Constraint Propagation: Inference in CSPs

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Overview of Constraint Propagation 约束传播概述

- Regular state-space search: 常规的状态空间搜索
  - can do only one thing, search.
    只能做一件事，搜索。
- CSPs: 约束满足问题
  - can do search, choose a new variable assignment from several possibilities,
    能做搜索，从若干可能性中选择一个新的变量赋值，
  - can also do a specific type of inference, called constraint propagation.
    还能做一种特殊类型的推理，称为约束传播。
- Constraint Propagation: 约束传播
  - uses the constraints to *reduce the number of legal values for a variable*, which in turn *can reduce the legal values for another variable*, and so on.
    采用约束来减少一个变量的合法值的数量，相应地可以减少其它变量的合法值，以此类推。

Overview of Constraint Propagation 约束传播概述

- Those techniques are used to modify a constraint satisfaction problem.
  这些技术被用于改变一个约束满足问题。
- More precisely, they enforce a form of local consistency, which are conditions related to the consistency of a group of variables and/or constraints.
  更精确地说，它们严格实施局部一致性，这种形式是一组变量和约束一致性相关的条件。
- Constraint propagation has various uses: 约束传播有各种用途：
  - 1) turns a problem into one that is equivalent but is usually simpler to solve,
    将其转转化成等价但通常更易于求解的问题。
  - 2) may prove satisfiability/unsatisfiability of problems.
    可以证明问题的可满足性/不可满足性。

  This always happens for some certain kinds of problems.
  对于某些特定类型的问题，这种情况总是发生。

Overview of Constraint Propagation 约束传播概述

- There are different types of local consistency:
  有不同类型的局部一致性：
  - Node consistency
    节点一致性
  - Arc consistency
    弧一致性
  - Path consistency
    路径一致性
  - k-consistency
    k一致性
- Most popular method is AC-3 algorithm which enforces arc consistency.
  最流行的约束传播方法是AC-3算法，它支持弧一致性。

  The name “AC-3” was used because it’s the 3rd version.

Node Consistency 节点一致性

- Definition 定义

  A single variable (node) is node-consistent, if all the values in the variable's domain satisfy the variable's unary constraints.

  如果变量的范畴中所有值满足变量的一元约束，则单个变量（节点）是节点一致的。

Example:

- in the variant of the Australia map-coloring, where South Australians (SA) dislike green, i.e.,

  在简化的澳大利亚着色问题中，南澳大利亚人（SA）不喜欢绿色，即：

$$\langle (SA), SA \neq green \rangle$$

leaving SA with the reduced domain $\{red, blue\}$.

留给SA的为缩减范畴{*red*, *blue*}。

Arc Consistency 弧一致性

- Definition 定义

  A variable is arc-consistent, if every value in its domain satisfies the variable's binary constraints.

  如果范畴中的每个值满足变量的二元约束，则该变量是弧一致的。

Example:

- in the variant of the Australia map-coloring, the constraint $SA \neq WA$, we can write this constraint explicitly as,

  在简化的澳大利亚着色问题中，该约束SA ≠ WA，我们可将这个约束显式地写成：

$$\langle (SA, WA), \{(red, green), (red, blue), (green, red), (green, blue), (blue, red), (blue, green)\}\rangle$$

Arc Consistency Algorithm AC-3 弧一致性算法AC-3

function AC-3(*csp*) **returns** false if an inconsistency is found and true otherwise
  inputs: *csp*, a binary CSP with components (X, D, C)
  local variables: *queue*, a queue of arcs, initially all the arcs in *csp*
  while *queue* is not empty **do**
    $(X_i, X_j) \leftarrow$ REMOVE-FIRST(*queue*)
    if REVISE(*csp*, X_i, X_j) **then**
      if size of $D_i = 0$ **then return** *false*
      for each X_k **in** X_i.NEIGHBORS - $\{X_j\}$ **do**
        add (X_k, X_i) to queue
  return *true*

---

function REVISE(*csp*, X_i, X_j) **returns** true iff we revise the domain of X_i
  revised ← *false*
  for each x **in** D_i **do**
    if no value y in D_j allows (x, y) to satisfy the constraint between X_i and X_j **then**
      delete x from D_i
  revised ← *true*
  return *revised*

Path Consistency 路径一致性

- Definition 定义

  A two-variable set $\{X_i, X_j\}$ is path-consistent with respect to a third variable X_m if, for every assignment $\{X_i = a, X_j = b\}$ consistent with the constraints on $\{X_i, X_j\}$, there is an assignment to X_m that satisfies the constraints on $\{X_i, X_m\}$ and $\{X_m, X_j\}$.

