Lecture 5: Search 4
Bandits and MCTS

Previously...

Path-based search

- Uninformed search
  - Depth-first, breadth first, uniform-cost search
- Informed search
  - Best-first, **A* search**

Adversarial search

- Alpha-Beta search

Beyond classical search

Bandit search

Tree search: Monte-Carlo Tree Search

Functions for pseudo-random numbers

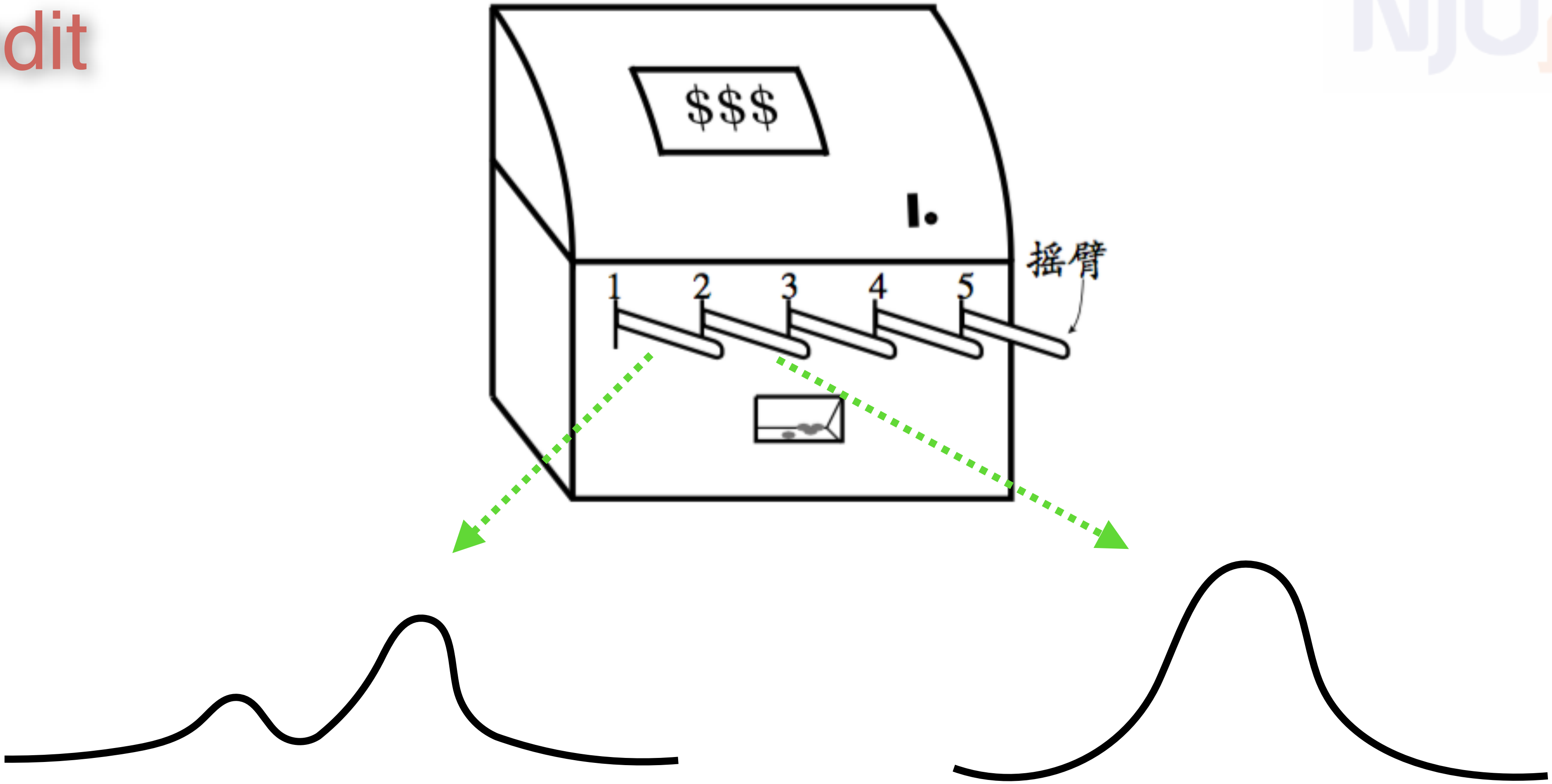
in C++

```
#include <stdlib.h>
srand(seed);
int r = rand();            0~RAND_MAX
```

in JAVA

```
import java.util.Random;
Random rnd = new Random(seed);
int r = rnd.nextInt(upper);            0~upper-1
```

Bandit



Multiple arms

Each arm has an expected reward,
but unknown, with an unknown distribution

Maximize your award in fixed trials

Simplest strategies

Two simplest strategies

Exploration-only:
for T trails and K arms, try each arm T/K times

problem? waste on suboptimal arms

Exploitation-only:

1. try each arm once
2. try the observed best arm T-K times

problem? risk of wrong best arm

ε-greedy

Balance the exploration and exploitation:

with ε probability, try a random arm
with 1-ε probability, try the best arm

ε controls the balance

输入: 摇臂数 K;
奖赏函数 R;
尝试次数 T;
探索概率 ϵ.

