443 Project

2022-11-06

Data Cleaning and Preprocessing

```
# read csv
df <- read.csv("Data_Group11.csv")
```

We first convert the data in to a more usable long format.

```
# convert month and day to MM-DD format
singledig <- which(df$X.1 < 10)
df$day <- as.character(df$X.1)
df$day[singledig] <- paste0("0", df$day[singledig])

df$month <- rep(c("01","02","03","04", "05","06","07","08","09","10","11","12"), times=c(31,29,31,30,31

df$MMDD <- paste0(df$month, "-", df$day)

# drop unnecessary columns
to_drop <- c("X", "X.1", "X.2", "X1981.2010.mean", "X1981.2010.median", "month", "day")
df <- subset(df, select=!(names(df) %in% to_drop))

# melt years to be one column
library(tidyr)
df <- pivot_longer(data=df, cols=!MMDD, names_to="year", values_to="extent")

# format year and creating a column with YY-MM-DD format
library(stringr)
df$year = str_replace(df$year, "X", "")
df$YYMMDD <- paste0(df$year, "-", df$MMDD)

# tell R YYMMDD is a date
df$YYMMDD = as.Date(df$YYMMDD) # also conveniently rids of non leap year Feb 29's

# drop unnecessary columns
to_drop <- c("MMDD", "year")
df <- subset(df, select=!(names(df) %in% to_drop))

# order data by date
df <- df[order(df$YYMMDD),]

head(df)
```

```
## # A tibble: 6 x 2
##   extent YYMMDD
##    <dbl> <date>
## 1     NA 1978-01-01
```

```
## 2     NA 1978-01-02
## 3     NA 1978-01-03
## 4     NA 1978-01-04
## 5     NA 1978-01-05
## 6     NA 1978-01-06
```

We then deal with NA values.

```r
# drop initial and ending NAs because we don't have data collected for these dates
library(zoo)
```

```
##
## Attaching package: 'zoo'

## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric
```

```r
df <- na.trim(df)
```

There seems to be a change point at 1988-01-13 where a new method of measurement may of been put in to place. Before this date, there is a long stretch of NA values, and all measurements previously were recorded every second day.

```r
# There is a period between 1987-12-03 and 1988-01-12 with a stretch of NAs. The following allows this p
df_subset <- subset(df, YYMMDD>as.Date("1987-12-01") & YYMMDD<as.Date("1988-01-15"))
print(df_subset)
```

```
## # A tibble: 44 x 2
##    extent YYMMDD
##     <dbl> <date>
##  1   12.6 1987-12-02
##  2   NA   1987-12-03
##  3   NA   1987-12-04
##  4   NA   1987-12-05
##  5   NA   1987-12-06
##  6   NA   1987-12-07
##  7   NA   1987-12-08
##  8   NA   1987-12-09
##  9   NA   1987-12-10
## 10   NA   1987-12-11
## # ... with 34 more rows
```

To deal with this long stretch of NAs and the NA values caused by measurement every second day, we propose two options. Either, we impute the missing data using cubic splines, or drop all observations before the possible changepoint.

```r
# impute missing values using cubic splines as one option
df_imputed <- df
df_imputed$extent <- na.spline(df_imputed$extent)

# drop all data before 1988-01-13 as another option
df <- subset(df, YYMMDD>=as.Date("1988-01-13"))
```

We are interested in overall trend, not day to day fluctuations, so we consider aggregating values by month.

```r
library(dplyr)
```

```
##
```

```
## Attaching package: 'dplyr'

## The following objects are masked from 'package:stats':
##
##     filter, lag

## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union
```

```
library(lubridate)
```

```
##
## Attaching package: 'lubridate'

## The following objects are masked from 'package:base':
##
##     date, intersect, setdiff, union
```

```
df_aggregated <- df %>%
  group_by(year=year(YYMMDD), month=month(YYMMDD)) %>%
  mutate(avg_extent = mean(extent)) %>%
  distinct(year, month, .keep_all=TRUE) %>%
  subset(select=c(year, month, avg_extent))
```

Now that our data is cleaned and processed, we may proceed with analysis.

```
library(huxtable)
```

```
##
## Attaching package: 'huxtable'

## The following object is masked from 'package:dplyr':
##
##     add_rownames
```

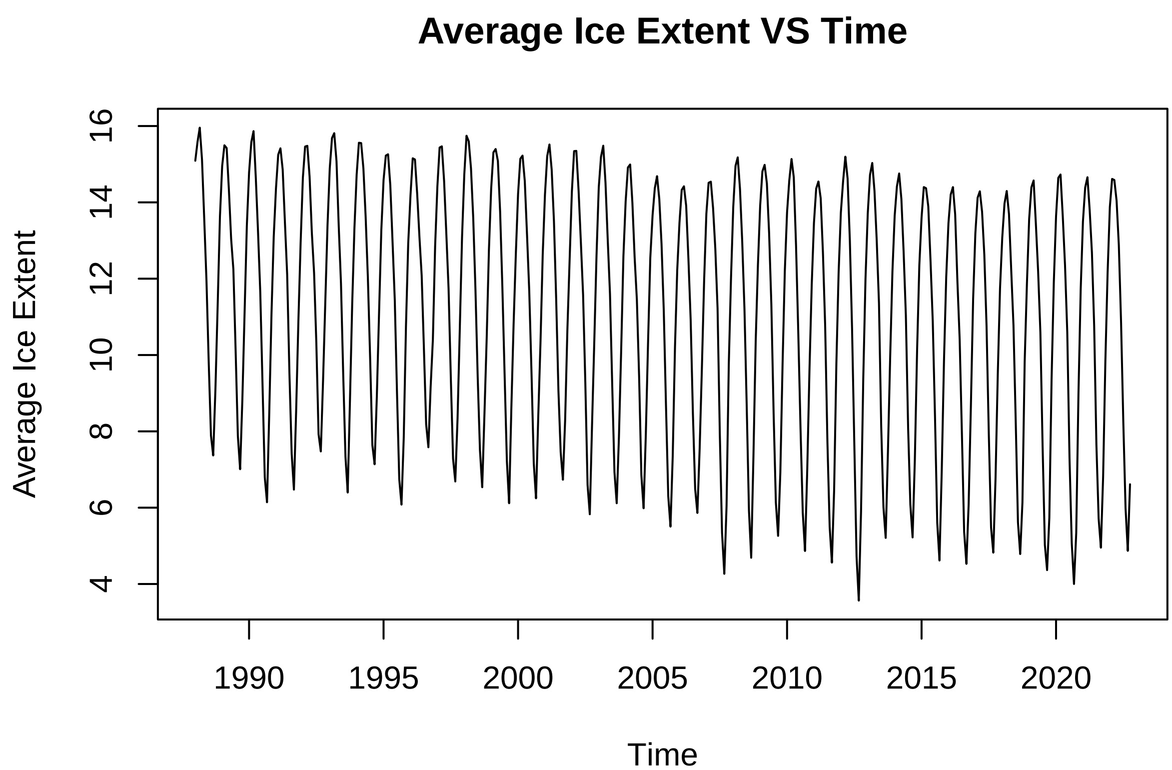
```
summary_Avg_Extent <- summary(df_aggregated$avg_extent)
summary_Extent <- summary(df$extent)
summary <- as.data.frame(rbind(matrix(summary_Avg_Extent,nrow=1), matrix(summary_Extent,nrow=1)),
                         row.names=c("Aggregated", "Unaggregated"))
colnames(summary) <- c("Min", "First Quartile", "Median", "Mean", "Third Quartile", "Max")

summary_table <- hux(summary, add_rownames = "")

summary_table %>%
  set_number_format(3) %>%
  set_align(everywhere, c(2,3,4,5,6,7), "center") %>%
  set_bottom_border(1, everywhere)
```

	Min	First Quartile	Median	Mean	Third Quartile	Max
Aggregated	3.566	8.276	11.914	11.150	14.166	15.957
Unaggregated	3.340	8.303	11.882	11.127	14.109	16.309

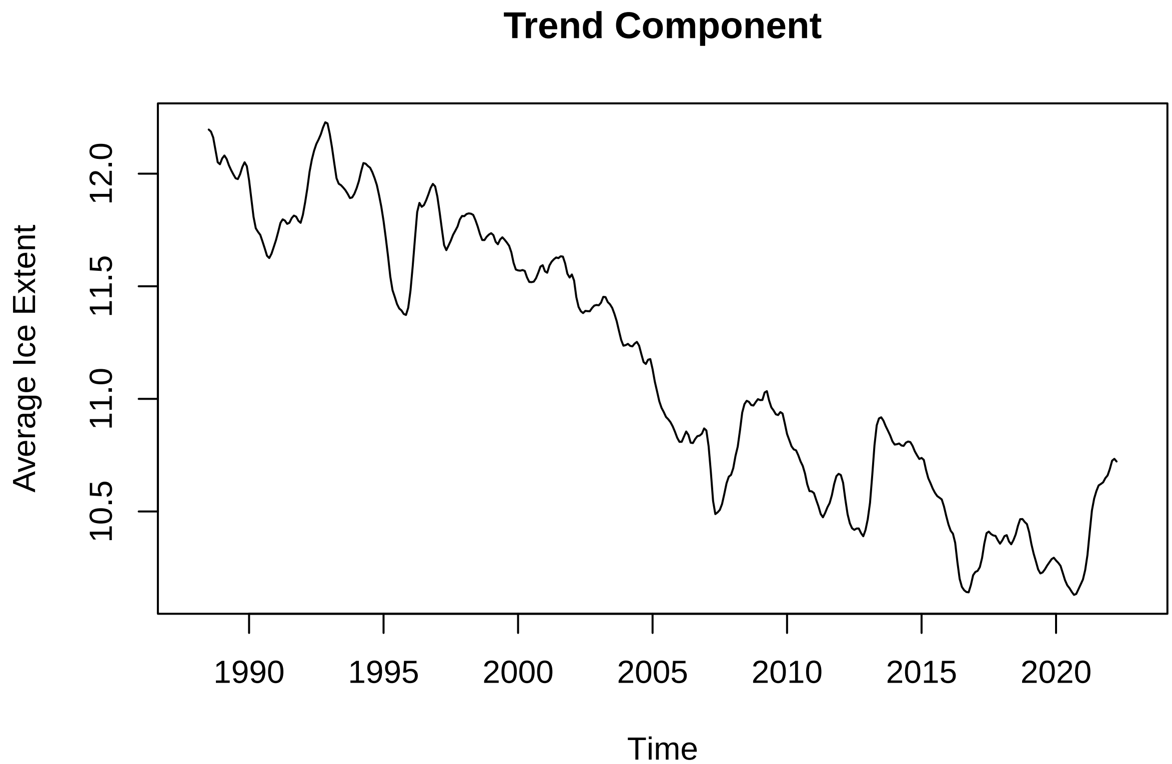
Aggregated Data

We analyze the monthly aggregated data.

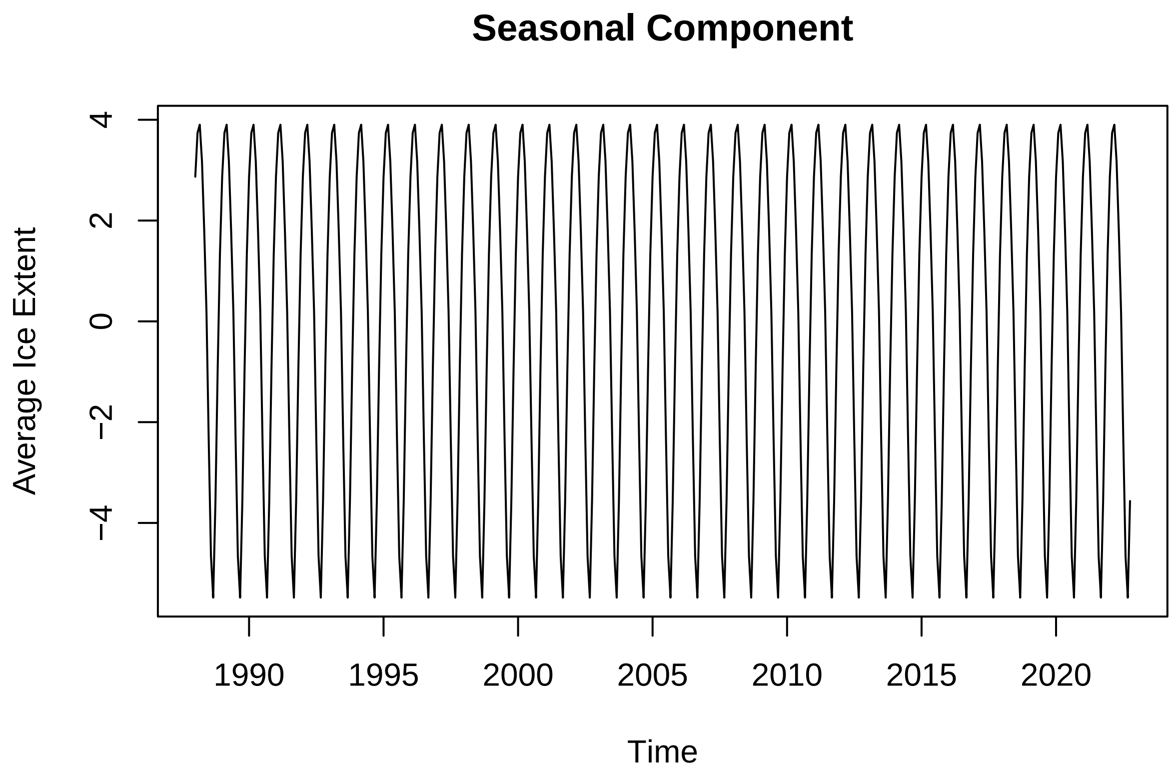
```
# make a ts object
Avg_ExtentTS <- ts(df_aggregated$avg_extent, frequency=12, start=year(df$YYMMDD[1]))
plot(Avg_ExtentTS, ylab="Average Ice Extent", main="Average Ice Extent VS Time")
```



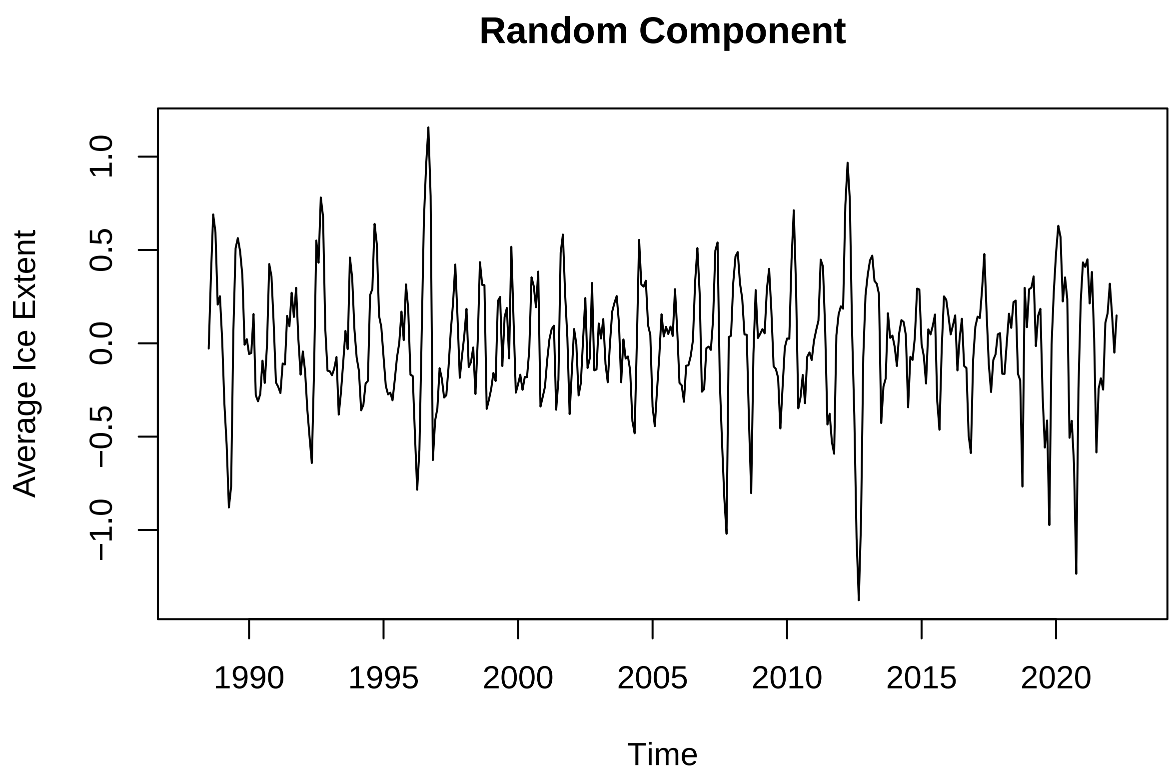

```
# Plotting Classical Decomposition
plot(decompose(Avg_ExtentTS)$trend, ylab="Average Ice Extent", main="Trend Component")
```




```
plot(decompose(Avg_ExtentTS)$season, ylab="Average Ice Extent", main="Seasonal Component")
```




```
plot(decompose(Avg_ExtentTS)$random, ylab="Average Ice Extent", main="Random Component")
```



From the decomposition, we see there is a significant seasonal pattern, and likely significant trend.

Variance

From the plot of the data, we see a clear seasonal pattern, and perhaps a decreasing linear trend.

It is unclear whether variance is constant. We test this using the Fligner-Killeen test.

```
# do Fligner test for constant variance.
segments = factor(c(rep(1:4, each=84), rep(5, times=82)))
fligner.test(Avg_ExtentTS, segments)
```

```
##
##  Fligner-Killeen test of homogeneity of variances
##
## data:  Avg_ExtentTS and segments
## Fligner-Killeen:med chi-squared = 5.8565, df = 4, p-value = 0.2101
```

```
segments = factor(c(rep(1:9, each=42), rep(10, times=40)))
fligner.test(Avg_ExtentTS, segments)
```

```
##
##  Fligner-Killeen test of homogeneity of variances
##
## data:  Avg_ExtentTS and segments
## Fligner-Killeen:med chi-squared = 8.2771, df = 9, p-value = 0.5065
```

```
segments = factor(c(rep(1:19, each=21), rep(20, times=19)))
fligner.test(Avg_ExtentTS, segments)
```

```
##
##  Fligner-Killeen test of homogeneity of variances
##
```

```
## data:  Avg_ExtentTS and segments
## Fligner-Killeen:med chi-squared = 13.213, df = 19, p-value = 0.8275
```

```
segments = factor(c(rep(1:34, each=12), rep(35, times=10))) # corresponds to number of years of data
fligner.test(Avg_ExtentTS, segments)
```

```
##
##  Fligner-Killeen test of homogeneity of variances
##
## data:  Avg_ExtentTS and segments
## Fligner-Killeen:med chi-squared = 12.564, df = 34, p-value = 0.9997
```

All give high p-value so may conclude that variance is relatively constant. This is against expectation, but perhaps this is because the change in variance is not significant over such a small time frame.

```
# define mse function for future use
mse <- function(y, yhat) {
  return(mean((as.vector(y)-as.vector(yhat))^2))
}
```

First, split the data, in to train and test set.

```
Avg_ExtentTS_Train <- window(Avg_ExtentTS, 1988, 2020+11/12)
Avg_ExtentTS_Test <- window(Avg_ExtentTS, 2021, 2022+9/12)
```

Regression

Try to remove non-stationarity using Regression (Multiple Linear, Ridge, Lasso, Elastic Net).

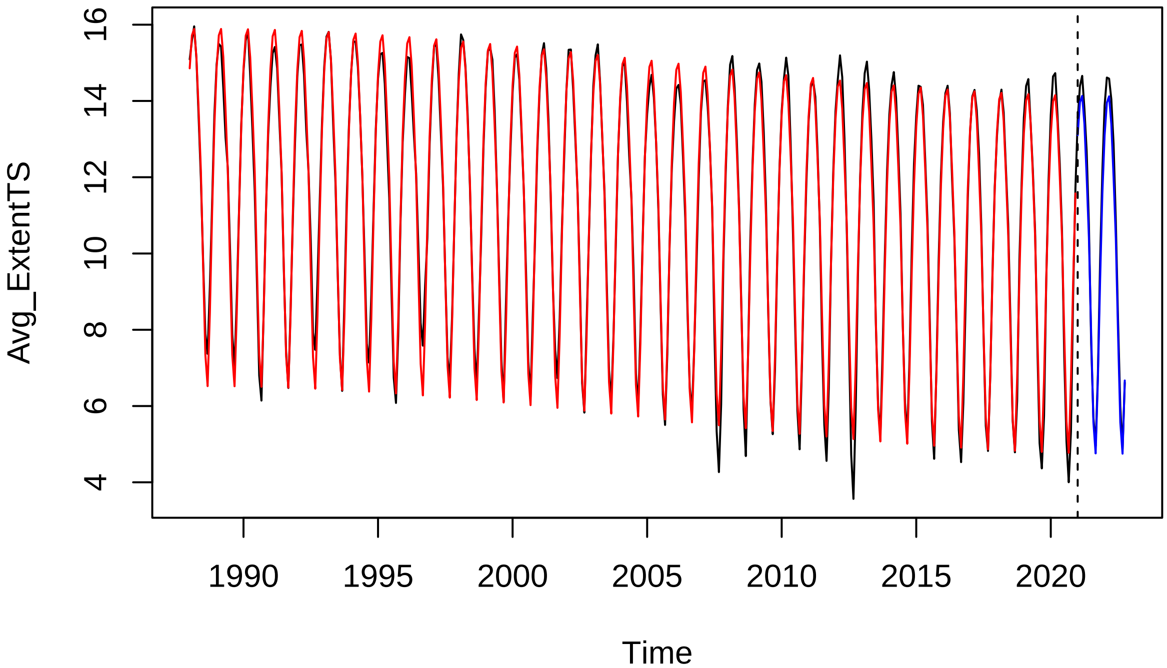
Multiple Linear Regression

```
tim <- as.vector(time(Avg_ExtentTS_Train))
season <- factor(cycle(Avg_ExtentTS_Train))

# degree 1 polynomial of time
mlr_train <- lm(Avg_ExtentTS_Train~tim+season)

new <- data.frame(tim=as.vector(time(Avg_ExtentTS_Test)), season=factor(cycle(Avg_ExtentTS_Test)))
pmse_mlr <- mse(Avg_ExtentTS_Test, predict.lm(mlr_train, new))

# degree 2 polynomial of time
mlr_train_2 <- lm(Avg_ExtentTS_Train~poly(tim,2)+season)
pmse_mlr_2 <- mse(Avg_ExtentTS_Test, predict.lm(mlr_train_2, new))

# degree 3 polynomial of time
mlr_train_3 <- lm(Avg_ExtentTS_Train~poly(tim,3)+season)
pmse_mlr_3 <- mse(Avg_ExtentTS_Test, predict.lm(mlr_train_3, new))
```

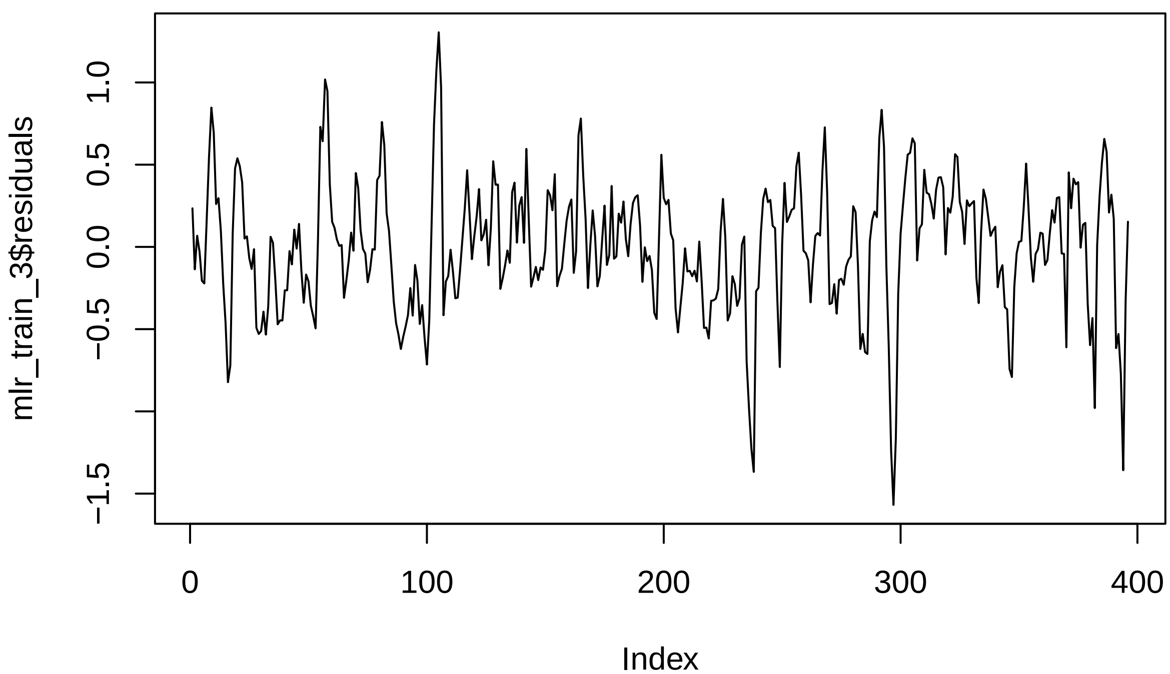
The cubic model performs best on the hold out set.

```
# plot of cubic data
plot(Avg_ExtentTS)

# plot of fit
points(time(Avg_ExtentTS_Train), predict.lm(mlr_train_3), type='l', col='red')

# plot of test set prediction
```

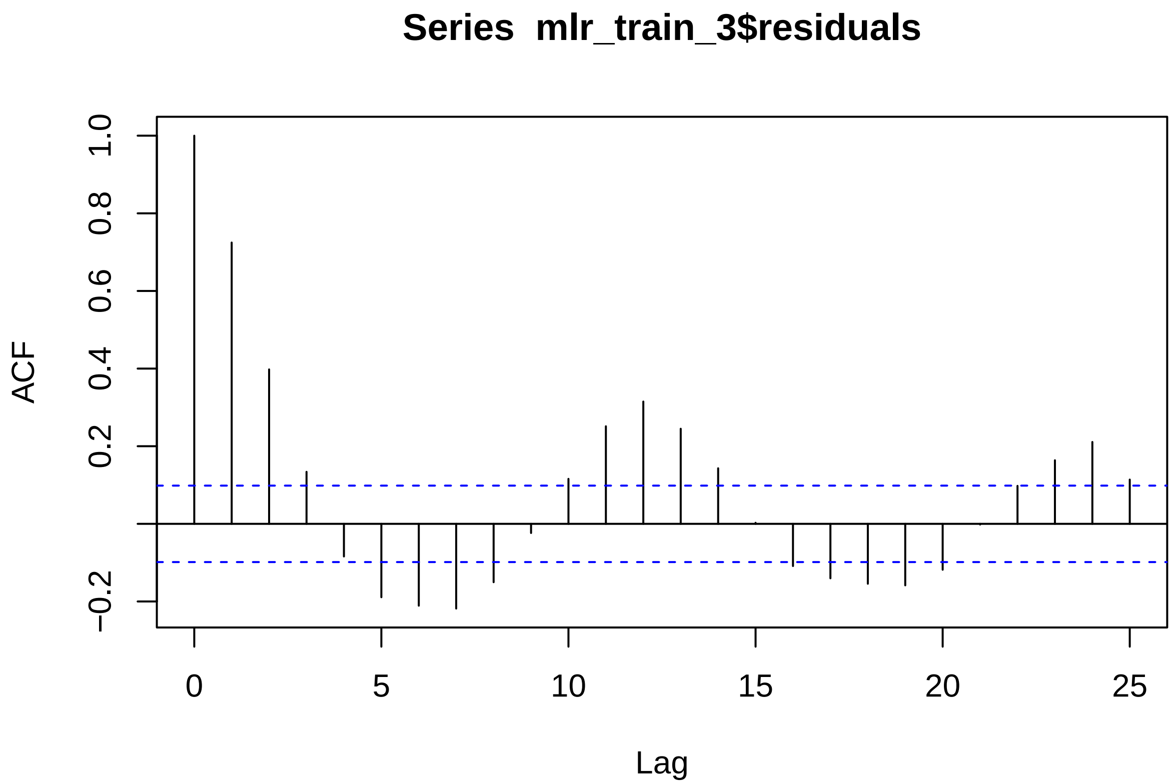
```
points(time(Avg_ExtentTS_Test), predict.lm(mlr_train_3, new), type='l', col='blue')

# plot line at test set cutoff
abline(v=2021, lty="dashed")
```




```
# plot of model residuals
plot(mlr_train_3$residuals, type="l")
```




```
# plot of acf of residuals
acf(mlr_train_3$residuals)
```

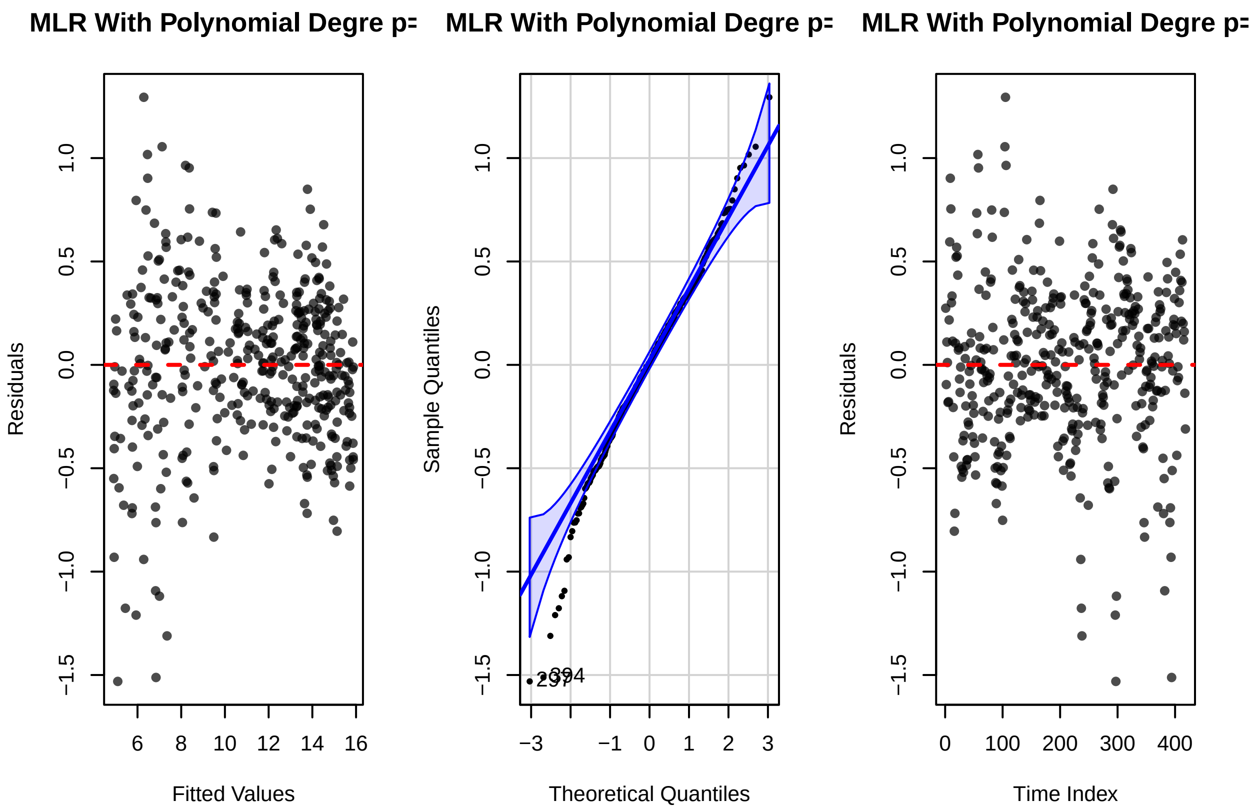


Checking normality of this model:

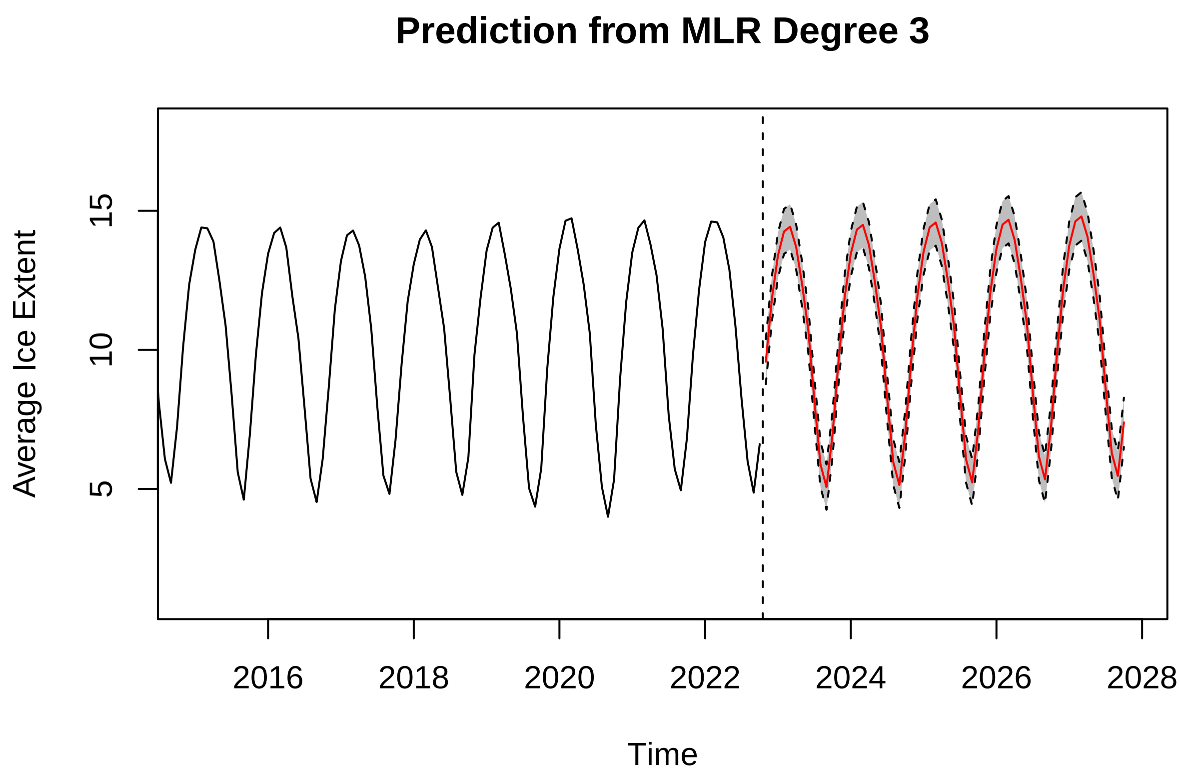
```
# training the cubic model on the entore data
tim <- as.vector(time(Avg_ExtentTS))
season <- factor(cycle(Avg_ExtentTS))
mlr_3 <- lm(Avg_ExtentTS~poly(tim,3)+season)

# model diagnostics
par(mfrow=c(1,3)) # Dividing the plotting page into 4 panels
plot(mlr_3$fitted, mlr_3$residuals , pch=16 , col=adjustcolor("black" , 0.7), xlab="Fitted Values", ylab
title(main = "MLR With Polynomial Degre p=3")
abline(h=0,lty=2 , lwd=2 , col="red") # plotting a horizontal line at 0
car::qqPlot(mlr_3$residuals , pch=16, xlab="Theoretical Quantiles", ylab="Sample Quantiles")
```

```
## [1] 297 394
```

```
title(main = "MLR With Polynomial Degre p=3")
plot(mlr_3$residuals, pch=16 , col=adjustcolor("black" , 0.7), xlab="Time Index", ylab="Residuals") # p
title(main = "MLR With Polynomial Degre p=3")
abline(h=0,lty=2 , lwd=2 , col="red") # plotting a horizontal line at 0
```




```
# Getting predictions for model and plotting
tim.new <- as.vector(seq(2022+10/12,2027+9/12,by=1/12))
season.new <- factor(c(11, 12, rep(1:12,4), 1:10))
new <- data.frame(tim=tim.new,season=season.new)
predict_mlr_3 <- predict.lm(mlr_3, new, interval='prediction')

par(mfrow=c(1,1))
plot(Avg_ExtentTS , xlim = c(2015 , 2027+10/12), ylim=c(1,18), ylab="Average Ice Extent", main="Predict

#The three lines below plot the prediction interval in a grey scale
x = c(tim.new , rev(tim.new))
y = c(predict_mlr_3[,"upr"] , rev(predict_mlr_3[,"lwr"]))
polygon(x, y, col="grey", border=NA)

#The three line below add the predicted values and highlight the borders of the prediction interval
lines(x=tim.new, y=predict_mlr_3[,"upr"], col="black" , lty=2)
lines(x=tim.new, y=predict_mlr_3[,"lwr"], col="black", lty=2)
lines(x=tim.new, y=predict_mlr_3[,"fit"] , col="red")
abline(v=2022+9.5/12, lty="dashed")
```




```
# Shapiro-Wilk Test of normality test
shapiro.test(mlr_3$residuals)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  mlr_3$residuals
## W = 0.98352, p-value = 0.0001077
```

Ridge

```
library(glmnet)
```

```
## Loading required package: Matrix
```

```
##
## Attaching package: 'Matrix'
```

```
## The following objects are masked from 'package:tidyr':
##
##     expand, pack, unpack
```

```
## Loaded glmnet 4.1-4
```

```
# fit to training data
tim <- as.vector(time(Avg_ExtentTS_Train))
season <- factor(cycle(Avg_ExtentTS_Train))
X <- model.matrix(as.vector(Avg_ExtentTS_Train)~tim+season)
X_2 <- model.matrix(as.vector(Avg_ExtentTS_Train)~poly(tim,2)+season)
X_3 <- model.matrix(as.vector(Avg_ExtentTS_Train)~poly(tim,3)+season)
ridge_train <- glmnet(X, as.vector(Avg_ExtentTS_Train), alpha=0)
ridge_train_2 <- glmnet(X_2, as.vector(Avg_ExtentTS_Train), alpha=0)
```

```r
ridge_train_3 <- glmnet(X_3, as.vector(Avg_ExtentTS_Train), alpha=0)

# compute mse on training data for each value of lambda
ridge_train_fitted <- predict(ridge_train, X)
ridge_train_fitted_2 <- predict(ridge_train_2, X_2)
ridge_train_fitted_3 <- predict(ridge_train_3, X_3)

mses <- c()
mses_2 <- c()
mses_3 <- c()
for(i in 1:100) {
  mses <- c(mses, mse(Avg_ExtentTS_Train, ridge_train_fitted[,i]))
  mses_2 <- c(mses_2, mse(Avg_ExtentTS_Train, ridge_train_fitted_2[,i]))
  mses_3 <- c(mses_3, mse(Avg_ExtentTS_Train, ridge_train_fitted_3[,i]))
}

min10_mses <- head(sort(mses), 10)
min10_mses_2 <- head(sort(mses_2), 10)
min10_mses_3 <- head(sort(mses_3), 10)

ridge_train_lambdas <- c()
ridge_train_lambdas_2 <- c()
ridge_train_lambdas_3 <- c()

for(m in min10_mses) {
  ridge_train_lambdas <- c(ridge_train_lambdas, ridge_train$lambda[which(mses==m)])
}
for(m in min10_mses_2) {
  ridge_train_lambdas_2 <- c(ridge_train_lambdas_2, ridge_train_2$lambda[which(mses_2==m)])
}
for(m in min10_mses_3) {
  ridge_train_lambdas_3 <- c(ridge_train_lambdas_3, ridge_train_3$lambda[which(mses_3==m)])
}
```

```r
# retrain using lambdas that gave the 10 best fits
ridge_train <- glmnet(X, as.vector(Avg_ExtentTS_Train), alpha=0, lambda=ridge_train_lambdas)
ridge_train_2 <- glmnet(X_2, as.vector(Avg_ExtentTS_Train), alpha=0, lambda=ridge_train_lambdas_2)
ridge_train_3 <- glmnet(X_3, as.vector(Avg_ExtentTS_Train), alpha=0, lambda=ridge_train_lambdas_3)

# predict the test set
tim <- as.vector(time(Avg_ExtentTS_Test))
season <- factor(cycle(Avg_ExtentTS_Test))
X <- model.matrix(as.vector(Avg_ExtentTS_Test)~tim+season)
X_2 <- model.matrix(as.vector(Avg_ExtentTS_Test)~poly(tim,2)+season)
X_3 <- model.matrix(as.vector(Avg_ExtentTS_Test)~poly(tim,3)+season)
ridge_predictions <- predict(ridge_train, X)
ridge_predictions_2 <- predict(ridge_train_2, X_2)
ridge_predictions_3 <- predict(ridge_train_3, X_3)

# compute pmse on test set
pmses <- c()
pmses_2 <- c()
pmses_3 <- c()
for(i in 1:10) {
```

```
  pmses <- c(pmses, mse(Avg_ExtentTS_Test, ridge_predictions[,i]))
  pmses_2 <- c(pmses_2, mse(Avg_ExtentTS_Test, ridge_predictions_2[,i]))
  pmses_3 <- c(pmses_3, mse(Avg_ExtentTS_Test, ridge_predictions_3[,i]))
}

lambda_ridge <- ridge_train$lambda[which.min(pmses)]
pmse_ridge <- pmses[which.min(pmses)]
lambda_ridge_2 <- ridge_train_2$lambda[which.min(pmses_2)]
pmse_ridge_2 <- pmses_2[which.min(pmses_2)]
lambda_ridge_3 <- ridge_train_3$lambda[which.min(pmses_3)] #which.min
pmse_ridge_3 <- pmses_3[which.min(pmses_3)]

lambda_ridge
```

```
## [1] 0.1653056
```

```
pmse_ridge
```

```
## [1] 0.5345
```

```
lambda_ridge_2
```

```
## [1] 0.3818774
```

```
pmse_ridge_2
```

```
## [1] 5.203632
```

```
lambda_ridge_3
```

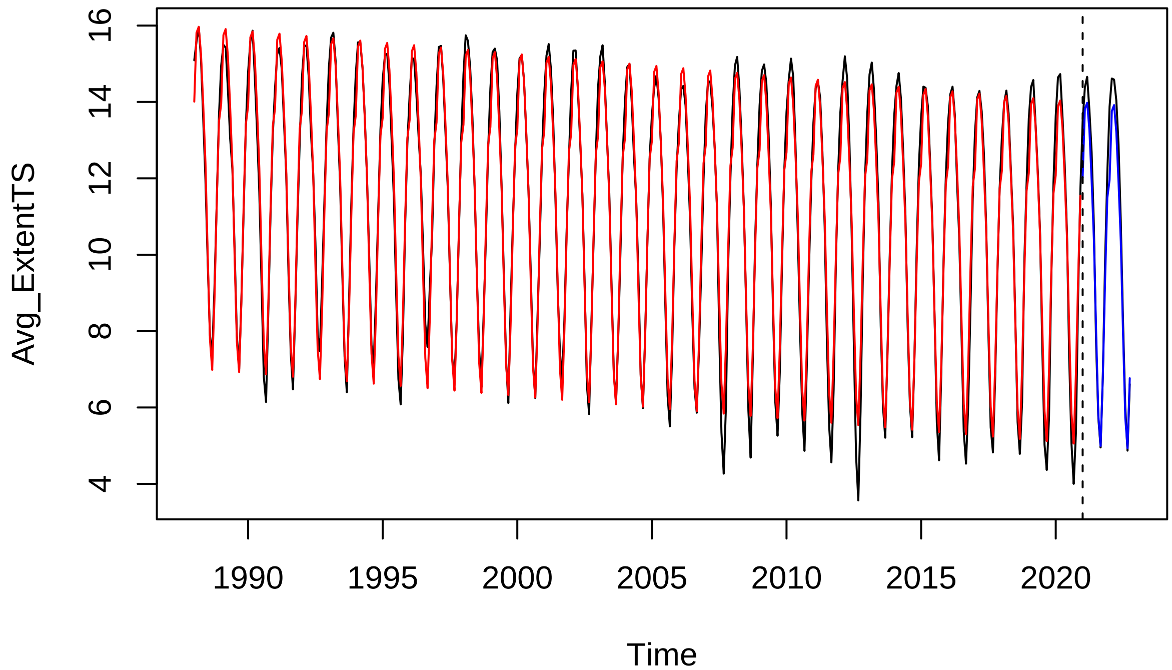
```
## [1] 0.3818774
```

```
pmse_ridge_3
```

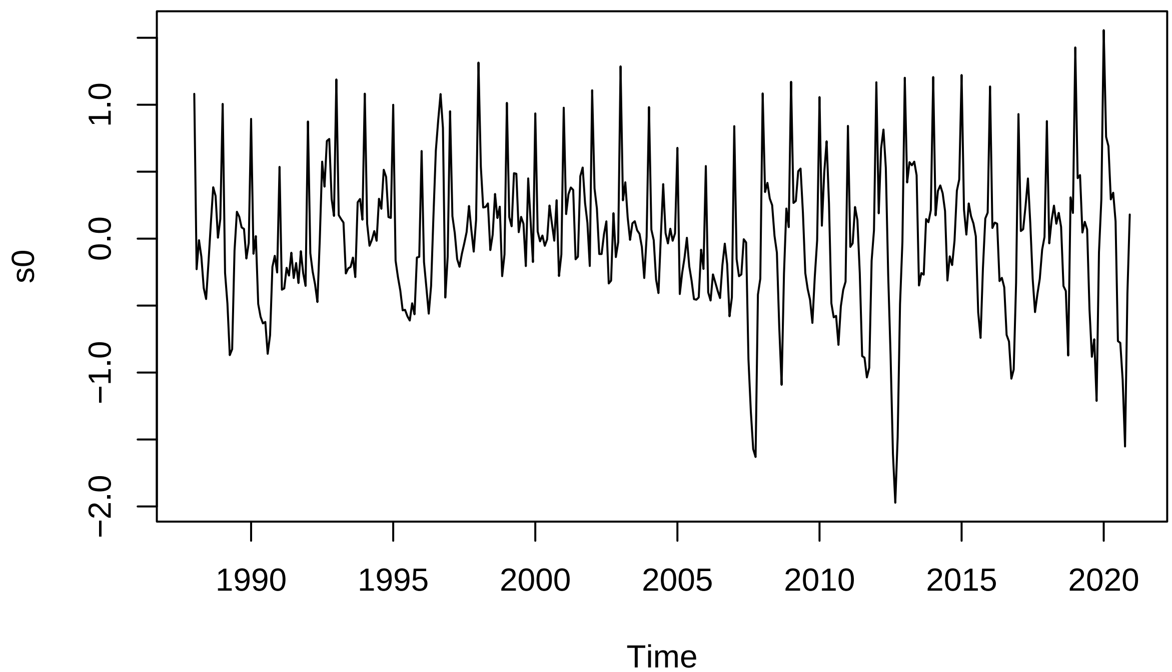
```
## [1] 5.395475
```

We see that the degree 1 polynomial gives the lowest MSE.

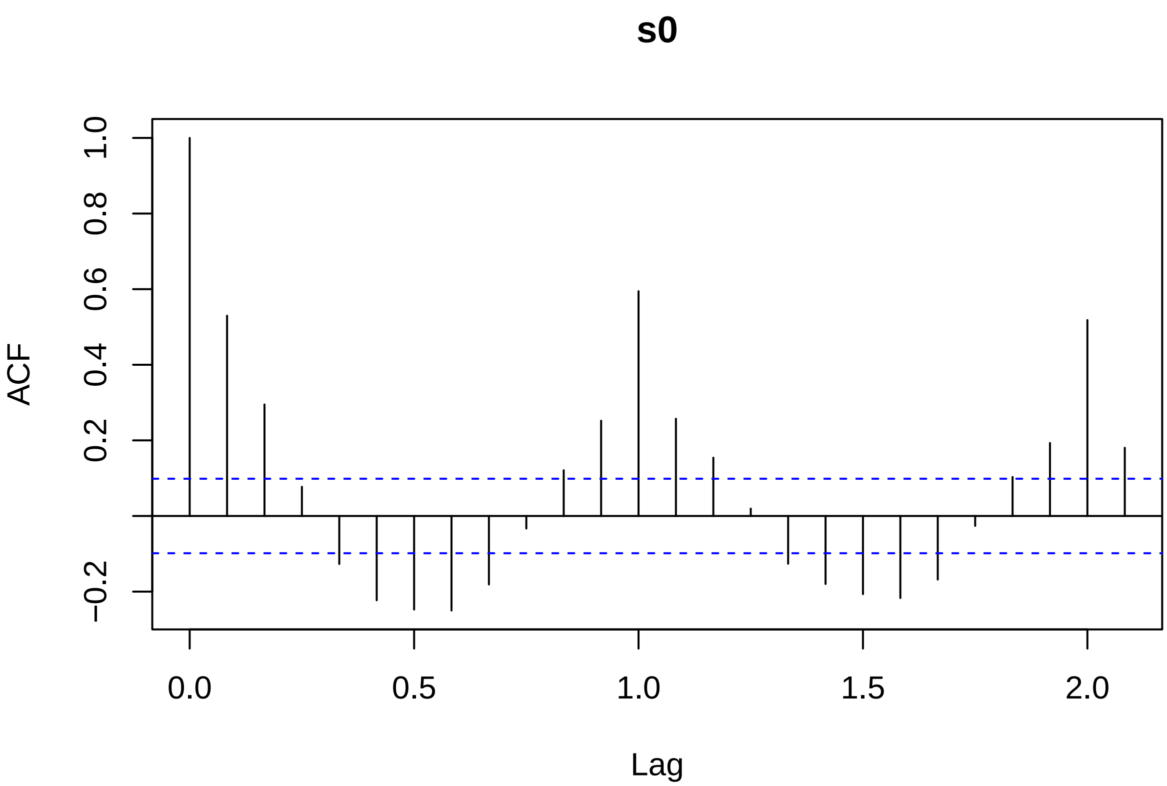
```
#degree 1 polynomial
tim <- as.vector(time(Avg_ExtentTS_Train))
season <- factor(cycle(Avg_ExtentTS_Train))
X <- model.matrix(as.vector(Avg_ExtentTS_Train)~tim+season)
newX <- model.matrix(as.vector(Avg_ExtentTS_Test)~as.vector(time(Avg_ExtentTS_Test))+factor(cycle(Avg_E

ridge <- glmnet(X, as.vector(Avg_ExtentTS_Train), alpha=0, lambda=0.1653056)
ridge_fitted <- predict(ridge, X)

plot(Avg_ExtentTS)
points(time(Avg_ExtentTS_Train), ridge_fitted, type='l', col='red')
points(time(Avg_ExtentTS_Test), predict(ridge, newX), type='l', col='blue')
abline(v=2021, lty="dashed")
```



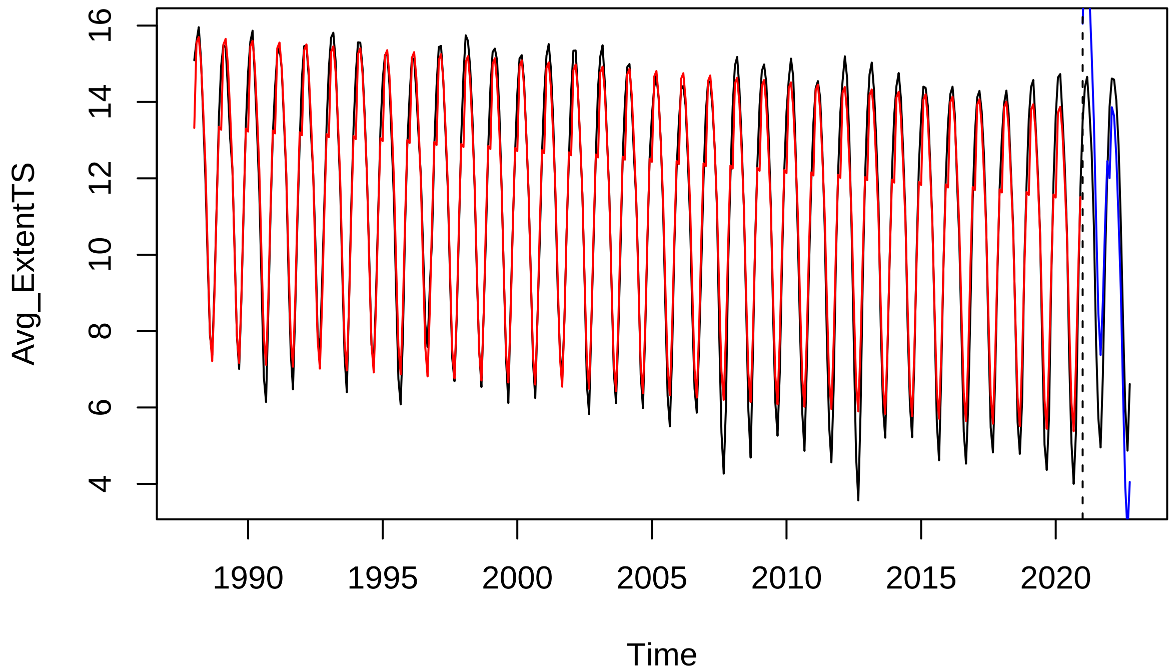

```r
ridge_residuals <- Avg_ExtentTS_Train - ridge_fitted
plot(ridge_residuals, type="l")
```



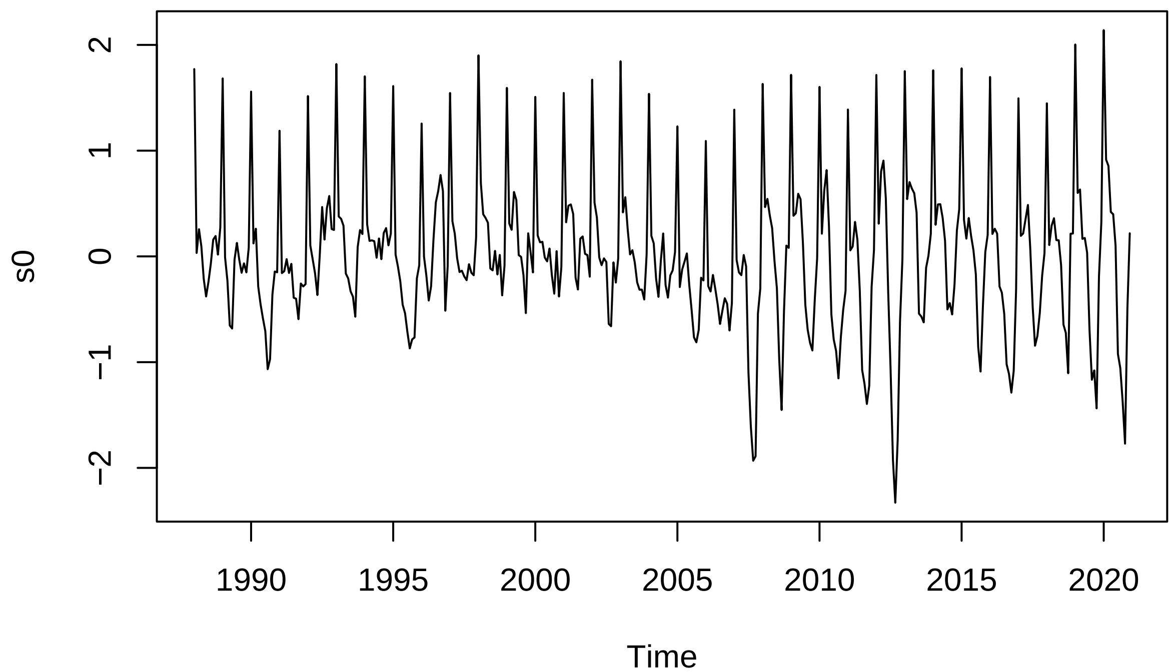

```r
acf(ridge_residuals)
```



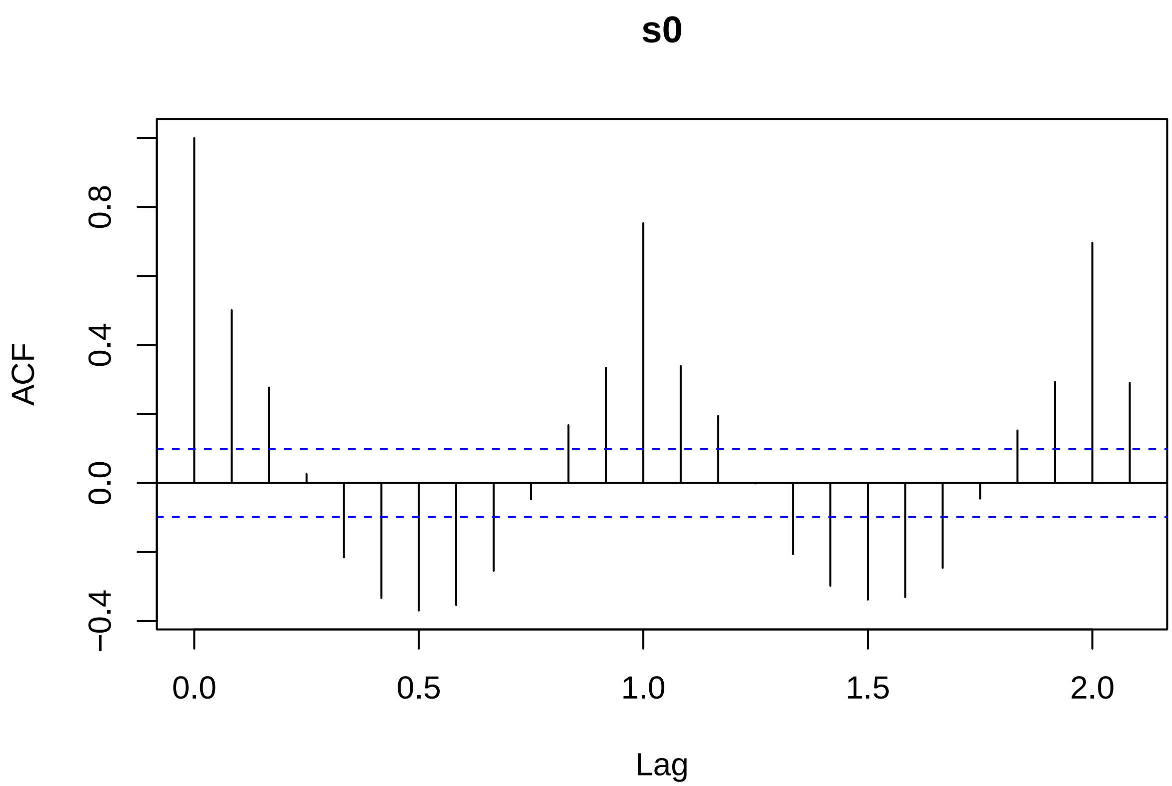

```
#degree 2 polynomial
tim <- as.vector(time(Avg_ExtentTS_Train))
season <- factor(cycle(Avg_ExtentTS_Train))
X <- model.matrix(as.vector(Avg_ExtentTS_Train)~poly(tim,2)+season)
newX <- model.matrix(as.vector(Avg_ExtentTS_Test)~poly(as.vector(time(Avg_ExtentTS_Test)),2)+factor(cycl

ridge <- glmnet(X, as.vector(Avg_ExtentTS_Train), alpha=0, lambda=0.3818774)
ridge_fitted <- predict(ridge, X)

plot(Avg_ExtentTS)
points(time(Avg_ExtentTS_Train), ridge_fitted, type='l', col='red')
points(time(Avg_ExtentTS_Test), predict(ridge, newX), type='l', col='blue')
abline(v=2021, lty="dashed")
```




```
ridge_residuals <- Avg_ExtentTS_Train - ridge_fitted
plot(ridge_residuals, type="l")
```




```
acf(ridge_residuals)
```



Lasso

```
library(glmnet)

# fit to training data
tim <- as.vector(time(Avg_ExtentTS_Train))
season <- factor(cycle(Avg_ExtentTS_Train))
X <- model.matrix(as.vector(Avg_ExtentTS_Train)~tim+season)
X_2 <- model.matrix(as.vector(Avg_ExtentTS_Train)~poly(tim,2)+season)
X_3 <- model.matrix(as.vector(Avg_ExtentTS_Train)~poly(tim,3)+season)
lasso_train <- glmnet(X, as.vector(Avg_ExtentTS_Train), alpha=1)
lasso_train_2 <- glmnet(X_2, as.vector(Avg_ExtentTS_Train), alpha=1)
lasso_train_3 <- glmnet(X_3, as.vector(Avg_ExtentTS_Train), alpha=1)

# compute mse on training data for each value of lambda
lasso_train_fitted <- predict(lasso_train, X)
lasso_train_fitted_2 <- predict(lasso_train_2, X_2)
lasso_train_fitted_3 <- predict(lasso_train_3, X_3)

mses <- c()
mses_2 <- c()
mses_3 <- c()
for(i in 1:67) {
  mses <- c(mses, mse(Avg_ExtentTS_Train, lasso_train_fitted[,i]))
}
for(i in 1:68) {
  mses_2 <- c(mses_2, mse(Avg_ExtentTS_Train, lasso_train_fitted_2[,i]))
  mses_3 <- c(mses_3, mse(Avg_ExtentTS_Train, lasso_train_fitted_3[,i]))
```

```
}

min10_mses <- head(sort(mses), 10)
min10_mses_2 <- head(sort(mses_2), 10)
min10_mses_3 <- head(sort(mses_3), 10)

lasso_train_lambdas <- c()
lasso_train_lambdas_2 <- c()
lasso_train_lambdas_3 <- c()

for(m in min10_mses) {
  lasso_train_lambdas <- c(lasso_train_lambdas, lasso_train$lambda[which(mses==m)])
}
for(m in min10_mses_2) {
  lasso_train_lambdas_2 <- c(lasso_train_lambdas_2, lasso_train_2$lambda[which(mses_2==m)])
}
for(m in min10_mses_3) {
  lasso_train_lambdas_3 <- c(lasso_train_lambdas_3, lasso_train_3$lambda[which(mses_3==m)])
}
```

```
# retrain using lambdas that gave the 10 best fits
lasso_train <- glmnet(X, as.vector(Avg_ExtentTS_Train), alpha=0, lambda=lasso_train_lambdas)
lasso_train_2 <- glmnet(X_2, as.vector(Avg_ExtentTS_Train), alpha=0, lambda=lasso_train_lambdas_2)
lasso_train_3 <- glmnet(X_3, as.vector(Avg_ExtentTS_Train), alpha=0, lambda=lasso_train_lambdas_3)

# predict the test set
tim <- as.vector(time(Avg_ExtentTS_Test))
season <- factor(cycle(Avg_ExtentTS_Test))
X <- model.matrix(as.vector(Avg_ExtentTS_Test)~tim+season)
X_2 <- model.matrix(as.vector(Avg_ExtentTS_Test)~poly(tim,2)+season)
X_3 <- model.matrix(as.vector(Avg_ExtentTS_Test)~poly(tim,3)+season)
lasso_predictions <- predict(lasso_train, X)
lasso_predictions_2 <- predict(lasso_train_2, X_2)
lasso_predictions_3 <- predict(lasso_train_3, X_3)

# compute pmse on test set
pmses <- c()
pmses_2 <- c()
pmses_3 <- c()
for(i in 1:10) {
  pmses <- c(pmses, mse(Avg_ExtentTS_Test, lasso_predictions[,i]))
  pmses_2 <- c(pmses_2, mse(Avg_ExtentTS_Test, lasso_predictions_2[,i]))
  pmses_3 <- c(pmses_3, mse(Avg_ExtentTS_Test, lasso_predictions_3[,i]))
}

lambda_lasso <- lasso_train$lambda[which.min(pmses)]
pmse_lasso <- pmses[which.min(pmses)]
lambda_lasso_2 <- lasso_train_2$lambda[which.min(pmses_2)]
pmse_lasso_2 <- pmses_2[which.min(pmses_2)]
lambda_lasso_3 <- lasso_train_3$lambda[which.min(pmses_3)]
pmse_lasso_3 <- pmses_3[which.min(pmses_3)]

lambda_lasso
```

```
## [1] 0.003561401
```

```
pmse_lasso
```

```
## [1] 0.368189
```

```
lambda_lasso_2
```

```
## [1] 0.007496408
```

```
pmse_lasso_2
```

```
## [1] 6.69867
```

```
lambda_lasso_3
```

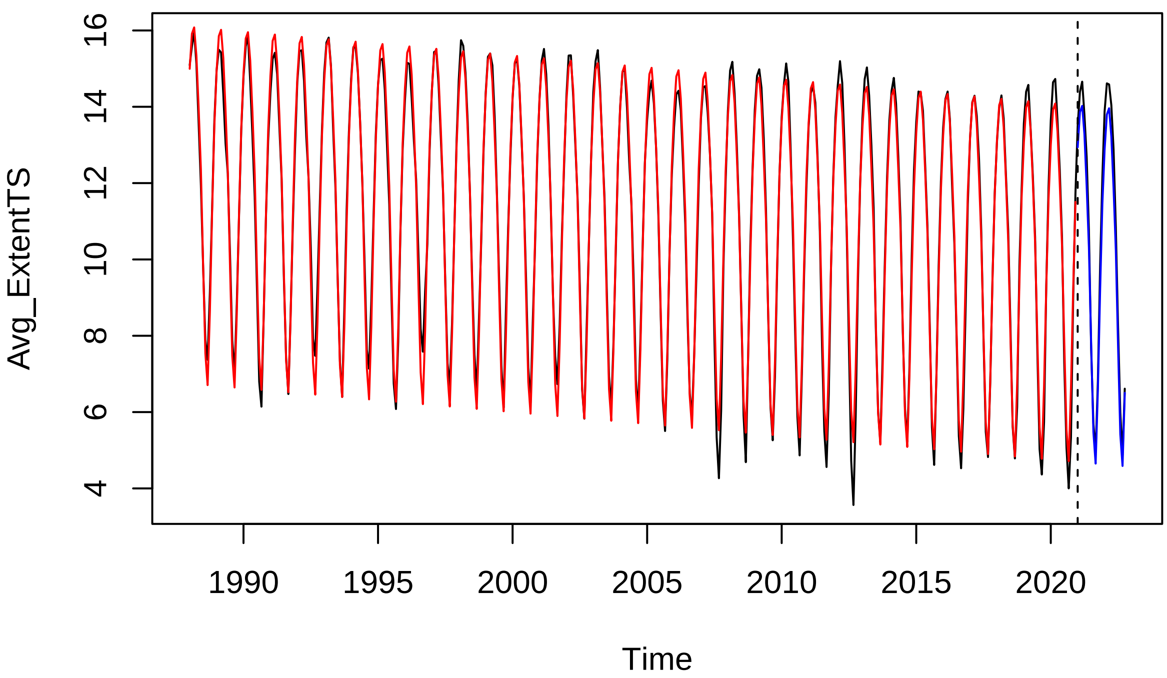
```
## [1] 0.007496408
```

```
pmse_lasso_3
```

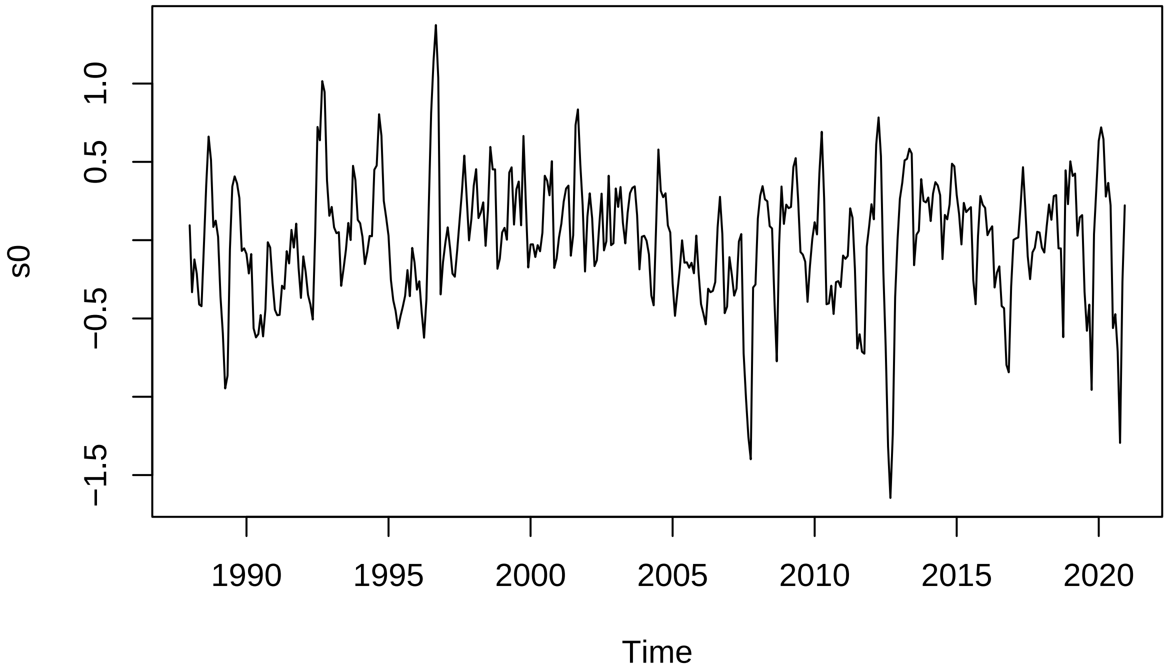
```
## [1] 6.85394
```

We see that the degree 1 polynomial gives the lowest MSE.

