Distributed linear SVM with the Alternating Direction Method of Multipliers

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Implementation of

C. Zhang, H. Lee, und K. G. Shin.
Efficient distributed linear classification algorithms via the alternating direction method of multipliers.
In *Proceedings of AISTATS 2012*, Seiten 1398–1406, 2012.

Outline

Consensus SVM

Dual coordinate descent

Implementation

Experiments

Consensus SVM: support vector machines

- Binary classification problem
- Dataset $\mathcal{D} = \{(x_i, y_i) \mid x_i \in \mathbb{R}^d, y_i \in \{-1, +1\}, i = 1, \ldots, m\}$
- Find hyperplane $H = \{x \mid w^T x + b = 0\}$ that separates classes with maximum margin
- Incorporate b into w:

$$x_i^T \leftarrow [1, x_i^T] \qquad w^T \leftarrow [b, w^T] \qquad d \leftarrow d + 1$$

- SVM can be formulated as an unconstrained optimization problem:

$$\min_w \frac{1}{2}\|w\|_2^2 + C \sum_{i=1}^{m} \ell(w, x_i, y_i)$$

- ℓ is a loss function. We use squared hinge loss (L2-SVM):

$$\min_{w} \frac{1}{2}\|w\|_2^2 + C\sum_{i=1}^{m} \max\left\{0, 1 - y_i w^T x_i\right\}^2$$

- Equivalent constrained formulation:

$$\begin{aligned} \min_{w,\xi} \quad & \frac{1}{2}\|w\|_2^2 + C\sum_{i=1}^{m} \xi_i^2 & \\ s.t. \quad & y_i w^T x_i \geq 1 - \xi_i & i = 1, \ldots, m \\ & \xi_i \geq 0 & i = 1, \ldots, m \end{aligned}$$

Consensus SVM: ADMM

- Framework for distributed optimization
- Optimization problems of type

$$\begin{aligned} \min_{w,z} \quad & f(w) + g(z) \\ s.t. \quad & Aw + Bz = c \end{aligned}$$

- Uses augmented Lagrangian:

$$\mathscr{L}_\rho(w, z, \lambda) = f(w) + g(z) + \lambda^T (Aw + Bz - c) + \frac{\rho}{2} \|Aw + Bz - c\|_2^2$$

- Update steps:

$$w \leftarrow \operatorname*{argmin}_w \mathscr{L}_\rho(w, z, \lambda) \tag{1}$$

$$z \leftarrow \operatorname*{argmin}_z \mathscr{L}_\rho(w, z, \lambda) \tag{2}$$

$$\lambda \leftarrow \lambda + \rho(Aw + Bz - b) \tag{3}$$

Consensus SVM

- Assume the dataset $\mathcal{D}$ is split across N nodes in a network. Let $B_i = \{j \mid (x_j, y_j) \in \mathcal{D} \text{ is stored in node } i\}$.
- Reformulate SVM as a consensus problem:

$$\begin{aligned} \min_{w,z} \quad & \frac{1}{2}\|z\|_2^2 + C\sum_{i=1}^{N}\sum_{j\in B_i} \max\{0, 1 - y_j\langle w_i, x_j\rangle\}^2 \\ s.t. \quad & w_i - z = 0 \quad i = 1, \ldots, N \end{aligned}$$

- Each node learns its own local w_i, consensus is reached via z
- Consensus problem can be solved in parallel using ADMM

Consensus SVM: ADMM updates

$$
\begin{aligned}
w_i &\leftarrow \operatorname*{argmin}_{w_i} \mathscr{L}_\rho(w, z, \lambda) \\
&= \operatorname*{argmin}_{w_i} C \sum_{j \in B_i} \max\{0, 1 - y_j \langle w_i, x_j \rangle\}^2 + \frac{\rho}{2} \|w_i - z\|_2^2 + \lambda_i (w_i - z) \\
z &\leftarrow \operatorname*{argmin}_{z} \mathscr{L}_\rho(w, z, \lambda) \\
\lambda_i &\leftarrow \lambda_i + \rho(w_i - z)
\end{aligned}
$$

