

Finding the h Parameter of a Ray-Ellipse Interaction

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1 Determining h

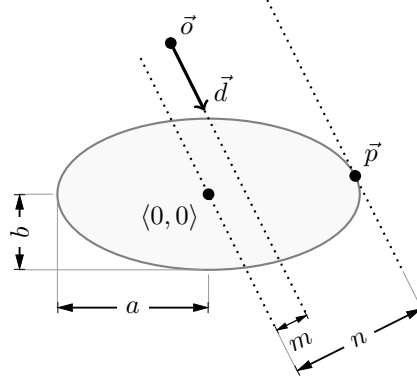
Given an ellipse centered at the origin with major and minor axes a and b defined by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (1)$$

and a ray with origin $\vec{o} = \langle o_x, o_y \rangle$ and direction $\vec{d} = \langle d_x, d_y \rangle$, the parameter h for the intersection of the ray and ellipse can be computed as:

$$h = \left| \frac{m}{n} \right|, \text{ where } m = \vec{o} \cdot \langle -d_y, d_x \rangle \text{ and } n = \vec{p} \cdot \langle -d_y, d_x \rangle \quad (2)$$

where \vec{p} is a point on the ellipse at which the tangent vector is parallel to \vec{d} .



1.1 Determining \vec{p}

Given a point (x, y) on the ellipse, the normalized tangent vector \vec{t} can be computed as:

$$\vec{t} = \frac{\langle ya^2, -xb^2 \rangle}{\sqrt{x^2b^4 + y^2a^4}}. \quad (3)$$

Solving for x and y , it can be determined that

$$\vec{p} = c \cdot \langle -a^2d_y, b^2d_x \rangle, \text{ where } c = \sqrt{\frac{1}{a^2d_y^2 + b^2d_x^2}}. \quad (4)$$

Substituting the value of \vec{p} in Equation 2 yields:

$$h = \left| \frac{o_yd_x - o_xd_y}{\sqrt{a^2d_y^2 + b^2d_x^2}} \right|. \quad (5)$$

An interactive tool can be found at <https://www.desmos.com/calculator/m1vsowmkvi>