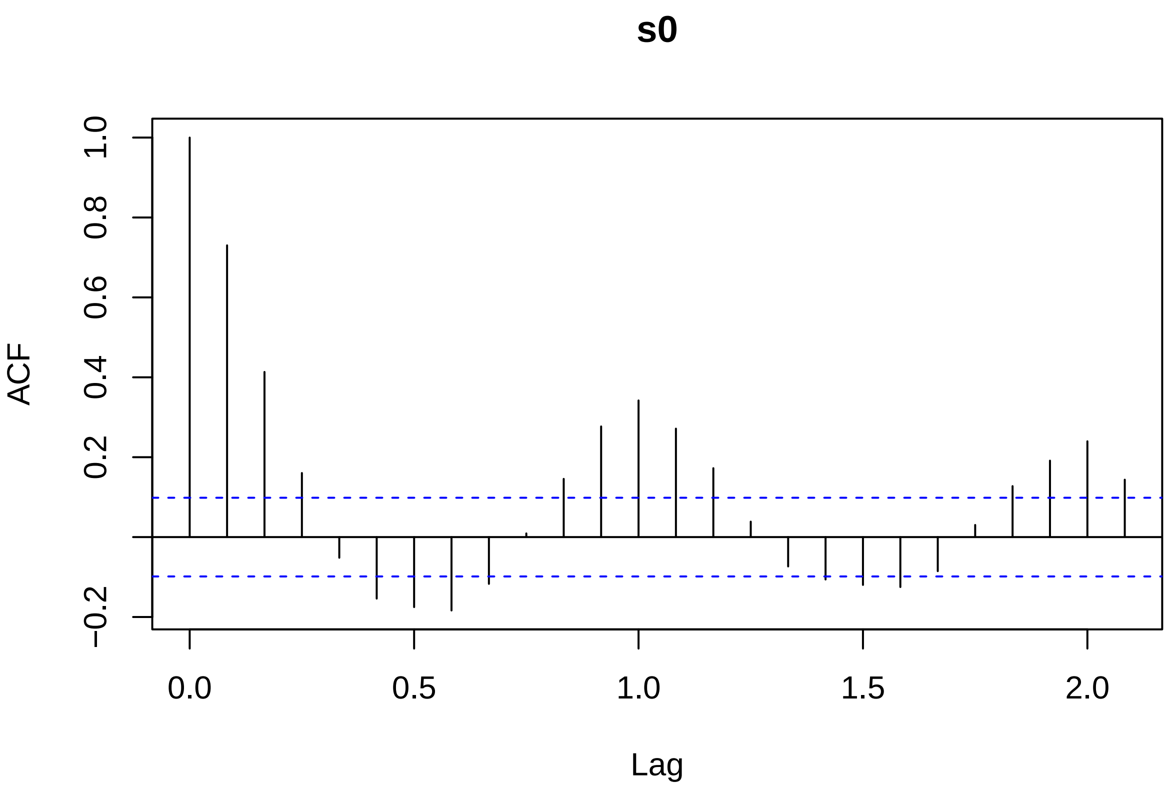
```
#degree 1 polynomial
tim <- as.vector(time(Avg_ExtentTS_Train))
season <- factor(cycle(Avg_ExtentTS_Train))
X <- model.matrix(as.vector(Avg_ExtentTS_Train)~tim+season)
newX <- model.matrix(as.vector(Avg_ExtentTS_Test)~as.vector(time(Avg_ExtentTS_Test))+factor(cycle(Avg_E

lasso <- glmnet(X, as.vector(Avg_ExtentTS_Train), alpha=1, lambda=0.003561401)
lasso_fitted <- predict(lasso, X)

plot(Avg_ExtentTS)
points(time(Avg_ExtentTS_Train), lasso_fitted, type='l', col='red')
points(time(Avg_ExtentTS_Test), predict(lasso, newX), type='l', col='blue')
abline(v=2021, lty="dashed")
```



```
lasso_residuals <- Avg_ExtentTS_Train - lasso_fitted
plot(lasso_residuals, type="l")
```




```
acf(lasso_residuals)
```



Elastic Net

```r
library(glmnet)
alpha_seq <- seq(0.1, 0.9, by=0.1)

en_train_min_lamdas <- c()
en_train_min_lamdas_2 <- c()
en_train_min_lamdas_3 <- c()
min_pmses <- c()
min_pmses_2 <- c()
min_pmses_3 <- c()

for(a in alpha_seq){
  # fit to training data
  tim <- as.vector(time(Avg_ExtentTS_Train))
  season <- factor(cycle(Avg_ExtentTS_Train))
  X <- model.matrix(as.vector(Avg_ExtentTS_Train)~tim+season)
  X_2 <- model.matrix(as.vector(Avg_ExtentTS_Train)~poly(tim,2)+season)
  X_3 <- model.matrix(as.vector(Avg_ExtentTS_Train)~poly(tim,3)+season)
  en_train <- glmnet(X, as.vector(Avg_ExtentTS_Train), alpha=a)
  en_train_2 <- glmnet(X_2, as.vector(Avg_ExtentTS_Train), alpha=a)
  en_train_3 <- glmnet(X_3, as.vector(Avg_ExtentTS_Train), alpha=a)

  # compute mse on training data for each value of lambda
  en_train_fitted <- predict(en_train, X)
  en_train_fitted_2 <- predict(en_train_2, X_2)
  en_train_fitted_3 <- predict(en_train_3, X_3)

  mses <- c()
  mses_2 <- c()
  mses_3 <- c()
  for(i in 1:length(en_train$lambda)) {
    mses <- c(mses, mse(Avg_ExtentTS_Train, en_train_fitted[,i]))
  }
  for(i in 1:length(en_train_2$lambda)) {
    mses_2 <- c(mses_2, mse(Avg_ExtentTS_Train, en_train_fitted_2[,i]))
  }
  for(i in 1:length(en_train_3$lambda)) {
    mses_3 <- c(mses_3, mse(Avg_ExtentTS_Train, en_train_fitted_3[,i]))
  }

  min10_mses <- head(sort(mses), 10)
  min10_mses_2 <- head(sort(mses_2), 10)
  min10_mses_3 <- head(sort(mses_3), 10)

  en_train_lambdas <- c()
  en_train_lambdas_2 <- c()
  en_train_lambdas_3 <- c()

  for(m in min10_mses) {
    en_train_lambdas <- c(en_train_lambdas, en_train$lambda[which(mses==m)])
  }
  for(m in min10_mses_2) {
    en_train_lambdas_2 <- c(en_train_lambdas_2, en_train_2$lambda[which(mses_2==m)])
```

```
  }
  for(m in min10_mses_3) {
    en_train_lambdas_3 <- c(en_train_lambdas_3, en_train_3$lambda[which(mses_3==m)])
  }


  # retrain using lambdas that gave the 10 best fits
  en_train <- glmnet(X, as.vector(Avg_ExtentTS_Train), alpha=a, lambda=en_train_lambdas)
  en_train_2 <- glmnet(X_2, as.vector(Avg_ExtentTS_Train), alpha=a, lambda=en_train_lambdas_2)
  en_train_3 <- glmnet(X_3, as.vector(Avg_ExtentTS_Train), alpha=a, lambda=en_train_lambdas_3)

  # predict the test set
  tim <- as.vector(time(Avg_ExtentTS_Test))
  season <- factor(cycle(Avg_ExtentTS_Test))
  X <- model.matrix(as.vector(Avg_ExtentTS_Test)~tim+season)
  X_2 <- model.matrix(as.vector(Avg_ExtentTS_Test)~poly(tim,2)+season)
  X_3 <- model.matrix(as.vector(Avg_ExtentTS_Test)~poly(tim,3)+season)
  en_predictions <- predict(en_train, X)
  en_predictions_2 <- predict(en_train_2, X_2)
  en_predictions_3 <- predict(en_train_3, X_3)

  # compute pmse on test set
  pmses <- c()
  pmses_2 <- c()
  pmses_3 <- c()
  for(i in 1:10) {
    pmses <- c(pmses, mse(Avg_ExtentTS_Test, en_predictions[,i]))
    pmses_2 <- c(pmses_2, mse(Avg_ExtentTS_Test, en_predictions_2[,i]))
    pmses_3 <- c(pmses_3, mse(Avg_ExtentTS_Test, en_predictions_3[,i]))
  }

  min_pmses <- c(min_pmses, pmses[which.min(pmses)])
  min_pmses_2 <- c(min_pmses_2, pmses_2[which.min(pmses_2)])
  min_pmses_3 <- c(min_pmses_3, pmses_3[which.min(pmses_3)])

  en_train_min_lamdas <- c(en_train_min_lamdas, en_train$lambda[which.min(pmses)])
  en_train_min_lamdas_2 <- c(en_train_min_lamdas_2, en_train_2$lambda[which.min(pmses_2)])
  en_train_min_lamdas_3 <- c(en_train_min_lamdas_3, en_train_3$lambda[which.min(pmses_3)])
}

pmse_en <- min_pmses[which.min(min_pmses)]
alpha_en <- alpha_seq[which.min(min_pmses)]
lambda_en <- en_train_min_lamdas[which.min(min_pmses)]
pmse_en_2 <- min_pmses_2[which.min(min_pmses_2)]
lambda_en_2 <- en_train_min_lamdas_2[which.min(min_pmses_2)]
alpha_en_2 <- alpha_seq[which.min(min_pmses_2)]
pmse_en_3 <- min_pmses_3[which.min(min_pmses_3)]
lambda_en_3 <- en_train_min_lamdas_3[which.min(min_pmses_3)]
alpha_en_3 <- alpha_seq[which.min(min_pmses_3)]

pmse_en
```

```
## [1] 0.3659694
```

```
lambda_en
```

```
## [1] 0.004342926
```

```
alpha_en
```

```
## [1] 0.9
```

```
pmse_en_2
```

```
## [1] 6.537295
```

```
lambda_en_2
```

```
## [1] 0.008329342
```

```
alpha_en_2
```

```
## [1] 0.9
```

```
pmse_en_3
```

```
## [1] 6.668906
```

```
lambda_en_3
```

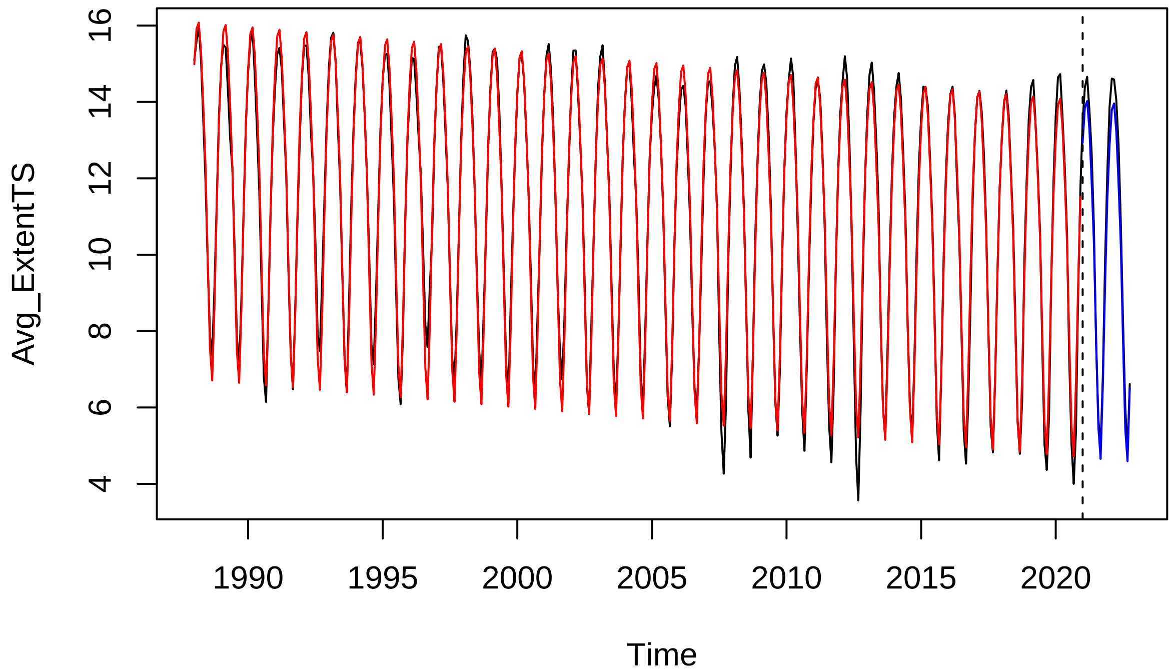
```
## [1] 0.008329342
```

```
alpha_en_3
```

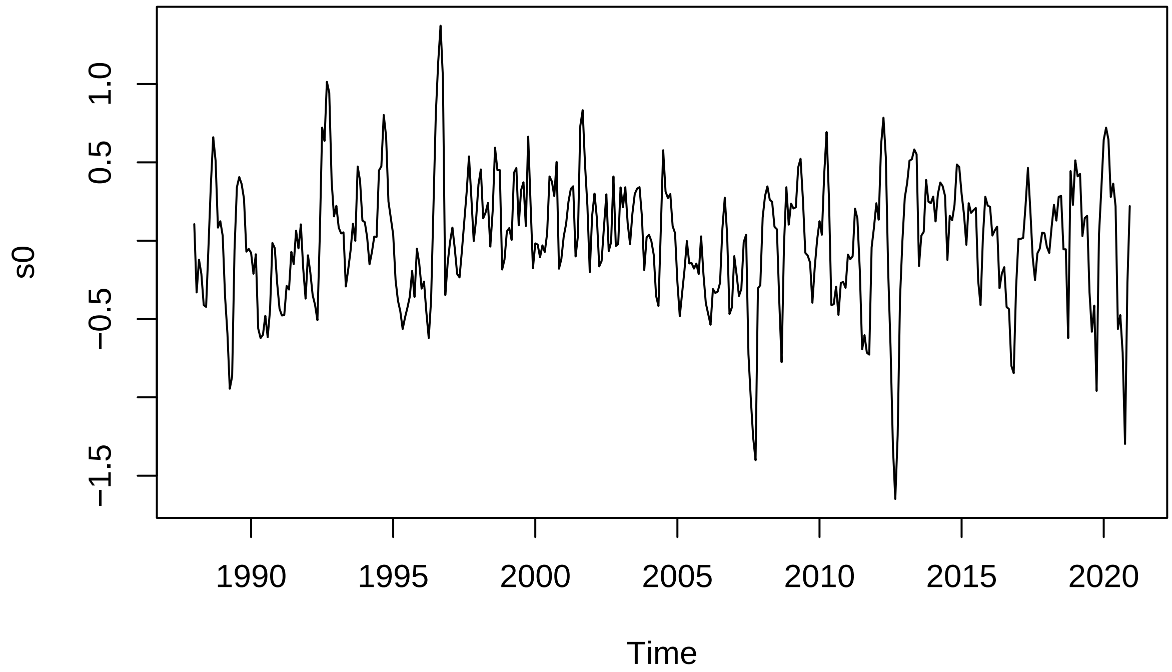
```
## [1] 0.9
```

We see that the degree 1 polynomial gives the lowest MSE.

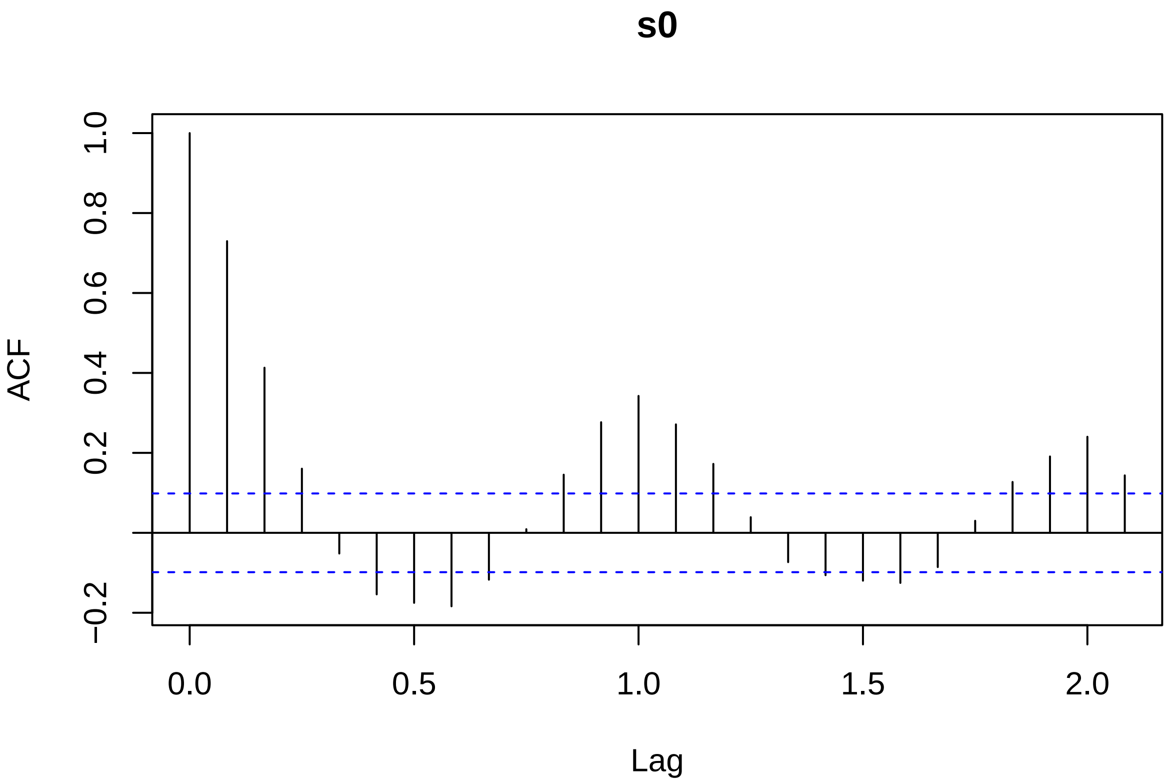
```
#degree 1 polynomial
tim <- as.vector(time(Avg_ExtentTS_Train))
season <- factor(cycle(Avg_ExtentTS_Train))
X <- model.matrix(as.vector(Avg_ExtentTS_Train)~tim+season)
newX <- model.matrix(as.vector(Avg_ExtentTS_Test)~as.vector(time(Avg_ExtentTS_Test))+factor(cycle(Avg_E

en <- glmnet(X, as.vector(Avg_ExtentTS_Train), alpha=0.9, lambda=0.004342926)
en_fitted <- predict(en, X)

plot(Avg_ExtentTS)
points(time(Avg_ExtentTS_Train), en_fitted, type='l', col='red')
points(time(Avg_ExtentTS_Test), predict(en, newX), type='l', col='blue')
abline(v=2021, lty="dashed")
```




```r
en_residuals <- Avg_ExtentTS_Train - en_fitted
plot(en_residuals, type="l")
```




```r
acf(en_residuals)
```




```
# creating a table to summarise regression results
pmse <- c(pmse_mlr, pmse_mlr_2, pmse_mlr_3, pmse_ridge, pmse_ridge_2, pmse_ridge_3, pmse_lasso, pmse_la
time_degree <- rep(c(1,2,3), 4)
lambda <- c("", "", "", lambda_ridge, lambda_ridge_2, lambda_ridge_3, lambda_lasso, lambda_lasso_2, laml
alpha <- c("", "", "", "", "", "", "", "", "", alpha_en, alpha_en_2, alpha_en_3)
regression <- as.data.frame(matrix(c(pmse, time_degree, lambda, alpha), ncol=4),
                         row.names=c("", "MLR", "", "", "Ridge", "", "", "Lasso", "", "", "Elastic Net"
colnames(regression) <- c("Prediction MSE", "Degree of Time Polynomial", "Lambda", "Alpha")

regression_table <- hux(regression, add_rownames = "")

regression_table %>%
  set_number_format(3) %>%
  set_align(everywhere, everywhere, "center") %>%
  set_bottom_border(c(1,4,7,10), everywhere)
```

Holt-Winters

Try to remove non-stationarity using exponential smoothing, double exponential smoothing, additive HW, and multiplicative HW.

Exponential Smoothing

```
es <- HoltWinters(Avg_ExtentTS_Train, gamma = FALSE , beta = FALSE)
predict_es = predict(es, n.ahead=22)
pmse_es <- mse(Avg_ExtentTS_Test, predict_es)
```

	Prediction MSE	Degree of Time Polynomial	Lambda	Alpha
X	0.366	1.000		
MLR	0.455	2.000		
X0.100	0.233	3.000		
X0.200	0.535	1.000	0.165	
Ridge	5.204	2.000	0.382	
X0.300	5.395	3.000	0.382	
X0.400	0.368	1.000	0.004	
Lasso	6.699	2.000	0.007	
X0.500	6.854	3.000	0.007	
X0.600	0.366	1.000	0.004	0.900
Elastic.Net	6.537	2.000	0.008	0.900
X0.700	6.669	3.000	0.008	0.900

Double Exponential Smoothing

```
des <- HoltWinters(Avg_ExtentTS_Train, gamma = FALSE)
predict_des = predict(des, n.ahead=22)
pmse_des <- mse(Avg_ExtentTS_Test, predict_des)
```

No Trend

```
no_trend <- HoltWinters(Avg_ExtentTS_Train, beta = FALSE)
predict_no_trend = predict(no_trend, n.ahead=22)
pmse_no_trend <- mse(Avg_ExtentTS_Test, predict_no_trend)
```

Additive Holt-Winters

```
additive <- HoltWinters(Avg_ExtentTS_Train, seasonal = "additive")
predict_additive = predict(additive, n.ahead=22)
pmse_additive <- mse(Avg_ExtentTS_Test, predict_additive)
```

Multiplicative Holt-Winters

```
multiplicative <- HoltWinters(Avg_ExtentTS_Train, seasonal = "multiplicative")
predict_multiplicative = predict(multiplicative, n.ahead=22)
pmse_multiplicative <- mse(Avg_ExtentTS_Test, predict_additive)
```

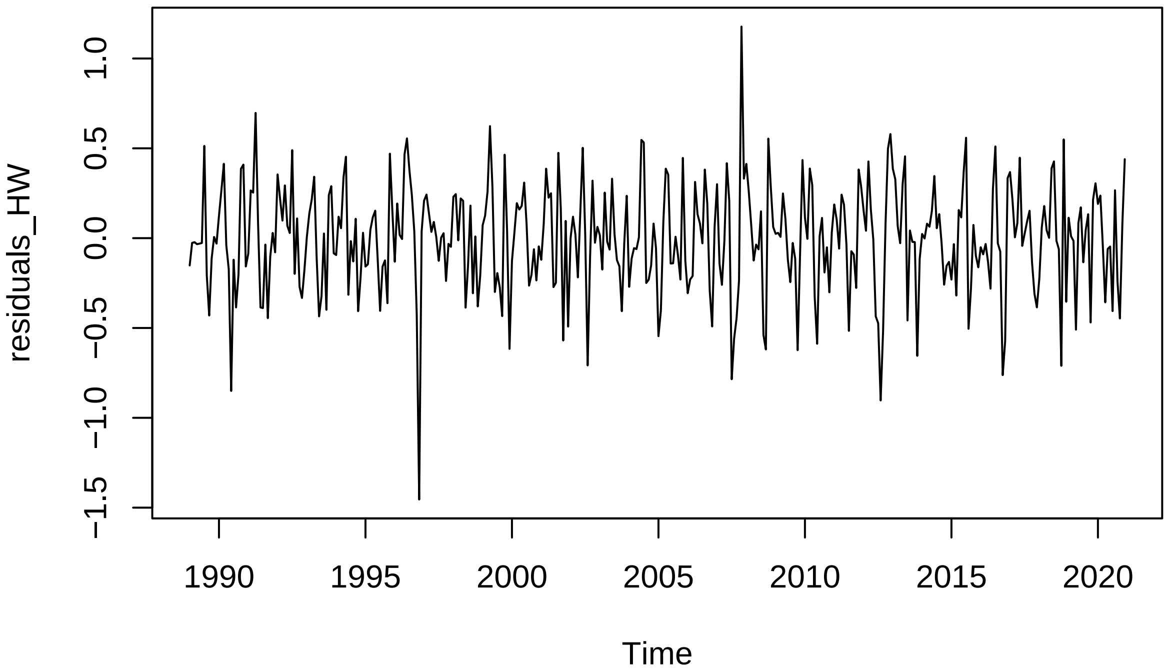
```
pmse <- c(pmse_es, pmse_des, pmse_no_trend, pmse_additive, pmse_multiplicative)
HW <- as.data.frame(matrix(pmse, nrow=1),
                         row.names=c("Prediction MSE"))
colnames(HW) <- c("\nExponential\nSmoothing", "Double\nExponential\nSmoothing", "HW\nWithout\nTrend", "
```

```r
HW_table <- hux(HW, add_rownames = "")

HW_table %>%
  set_number_format(3) %>%
  set_align(everywhere, everywhere, "center") %>%
  set_bottom_border(1, everywhere)
```

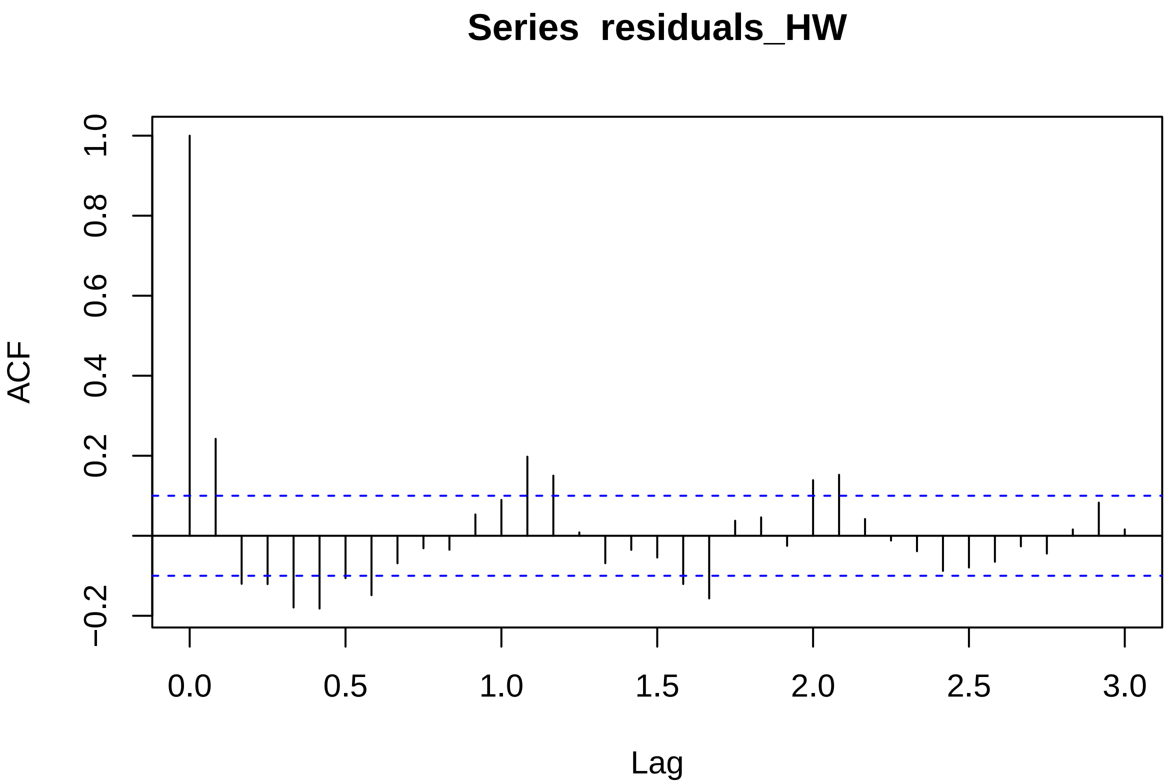
	ExponentialSmoothing	DoubleExponentialSmoothing	HWWithoutTrend	AdditiveHW	MultiplicativeHW
Prediction MSE	13.561	14.468	0.459	0.778	0.778

Best of HW models seems to be model with no trend. We fit this model to the entire data.

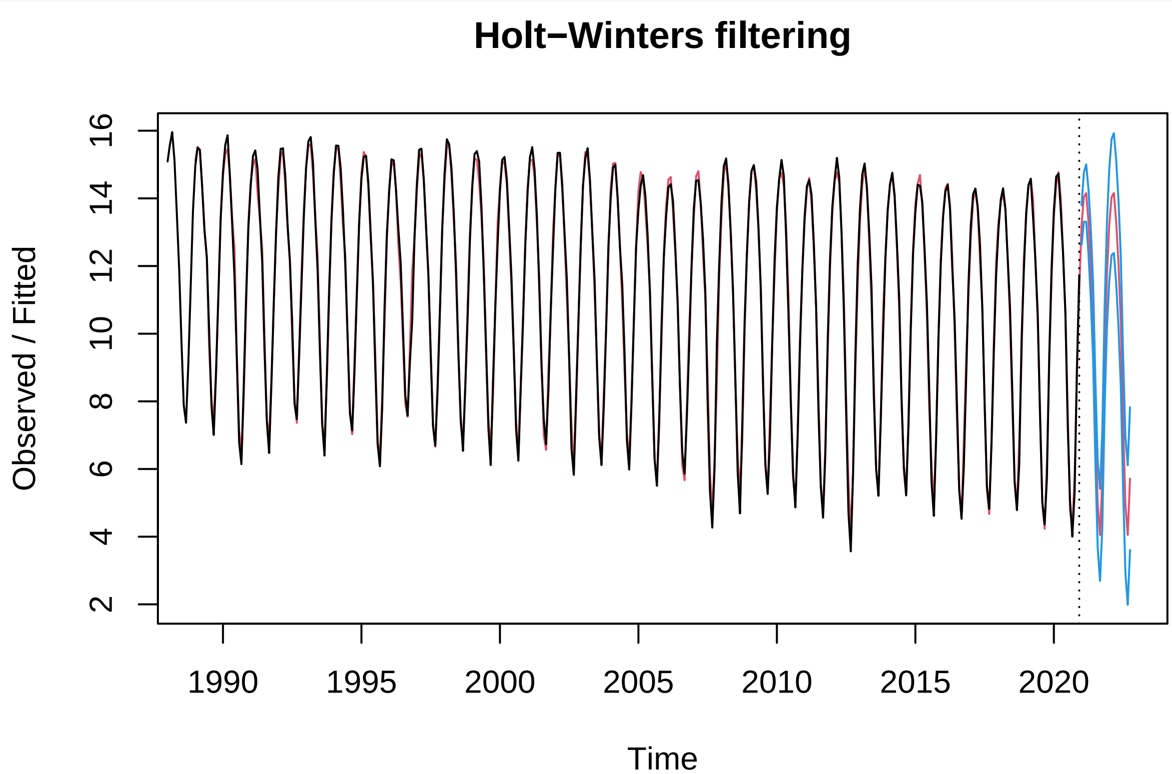
```r
residuals_HW <- as.vector(Avg_ExtentTS_Train[which(time(Avg_ExtentTS_Train)>=1989)]) - no_trend$fitted[
plot(residuals_HW, type="l")
```



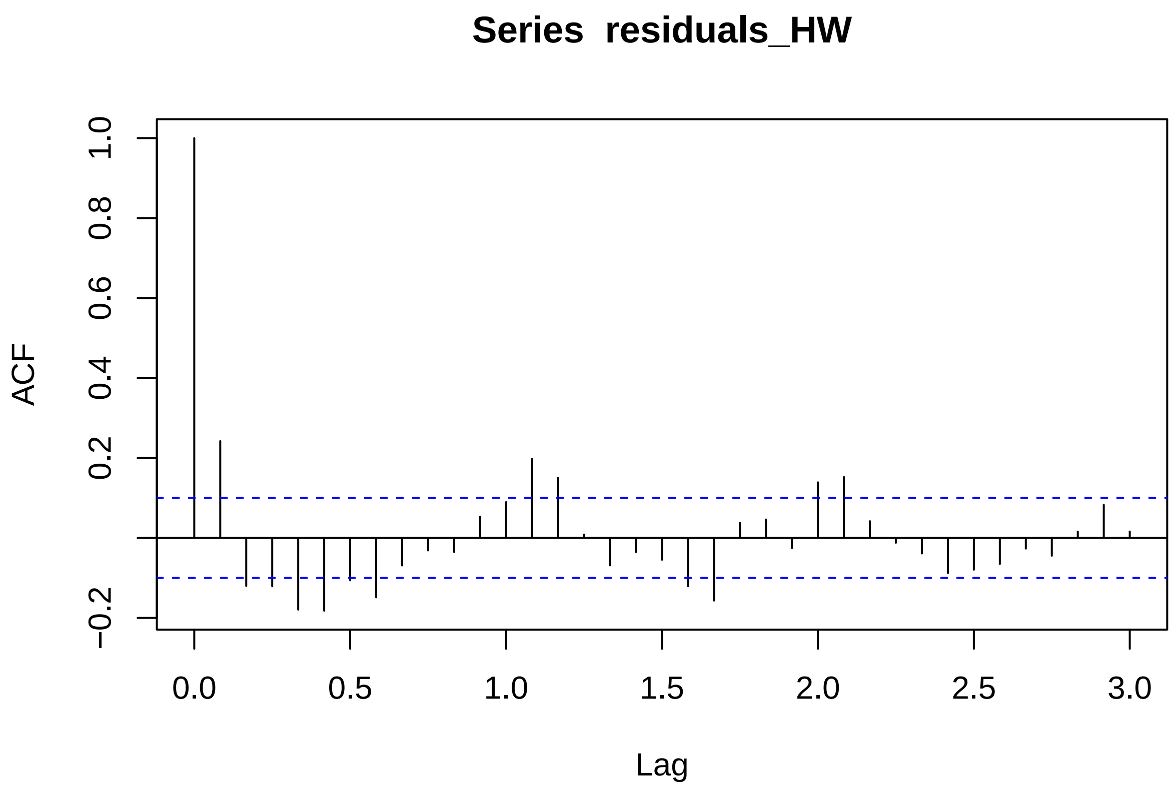

```r
acf(residuals_HW, lag.max=36)
```



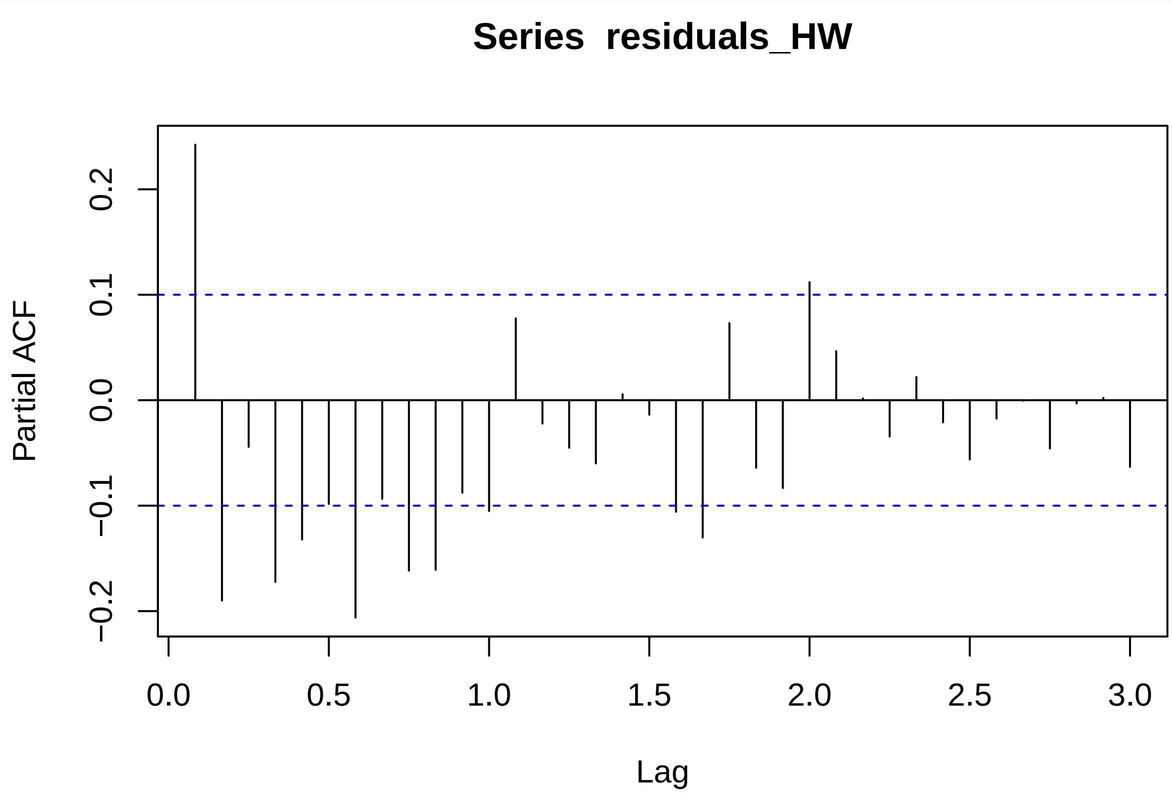

```
predict_no_trend <- predict(no_trend, n.ahead=22, prediction.interval = TRUE , level=0.95)
plot(no_trend, predict_no_trend)
```



```
acf(residuals_HW, lag.max=36)
```



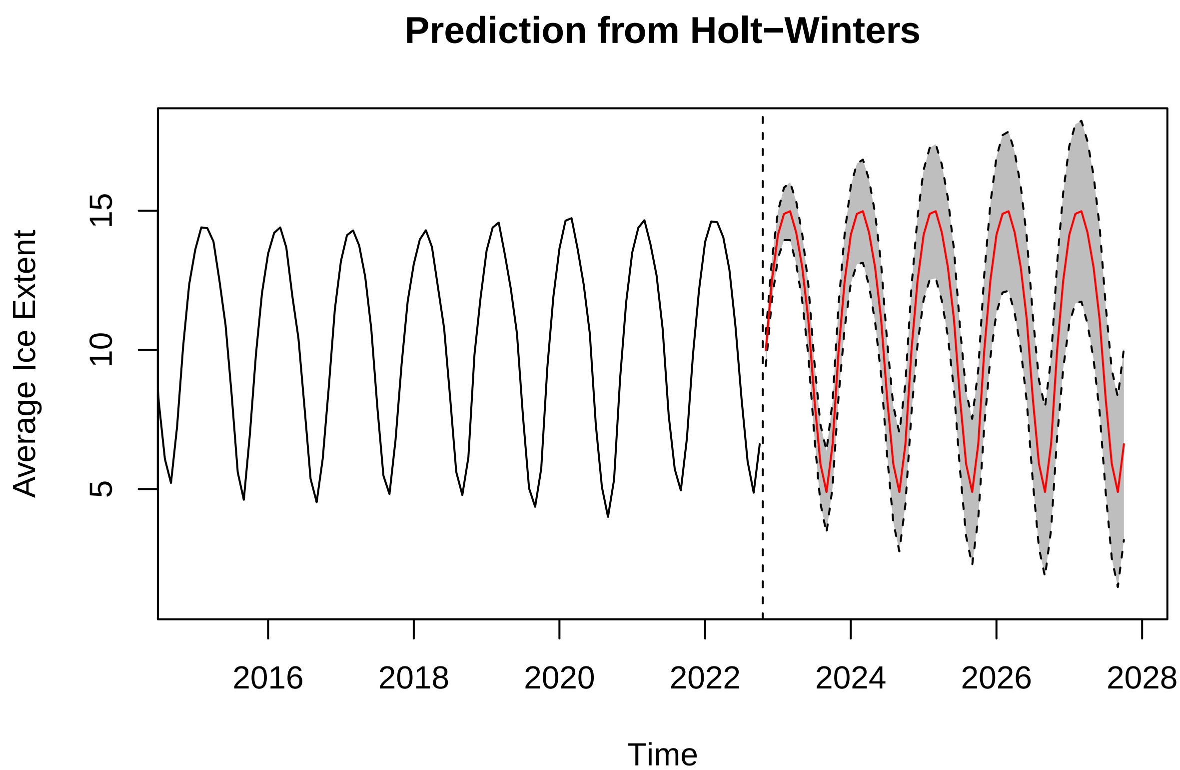

```
pacf(residuals_HW, lag.max=36)
```



Note here that a more thoporough analysis might have applied Box-Jenkins on the HW residuals, because a case could be made that they are stationary.

Prediction With best HW:

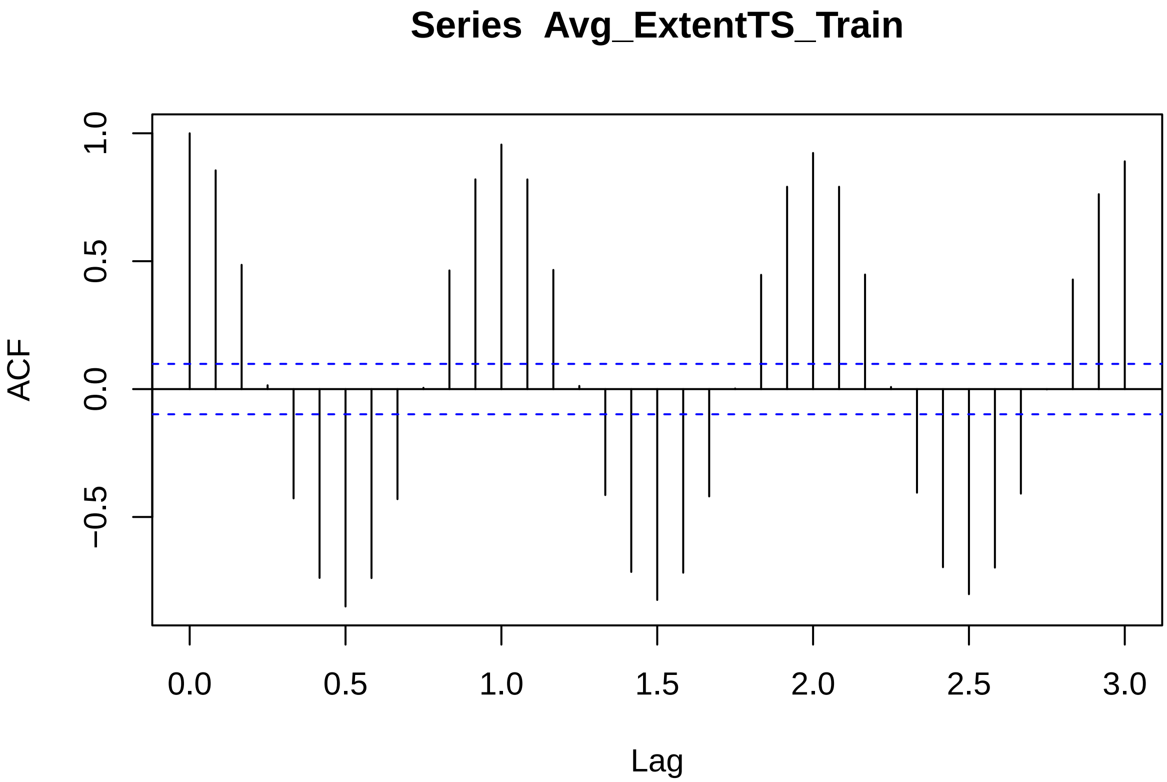
```
# Getting predictions for HW and plotting
HW <- HoltWinters(Avg_ExtentTS, beta = FALSE)
predict_HW <- predict(HW, n.ahead=60, prediction.interval = TRUE , level=0.95)

plot(Avg_ExtentTS , xlim = c(2015 , 2027+10/12), ylim=c(1,18), ylab="Average Ice Extent", main="Predict

#The three lines below plot the prediction interval in a grey scale
x = c(time(predict_HW[,"upr"]) , rev(time(predict_HW[,"upr"])))
y = c(predict_HW[,"upr"] , rev(predict_HW[,"lwr"]))
polygon(x, y, col="grey", border=NA)

#The three line below add the predicted values and highlight the borders of the prediction interval
lines(predict_HW[,"upr"], col="black" , lty=2)
lines(predict_HW[,"lwr"], col="black", lty=2)
lines(predict_HW[, "fit"] , col="red")
abline(v=2022+9.5/12, lty="dashed")
```



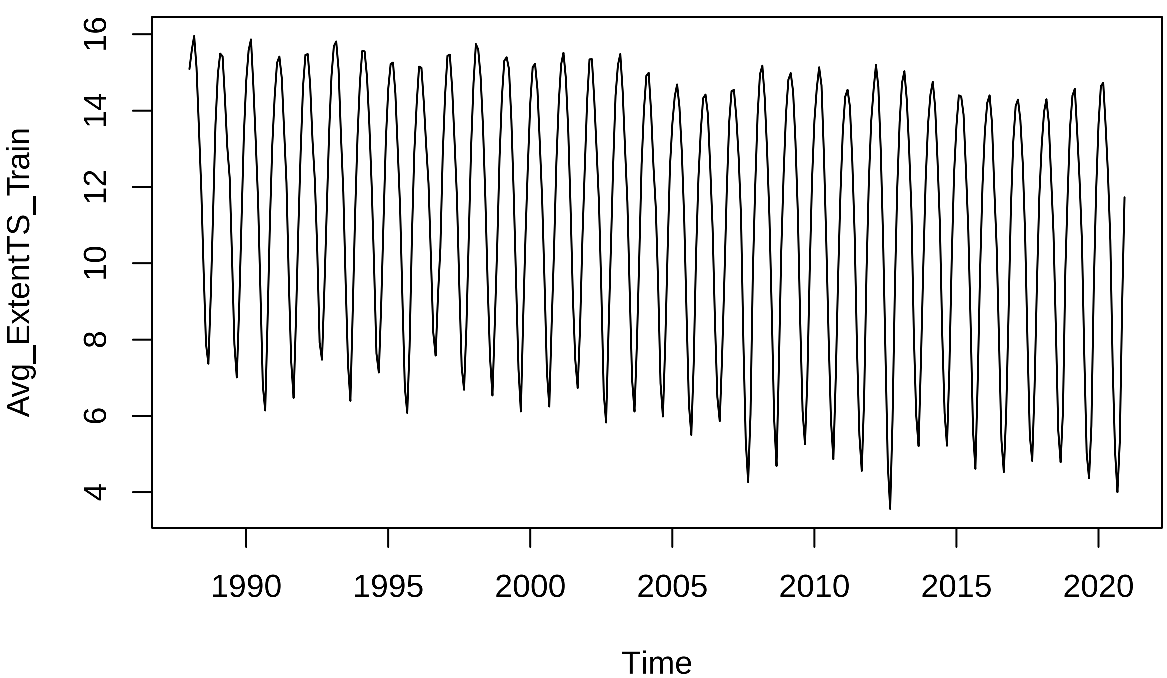
Differencing on Train Data

Try differencing to remove non-stationarity.

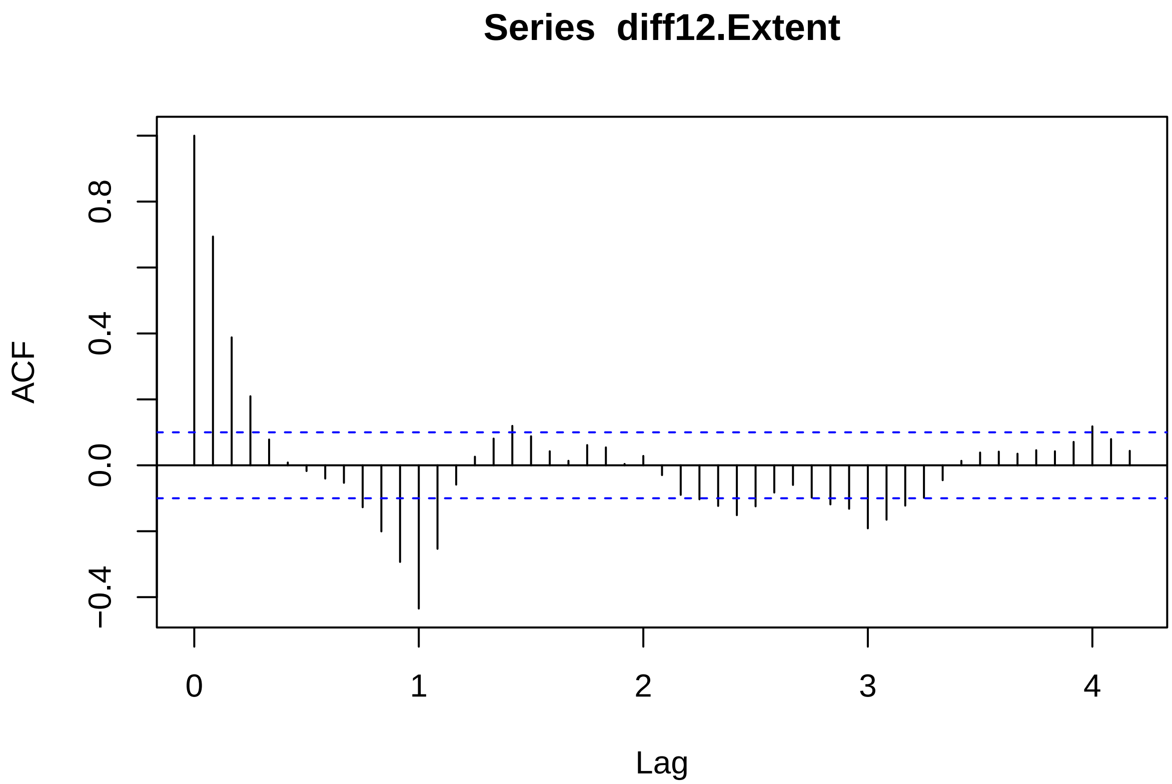
```r
acf(Avg_ExtentTS_Train, lag.max=36)
```



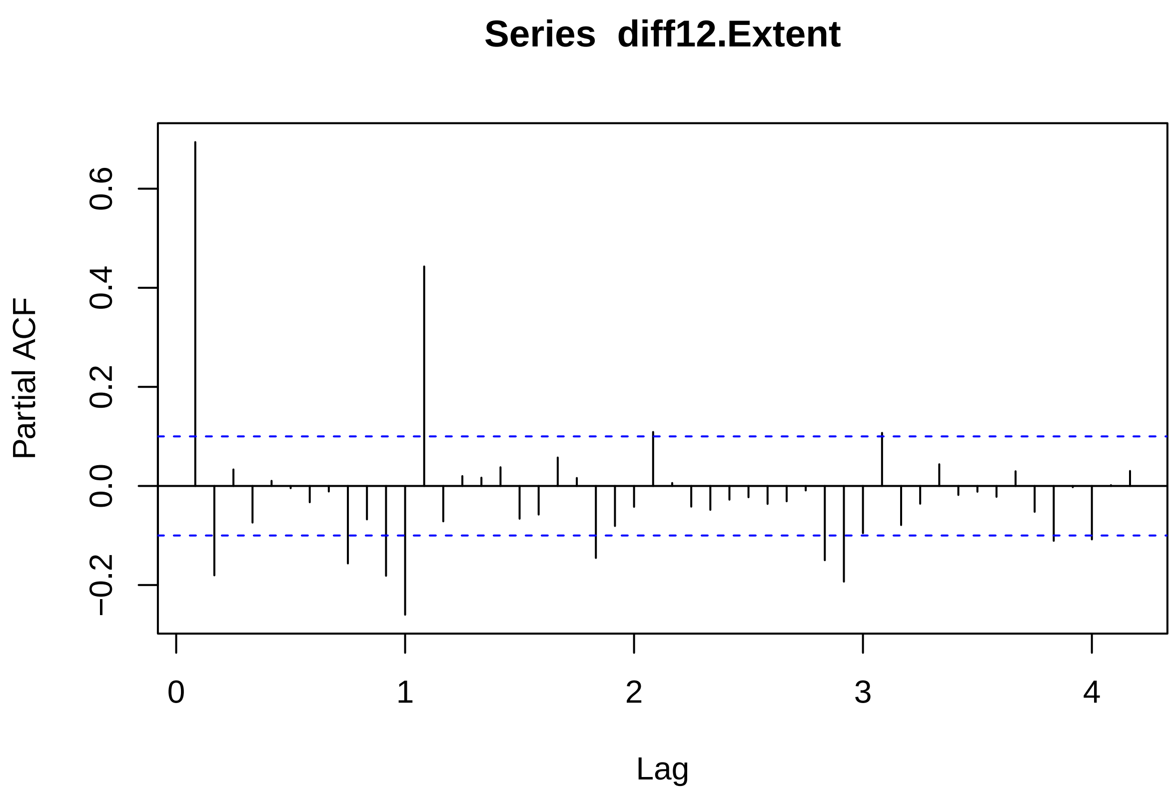

```r
plot(Avg_ExtentTS_Train)
```



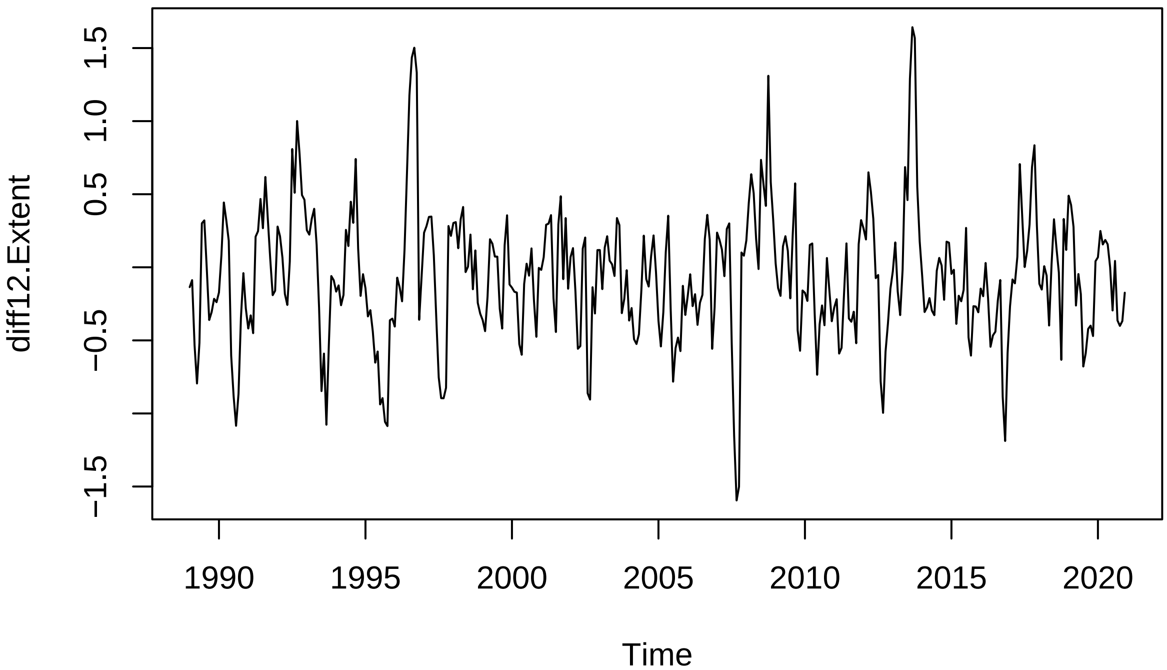

```r
#differencing in lag of season
diff12.Extent=diff(Avg_ExtentTS_Train, lag=12)
acf(diff12.Extent, lag.max=50)
```



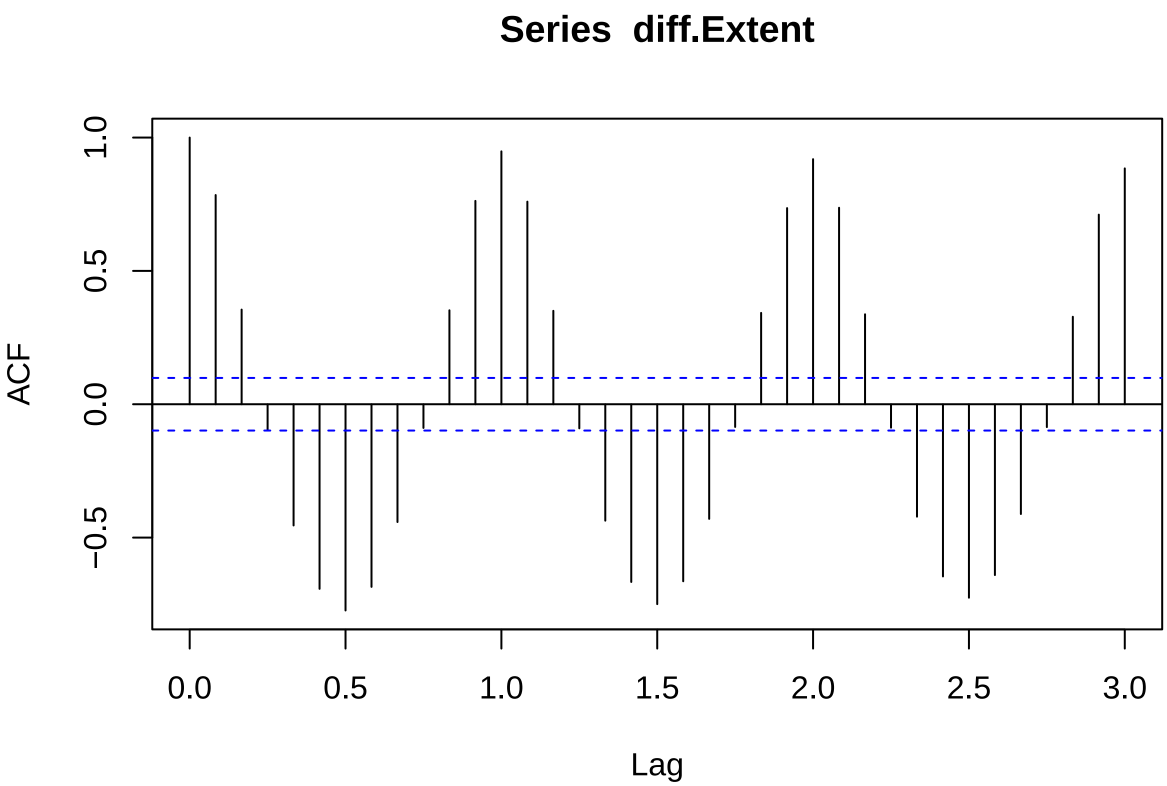

```
pacf(diff12.Extent, lag.max=50)
```



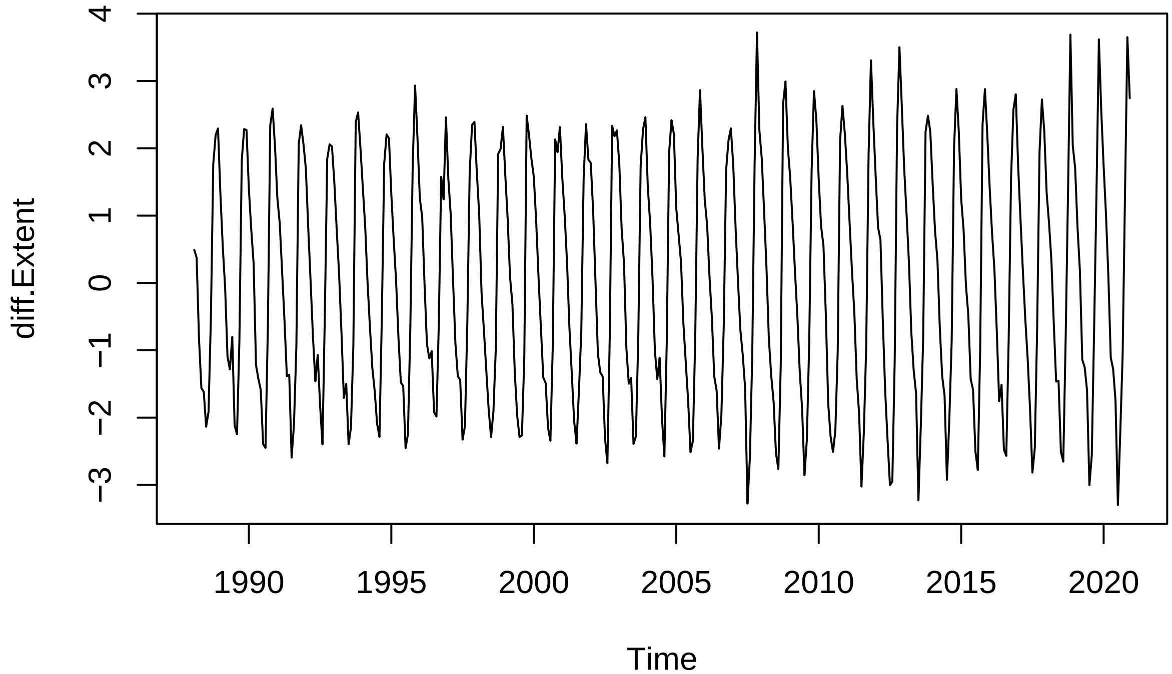

```
plot(diff12.Extent)
```



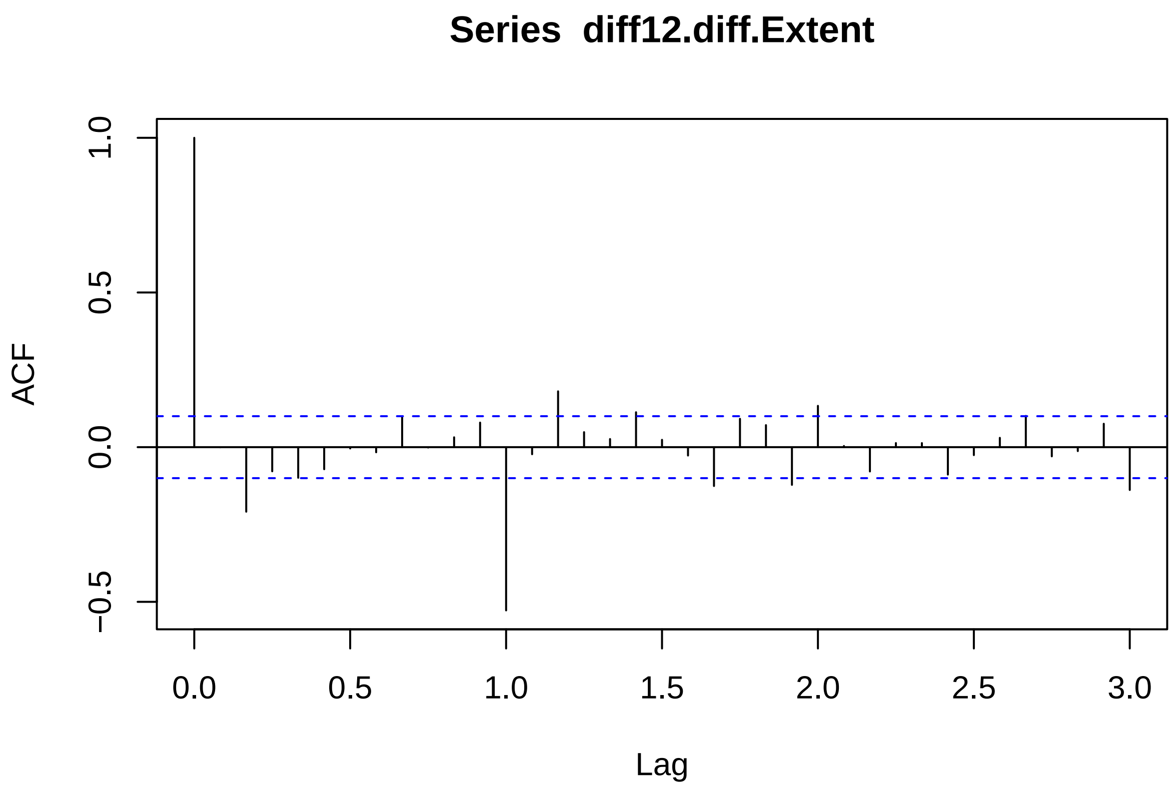

```
#regular differencing
diff.Extent=diff(Avg_ExtentTS_Train)
acf(diff.Extent, lag.max=36)
```



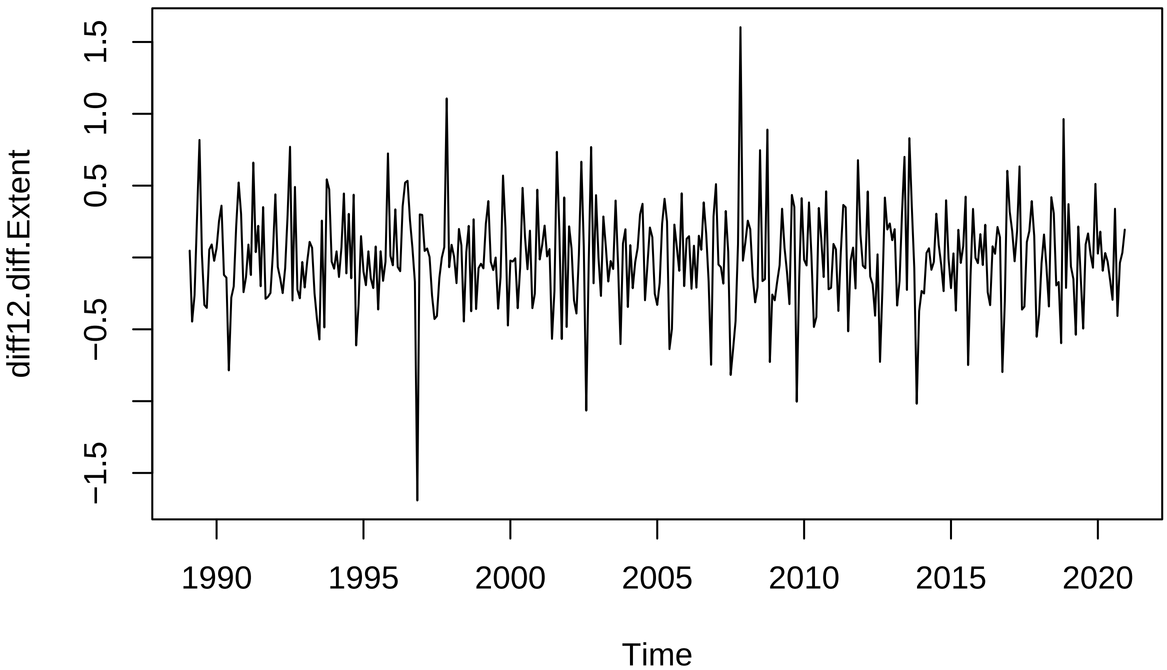

```
plot(diff.Extent)
```




```
# seasonal+regular differencing
diff12.diff.Extent=diff(diff12.Extent)
acf(diff12.diff.Extent, lag.max=36)
```

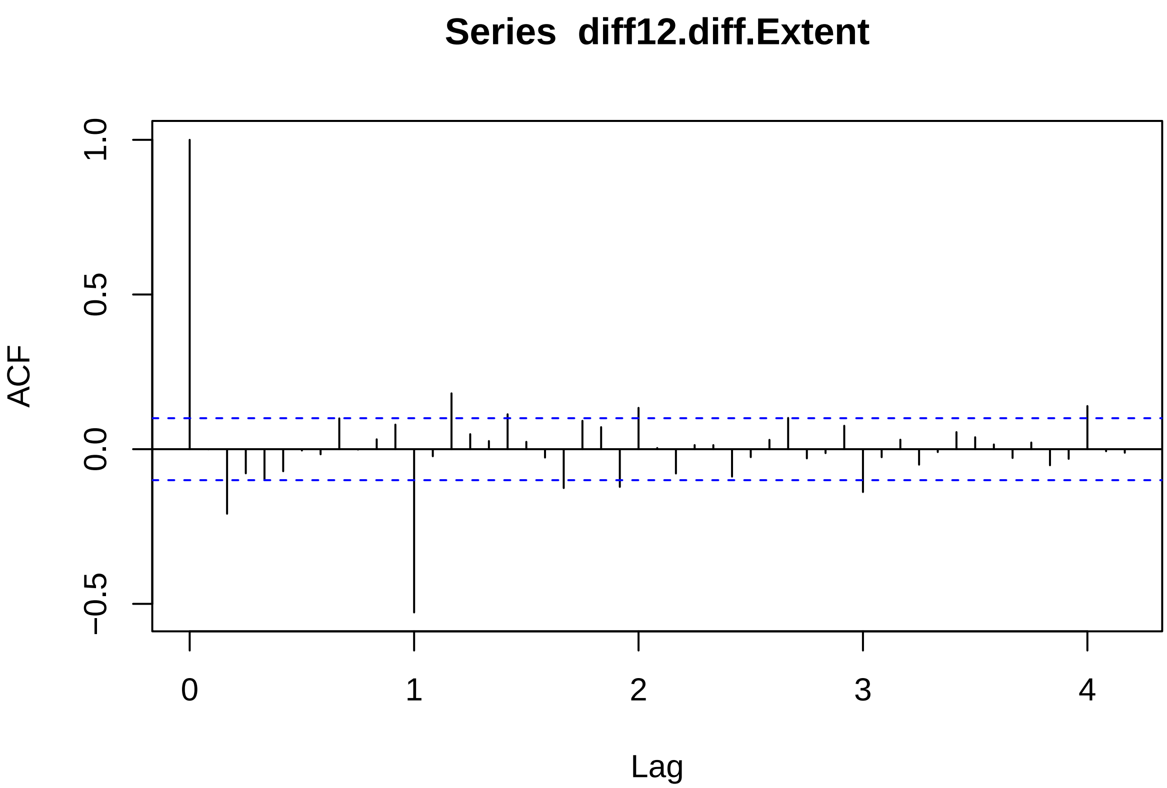



```
plot(diff12.diff.Extent)
```

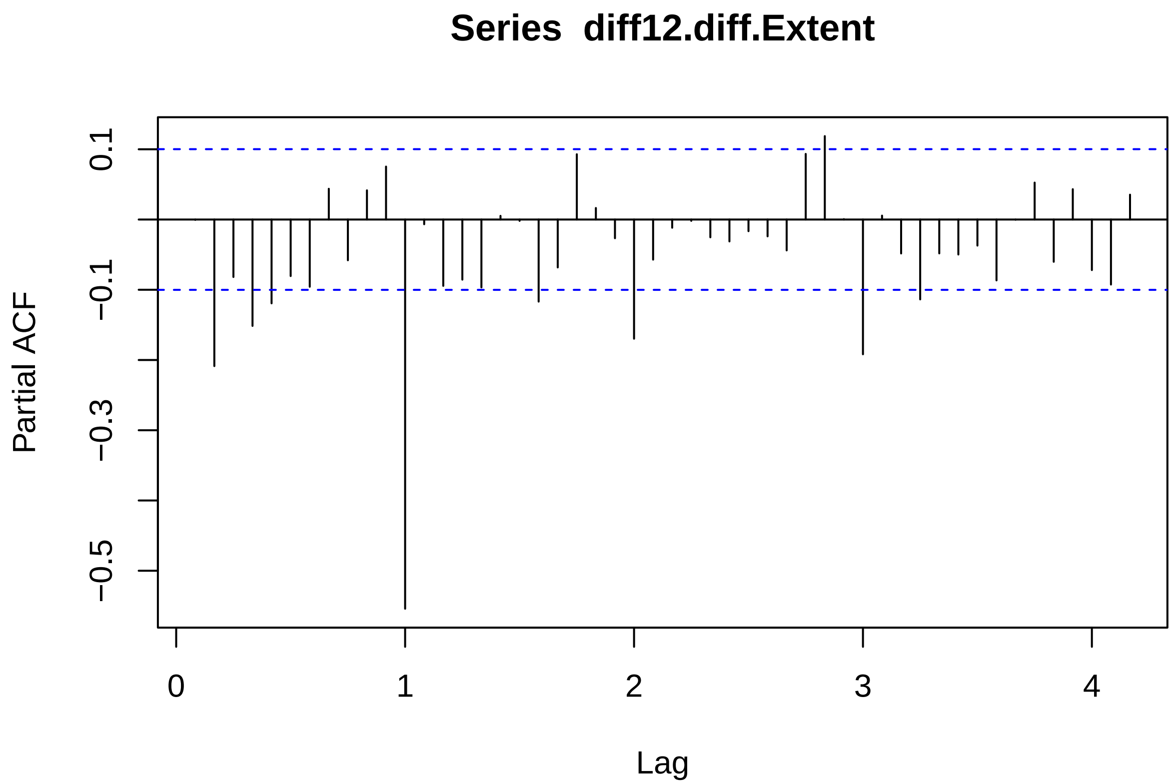


It seems that regression performs poorly in terms of removing non-stationarity compared to HW and Differencing. HW and differencing seem to perform similarly. For simplicity, we proceed with differencing. #TODO reword this

```
acf(diff12.diff.Extent, lag.max=50)
```



```r
pacf(diff12.diff.Extent, lag.max=50)
```