过程:

$r = 0$;
$\forall i = 1, 2, \ldots K: \ Q(i) = 0, \ \mathrm{count}(i) = 0$;
for $t = 1, 2, \ldots, T$ **do**
  if $\mathrm{rand}() < \epsilon$ **then**
    $k =$ 从 $1, 2, \ldots, K$ 中以均匀分布随机选取
  else
    $k = \arg\max_i Q(i)$
  end if
  $v = R(k)$;
  $r = r + v$;
  $Q(k) = \frac{Q(k) \times \mathrm{count}(k) + v}{\mathrm{count}(k) + 1}$;
  $\mathrm{count}(k) = \mathrm{count}(k) + 1$;
end for

输出: 累积奖赏 r

Softmax

Balance the exploration and exploitation:

Choose arm with probability

$$P(k) = \frac{e^{\frac{Q(k)}{\tau}}}{\sum\limits_{i=1}^{K} e^{\frac{Q(i)}{\tau}}}, \tag{16.4}$$

τ controls the balance

输入: 摇臂数 K;
奖赏函数 R;
尝试次数 T;
温度参数 τ.

过程:

```
r = 0;
∀i = 1, 2, ... K : Q(i) = 0, count(i) = 0;
for t = 1, 2, ..., T do
    k = 从1, 2, ..., K 中根据式(16.4)随机选取
    v = R(k);
    r = r + v;
    Q(k) = (Q(k) × count(k) + v) / (count(k) + 1);
    count(k) = count(k) + 1;
end for
```

输出: 累积奖赏 r

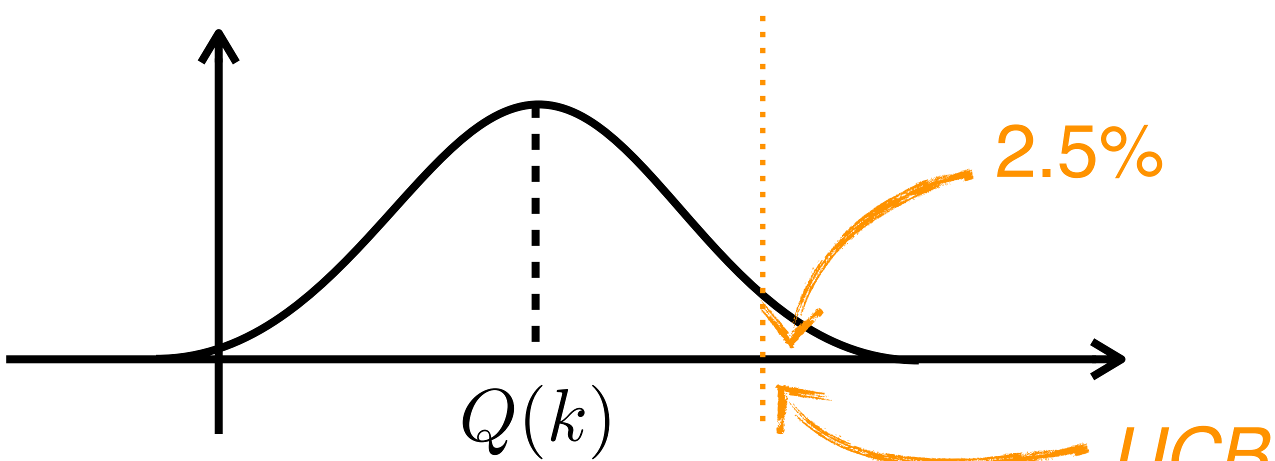
Upper-confidence bound

Balance the exploration and exploitation:

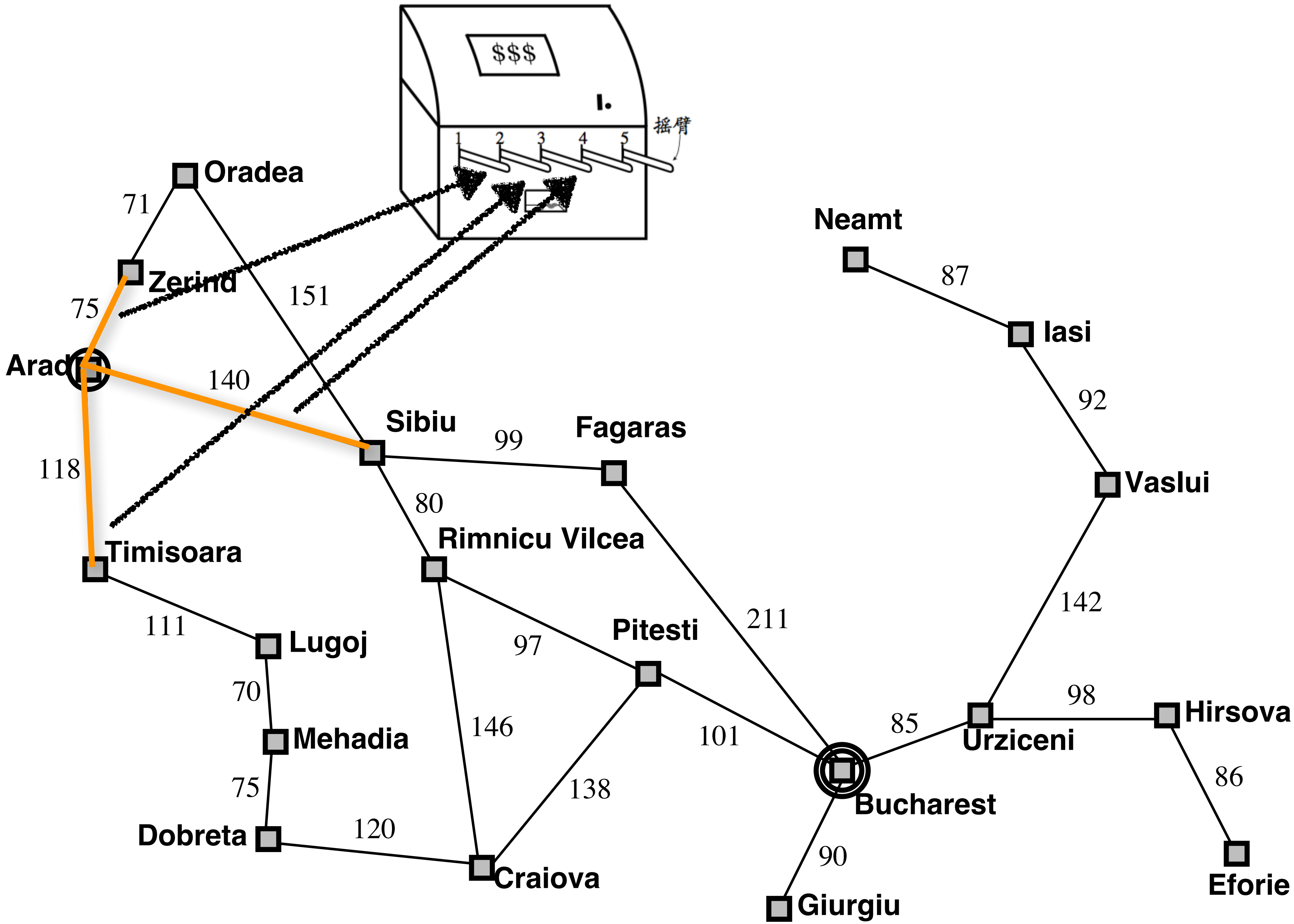
Choose arm with the largest value of

average reward + upper confidence bound

$$Q(k) + \sqrt{\frac{2 \ln n}{n_k}},$$

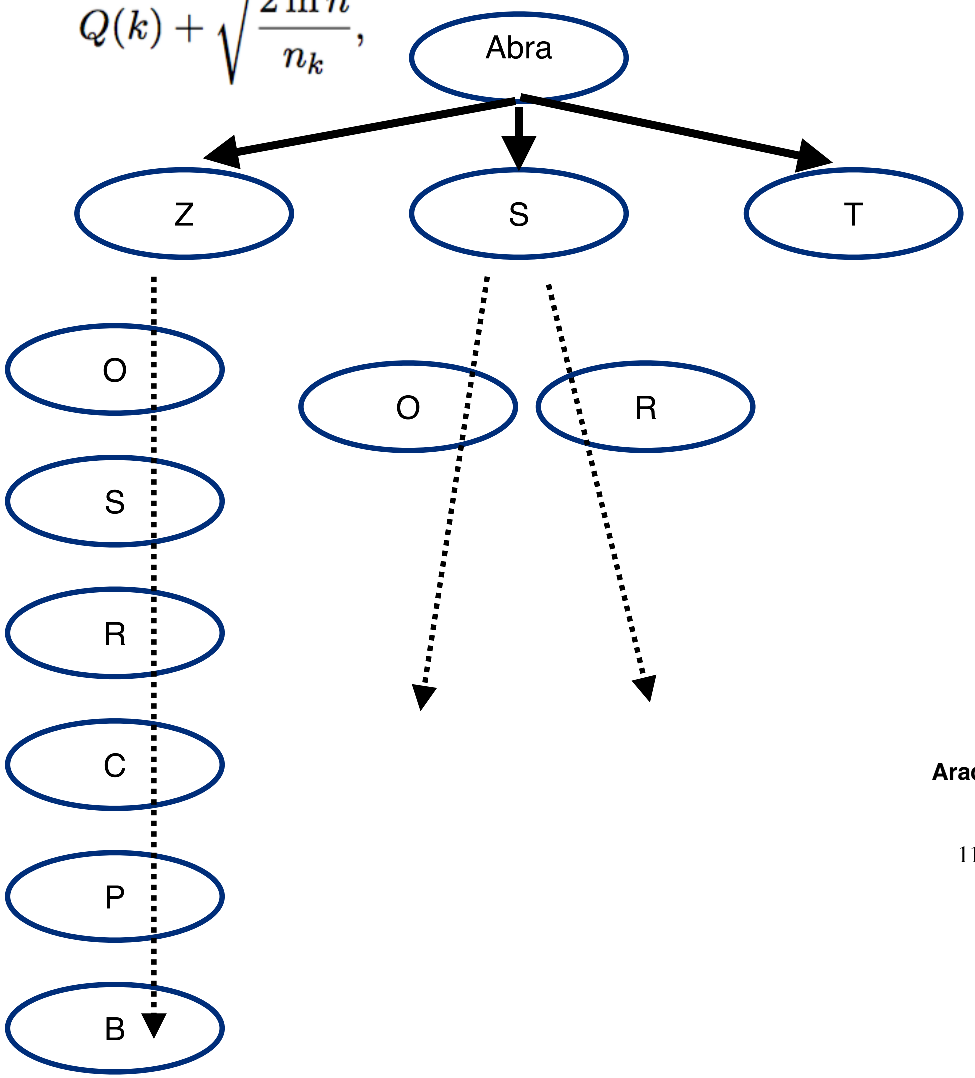


Use bandit to search