- w-update and λ-update can be computed in parallel
- z-update has nice closed form solution

$$
z = \frac{\sum_{i=1}^N (w_i + \lambda_i)}{1 + \rho N}
$$

Consensus SVM: Reformulation

We can set $\mu_i = \frac{\lambda_i}{\rho}$ to obtain a simpler formulation:

$$w_i \leftarrow \operatorname*{argmin}_{w_i} C \sum_{j \in B_i} \max\{0, 1 - y_j \langle w_i, x_j \rangle\}^2 + \frac{\rho}{2} \|w_i - z - \mu_i\|_2^2$$

$$z \leftarrow \frac{\sum_{i=1}^{N} (w_i + \mu_i)}{N + 1/\rho}$$

$$\mu_i \leftarrow \mu_i + w_i - z$$

Dual coordinate descent

- We need to find a way to compute w-update efficiently

$$\underset{w_i}{\operatorname{argmin}} \frac{\rho}{2}\|w - v\|_2^2 + C\sum_{j=1}^{s} \max\{0, 1 - y_j w_i^T x_j\}^2$$

where $(x_1, y_1), \ldots, (x_s, y_s)$ are data on machine i and $v = z - \mu_i$

- Equivalent constrained problem:

$$\begin{aligned} \min_{w,\xi} \quad & \frac{\rho}{2}\|w - v\|_2^2 + C\sum_{i=1}^{s} \xi_i^2 & \\ s.t. \quad & y_i w^T x_i \geq 1 - \xi_i & i = 1, \ldots, s \\ & \xi_i \geq 0 & i = 1, \ldots, s \end{aligned}$$

Dual coordinate descent: duality

$$\begin{aligned} \min_{\alpha} \quad & \frac{1}{2\rho}\alpha^T(Q+D)\alpha - b^T\alpha \\ s.t. \quad & \alpha_i \geq 0 \qquad\qquad\qquad\qquad i = 1, \ldots, s \end{aligned}$$

- $\alpha \in \mathbb{R}^s$
- $Q_{ij} = y_i y_j x_i^T x_j$
- D: diagonal matrix with $D_{ii} = \frac{\rho}{2C}$
- $b = [1 - y_1 v^T x_1, \ldots, 1 - y_s v^T x_s]^T$

Dual coordinate descent (DCD)

- Outer loop: update α in each iteration
- Inner loop: update each α_i separately
- Optimize one α_i at a time and then circularly move to the next variable
- The optimization for α_i has a closed form solution! 🙂

$$\alpha_i = \max\left\{0, \alpha_i - \frac{\rho}{(Q+D)_{ii}}\nabla_i\right\}$$

where ∇_i is the partial derivative w.r.t. α_i

DCD: compute partial derivative

- From the derivation of the dual we know

$$w = v + \frac{1}{\rho} \sum_{j=1}^{s} \alpha_j y_j x_j$$

- To get ∇_i we can first calculate w and then

$$\nabla_i = y_i w^T x_i + \alpha_i D_{ii} - 1$$

- We can update w easily once we have the new α_i:

$$w^{(t+1)} = w^{(t)} + (\alpha_i^{(t+1)} - \alpha_i^{(t)}) y_i x_i$$

DCD: projected partial derivative

- If $\nabla_i = 0$ and $\alpha_i > 0$, we do not need to update α_i
- Also if $\alpha_i = 0$ and $\nabla_i > 0$
- Projected partial derivative:

$$\tilde{\nabla}_i = \begin{cases} \min\{0, \nabla_i\} & \text{if } \alpha_i = 0, \\ \nabla_i & \text{otherwise} \end{cases}$$

$\Rightarrow$ Don't update α_i if $|\tilde{\nabla}_i| = 0$

Implementation: parallel computation

- Using Julias built-in parallelization framework
- processes are called workers

```
addprocs(N)
```

- Data are transformed into a `DistributedArray`

```
x = distribute(x, dist=[N,1])
y = distribute(y)
```

- Each worker has its local part on which it runs DCD

```
dcd(localpart(x), localpart(y), ...)
```

Implementation: program structure

main.jl Creates workers, preprocesses data and invokes ADMM.

admm.jl Implements the ADMM framework. Distributes code and data to the workers. Computes z and μ. Manages parallel computation of w_j and collects the results from the workers.

coord.jl Implements the dual coordinate descent. Must be available to all workers.