We propose the following models:

Box-Jenkins on Seasonally differenced data

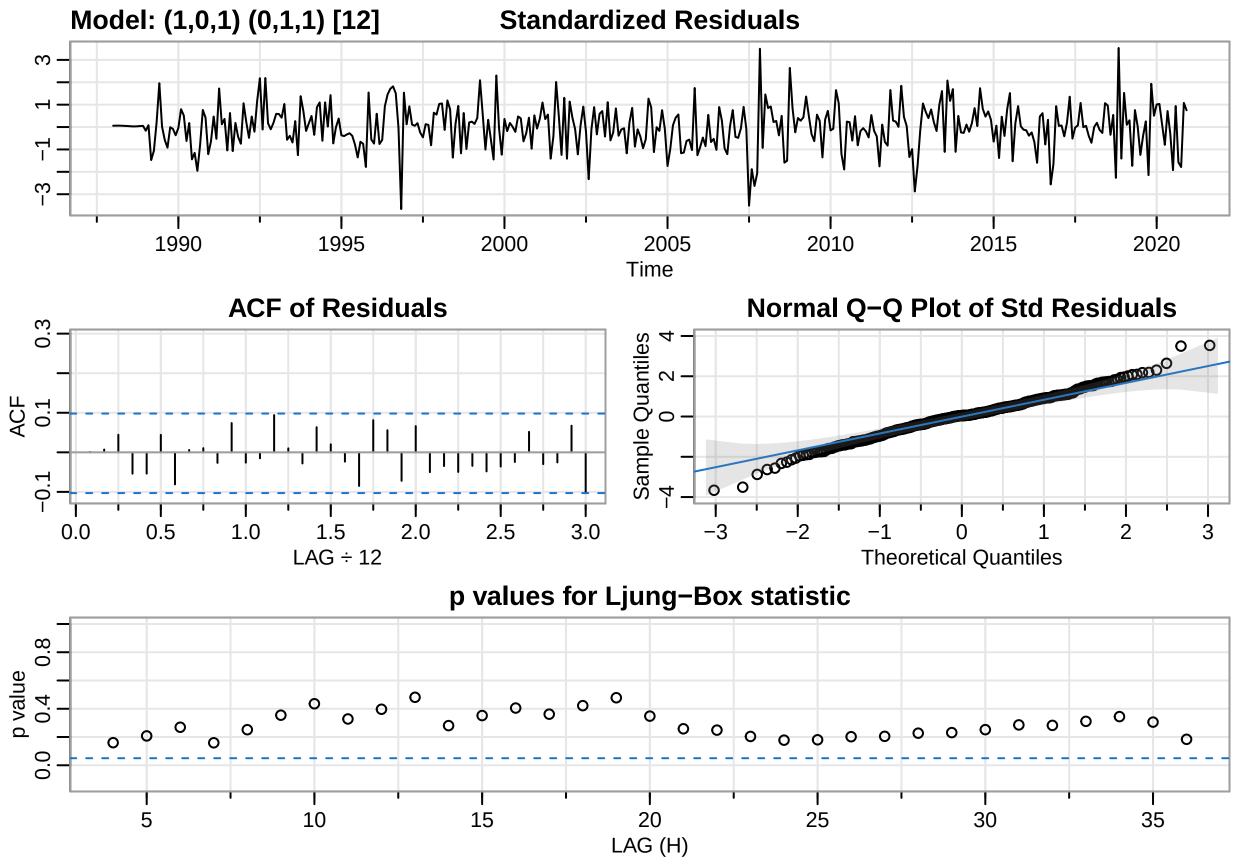
Seasonal differencing of the data seems to be enough to achieve stationarity, so we proceed with that. See appendix for analysis of seasonal + regular differencing

```r
library(astsa)
```

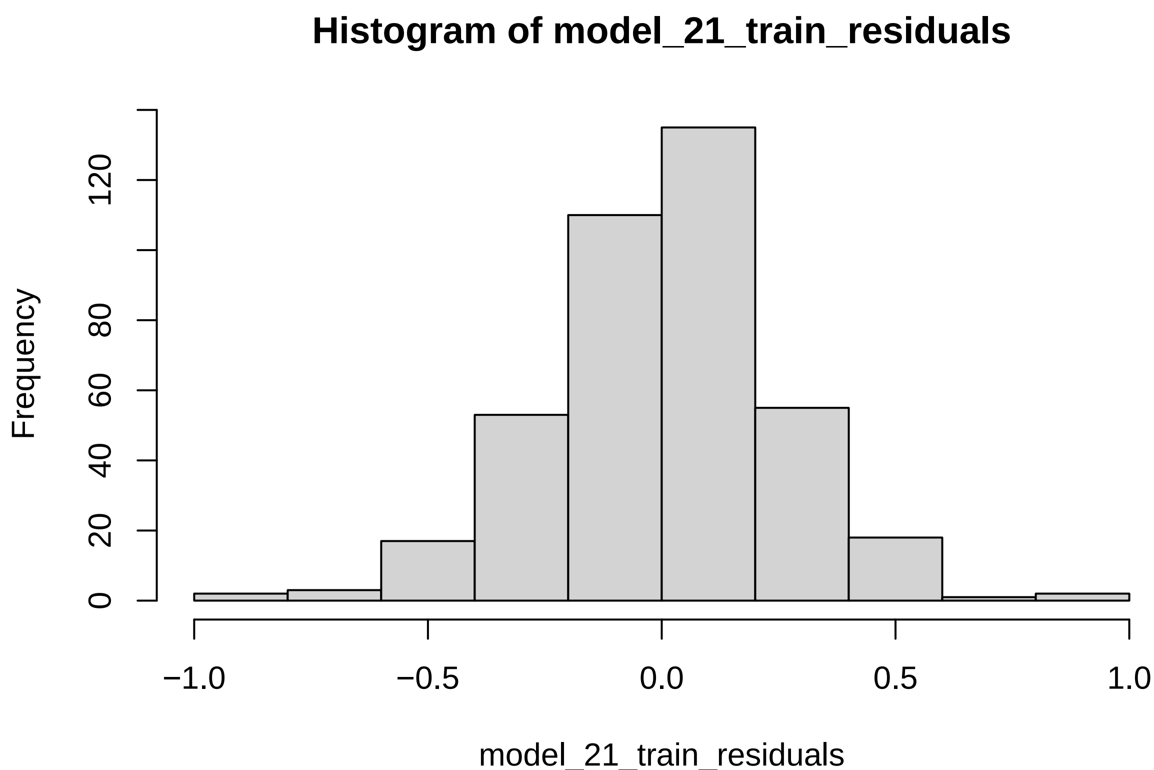
```r
#SARIMA(1,0,1)x(0,1,1)_12
model_21_train <- sarima(Avg_ExtentTS_Train, p=1, d=0, q=1, P=0, D=1, Q=1, S=12 , details = TRUE)
```

```
## initial  value -0.801086
## iter   2 value -1.133503
## iter   3 value -1.361046
## iter   4 value -1.364659
## iter   5 value -1.367122
## iter   6 value -1.367636
## iter   7 value -1.367710
## iter   8 value -1.367744
## iter   9 value -1.367751
## iter  10 value -1.367751
## iter  10 value -1.367751
## final  value -1.367751
## converged
## initial  value -1.365232
## iter   2 value -1.365266
## iter   3 value -1.365293
```

```
## iter   4 value -1.365301
## iter   5 value -1.365301
## iter   6 value -1.365301
## iter   6 value -1.365301
## iter   6 value -1.365301
## final  value -1.365301
## converged
```




```
model_21_train_residuals = resid(model_21_train$fit)
hist(model_21_train_residuals)
```



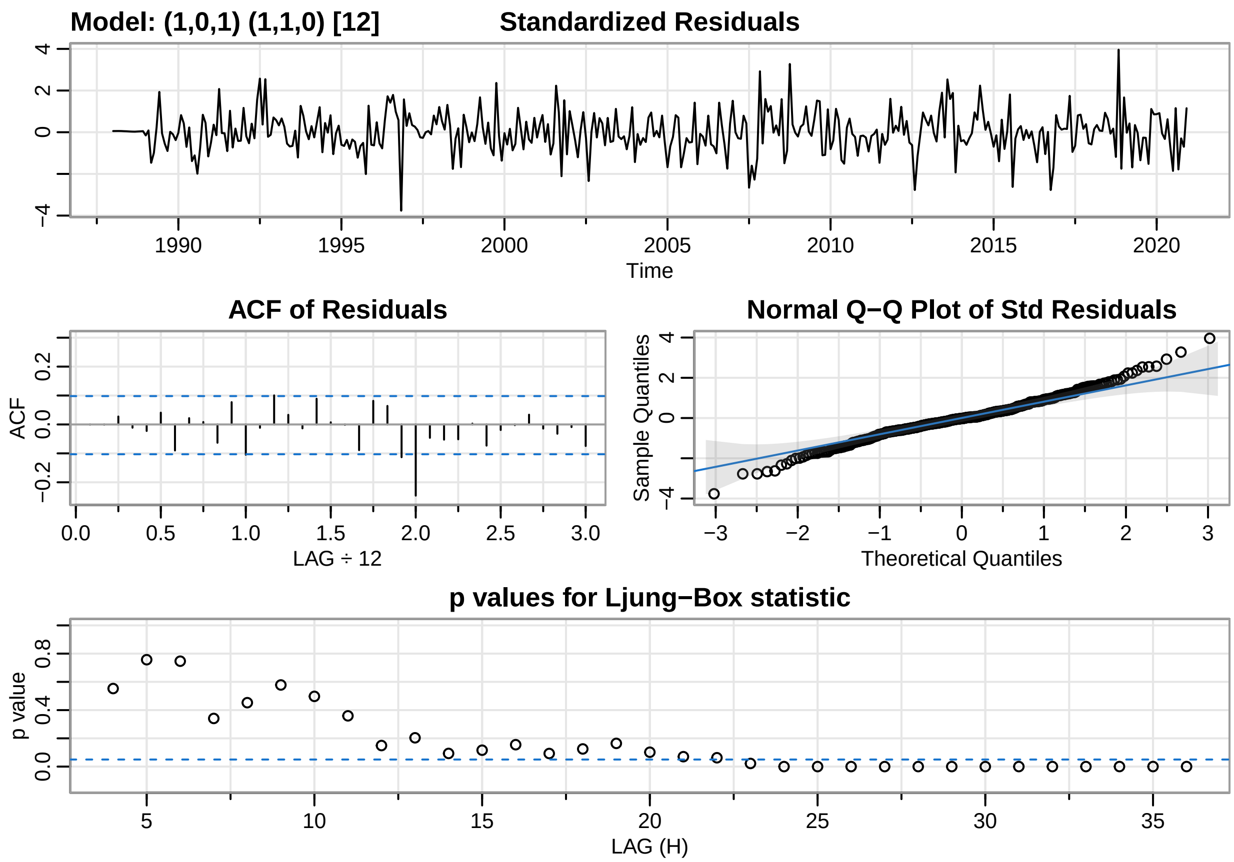

```
shapiro.test(model_21_train_residuals)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  model_21_train_residuals
## W = 0.98802, p-value = 0.002432
```

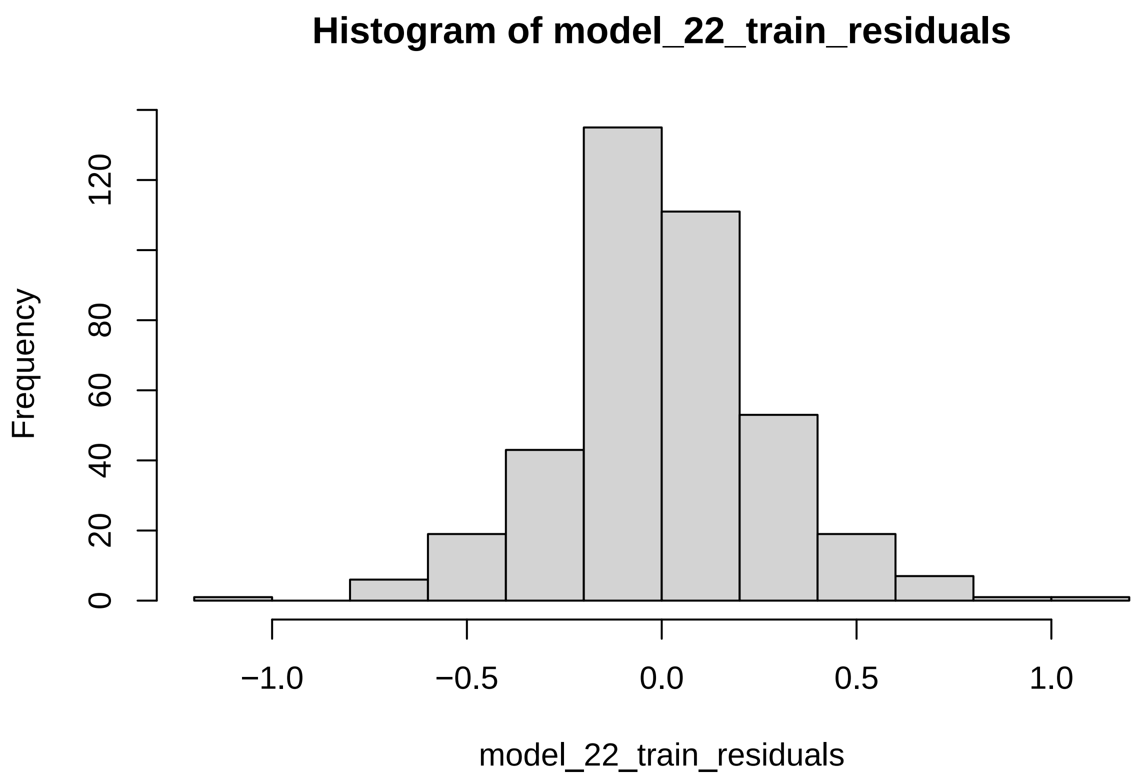
```
#SARIMA(1,0,1)x(1,1,0)_12
model_22_train <- sarima(Avg_ExtentTS_Train, p=1, d=0, q=1, P=1, D=1, Q=0, S=12 , details = TRUE)
```

```
## initial  value -0.794710
## iter   2 value -1.125638
## iter   3 value -1.297146
## iter   4 value -1.306046
## iter   5 value -1.306798
## iter   6 value -1.306962
## iter   7 value -1.307040
## iter   8 value -1.307043
## iter   9 value -1.307044
## iter  10 value -1.307044
## iter  11 value -1.307044
## iter  12 value -1.307044
## iter  12 value -1.307044
## iter  12 value -1.307044
## final  value -1.307044
## converged
## initial  value -1.307068
## iter   2 value -1.307118
## iter   3 value -1.307150
```

```
## iter   4 value -1.307158
## iter   5 value -1.307161
## iter   6 value -1.307161
## iter   7 value -1.307161
## iter   8 value -1.307161
## iter   8 value -1.307161
## iter   8 value -1.307161
## final  value -1.307161
## converged
```




```
model_22_train_residuals = resid(model_22_train$fit)
hist(model_22_train_residuals)
```



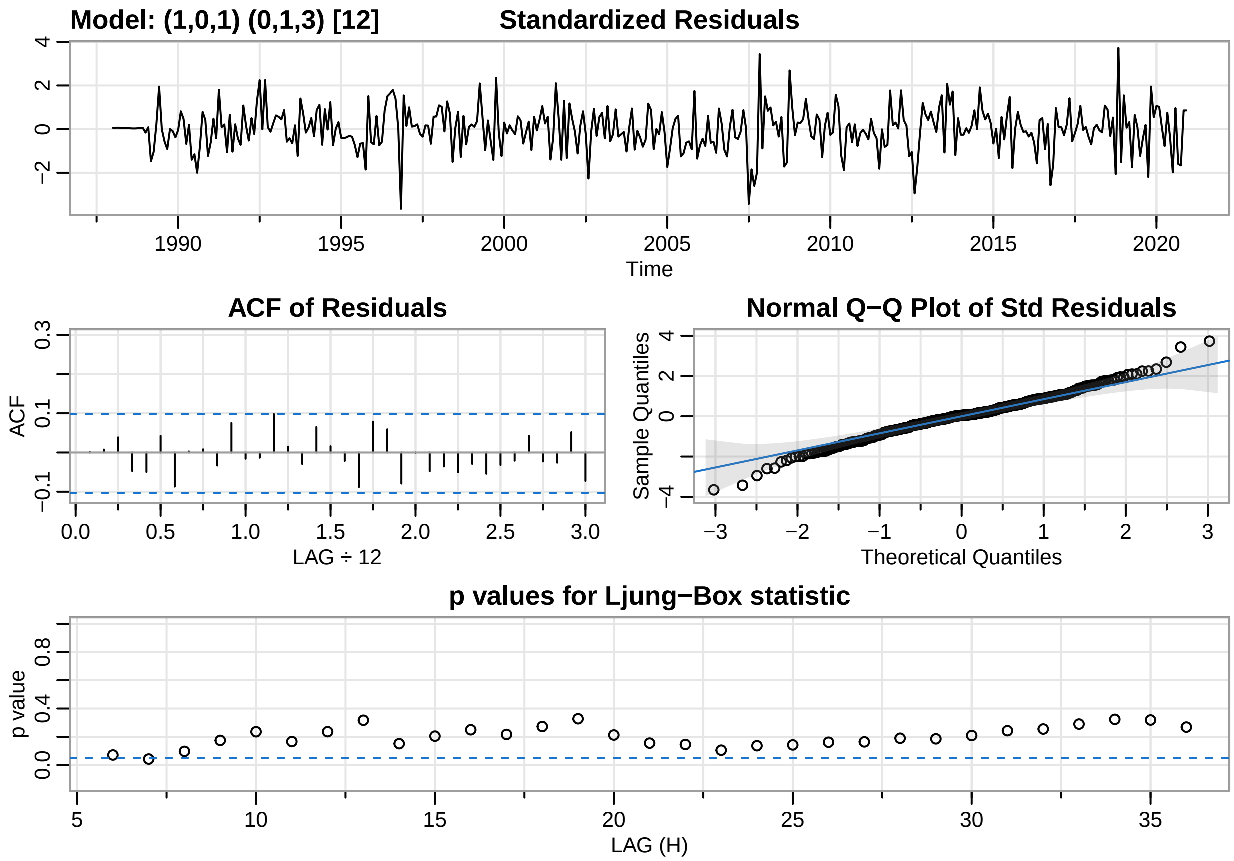

```
shapiro.test(model_22_train_residuals)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  model_22_train_residuals
## W = 0.98428, p-value = 0.0002677
```

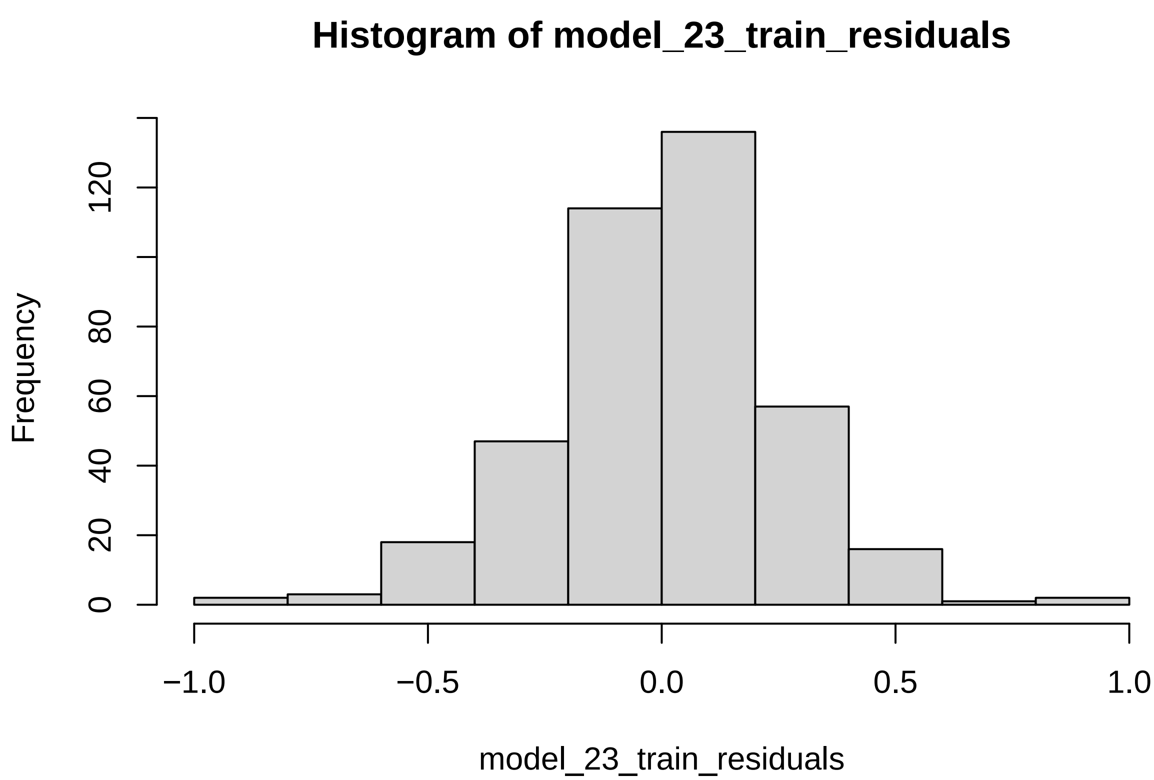
```
#SARIMA(1,0,1)x(0,1,3)_12
model_23_train <- sarima(Avg_ExtentTS_Train, p=1, d=0, q=1, P=0, D=1, Q=3, S=12 , details = TRUE)
```

```
## initial  value -0.801086
## iter   2 value -1.144412
## iter   3 value -1.344981
## iter   4 value -1.357046
## iter   5 value -1.368996
## iter   6 value -1.371115
## iter   7 value -1.371624
## iter   8 value -1.371650
## iter   9 value -1.371662
## iter  10 value -1.371662
## iter  11 value -1.371662
## iter  11 value -1.371662
## iter  11 value -1.371662
## final  value -1.371662
## converged
## initial  value -1.368723
## iter   2 value -1.368752
## iter   3 value -1.368772
## iter   4 value -1.368806
```

```
## iter   5 value -1.368811
## iter   6 value -1.368813
## iter   7 value -1.368813
## iter   7 value -1.368813
## iter   7 value -1.368813
## final  value -1.368813
## converged
```




```
model_23_train_residuals = resid(model_23_train$fit)
hist(model_23_train_residuals)
```



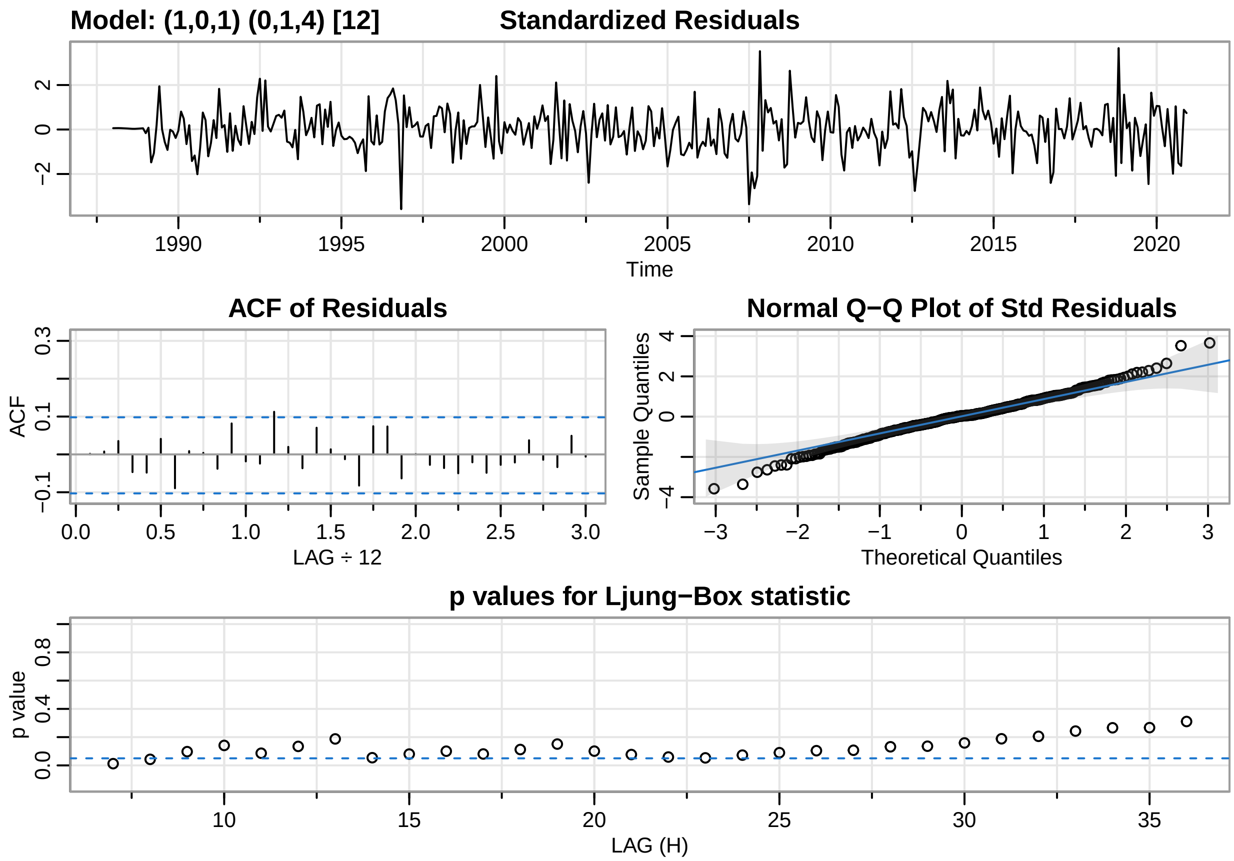

```
shapiro.test(model_23_train_residuals)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  model_23_train_residuals
## W = 0.98706, p-value = 0.001346
```

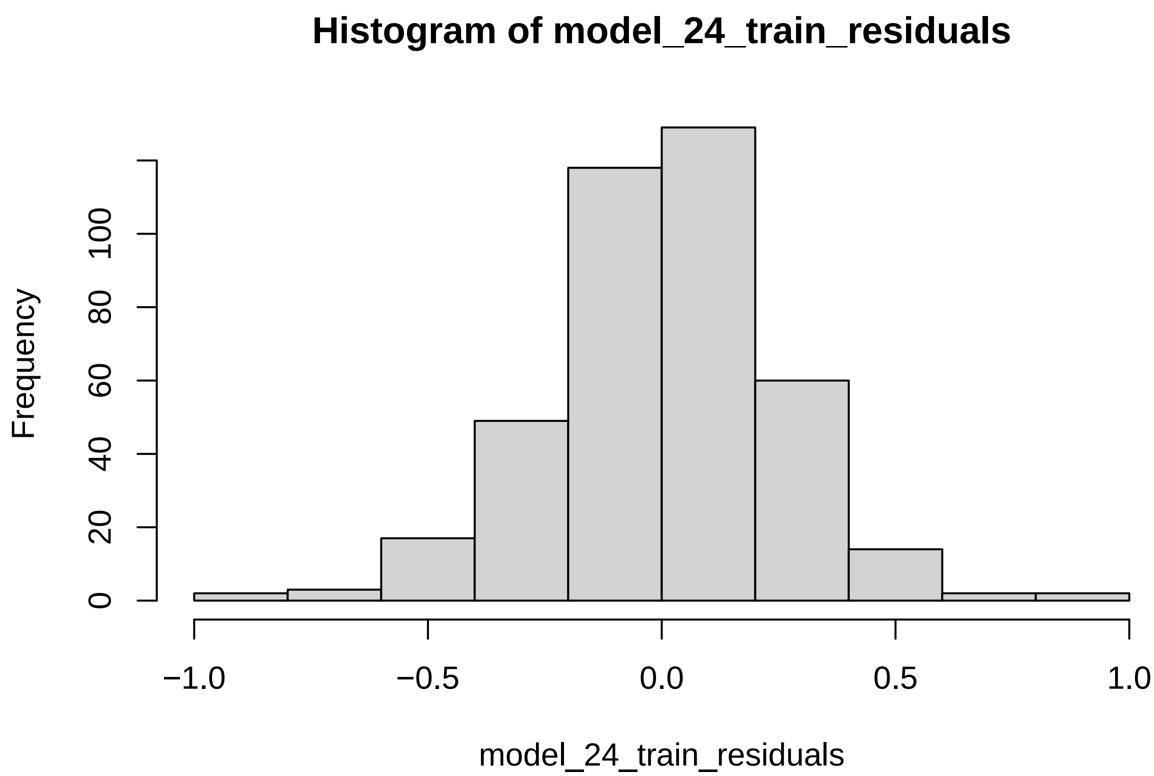
```
#SARIMA(1,0,1)x(0,1,4)_12
model_24_train <- sarima(Avg_ExtentTS_Train, p=1, d=0, q=1, P=0, D=1, Q=4, S=12 , details = TRUE)
```

```
## initial  value -0.801086
## iter   2 value -1.140892
## iter   3 value -1.368903
## iter   4 value -1.371731
## iter   5 value -1.373468
## iter   6 value -1.375135
## iter   7 value -1.375394
## iter   8 value -1.375462
## iter   9 value -1.375466
## iter  10 value -1.375469
## iter  11 value -1.375469
## iter  11 value -1.375469
## iter  11 value -1.375469
## final  value -1.375469
## converged
## initial  value -1.372802
## iter   2 value -1.372833
## iter   3 value -1.372871
## iter   4 value -1.372893
```

```
## iter   5 value -1.372905
## iter   6 value -1.372905
## iter   6 value -1.372905
## iter   6 value -1.372905
## final  value -1.372905
## converged
```




```
model_24_train_residuals = resid(model_24_train$fit)
hist(model_24_train_residuals)
```



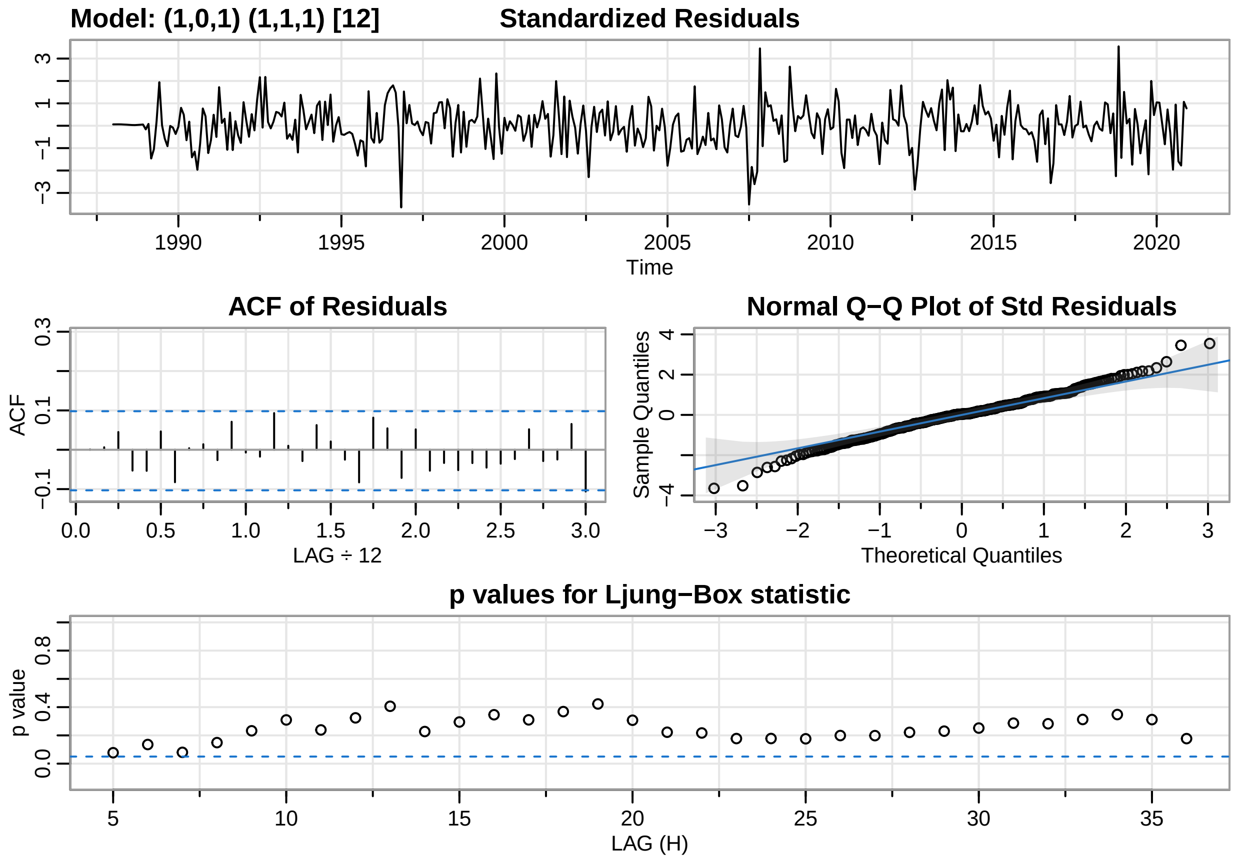

```
shapiro.test(model_24_train_residuals)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  model_24_train_residuals
## W = 0.98849, p-value = 0.003246
```

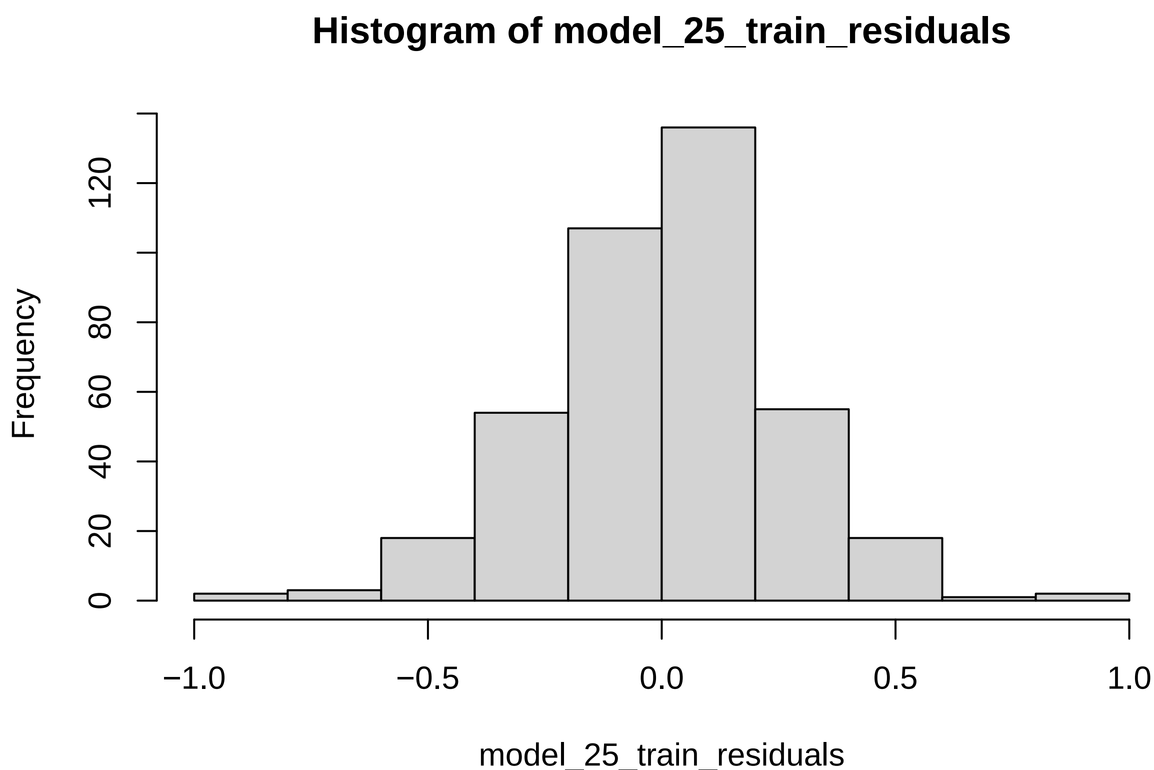
```
#SARIMA(1,0,1)x(1,1,1)_12
model_25_train <- sarima(Avg_ExtentTS_Train, p=1, d=0, q=1, P=1, D=1, Q=1, S=12 , details = TRUE)
```

```
## initial  value -0.794710
## iter   2 value -1.152892
## iter   3 value -1.325426
## iter   4 value -1.345857
## iter   5 value -1.352890
## iter   6 value -1.359234
## iter   7 value -1.360827
## iter   8 value -1.361190
## iter   9 value -1.361198
## iter  10 value -1.361203
## iter  11 value -1.361205
## iter  12 value -1.361205
## iter  12 value -1.361205
## final  value -1.361205
## converged
## initial  value -1.364327
## iter   2 value -1.364958
## iter   3 value -1.365484
## iter   4 value -1.365574
```

```
## iter   5 value -1.365589
## iter   6 value -1.365590
## iter   7 value -1.365590
## iter   8 value -1.365590
## iter   8 value -1.365590
## iter   8 value -1.365590
## final  value -1.365590
## converged
```




```
model_25_train_residuals = resid(model_25_train$fit)
hist(model_25_train_residuals)
```



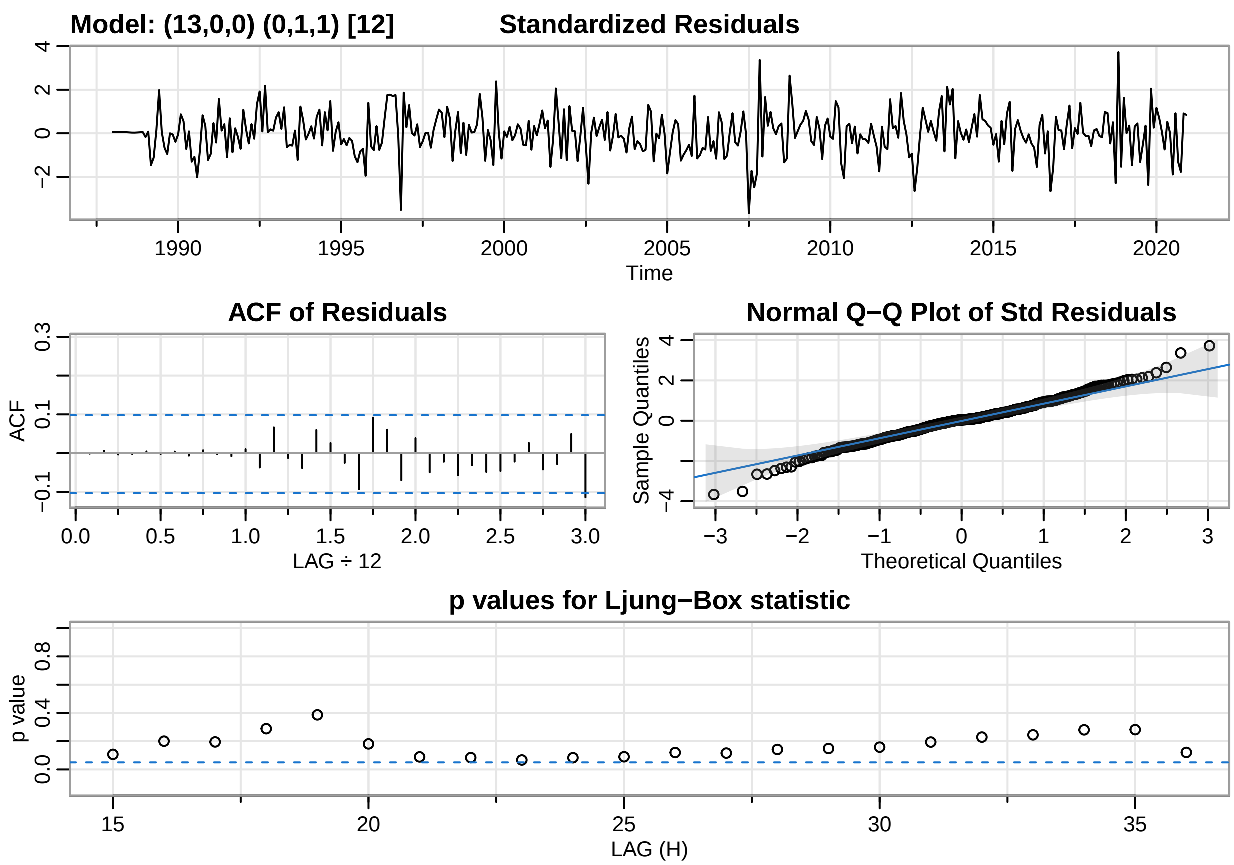

```
shapiro.test(model_25_train_residuals)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  model_25_train_residuals
## W = 0.98841, p-value = 0.00309
```

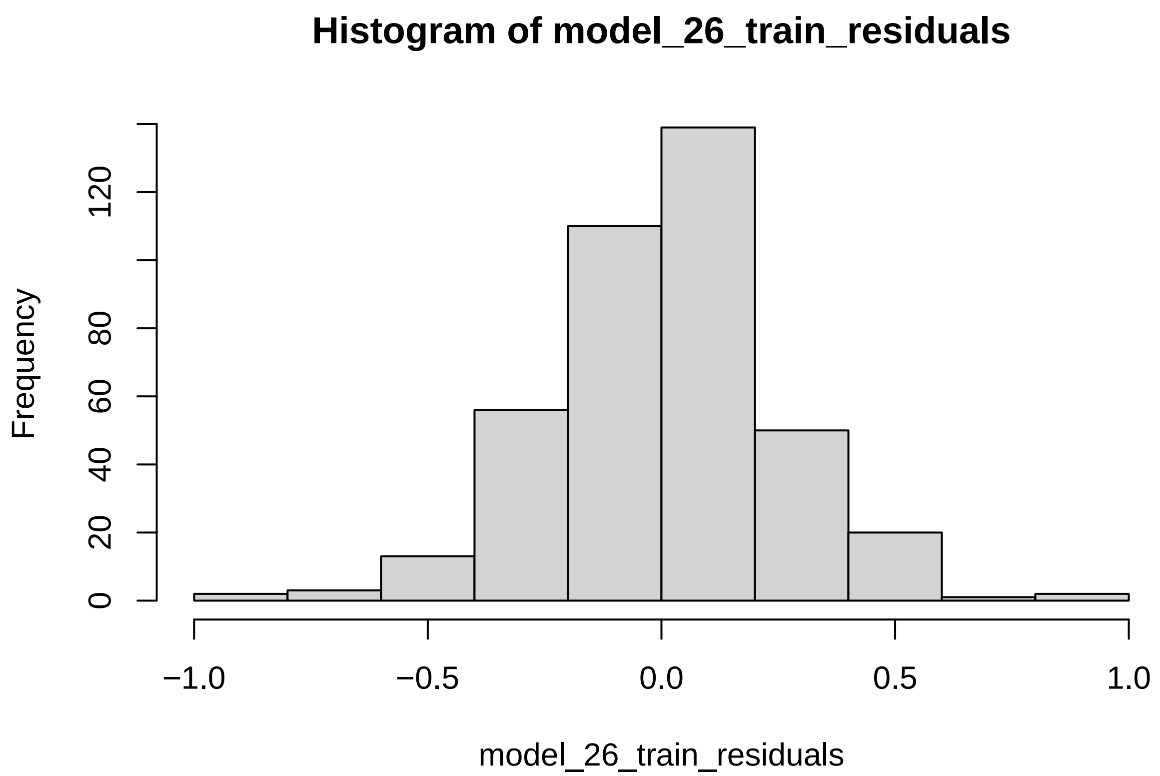
```
#SARIMA(13,0,0)x(0,1,1)_12
model_26_train <- sarima(Avg_ExtentTS_Train, p=13, d=0, q=0, P=0, D=1, Q=1, S=12 , details = TRUE)
```

```
## initial  value -0.794710
## iter   2 value -1.008413
## iter   3 value -1.149498
## iter   4 value -1.247785
## iter   5 value -1.312226
## iter   6 value -1.340321
## iter   7 value -1.362277
## iter   8 value -1.368586
## iter   9 value -1.372893
## iter  10 value -1.376137
## iter  11 value -1.377199
## iter  12 value -1.377346
## iter  13 value -1.377423
## iter  14 value -1.377445
## iter  15 value -1.377456
## iter  16 value -1.377457
## iter  17 value -1.377457
## iter  18 value -1.377458
## iter  19 value -1.377458
```

```
## iter  19 value -1.377458
## iter  19 value -1.377458
## final  value -1.377458
## converged
## initial  value -1.378066
## iter   2 value -1.378820
## iter   3 value -1.379100
## iter   4 value -1.379263
## iter   5 value -1.379282
## iter   6 value -1.379285
## iter   7 value -1.379285
## iter   8 value -1.379285
## iter   8 value -1.379285
## iter   8 value -1.379285
## final  value -1.379285
## converged
```




```
model_26_train_residuals = resid(model_26_train$fit)
hist(model_26_train_residuals)
```



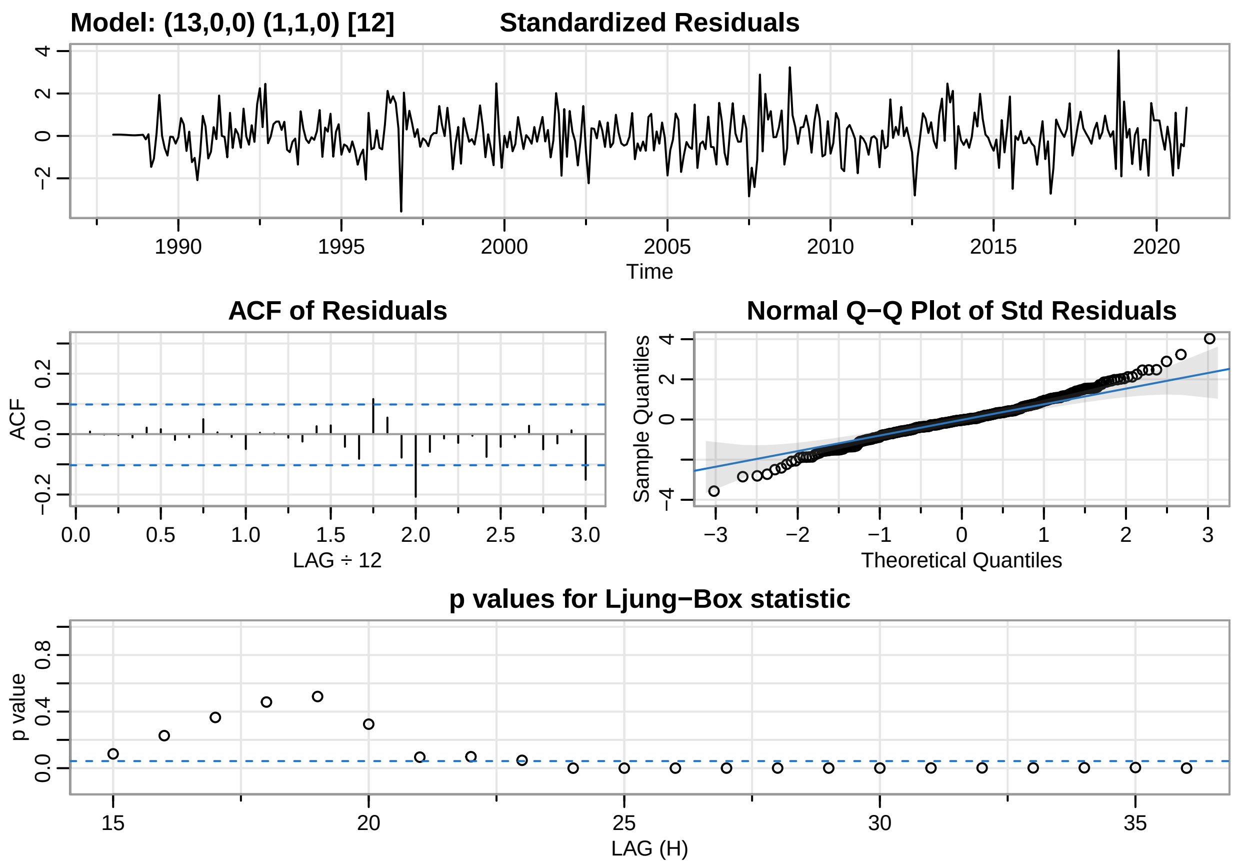

```
shapiro.test(model_26_train_residuals)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  model_26_train_residuals
## W = 0.98804, p-value = 0.002459
```

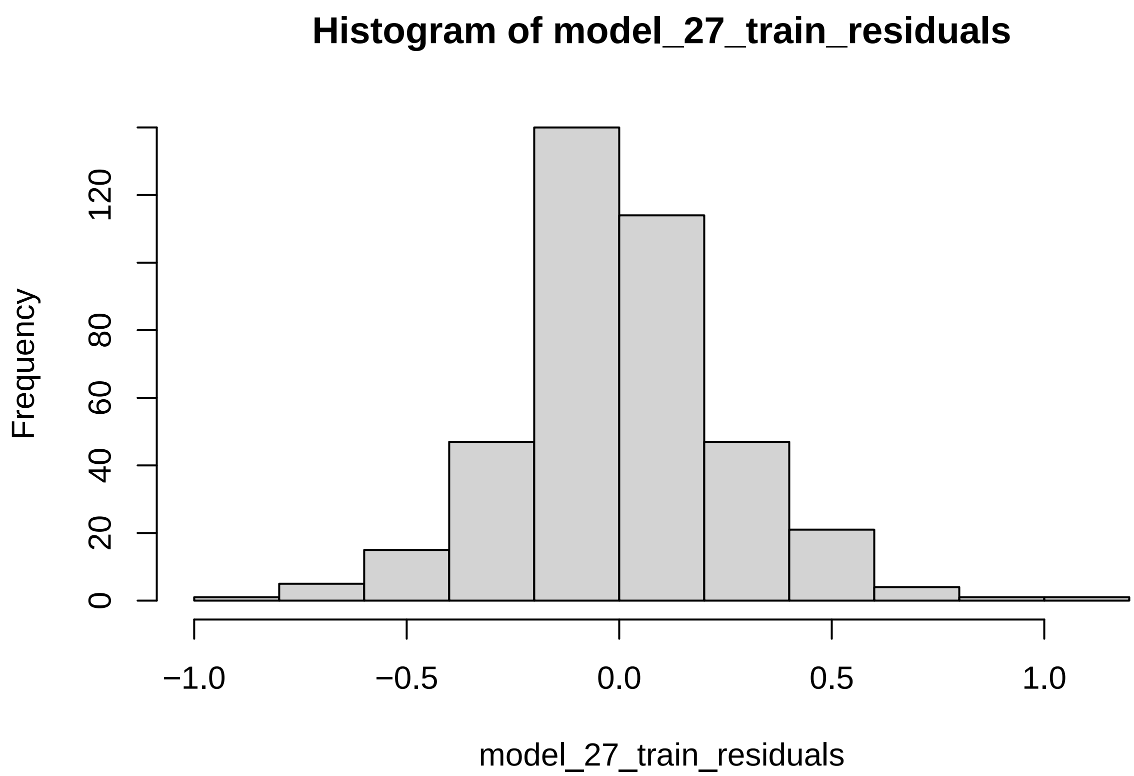
```
#SARIMA(13,0,0)x(1,1,0)_12
model_27_train <- sarima(Avg_ExtentTS_Train, p=13, d=0, q=0, P=1, D=1, Q=0, S=12 , details = TRUE)
```

```
## initial  value -0.801164
## iter   2 value -1.017847
## iter   3 value -1.143844
## iter   4 value -1.232274
## iter   5 value -1.284447
## iter   6 value -1.311521
## iter   7 value -1.320540
## iter   8 value -1.328806
## iter   9 value -1.334262
## iter  10 value -1.338005
## iter  11 value -1.339163
## iter  12 value -1.339774
## iter  13 value -1.340389
## iter  14 value -1.340593
## iter  15 value -1.340695
## iter  16 value -1.340719
## iter  17 value -1.340730
## iter  18 value -1.340734
## iter  19 value -1.340734
```

```
## iter  20 value -1.340735
## iter  20 value -1.340735
## iter  20 value -1.340735
## final  value -1.340735
## converged
## initial  value -1.338080
## iter   2 value -1.338274
## iter   3 value -1.338681
## iter   4 value -1.338714
## iter   5 value -1.338795
## iter   6 value -1.338818
## iter   7 value -1.338842
## iter   8 value -1.338848
## iter   9 value -1.338850
## iter  10 value -1.338850
## iter  11 value -1.338850
## iter  11 value -1.338850
## iter  11 value -1.338850
## final  value -1.338850
## converged
```




```
model_27_train_residuals = resid(model_27_train$fit)
hist(model_27_train_residuals)
```



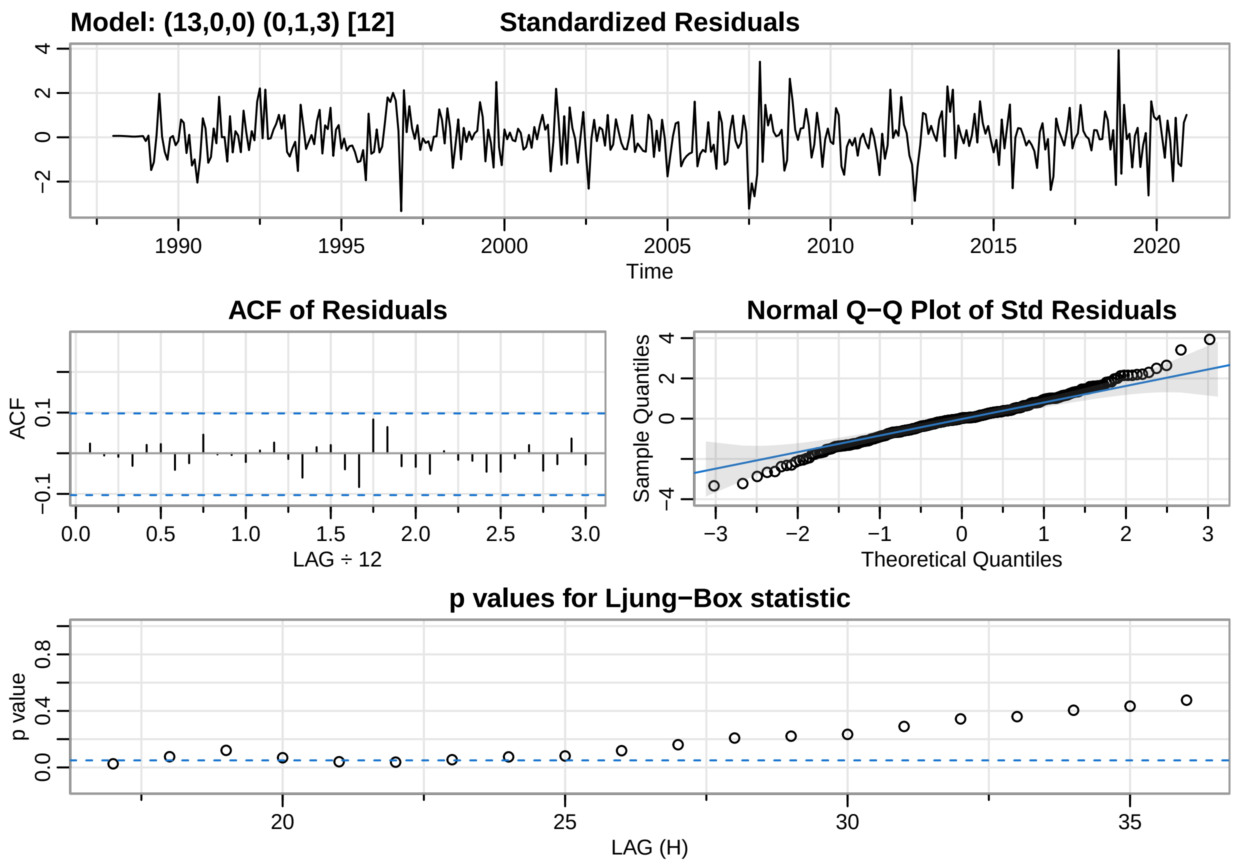

```
shapiro.test(model_27_train_residuals)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  model_27_train_residuals
## W = 0.98468, p-value = 0.0003358
```

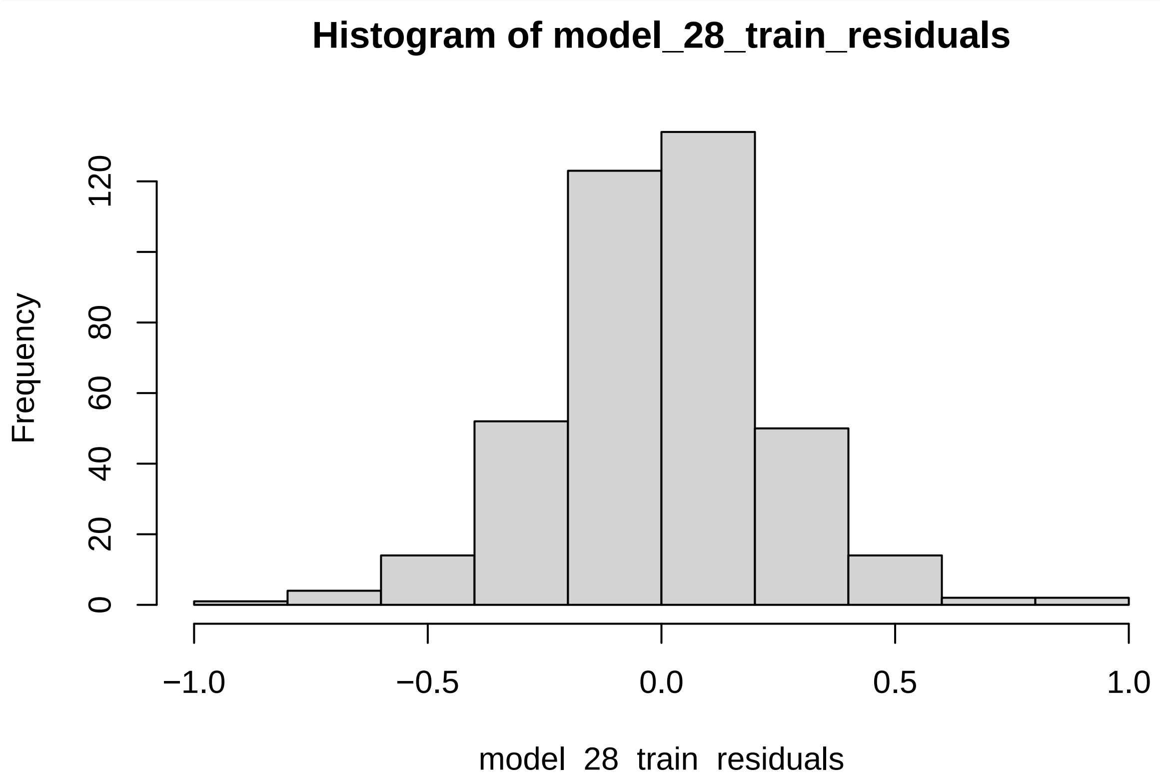
```
#SARIMA(13,0,0)x(0,1,3)_12
model_28_train <- sarima(Avg_ExtentTS_Train, p=13, d=0, q=0, P=0, D=1, Q=3, S=12 , details = TRUE)
```

```
## initial  value -0.794710
## iter   2 value -1.013936
## iter   3 value -1.173142
## iter   4 value -1.265645
## iter   5 value -1.324556
## iter   6 value -1.329399
## iter   7 value -1.354647
## iter   8 value -1.368673
## iter   9 value -1.381611
## iter  10 value -1.389125
## iter  11 value -1.391039
## iter  12 value -1.391755
## iter  13 value -1.392861
## iter  14 value -1.394528
## iter  15 value -1.395490
## iter  16 value -1.396737
## iter  17 value -1.396990
## iter  18 value -1.397072
## iter  19 value -1.397085
```

```
## iter  20 value -1.397089
## iter  21 value -1.397089
## iter  22 value -1.397089
## iter  22 value -1.397089
## iter  22 value -1.397089
## final  value -1.397089
## converged
## initial  value -1.392610
## iter   2 value -1.392693
## iter   3 value -1.392839
## iter   4 value -1.392864
## iter   5 value -1.392891
## iter   6 value -1.392893
## iter   7 value -1.392897
## iter   8 value -1.392898
## iter   9 value -1.392903
## iter  10 value -1.392904
## iter  11 value -1.392908
## iter  12 value -1.392910
## iter  13 value -1.392911
## iter  14 value -1.392912
## iter  15 value -1.392912
## iter  16 value -1.392912
## iter  16 value -1.392912
## iter  16 value -1.392912
## final  value -1.392912
## converged
```




```
model_28_train_residuals = resid(model_28_train$fit)
hist(model_28_train_residuals)
```



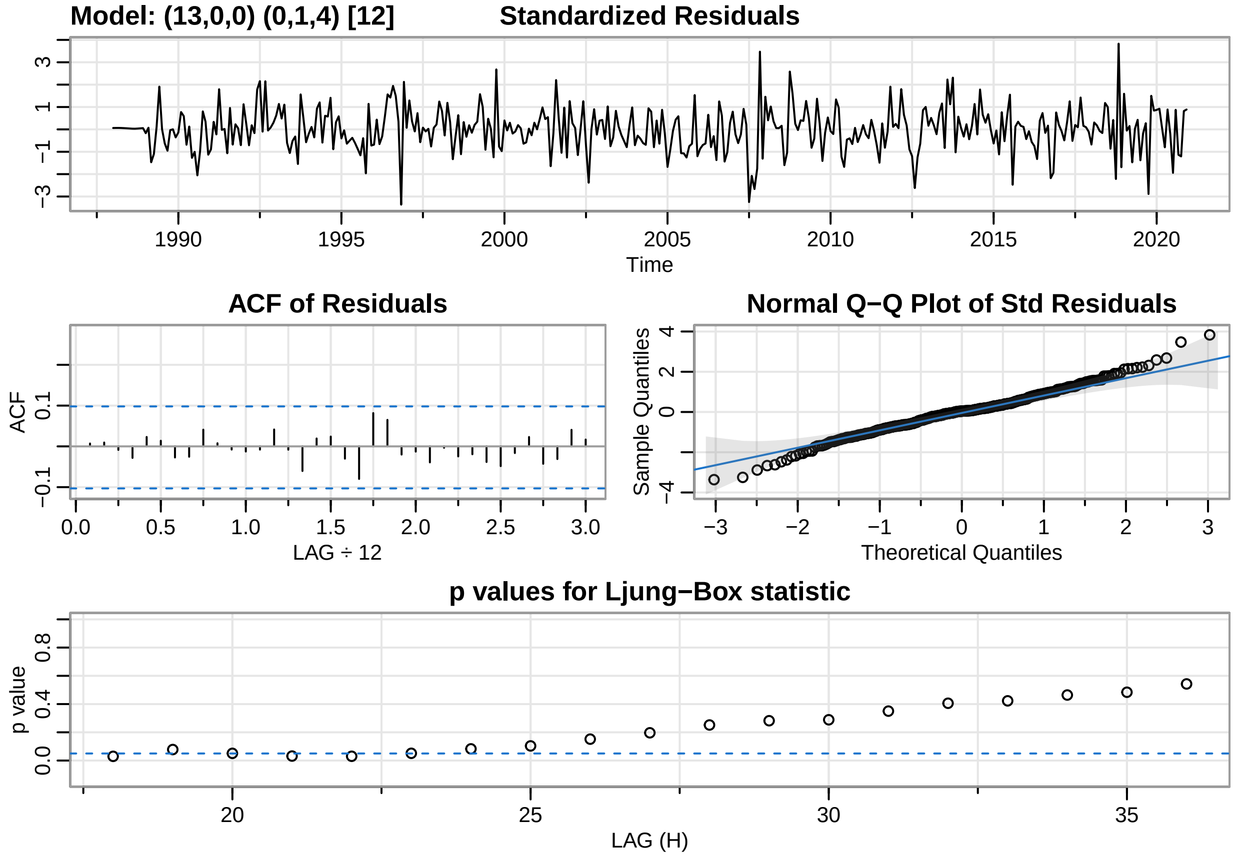
```r
shapiro.test(model_28_train_residuals)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  model_28_train_residuals
## W = 0.9863, p-value = 0.0008591
```

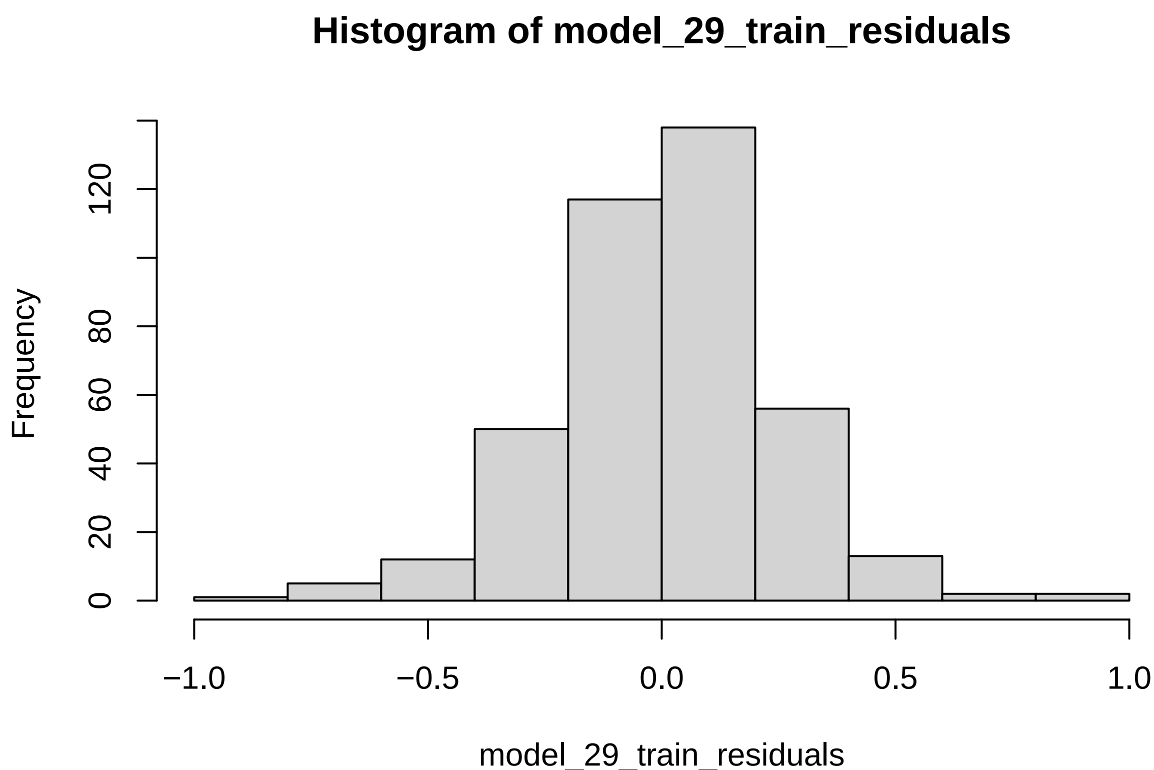
```r
#SARIMA(13,0,0)x(0,1,4)_12
model_29_train <- sarima(Avg_ExtentTS_Train, p=13, d=0, q=0, P=0, D=1, Q=4, S=12 , details = TRUE)
```

```
## initial  value -0.794710
## iter   2 value -1.018370
## iter   3 value -1.185015
## iter   4 value -1.269162
## iter   5 value -1.338386
## iter   6 value -1.361375
## iter   7 value -1.373123
## iter   8 value -1.389868
## iter   9 value -1.390417
## iter  10 value -1.395151
## iter  11 value -1.396699
## iter  12 value -1.398587
## iter  13 value -1.399541
## iter  14 value -1.401027
## iter  15 value -1.401975
## iter  16 value -1.402455
## iter  17 value -1.402620
## iter  18 value -1.402660
## iter  19 value -1.402671
## iter  20 value -1.402672
## iter  21 value -1.402673
## iter  22 value -1.402673
## iter  22 value -1.402673
## iter  22 value -1.402673
## final  value -1.402673
## converged
## initial  value -1.397598
## iter   2 value -1.397676
## iter   3 value -1.397908
## iter   4 value -1.397918
## iter   5 value -1.397948
## iter   6 value -1.397953
## iter   7 value -1.397964
## iter   8 value -1.397968
## iter   9 value -1.397974
## iter  10 value -1.397981
## iter  11 value -1.397982
## iter  12 value -1.397982
## iter  13 value -1.397983
## iter  14 value -1.397983
## iter  14 value -1.397983
## iter  14 value -1.397983
## final  value -1.397983
```

```
## converged
```




```
model_29_train_residuals = resid(model_29_train$fit)
hist(model_29_train_residuals)
```