  对于与 $\{X_i, X_j\}$ 约束一致的每个赋值 $\{X_i = a, X_j = b\}$ ，如果存在一个满足对$\{X_i, X_m\}$和$\{X_m, X_j\}$约束的X_m 赋值，则二元变量集 $\{X_i, X_j\}$ 对于第三个变量X_m 是路径一致的。

- Why called path consistency

  为什么称为路径一致

  - because it as looking at a path from X_i to X_j with X_m in the middle.

    因为它看起来是一条从X_i到X_j、中间为X_m的路径。

k-Consistency k—致性

- **Definition** 定义

  A CSP is k-consistent if, for any set of $k-1$ variables and for any consistent assignment to those variables, a consistent value can always be assigned to any k^{th} variable.

  对于任意k − 1个变量的集合以及对于这些变量的任意一致性赋值，如果某个一致性值总是能够被赋值于任意第k个变量，则该CSP是k—致性的。

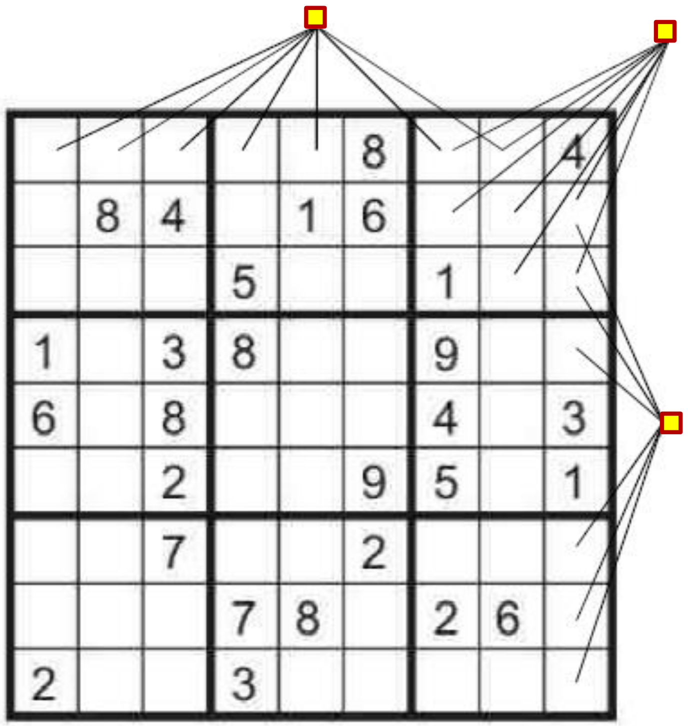
- The notion of k-consistency is the stronger forms of propagation.

  k—致性的概念更强的传播形式。

- Features of k-consistency k—致性的特点
  - $k = 1$: same as *node consistency*. k=1： 等同于节点一致性
  - $k = 2$: same as *arc consistency*. k=2： 等同于弧一致性
  - $k = 3$: same as *path consistency*. k=3： 等同于路径一致性

Example: Sudoku 数独

- Sudoku is a logic based, combinatorial number placement puzzle.
  数独是一种基于逻辑的组合数填空难题。
- The objective is to fill a 9×9 grid with digits, so that each column, each row, and each of the nine 3×3 sub-grids that compose the grid contains all of the digits from 1 to 9.
  其目的是用数字填满9x9个网格，使得每一行、每一列、以及9个3x3的子网格的每一个都包含从1到9的所有数字。



History

- Number puzzles appeared in French in late 19th century.
  19世纪后叶，数字谜题出现在法国
- It was introduced by Japan in April 1984.
  日本于1984年4月引入该数字谜题。
- London began featuring Sudoku in 2004.
  伦敦于2004年开始关注数独。

Example: Sudoku 数独

	1	2	3	4	5	6	7	8	9
A			3		2		6		
B	9			3		5			1
C			1	8		6	4		
D			8	1		2	9		
E	7								8
F			6	7		8	2		
G			2	6		9	5		
H	8			2		3			9
I			5		1		3		

- Variables: 变量：

$$A_1, A_2, \ldots, A_9$$
$$B_1, B_2, \ldots, B_9$$
$$\ldots$$
$$I_1, I_2, \ldots, I_9$$

- Domains: 范畴：

$$\{1, 2, \ldots, 9\}$$

- Constraints: 约束：

row ： 行：$Alldiff\,(A_1, A_2, \ldots, A_9), \ldots, Alldiff\,(I_1, I_2, \ldots, I_9)$

column：列：$Alldiff\,(A_1, B_1, \ldots, I_1), \ldots, Alldiff\,(A_9, B_9, \ldots, I_9)$

region ：区域：$Alldiff\,(A_1, A_2, A_3, \ldots, C_3), \ldots, Alldiff\,(G_7, G_8, G_9, \ldots, I_9)$

