geq$ ), and logical operators (AND, OR, NOT).





#### **Emp Relation**

| eno | ename     | title | salary |
|-----|-----------|-------|--------|
| E1  | J. Doe    | EE    | 30000  |
| E2  | M. Smith  | SA    | 50000  |
| E3  | A. Lee    | ME    | 40000  |
| E4  | J. Miller | PR    | 20000  |
| E5  | B. Casey  | SA    | 50000  |
| E6  | L. Chu    | EE    | 30000  |
| E7  | R. Davis  | ME    | 40000  |
| E8  | J. Jones  | SA    | 50000  |

### $\sigma_{title = 'EE'}(Emp)$

| eno | ename  | title | salary |
|-----|--------|-------|--------|
| E1  | J. Doe | EE    | 30000  |
| E6  | L. Chu | EE    | 30000  |

# $\sigma_{salary > 35000 \ OR \ title = 'PR'}(Emp)$

| eno | ename     | title | salary |
|-----|-----------|-------|--------|
| E2  | M. Smith  | SA    | 50000  |
| E3  | A. Lee    | ME    | 40000  |
| E4  | J. Miller | PR    | 20000  |
| E5  | B. Casey  | SA    | 50000  |
| E7  | R. Davis  | ME    | 40000  |
| E8  | J. Jones  | SA    | 50000  |





**Question:** Given this table and the query:

```
\sigma_{salary} > 50000 or title='PR' (Emp)
```

How many rows are returned?

- **A)** 0
- B) 1
- **C)** 2

**D)** 3

#### **Emp Relation**

| <u>eno</u> | ename     | title | salary |
|------------|-----------|-------|--------|
| E1         | J. Doe    | EE    | 30000  |
| E2         | M. Smith  | SA    | 50000  |
| E3         | A. Lee    | ME    | 40000  |
| E4         | J. Miller | PR    | 20000  |
| E5         | B. Casey  | SA    | 50000  |
| E6         | L. Chu    | EE    | 30000  |
| E7         | R. Davis  | ME    | 40000  |
| E8         | J. Jones  | SA    | 50000  |





**Question:** Given this table and the query:

```
\sigma_{salary} > 50000 or title='PR' (Emp)
```

How many columns are returned?

- **A)** 0
- **B)** 2
- **C)** 3

**D)** 4

#### **Emp Relation**

| <u>eno</u> | ename     | title | salary |
|------------|-----------|-------|--------|
| E1         | J. Doe    | EE    | 30000  |
| E2         | M. Smith  | SA    | 50000  |
| E3         | A. Lee    | ME    | 40000  |
| E4         | J. Miller | PR    | 20000  |
| E5         | B. Casey  | SA    | 50000  |
| E6         | L. Chu    | EE    | 30000  |
| E7         | R. Davis  | ME    | 40000  |
| E8         | J. Jones  | SA    | 50000  |





The *projection operation* takes a relation as input and returns a new relation as output that contains a subset of the attributes of the input relation and all non-duplicate tuples.

- The output relation has the same number of tuples as the input relation unless removing the attributes caused duplicates to be present.
- Question: When are we guaranteed to never have duplicates when performing a projection operation?
   Is a super key to the relation

Projection on relation R with output attributes  $A_1,...,A_m$  is denoted by  $\prod_{A_1,...,A_m} (R)$ .

- Order of  $A_1,...,A_m$  is significant in the result.
- Cardinality of  $\Pi_{A_1,...,A_m}(R)$  may not be the same as R due to duplicate removal.

# **Projection Example**



#### **Emp Relation**

| <u>eno</u> | ename     | title | salary |
|------------|-----------|-------|--------|
| E1         | J. Doe    | EE    | 30000  |
| E2         | M. Smith  | SA    | 50000  |
| E3         | A. Lee    | ME    | 40000  |
| E4         | J. Miller | PR    | 20000  |
| E5         | B. Casey  | SA    | 50000  |
| E6         | L. Chu    | EE    | 30000  |
| E7         | R. Davis  | ME    | 40000  |
| E8         | J. Jones  | SA    | 50000  |

# $\Pi_{eno,ename}$ (Emp)

| <u>eno</u> | ename     |
|------------|-----------|
| E1         | J. Doe    |
| E2         | M. Smith  |
| E3         | A. Lee    |
| E4         | J. Miller |
| E5         | B. Casey  |
| E6         | L. Chu    |
| E7         | R. Davis  |
| E8         | J. Jones  |