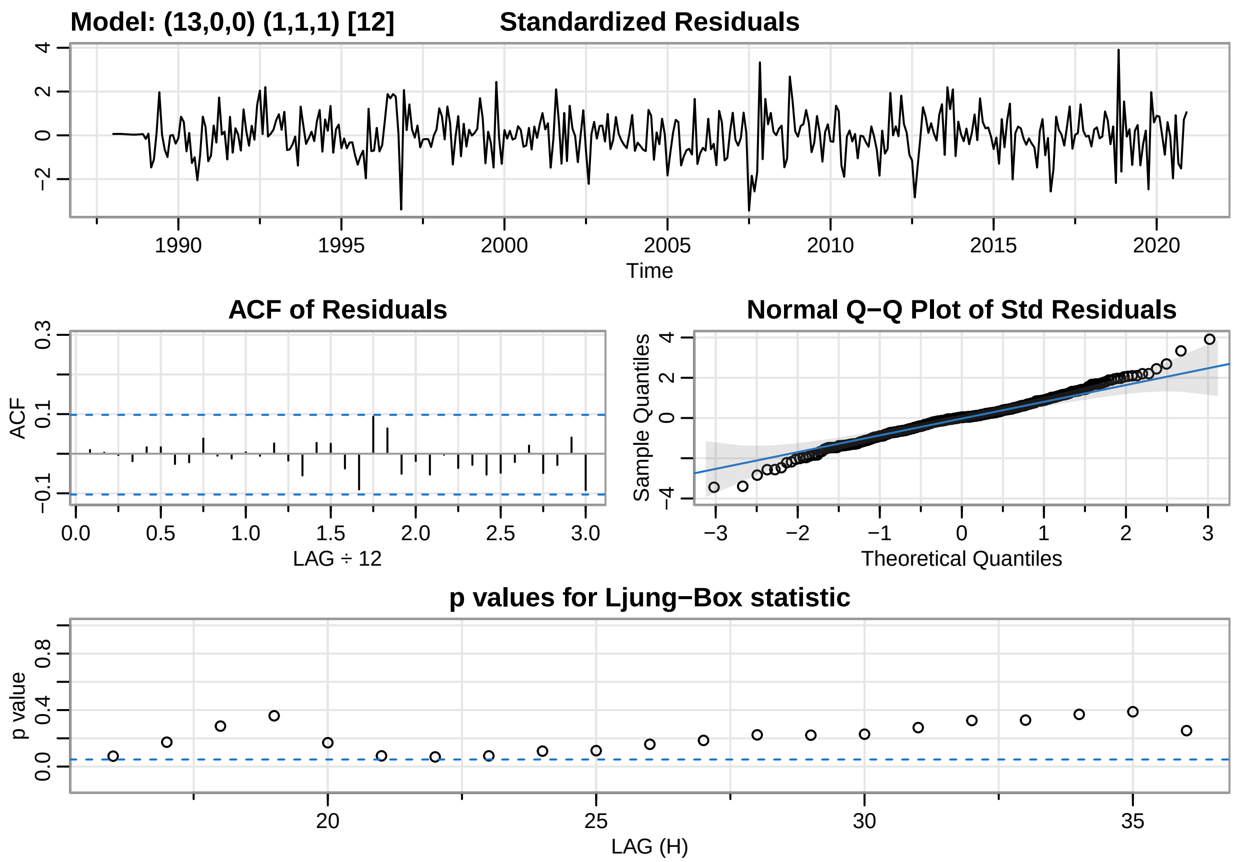

```
shapiro.test(model_29_train_residuals)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  model_29_train_residuals
## W = 0.98626, p-value = 0.0008373
```

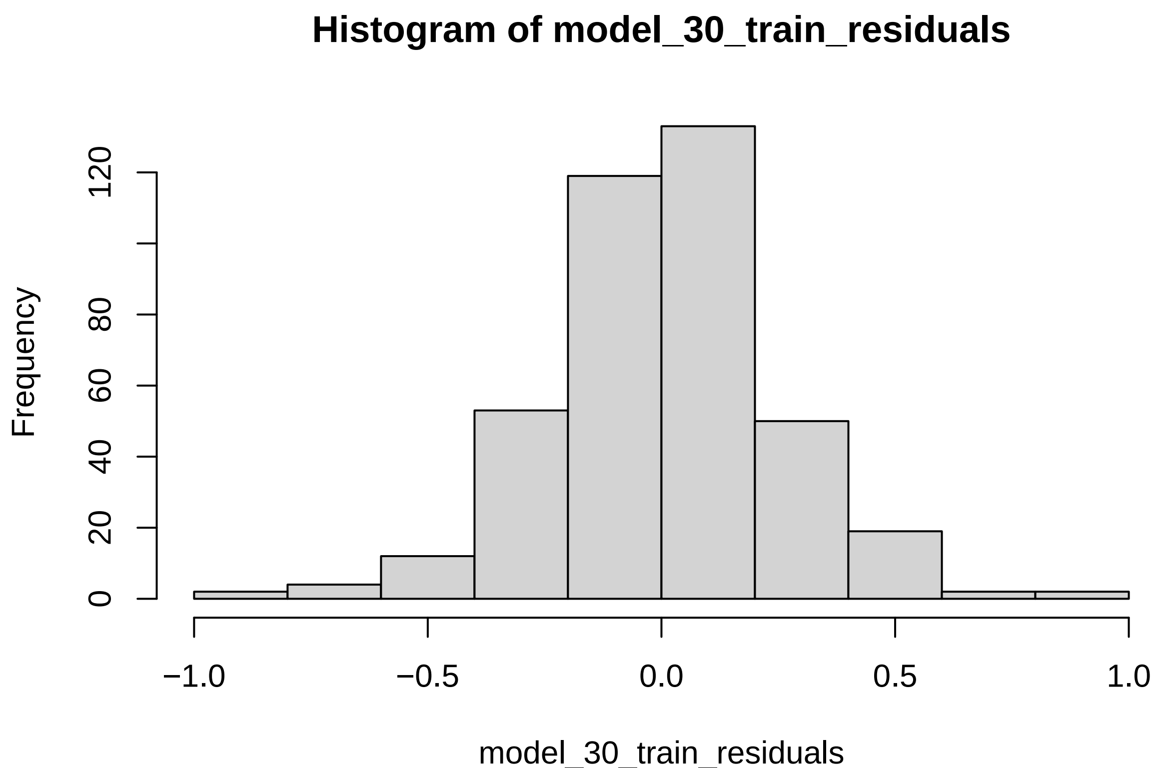
```
#SARIMA(13,0,0)x(1,1,1)_12
model_30_train <- sarima(Avg_ExtentTS_Train, p=13, d=0, q=0, P=1, D=1, Q=1, S=12 , details = TRUE)
```

```
## initial  value -0.801164
## iter   2 value -1.035924
## iter   3 value -1.163984
## iter   4 value -1.273600
## iter   5 value -1.313108
## iter   6 value -1.345098
## iter   7 value -1.350268
## iter   8 value -1.361690
## iter   9 value -1.364074
## iter  10 value -1.364704
## iter  11 value -1.365211
## iter  12 value -1.365454
## iter  13 value -1.365788
## iter  14 value -1.366112
## iter  15 value -1.366710
## iter  16 value -1.367095
## iter  17 value -1.367549
## iter  18 value -1.367600
## iter  19 value -1.367614
```

```
## iter  20 value -1.367619
## iter  21 value -1.367620
## iter  22 value -1.367621
## iter  23 value -1.367621
## iter  23 value -1.367621
## iter  23 value -1.367621
## final  value -1.367621
## converged
## initial  value -1.378413
## iter   2 value -1.381305
## iter   3 value -1.381612
## iter   4 value -1.383730
## iter   5 value -1.384338
## iter   6 value -1.385117
## iter   7 value -1.385893
## iter   8 value -1.386815
## iter   9 value -1.387004
## iter  10 value -1.387072
## iter  11 value -1.387079
## iter  12 value -1.387081
## iter  13 value -1.387081
## iter  14 value -1.387081
## iter  14 value -1.387081
## iter  14 value -1.387081
## final  value -1.387081
## converged
```



```r
model_30_train_residuals = resid(model_30_train$fit)
hist(model_30_train_residuals)
```



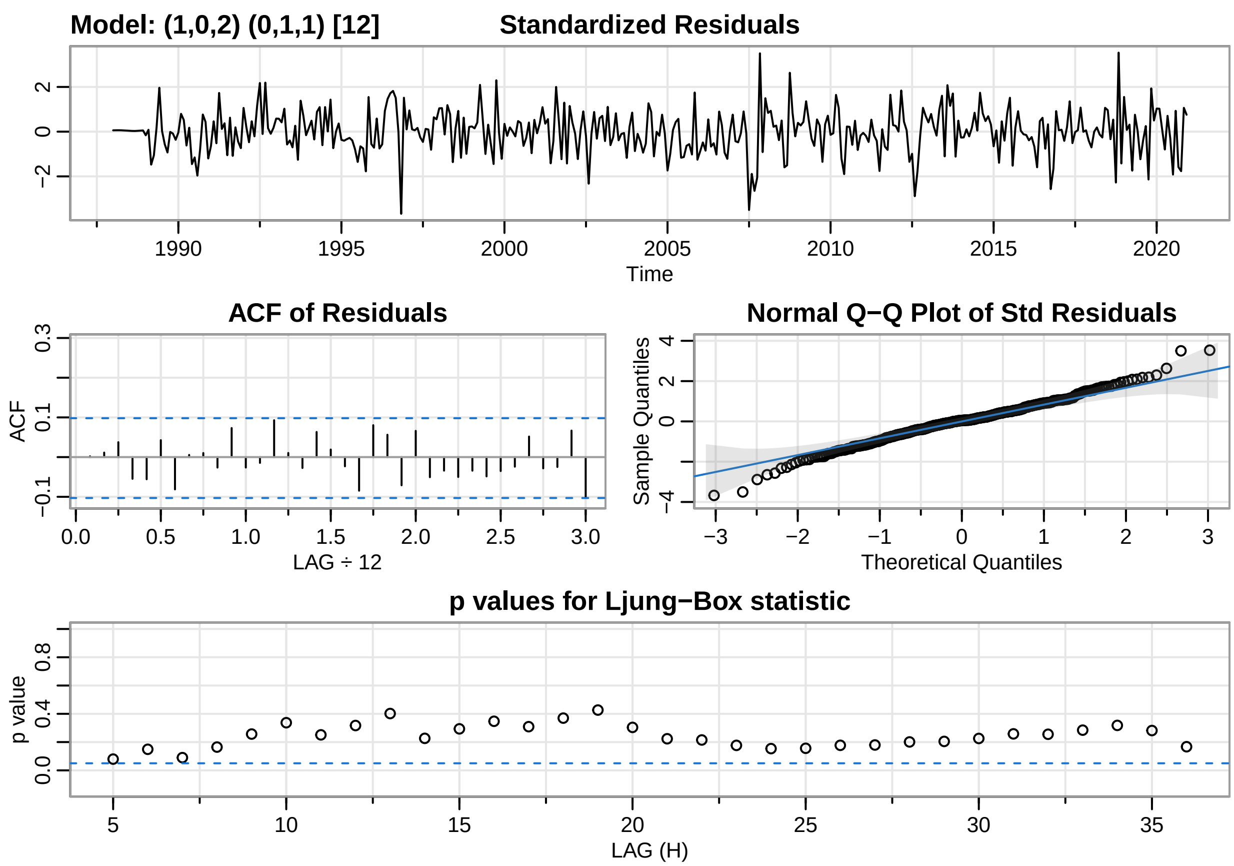

```r
shapiro.test(model_30_train_residuals)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  model_30_train_residuals
## W = 0.98623, p-value = 0.00082
```

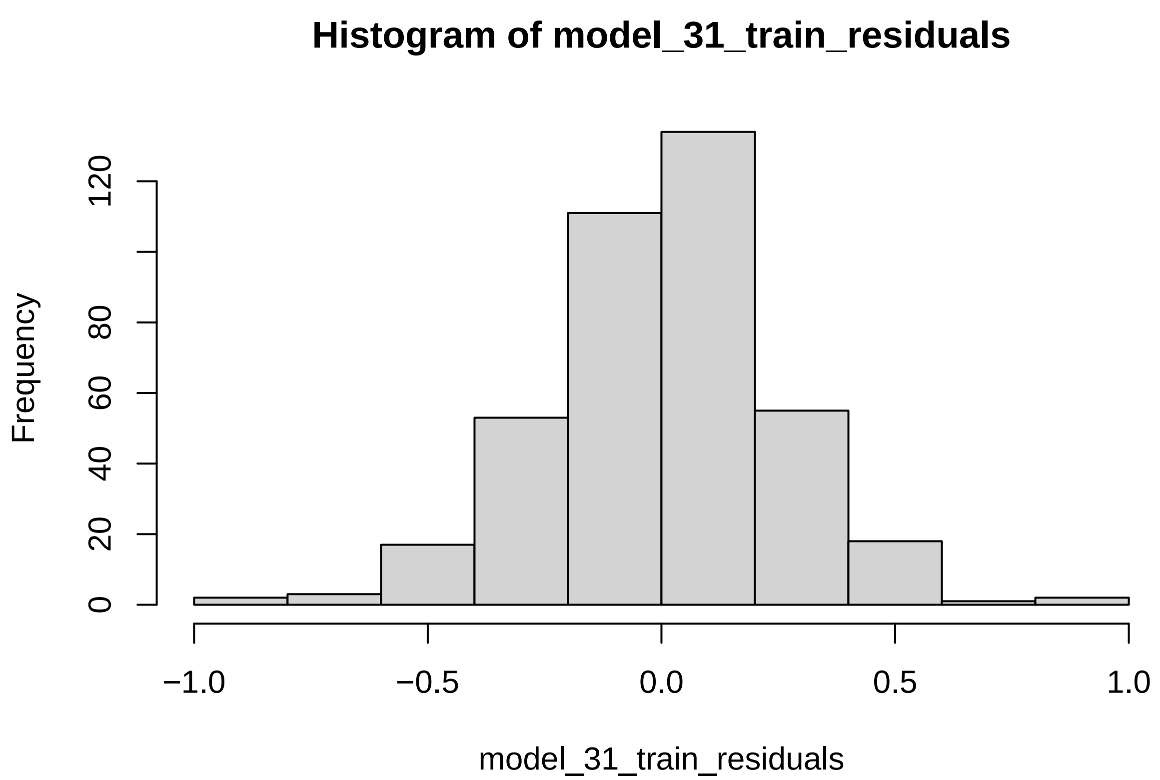
```r
#SARIMA(1,0,2)x(0,1,1)_12
model_31_train <- sarima(Avg_ExtentTS_Train, p=1, d=0, q=2, P=0, D=1, Q=1, S=12 , details = TRUE)
```

```
## initial  value -0.801086
## iter   2 value -1.138952
## iter   3 value -1.342237
## iter   4 value -1.355399
## iter   5 value -1.360693
## iter   6 value -1.362722
## iter   7 value -1.364221
## iter   8 value -1.365966
## iter   9 value -1.367305
## iter  10 value -1.367779
## iter  11 value -1.367834
## iter  12 value -1.367835
## iter  12 value -1.367835
## final  value -1.367835
## converged
## initial  value -1.365277
```

```
## iter   2 value -1.365339
## iter   3 value -1.365342
## iter   4 value -1.365342
## iter   5 value -1.365343
## iter   6 value -1.365344
## iter   7 value -1.365344
## iter   7 value -1.365344
## iter   7 value -1.365344
## final  value -1.365344
## converged
```




```
model_31_train_residuals = resid(model_31_train$fit)
hist(model_31_train_residuals)
```



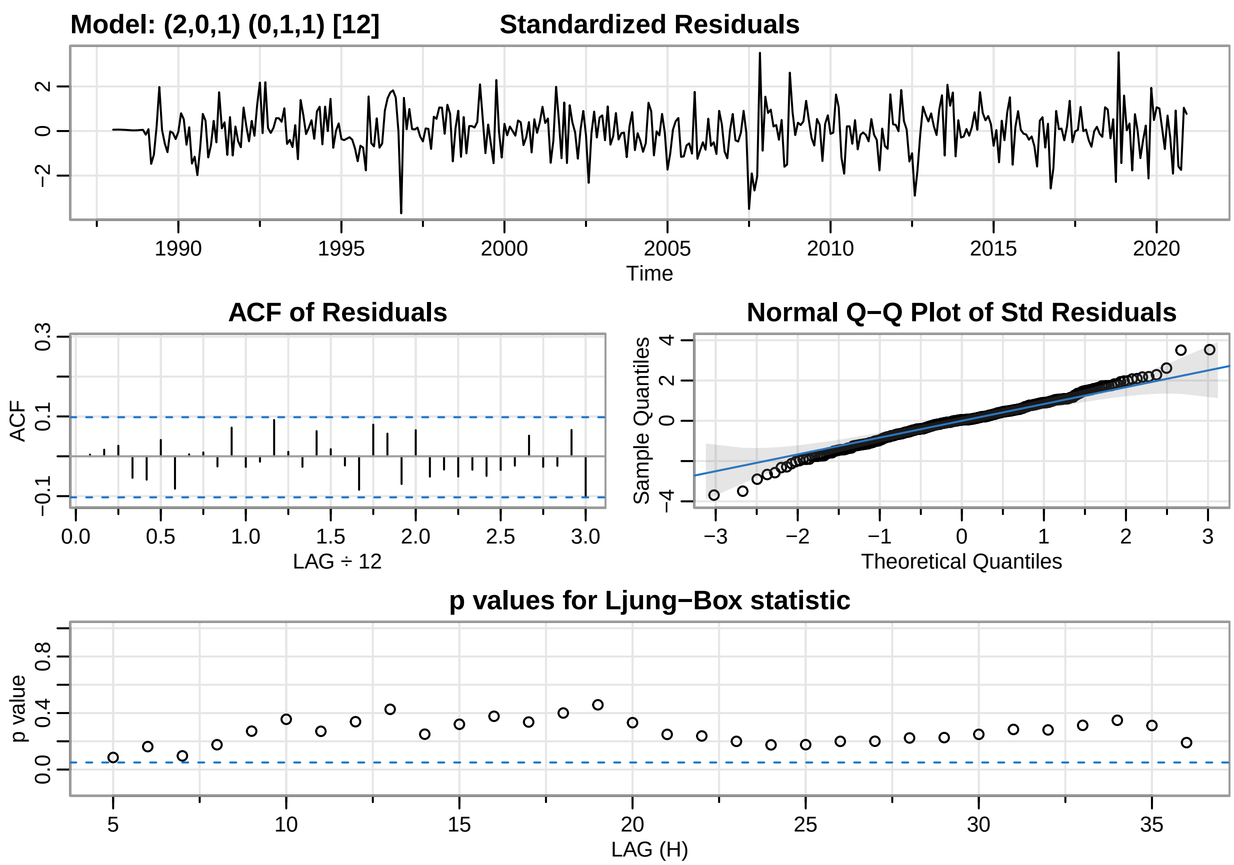

```
shapiro.test(model_31_train_residuals)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  model_31_train_residuals
## W = 0.98783, p-value = 0.002153
```

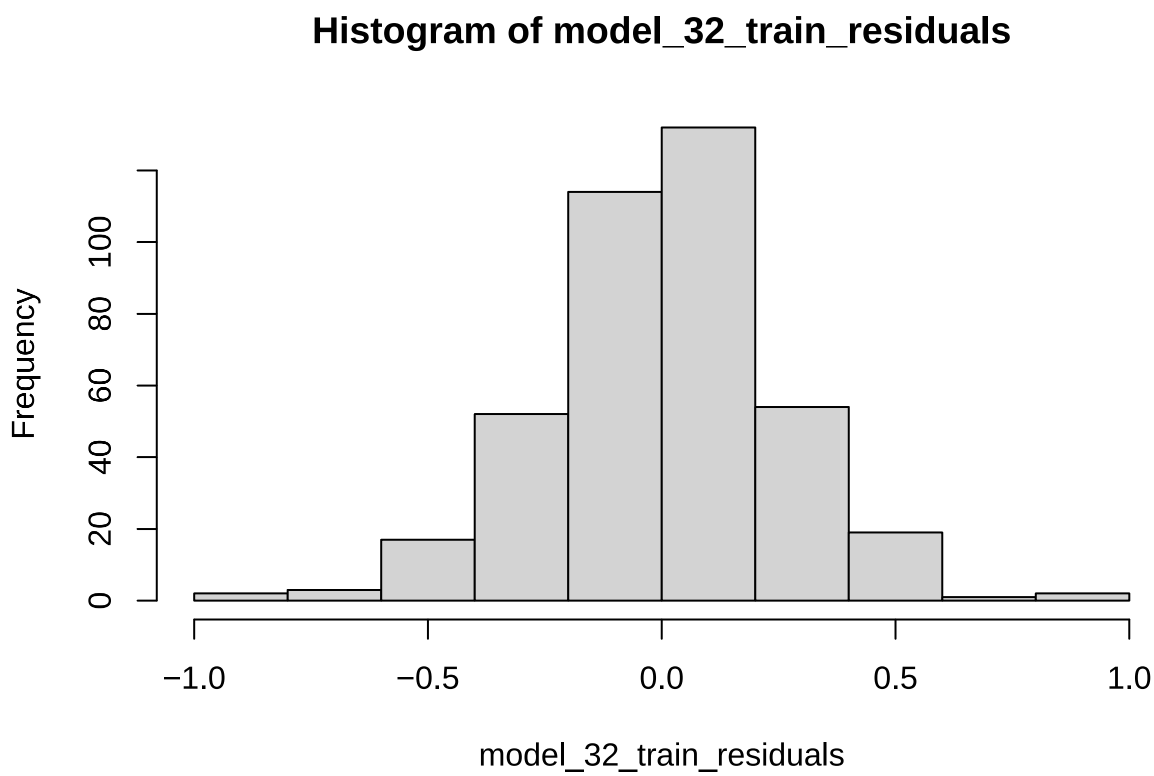
```
#SARIMA(2,0,1)x(0,1,1)_12
model_32_train <- sarima(Avg_ExtentTS_Train, p=2, d=0, q=1, P=0, D=1, Q=1, S=12 , details = TRUE)
```

```
## initial  value -0.799783
## iter   2 value -1.147624
## iter   3 value -1.332335
## iter   4 value -1.355514
## iter   5 value -1.366626
## iter   6 value -1.366675
## iter   7 value -1.366841
## iter   8 value -1.366857
## iter   9 value -1.366899
## iter  10 value -1.366998
## iter  11 value -1.367081
## iter  12 value -1.367109
## iter  13 value -1.367111
## iter  14 value -1.367112
## iter  14 value -1.367112
## iter  14 value -1.367112
## final  value -1.367112
## converged
## initial  value -1.365359
```

```
## iter   2 value -1.365381
## iter   3 value -1.365409
## iter   4 value -1.365409
## iter   5 value -1.365410
## iter   6 value -1.365412
## iter   7 value -1.365416
## iter   8 value -1.365420
## iter   9 value -1.365421
## iter  10 value -1.365421
## iter  10 value -1.365421
## iter  10 value -1.365421
## final  value -1.365421
## converged
```




```
model_32_train_residuals = resid(model_32_train$fit)
hist(model_32_train_residuals)
```



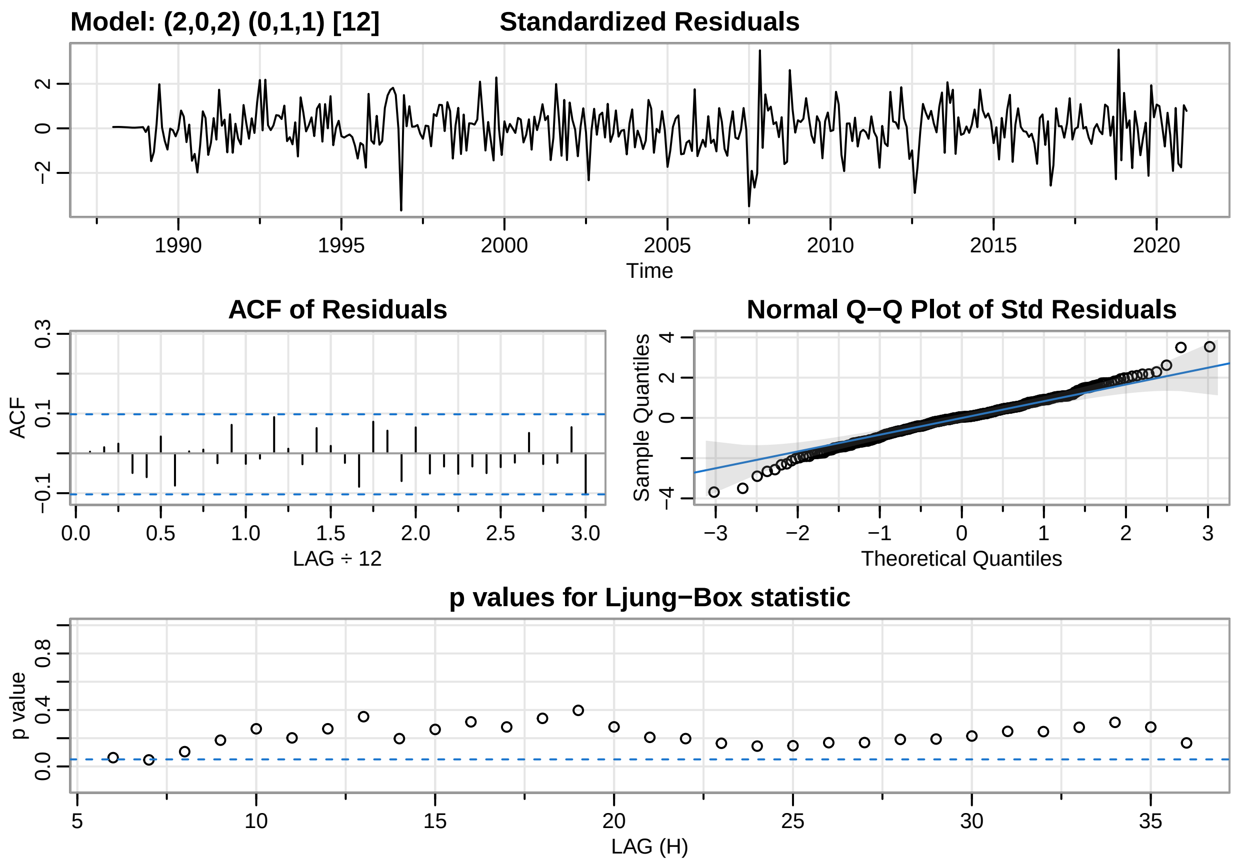

```
shapiro.test(model_32_train_residuals)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  model_32_train_residuals
## W = 0.98755, p-value = 0.001822
```

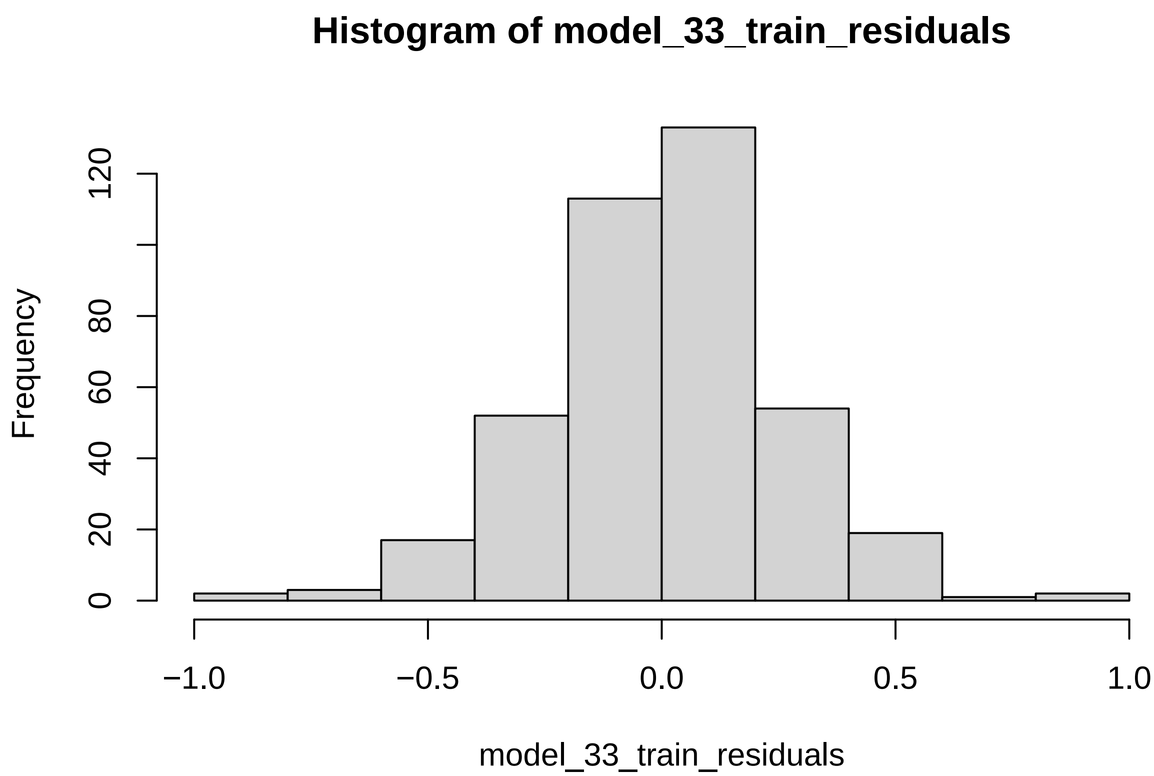
```
#SARIMA(2,0,2)x(0,1,1)_12
model_33_train <- sarima(Avg_ExtentTS_Train, p=2, d=0, q=2, P=0, D=1, Q=1, S=12 , details = TRUE)
```

```
## initial  value -0.799783
## iter   2 value -1.066061
## iter   3 value -1.228003
## iter   4 value -1.328895
## iter   5 value -1.356060
## iter   6 value -1.366108
## iter   7 value -1.366516
## iter   8 value -1.367171
## iter   9 value -1.367298
## iter  10 value -1.367313
## iter  11 value -1.367313
## iter  11 value -1.367313
## iter  11 value -1.367313
## final  value -1.367313
## converged
## initial  value -1.365536
## iter   2 value -1.365571
## iter   3 value -1.365588
## iter   4 value -1.365590
```

```
## iter   5 value -1.365591
## iter   6 value -1.365591
## iter   6 value -1.365591
## iter   6 value -1.365591
## final  value -1.365591
## converged
```




```
model_33_train_residuals = resid(model_33_train$fit)
hist(model_33_train_residuals)
```



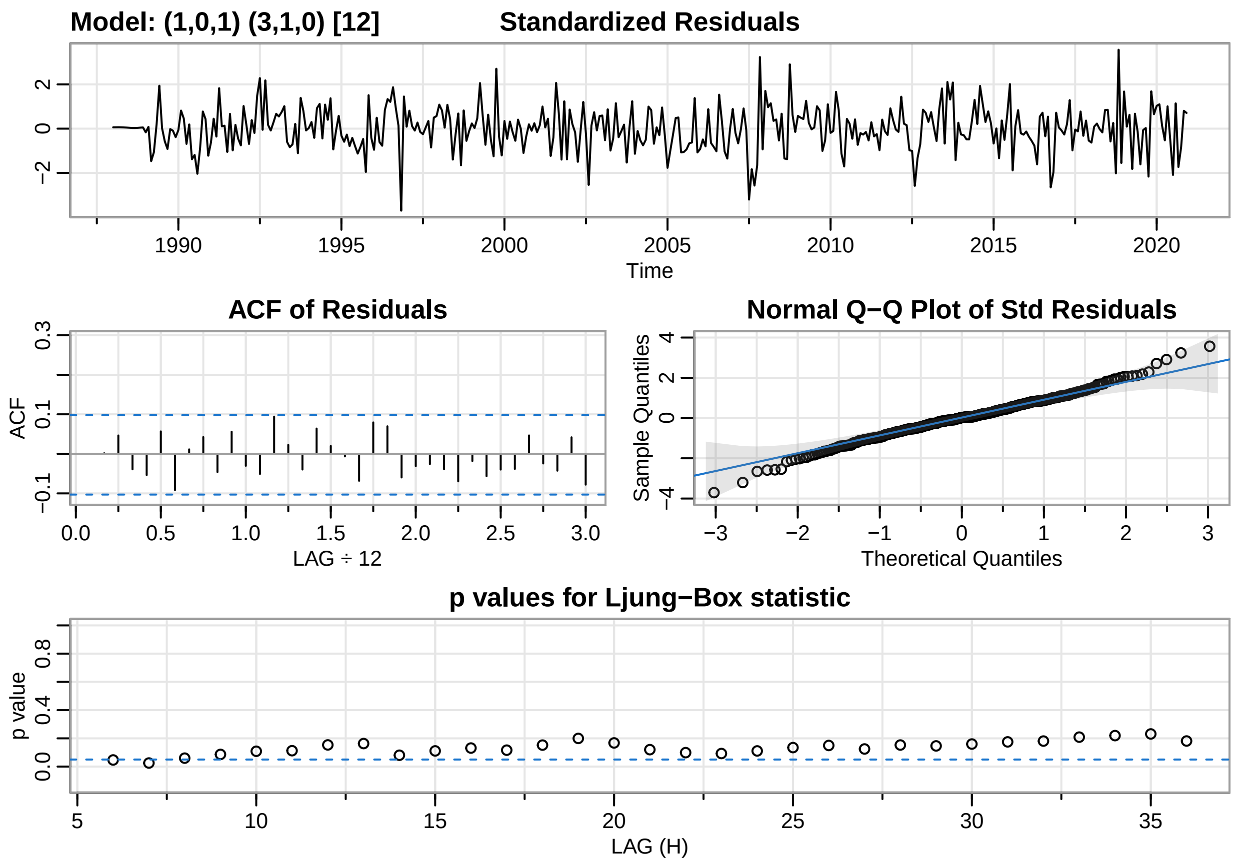

```
shapiro.test(model_33_train_residuals)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  model_33_train_residuals
## W = 0.98761, p-value = 0.001885
```

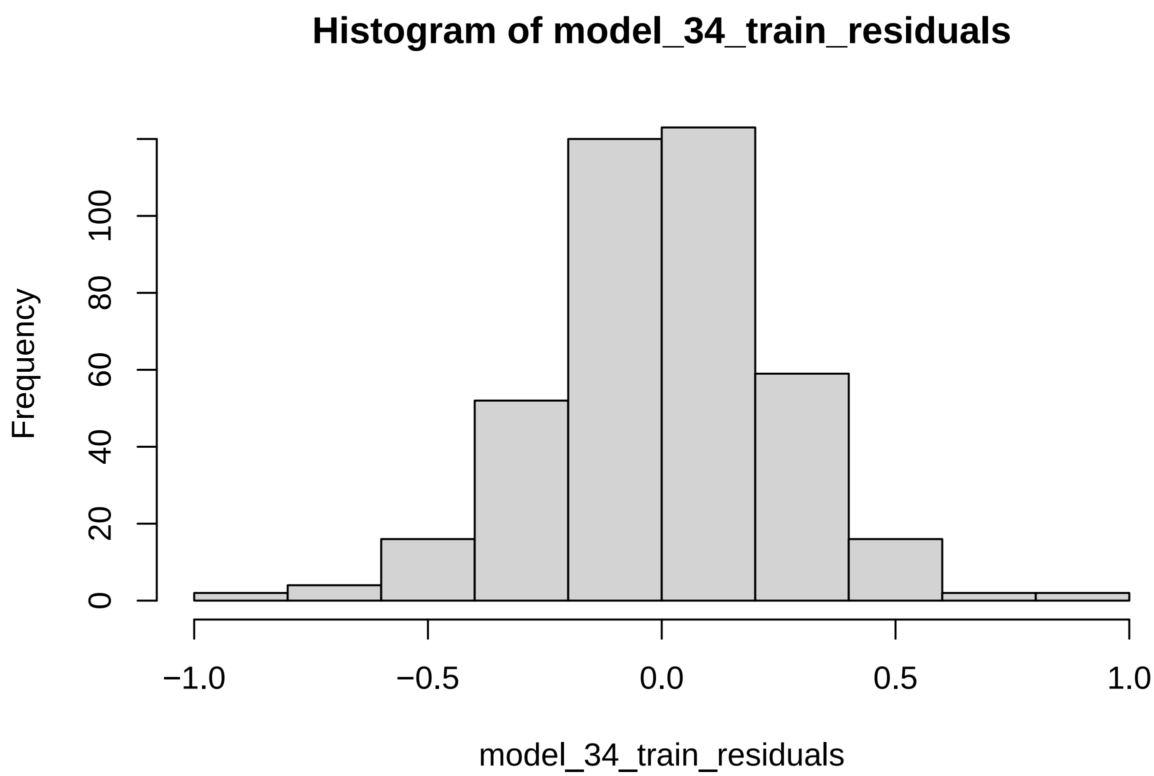
```
# Weird combo of optimal parameters for d=0, and d=1 (because had good results with d=D=1)
#SARIMA(1,0,1)x(3,1,0)_12
model_34_train <- sarima(Avg_ExtentTS_Train, p=1, d=0, q=1, P=3, D=1, Q=0, S=12 , details = TRUE)
```

```
## initial  value -0.795069
## iter   2 value -1.107339
## iter   3 value -1.327476
## iter   4 value -1.351082
## iter   5 value -1.353717
## iter   6 value -1.360317
## iter   7 value -1.360498
## iter   8 value -1.360526
## iter   9 value -1.360547
## iter  10 value -1.360548
## iter  11 value -1.360548
## iter  11 value -1.360548
## iter  11 value -1.360548
## final  value -1.360548
## converged
## initial  value -1.361106
## iter   2 value -1.361247
## iter   3 value -1.361301
```

```
## iter   4 value -1.361318
## iter   5 value -1.361322
## iter   6 value -1.361323
## iter   7 value -1.361324
## iter   8 value -1.361324
## iter   8 value -1.361324
## iter   8 value -1.361324
## final  value -1.361324
## converged
```




```
model_34_train_residuals = resid(model_34_train$fit)
hist(model_34_train_residuals)
```



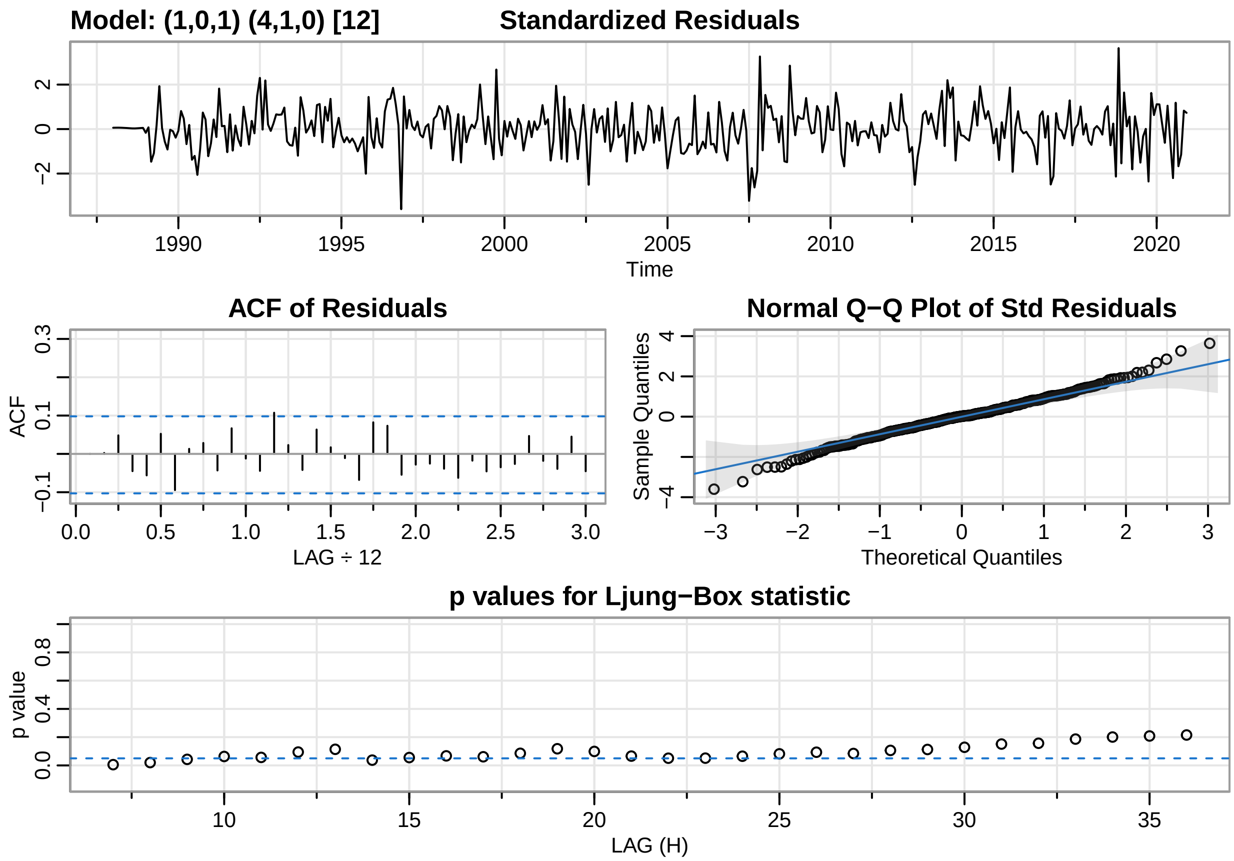

```
shapiro.test(model_34_train_residuals)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  model_34_train_residuals
## W = 0.99035, p-value = 0.01064
```

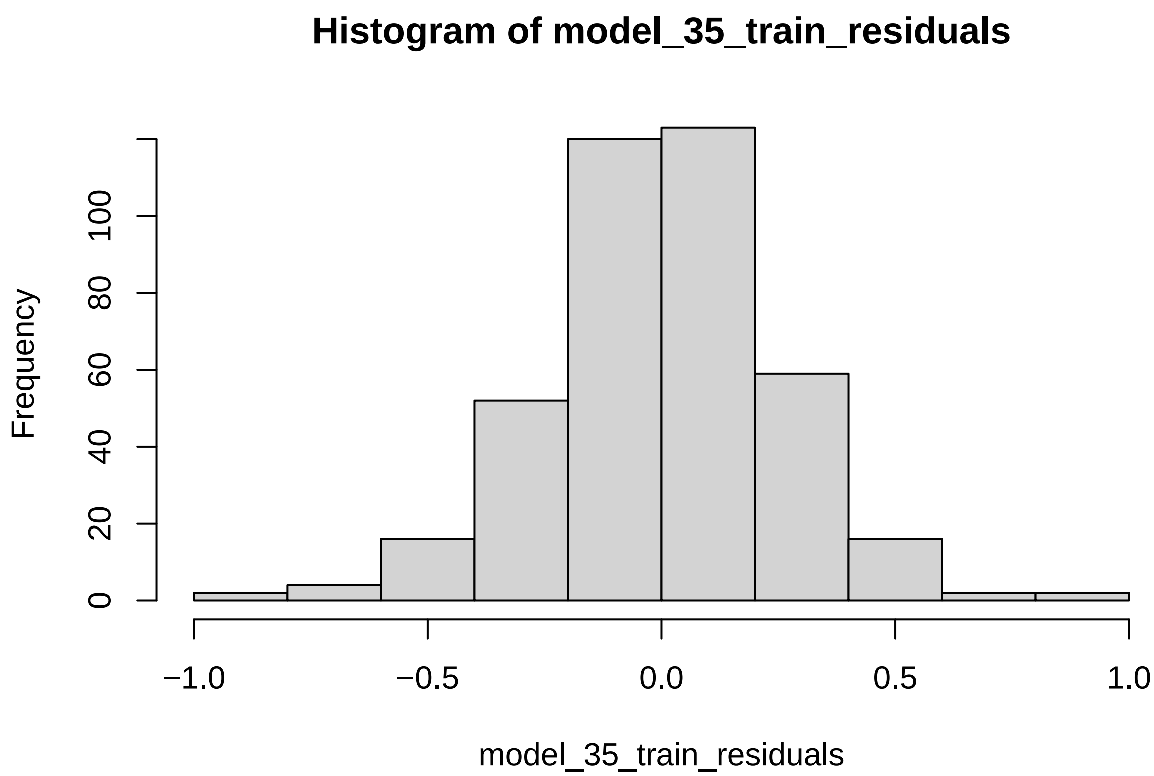
```
# Weird combo of optimal parameters for d=0, and d=1 (because had good results with d=D=1)
#SARIMA(1,0,1)x(3,1,0)_12
model_35_train <- sarima(Avg_ExtentTS_Train, p=1, d=0, q=1, P=4, D=1, Q=0, S=12 , details = TRUE)
```

```
## initial  value -0.804892
## iter   2 value -1.078475
## iter   3 value -1.326207
## iter   4 value -1.348467
## iter   5 value -1.357442
## iter   6 value -1.366662
## iter   7 value -1.367642
## iter   8 value -1.367689
## iter   9 value -1.367707
## iter  10 value -1.367713
## iter  11 value -1.367715
## iter  12 value -1.367715
## iter  12 value -1.367715
## iter  12 value -1.367715
## final  value -1.367715
## converged
## initial  value -1.365081
## iter   2 value -1.365423
```

```
## iter   3 value -1.365564
## iter   4 value -1.365571
## iter   5 value -1.365575
## iter   6 value -1.365577
## iter   7 value -1.365577
## iter   8 value -1.365577
## iter   8 value -1.365577
## iter   8 value -1.365577
## final  value -1.365577
## converged
```




```
model_35_train_residuals = resid(model_34_train$fit)
hist(model_35_train_residuals)
```




```
shapiro.test(model_35_train_residuals)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  model_35_train_residuals
## W = 0.99035, p-value = 0.01064
```

Summarize the fit of these models in a table.

```
library(huxtable)
goodness_of_fit <- hux(
        Model = c('SARIMA(1,0,1)x(0,1,1)_12', 'SARIMA(1,0,1)x(1,1,0)_12', 'SARIMA(1,0,1)x(0,1,3)_12', 'S
                  'SARIMA(13,0,0)x(0,1,1)_12', 'SARIMA(13,0,0)x(1,1,0)_12', 'SARIMA(13,0,0)x(0,1,3)_12'
                  'SARIMA(1,0,2)x(0,1,1)_12', 'SARIMA(2,0,1)x(0,1,1)_12', 'SARIMA(2,0,2)x(0,1,1)_12', 'S
        AIC = c(model_21_train$AIC, model_22_train$AIC, model_23_train$AIC, model_24_train$AIC, model_25
                model_26_train$AIC, model_27_train$AIC, model_28_train$AIC, model_29_train$AIC, model_30
                model_31_train$AIC, model_32_train$AIC, model_33_train$AIC, model_34_train$AIC, model_35
        AICc = c(model_21_train$AICc, model_22_train$AICc, model_23_train$AICc, model_24_train$AICc, mod
                model_26_train$AICc, model_27_train$AICc, model_28_train$AICc, model_29_train$AICc, mode
                model_31_train$AICc, model_32_train$AICc, model_33_train$AICc, model_34_train$AICc, mode
        BIC = c(model_21_train$BIC, model_22_train$BIC, model_23_train$BIC, model_24_train$BIC, model_25
                model_26_train$BIC, model_27_train$BIC, model_28_train$BIC, model_29_train$BIC, model_30
                model_31_train$BIC, model_32_train$BIC, model_33_train$BIC, model_34_train$BIC, model_35
        MSE = c(mean(model_21_train_residuals^2), mean(model_22_train_residuals^2), mean(model_23_train_
                mean(model_26_train_residuals^2), mean(model_27_train_residuals^2), mean(model_28_train_
                mean(model_31_train_residuals^2), mean(model_32_train_residuals^2), mean(model_33_train_
      )

goodness_of_fit %>%
  set_number_format(col=c(2,3,4,5), value=3) %>%
```

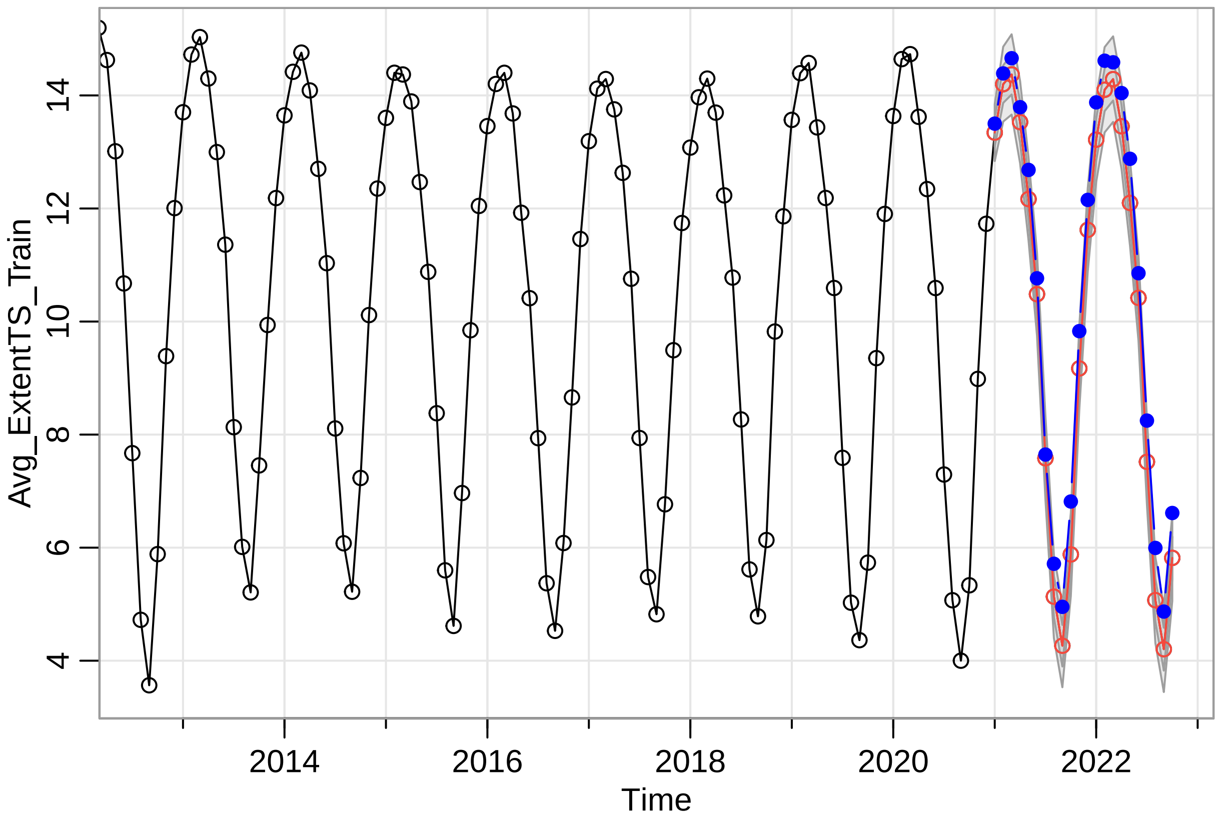
```r
set_bottom_border(c(1,11,14), everywhere) %>%
set_bold(c(2,4,5,6), everywhere) %>%
set_background_color(evens, everywhere, "grey95")
```

Model	AIC	AICc	BIC	MSE
SARIMA(1,0,1)x(0,1,1)__12	**0.133**	**0.134**	**0.185**	**0.061**
SARIMA(1,0,1)x(1,1,0)__12	0.250	0.250	0.301	0.070
SARIMA(1,0,1)x(0,1,3)__12	**0.137**	**0.137**	**0.209**	**0.061**
SARIMA(1,0,1)x(0,1,4)__12	**0.134**	**0.135**	**0.216**	**0.060**
SARIMA(1,0,1)x(1,1,1)__12	**0.138**	**0.138**	**0.200**	**0.061**
SARIMA(13,0,0)x(0,1,1)__12	0.163	0.166	0.327	0.060
SARIMA(13,0,0)x(1,1,0)__12	0.244	0.247	0.408	0.066
SARIMA(13,0,0)x(0,1,3)__12	0.146	0.150	0.331	0.058
SARIMA(13,0,0)x(0,1,4)__12	0.141	0.146	0.336	0.057
SARIMA(13,0,0)x(1,1,1)__12	0.152	0.156	0.327	0.059
SARIMA(1,0,2)x(0,1,1)__12	0.138	0.139	0.200	0.061
SARIMA(2,0,1)x(0,1,1)__12	0.138	0.139	0.200	0.061
SARIMA(2,0,2)x(0,1,1)__12	0.143	0.144	0.215	0.061
SARIMA(1,0,1)x(3,1,0)__12	0.152	0.152	0.224	0.062
SARIMA(1,0,1)x(4,1,0)__12	0.148	0.149	0.231	0.062

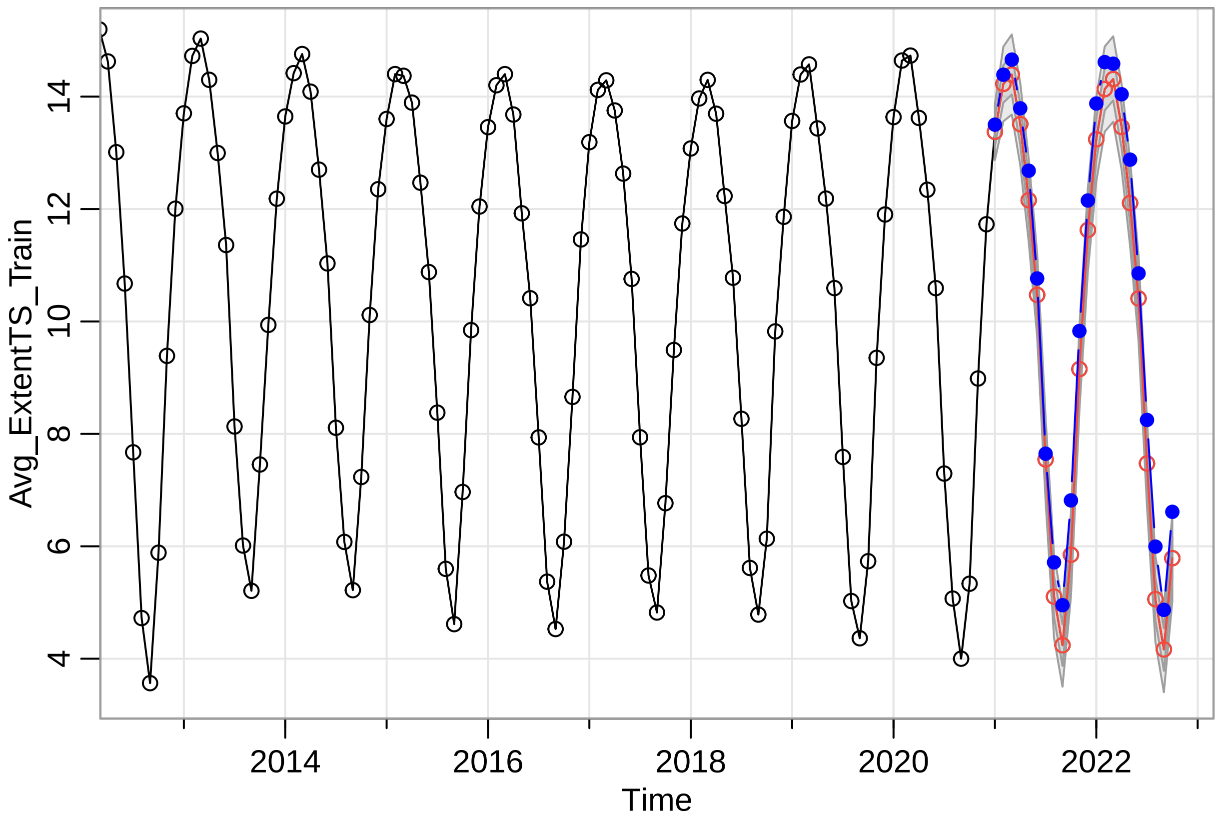
Model Selection

We evaluate performance on the test set of a few of the models which gave the best fit.

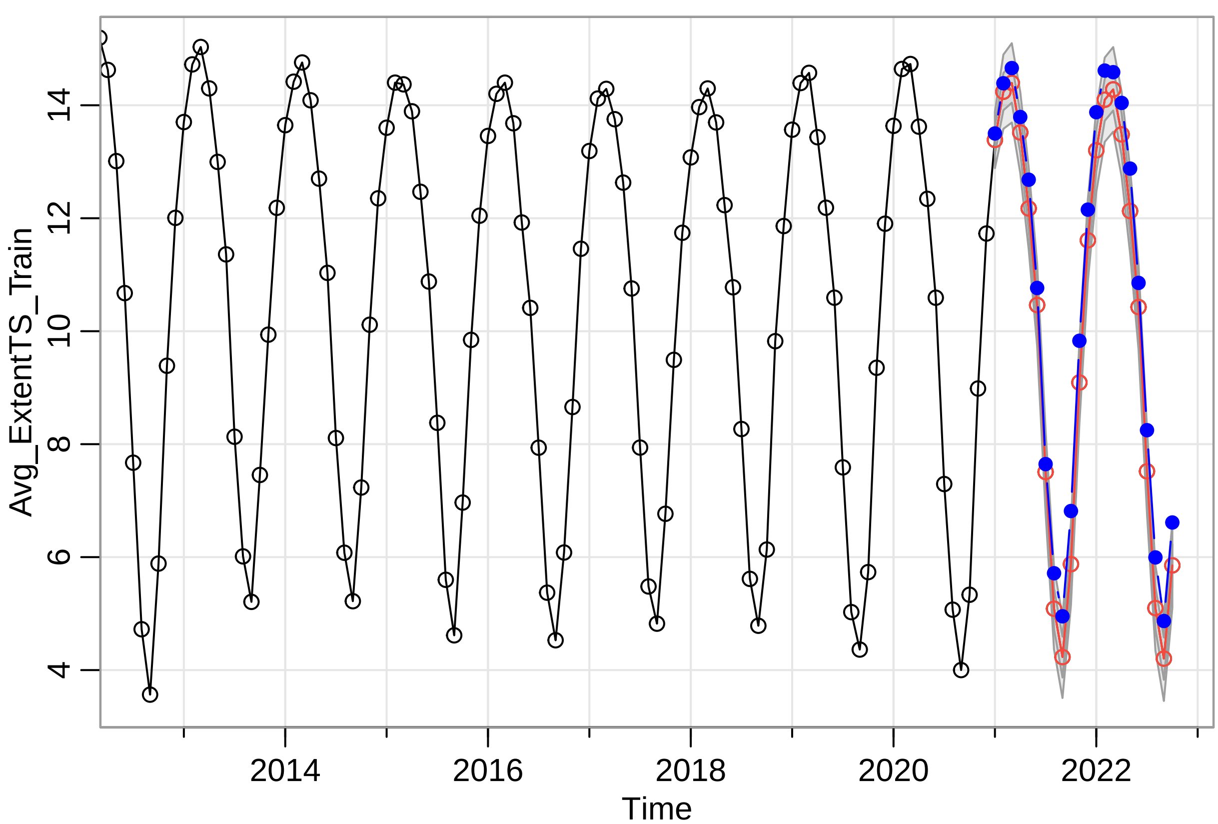
```
model_21_train_forecast<- sarima.for(Avg_ExtentTS_Train, n.ahead=22, p=1,d=0,q=1,P=0,D=1,Q=1,S=12)
lines(Avg_ExtentTS_Test,col='blue',type='b',pch=16)
```



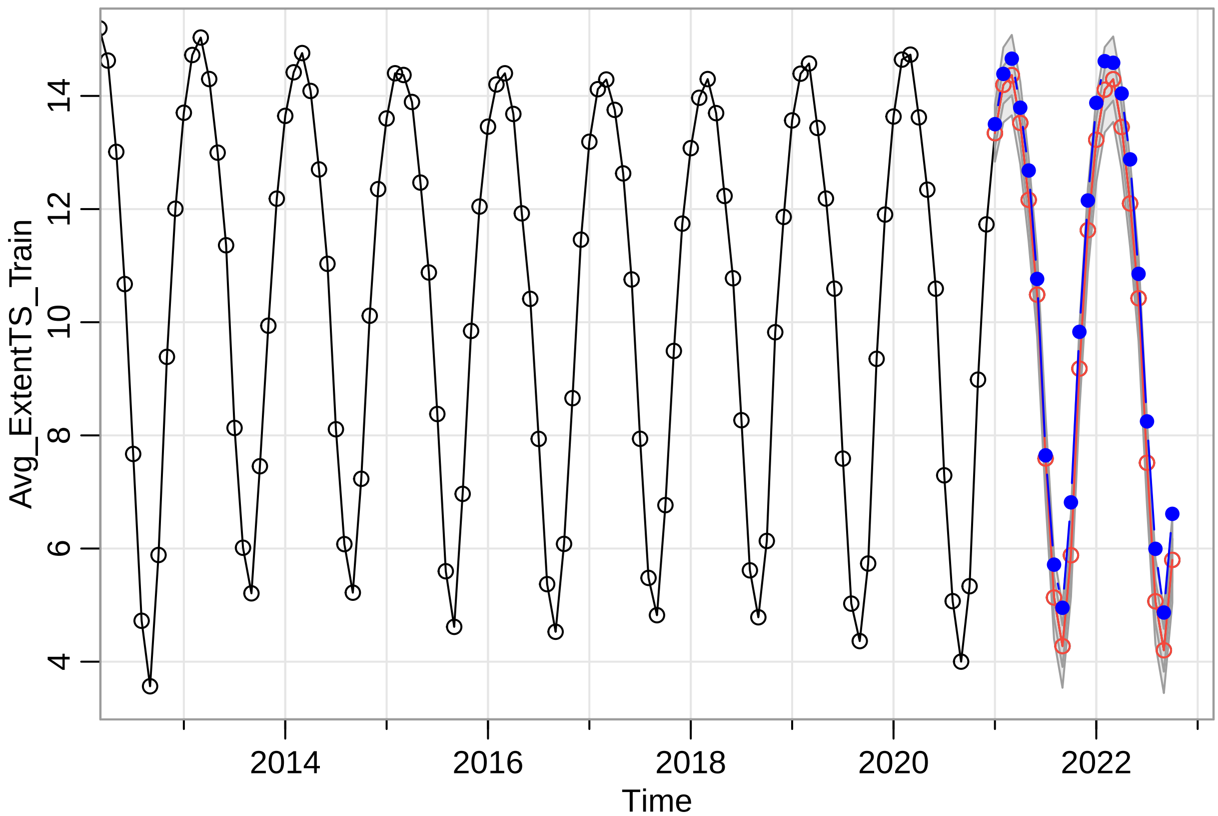

```
model_23_train_forecast<- sarima.for(Avg_ExtentTS_Train, n.ahead=22, p=1,d=0,q=1,P=0,D=1,Q=3,S=12)
lines(Avg_ExtentTS_Test,col='blue',type='b',pch=16)
```




```r
model_24_train_forecast<- sarima.for(Avg_ExtentTS_Train, n.ahead=22, p=1,d=0,q=1,P=0,D=1,Q=4,S=12)
lines(Avg_ExtentTS_Test,col='blue',type='b',pch=16)
```



```r
model_25_train_forecast<- sarima.for(Avg_ExtentTS_Train, n.ahead=22, p=1,d=0,q=1,P=1,D=1,Q=1,S=12)
lines(Avg_ExtentTS_Test,col='blue',type='b',pch=16)
```




```r
mean((model_21_train_forecast$pred-Avg_ExtentTS_Test)^2)
```

```
## [1] 0.3343123
```

```r
mean((model_23_train_forecast$pred-Avg_ExtentTS_Test)^2)
```

```
## [1] 0.3455438
```

```r
mean((model_24_train_forecast$pred-Avg_ExtentTS_Test)^2)
```

```
## [1] 0.3370083
```

```r
mean((model_25_train_forecast$pred-Avg_ExtentTS_Test)^2)
```

```
## [1] 0.3340769
```

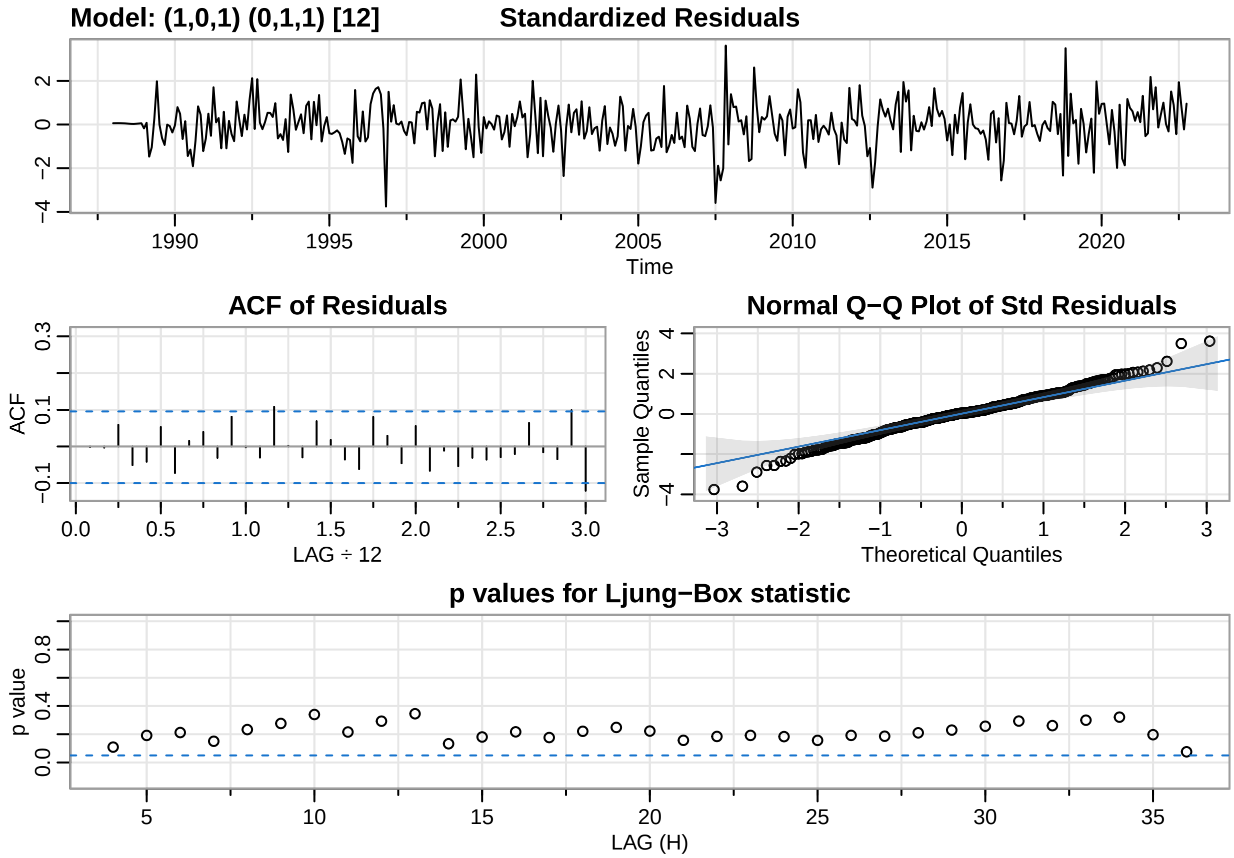
Summarize these results in a table.

```r
sarima_prediction <- hux(
        Model = c('SARIMA(1,0,1)x(0,1,1)_12', 'SARIMA(1,0,1)x(0,1,3)_12', 'SARIMA(1,0,1)x(0,1,4)_12', 'S
        PMSE = c(mean((model_21_train_forecast$pred-Avg_ExtentTS_Test)^2), mean((model_23_train_forecas
                 mean((model_24_train_forecast$pred-Avg_ExtentTS_Test)^2), mean((model_25_train_forecas

sarima_prediction %>%
  set_number_format(col=2, value=3) %>%
  set_bottom_border(1, everywhere) %>%
  set_background_color(evens, everywhere, "grey95")
```

Model	PMSE
SARIMA(1,0,1)x(0,1,1)_12	0.334
SARIMA(1,0,1)x(0,1,3)_12	0.346
SARIMA(1,0,1)x(0,1,4)_12	0.337
SARIMA(1,0,1)x(1,1,1)_12	0.334

```
# Best SARIMA model
model_21 <- sarima(Avg_ExtentTS, p=1, d=0, q=1, P=0, D=1, Q=1, S=12 , details = TRUE)
```

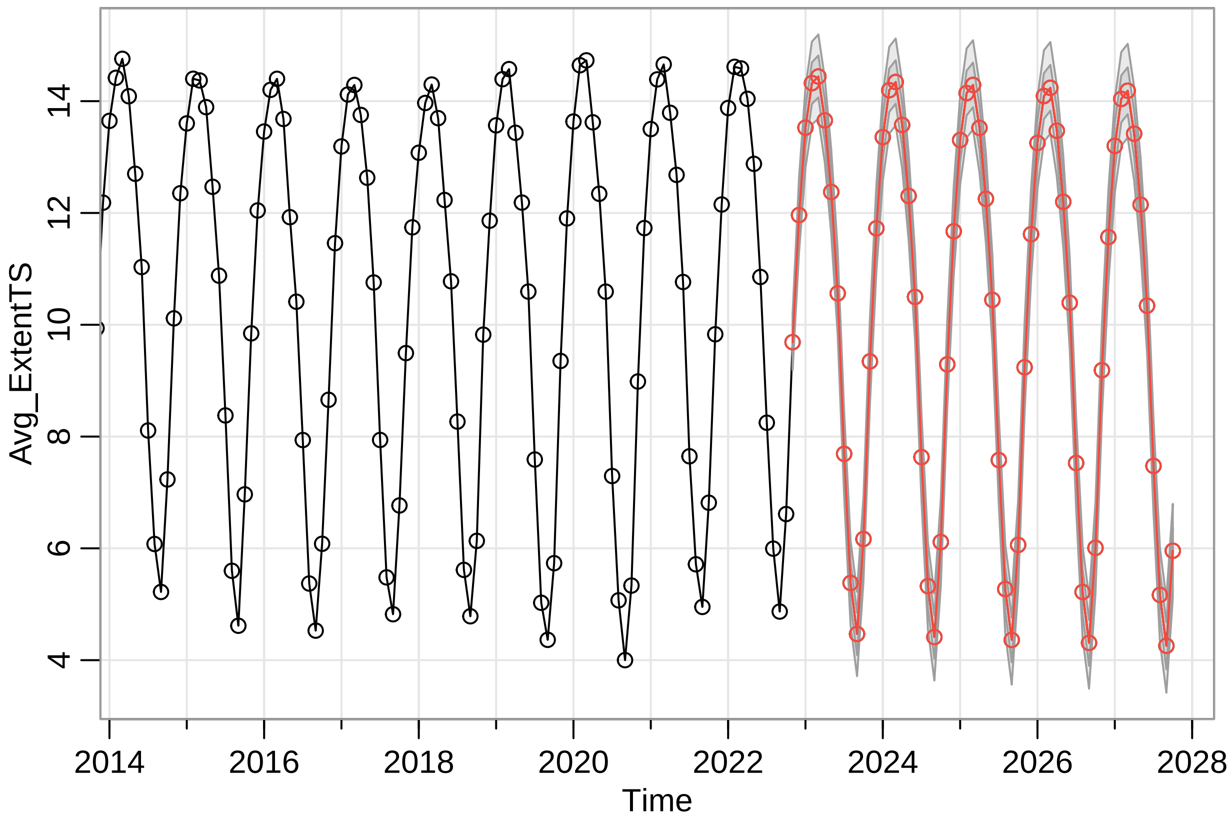
```
## initial  value -0.789422
## iter   2 value -1.114422
## iter   3 value -1.355131
## iter   4 value -1.356779
## iter   5 value -1.365174
## iter   6 value -1.367226
## iter   7 value -1.367725
## iter   8 value -1.367797
## iter   9 value -1.367800
## iter  10 value -1.367803
## iter  11 value -1.367803
## iter  12 value -1.367804
## iter  12 value -1.367804
## iter  12 value -1.367804
## final  value -1.367804
## converged
## initial  value -1.365801
## iter   2 value -1.365834
## iter   3 value -1.365859
## iter   4 value -1.365875
## iter   5 value -1.365876
## iter   5 value -1.365876
## iter   5 value -1.365876
## final  value -1.365876
## converged
```



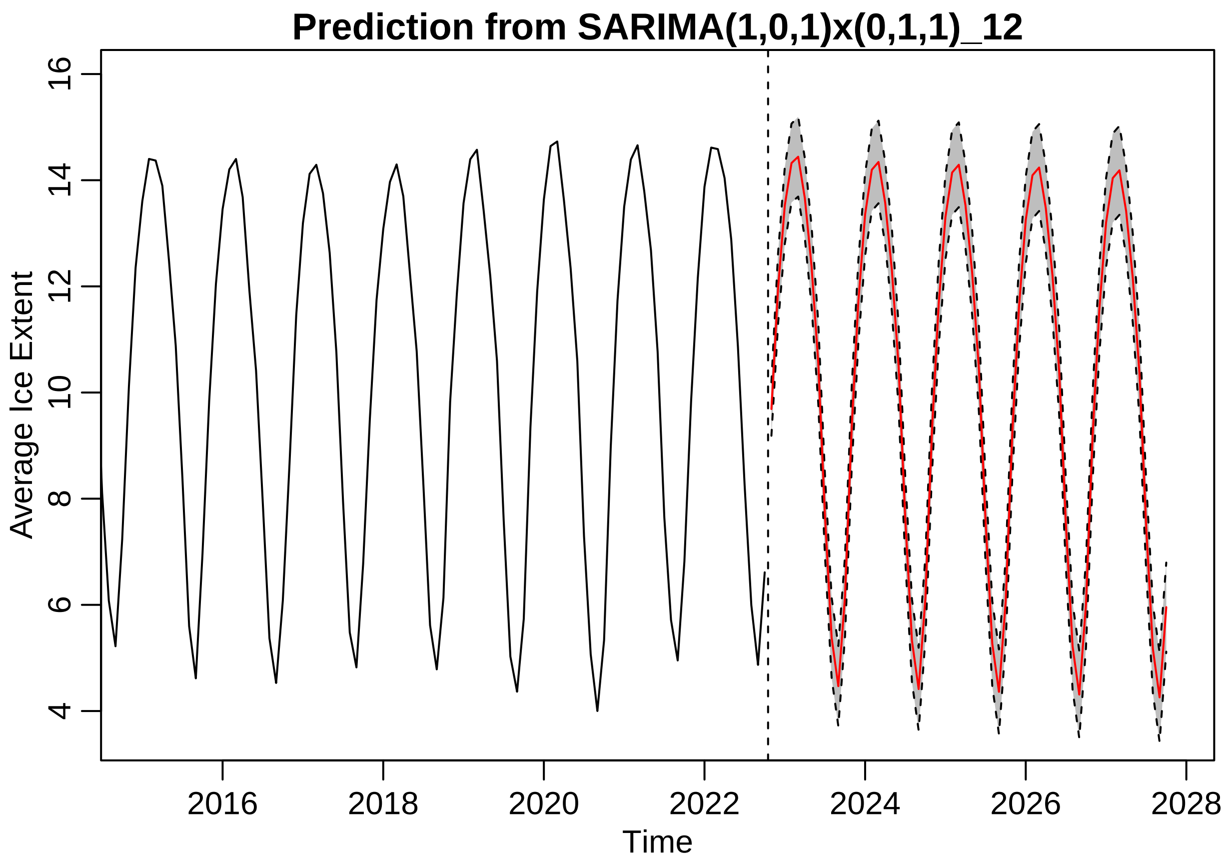

```r
model_21_residuals = resid(model_21$fit)
shapiro.test(model_21_residuals)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  model_21_residuals
## W = 0.9883, p-value = 0.001968
```

```r
# Getting predictions for best SARIMA and plotting
pred_model_21 = sarima.for(Avg_ExtentTS, n.ahead=60, p=1, d=0, q=1 , P=0, D=1, Q=1, S=12)
```




```
Upper_Limit = pred_model_21$pred + 2*pred_model_21$se # upper prediction band
Lower_Limit = pred_model_21$pred - 2*pred_model_21$se # lower prediction band
plot(Avg_ExtentTS , xlim = c(2015 , 2027+10/12), ylab="Average Ice Extent", main="Prediction from SARIMA

#The three lines below plot the prediction interval in a grey scale
x = c(time(Upper_Limit) , rev(time(Upper_Limit)))
y = c(Upper_Limit , rev(Lower_Limit))
polygon(x, y, col="grey", border=NA)

#The three line below add the predicted values and highlight the borders of the prediction interval
lines(Upper_Limit, col="black" , lty=2)
lines(Lower_Limit, col="black", lty=2)
lines(pred_model_21$pred , col="red")
abline(v=2022+9.5/12, lty="dashed")
```




```r
model_21_residuals = resid(model_21$fit)
shapiro.test(model_21_residuals)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  model_21_residuals
## W = 0.9883, p-value = 0.001968
```

Summary of prediction from best sarima model.

```r
summary_prediction <- summary(pred_model_21$pred)
summary <- as.data.frame(rbind(matrix(summary_prediction,nrow=1)))
colnames(summary) <- c("Min", "First Quartile", "Median", "Mean", "Third Quartile", "Max")

summary_table <- hux(summary)

summary_table %>%
  set_number_format(3) %>%
  set_align(everywhere, everywhere, "center") %>%
  set_bottom_border(1, everywhere) %>%
  set_bold(1, everywhere)
```

Min	**First Quartile**	**Median**	**Mean**	**Third Quartile**	**Max**
4.257	7.148	11.065	10.197	13.431	14.442

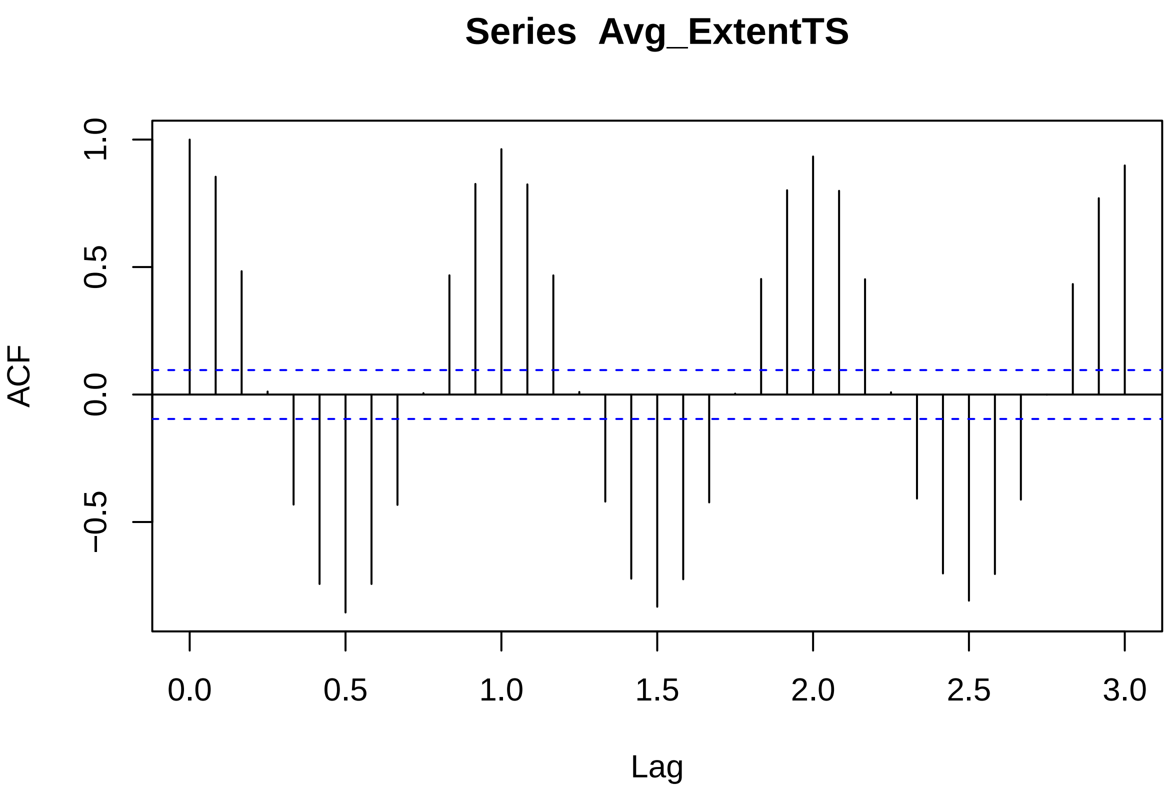
First year of prediction from best sarima model.