Implementation: stopping criteria for DCD

- Use the projected partial derivative as stopping criterion:

$$\max_i |\tilde{\nabla}_i| < \varepsilon$$

  Somehow this doesn't work, because $\tilde{\nabla}_i$ values get larger
- Alternative: use duality gap, but expensive to compute 😕
- Had to use maximum number of iterations as a stopping criterion

Experiments: spam detection

- Important application
- Typical linear classification task
- Users want better spam filters, but don't want to share their private emails

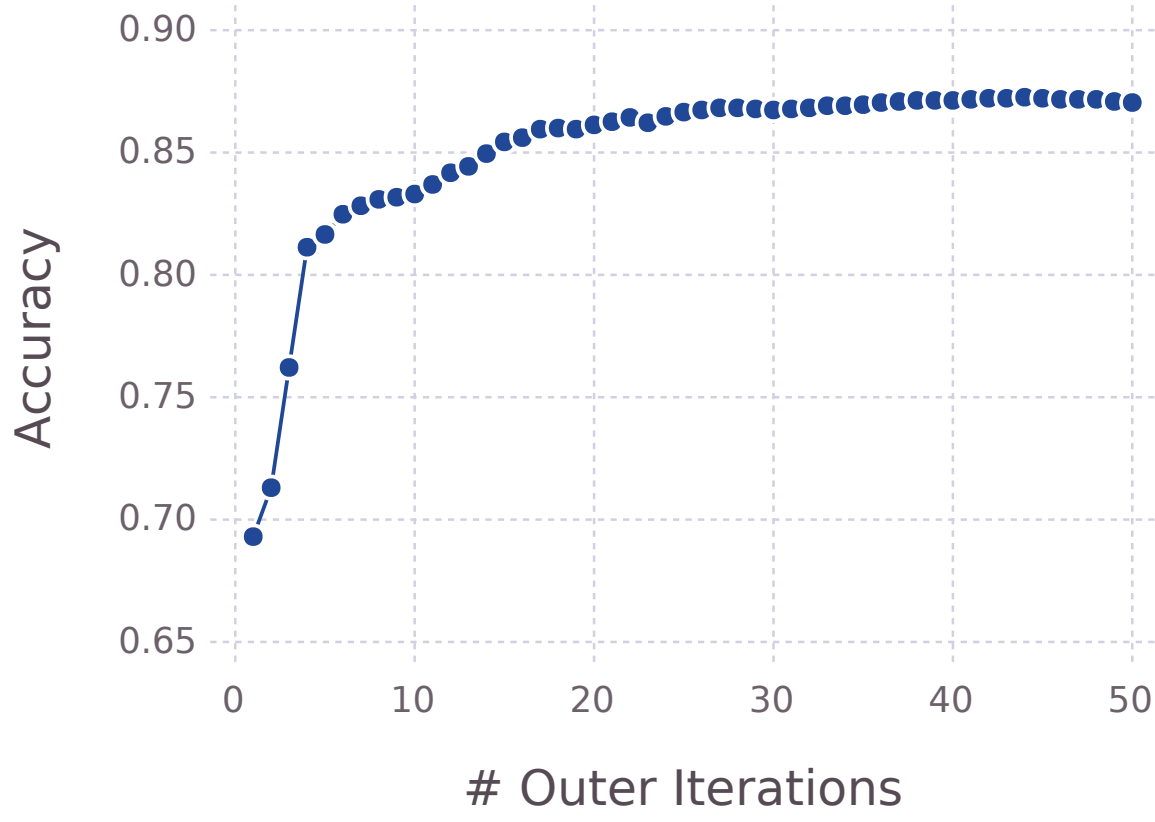
Experiments: dataset