 $\Pi_{title}(Emp)$ 







### **Question:** Given this table and the query:

$$\Pi_{title}$$
 (Emp)

### How many rows are returned?

- **A)** 0
- **B)** 2
- **C)** 4

**D)** 8

#### **Emp Relation**

| <u>eno</u> | ename     | title | salary |
|------------|-----------|-------|--------|
| E1         | J. Doe    | EE    | 30000  |
| E2         | M. Smith  | SA    | 50000  |
| E3         | A. Lee    | ME    | 40000  |
| E4         | J. Miller | PR    | 20000  |
| E5         | B. Casey  | SA    | 50000  |
| E6         | L. Chu    | EE    | 30000  |
| E7         | R. Davis  | ME    | 40000  |
| E8         | J. Jones  | SA    | 50000  |





#### WorksOn Relation

| eno | pno | resp       | hours |
|-----|-----|------------|-------|
| E1  | P1  | Manager    | 12    |
| E2  | P1  | Analyst    | 24    |
| E2  | P2  | Analyst    | 6     |
| E3  | P3  | Consultant | 10    |
| E3  | P4  | Engineer   | 48    |
| E4  | P2  | Programmer | 18    |
| E5  | P2  | Manager    | 24    |
| E6  | P4  | Manager    | 48    |
| E7  | P3  | Engineer   | 36    |
| E7  | P5  | Engineer   | 23    |
| E8  | P3  | Manager    | 40    |

#### Write the relational algebra expression that:

- 1) Returns all rows with an employee working on project P2.
- 2) Returns all rows with an employee who is working as a manager on a project.
- 3) Returns all rows with an employee working as a manager for more than 40 hours.
- 4) Returns only attributes resp and hours.
- 5) Returns only *eno*.
- 6) Returns only *pno*.

How many tuples in each output relation?

## Union



*Union* takes two relations R and S as input and produces an output relation that includes all tuples that are either in R, or in S, or in both R and S. Duplicate tuples are eliminated. Syntax:  $R \cup S$ 

### R and S must be union-compatible:

- 1) Both relations have same number of attributes.
- 2) Each attribute pair,  $R_i$  and  $S_i$ , have compatible data types for all attribute indexes i.
- Note that attributes do not need to have the same name.
- Result has attribute names of first relation.

# **Union Example**



#### Emp

| <u>eno</u> | ename     | title | salary |
|------------|-----------|-------|--------|
| E1         | J. Doe    | EE    | 30000  |
| E2         | M. Smith  | SA    | 50000  |
| E3         | A. Lee    | ME    | 40000  |
| E4         | J. Miller | PR    | 20000  |
| E5         | B. Casey  | SA    | 50000  |
| E6         | L. Chu    | EE    | 30000  |
| E7         | R. Davis  | ME    | 40000  |
| E8         | J. Jones  | SA    | 50000  |

#### WorksOn

| <u>eno</u> | <u>pno</u> | resp     | hours |
|------------|------------|----------|-------|
| E1         | P1         | Manager  | 12    |
| E2         | P1         | Analyst  | 24    |
| E2         | P2         | Analyst  | 6     |
| E3         | P4         | Engineer | 48    |
| E5         | P2         | Manager  | 24    |
| E6         | P4         | Manager  | 48    |
| E7         | P3         | Engineer | 36    |
| E7         | P5         | Engineer | 23    |

# $\Pi_{eno}(\text{Emp}) \cup \Pi_{eno}(\text{WorksOn})$

| eno |  |
|-----|--|
| E1  |  |
| E2  |  |
| E3  |  |
| E4  |  |
| E5  |  |
| E6  |  |
| E7  |  |
| E8  |  |





**Set difference** takes two relations *R* and *S* as input and produces an output relation that contains all the tuples of *R* that are not in *S*.

Syntax: R - S

#### Note:

- $R S \neq S R$
- R and S must be union compatible.