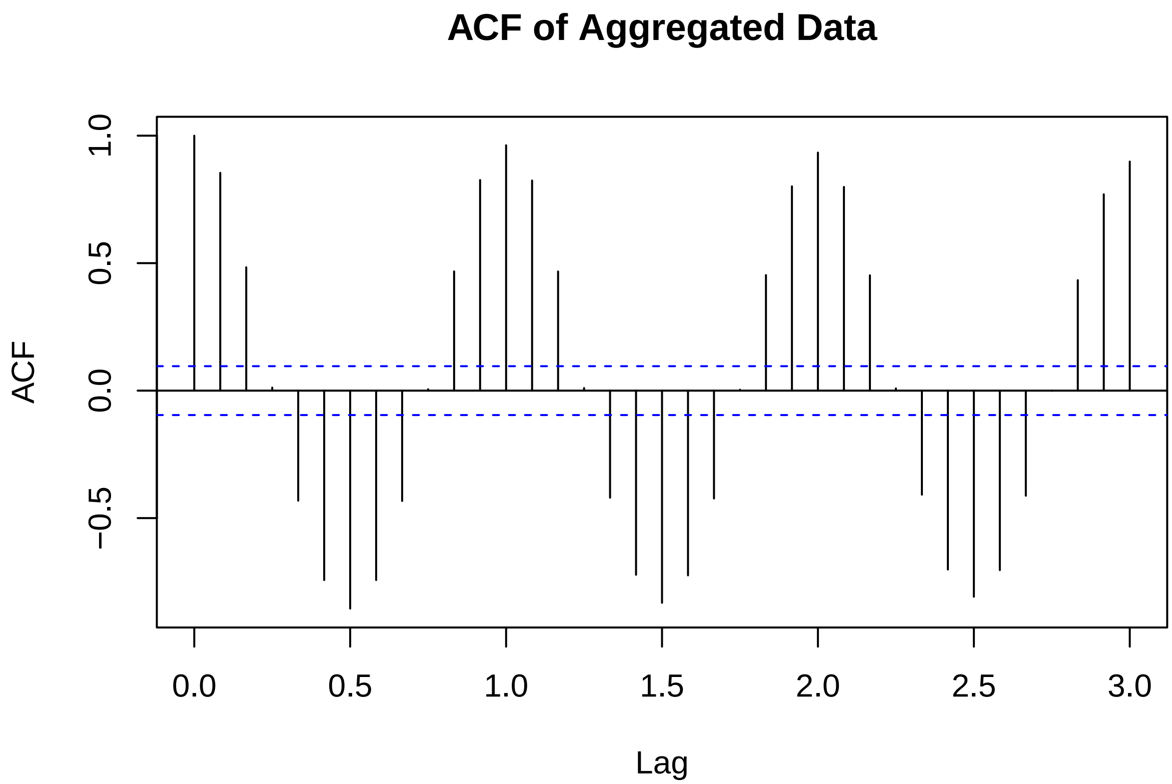
```
prediction <- as.data.frame(rbind(matrix(head(pred_model_21$pred, 12),nrow=1)))
colnames(prediction) <- c("Nov.", "Dec.", "Jan.", "Feb.", "Mar.", "Apr.", "May", "Jun.","Jul.", "Aug.",

prediction_table <- hux(prediction)

prediction_table %>%
  set_number_format(3) %>%
  set_align(everywhere, everywhere, "center") %>%
  set_bottom_border(1, everywhere) %>%
  set_bold(1, everywhere) %>%
  print_latex(tabular_only=TRUE)
```

```
##
##
## ```{=latex}
##
##   \providecommand{\huxb}[2]{\arrayrulecolor[RGB]{#1}\global\arrayrulewidth=#2pt}
##   \providecommand{\huxvb}[2]{\color[RGB]{#1}\vrule width #2pt}
##   \providecommand{\huxtpad}[1]{\rule{0pt}{#1}}
##   \providecommand{\huxbpad}[1]{\rule[-#1]{0pt}{#1}}
## \begin{tabular}{l l l l l l l l l l l l}
##
##
## \hhline{}
## \arrayrulecolor{black}
##
## \multicolumn{1}{!{\huxvb{0, 0, 0}{0}}c!{\huxvb{0, 0, 0}{0}}}{\huxtpad{6pt + 1em}\centering \hspace{6
## \multicolumn{1}{c!{\huxvb{0, 0, 0}{0}}}{\huxtpad{6pt + 1em}\centering \hspace{6pt} \textbf{Dec.} \hs
## \multicolumn{1}{c!{\huxvb{0, 0, 0}{0}}}{\huxtpad{6pt + 1em}\centering \hspace{6pt} \textbf{Jan.} \hs
## \multicolumn{1}{c!{\huxvb{0, 0, 0}{0}}}{\huxtpad{6pt + 1em}\centering \hspace{6pt} \textbf{Feb.} \hs
## \multicolumn{1}{c!{\huxvb{0, 0, 0}{0}}}{\huxtpad{6pt + 1em}\centering \hspace{6pt} \textbf{Mar.} \hs
## \multicolumn{1}{c!{\huxvb{0, 0, 0}{0}}}{\huxtpad{6pt + 1em}\centering \hspace{6pt} \textbf{Apr.} \hs
## \multicolumn{1}{c!{\huxvb{0, 0, 0}{0}}}{\huxtpad{6pt + 1em}\centering \hspace{6pt} \textbf{May} \hsp
## \multicolumn{1}{c!{\huxvb{0, 0, 0}{0}}}{\huxtpad{6pt + 1em}\centering \hspace{6pt} \textbf{Jun.} \hs
## \multicolumn{1}{c!{\huxvb{0, 0, 0}{0}}}{\huxtpad{6pt + 1em}\centering \hspace{6pt} \textbf{Jul.} \hs
## \multicolumn{1}{c!{\huxvb{0, 0, 0}{0}}}{\huxtpad{6pt + 1em}\centering \hspace{6pt} \textbf{Aug.} \hs
## \multicolumn{1}{c!{\huxvb{0, 0, 0}{0}}}{\huxtpad{6pt + 1em}\centering \hspace{6pt} \textbf{Sep.} \hs
## \multicolumn{1}{c!{\huxvb{0, 0, 0}{0}}}{\huxtpad{6pt + 1em}\centering \hspace{6pt} \textbf{Oct.} \hs
##
##
## \hhline{>{\huxb{0, 0, 0}{0.4}}->{\huxb{0, 0, 0}{0.4}}->{\huxb{0, 0, 0}{0.4}}->{\huxb{0, 0, 0}{0.4}}-
## \arrayrulecolor{black}
##
## \multicolumn{1}{!{\huxvb{0, 0, 0}{0}}c!{\huxvb{0, 0, 0}{0}}}{\huxtpad{6pt + 1em}\centering \hspace{6
## \multicolumn{1}{c!{\huxvb{0, 0, 0}{0}}}{\huxtpad{6pt + 1em}\centering \hspace{6pt} 11.962 \hspace{6p
## \multicolumn{1}{c!{\huxvb{0, 0, 0}{0}}}{\huxtpad{6pt + 1em}\centering \hspace{6pt} 13.526 \hspace{6p
## \multicolumn{1}{c!{\huxvb{0, 0, 0}{0}}}{\huxtpad{6pt + 1em}\centering \hspace{6pt} 14.323 \hspace{6p
## \multicolumn{1}{c!{\huxvb{0, 0, 0}{0}}}{\huxtpad{6pt + 1em}\centering \hspace{6pt} 14.442 \hspace{6p
## \multicolumn{1}{c!{\huxvb{0, 0, 0}{0}}}{\huxtpad{6pt + 1em}\centering \hspace{6pt} 13.656 \hspace{6p
## \multicolumn{1}{c!{\huxvb{0, 0, 0}{0}}}{\huxtpad{6pt + 1em}\centering \hspace{6pt} 12.376 \hspace{6p
## \multicolumn{1}{c!{\huxvb{0, 0, 0}{0}}}{\huxtpad{6pt + 1em}\centering \hspace{6pt} 10.562 \hspace{6p
## \multicolumn{1}{c!{\huxvb{0, 0, 0}{0}}}{\huxtpad{6pt + 1em}\centering \hspace{6pt} 7.691 \hspace{6pt
## \multicolumn{1}{c!{\huxvb{0, 0, 0}{0}}}{\huxtpad{6pt + 1em}\centering \hspace{6pt} 5.381 \hspace{6pt
```

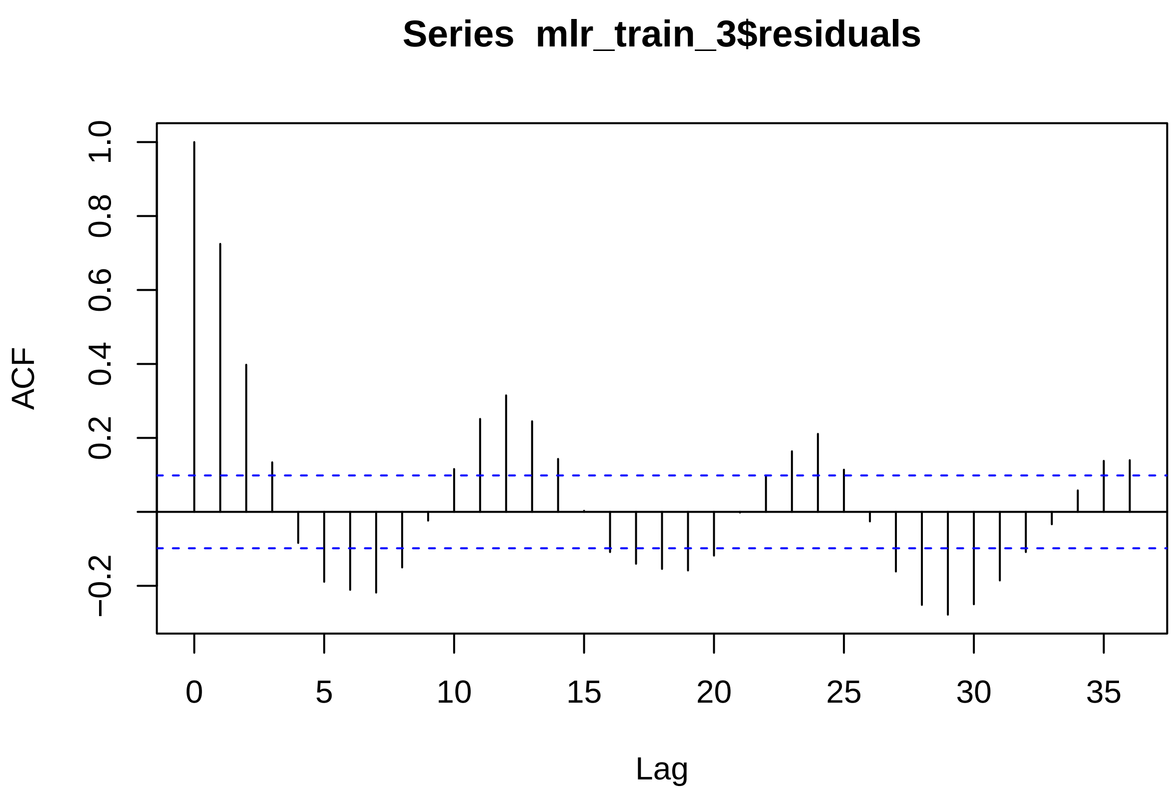
```
## \multicolumn{1}{c!{\huxvb{0, 0, 0}{0}}}{\huxtpad{6pt + 1em}\centering \hspace{6pt} 4.468 \hspace{6pt
## \multicolumn{1}{c!{\huxvb{0, 0, 0}{0}}}{\huxtpad{6pt + 1em}\centering \hspace{6pt} 6.168 \hspace{6pt
##
##
## \hhline{}
## \arrayrulecolor{black}
## \end{tabular}
## ```
```

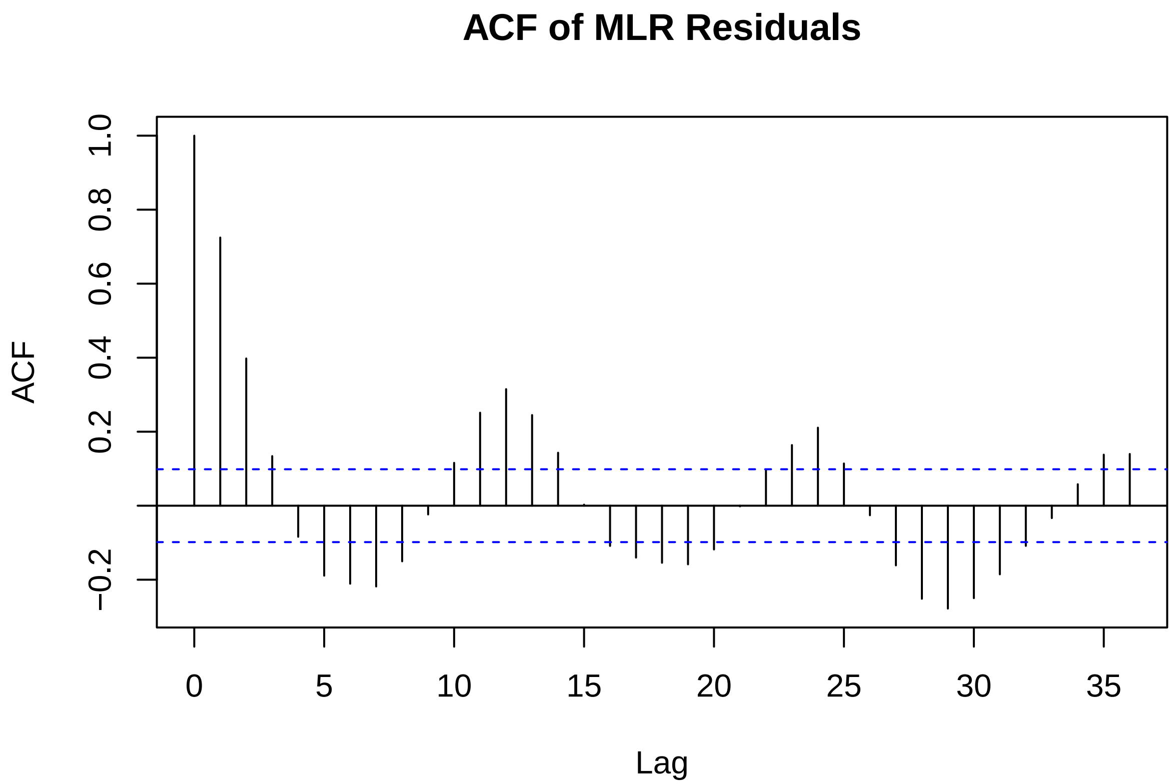
```
# code used for additional plots in report
plot(acf(Avg_ExtentTS, lag.max=36), main="ACF of Aggregated Data")
```



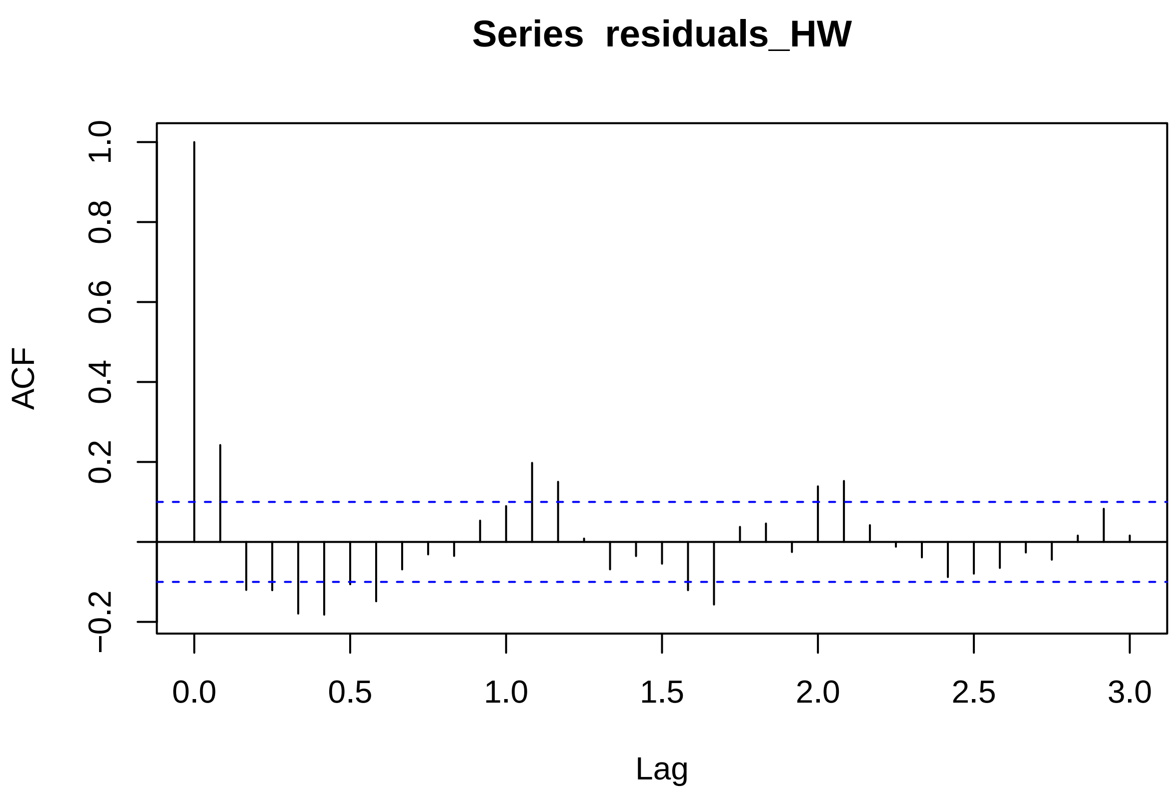


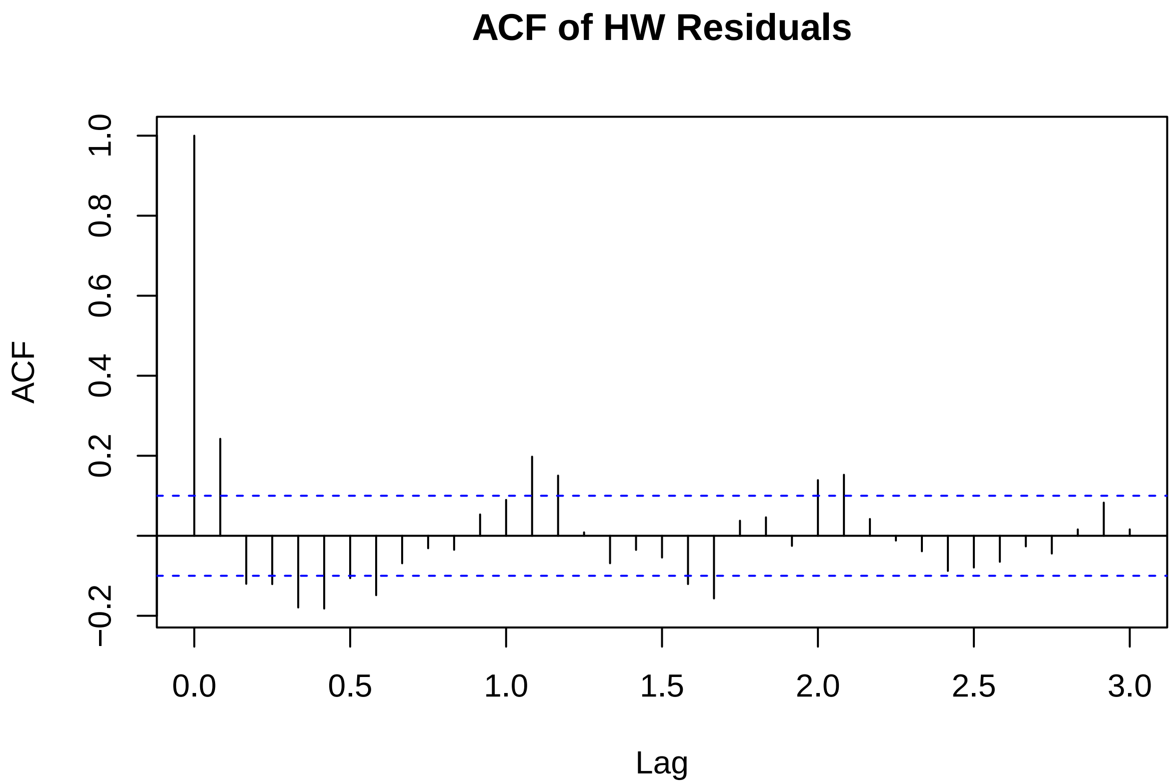

```
plot(acf(mlr_train_3$residuals, lag.max=36), main="ACF of MLR Residuals")
```



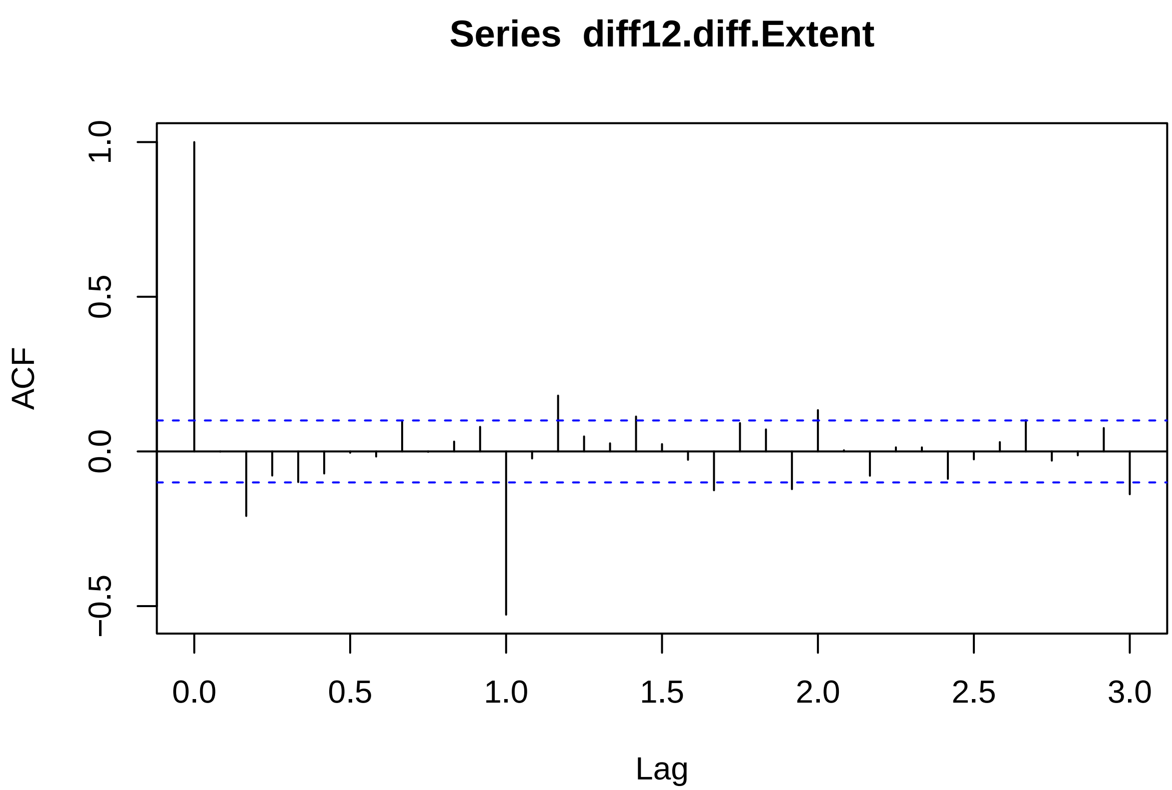



```r
plot(acf(residuals_HW, lag.max=36), main="ACF of HW Residuals")
```

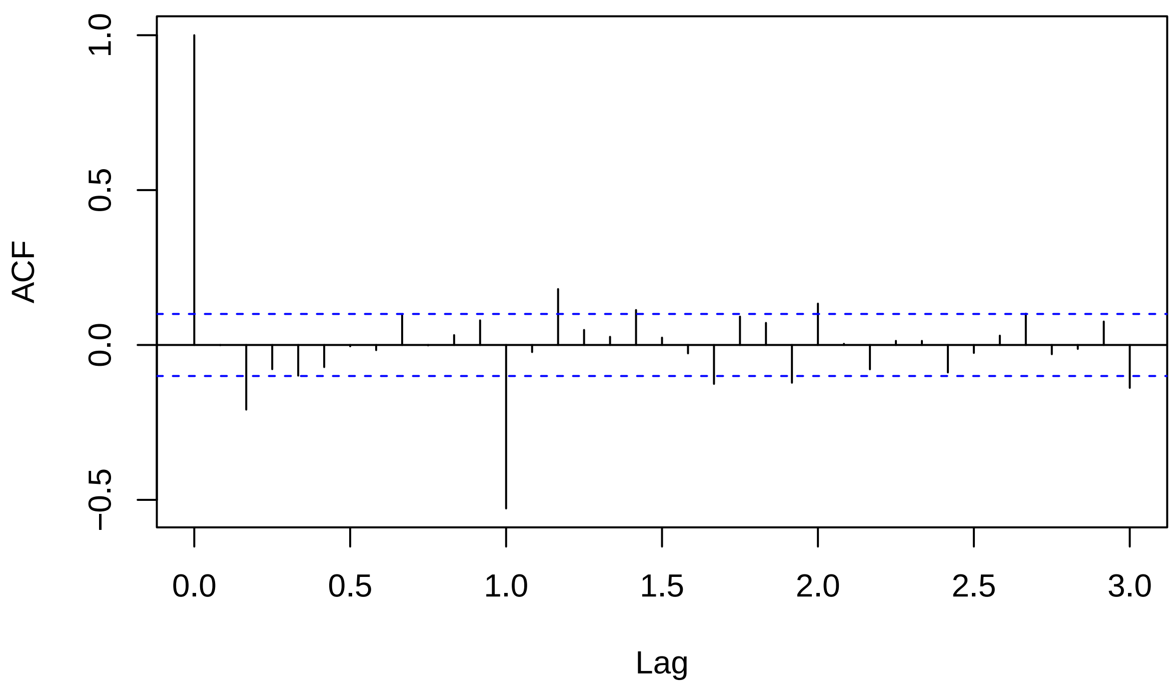





```
plot(acf(diff12.diff.Extent, lag.max=36), main="ACF of Differenced Data (Seasonal+Regular)")
```

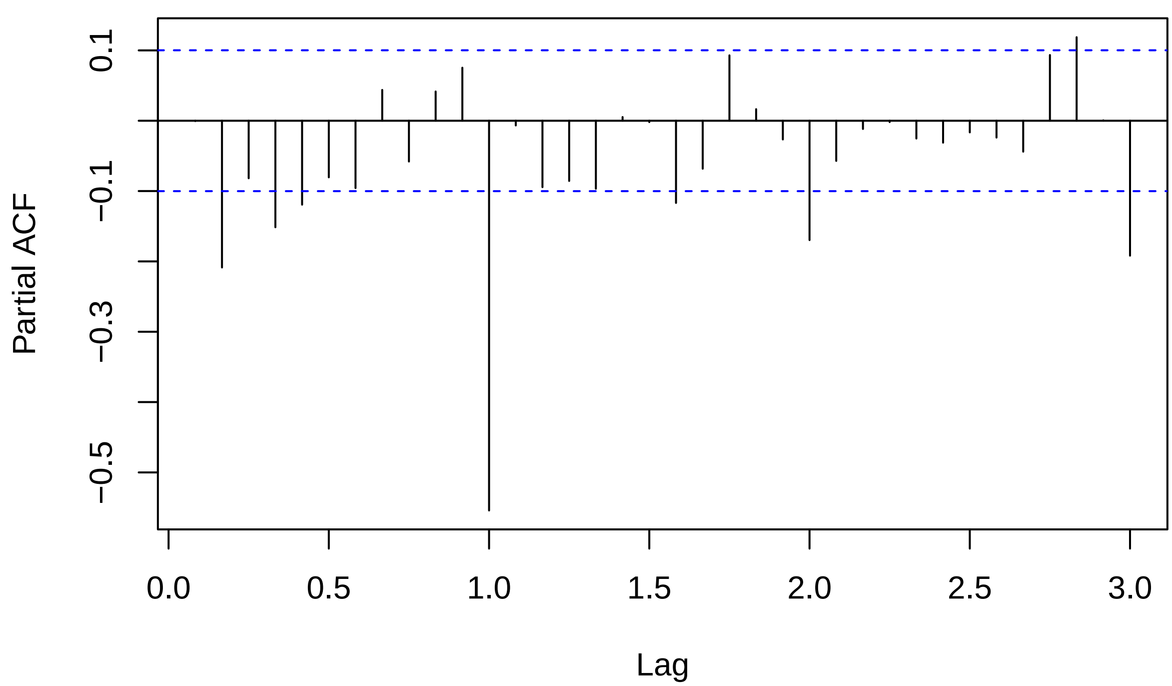


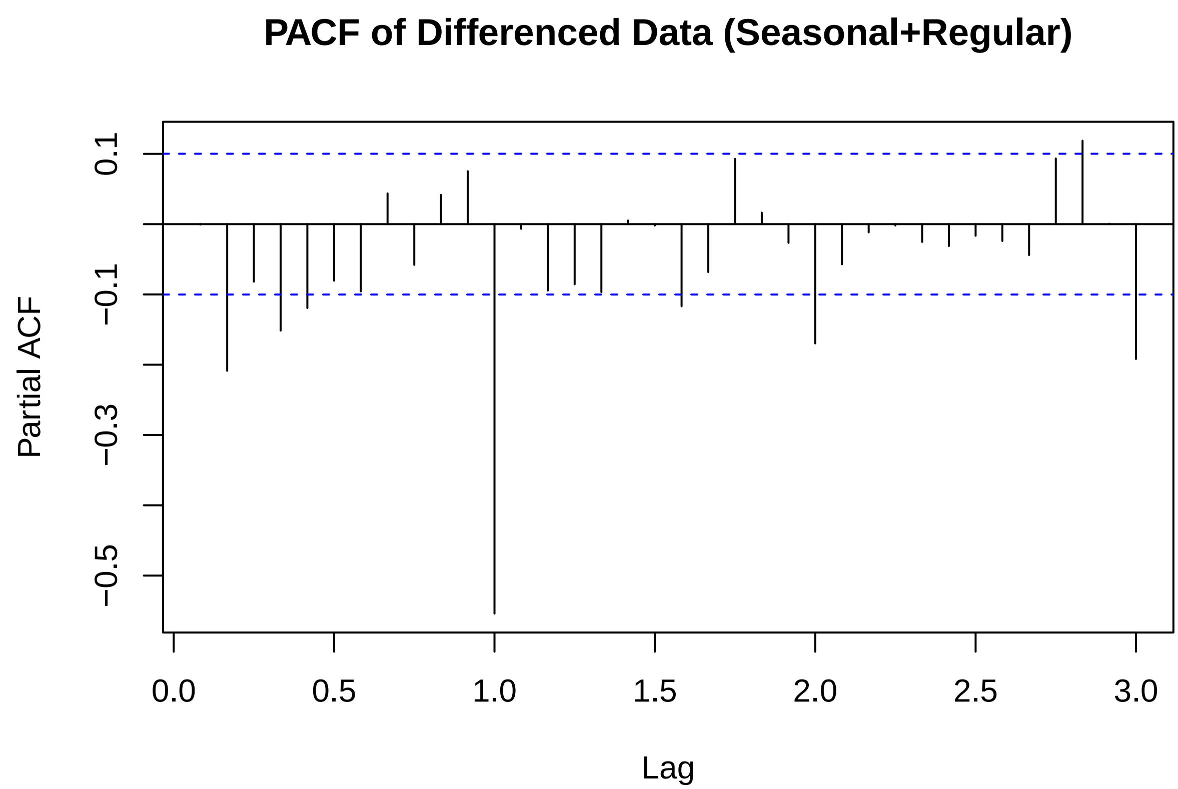
ACF of Differenced Data (Seasonal+Regular)



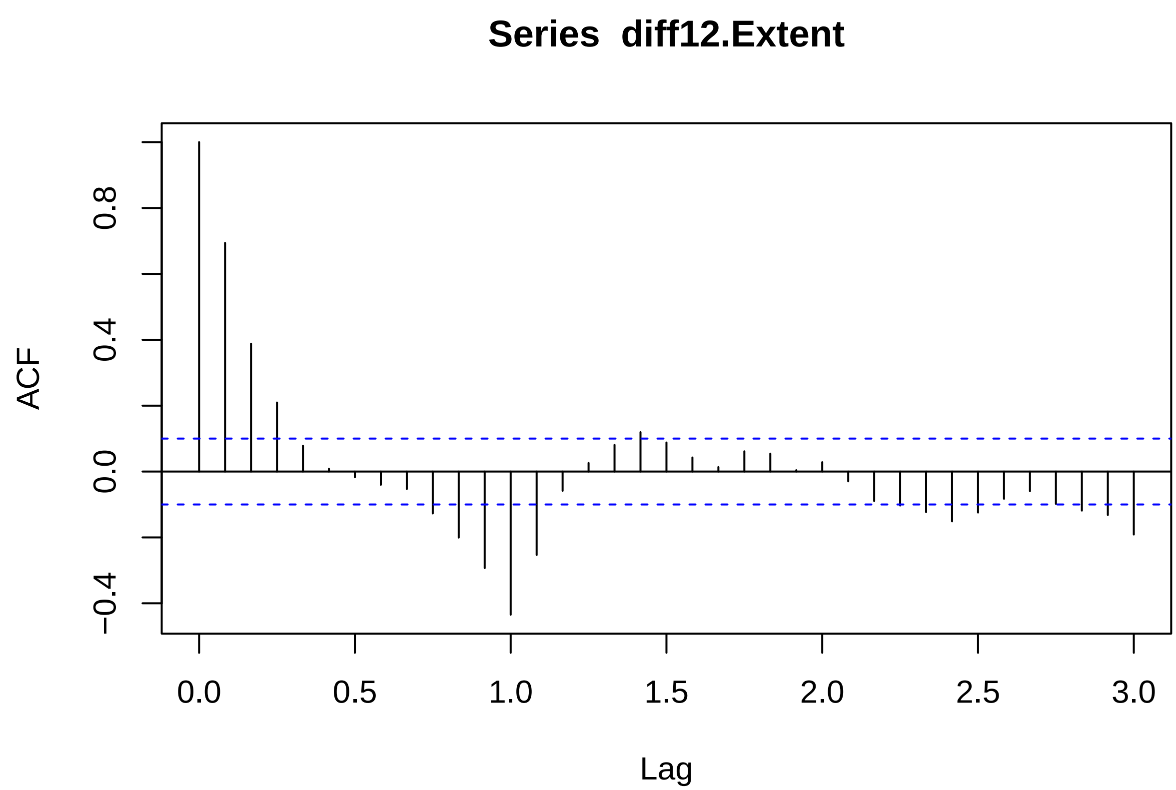

```r
plot(pacf(diff12.diff.Extent, lag.max=36), main="PACF of Differenced Data (Seasonal+Regular)")
```

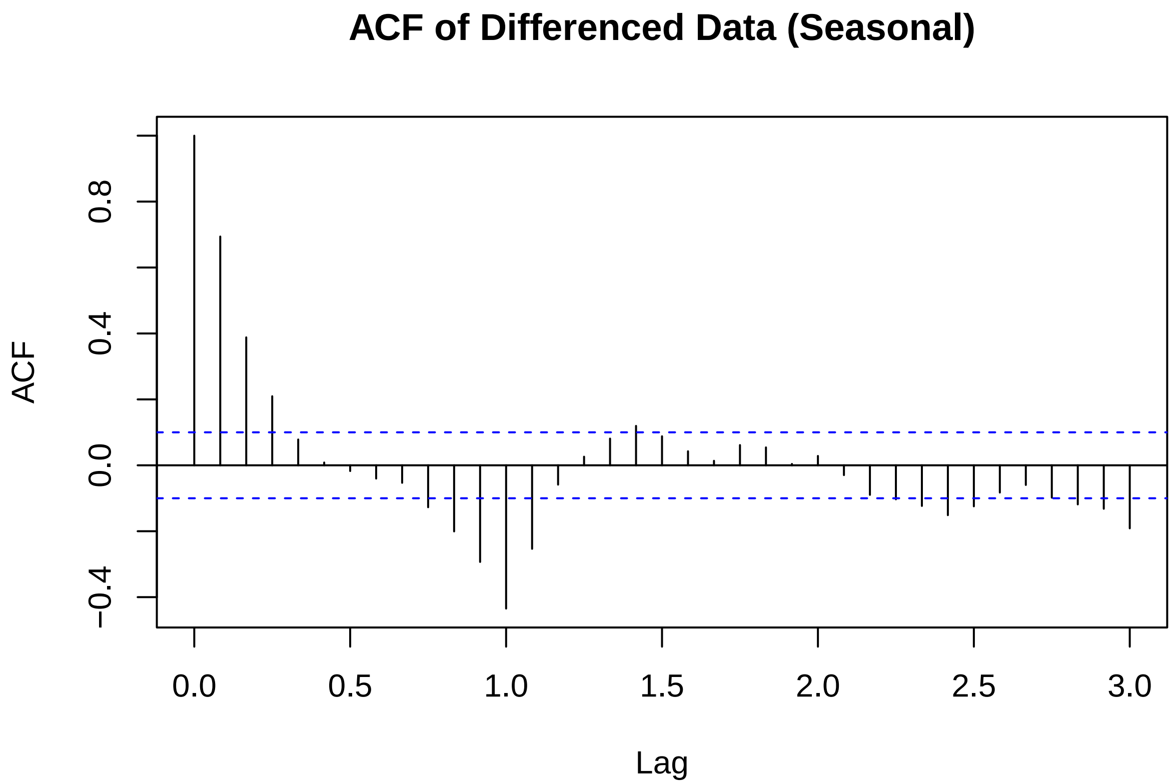
Series diff12.diff.Extent



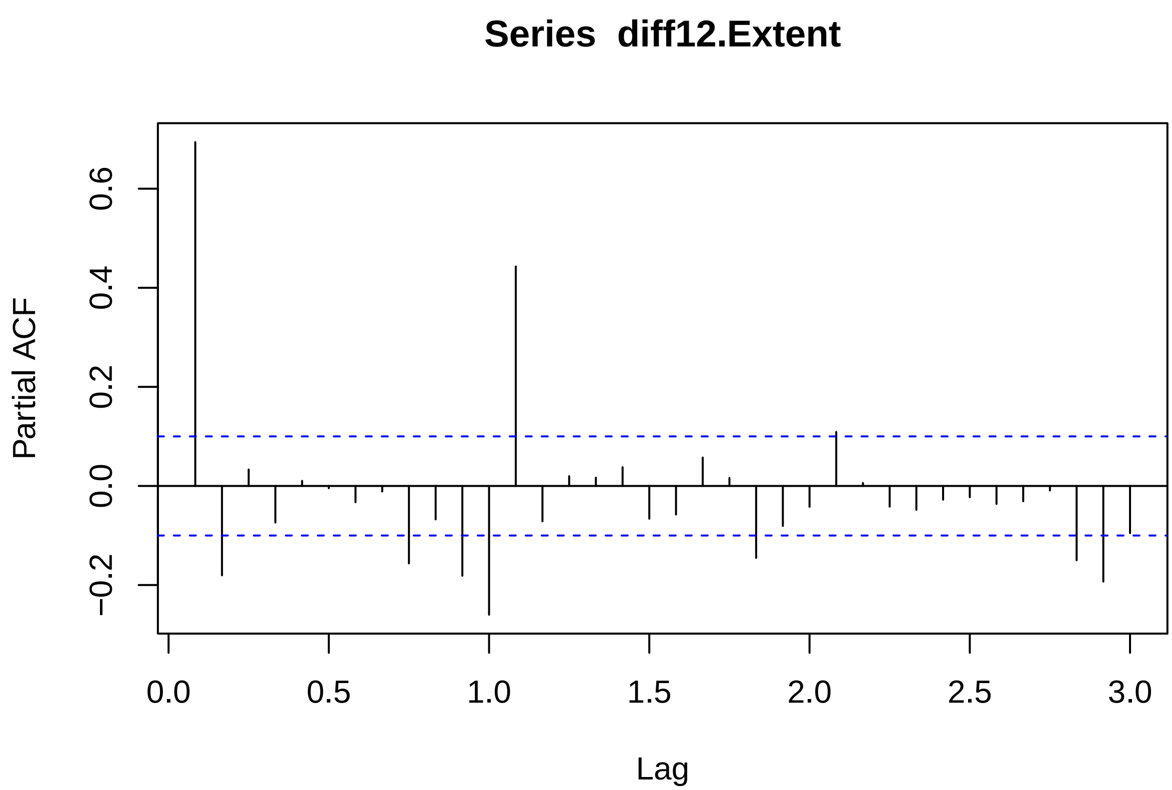


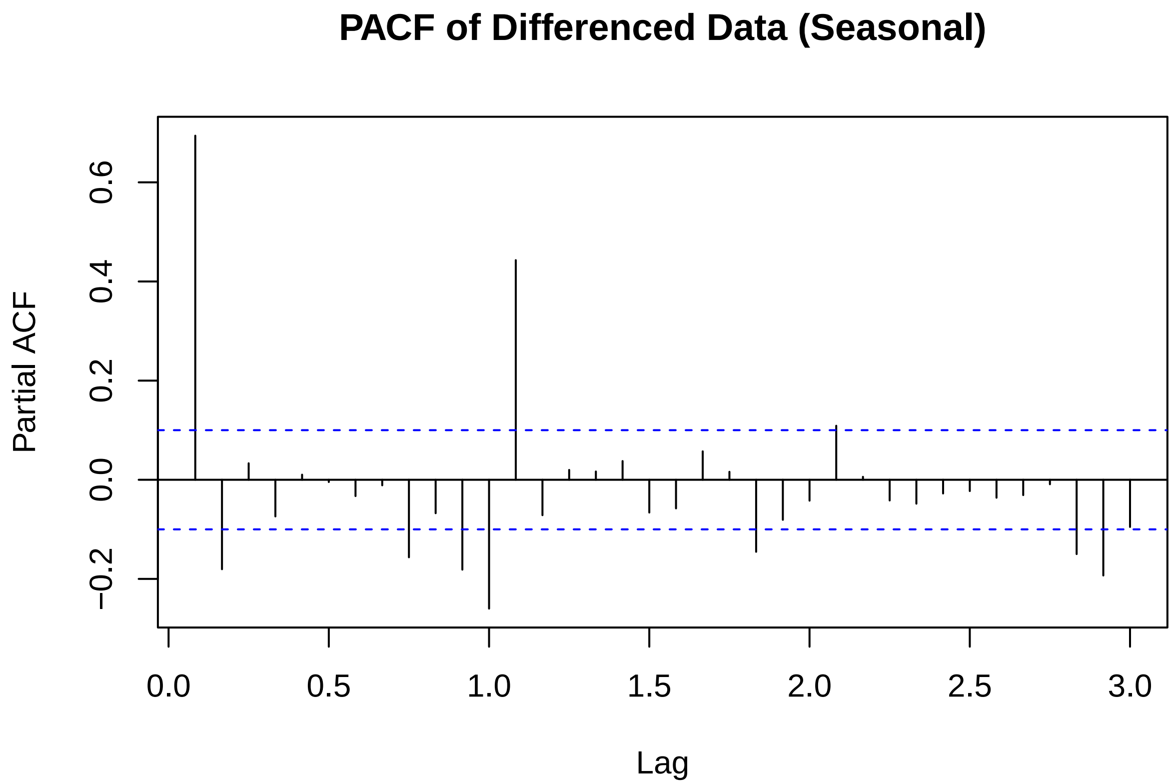

```
plot(acf(diff12.Extent, lag.max=36), main="ACF of Differenced Data (Seasonal)")
```






```
plot(pacf(diff12.Extent, lag.max=36), main="PACF of Differenced Data (Seasonal)")
```





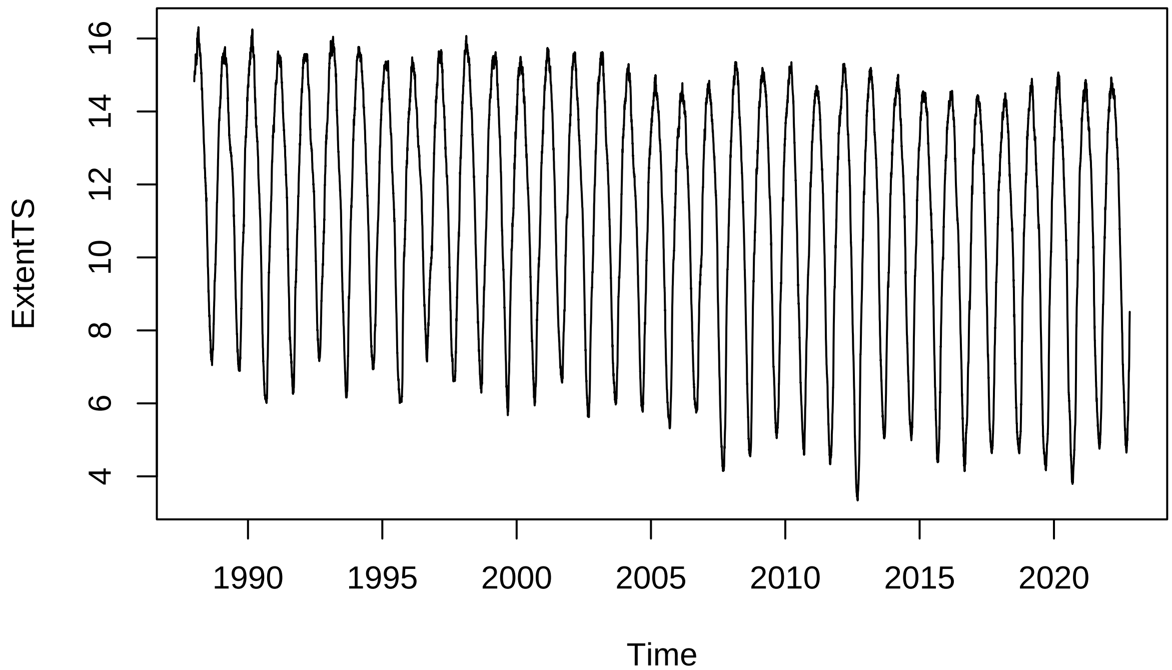
Things we tried but didn't make the report

Unaggregated Data

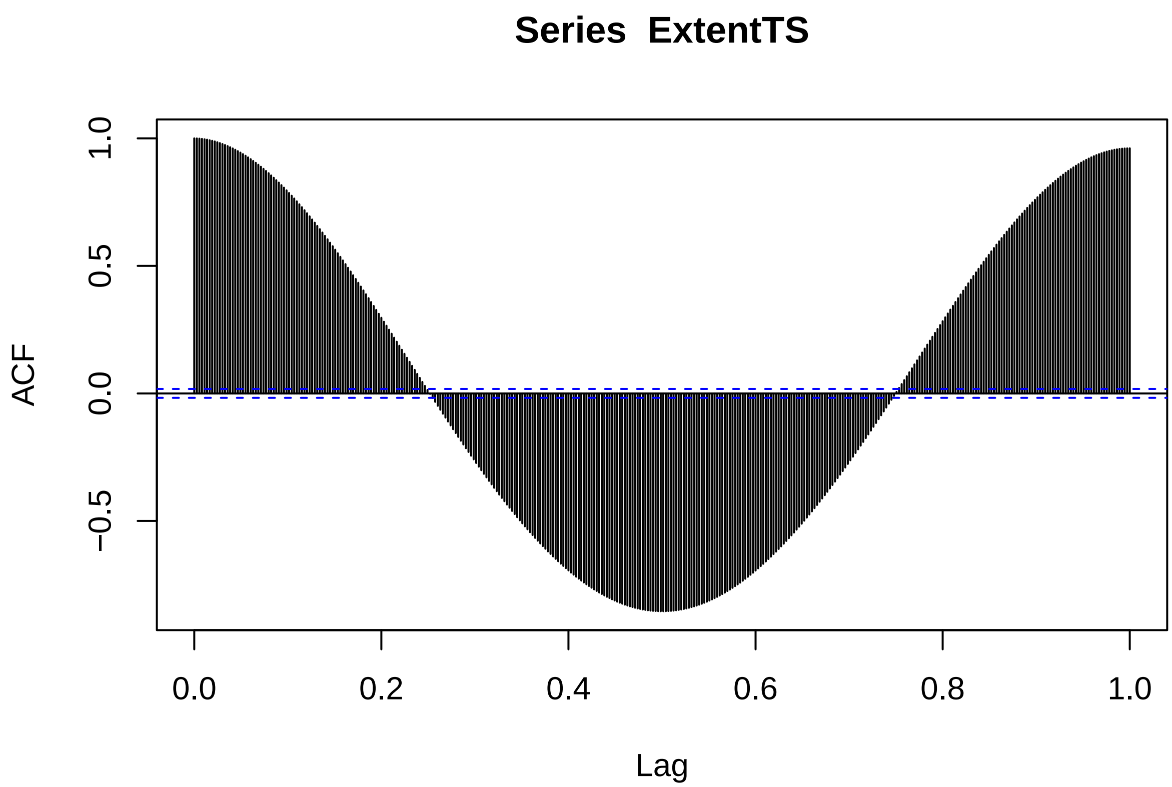
We analyze the unaggregated data.

```
# make a TS object
ExtentTS <- ts(df$extent, frequency=365, start=year(df$YYMMDD[1]))
```

```
plot(ExtentTS)
```




```r
acf(ExtentTS, lag.max=365)
```



Variance

From the plot, we see a clear seasonal pattern, and perhaps a decreasing linear trend.

It is unclear whether variance is constant. We test this using the Fligner-Keileen test.

```r
# do Fligner test for constant variance.
segments = factor(c(rep(1:4, each=2542), rep(5, times=2543)))
fligner.test(ExtentTS, segments)
```

```
##
##  Fligner-Killeen test of homogeneity of variances
##
## data:  ExtentTS and segments
## Fligner-Killeen:med chi-squared = 170.73, df = 4, p-value < 2.2e-16
```

```r
segments = factor(c(rep(1:9, each=1271), rep(10, times=1272)))
fligner.test(ExtentTS, segments)
```

```
##
##  Fligner-Killeen test of homogeneity of variances
##
## data:  ExtentTS and segments
## Fligner-Killeen:med chi-squared = 219.12, df = 9, p-value < 2.2e-16
```

```r
segments = factor(c(rep(1:49, each=254), rep(50, times=265)))
fligner.test(ExtentTS, segments)
```

```
##
##  Fligner-Killeen test of homogeneity of variances
##
## data:  ExtentTS and segments
## Fligner-Killeen:med chi-squared = 1326.7, df = 49, p-value < 2.2e-16
```

```r
segments = factor(c(rep(1:99, each=127), rep(100, times=138)))
fligner.test(ExtentTS, segments)
```

```
##
##  Fligner-Killeen test of homogeneity of variances
##
## data:  ExtentTS and segments
## Fligner-Killeen:med chi-squared = 3386.9, df = 99, p-value < 2.2e-16
```

```r
segments = factor(c(rep(1:34, each=364), rep(35, times=335))) # corresponds more closely to each "wave"
fligner.test(ExtentTS, segments)
```

```
##
##  Fligner-Killeen test of homogeneity of variances
##
## data:  ExtentTS and segments
## Fligner-Killeen:med chi-squared = 368.79, df = 34, p-value < 2.2e-16
```

All give really low p-value so may conclude that variance is not constant. However, this could be due to the amount of data we have.

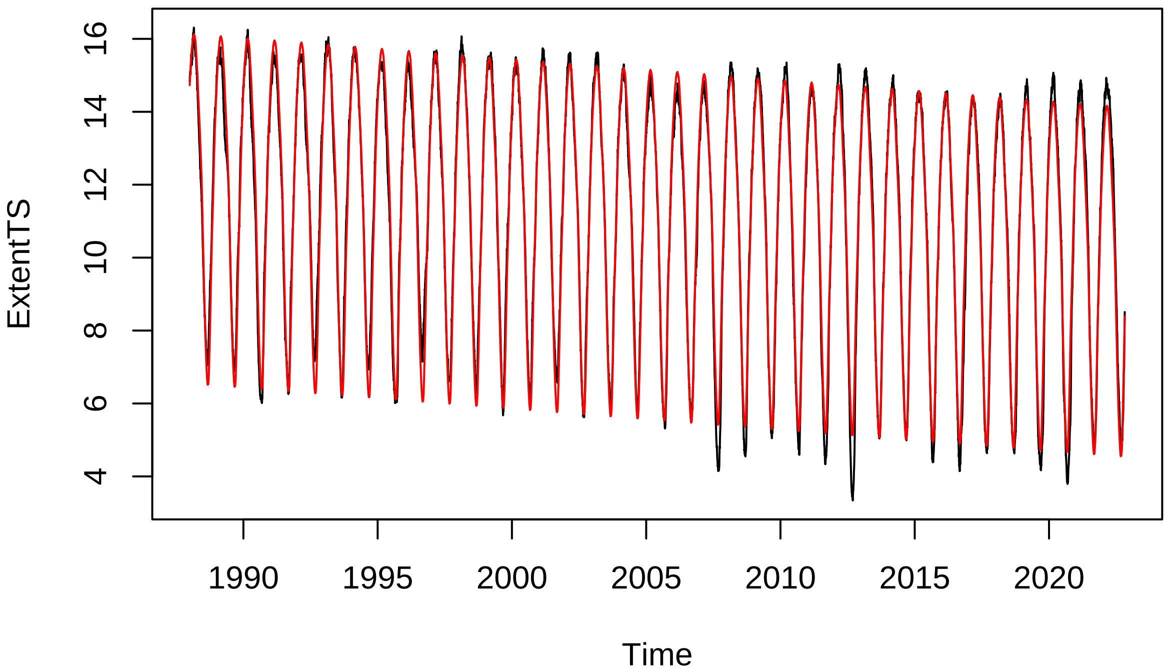
Regression

Try to remove non-stationarity using Regression (Multiple Linear, Ridge, Lasso, Elastic Net).

Multiple Linear Regression

```r
mlr <- lm(ExtentTS~time(ExtentTS)+factor(cycle(ExtentTS)))
#summary(mlr) #verrrryyyyy long output and complicated model.
```

```
plot(ExtentTS)
points(time(ExtentTS),predict.lm(mlr),type='l',col='red')
```

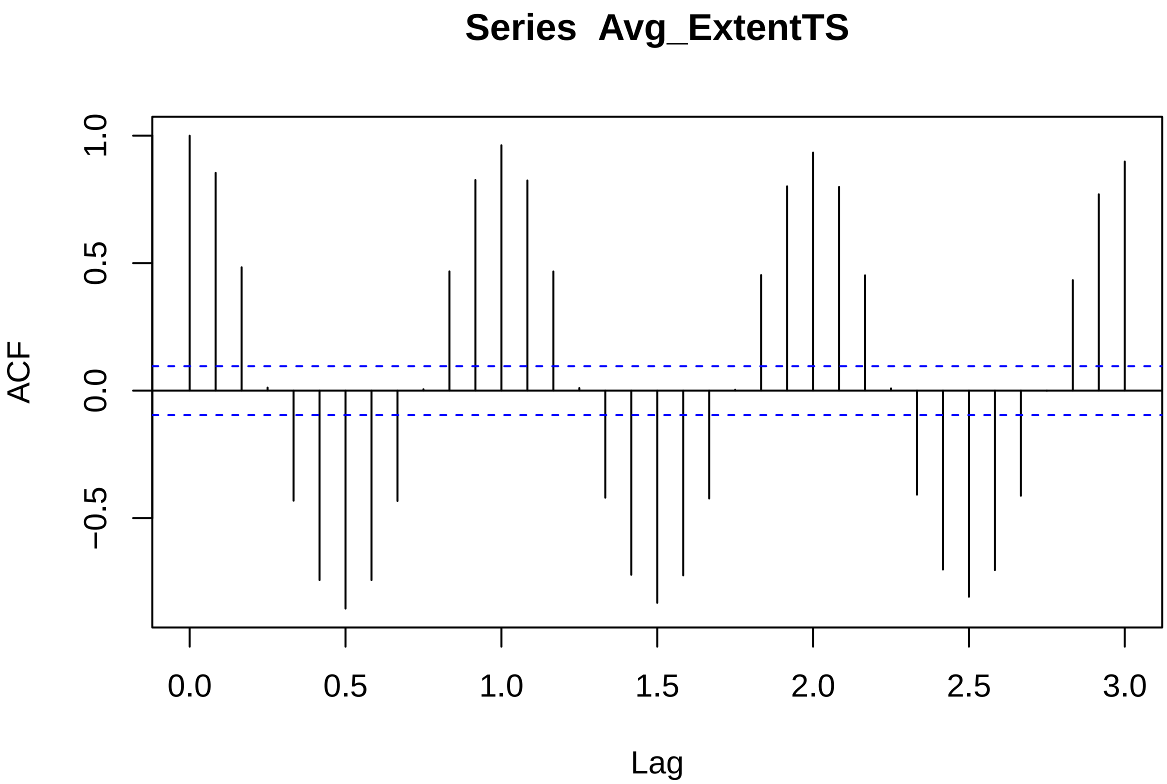


We see from the regression, that including daily data leads to a very complicated regression model, and acf plot which has to go way beyond recommended lag to observe an entire period. For this reason, and because we care mostly about overall trend and not daily fluctuation, we proceed with the aggregated data.

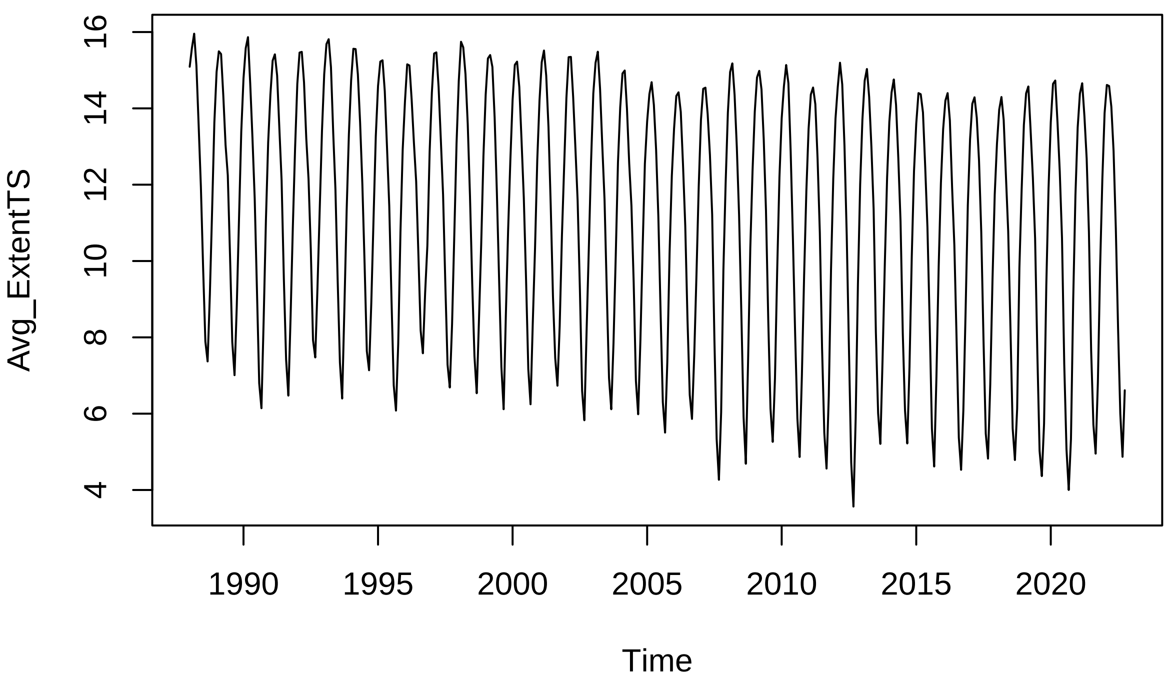
Differencing on Entire Data

Try differencing to remove non-stationarity.