Use bandit to search

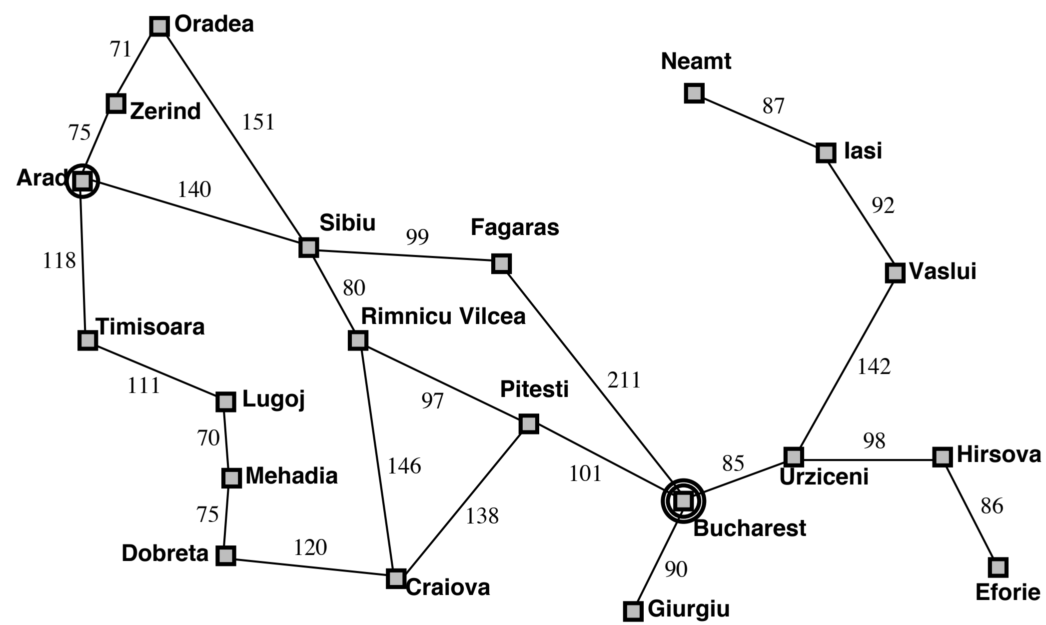
$$Q(k) + \sqrt{\frac{2\ln n}{n_k}},$$



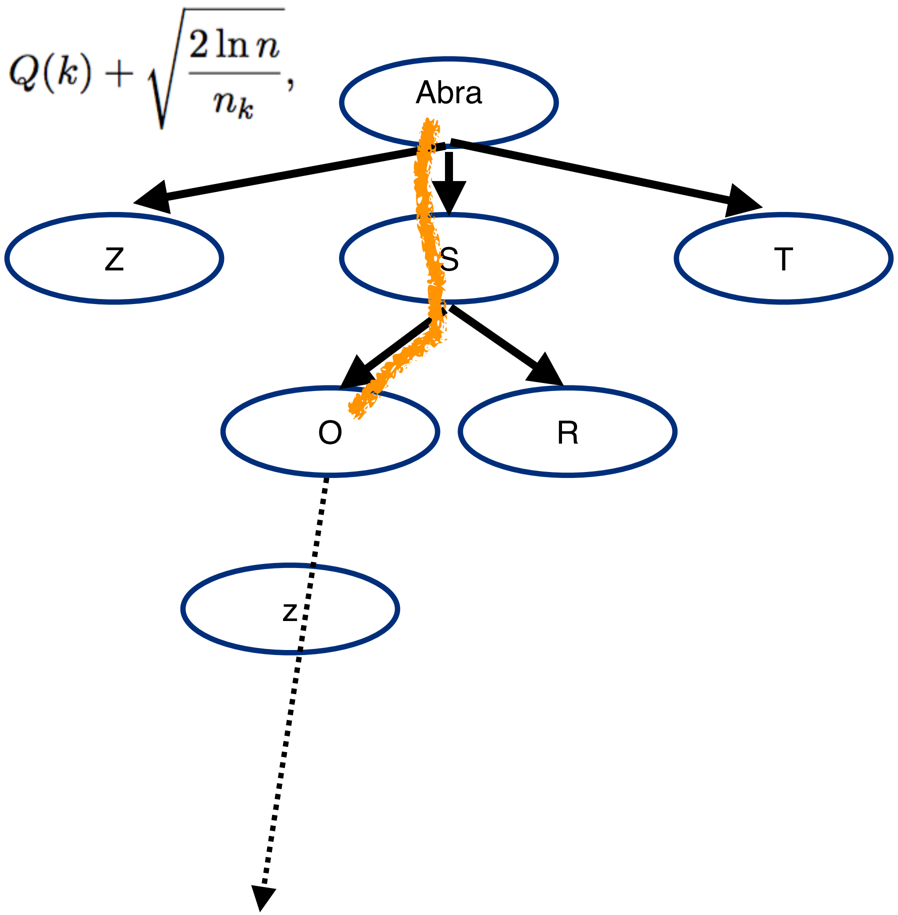
762 a roll-out

use many roll-outs to estimate the average cost of each arm

arm selection: UCB



From bandit to tree



grow a tree

update the values along the path

Monte-Carlo Tree Search

also called Upper-Confidence Tree (UCT)

Kocsis Szepesvári, 06

Gradually grow the search tree:

- Iterate Tree-Walk
  - Building Blocks
    - Select next action

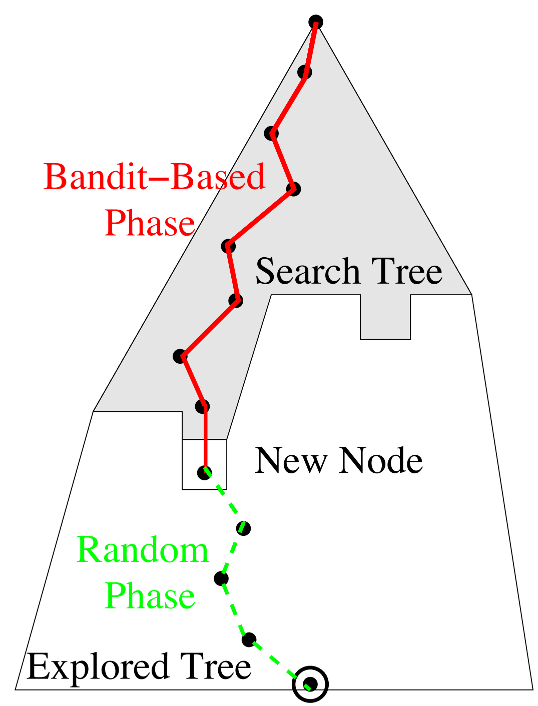
      Bandit phase
    - Add a node

      Grow a leaf of the search tree
    - Select next action bis

      Random phase, roll-out
    - Compute instant reward

      Evaluate
    - Update information in visited nodes

      Propagate
- Returned solution:
  - Path visited most often



Monte-Carlo Tree Search

```
private TreeNode select() {
    TreeNode selected = null;
    double bestValue = Double.MIN_VALUE;
    for (TreeNode c : children) {
        double uctValue = c.totValue / (c.nVisits + epsilon) +
                Math.sqrt(Math.log(nVisits+1) / (c.nVisits + epsilon)) +
                    r.nextDouble() * epsilon;
        // small random number to break ties randomly in unexpanded nodes
        if (uctValue > bestValue) {
            selected = c;
            bestValue = uctValue;
        }
    }
    return selected;
}
```

```
public
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    Tr
    do

    pu
            cur = cur.select();
            visited.add(cur);
        }
        cur.expand();
        TreeNode newNode = cur.select();
        visited.add(newNode);
        double value = rollOut(newNode);
        for (TreeNode node : visited) {
            // would need extra logic for n-player game
            node.updateStats(value);
        }
    }
```
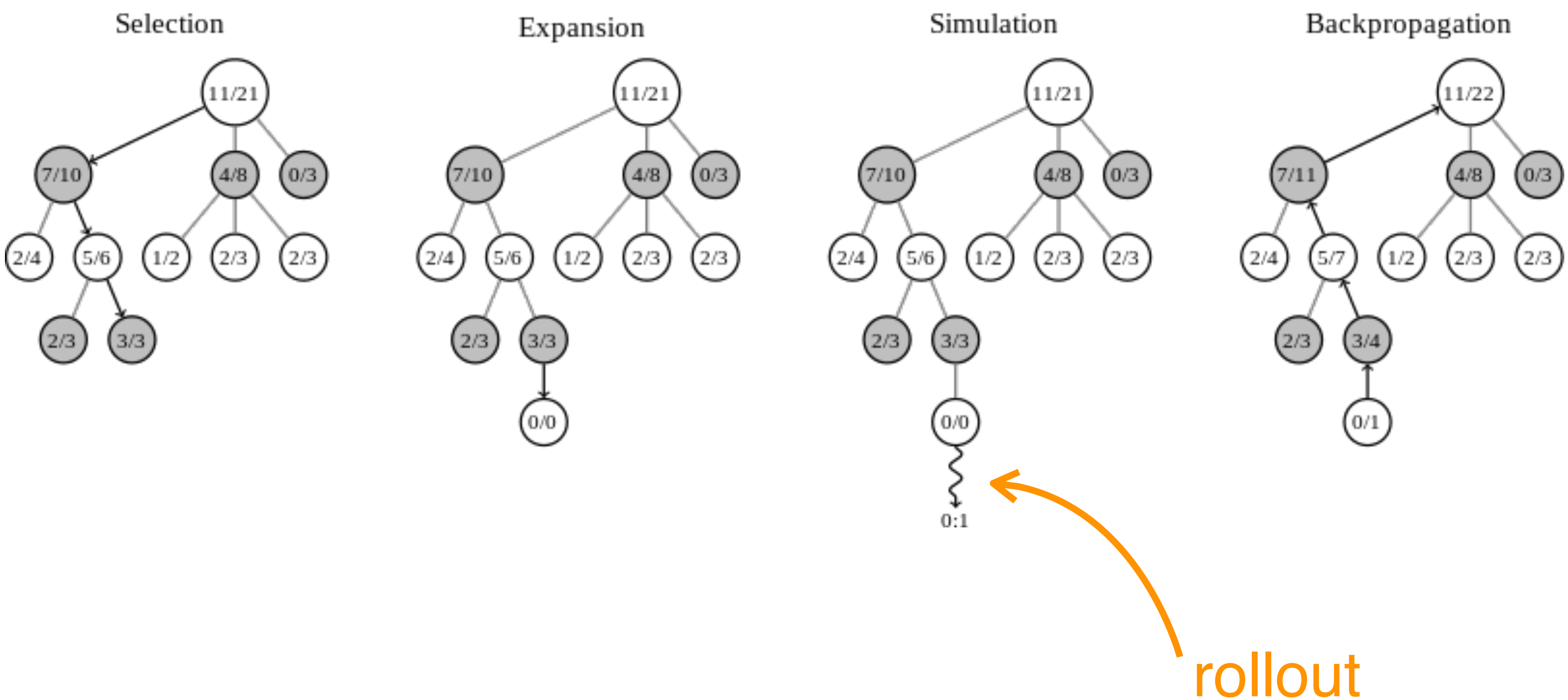

```
        totValue += value;
    }
```

codes from http://mcts.ai/code/java.html

Monte-Carlo Tree Search

Example:



Pic from https://en.wikipedia.org/wiki/Monte_Carlo_tree_search#cite_note-Kocsis-Szepesvari-5

Monte-Carlo Tree Search

optimal? Yes, after infinite tries

compare with alpha-beta pruning

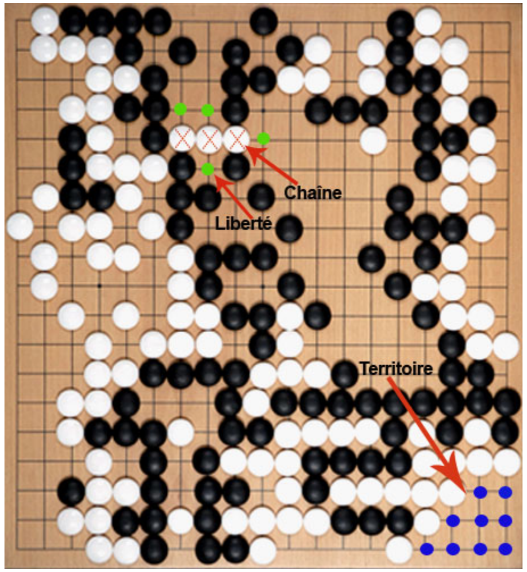
no need of heuristic function

Monte-Carlo Tree Search

Improving random rollout

Monte-Carlo-based   Brügman 93

1. Until the goban is filled,
   add a stone (black or white in turn)
   at a uniformly selected empty position
2. Compute $r = \text{Win(black)}$
3. The outcome of the tree-walk is r