# **Set Difference Example**



#### **Emp Relation**

| eno | ename     | title | salary |
|-----|-----------|-------|--------|
| E1  | J. Doe    | EE    | 30000  |
| E2  | M. Smith  | SA    | 50000  |
| E3  | A. Lee    | ME    | 40000  |
| E4  | J. Miller | PR    | 20000  |
| E5  | B. Casey  | SA    | 50000  |
| E6  | L. Chu    | EE    | 30000  |
| E7  | R. Davis  | ME    | 40000  |
| E8  | J. Jones  | SA    | 50000  |

#### WorksOn Relation

| <u>eno</u> | <u>pno</u> | resp     | hours |
|------------|------------|----------|-------|
| E1         | P1         | Manager  | 12    |
| E2         | P1         | Analyst  | 24    |
| E2         | P2         | Analyst  | 6     |
| E3         | P4         | Engineer | 48    |
| E5         | P2         | Manager  | 24    |
| E6         | P4         | Manager  | 48    |
| E7         | P3         | Engineer | 36    |
| E7         | P5         | Engineer | 23    |

 $\overline{\Pi_{eno}}(\text{Emp}) - \overline{\Pi_{eno}}(\text{WorksOn})$ 

Question: What is the meaning of this query?

Question: What is  $\Pi_{eno}(WorksOn) - \Pi_{eno}(Emp)$ ?

eno

E4

E8





*Intersection* takes two relations *R* and *S* as input and produces an output relation which contains all tuples that are in both *R* and *S*.

• *R* and *S* must be union-compatible.

Syntax: *R* ∩ *S* 

Note that 
$$R \cap S = R - (R - S) = S - (S - R)$$
.

# **Intersection Example**



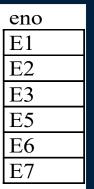
#### Emp

| eno | ename     | title | salary |
|-----|-----------|-------|--------|
| E1  | J. Doe    | EE    | 30000  |
| E2  | M. Smith  | SA    | 50000  |
| E3  | A. Lee    | ME    | 40000  |
| E4  | J. Miller | PR    | 20000  |
| E5  | B. Casey  | SA    | 50000  |
| E6  | L. Chu    | EE    | 30000  |
| E7  | R. Davis  | ME    | 40000  |
| E8  | J. Jones  | SA    | 50000  |

# $\Pi_{eno}(\text{Emp}) \cap \Pi_{eno}(\text{WorksOn})$

#### WorksOn

| <u>eno</u> | <u>pno</u> | resp     | hours |
|------------|------------|----------|-------|
| E1         | P1         | Manager  | 12    |
| E2         | P1         | Analyst  | 24    |
| E2         | P2         | Analyst  | 6     |
| E3         | P4         | Engineer | 48    |
| E5         | P2         | Manager  | 24    |
| E6         | P4         | Manager  | 48    |
| E7         | P3         | Engineer | 36    |
| E7         | P5         | Engineer | 23    |





# **Set Operations: Union-compatible Question**

Question: Two tables have the same number of fields in the same order with the same types, but the names of some fields are different. True or false: The two tables are union-compatible.

A) true

B) false

### **Cartesian Product**



Cartesian product of relations R (of degree  $k_1$ ) and S (of degree  $k_2$ ) is a relation of degree ( $k_1 + k_2$ ) that consists of all ( $k_1 + k_2$ )-tuples where each tuple is a concatenation of one tuple of R with one tuple of S. Syntax:  $R \times S$ 

The cardinality of R  $\times$  S is |R| \* |S|.

The Cartesian product is also known as cross product.





#### Emp Relation

| <u>eno</u> | ename     | title | salary |
|------------|-----------|-------|--------|
| E1         | J. Doe    | EE    | 30000  |
| E2         | M. Smith  | SA    | 50000  |
| E3         | A. Lee    | ME    | 40000  |
| E4         | J. Miller | PR    | 20000  |

#### Proj Relation

| <u>pno</u> | pname       | budget |
|------------|-------------|--------|
| P1         | Instruments | 150000 |
| P2         | DB Develop  | 135000 |
| P3         | CAD/CAM     | 250000 |