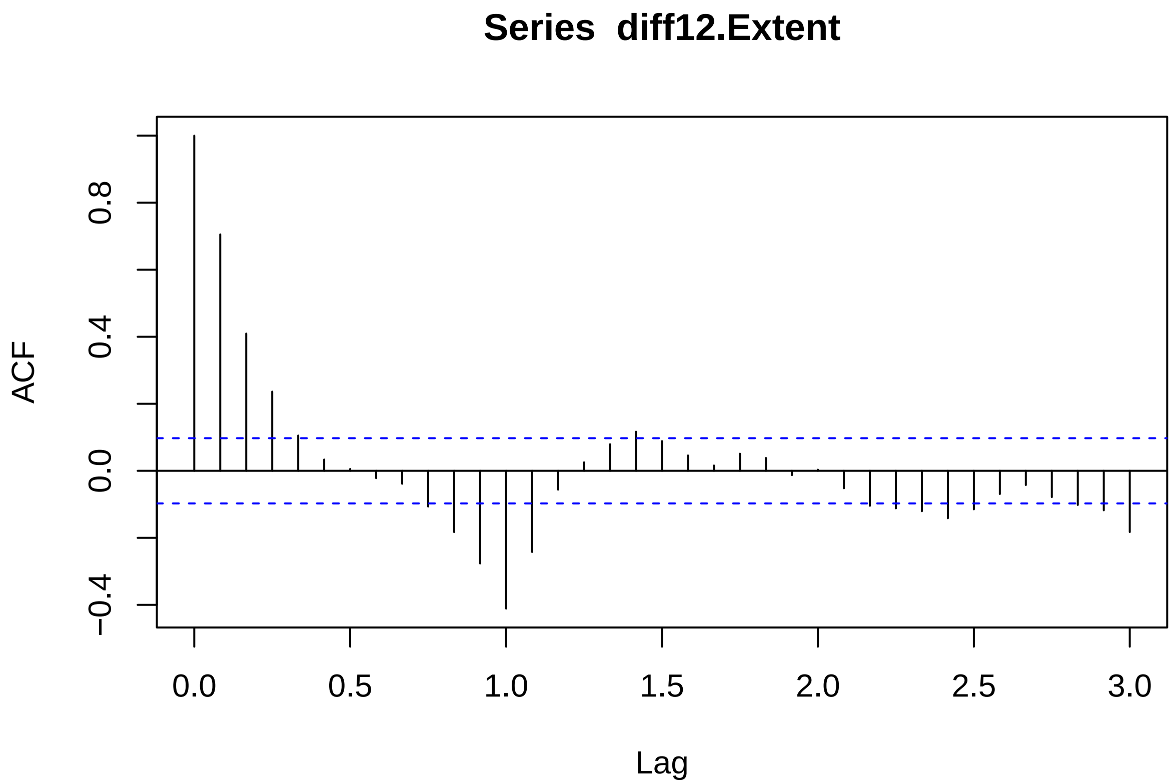
```
acf(Avg_ExtentTS, lag.max=36)
```



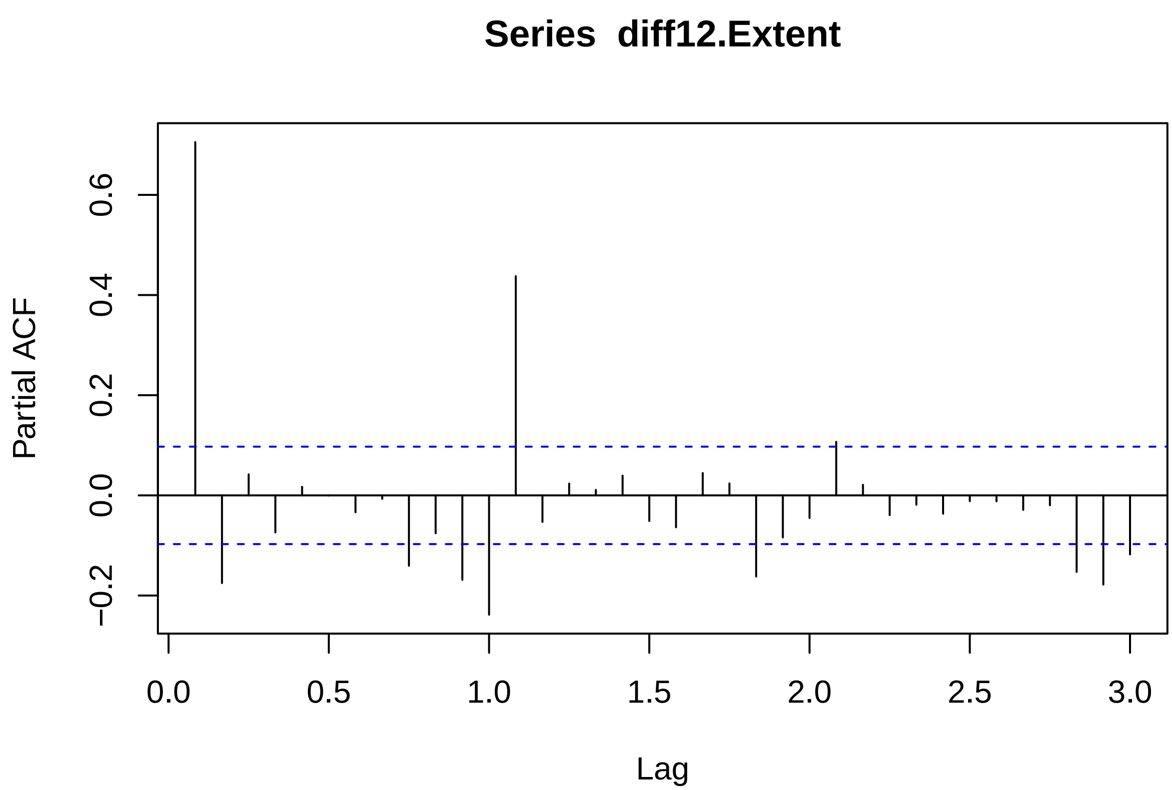

```r
plot(Avg_ExtentTS)
```



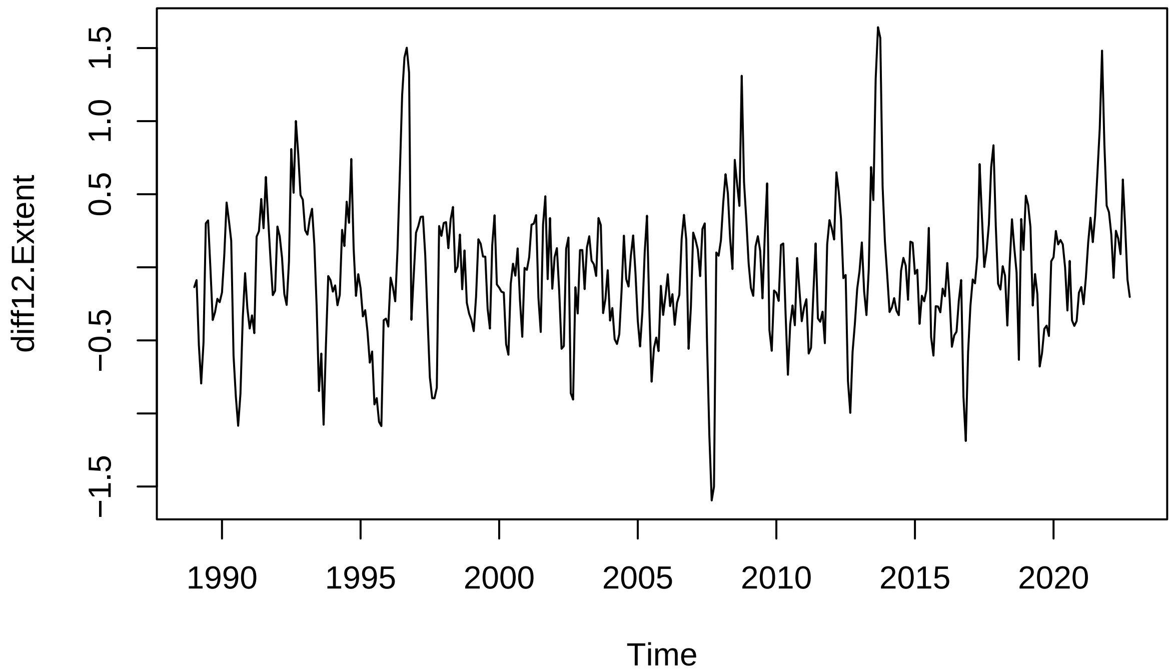

```r
#differencing in lag of season
diff12.Extent=diff(Avg_ExtentTS, lag=12)
acf(diff12.Extent, lag.max=36)
```



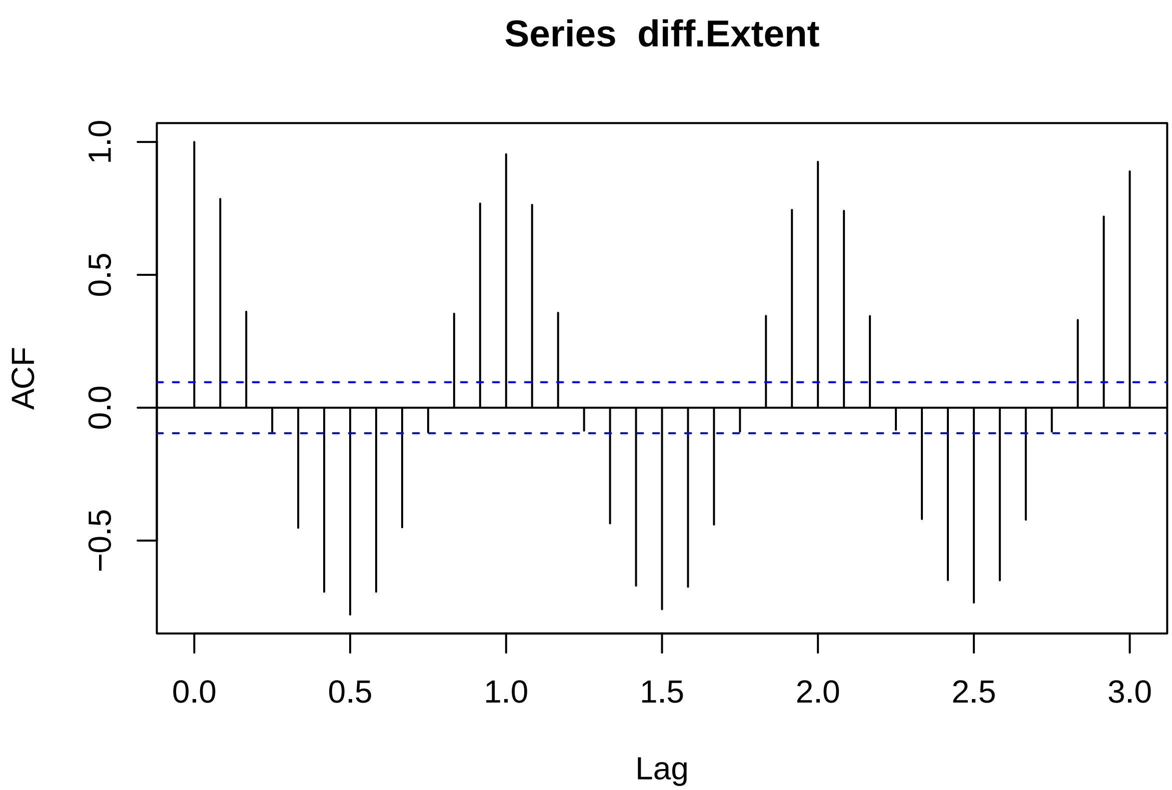

```
pacf(diff12.Extent, lag.max=36)
```



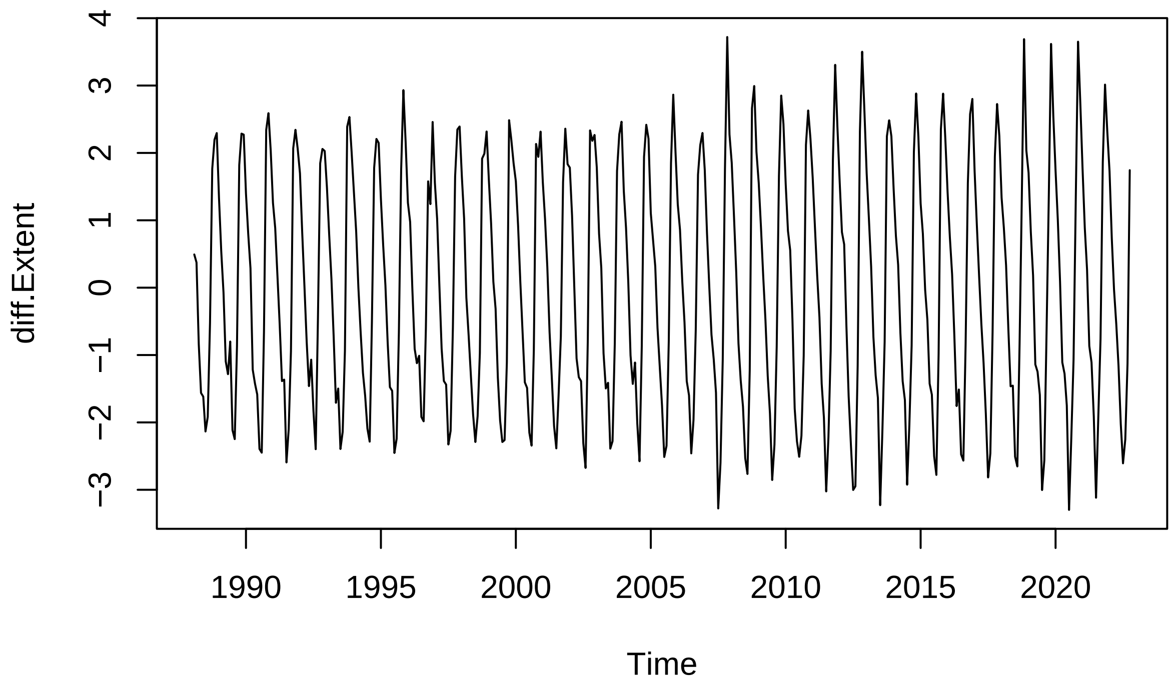

```
plot(diff12.Extent)
```



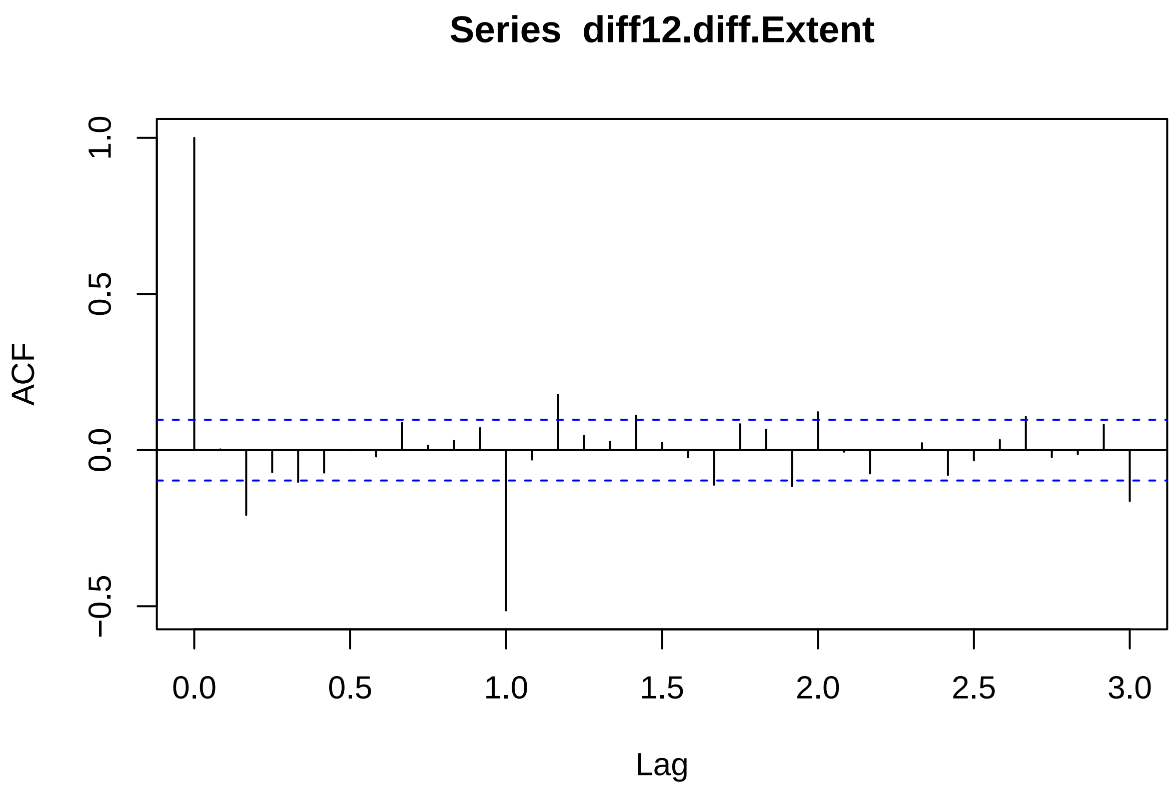

```r
#regular differencing
diff.Extent=diff(Avg_ExtentTS)
acf(diff.Extent, lag.max=36)
```



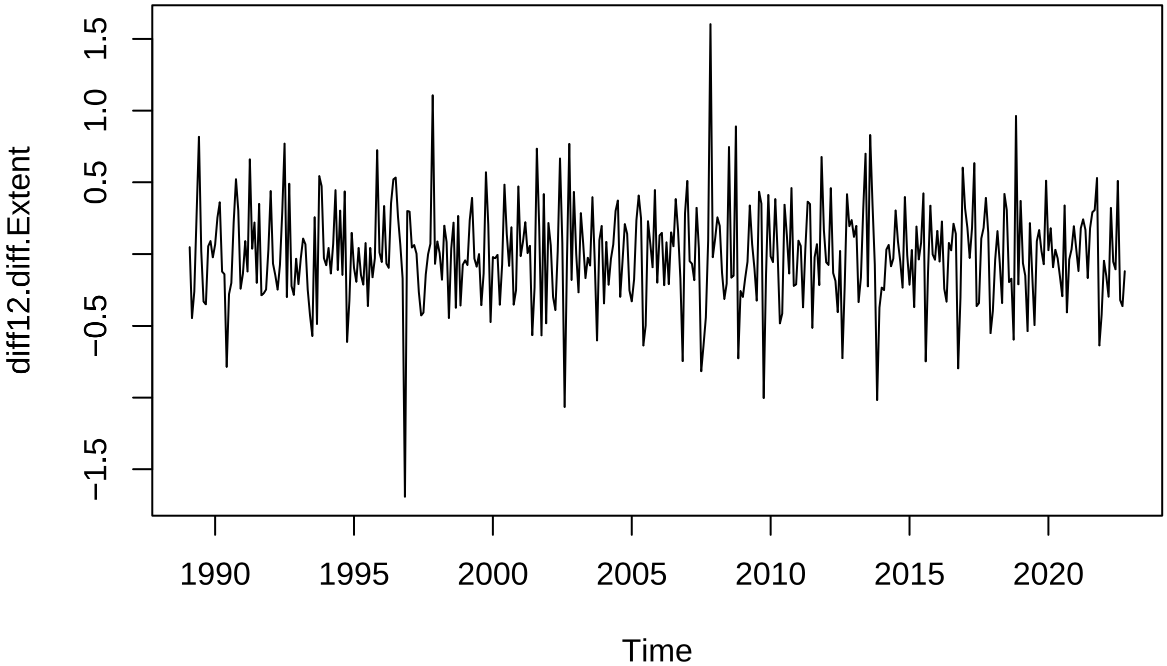

```r
plot(diff.Extent)
```



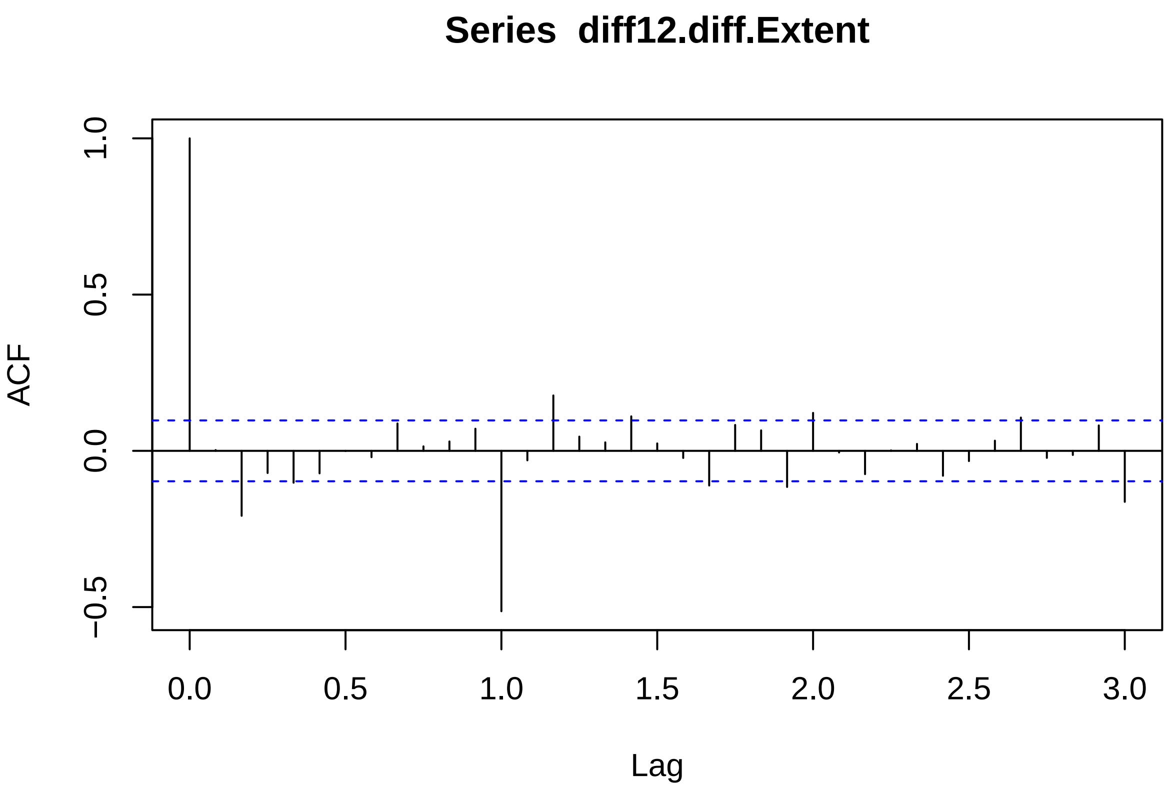

```
#seasonal+regular differencing
diff12.diff.Extent=diff(diff12.Extent)
acf(diff12.diff.Extent, lag.max=36)
```



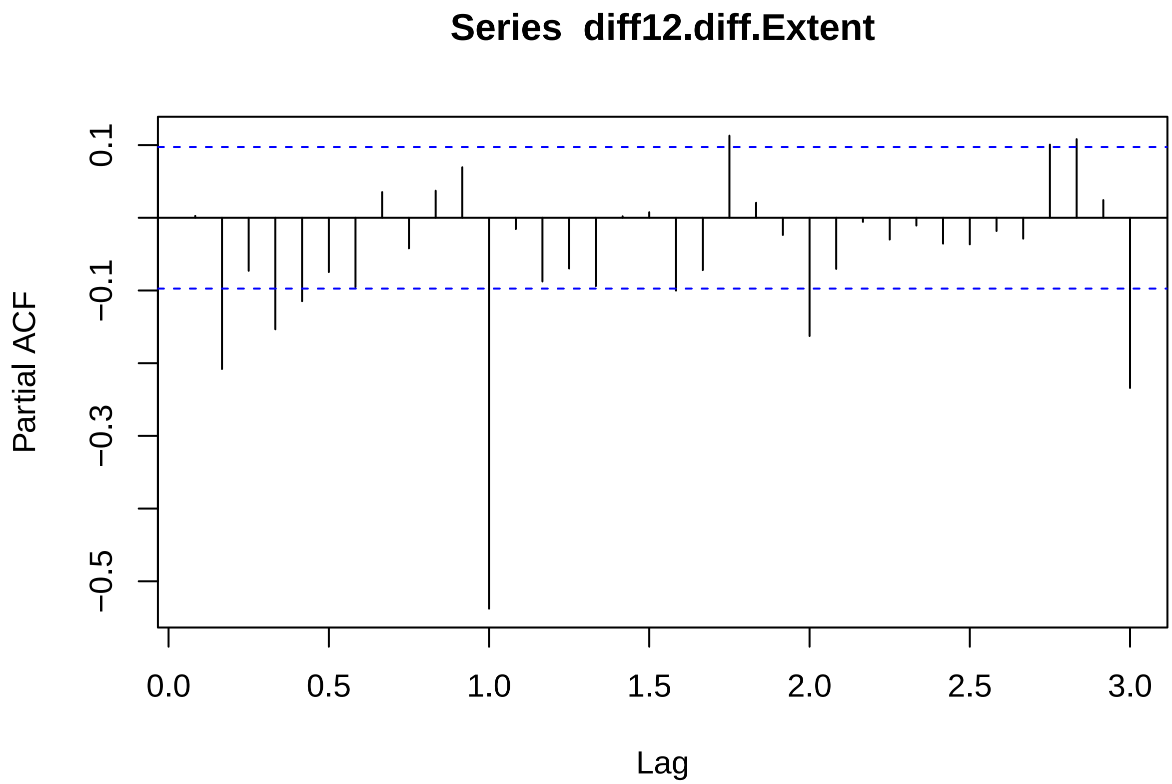

```
plot(diff12.diff.Extent)
```




```
acf(diff12.diff.Extent, lag.max=36)
```




```
pacf(diff12.diff.Extent, lag.max=36)
```

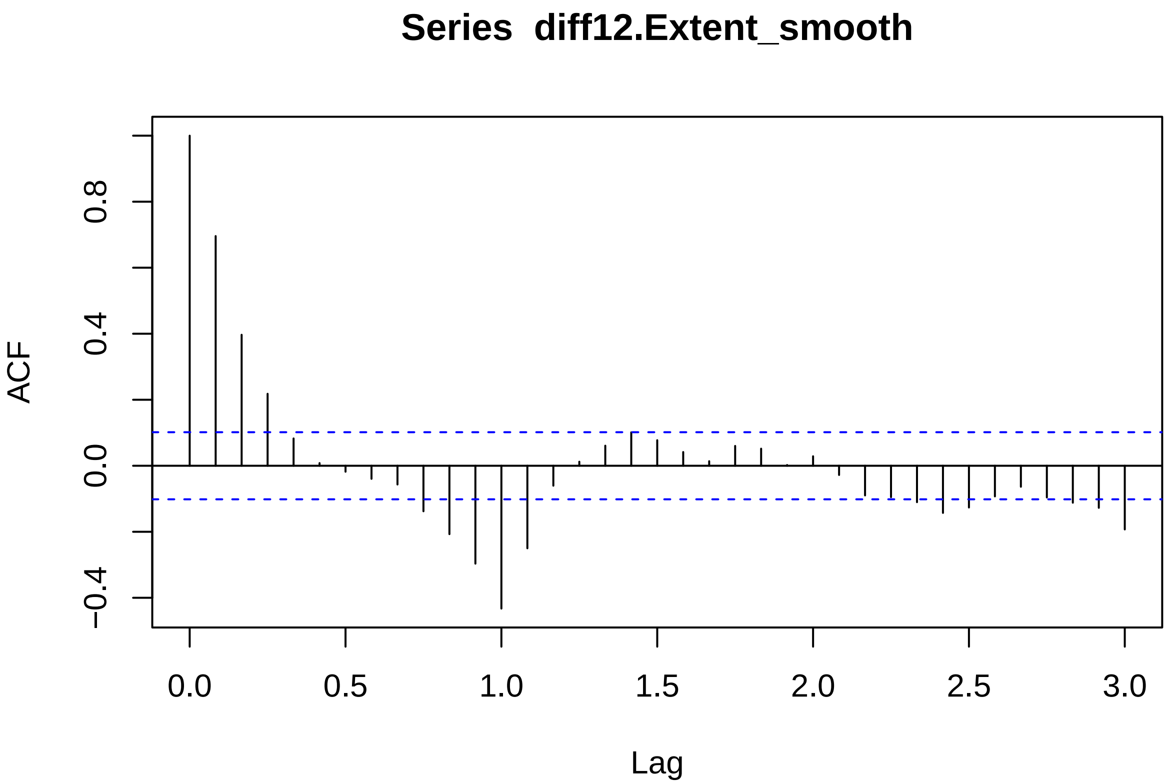


Noted after that differencing should have been done on training data.

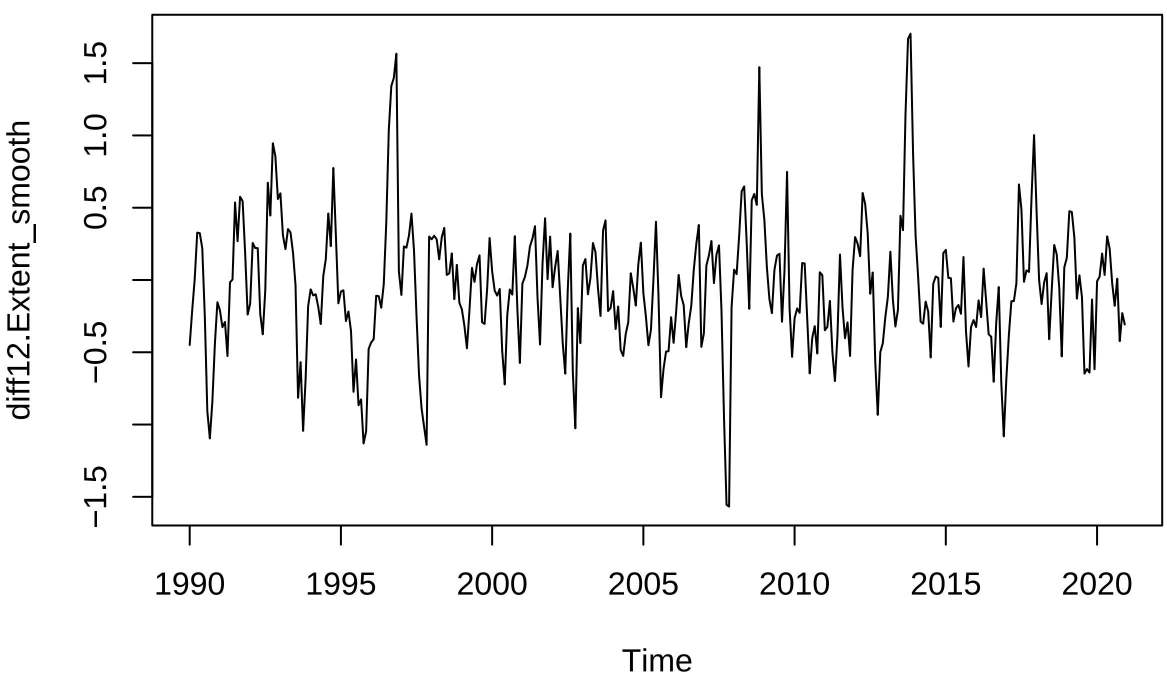
Smoothing, followed by differencing

Try smoothing before differencing to see effect on acf

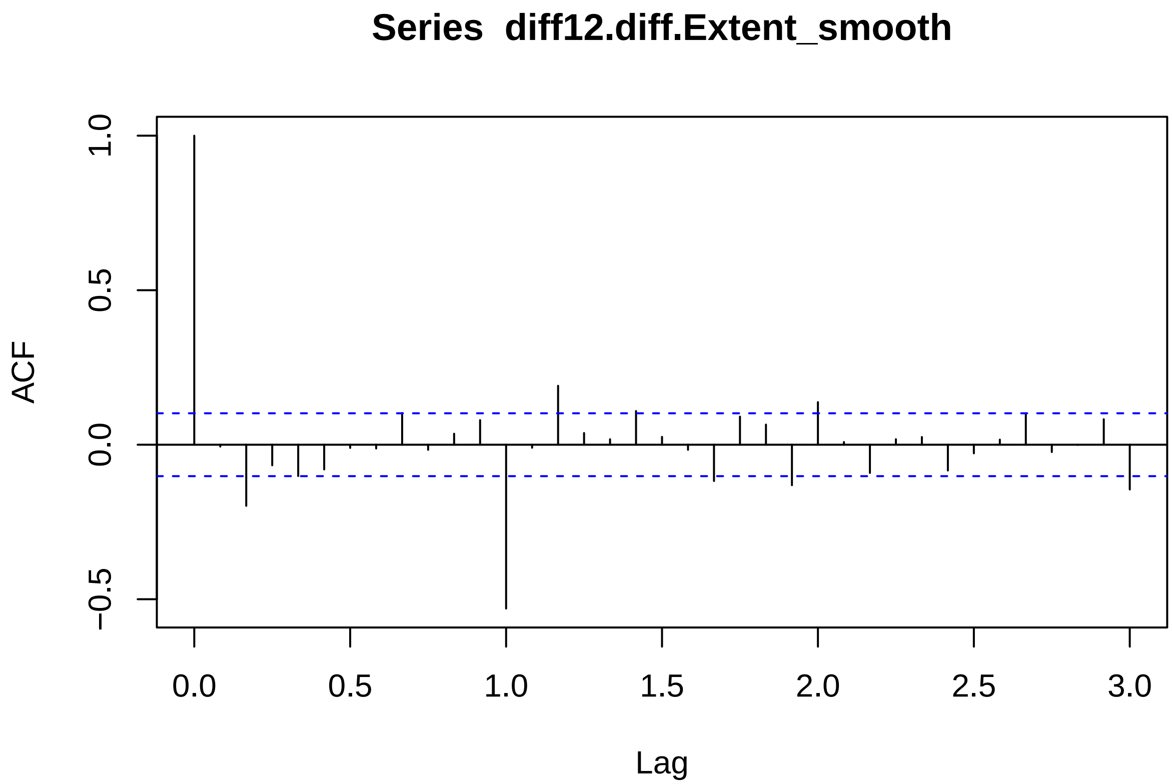
```
smoothing <- HoltWinters(Avg_ExtentTS_Train, season="additive")
smoothed <- smoothing$fitted[,1]
diff12.Extent_smooth=diff(smoothed, lag=12)
acf(diff12.Extent_smooth, lag.max=36)
```



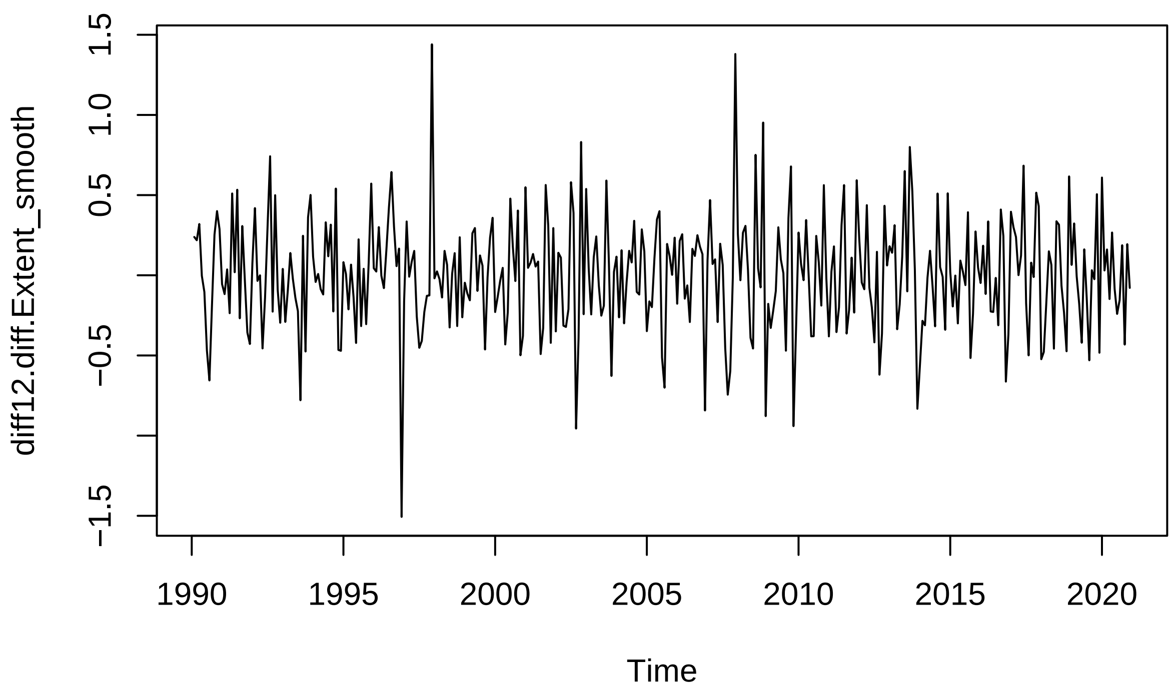

```
plot(diff12.Extent_smooth)
```



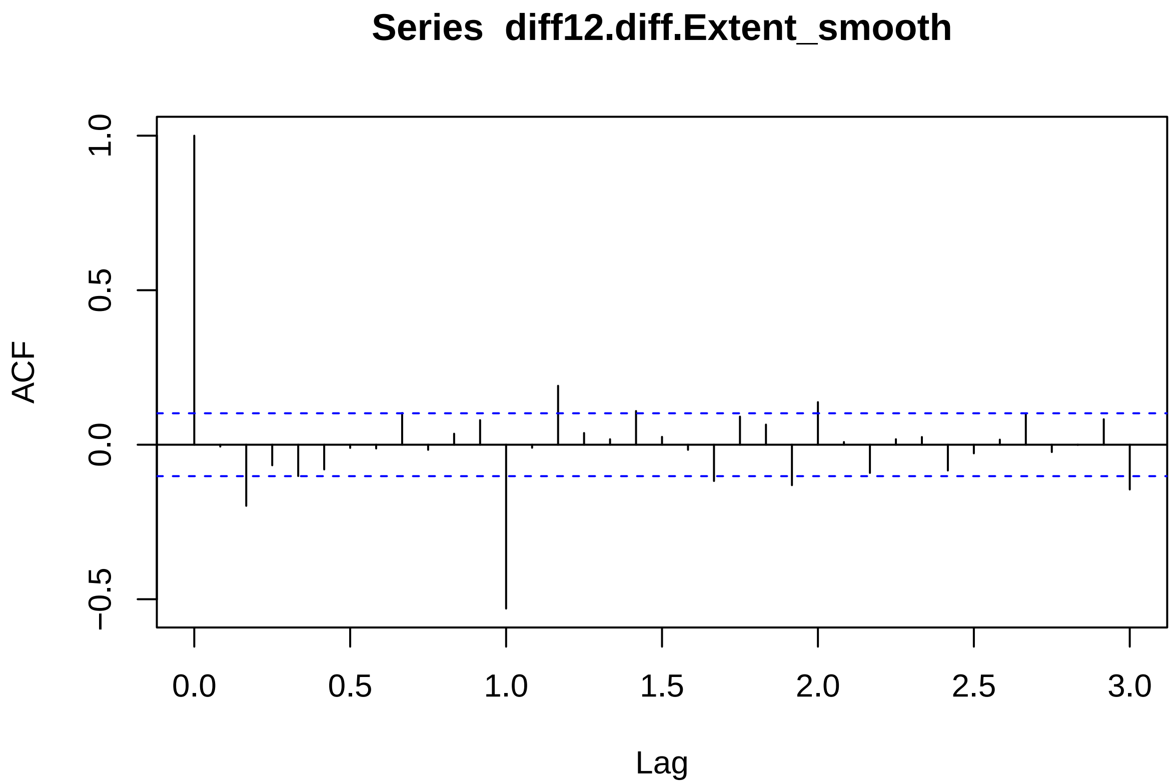

```
diff12.diff.Extent_smooth=diff(diff12.Extent_smooth)
acf(diff12.diff.Extent_smooth, lag.max=36)
```



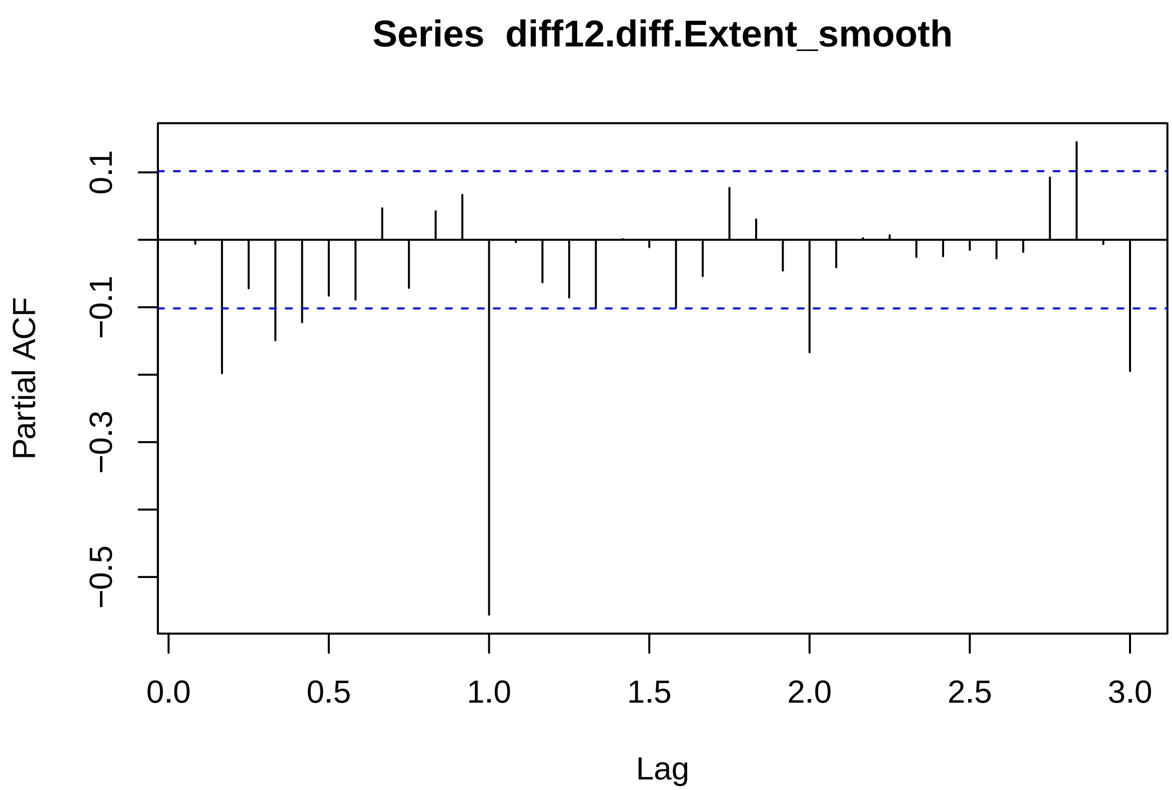

```
plot(diff12.diff.Extent_smooth)
```




```
acf(diff12.diff.Extent_smooth, lag.max=36)
```




```
pacf(diff12.diff.Extent_smooth, lag.max=36)
```



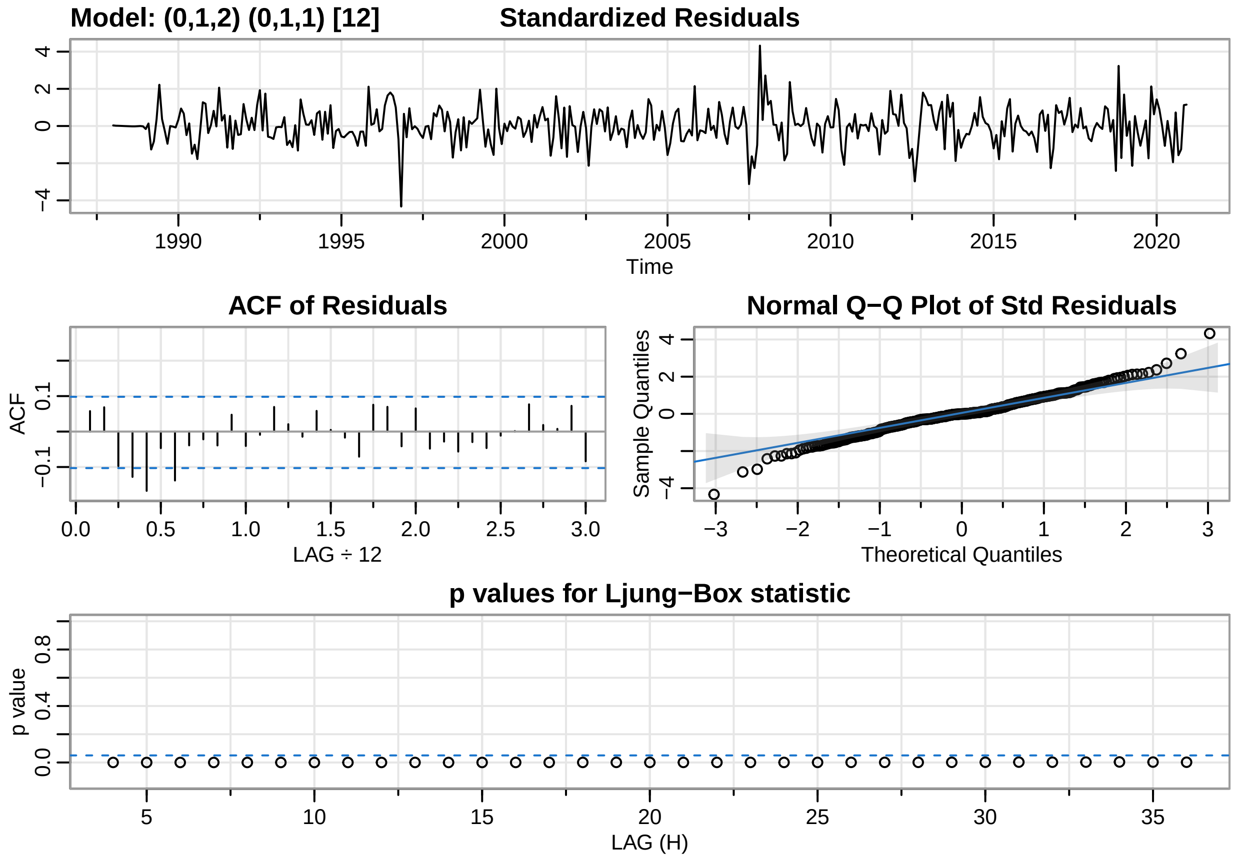
This didn't change anything.

Model Fitting

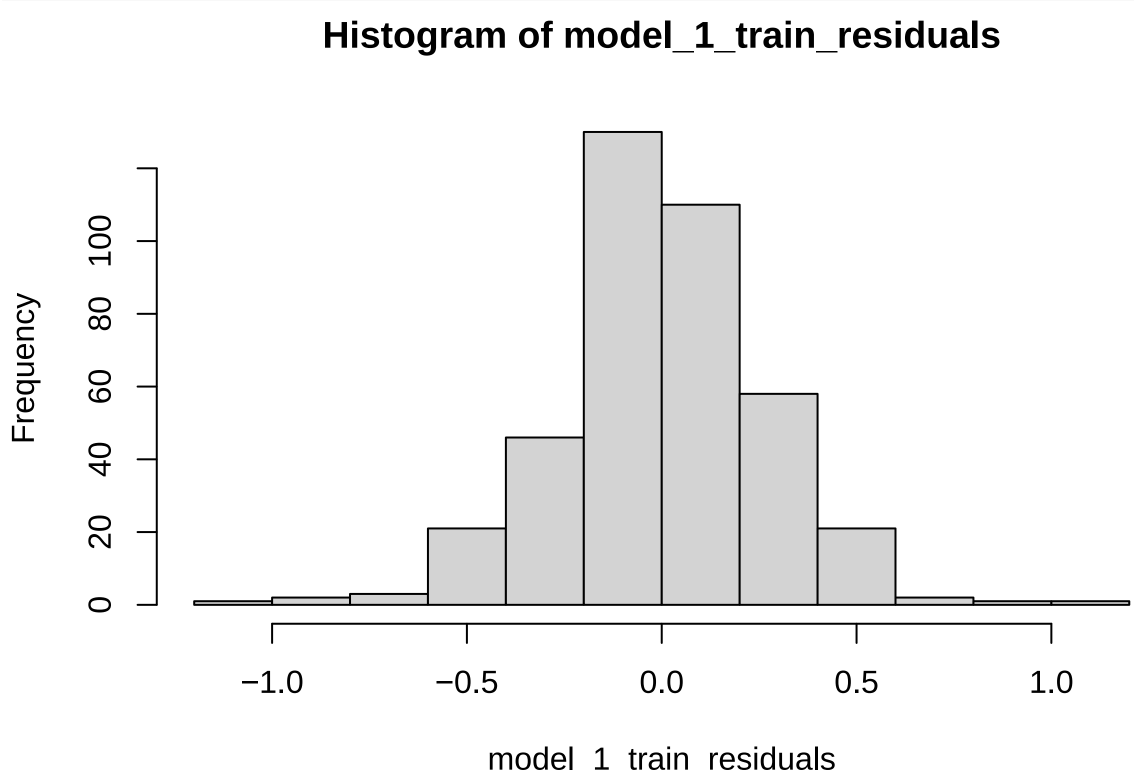
Seasonal + regular differencing.

```
#SARIMA(0,1,2)x(0,1,1)_12
model_1_train <- sarima(Avg_ExtentTS_Train, p=0, d=1, q=2, P=0, D=1, Q=1, S=12 , details = TRUE)
```

```
## initial  value -1.046835
## iter   2 value -1.270895
## iter   3 value -1.286671
## iter   4 value -1.288786
## iter   5 value -1.299437
## iter   6 value -1.301999
## iter   7 value -1.302786
## iter   8 value -1.302845
## iter   9 value -1.302850
## iter  10 value -1.302854
## iter  11 value -1.302854
## iter  11 value -1.302854
## iter  11 value -1.302854
## final  value -1.302854
## converged
## initial  value -1.299104
## iter   2 value -1.299174
## iter   3 value -1.299223
## iter   4 value -1.299225
## iter   5 value -1.299225
## iter   5 value -1.299225
## iter   5 value -1.299225
## final  value -1.299225
## converged
```




```
model_1_train_residuals = resid(model_1_train$fit)
hist(model_1_train_residuals)
```

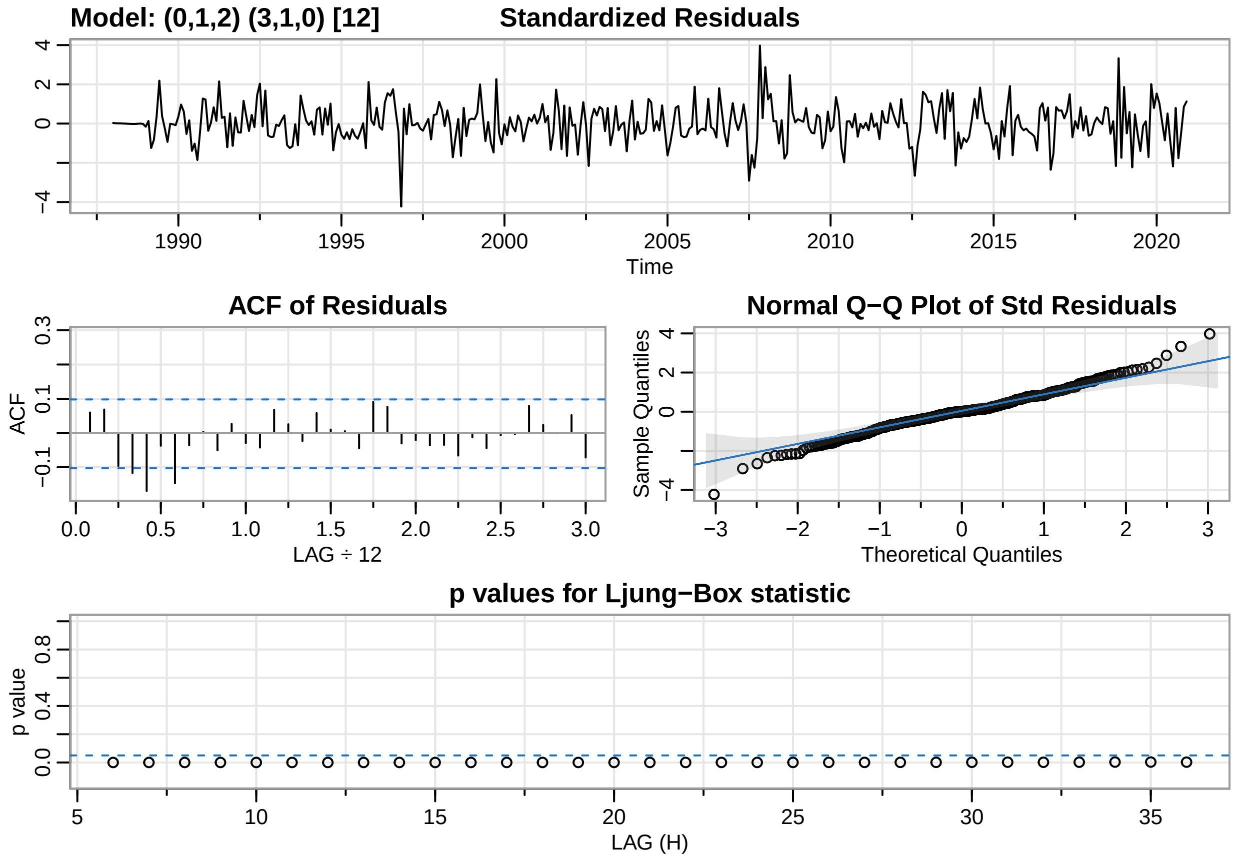


```r
shapiro.test(model_1_train_residuals)
```

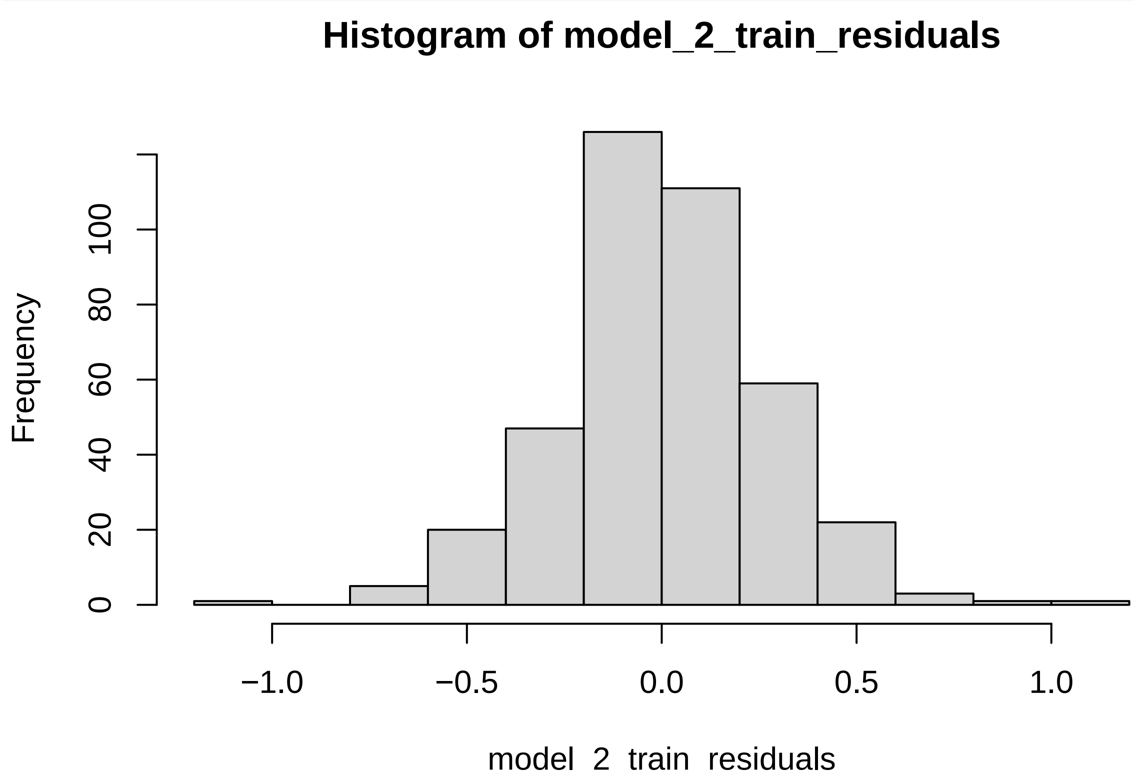
```
##
##  Shapiro-Wilk normality test
##
## data:  model_1_train_residuals
## W = 0.98032, p-value = 3.21e-05
```

```r
#SARIMA(0,1,2)x(3,1,0)_12
model_2_train <- sarima(Avg_ExtentTS_Train, p=0, d=1, q=2, P=3, D=1, Q=0, S=12 , details = TRUE)
```

```
## initial  value -1.039490
## iter   2 value -1.176658
## iter   3 value -1.269525
## iter   4 value -1.294457
## iter   5 value -1.299214
## iter   6 value -1.299421
## iter   7 value -1.299429
## iter   8 value -1.299429
## iter   8 value -1.299429
## iter   8 value -1.299429
## final  value -1.299429
## converged
## initial  value -1.297378
## iter   2 value -1.297439
## iter   3 value -1.297448
## iter   4 value -1.297451
## iter   5 value -1.297451
## iter   6 value -1.297451
## iter   6 value -1.297451
## iter   6 value -1.297451
## final  value -1.297451
## converged
```




```
model_2_train_residuals = resid(model_2_train$fit)
hist(model_2_train_residuals)
```

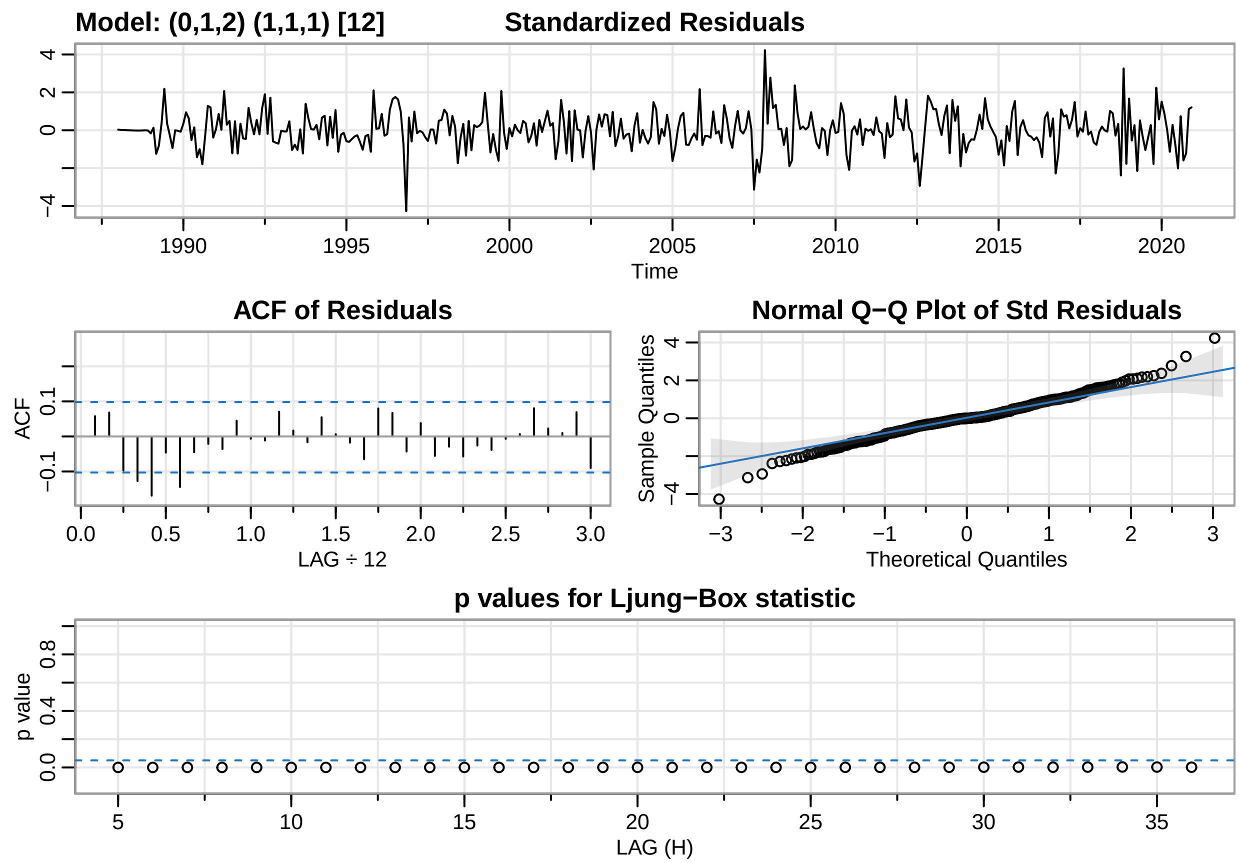


```r
shapiro.test(model_2_train_residuals)
```

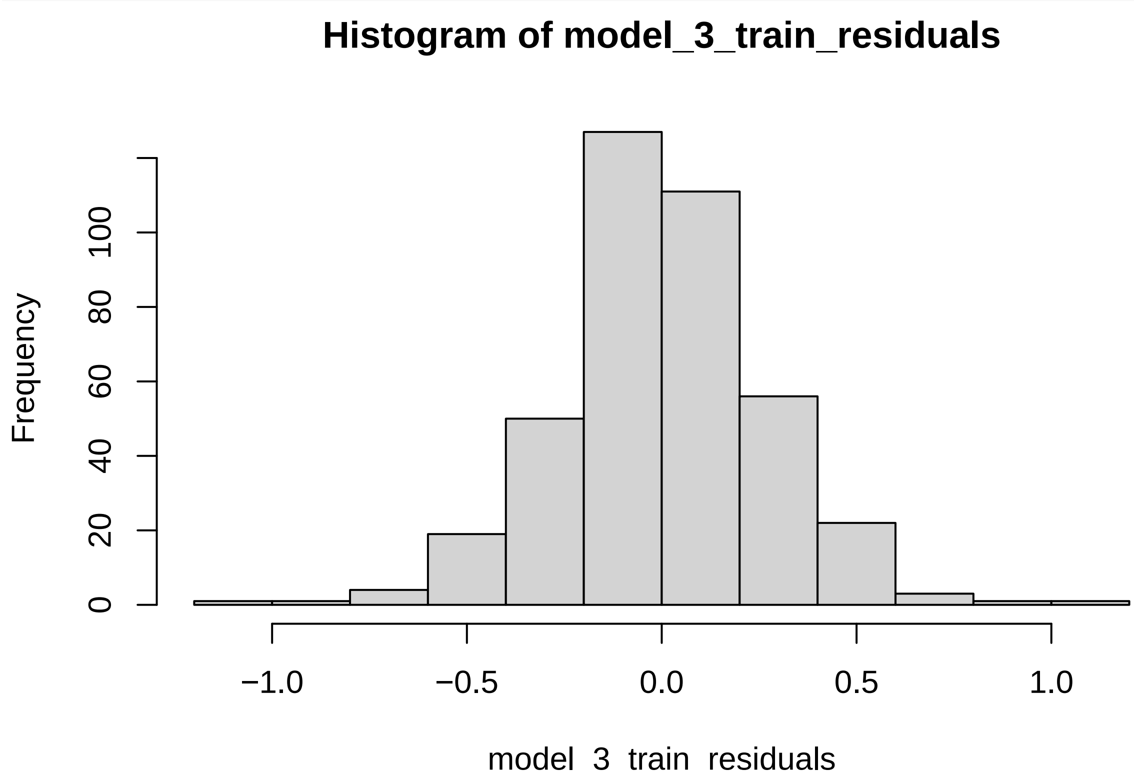
```
## 
##  Shapiro-Wilk normality test
## 
## data:  model_2_train_residuals
## W = 0.98446, p-value = 0.0002963
```

```r
#SARIMA(0,1,2)x(1,1,1)_12
model_3_train <- sarima(Avg_ExtentTS_Train, p=0, d=1, q=2, P=1, D=1, Q=1, S=12 , details = TRUE)
```

```
## initial  value -1.044499 
## iter   2 value -1.237721
## iter   3 value -1.282334
## iter   4 value -1.287558
## iter   5 value -1.293725
## iter   6 value -1.293955
## iter   7 value -1.294047
## iter   8 value -1.294078
## iter   9 value -1.294083
## iter  10 value -1.294083
## iter  10 value -1.294083
## final  value -1.294083 
## converged
## initial  value -1.299017 
## iter   2 value -1.299579
## iter   3 value -1.300082
## iter   4 value -1.300197
## iter   5 value -1.300210
## iter   6 value -1.300211
## iter   7 value -1.300211
## iter   8 value -1.300211
## iter   8 value -1.300211
## iter   8 value -1.300211
## final  value -1.300211 
## converged
```




```
model_3_train_residuals = resid(model_3_train$fit)
hist(model_3_train_residuals)
```

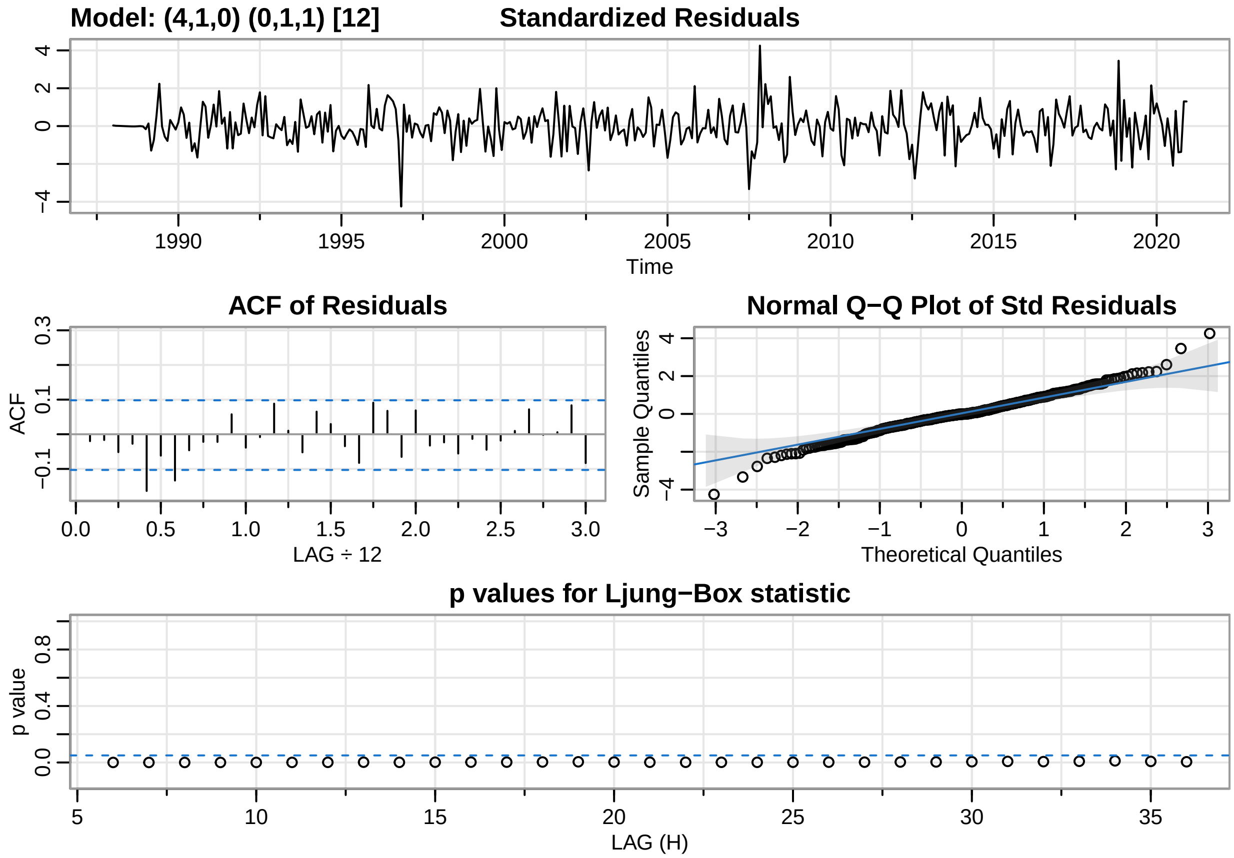


```r
shapiro.test(model_3_train_residuals)
```

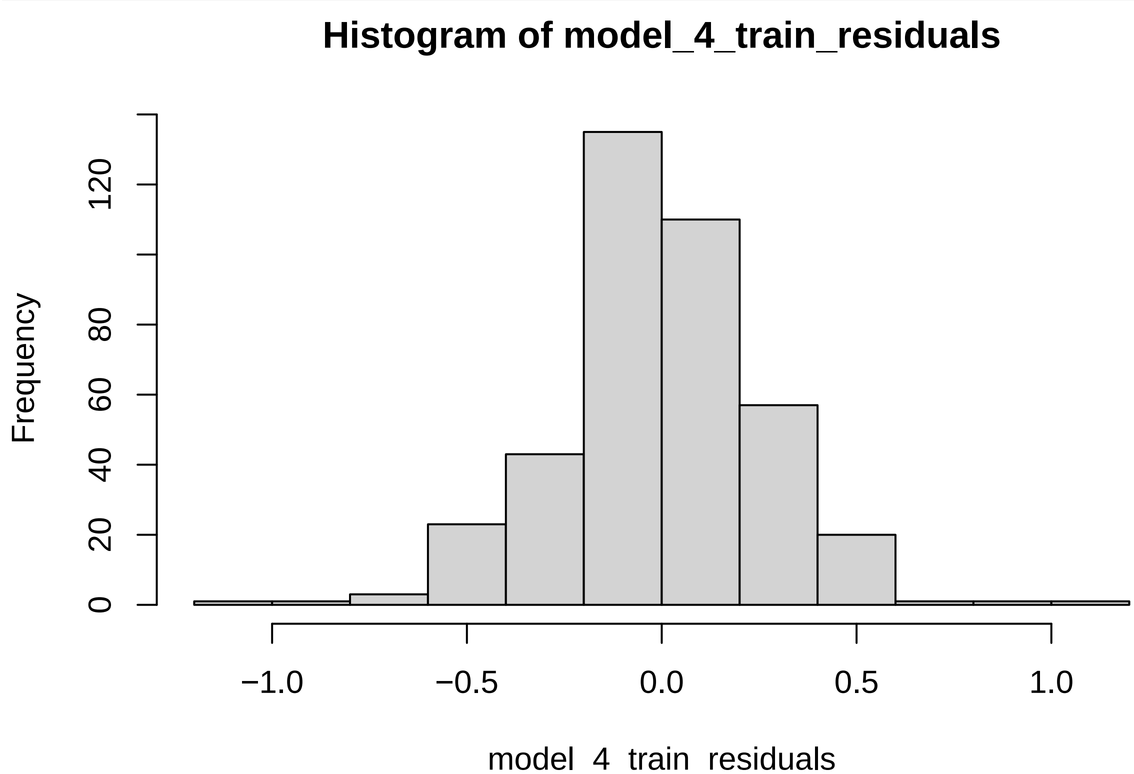
```
##
##  Shapiro-Wilk normality test
##
## data:  model_3_train_residuals
## W = 0.98178, p-value = 6.856e-05
```

```r
#SARIMA(4,1,0)x(0,1,1)_12
model_4_train <- sarima(Avg_ExtentTS_Train, p=4, d=1, q=0, P=0, D=1, Q=1, S=12 , details = TRUE)
```

```
## initial  value -1.045273
## iter   2 value -1.275376
## iter   3 value -1.289687
## iter   4 value -1.291649
## iter   5 value -1.300383
## iter   6 value -1.302661
## iter   7 value -1.303220
## iter   8 value -1.303284
## iter   9 value -1.303285
## iter   9 value -1.303285
## iter   9 value -1.303285
## final  value -1.303285
## converged
## initial  value -1.305545
## iter   2 value -1.305718
## iter   3 value -1.305759
## iter   4 value -1.305764
## iter   5 value -1.305764
## iter   5 value -1.305764
## iter   5 value -1.305764
## final  value -1.305764
## converged
```




```r
model_4_train_residuals = resid(model_4_train$fit)
hist(model_4_train_residuals)
```