Example: Sudoku 数独

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

5	3	4	6	7	8	9	1	2
6	7	2	1	9	5	3	4	8
1	9	8	3	4	2	5	6	7
8	5	9	7	6	1	4	2	3
4	2	6	8	5	3	7	9	1
7	1	3	9	2	4	8	5	6
9	6	1	5	3	7	2	8	4
2	8	7	4	1	9	6	3	5
3	4	5	2	8	6	1	7	9

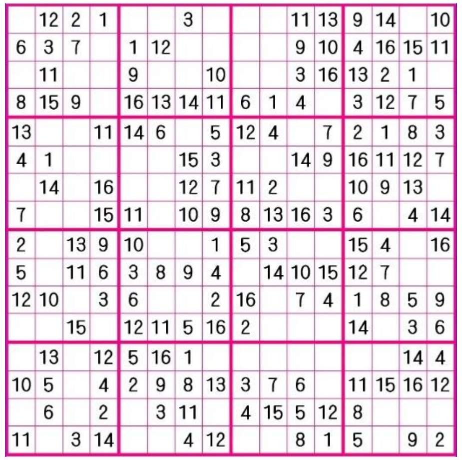
A Sudoku puzzle (9×9) and its solution

一个数独的谜题（9×9）和它的谜底

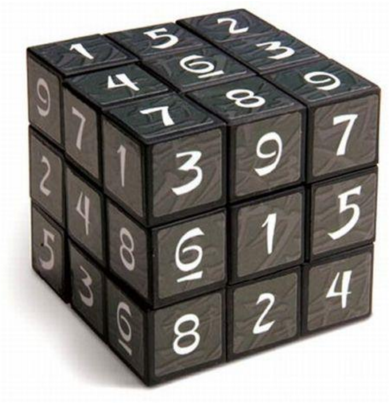
Example: Variants of Sudoku 数独的变体

1	3	4	5	2	6
2	5	6	1	4	3
5	1	2	6	3	4
6	4	3	2	5	1
3	6	5	4	1	2
4	2	1	3	6	5

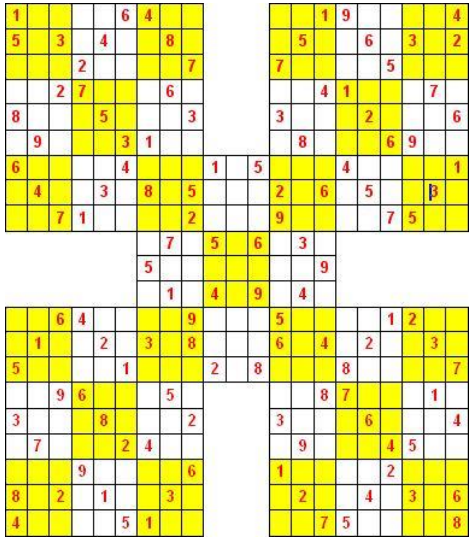
6×6



16×16



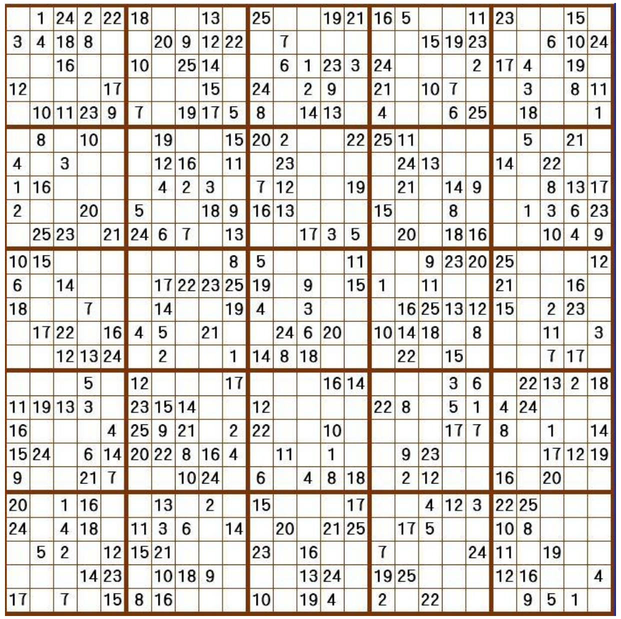
9×9



9×9×5

3	5	8	1	9	6	2	7	4
4	9	2	5	6	7	1	3	8
6	1	3	9	7	8	4	2	5
1	7	5	8	4	2	6	9	3
8	2	6	4	5	3	7	1	9
2	4	9	7	3	1	8	5	6
9	8	7	3	2	4	5	6	1
7	3	4	6	1	5	9	8	2
5	6	1	2	8	9	3	4	7

9×9



25×25

Thank you for your attention!

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