#### $\overline{Emp \times Proj}$

| eno | ename     | title | salary | pno | pname       | budget |
|-----|-----------|-------|--------|-----|-------------|--------|
| E1  | J. Doe    | EE    | 30000  | P1  | Instruments | 150000 |
| E2  | M. Smith  | SA    | 50000  | P1  | Instruments | 150000 |
| E3  | A. Lee    | ME    | 40000  | P1  | Instruments | 150000 |
| E4  | J. Miller | PR    | 20000  | P1  | Instruments | 150000 |
| E1  | J. Doe    | EE    | 30000  | P2  | DB Develop  | 135000 |
| E2  | M. Smith  | SA    | 50000  | P2  | DB Develop  | 135000 |
| E3  | A. Lee    | ME    | 40000  | P2  | DB Develop  | 135000 |
| E4  | J. Miller | PR    | 20000  | P2  | DB Develop  | 135000 |
| E1  | J. Doe    | EE    | 30000  | P3  | CAD/CAM     | 250000 |
| E2  | M. Smith  | SA    | 50000  | P3  | CAD/CAM     | 250000 |
| E3  | A. Lee    | ME    | 40000  | P3  | CAD/CAM     | 250000 |
| E4  | J. Miller | PR    | 20000  | P3  | CAD/CAM     | 250000 |
|     |           |       |        |     |             |        |





**Question:** R is a relation with 10 rows and 5 columns. S is a relation with 8 rows and 3 columns.

What is the degree and cardinality of the Cartesian product?

A) degree = 8, cardinality = 80

B) degree = 80, cardinality = 8

C) degree = 15, cardinality = 80

D) degree = 8, cardinality = 18

# θ-Join



Theta  $(\theta)$  join is a derivative of the Cartesian product. Instead of taking all combinations of tuples from R and S, we only take a subset of those tuples that match a given condition F. Syntax:  $R \bowtie_F S$ 

Note that  $R \bowtie_F S = \sigma_F(R \times S)$ .

# **θ-Join Example**



#### WorksOn Relation

| eno | <u>pno</u> | resp     | hours |
|-----|------------|----------|-------|
| E1  | P1         | Manager  | 12    |
| E2  | P1         | Analyst  | 24    |
| E2  | P2         | Analyst  | 6     |
| E3  | P4         | Engineer | 48    |
| E5  | P2         | Manager  | 24    |
| E6  | P4         | Manager  | 48    |
| E7  | P3         | Engineer | 36    |
| E7  | P4         | Engineer | 23    |

#### Proj Relation

| pno | pname       | budget |
|-----|-------------|--------|
| P1  | Instruments | 150000 |
| P2  | DB Develop  | 135000 |
| P3  | CAD/CAM     | 250000 |
| P4  | Maintenance | 310000 |
| P5  | CAD/CAM     | 500000 |

# WorksOn ⋈ hours\*10000 > budget Proj

| eno | pno | resp     | hours | P.pno | pname       | budget |
|-----|-----|----------|-------|-------|-------------|--------|
| E2  | P1  | Analyst  | 24    | P1    | Instruments | 150000 |
| E2  | P1  | Analyst  | 24    | P2    | DB Develop  | 135000 |
| E3  | P4  | Engineer | 48    | P1    | Instruments | 150000 |
| E3  | P4  | Engineer | 48    | P2    | DB Develop  | 135000 |
| E3  | P4  | Engineer | 48    | P3    | CAD/CAM     | 250000 |
| E3  | P4  | Engineer | 48    | P4    | Maintenance | 310000 |
| E5  | P2  | Manager  | 24    | P1    | Instruments | 150000 |
| E5  | P2  | Manager  | 24    | P2    | DB Develop  | 135000 |
| E6  | P4  | Manager  | 48    | P1    | Instruments | 150000 |
| E6  | P4  | Manager  | 48    | P2    | DB Develop  | 135000 |
| E6  | P4  | Manager  | 48    | P3    | CAD/CAM     | 250000 |
| E6  | P4  | Manager  | 48    | P4    | Maintenance | 310000 |
| E7  | P3  | Engineer | 36    | P1    | Instruments | 150000 |
| E7  | P3  | Engineer | 36    | P2    | DB Develop  | 135000 |
| E7  | P3  | Engineer | 36    | P3    | CAD/CAM     | 250000 |
| E7  | P4  | Engineer | 23    | P1    | Instruments | 150000 |
| E7  | P4  | Engineer | 23    | P2    | DB Develop  | 135000 |

# Types of Joins



The  $\theta$ -Join is a general join that allows any expression for condition F. However, there are more specialized joins that are frequently used.

A *equijoin* only contains the equality operator (=) in formula *F*.