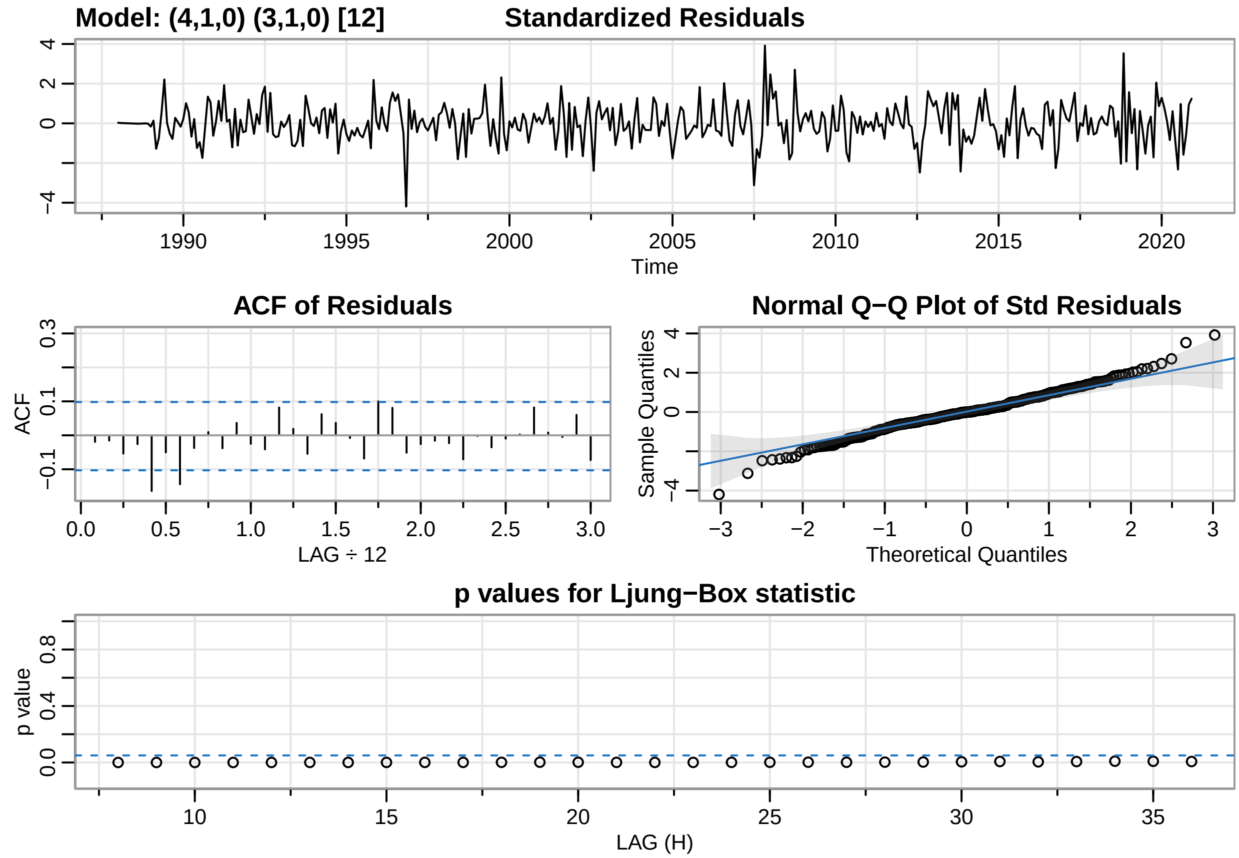


```r
shapiro.test(model_4_train_residuals)
```

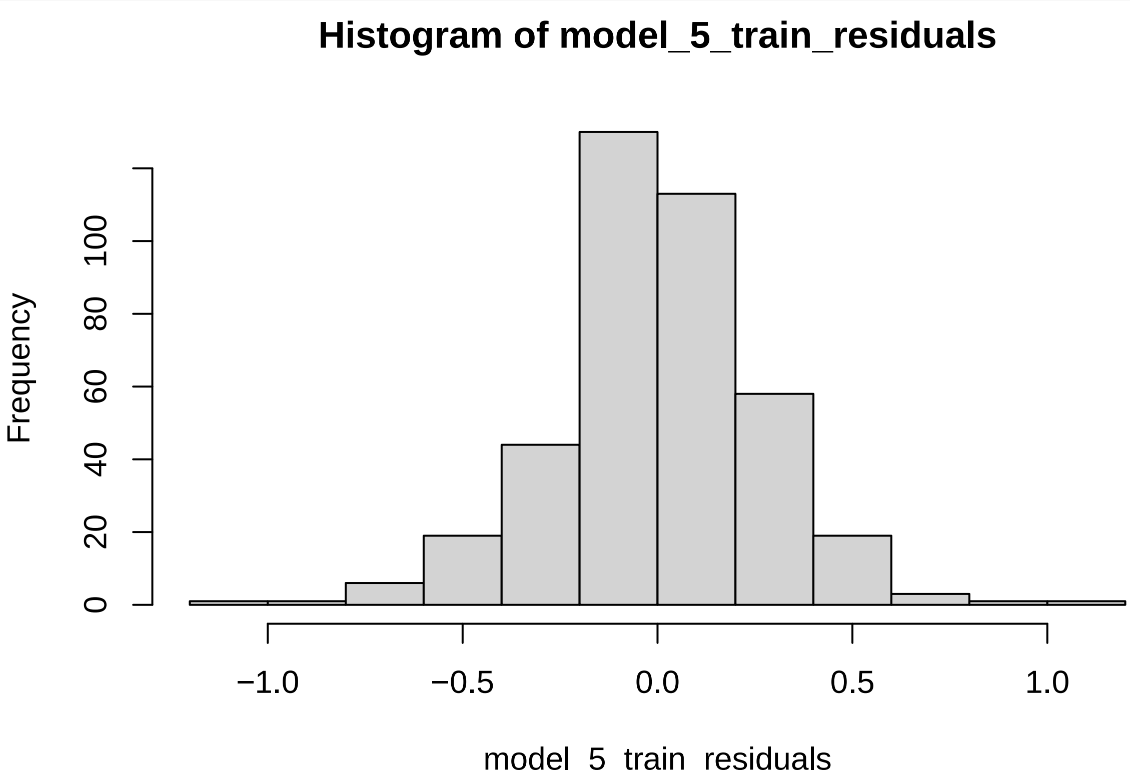
```
##
##  Shapiro-Wilk normality test
##
## data:  model_4_train_residuals
## W = 0.98293, p-value = 0.0001267
```

```r
#SARIMA(4,1,0)x(3,1,0)_12
model_5_train <- sarima(Avg_ExtentTS_Train, p=4, d=1, q=0, P=3, D=1, Q=0, S=12 , details = TRUE)
```

```
## initial  value -1.034768
## iter   2 value -1.176278
## iter   3 value -1.267070
## iter   4 value -1.292224
## iter   5 value -1.298051
## iter   6 value -1.298311
## iter   7 value -1.298325
## iter   8 value -1.298325
## iter   8 value -1.298325
## iter   8 value -1.298325
## final  value -1.298325
## converged
## initial  value -1.301857
## iter   2 value -1.302141
## iter   3 value -1.302152
## iter   4 value -1.302153
## iter   5 value -1.302153
## iter   5 value -1.302153
## iter   5 value -1.302153
## final  value -1.302153
## converged
```




```
model_5_train_residuals = resid(model_5_train$fit)
hist(model_5_train_residuals)
```

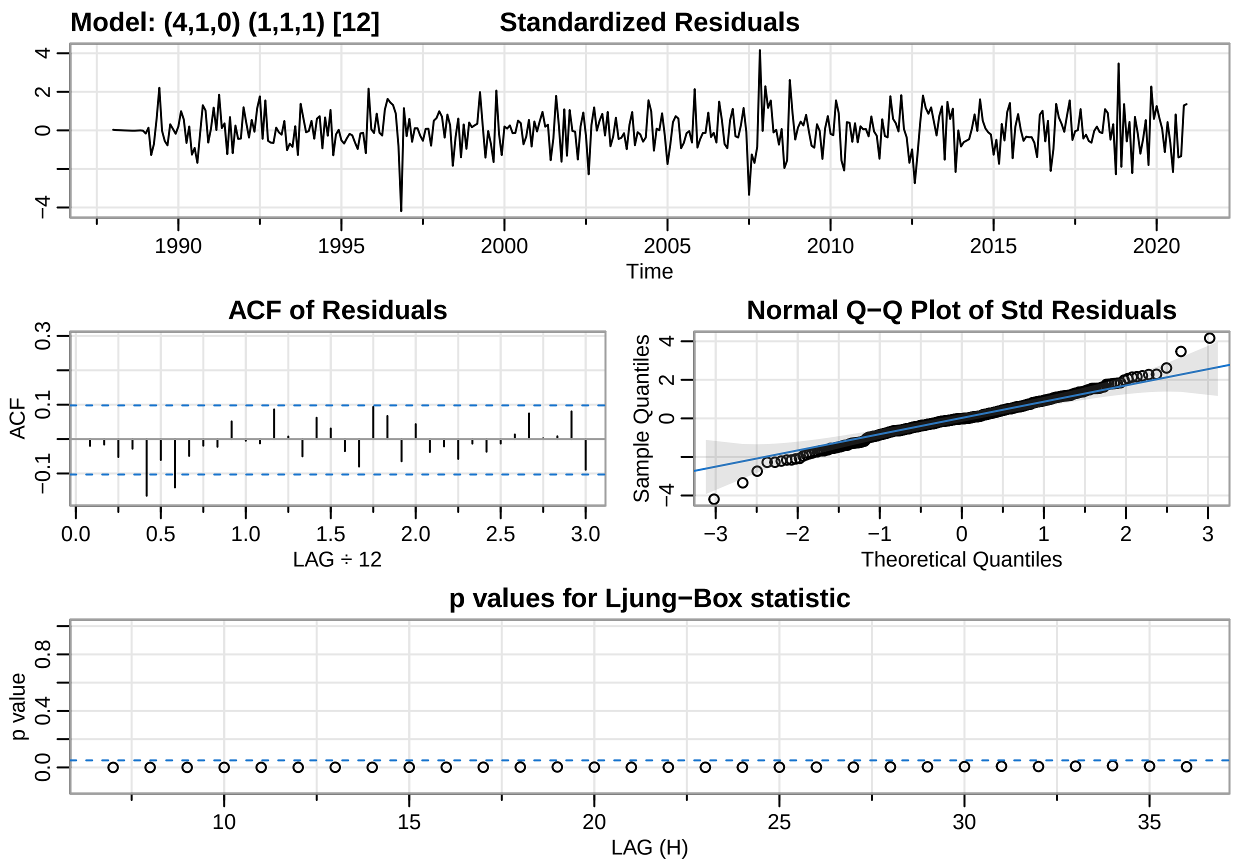


```r
shapiro.test(model_5_train_residuals)
```

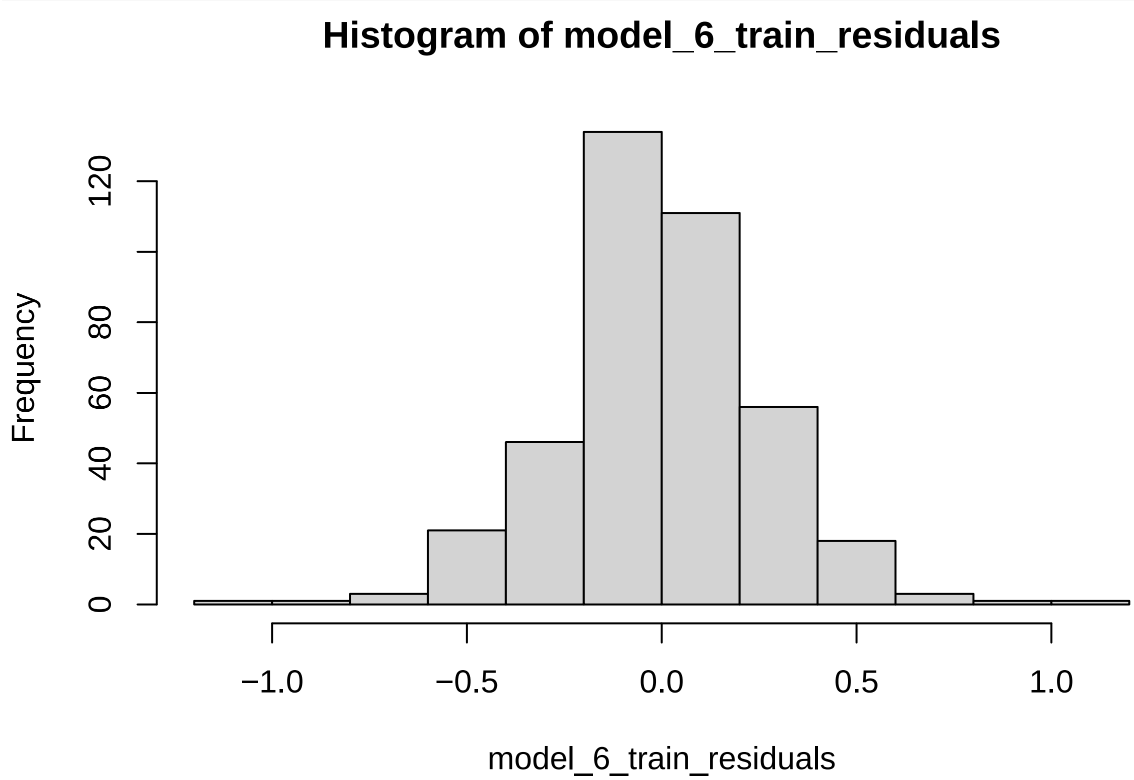
```
##
##  Shapiro-Wilk normality test
##
## data:  model_5_train_residuals
## W = 0.98537, p-value = 0.0004979
```

```r
#SARIMA(4,1,0)x(1,1,1)_12
model_6_train <- sarima(Avg_ExtentTS_Train, p=4, d=1, q=0, P=1, D=1, Q=1, S=12 , details = TRUE)
```

```
## initial  value -1.041584
## iter   2 value -1.241588
## iter   3 value -1.287381
## iter   4 value -1.292821
## iter   5 value -1.300492
## iter   6 value -1.300777
## iter   7 value -1.300797
## iter   8 value -1.300801
## iter   9 value -1.300802
## iter   9 value -1.300802
## iter   9 value -1.300802
## final  value -1.300802
## converged
## initial  value -1.305672
## iter   2 value -1.306365
## iter   3 value -1.306596
## iter   4 value -1.306681
## iter   5 value -1.306698
## iter   6 value -1.306701
## iter   7 value -1.306701
## iter   7 value -1.306701
## iter   7 value -1.306701
## final  value -1.306701
## converged
```




```r
model_6_train_residuals = resid(model_6_train$fit)
hist(model_6_train_residuals)
```

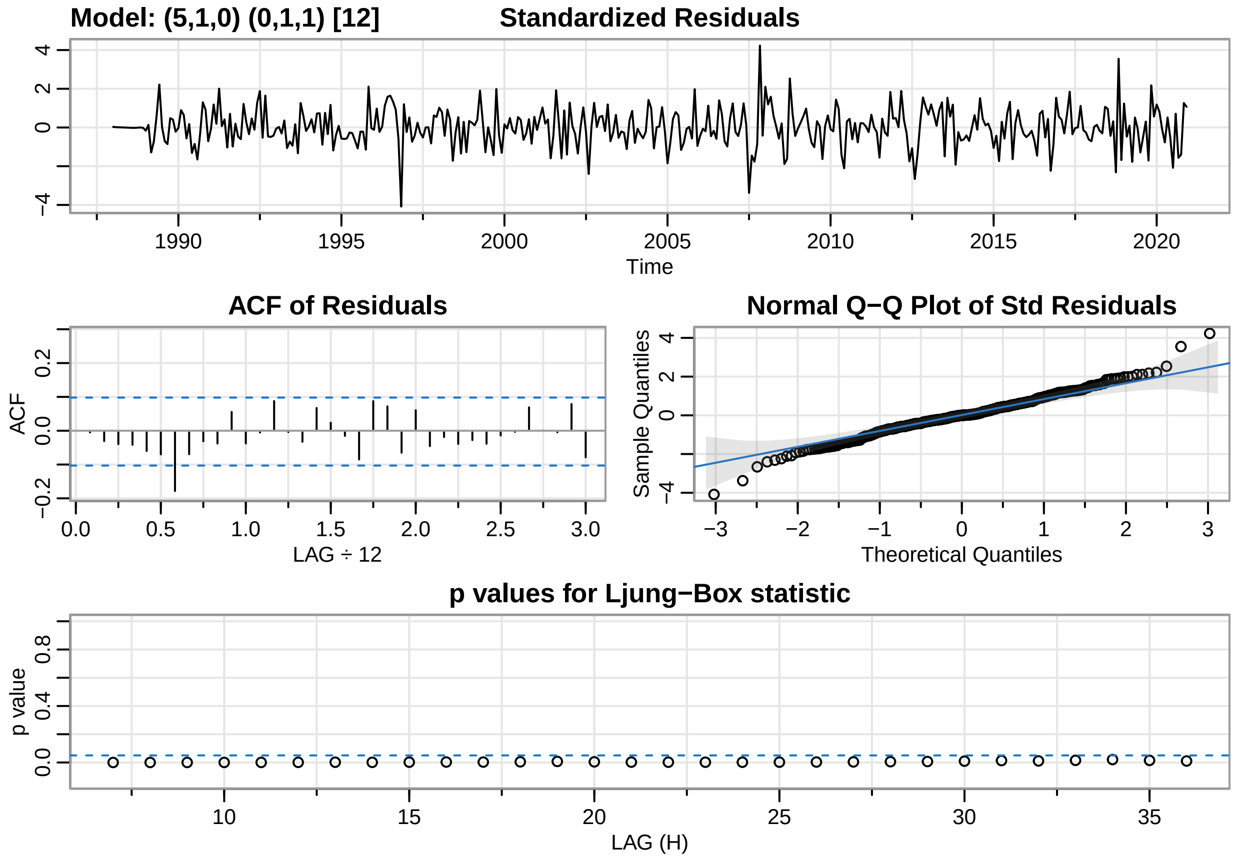


```r
shapiro.test(model_6_train_residuals)
```

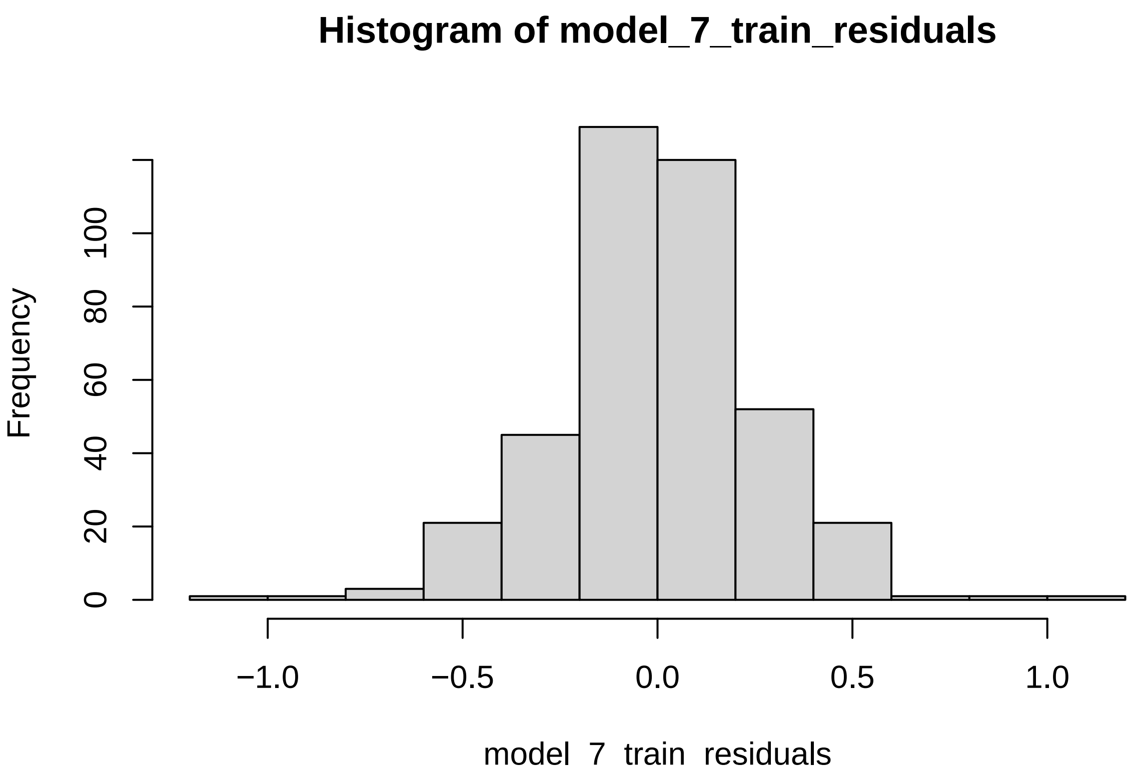
```
##
##  Shapiro-Wilk normality test
##
## data:  model_6_train_residuals
## W = 0.98408, p-value = 0.0002389
```

```r
#SARIMA(5,1,0)x(0,1,1)_12
model_7_train <- sarima(Avg_ExtentTS_Train, p=5, d=1, q=0, P=0, D=1, Q=1, S=12 , details = TRUE)
```

```
## initial  value -1.051141
## iter   2 value -1.280688
## iter   3 value -1.294984
## iter   4 value -1.299314
## iter   5 value -1.306323
## iter   6 value -1.308043
## iter   7 value -1.308352
## iter   8 value -1.308378
## iter   9 value -1.308378
## iter   9 value -1.308378
## iter   9 value -1.308378
## final  value -1.308378
## converged
## initial  value -1.312741
## iter   2 value -1.313183
## iter   3 value -1.313486
## iter   4 value -1.313497
## iter   5 value -1.313500
## iter   6 value -1.313500
## iter   6 value -1.313500
## iter   6 value -1.313500
## final  value -1.313500
## converged
```




```
model_7_train_residuals = resid(model_7_train$fit)
hist(model_7_train_residuals)
```

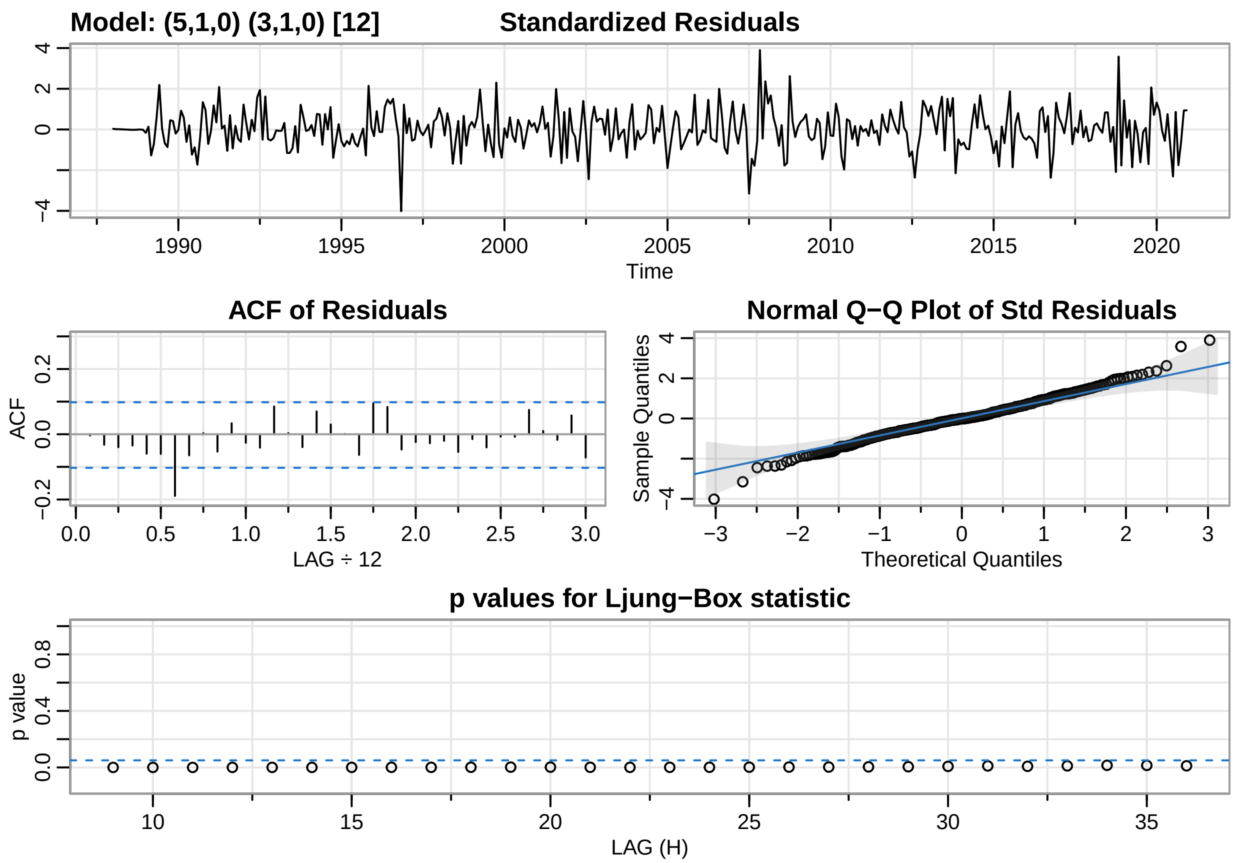


```r
shapiro.test(model_7_train_residuals)
```

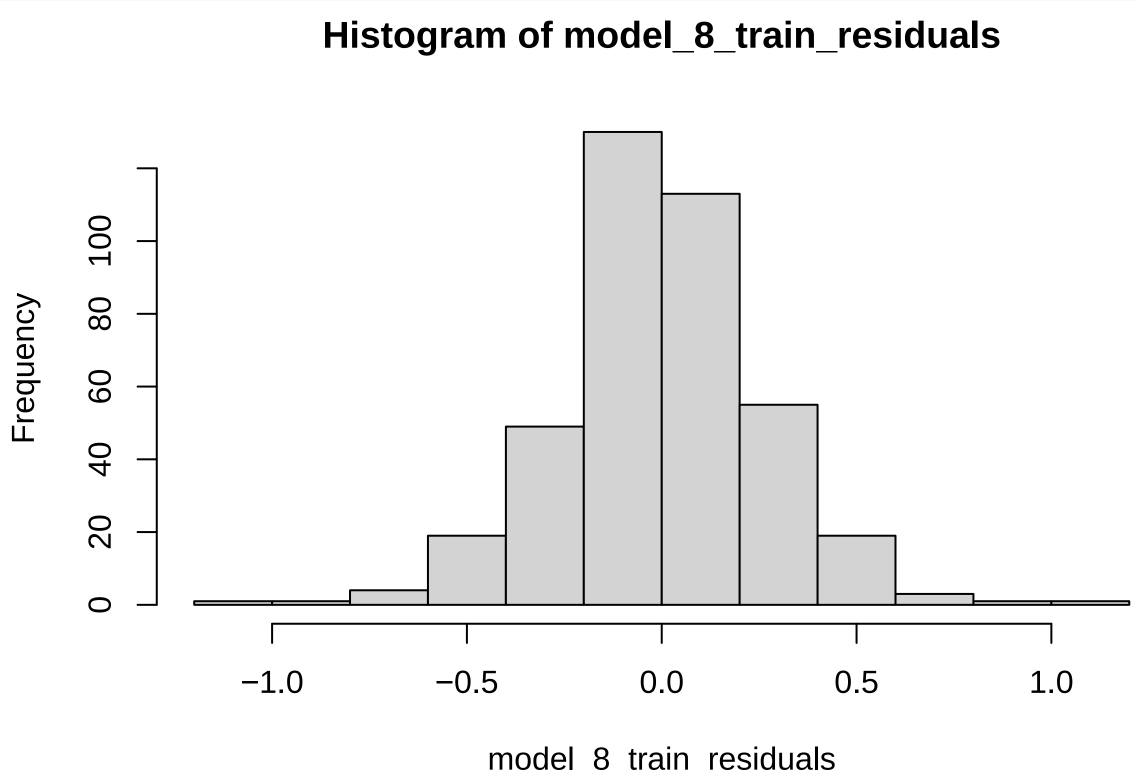
```
##
##  Shapiro-Wilk normality test
##
## data:  model_7_train_residuals
## W = 0.98416, p-value = 0.0002503
```

```r
#SARIMA(5,1,0)x(3,1,0)_12
model_8_train <- sarima(Avg_ExtentTS_Train, p=5, d=1, q=0, P=3, D=1, Q=0, S=12 , details = TRUE)
```

```
## initial  value -1.034323
## iter   2 value -1.181171
## iter   3 value -1.276196
## iter   4 value -1.303175
## iter   5 value -1.309458
## iter   6 value -1.309745
## iter   7 value -1.309762
## iter   8 value -1.309762
## iter   8 value -1.309762
## iter   8 value -1.309762
## final  value -1.309762
## converged
## initial  value -1.309715
## iter   2 value -1.310249
## iter   3 value -1.310286
## iter   4 value -1.310300
## iter   5 value -1.310300
## iter   6 value -1.310300
## iter   6 value -1.310300
## iter   6 value -1.310300
## final  value -1.310300
## converged
```




```
model_8_train_residuals = resid(model_8_train$fit)
hist(model_8_train_residuals)
```

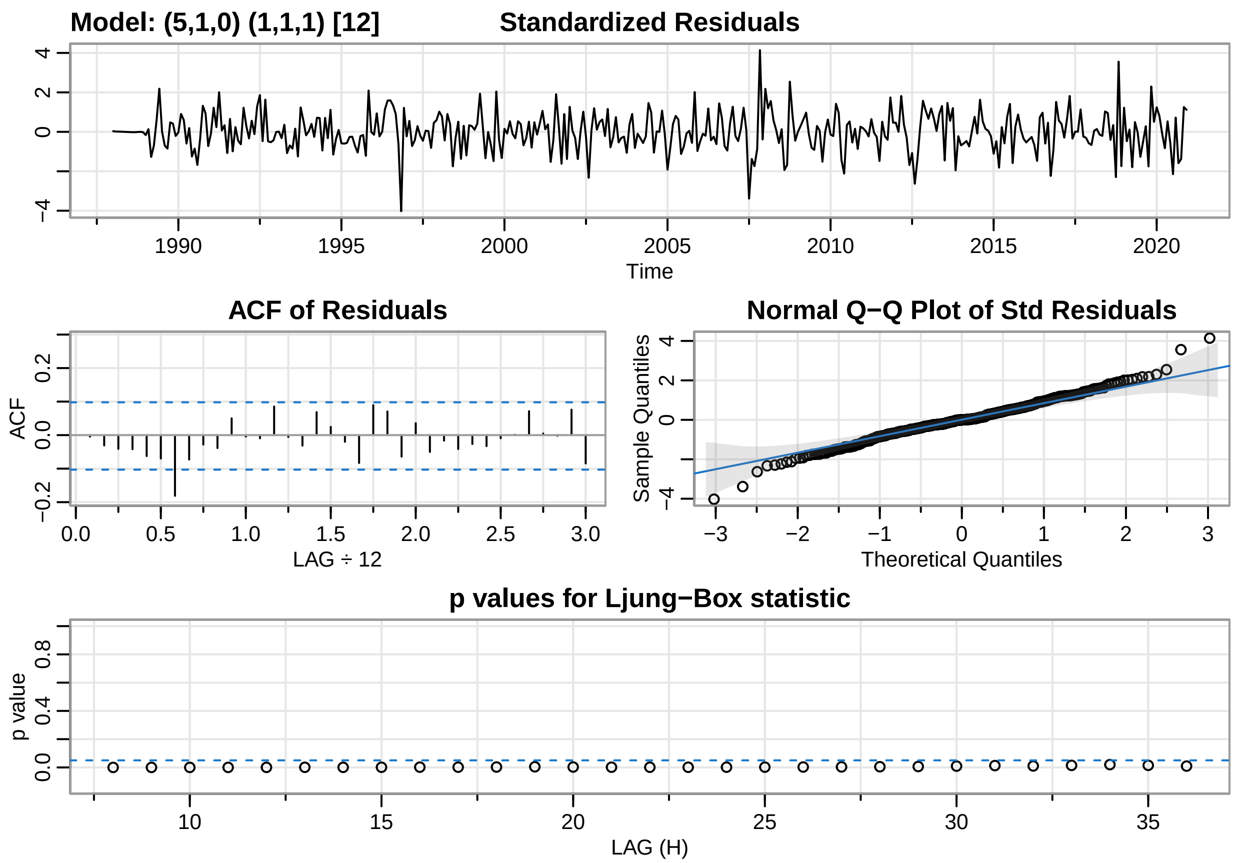


```r
shapiro.test(model_8_train_residuals)
```

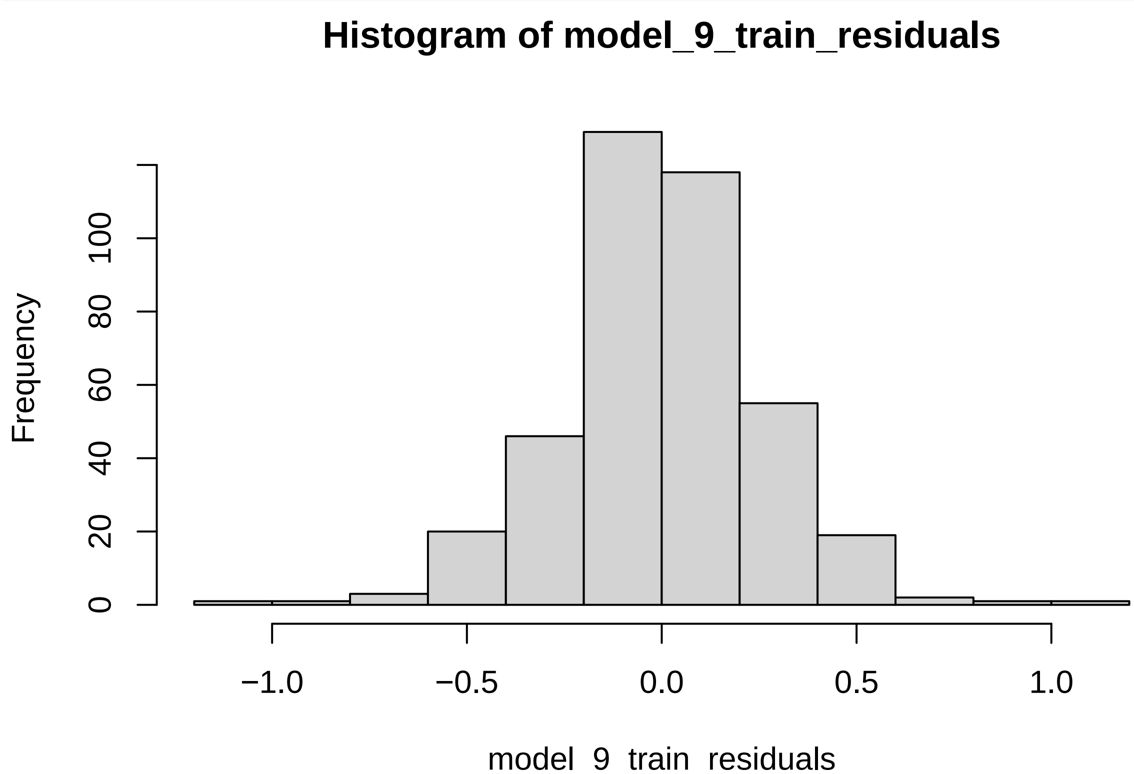
```
##
##  Shapiro-Wilk normality test
##
## data:  model_8_train_residuals
## W = 0.98765, p-value = 0.00193
```

```r
#SARIMA(5,1,0)x(1,1,1)_12
model_9_train <- sarima(Avg_ExtentTS_Train, p=5, d=1, q=0, P=1, D=1, Q=1, S=12 , details = TRUE)
```

```
## initial  value -1.047019
## iter   2 value -1.250977
## iter   3 value -1.298091
## iter   4 value -1.305471
## iter   5 value -1.315008
## iter   6 value -1.315491
## iter   7 value -1.315527
## iter   8 value -1.315540
## iter   9 value -1.315541
## iter   9 value -1.315541
## iter   9 value -1.315541
## final  value -1.315541
## converged
## initial  value -1.313961
## iter   2 value -1.314344
## iter   3 value -1.314392
## iter   4 value -1.314401
## iter   5 value -1.314406
## iter   6 value -1.314406
## iter   7 value -1.314406
## iter   7 value -1.314406
## iter   7 value -1.314406
## final  value -1.314406
## converged
```




```
model_9_train_residuals = resid(model_9_train$fit)
hist(model_9_train_residuals)
```



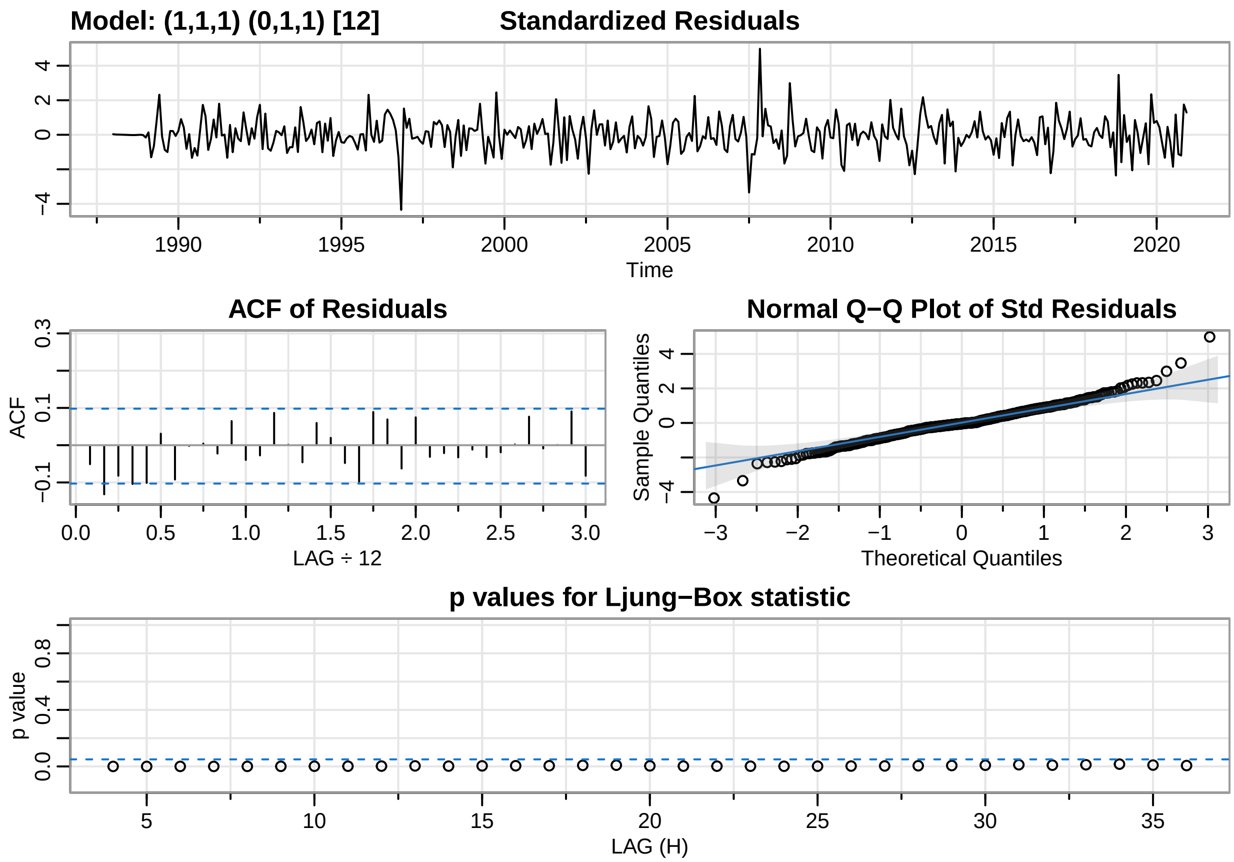
```r
shapiro.test(model_9_train_residuals)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  model_9_train_residuals
## W = 0.98544, p-value = 0.0005193
```

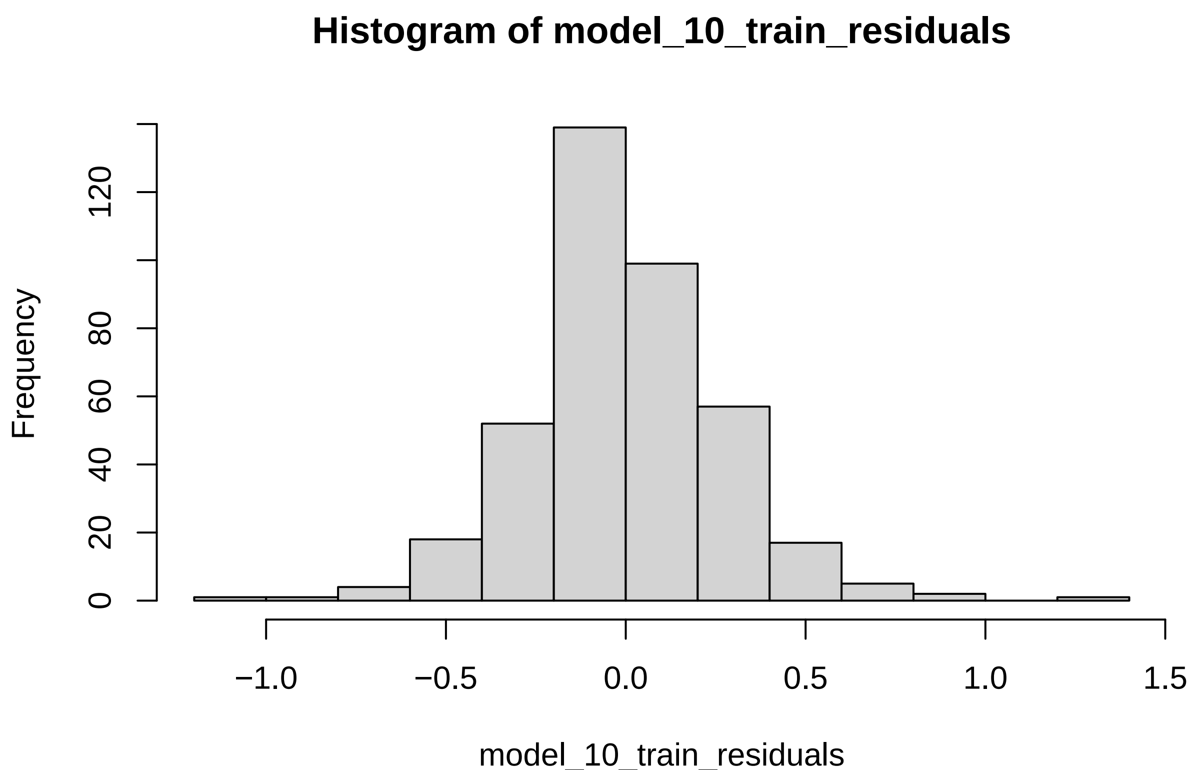
```r
#SARIMA(1,1,1)x(0,1,1)_12
model_10_train <- sarima(Avg_ExtentTS_Train, p=1, d=1, q=1, P=0, D=1, Q=1, S=12 , details = TRUE)
```

```
## initial  value -1.045552
## iter   2 value -1.242919
## iter   3 value -1.258615
## iter   4 value -1.265606
## iter   5 value -1.269085
## iter   6 value -1.271594
## iter   7 value -1.272018
## iter   8 value -1.272061
## iter   9 value -1.272075
## iter  10 value -1.272103
## iter  11 value -1.273789
## iter  12 value -1.274147
## iter  13 value -1.274999
## iter  14 value -1.275080
## iter  15 value -1.276771
## iter  16 value -1.276973
## iter  17 value -1.277440
## iter  18 value -1.277524
## iter  19 value -1.277528
## iter  20 value -1.277529
## iter  21 value -1.277529
## iter  22 value -1.277530
## iter  23 value -1.277530
## iter  24 value -1.277530
## iter  25 value -1.277531
## iter  26 value -1.277531
## iter  26 value -1.277531
## iter  26 value -1.277531
## final  value -1.277531
## converged
## initial  value -1.276765
## iter   2 value -1.276787
## iter   3 value -1.276795
## iter   4 value -1.276795
## iter   5 value -1.276795
## iter   6 value -1.276796
## iter   7 value -1.276797
## iter   8 value -1.276797
## iter   9 value -1.276798
## iter  10 value -1.276798
## iter  10 value -1.276798
## iter  10 value -1.276798
## final  value -1.276798
```

```
## converged
```




```
model_10_train_residuals = resid(model_10_train$fit)
hist(model_10_train_residuals)
```



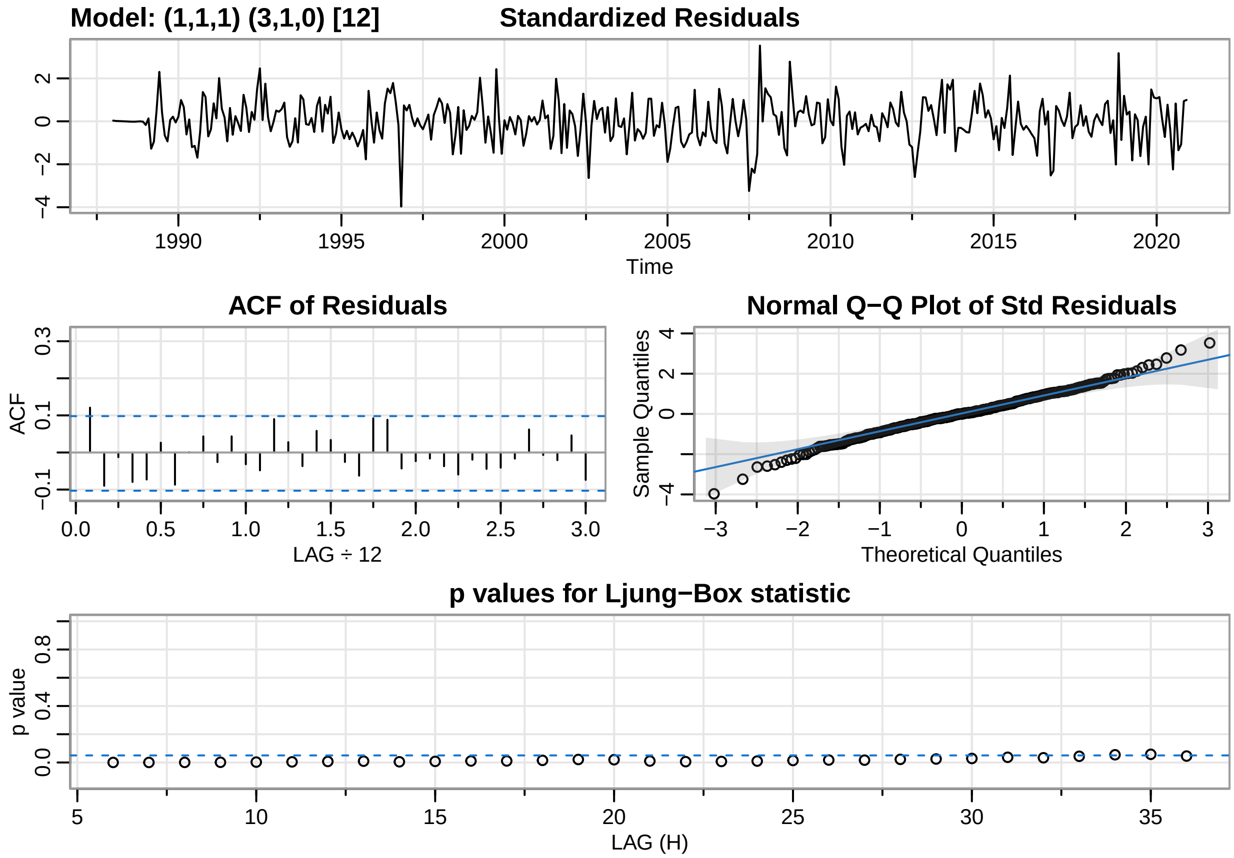

```
shapiro.test(model_10_train_residuals)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  model_10_train_residuals
## W = 0.97359, p-value = 1.319e-06
```

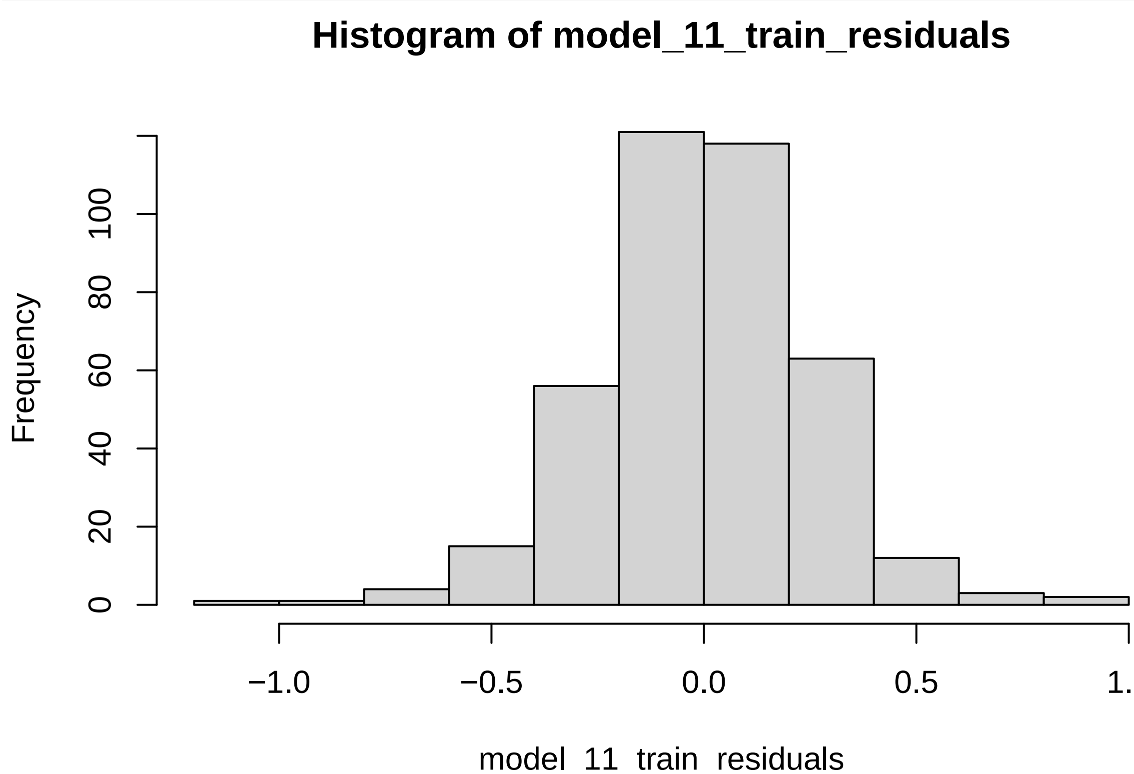
```
#SARIMA(1,1,1)x(3,1,0)_12
model_11_train <- sarima(Avg_ExtentTS_Train, p=1, d=1, q=1, P=3, D=1, Q=0, S=12 , details = TRUE)
```

```
## initial  value -1.038097
## iter   2 value -1.144641
## iter   3 value -1.239216
## iter   4 value -1.265137
## iter   5 value -1.269506
## iter   6 value -1.269577
## iter   7 value -1.269587
## iter   8 value -1.269588
## iter   9 value -1.269590
## iter  10 value -1.269601
## iter  11 value -1.269726
## iter  12 value -1.269928
## iter  13 value -1.269990
## iter  14 value -1.270074
## iter  15 value -1.270123
## iter  16 value -1.270398
## iter  17 value -1.274020
## iter  18 value -1.276115
## iter  19 value -1.281384
```

```
## iter  20 value -1.292576
## iter  21 value -1.304600
## iter  22 value -1.318963
## iter  23 value -1.331206
## iter  24 value -1.334219
## iter  25 value -1.334457
## iter  26 value -1.334997
## iter  27 value -1.336065
## iter  28 value -1.336081
## iter  29 value -1.336093
## iter  30 value -1.336095
## iter  30 value -1.336095
## iter  30 value -1.336095
## final  value -1.336095
## converged
## initial  value -1.332968
## iter   2 value -1.334981
## iter   3 value -1.336741
## iter   4 value -1.336979
## iter   5 value -1.337236
## iter   6 value -1.337267
## iter   7 value -1.337272
## iter   8 value -1.337273
## iter   9 value -1.337273
## iter   9 value -1.337273
## iter   9 value -1.337273
## final  value -1.337273
## converged
```




```
model_11_train_residuals = resid(model_11_train$fit)
hist(model_11_train_residuals)
```



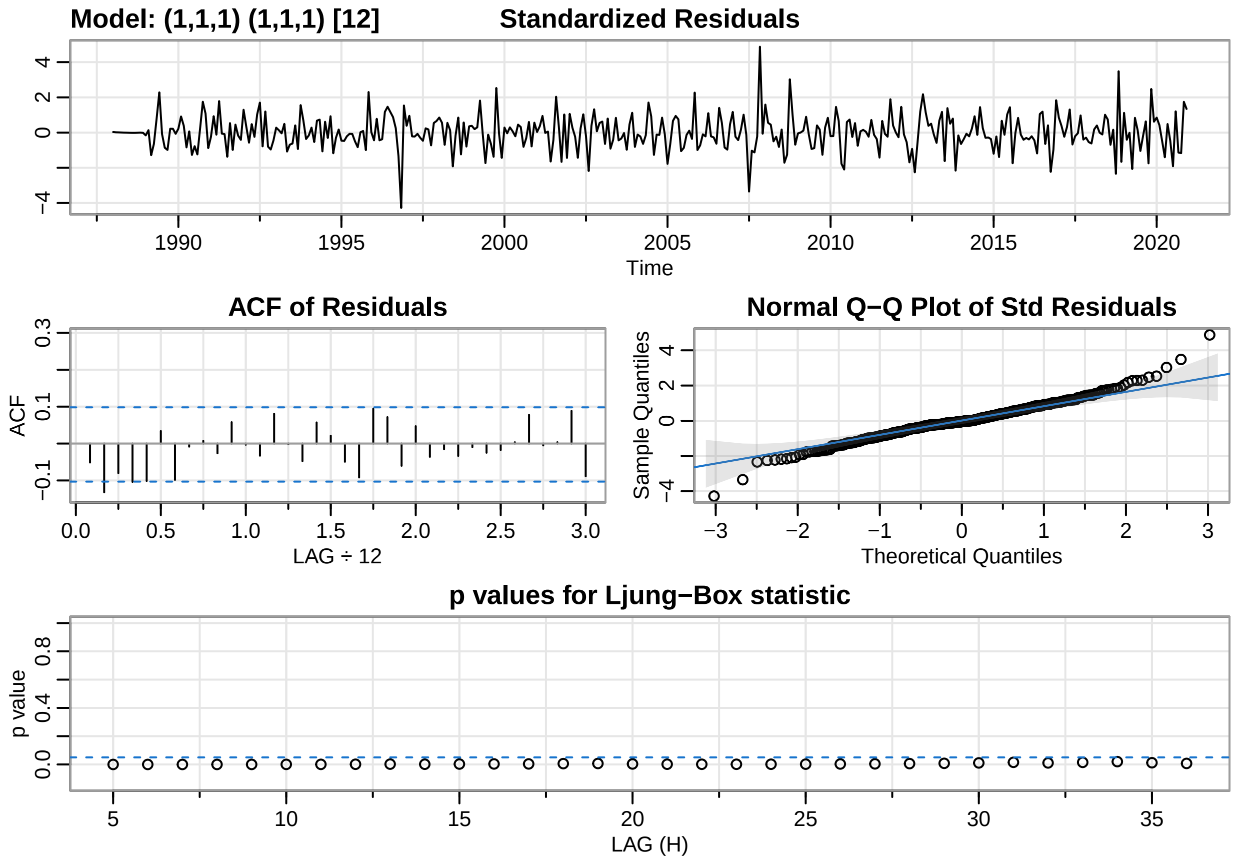
```r
shapiro.test(model_11_train_residuals)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  model_11_train_residuals
## W = 0.99044, p-value = 0.01131
```

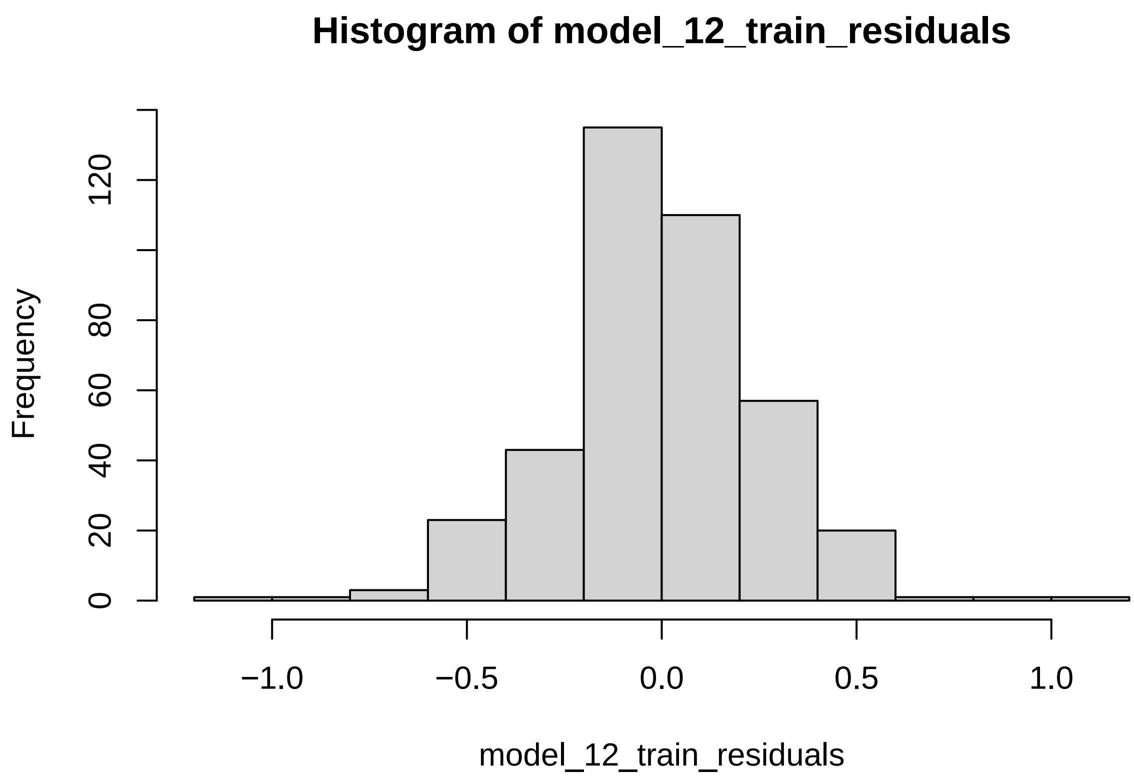
```r
#SARIMA(1,1,1)x(1,1,1)_12
model_12_train <- sarima(Avg_ExtentTS_Train, p=1, d=1, q=1, P=1, D=1, Q=1, S=12 , details = TRUE)
```

```
## initial  value -1.043857
## iter   2 value -1.216764
## iter   3 value -1.259588
## iter   4 value -1.264738
## iter   5 value -1.271016
## iter   6 value -1.271155
## iter   7 value -1.271164
## iter   8 value -1.271165
## iter   9 value -1.271166
## iter  10 value -1.271174
## iter  11 value -1.271177
## iter  12 value -1.271179
## iter  13 value -1.271182
## iter  14 value -1.271192
## iter  15 value -1.271226
## iter  16 value -1.271350
## iter  17 value -1.271513
## iter  18 value -1.271902
## iter  19 value -1.272301
## iter  20 value -1.272695
## iter  21 value -1.272800
## iter  22 value -1.272856
## iter  23 value -1.272922
## iter  24 value -1.273079
## iter  25 value -1.273307
## iter  26 value -1.273488
## iter  27 value -1.273600
## iter  28 value -1.273602
## iter  29 value -1.273604
## iter  30 value -1.273604
## iter  31 value -1.273604
## iter  31 value -1.273604
## iter  31 value -1.273604
## final  value -1.273604
## converged
## initial  value -1.276218
## iter   2 value -1.276807
## iter   3 value -1.277527
## iter   4 value -1.277549
## iter   5 value -1.277577
## iter   6 value -1.277709
## iter   7 value -1.277802
## iter   8 value -1.277869
```

```
## iter   9 value -1.277873
## iter  10 value -1.277874
## iter  10 value -1.277874
## final  value -1.277874
## converged
```




```
model_12_train_residuals = resid(model_4_train$fit)
hist(model_12_train_residuals)
```



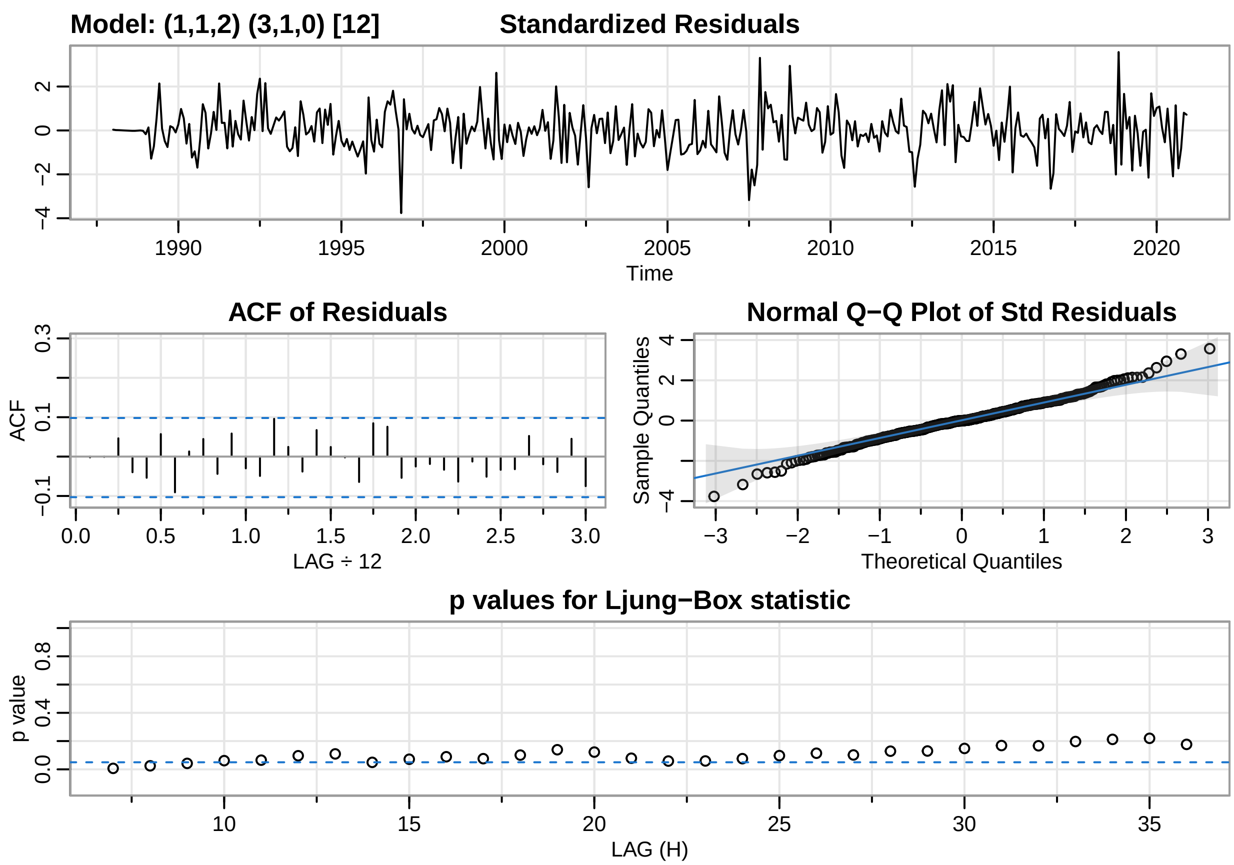

```
shapiro.test(model_12_train_residuals)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  model_12_train_residuals
## W = 0.98293, p-value = 0.0001267
```

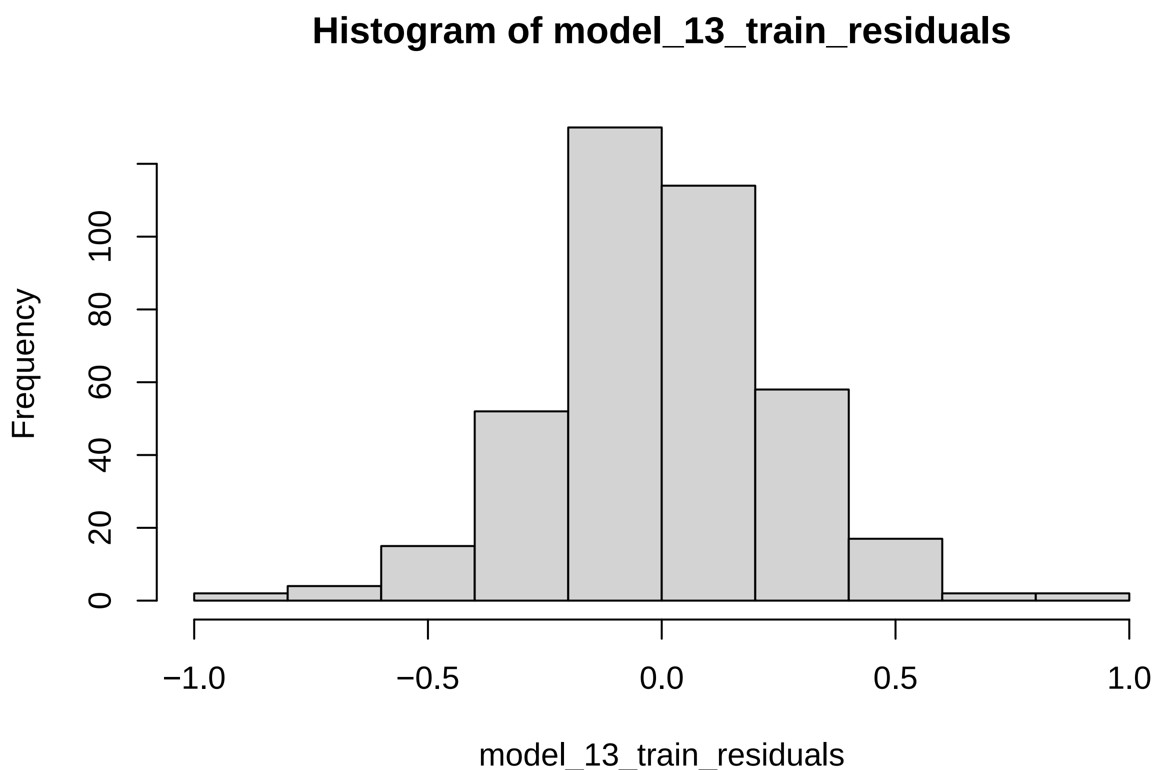
```
#SARIMA(1,1,2)x(3,1,0)_12
model_13_train <- sarima(Avg_ExtentTS_Train, p=1, d=1, q=2, P=3, D=1, Q=0, S=12 , details = TRUE)
```

```
## initial  value -1.038097
## iter   2 value -1.174147
## iter   3 value -1.267295
## iter   4 value -1.292320
## iter   5 value -1.298557
## iter   6 value -1.301947
## iter   7 value -1.306400
## iter   8 value -1.321210
## iter   9 value -1.330951
## iter  10 value -1.335397
## iter  11 value -1.337789
## iter  12 value -1.340572
## iter  13 value -1.347191
## iter  14 value -1.348937
## iter  15 value -1.350213
## iter  16 value -1.351118
## iter  17 value -1.351773
## iter  18 value -1.352098
## iter  19 value -1.352148
```

```
## iter  20 value -1.352151
## iter  21 value -1.352151
## iter  21 value -1.352151
## final  value -1.352151
## converged
## initial  value -1.349885
## iter   2 value -1.351590
## iter   3 value -1.352561
## iter   4 value -1.352788
## iter   5 value -1.352968
## iter   6 value -1.352987
## iter   7 value -1.353011
## iter   8 value -1.353019
## iter   9 value -1.353020
## iter  10 value -1.353021
## iter  10 value -1.353021
## iter  10 value -1.353021
## final  value -1.353021
## converged
```




```
model_13_train_residuals = resid(model_13_train$fit)
hist(model_13_train_residuals)
```