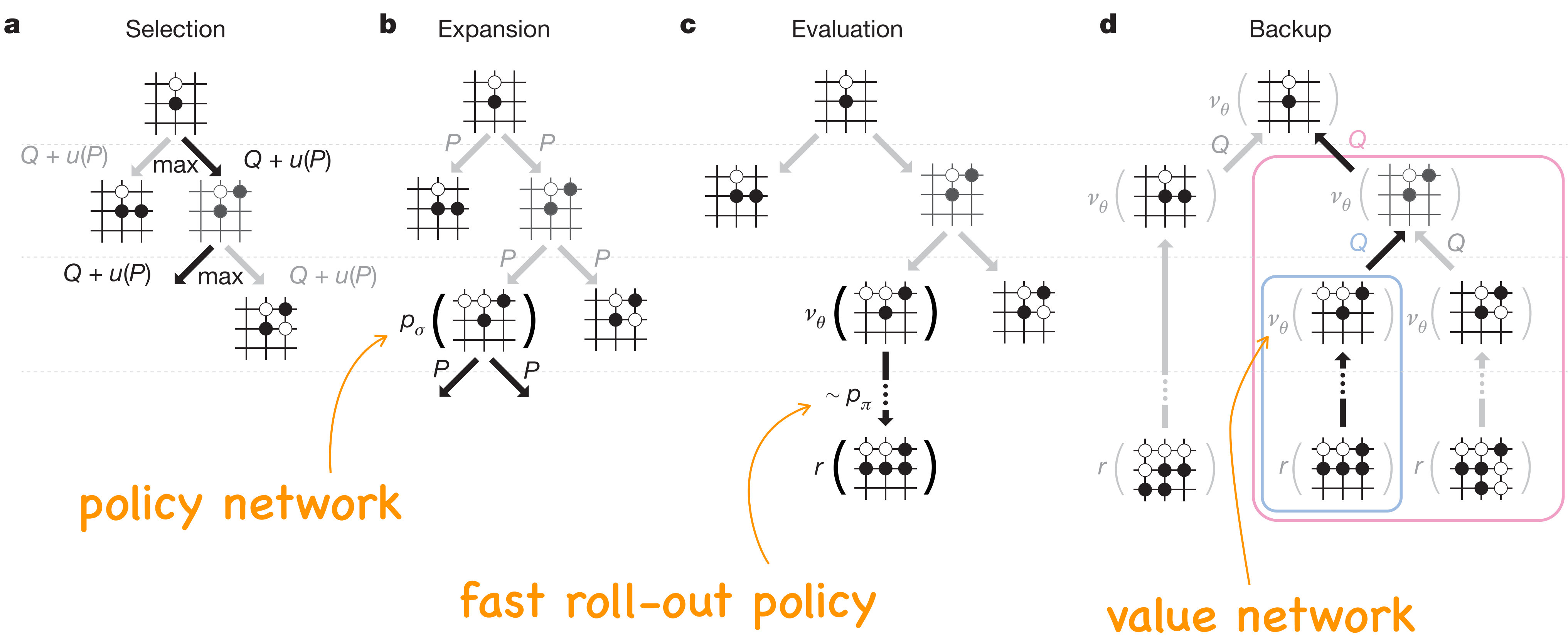


Improvements ?

- Put stones randomly in the neighborhood of a previous stone
- Put stones matching patterns   prior knowledge
- Put stones optimizing a value function   Silver et al. 07

AlphaGo

A combination of tree search, deep neural networks and reinforcement learning

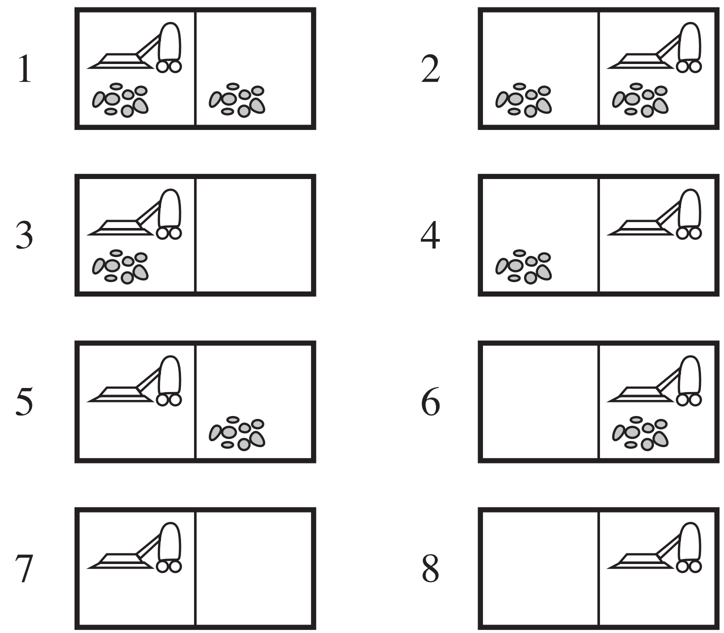


Different Environment Properties

Nondeterministic actions

In the **erratic vacuum world**, the *Suck* action works as follows:

- When applied to a dirty square the action cleans the square and sometimes cleans up dirt in an adjacent square, too.
- When applied to a clean square the action sometimes deposits dirt on the carpet.