• e.g. WorksOn ⋈ <sub>WorksOn.pno = Proj.pno</sub> Proj

A *natural join* over two relations R and S denoted by  $R \bowtie S$  is the equijoin of R and S over a set of attributes common to both R and S.

- It removes the "extra copies" of the join attributes.
- The attributes must have the same name in both relations.

# **Equijoin Example**



#### WorksOn Relation

| <u>eno</u> | <u>pno</u> | resp     | hours |
|------------|------------|----------|-------|
| E1         | P1         | Manager  | 12    |
| E2         | P1         | Analyst  | 24    |
| E2         | P2         | Analyst  | 6     |
| E3         | P4         | Engineer | 48    |
| E5         | P2         | Manager  | 24    |
| E6         | P4         | Manager  | 48    |
| E7         | P3         | Engineer | 36    |
| E7         | P4         | Engineer | 23    |
|            |            |          |       |

### WorksOn $\bowtie$ WorksOn.pno = Proj.pno Proj

| eno | pno | resp     | hours | P.pno | pname       | budget |
|-----|-----|----------|-------|-------|-------------|--------|
| E1  | P1  | Manager  | 12    | P1    | Instruments | 150000 |
| E2  | P1  | Analyst  | 24    | P1    | Instruments | 150000 |
| E2  | P2  | Analyst  | 6     | P2    | DB Develop  | 135000 |
| E3  | P4  | Engineer | 48    | P4    | Maintenance | 310000 |
| E5  | P2  | Manager  | 24    | P2    | DB Develop  | 135000 |
| E6  | P4  | Manager  | 48    | P4    | Maintenance | 310000 |
| E7  | P3  | Engineer | 36    | P3    | CAD/CAM     | 250000 |
| E7  | P4  | Engineer | 23    | P4    | Maintenance | 310000 |

#### Proj Relation

| <u>pno</u> | pname       | budget |
|------------|-------------|--------|
| P1         | Instruments | 150000 |
| P2         | DB Develop  | 135000 |
| P3         | CAD/CAM     | 250000 |
| P4         | Maintenance | 310000 |
| P5         | CAD/CAM     | 500000 |

### What is the meaning of this join?





#### WorksOn Relation

| <u>pno</u> | resp   | hours  |
|------------|--|--|
| P1         | Manager  | 12   |
| P1         | Analyst  | 24   |
| P2         | Analyst  | 6  |
| P4         | Engineer   | 48   |
| P2         | Manager  | 24   |
| P4         | Manager  | 48   |
| P3         | Engineer   | 36   |
| P4         | Engineer   | 23   |
|            | P1<br>P1<br>P2<br>P4<br>P2<br>P4<br>P2<br>P4<br>P3 | P1 Manager P1 Analyst P2 Analyst P4 Engineer P2 Manager P4 Manager P3 Engineer |

#### Proj Relation

| pno | pname       | budget |
|-----|-------------|--------|
| P1  | Instruments | 150000 |
| P2  | DB Develop  | 135000 |
| P3  | CAD/CAM     | 250000 |
| P4  | Maintenance | 310000 |
| P5  | CAD/CAM     | 500000 |

#### WorksOn ⋈ Proj

| eno | pno | resp     | hours | pname       | budget |
|-----|-----|----------|-------|-------------|--------|
| E1  | P1  | Manager  | 12    | Instruments | 150000 |
| E2  | P1  | Analyst  | 24    | Instruments | 150000 |
| E2  | P2  | Analyst  | 6     | DB Develop  | 135000 |
| E3  | P4  | Engineer | 48    | Maintenance | 310000 |
| E5  | P2  | Manager  | 24    | DB Develop  | 135000 |
| E6  | P4  | Manager  | 48    | Maintenance | 310000 |
| E7  | P3  | Engineer | 36    | CAD/CAM     | 250000 |
| E7  | P4  | Engineer | 23    | Maintenance | 310000 |

Natural join is performed by comparing *pno* in both relations.