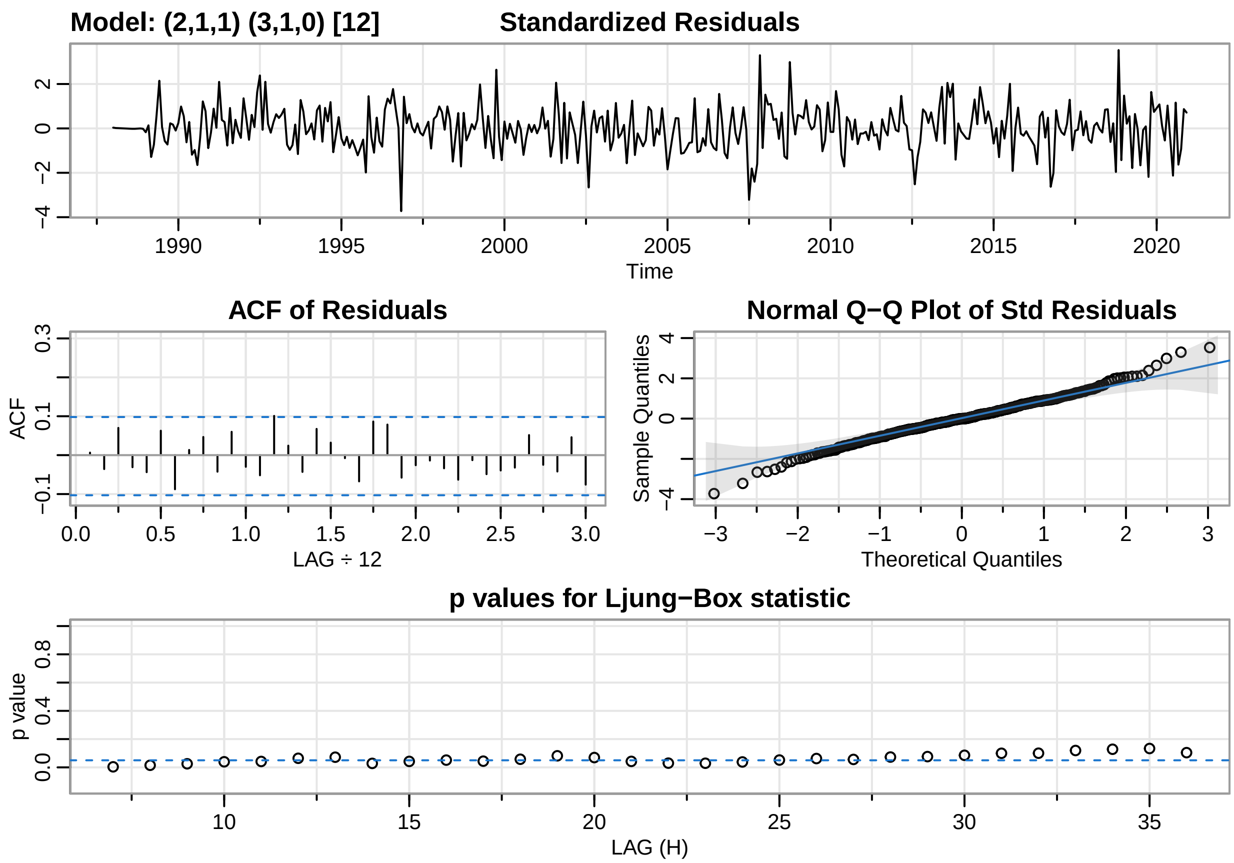

```
shapiro.test(model_13_train_residuals)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  model_13_train_residuals
## W = 0.98949, p-value = 0.006128
```

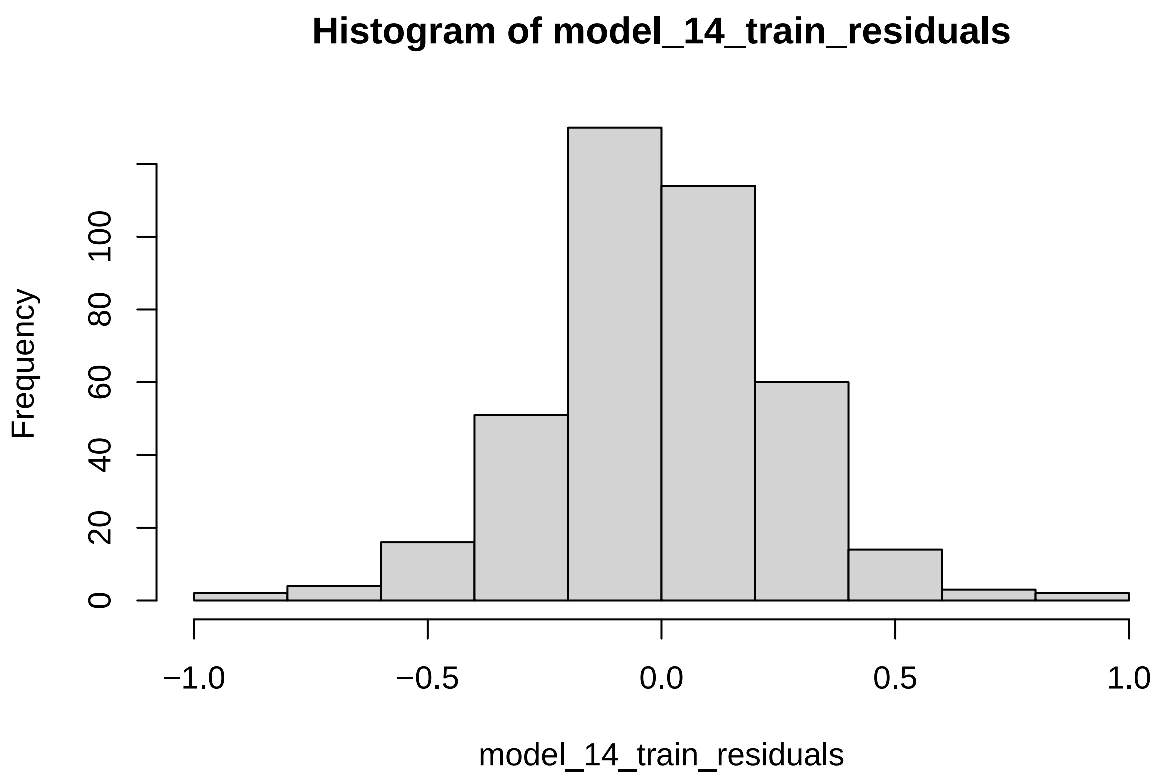
```
#SARIMA(2,1,1)x(3,1,0)_12
model_14_train <- sarima(Avg_ExtentTS_Train, p=2, d=1, q=1, P=3, D=1, Q=0, S=12 , details = TRUE)
```

```
## initial  value -1.036901
## iter   2 value -1.166690
## iter   3 value -1.255867
## iter   4 value -1.280740
## iter   5 value -1.285485
## iter   6 value -1.286803
## iter   7 value -1.286902
## iter   8 value -1.286964
## iter   9 value -1.287912
## iter  10 value -1.294229
## iter  11 value -1.298570
## iter  12 value -1.305005
## iter  13 value -1.310441
## iter  14 value -1.314638
## iter  15 value -1.315596
## iter  16 value -1.329652
## iter  17 value -1.330212
## iter  18 value -1.330363
## iter  19 value -1.330578
```

```
## iter  20 value -1.330666
## iter  21 value -1.330705
## iter  22 value -1.330707
## iter  23 value -1.330707
## iter  24 value -1.330707
## iter  24 value -1.330707
## iter  24 value -1.330707
## final  value -1.330707
## converged
## initial  value -1.343351
## iter   2 value -1.348834
## iter   3 value -1.350278
## iter   4 value -1.351392
## iter   5 value -1.351635
## iter   6 value -1.351709
## iter   7 value -1.351762
## iter   8 value -1.351781
## iter   9 value -1.351785
## iter  10 value -1.351785
## iter  10 value -1.351785
## iter  10 value -1.351785
## final  value -1.351785
## converged
```




```
model_14_train_residuals = resid(model_14_train$fit)
hist(model_14_train_residuals)
```




```
shapiro.test(model_14_train_residuals)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  model_14_train_residuals
## W = 0.99044, p-value = 0.01131
```

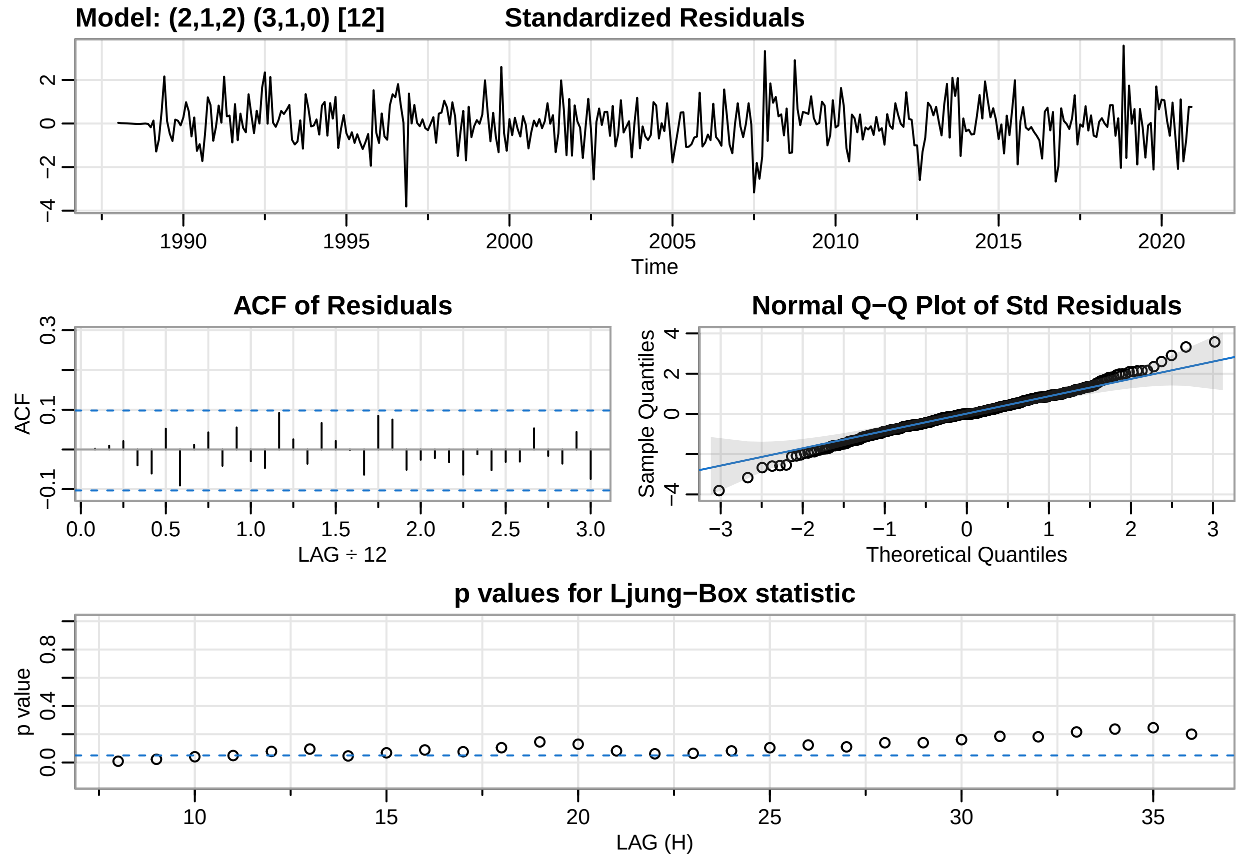
```
#SARIMA(2,1,2)x(3,1,0)_12
model_15_train <- sarima(Avg_ExtentTS_Train, p=2, d=1, q=2, P=3, D=1, Q=0, S=12 , details = TRUE)
```

```
## initial  value -1.036901
## iter   2 value -1.157349
## iter   3 value -1.257467
## iter   4 value -1.285105
## iter   5 value -1.293434
## iter   6 value -1.295560
## iter   7 value -1.300881
## iter   8 value -1.302705
## iter   9 value -1.311610
## iter  10 value -1.325367
## iter  11 value -1.333384
## iter  12 value -1.335509
## iter  13 value -1.335519
## iter  14 value -1.337033
## iter  15 value -1.338102
## iter  16 value -1.339632
## iter  17 value -1.340943
## iter  18 value -1.341232
## iter  19 value -1.343535
```

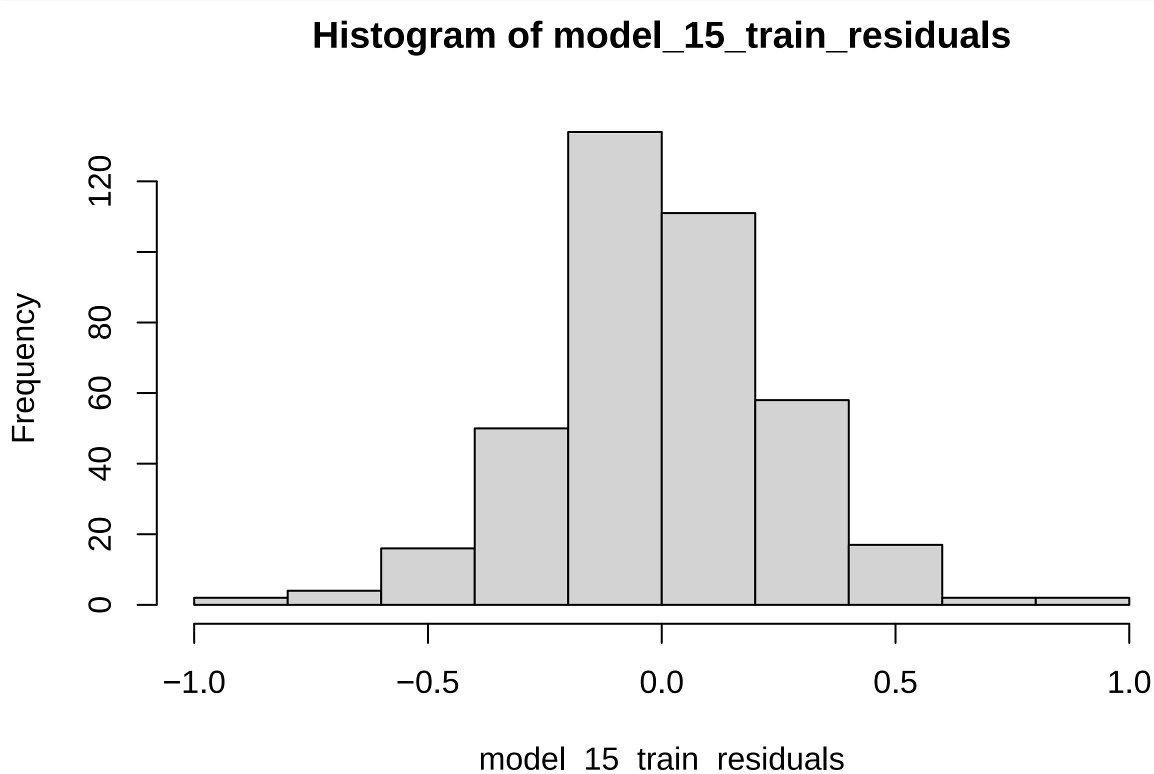
```
## iter  20 value -1.345330
## iter  21 value -1.346789
## iter  22 value -1.346934
## iter  23 value -1.347039
## iter  24 value -1.347063
## iter  25 value -1.347078
## iter  26 value -1.347086
## iter  27 value -1.347112
## iter  28 value -1.347139
## iter  29 value -1.347147
## iter  30 value -1.347149
## iter  31 value -1.347149
## iter  32 value -1.347149
## iter  32 value -1.347149
## iter  32 value -1.347149
## final  value -1.347149
## converged
## initial  value -1.348512
## iter   2 value -1.349569
## iter   3 value -1.349586
## iter   4 value -1.350848
## iter   5 value -1.350982
## iter   6 value -1.351193
## iter   7 value -1.352121
## iter   8 value -1.352601
## iter   9 value -1.353193
## iter  10 value -1.353272
## iter  11 value -1.353333
## iter  12 value -1.353339
## iter  13 value -1.353341
## iter  14 value -1.353341
## iter  15 value -1.353341
## iter  16 value -1.353342
## iter  16 value -1.353341
## iter  16 value -1.353341
## final  value -1.353342
## converged
```

```
## Warning in sqrt(diag(fitit$var.coef)): NaNs produced
```

```
## Warning in sqrt(diag(fitit$var.coef)): NaNs produced
```




```
model_15_train_residuals = resid(model_15_train$fit)
hist(model_15_train_residuals)
```

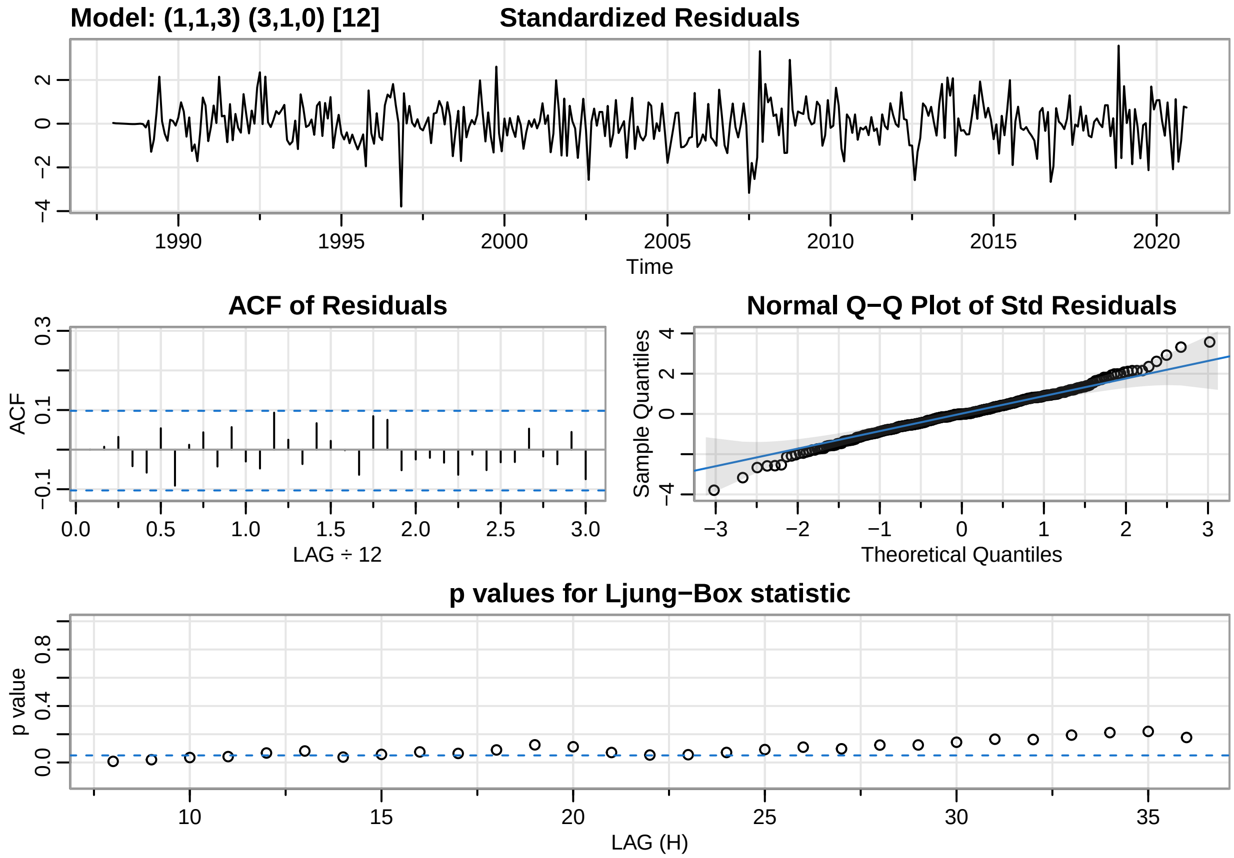


```r
shapiro.test(model_15_train_residuals)
```

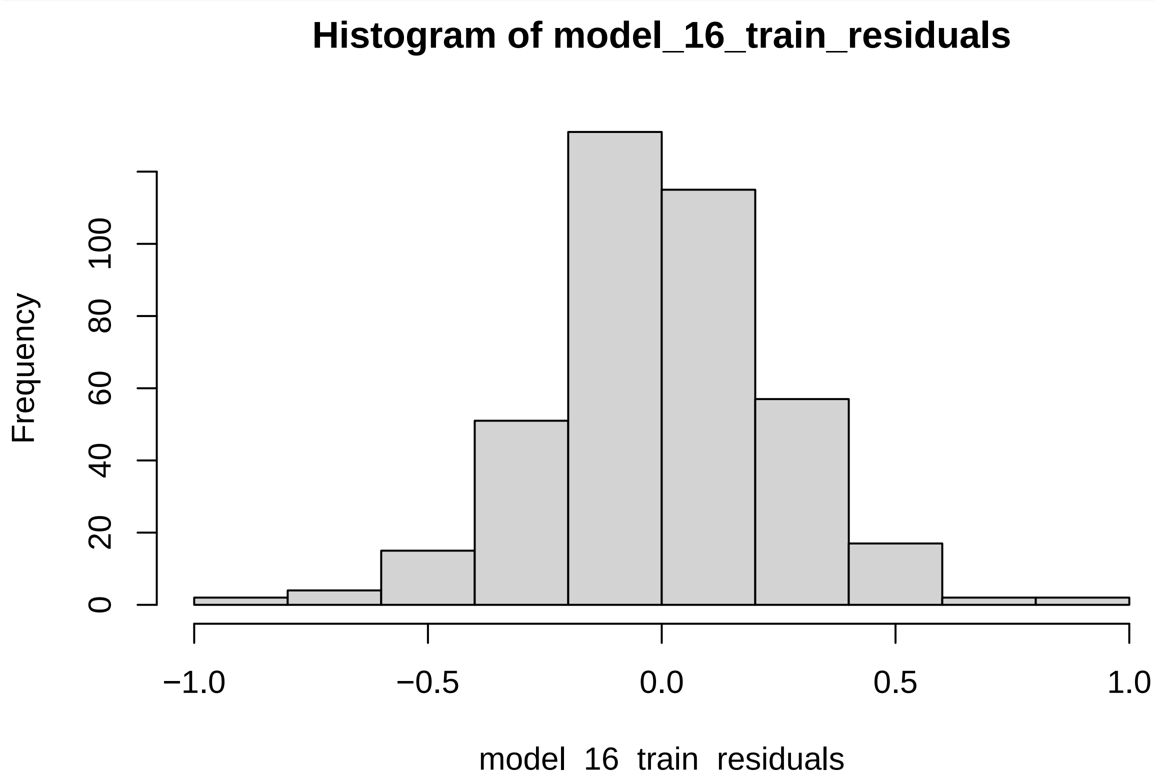
```
##
##  Shapiro-Wilk normality test
##
## data:  model_15_train_residuals
## W = 0.98876, p-value = 0.003849
```

```r
#SARIMA(1,1,3)x(3,1,0)_12
model_16_train <- sarima(Avg_ExtentTS_Train, p=1, d=1, q=3, P=3, D=1, Q=0, S=12 , details = TRUE)
```

```
## initial  value -1.038097
## iter   2 value -1.179338
## iter   3 value -1.282734
## iter   4 value -1.316080
## iter   5 value -1.323583
## iter   6 value -1.325829
## iter   7 value -1.327176
## iter   8 value -1.330400
## iter   9 value -1.336051
## iter  10 value -1.343009
## iter  11 value -1.345708
## iter  12 value -1.349993
## iter  13 value -1.351352
## iter  14 value -1.351698
## iter  15 value -1.352012
## iter  16 value -1.352157
## iter  17 value -1.352161
## iter  18 value -1.352172
## iter  19 value -1.352179
## iter  20 value -1.352186
## iter  21 value -1.352188
## iter  22 value -1.352189
## iter  23 value -1.352190
## iter  24 value -1.352190
## iter  24 value -1.352190
## iter  24 value -1.352190
## final  value -1.352190
## converged
## initial  value -1.349943
## iter   2 value -1.352612
## iter   3 value -1.352925
## iter   4 value -1.353025
## iter   5 value -1.353075
## iter   6 value -1.353123
## iter   7 value -1.353167
## iter   8 value -1.353192
## iter   9 value -1.353198
## iter  10 value -1.353199
## iter  11 value -1.353199
## iter  11 value -1.353199
## iter  11 value -1.353199
## final  value -1.353199
## converged
```




```r
model_16_train_residuals = resid(model_16_train$fit)
hist(model_16_train_residuals)
```



```r
shapiro.test(model_16_train_residuals)
```

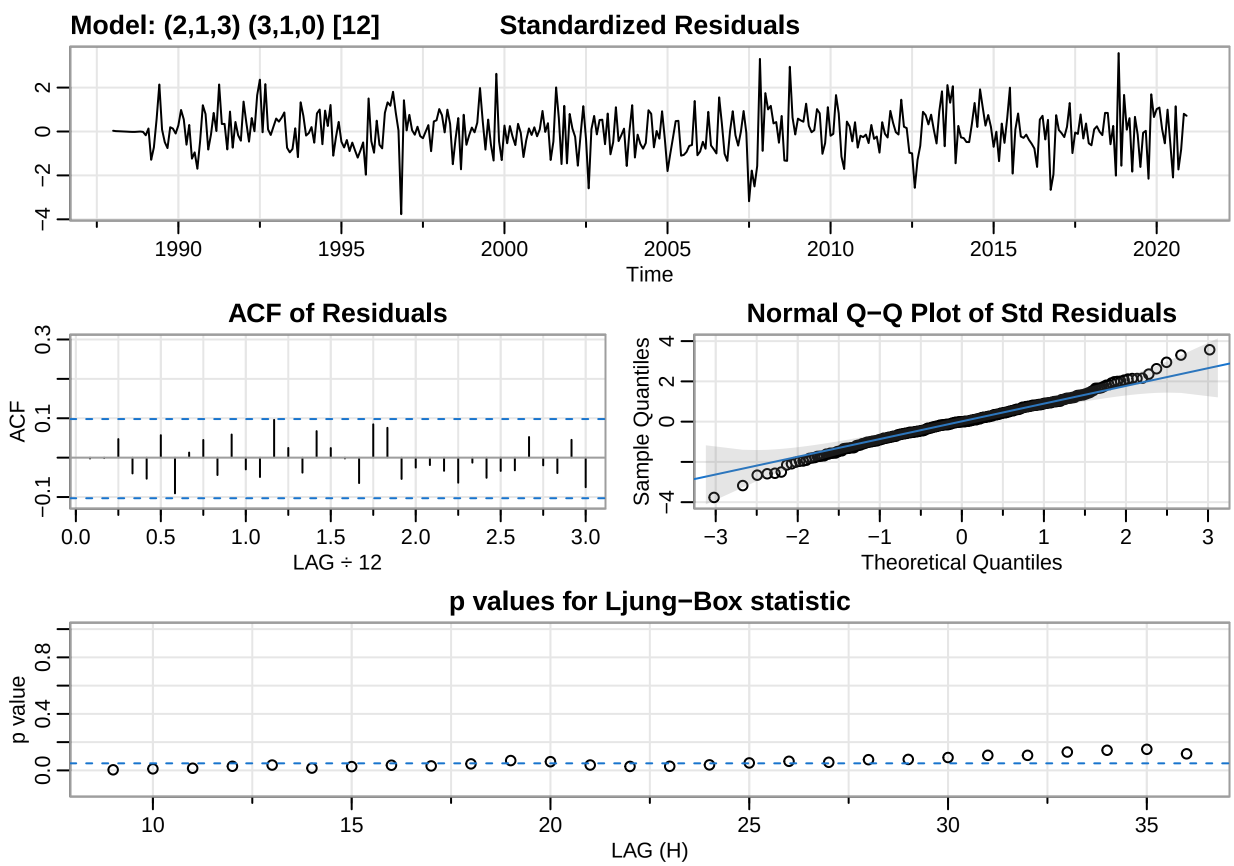
```
##
##  Shapiro-Wilk normality test
##
## data:  model_16_train_residuals
## W = 0.98903, p-value = 0.004551
```

```r
# This is commented out because it results in an error
#SARIMA(3,1,1)x(3,1,0)_12
#model_17_train <- sarima(Avg_ExtentTS_Train, p=3, d=1, q=1, P=3, D=1, Q=0, S=12, details = TRUE)
#model_17_train_residuals = resid(model_17_train$fit)
#hist(model_17_train_residuals)
#shapiro.test(model_17_train_residuals)
#gives an error when run
```

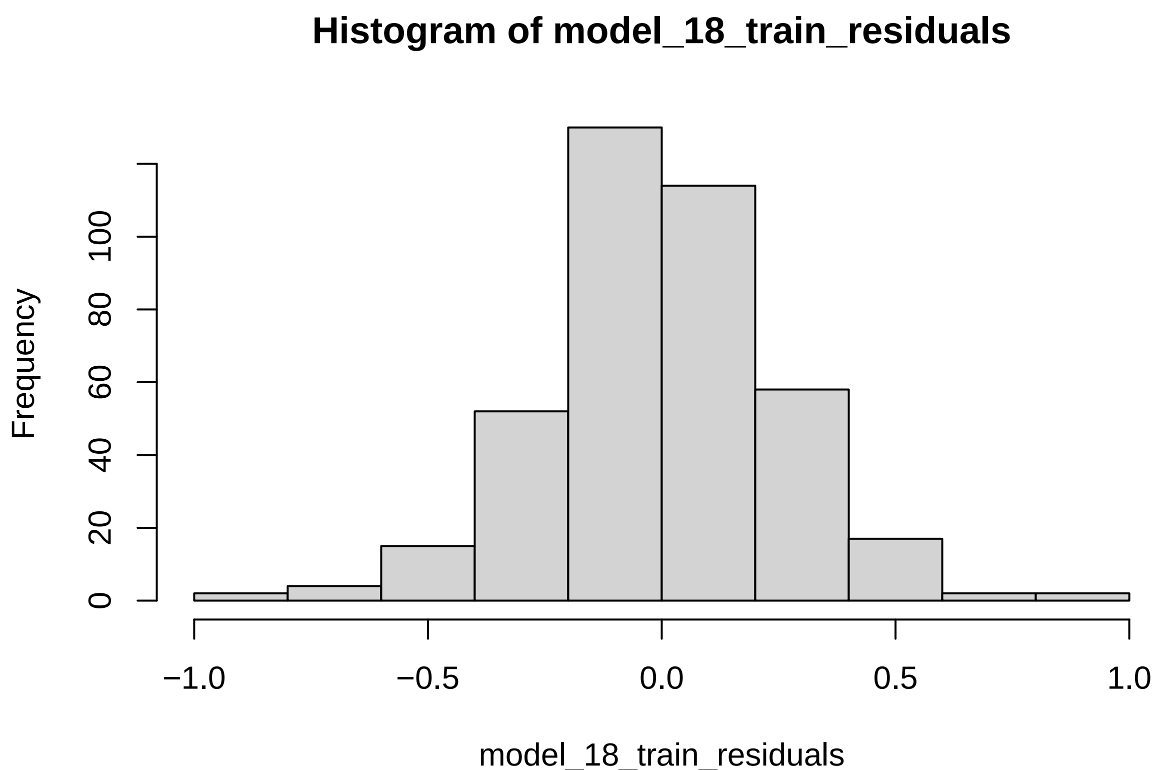
```r
#SARIMA(2,1,3)x(3,1,0)_12
model_18_train <- sarima(Avg_ExtentTS_Train, p=2, d=1, q=3, P=3, D=1, Q=0, S=12 , details = TRUE)
```

```
## initial  value -1.036901
## iter   2 value -1.163257
## iter   3 value -1.266869
## iter   4 value -1.296361
## iter   5 value -1.310632
## iter   6 value -1.321697
## iter   7 value -1.325417
## iter   8 value -1.332240
## iter   9 value -1.336089
## iter  10 value -1.340935
## iter  11 value -1.350055
## iter  12 value -1.350337
## iter  13 value -1.350570
## iter  14 value -1.350839
## iter  15 value -1.350916
## iter  16 value -1.351018
## iter  17 value -1.351217
## iter  18 value -1.351605
## iter  19 value -1.351656
## iter  20 value -1.351764
## iter  21 value -1.351810
## iter  22 value -1.351859
## iter  23 value -1.351932
## iter  24 value -1.352073
## iter  25 value -1.352252
## iter  26 value -1.352391
## iter  27 value -1.352416
## iter  28 value -1.352463
## iter  29 value -1.352465
## iter  30 value -1.352465
## iter  31 value -1.352466
## iter  32 value -1.352466
## iter  32 value -1.352466
## iter  33 value -1.352466
## iter  33 value -1.352466
## iter  33 value -1.352466
```

```
## final  value -1.352466
## converged
## initial  value -1.352034
## iter   2 value -1.352248
## iter   3 value -1.352661
## iter   4 value -1.352988
## iter   5 value -1.353009
## iter   6 value -1.353020
## iter   7 value -1.353021
## iter   8 value -1.353021
## iter   9 value -1.353021
## iter   9 value -1.353021
## iter   9 value -1.353021
## final  value -1.353021
## converged

## Warning in sqrt(diag(fitit$var.coef)): NaNs produced

## Warning in sqrt(diag(fitit$var.coef)): NaNs produced
```




```
model_18_train_residuals = resid(model_18_train$fit)
hist(model_18_train_residuals)
```



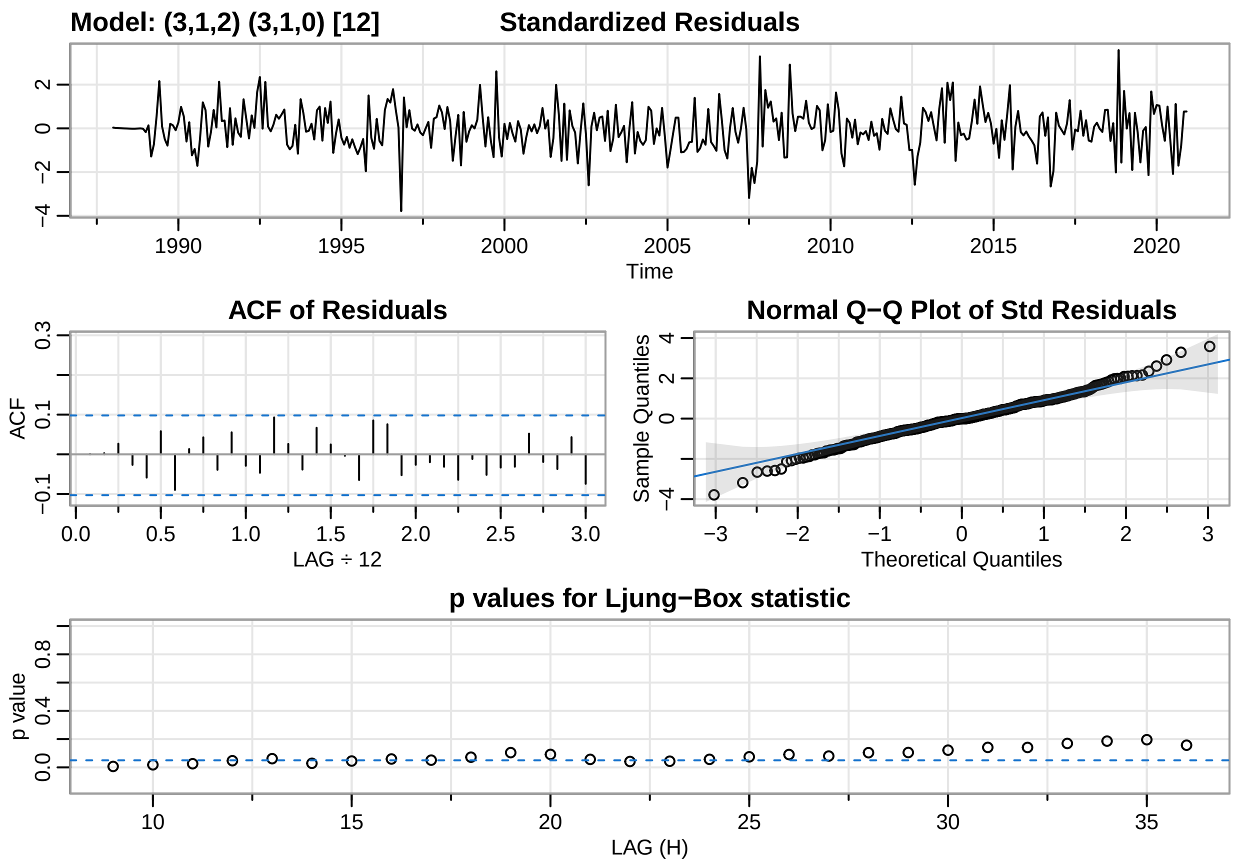

```r
shapiro.test(model_18_train_residuals)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  model_18_train_residuals
## W = 0.9895, p-value = 0.00616
```

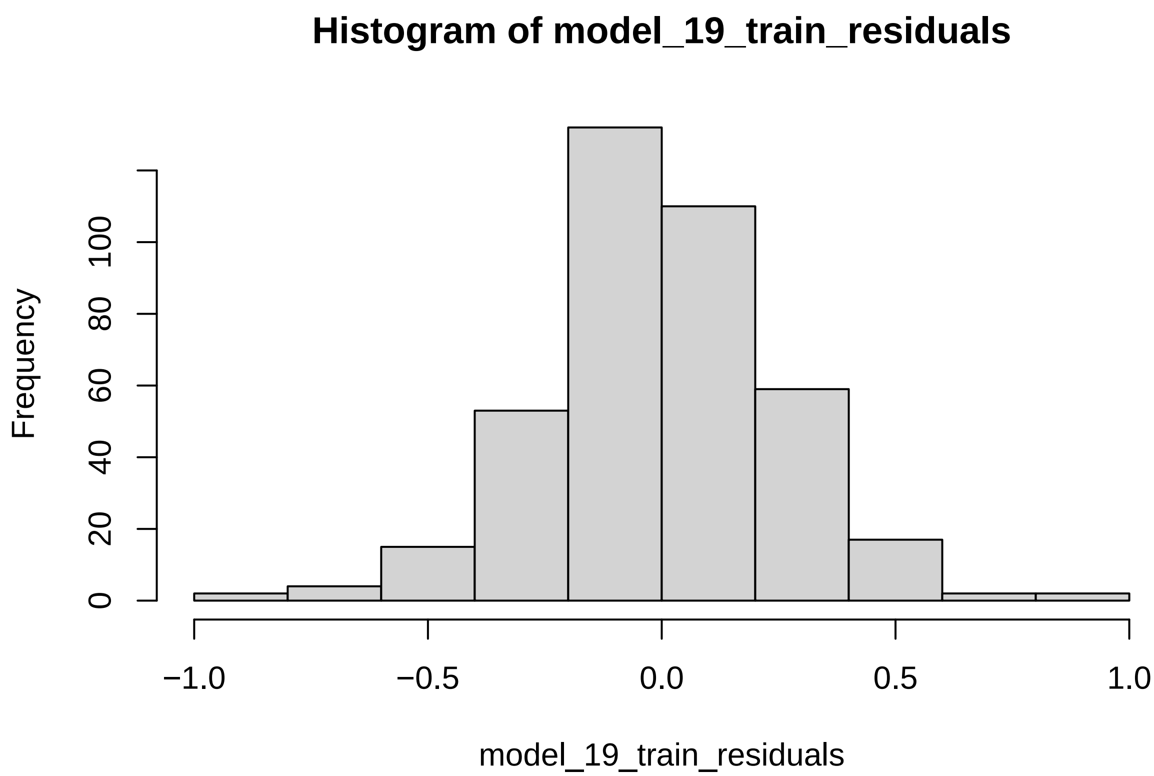
```r
#SARIMA(3,1,2)x(3,1,0)_12
model_19_train <- sarima(Avg_ExtentTS_Train, p=3, d=1, q=2, P=3, D=1, Q=0, S=12 , details = TRUE)
```

```
## initial  value -1.036159
## iter   2 value -1.160007
## iter   3 value -1.259848
## iter   4 value -1.286934
## iter   5 value -1.296300
## iter   6 value -1.299039
## iter   7 value -1.331452
## iter   8 value -1.341032
## iter   9 value -1.349800
## iter  10 value -1.351699
## iter  11 value -1.352268
## iter  12 value -1.352842
## iter  13 value -1.354250
## iter  14 value -1.357163
## iter  15 value -1.359003
## iter  16 value -1.359630
## iter  17 value -1.360347
## iter  18 value -1.361172
## iter  18 value -1.361172
```

```
## iter  18 value -1.361172
## final  value -1.361172
## converged
## initial  value -1.349476
## iter   2 value -1.350545
## iter   3 value -1.352126
## iter   4 value -1.352892
## iter   5 value -1.353195
## iter   6 value -1.353227
## iter   7 value -1.353242
## iter   8 value -1.353248
## iter   9 value -1.353255
## iter  10 value -1.353260
## iter  11 value -1.353313
## iter  12 value -1.353376
## iter  13 value -1.353392
## iter  14 value -1.353425
## iter  15 value -1.353444
## iter  16 value -1.353444
## iter  17 value -1.353446
## iter  17 value -1.353445
## final  value -1.353446
## converged
```




```
model_19_train_residuals = resid(model_19_train$fit)
hist(model_19_train_residuals)
```




```
shapiro.test(model_19_train_residuals)
```

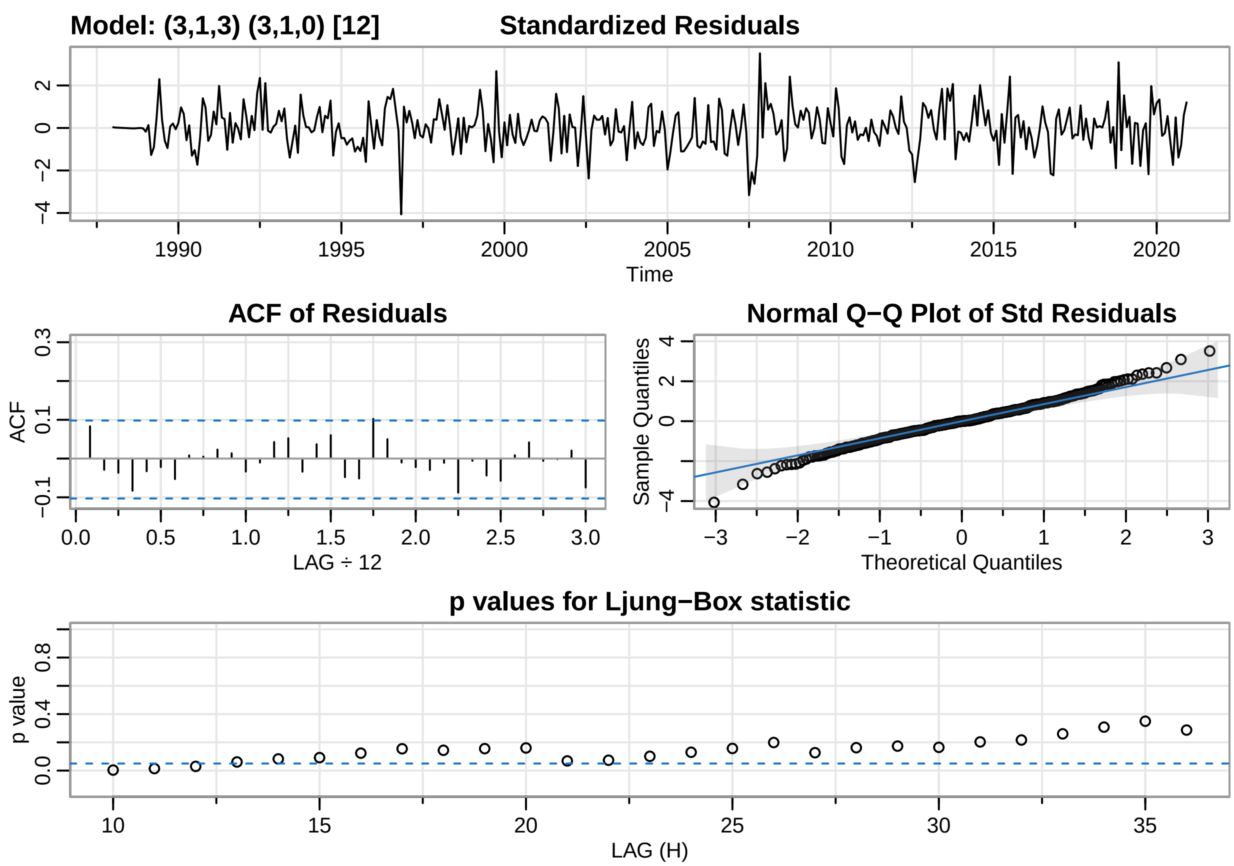
```
##
##  Shapiro-Wilk normality test
##
## data:  model_19_train_residuals
## W = 0.98931, p-value = 0.005438
```

```
#SARIMA(3,1,3)x(3,1,0)_12
model_20_train <- sarima(Avg_ExtentTS_Train, p=3, d=1, q=3, P=3, D=1, Q=0, S=12 , details = TRUE)
```

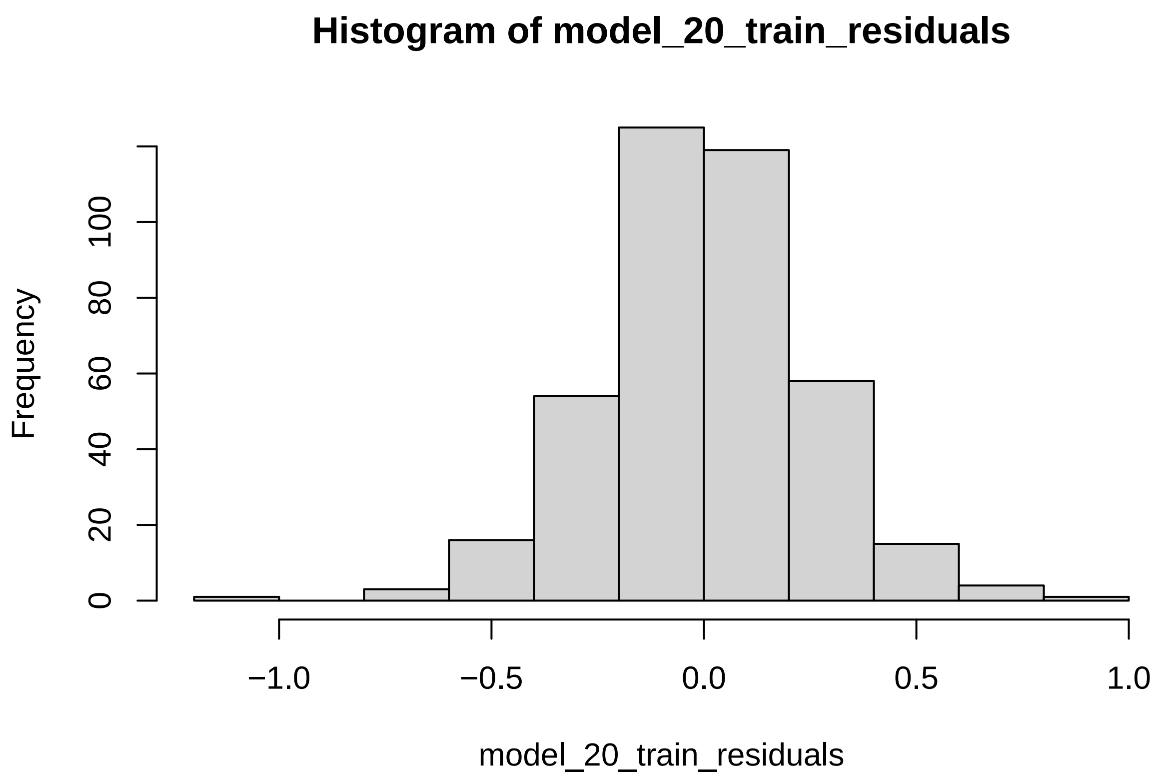
```
## initial  value -1.036159
## iter   2 value -1.164260
## iter   3 value -1.261457
## iter   4 value -1.294471
## iter   5 value -1.310799
## iter   6 value -1.331362
## iter   7 value -1.332748
## iter   8 value -1.340710
## iter   9 value -1.342401
## iter  10 value -1.344768
## iter  11 value -1.356444
## iter  12 value -1.358054
## iter  13 value -1.361687
## iter  14 value -1.361893
## iter  15 value -1.361951
## iter  16 value -1.362041
## iter  16 value -1.362041
## iter  17 value -1.362048
## iter  17 value -1.362048
```

```
## iter  18 value -1.362055
## iter  18 value -1.362055
## iter  19 value -1.362057
## iter  19 value -1.362057
## iter  19 value -1.362057
## final  value -1.362057
## converged
## initial  value -1.349245
## iter   2 value -1.350423
## iter   3 value -1.351430
## iter   4 value -1.352904
## iter   5 value -1.352954
## iter   6 value -1.353039
## iter   7 value -1.353175
## iter   8 value -1.353367
## iter   9 value -1.353555
## iter  10 value -1.353680
## iter  11 value -1.353702
## iter  12 value -1.353712
## iter  13 value -1.353734
## iter  14 value -1.353767
## iter  15 value -1.353868
## iter  16 value -1.353965
## iter  17 value -1.354067
## iter  18 value -1.354333
## iter  19 value -1.354582
## iter  20 value -1.354746
## iter  21 value -1.354794
## iter  22 value -1.354799
## iter  23 value -1.354846
## iter  24 value -1.354906
## iter  25 value -1.355108
## iter  26 value -1.356058
## iter  27 value -1.356158
## iter  28 value -1.356195
## iter  29 value -1.357452
## iter  30 value -1.358237
## iter  31 value -1.358849
## iter  32 value -1.359578
## iter  33 value -1.360331
## iter  34 value -1.361291
## iter  35 value -1.361471
## iter  36 value -1.362078
## iter  37 value -1.362381
## iter  38 value -1.362544
## iter  39 value -1.362731
## iter  40 value -1.362773
## iter  41 value -1.362816
## iter  42 value -1.362843
## iter  43 value -1.362863
## iter  44 value -1.362906
## iter  45 value -1.362946
## iter  46 value -1.363173
## iter  47 value -1.363342
```

```
## iter  48 value -1.363621
## iter  49 value -1.363728
## iter  50 value -1.363844
## iter  51 value -1.363959
## iter  52 value -1.364065
## iter  53 value -1.364278
## iter  54 value -1.364305
## iter  55 value -1.364463
## iter  56 value -1.364505
## iter  57 value -1.364535
## iter  58 value -1.364554
## iter  59 value -1.364565
## iter  60 value -1.364576
## iter  61 value -1.364584
## iter  62 value -1.364587
## iter  63 value -1.364587
## iter  64 value -1.364587
## iter  64 value -1.364587
## iter  64 value -1.364587
## final  value -1.364587
## converged
```




```
model_20_train_residuals = resid(model_20_train$fit)
hist(model_20_train_residuals)
```




```
shapiro.test(model_20_train_residuals)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  model_20_train_residuals
## W = 0.99038, p-value = 0.01088
```

Summarize the fits of these models in a table.

```
library(huxtable)
goodness_of_fit <- hux(
        Model = c('SARIMA(0,1,2)x(0,1,1)_12', 'SARIMA(0,1,2)x(3,1,0)_12', 'SARIMA(0,1,2)x(1,1,1)_12',
                  'SARIMA(4,1,0)x(0,1,1)_12', 'SARIMA(4,1,0)x(3,1,0)_12', 'SARIMA(4,1,0)x(1,1,1)_12',
                  'SARIMA(5,1,0)x(0,1,1)_12', 'SARIMA(5,1,0)x(3,1,0)_12', 'SARIMA(5,1,0)x(1,1,1)_12',
                  'SARIMA(1,1,1)x(0,1,1)_12', 'SARIMA(1,1,1)x(3,1,0)_12', 'SARIMA(1,1,1)x(1,1,1)_12',
                  'SARIMA(1,1,2)x(3,1,0)_12', 'SARIMA(2,1,1)x(3,1,0)_12', 'SARIMA(2,1,2)x(3,1,0)_12',
                  'SARIMA(1,1,3)x(3,1,0)_12', 'SARIMA(2,1,3)x(3,1,0)_12', 'SARIMA(3,1,2)x(3,1,0)_12',
                  'SARIMA(3,1,3)x(3,1,0)_12'),
        AIC = c(model_1_train$AIC, model_2_train$AIC, model_3_train$AIC,
                model_4_train$AIC, model_5_train$AIC, model_6_train$AIC,
                model_7_train$AIC, model_8_train$AIC, model_9_train$AIC,
                model_10_train$AIC, model_11_train$AIC, model_12_train$AIC,
                model_13_train$AIC, model_14_train$AIC, model_15_train$AIC,
                model_16_train$AIC, model_18_train$AIC, model_19_train$AIC,
                model_20_train$AIC),
        AICc = c(model_1_train$AICc, model_2_train$AICc, model_3_train$AICc,
                model_4_train$AICc, model_5_train$AICc, model_6_train$AICc,
                model_7_train$AICc, model_8_train$AICc, model_9_train$AICc,
                model_10_train$AICc, model_11_train$AICc, model_12_train$AICc,
                model_13_train$AICc, model_14_train$AICc, model_15_train$AICc,
```

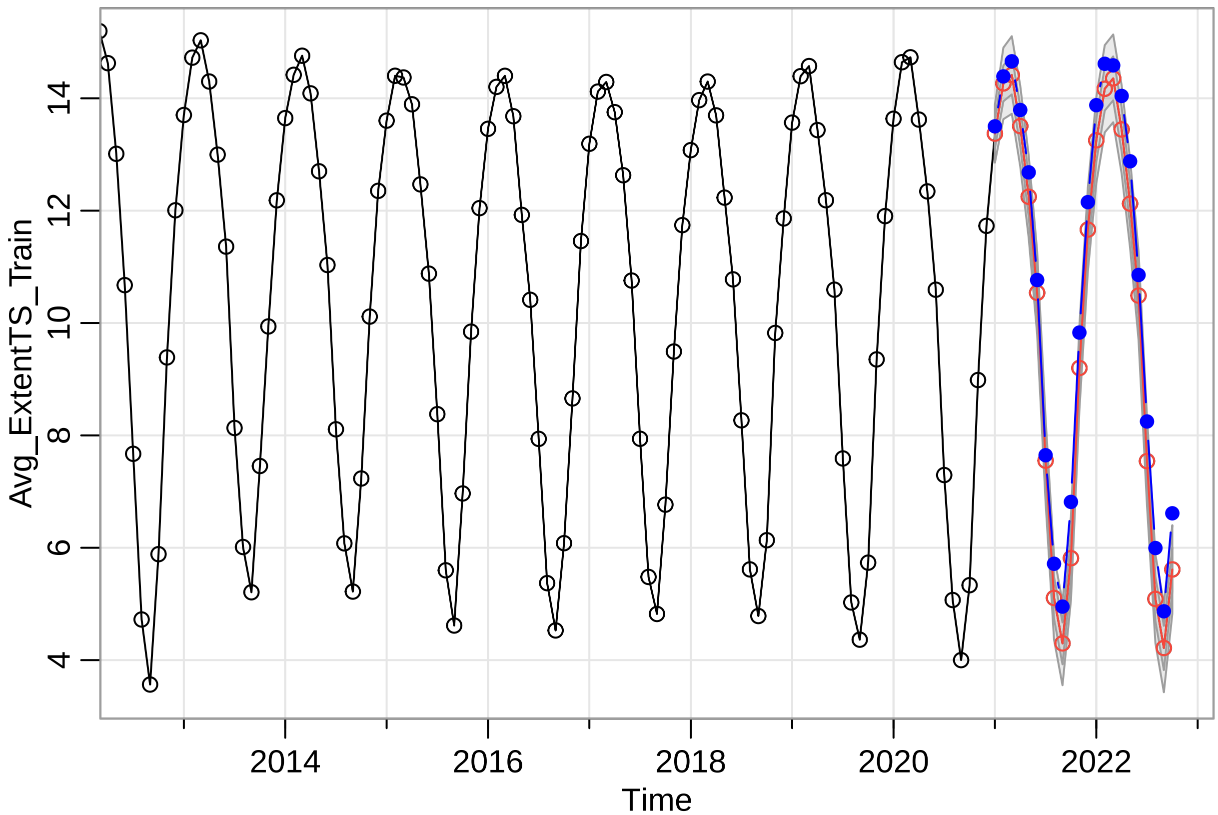
```
                model_16_train$AICc, model_18_train$AICc, model_19_train$AICc,
                model_20_train$AICc),
        BIC = c(model_1_train$BIC, model_2_train$BIC, model_3_train$BIC,
                model_4_train$BIC, model_5_train$BIC, model_6_train$BIC,
                model_7_train$BIC, model_8_train$BIC, model_9_train$BIC,
                model_10_train$BIC, model_11_train$BIC, model_12_train$BIC,
                model_13_train$BIC, model_14_train$BIC, model_15_train$BIC,
                model_16_train$BIC, model_18_train$BIC, model_19_train$BIC,
                model_20_train$BIC),
        MSE = c(mean(model_1_train_residuals^2), mean(model_2_train_residuals^2), mean(model_3_train_res
                mean(model_4_train_residuals^2), mean(model_5_train_residuals^2), mean(model_6_train_res
                mean(model_7_train_residuals^2), mean(model_8_train_residuals^2), mean(model_9_train_res
                mean(model_10_train_residuals^2), mean(model_11_train_residuals^2), mean(model_12_train_
                mean(model_13_train_residuals^2), mean(model_14_train_residuals^2), mean(model_15_train_
                mean(model_16_train_residuals^2), mean(model_18_train_residuals^2), mean(model_19_train_
                mean(model_20_train_residuals^2))
      )

goodness_of_fit %>%
  set_number_format(col=c(2,3,4,5), value=3) %>%
  set_bottom_border(c(1,13,16), everywhere) %>%
  set_bold(c(12,14,15,16), everywhere) %>%
  set_background_color(evens, everywhere, "grey95")
```

Model	AIC	AICc	BIC	MSE
SARIMA(0,1,2)x(0,1,1)_12	0.260	0.260	0.302	0.070
SARIMA(0,1,2)x(3,1,0)_12	0.274	0.275	0.336	0.071
SARIMA(0,1,2)x(1,1,1)_12	0.264	0.264	0.315	0.070
SARIMA(4,1,0)x(0,1,1)_12	0.258	0.258	0.320	0.069
SARIMA(4,1,0)x(3,1,0)_12	0.275	0.276	0.358	0.070
SARIMA(4,1,0)x(1,1,1)_12	0.261	0.262	0.333	0.069
SARIMA(5,1,0)x(0,1,1)_12	0.247	0.248	0.320	0.068
SARIMA(5,1,0)x(3,1,0)_12	0.264	0.265	0.357	0.069
SARIMA(5,1,0)x(1,1,1)_12	0.251	0.252	0.333	0.068
SARIMA(1,1,1)x(0,1,1)_12	0.305	0.305	0.346	0.073
SARIMA(1,1,1)x(3,1,0)_12	**0.195**	**0.195**	**0.257**	**0.064**
SARIMA(1,1,1)x(1,1,1)_12	0.308	0.309	0.360	0.069
SARIMA(1,1,2)x(3,1,0)_12	**0.168**	**0.169**	**0.241**	**0.062**
SARIMA(2,1,1)x(3,1,0)_12	**0.171**	**0.171**	**0.243**	**0.062**
SARIMA(2,1,2)x(3,1,0)_12	**0.173**	**0.174**	**0.255**	**0.062**
SARIMA(1,1,3)x(3,1,0)_12	0.173	0.174	0.256	0.062
SARIMA(2,1,3)x(3,1,0)_12	0.179	0.180	0.272	0.062
SARIMA(3,1,2)x(3,1,0)_12	0.178	0.179	0.271	0.062
SARIMA(3,1,3)x(3,1,0)_12	0.161	0.162	0.264	0.060

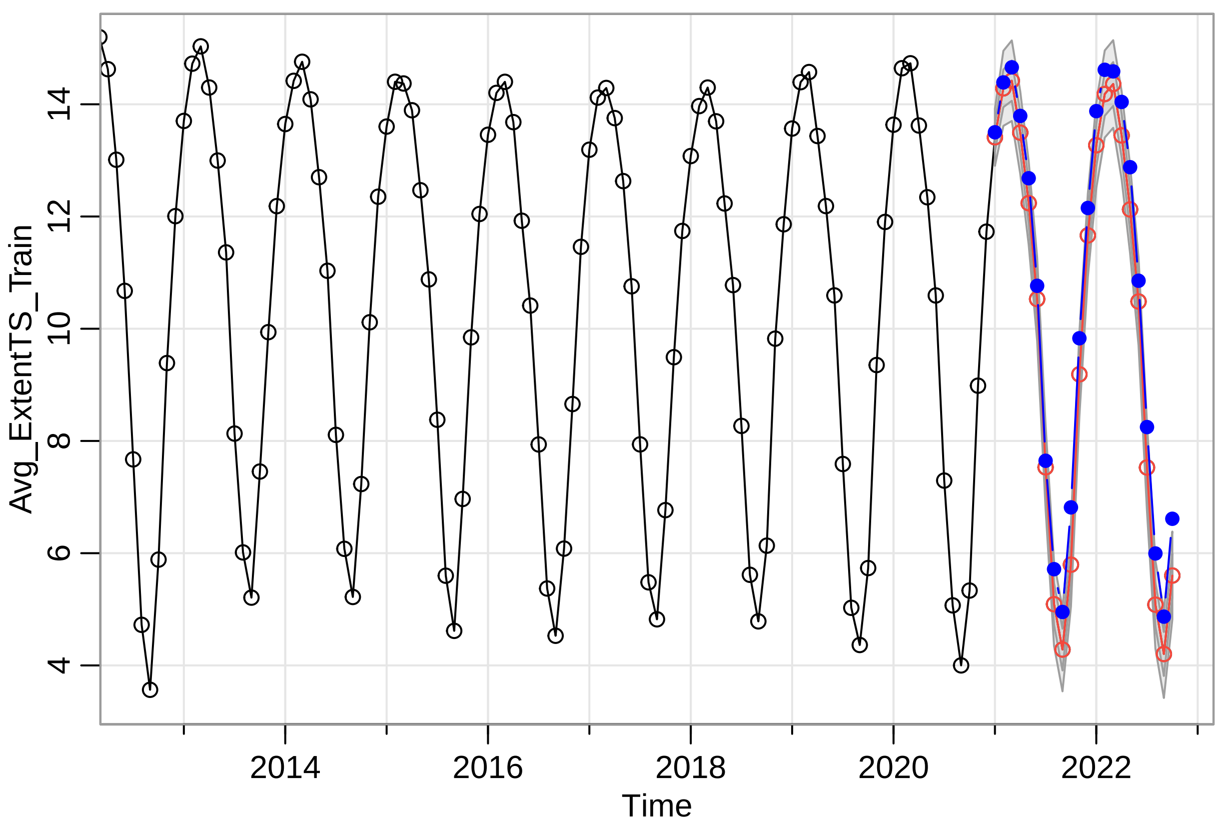
Model Selection

We evaluate performance on the test set of a few of the models which gave the best fit.

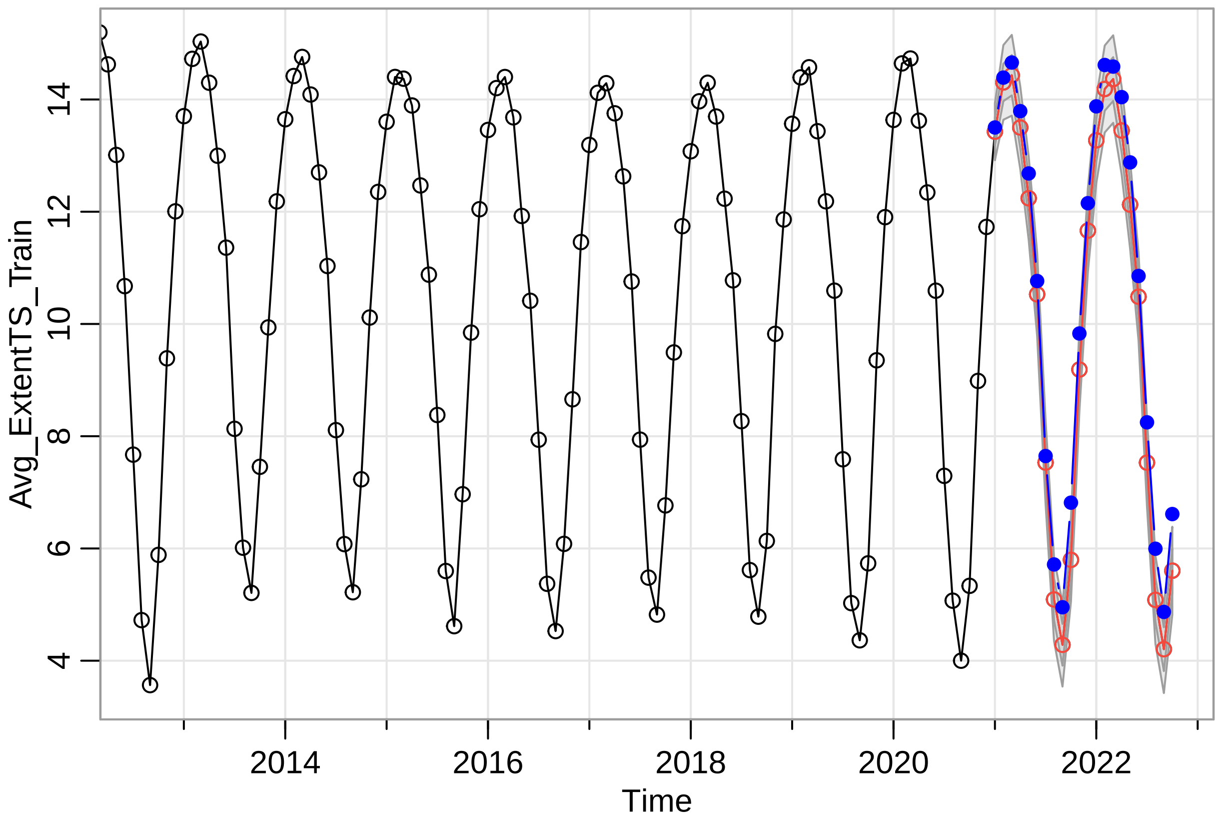
```
model_11_train_forecast<- sarima.for(Avg_ExtentTS_Train, n.ahead=22, p=1,d=1,q=1,P=3,D=1,Q=0,S=12)
lines(Avg_ExtentTS_Test,col='blue',type='b',pch=16)
```



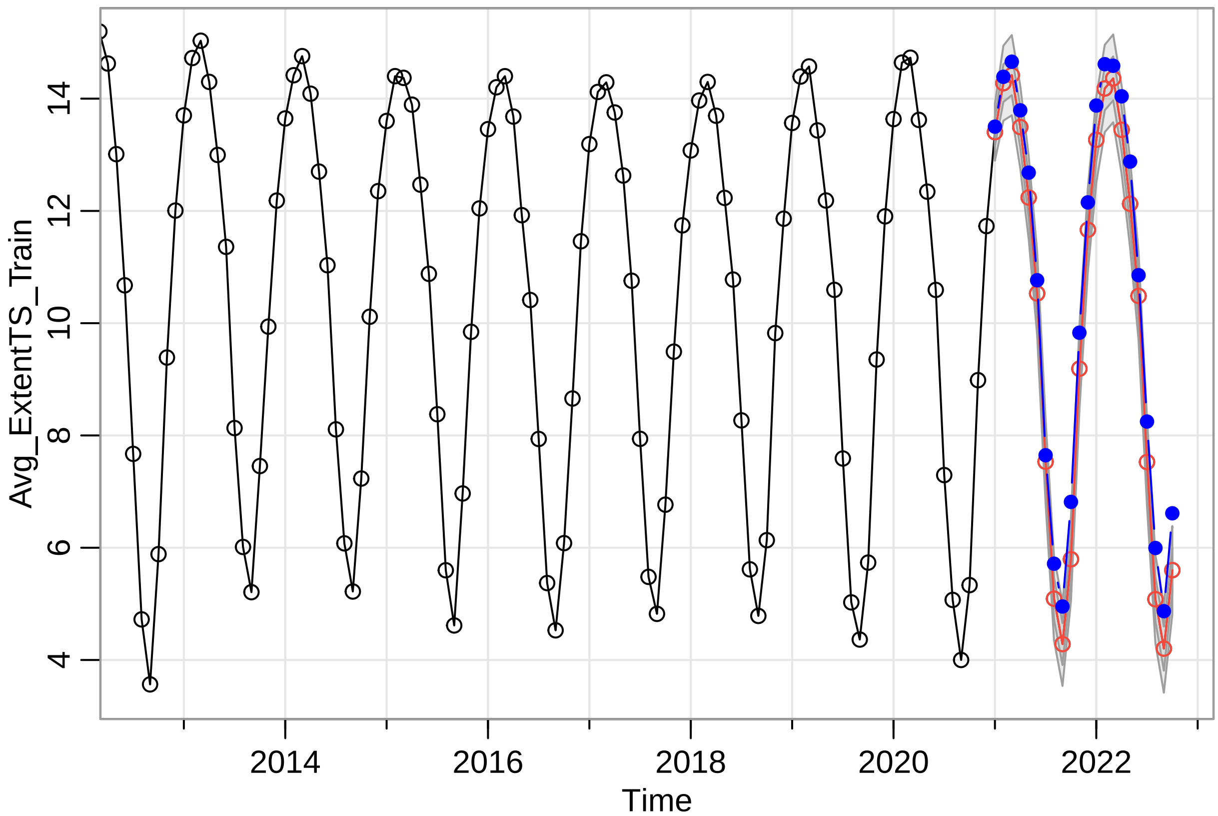

```
model_13_train_forecast<- sarima.for(Avg_ExtentTS_Train, n.ahead=22, p=1,d=1,q=2,P=3,D=1,Q=0,S=12)
lines(Avg_ExtentTS_Test,col='blue',type='b',pch=16)
```



```r
model_14_train_forecast<- sarima.for(Avg_ExtentTS_Train, n.ahead=22, p=2,d=1,q=1,P=3,D=1,Q=0,S=12)
lines(Avg_ExtentTS_Test,col='blue',type='b',pch=16)
```




```r
model_15_train_forecast<- sarima.for(Avg_ExtentTS_Train, n.ahead=22, p=2,d=1,q=2,P=3,D=1,Q=0,S=12)
lines(Avg_ExtentTS_Test,col='blue',type='b',pch=16)
```




```
mean((model_11_train_forecast$pred-Avg_ExtentTS_Test)^2)
```

```
## [1] 0.3321616
```

```
mean((model_13_train_forecast$pred-Avg_ExtentTS_Test)^2)
```

```
## [1] 0.3385571
```

```
mean((model_14_train_forecast$pred-Avg_ExtentTS_Test)^2)
```

```
## [1] 0.3363344
```

```
mean((model_15_train_forecast$pred-Avg_ExtentTS_Test)^2)
```

```
## [1] 0.3388543
```

Summarize these results in a table.

```
sarima_prediction <- hux(
        Model = c('SARIMA(1,1,1)x(3,1,0)_12', 'SARIMA(1,1,2)x(3,1,0)_12', 'SARIMA(2,1,1)x(3,1,0)_12', 'S
        PMSE = c(mean((model_11_train_forecast$pred-Avg_ExtentTS_Test)^2), mean((model_13_train_forecast
                 mean((model_14_train_forecast$pred-Avg_ExtentTS_Test)^2), mean((model_15_train_forecast

sarima_prediction %>%
  set_number_format(col=2, value=3) %>%
  set_bottom_border(1, everywhere) %>%
  set_background_color(evens, everywhere, "grey95")
```

Model	PMSE
SARIMA(1,1,1)x(3,1,0)_12	0.332
SARIMA(1,1,2)x(3,1,0)_12	0.339
SARIMA(2,1,1)x(3,1,0)_12	0.336
SARIMA(2,1,2)x(3,1,0)_12	0.339