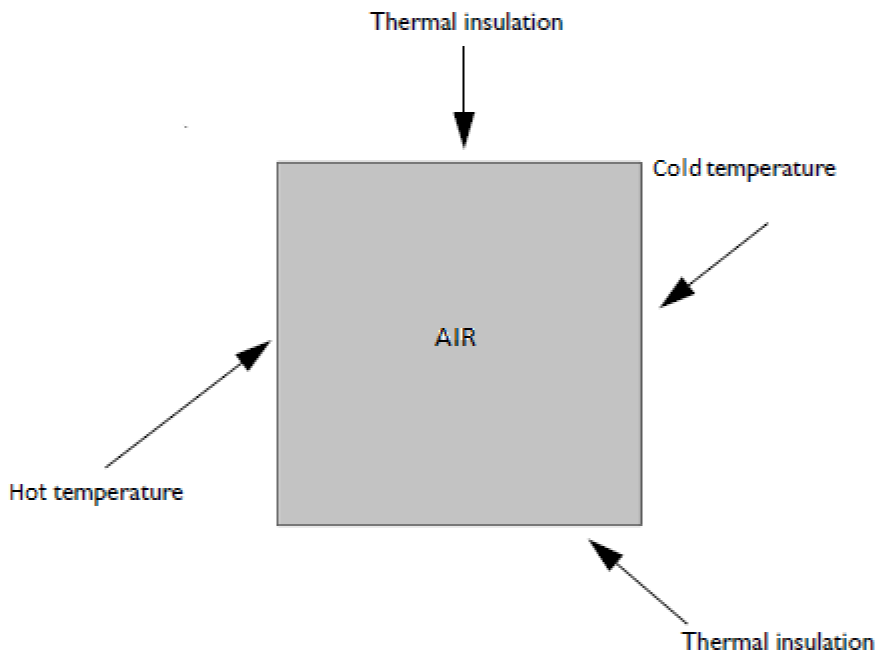

Figure 1 Domain Geometry and boundary conditions

Governing Equations

In order to describe the hydrodynamics of the fluid enclosed inside a square domain as shown in the Fig. 1 under the influence of heated wall, Navier-Stokes equations along with energy equation are employed. The fluid is considered to be incompressible and Newtonian in nature. Here, in this problem, stream function- vorticity formulation is being used to develop numerical tool to study the physics of Natural convection. Thermo-physical properties of the fluid in the flow field are assumed to be constant except the density variations causing a body force equation in the vorticity equation. The governing equations are non-dimensionalized by suitable reference scale. The dimensional forms of stream function and vorticity equations are given below.

Vorticity Equation :

$$\frac{\partial \omega^{*2}}{\partial t^{*2}} + \frac{\partial u^* \omega^*}{\partial x^*} + \frac{\partial v^* \omega^*}{\partial y^*} = \nu\left(\frac{\partial^2 \omega^*}{\partial x^{*2}} + \frac{\partial^2 \omega^*}{\partial y^{*2}}\right) + g\beta \frac{\partial (T^* - T_c)}{\partial x^*} \qquad \ldots\ldots. (1)$$

Energy Equation :

$$\frac{\partial T^*}{\partial t^*} + \frac{\partial u^* T^*}{\partial x^*} + \frac{\partial v^* T^*}{\partial y^*} = \alpha\left(\frac{\partial^2 T^*}{\partial x^{*2}} + \frac{\partial^2 T^*}{\partial y^{*2}}\right) \qquad \ldots\ldots.. (2)$$

$$\frac{\partial^2 \psi^*}{\partial {x^*}^2} + \frac{\partial^2 \psi^*}{\partial {y^*}^2} = -\omega^* \qquad \text{…….. (3)}$$

$$u^* = \frac{\partial \psi^*}{\partial y^*},\ v^* = -\frac{\partial \psi^*}{\partial x^*} \qquad \text{…….. (4)}$$

For non-dimensionalizing the governing equations, the given below scale parameters are considered :

$$x = \frac{x^*}{L},\ y = \frac{y^*}{L},\ u = \frac{u^* L}{\alpha},\ \ v = \frac{v^* L}{\alpha},\ T = \frac{T^* - T_c^*}{T_h^* - T_c^*},\ p = \frac{p^* L^2}{\rho \alpha^2},$$

$$\omega = \frac{\omega^* L^2}{\alpha}, t = \frac{\alpha t^*}{L^2},\ Ra = \frac{g\beta(\ T^* - T_c^*) L^3 Pr}{\nu^2},\ \ \psi = \frac{\psi^*}{\alpha} \qquad \text{………. (5)}$$

Where, L is the length of the square cavity, α is the thermal diffusivity, g is gravitational acceleration, β is volumetric thermal expansion coefficient, ν is kinematic viscosity, Pr is Prandtl Number and Ra is Rayleigh Number. The non-dimensional governing equations are :

Vorticity equation :

$$\frac{\partial \omega}{\partial t} + \frac{\partial u\omega}{\partial x} + \frac{\partial v\omega}{\partial y} = Pr\left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2}\right) + RaPr\frac{\partial (T)}{\partial x} \qquad \text{………… (6)}$$

Energy equation :

$$\frac{\partial T}{\partial t} + \frac{\partial uT}{\partial x} + \frac{\partial vT}{\partial y} = \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \qquad \text{…………. (7)}$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\omega \qquad \text{…………. (8)}$$

$$u = \frac{\partial \psi}{\partial y},\ v = -\frac{\partial \psi}{\partial x} \qquad \text{………….. (9)}$$

The above governing equations are solved with the help of numerical tool. code employing discretization of all equations using second order Finite Difference Method (FDM) and taking Pr = 0.71 for air and producing results for different Rayleigh number. No slip and impermeable boundary conditions are considered in two directions (u = v = 0) and the value of stream function has been taken as zero (ψ = 0) at all solid boundaries. The boundary conditions for temperature are given below.