# **Join Practice Questions**



#### **Emp Relation**

|            | -         |       |        |
|------------|-----------|-------|--------|
| <u>eno</u> | ename     | title | salary |
| E1         | J. Doe    | EE    | 30000  |
| E2         | M. Smith  | SA    | 50000  |
| E3         | A. Lee    | ME    | 40000  |
| E4         | J. Miller | PR    | 20000  |
| E5         | B. Casey  | SA    | 50000  |
| E6         | L. Chu    | EE    | 30000  |
| E7         | R. Davis  | ME    | 40000  |
| E8         | J. Jones  | SA    | 50000  |

#### Proj Relation

| pno | pname       | budget |
|-----|-------------|--------|
| P1  | Instruments | 150000 |
| P2  | DB Develop  | 135000 |
| P3  | CAD/CAM     | 250000 |
| P4  | Maintenance | 310000 |
| P5  | CAD/CAM     | 500000 |

#### WorksOn Relation

| <u>eno</u> | <u>pno</u> | resp       | hours |
|------------|------------|------------|-------|
| E1         | P1         | Manager    | 12    |
| E2         | P1         | Analyst    | 24    |
| E2         | P2         | Analyst    | 6     |
| E3         | P3         | Consultant | 10    |
| E3         | P4         | Engineer   | 48    |
| E4         | P2         | Programmer | 18    |
| E5         | P2         | Manager    | 24    |
| E6         | P4         | Manager    | 48    |
| E7         | P3         | Engineer   | 36    |
| E7         | P5         | Engineer   | 23    |
| E8         | P3         | Manager    | 40    |

#### Compute the following joins (counts only):

- 1) Emp  $\bowtie$  title='EE' and budget > 400000 Proj
- 2) Emp ⋈ WorksOn
- 3) Emp ⋈ WorksOn ⋈ Proj
- 4) Proj<sub>1</sub>  $\bowtie$  Proj<sub>1.budget > Proj<sub>2.budget</sub> Proj<sub>2</sub></sub>





Outer joins are used in cases where performing a join "loses" some tuples of the relations. These are called *dangling tuples*.

There are three types of outer joins: equijoin + not matched item

- 1) Left outer join  $R \supset S$  The output contains all tuples of R that match with tuples of S. If there is a tuple in R that matches with no tuple in S, the tuple is included in the final result and is padded with nulls for the attributes of S.
- 2) *Right outer join*  $R \bowtie S$  The output contains all tuples of S that match with tuples of R. If there is a tuple in S that matches with no tuple in R, the tuple is included in the final result and is padded with nulls for the attributes of R.
- 3) Full outer join  $R \supset S$  All tuples of R and S are included in the result whether or not they have a matching tuple in the other relation.





#### WorksOn Relation

| eno | <u>pno</u> | resp     | hours |
|-----|------------|----------|-------|
| E1  | P1         | Manager  | 12    |
| E2  | P1         | Analyst  | 24    |
| E2  | P2         | Analyst  | 6     |
| E3  | P4         | Engineer | 48    |
| E5  | P2         | Manager  | 24    |
| E6  | P4         | Manager  | 48    |
| E7  | P3         | Engineer | 36    |
| E7  | P4         | Engineer | 23    |

#### Proj Relation

| <u>pno</u> | pname       | budget |
|------------|-------------|--------|
| P1         | Instruments | 150000 |
| P2         | DB Develop  | 135000 |
| P3         | CAD/CAM     | 250000 |
| P4         | Maintenance | 310000 |
| P5         | CAD/CAM     | 500000 |

### $WorksOn \bowtie_{WorksOn.pno = Proj.pno} Proj$

| /    |      | //       | 7     | ·     |             |        |
|------|------|----------|-------|-------|-------------|--------|
| eno  | pno  | resp     | hours | P.pno | pname       | budget |
| E1   | P1   | Manager  | 12    | P1    | Instruments | 150000 |
| E2   | P1   | Analyst  | 24    | P1    | Instruments | 150000 |
| E2   | P2   | Analyst  | 6     | P2    | DB Develop  | 135000 |
| E3   | P4   | Engineer | 48    | P4    | Maintenance | 310000 |
| E5   | P2   | Manager  | 24    | P2    | DB Develop  | 135000 |
| E6   | P4   | Manager  | 48    | P4    | Maintenance | 310000 |
| E7   | P3   | Engineer | 36    | P3    | CAD/CAM     | 250000 |
| E7   | P4   | Engineer | 23    | P4    | Maintenance | 310000 |
| null | null | null     | null  | P5    | CAD/CAM     | 500000 |
|      |      |          |       |       |             |        |





### **Question:** Given this table and the query:

WorksOn → WorksOn.pno = Proj.pno Proj

How many rows are returned?