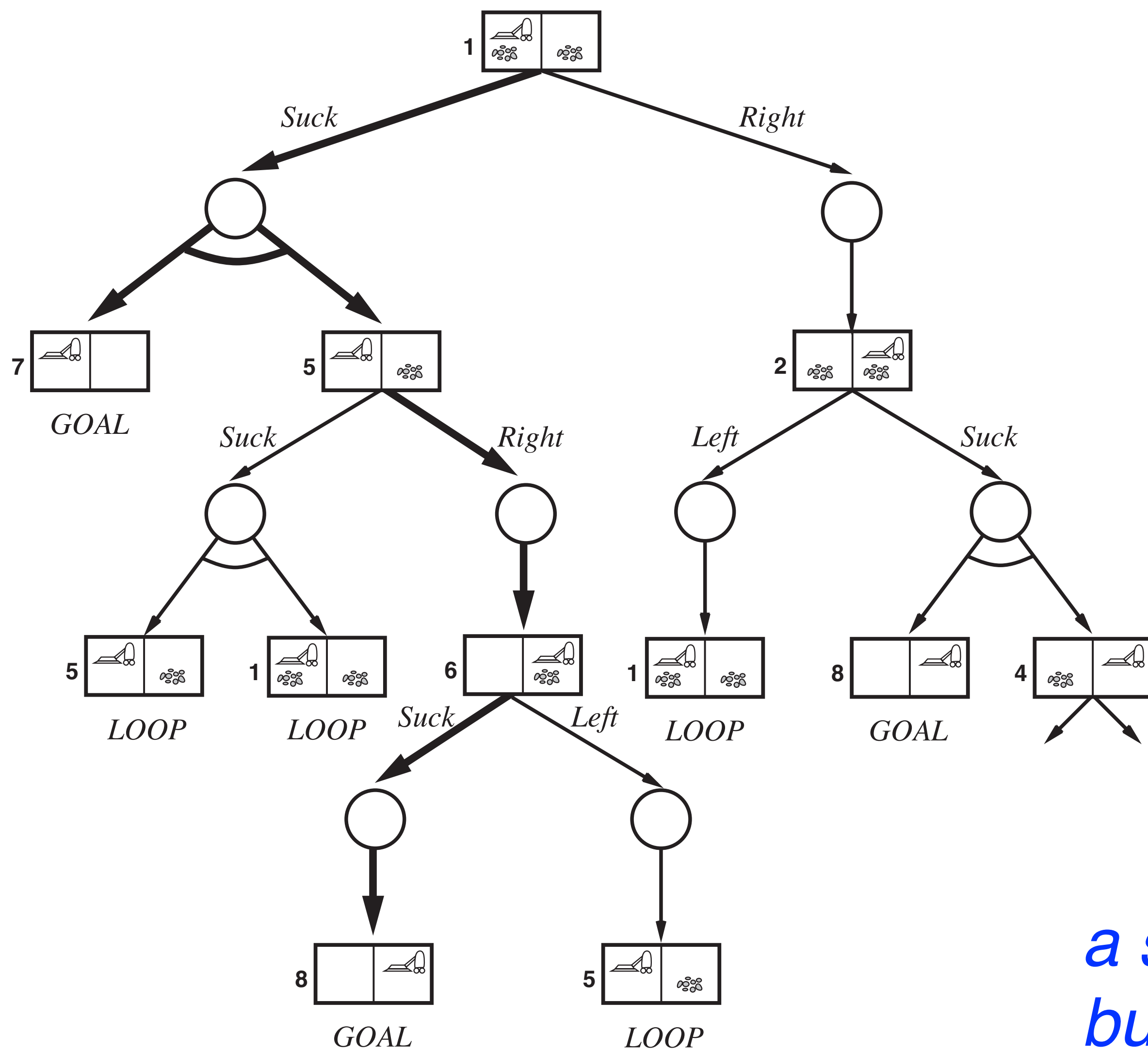


almost all real-world problems are nondeterministic

how do you solve this problem?

AND-OR tree search

OR node: different actions (as usual)
AND node: different transitions



a solution is not a path
but a tree

Depth-first AND-OR tree search

function AND-OR-GRAPH-SEARCH($problem$) **returns** *a conditional plan, or failure*
    OR-SEARCH($problem$.INITIAL-STATE, $problem$, [ ])

---

function OR-SEARCH($state, problem, path$) **returns** *a conditional plan, or failure*
    if $problem$.GOAL-TEST($state$) **then return** the empty plan
    if $state$ is on $path$ **then return** $failure$
    for each $action$ **in** $problem$.ACTIONS($state$) **do**
        $plan \leftarrow$ AND-SEARCH(RESULTS($state, action$), $problem$, $[state \mid path]$)
        if $plan \neq failure$ **then return** $[action \mid plan]$
    return $failure$

---

function AND-SEARCH($states, problem, path$) **returns** *a conditional plan, or failure*
    for each s_i **in** $states$ **do**
        $plan_i \leftarrow$ OR-SEARCH($s_i, problem, path$)
        if $plan_i = failure$ **then return** $failure$
    return [**if** s_1 **then** $plan_1$ **else if** s_2 **then** $plan_2$ **else** $\ldots$ **if** s_{n-1} **then** $plan_{n-1}$ **else** $plan_n$]

Search with no observations

search in **belief (in agent's mind)**