Finding the h Parameter of a Ray-Ellipse Interaction

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1 Determining h

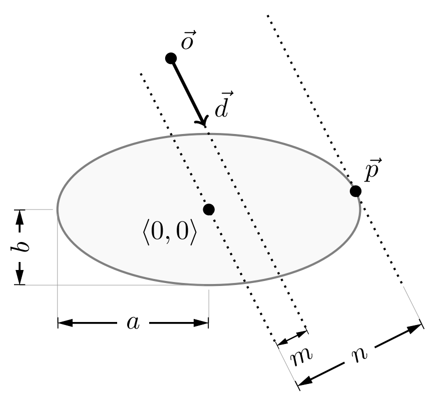
Given an ellipse centered at the origin with major and minor axes a and b defined by

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{1}$$

and a ray with origin $\vec{o} = \langle o_x, o_y \rangle$ and direction $\vec{d} = \langle d_x, d_y \rangle$, the parameter h for the intersection of the ray and ellipse can be computed as:

$$h = \frac{m}{n}, \text{ where } m = \vec{o} \cdot \langle -d_y, d_x \rangle \text{ and } n = \vec{p} \cdot \langle -d_y, d_x \rangle \tag{2}$$

where $\vec{p}$ is a point on the ellipse at which the tangent vector is parallel to $\vec{d}$.



1.1 Determining $\vec{p}$

Given a point (x, y) on the ellipse, the normalized tangent vector $\vec{t}$ can be computed as:

$$\vec{t} = \frac{\langle ya^2, -xb^2 \rangle}{\sqrt{x^2b^4 + y^2a^4}}. \tag{3}$$

Solving for x and y, it can be determined that

$$\vec{p} = c \cdot \left\langle -a^2 d_y, b^2 d_x \right\rangle, \text{ where } c = \sqrt{\frac{1}{a^2 d_y^2 + b^2 d_x^2}}. \tag{4}$$

Substituting the value of $\vec{p}$ in Equation 2 yields:

$$h = \frac{o_y d_x - o_x d_y}{\sqrt{a^2 d_y^2 + b^2 d_x^2}}. \tag{5}$$

An interactive tool can be found at https://www.desmos.com/calculator/m1vsowmkvi