A) 10

**B)** 9

**C)** 8

D) 7

#### WorksOn Relation

| eno       | pno | resp     | hours |
|-----------|-----|----------|-------|
| <u>E1</u> | P1  | Manager  | 12    |
| E2        | P1  | Analyst  | 24    |
| E2        | P2  | Analyst  | 6     |
| E3        | P4  | Engineer | 48    |
| E5        | P2  | Manager  | 24    |
| E6        | P4  | Manager  | 48    |
| E7        | P4  | Engineer | 36    |
| E7        | P4  | Engineer | 23    |

#### Proj Relation

| pno | pname       | budget |
|-----|-------------|--------|
| P1  | Instruments | 150000 |
| P2  | DB Develop  | 135000 |
| P3  | CAD/CAM     | 250000 |
| P4  | Maintenance | 310000 |
| P5  | CAD/CAM     | 500000 |





A *semi-join* between tables returns rows from the first table where one or more matches are found in the second table.

• Semi-joins are used in EXISTS and IN constructs in SQL.

An *anti-join* between two tables returns rows from the first table where *no* matches are found in the second table.

- Anti-joins are used with NOT EXISTS, NOT IN, and FOR ALL.
- Anti-join is the complement of semi-join:  $R \triangleright S = R R \bowtie S$

# **Semi-Join Example**



#### WorksOn Relation

| <u>eno</u> | <u>pno</u> | resp     | hours |
|------------|------------|----------|-------|
| E1         | P1         | Manager  | 12    |
| E2         | P1         | Analyst  | 24    |
| E2         | P2         | Analyst  | 6     |
| E3         | P4         | Engineer | 48    |
| E5         | P2         | Manager  | 24    |
| E6         | P4         | Manager  | 48    |
| E7         | P3         | Engineer | 36    |
| E7         | P4         | Engineer | 23    |

#### Proj Relation

| <u>pno</u> | pname       | budget |
|------------|-------------|--------|
| P1         | Instruments | 150000 |
| P2         | DB Develop  | 135000 |
| P3         | CAD/CAM     | 250000 |
| P4         | Maintenance | 310000 |
| P5         | CAD/CAM     | 500000 |

### Proj ⋉<sub>Proj.pno = WorksOn.pno</sub> WorksOn

| pno | pname       | budget |
|-----|-------------|--------|
| P1  | Instruments | 150000 |
| P2  | DB Develop  | 135000 |
| P3  | CAD/CAM     | 250000 |
| P4  | Maintenance | 310000 |

# **Anti-Join Example**



#### WorksOn Relation

| <u>eno</u> | <u>pno</u> | resp     | hours |
|------------|------------|----------|-------|
| E1         | P1         | Manager  | 12    |
| E2         | P1         | Analyst  | 24    |
| E2         | P2         | Analyst  | 6     |
| E3         | P4         | Engineer | 48    |
| E5         | P2         | Manager  | 24    |
| E6         | P4         | Manager  | 48    |
| E7         | P3         | Engineer | 36    |
| E7         | P4         | Engineer | 23    |

#### Proj Relation

| pno | pname       | budget |
|-----|-------------|--------|
| P1  | Instruments | 150000 |
| P2  | DB Develop  | 135000 |
| P3  | CAD/CAM     | 250000 |
| P4  | Maintenance | 310000 |
| P5  | CAD/CAM     | 500000 |

### Proj > Proj.pno = WorksOn.pno WorksOn

| pno | pname   | budget |
|-----|---------|--------|
| P5  | CAD/CAM | 500000 |





Relational algebra operations can be combined in one expression by nesting them:

$$\Pi_{eno,pno,hours}(\sigma_{ename='J.\ Doe'}(Emp) \bowtie \sigma_{hours>16}(WorksOn))$$

 Return the eno, pno, and hours for employee 'J. Doe' when he has worked on a project for more than 16 months.

Can also use a temporary relation variables for intermediate results.

• Use the assignment operator ← for indicating that the result of an operation is assigned to a temporary relation.

```
empdoe \leftarrow \sigma_{ename='J.\ Doe'}(Emp)
wohours \leftarrow \sigma_{hours>16}(WorksOn)
empwo \leftarrow empdoe \bowtie wohours
result \leftarrow \Pi_{eno,pno,hours}(empwo)
```





Renaming can be applied when assigning a result:

result(EmployeeNum, ProjectNum, Hours)  $\leftarrow \Pi_{eno,pno,hours}(empwo)$ 

Or by using the rename operator  $\rho$  (rho):

Presult(EmployeeName, ProjectNum, Hours)(empwo)





Just like mathematical operators, the relational operators have precedence.

The precedence of operators from highest to lowest is:

- unary operators  $\sigma$ ,  $\Pi$ ,  $\rho$
- Cartesian product and joins X, ⋈ , division
- intersection
- union and set difference

Parentheses can be used to changed the order of operations.

It is a good idea to *always* use parentheses around the argument for both unary and binary operators.

# BEC

# **Complete Set of Relational Algebra Operators**

It has been shown that the relational operators  $\{\sigma, \Pi, \times, \cup, -\}$  form a complete set of operators.

• That is, any of the other operators can be derived from a combination of these 5 basic operators.

### Examples:

- Intersection R  $\cap$  S  $\equiv$  R  $\cup$  S ((R S)  $\cup$  (S R))
- We have also seen how a join is a combination of a Cartesian product followed by a selection.





Consider the database schema

Emp (eno, ename, title, salary)

Proj (pno, pname, budget)

WorksOn (eno, pno, resp, hours)

#### Queries:

- List the names of all employees.
  - $\Pi_{\text{ename}}(\text{Emp})$
- Find the names of projects with budgets over \$100,000.
  - $\Pi_{\text{pname}}(\sigma_{\text{budget}>100000}(\text{Proj}))$





#### Relational database schema:

```
branch (<u>bname</u>, address, city, assets)
customer (<u>cname</u>, street, city)
deposit (<u>accnum</u>, cname, bname, balance)
borrow (accnum, cname, bname, amount)
```

- 1) List the names of all branches of the bank.
- 2) List the names of all deposit customers together with their account numbers.
- 3) Find all cities where at least one customer lives.
- 4) Find all cities with at least one branch.
- 5) Find all cities with at least one branch or customer.
- 6) Find all cities that have a branch but no customers who live in that city.





```
branch (<u>bname</u>, address, city, assets)
customer (<u>cname</u>, street, city)
deposit (<u>accnum</u>, cname, bname, balance)
borrow (<u>accnum</u>, cname, bname, amount)
```

- 1) Find the names of all branches with assets greater than \$2,500,000.
- 2) List the name and cities of all customers who have an account with balance greater than \$2,000.
- 3) List all the cities with at least one customer but without any bank branches.
- 4) Find the name of all the customers who live in a city with no bank branches.

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```
branch (<u>bname</u>, address, city, assets)
customer (<u>cname</u>, street, city)
deposit (<u>accnum</u>, cname, bname, balance)
borrow (<u>accnum</u>, cname, bname, amount)
```

- 1) Find all the cities that have both customers and bank branches.
- 2) List the customer name and loan and deposit amounts, who have a loan larger than a deposit account at the same branch.
- 3) Find the name and assets of all branches which have deposit customers living in Vancouver.
- 4) Find all the customers who have both a deposit account and a loan at the branch with name CalgaryCentral.
- 5) Your own?



# **Other Relational Algebra Operators**

There are other relational algebra operators that we will not discuss. Most notably, we often need aggregate operations that compute functions on the data.

For example, given the current operators, we cannot answer the query:

• What is the total amount of deposits at the Kelowna branch?

We will see how to answer these queries when we study SQL.





**Relational algebra** is a set of operations for answering queries on data stored in the relational model.

| Operator          | Symbol | Purpose   |
|-------------------|--------|---|
| Selection         | σ      | Filter rows                                     |
| Projection        | П      | Keep only certain columns                       |
| Cartesian Product | ×      | Combine two tables in all possible ways         |
| Join              |        | Combine two tables based on a condition         |
| Union             | U      | Keep rows in either of two tables               |
| Difference        | -      | Keep rows in first table that are not in second |
| Intersection      | $\cap$ | Keep rows that are in both tables               |

By combining relational operators, queries can be answered over the base relations.

# **Objectives**



Define: relational algebra, query language

Define and perform all relational algebra operators.

List the operators which form the complete set of operators.



Given a relational schema and instance be able to translate English queries into relational algebra and show the resulting relation.