UCI Spambase Data Set

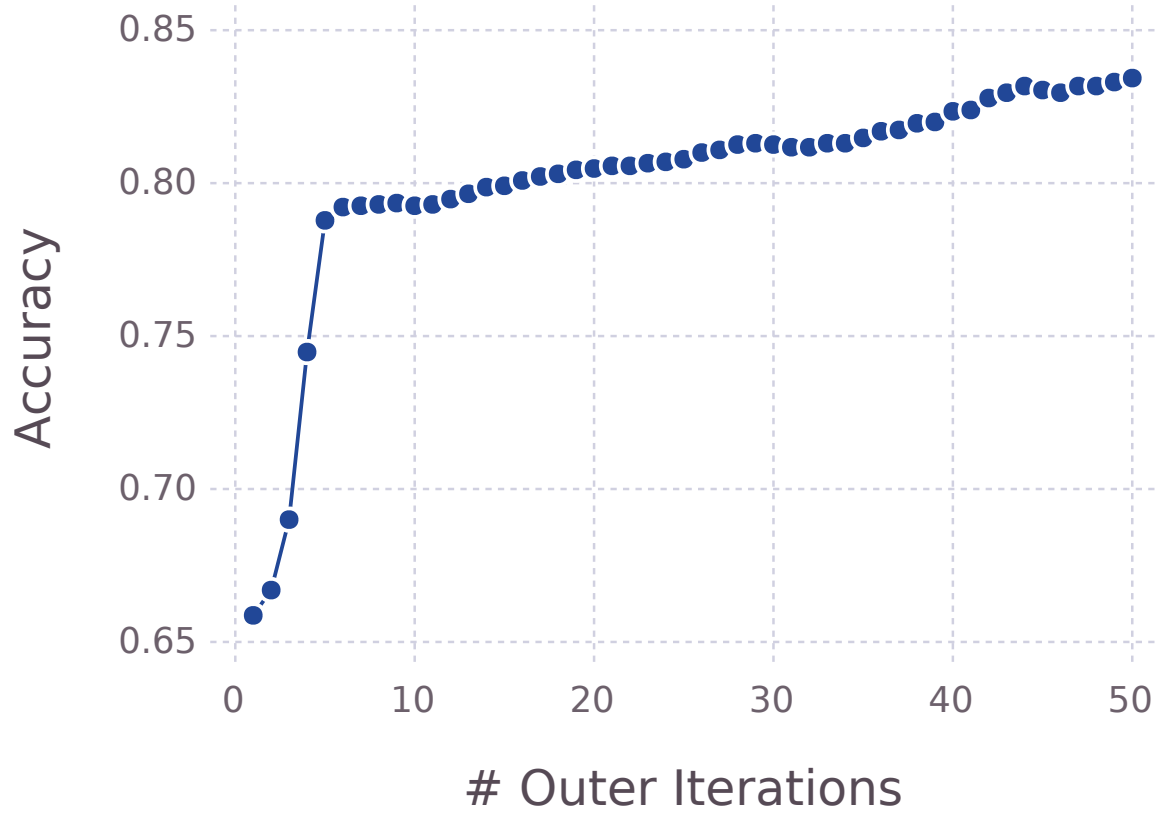
- 4600 data points (1813 Spam)
- 57 attributes + labels
- 48 attributes are frequencies of specific words
- Others include length of longest sequence of capital letters and total number of capital letter

Experiments: preprocessing

- Randomly permutate the rows of the data set
- Split into attributes and labels
- Convert labels to $\{-1, 1\}$ format
- Add a column of ones to the data
- Split the dataset equally into training and testing set



Accuracy when number of outer iterations is increased (number of inner iterations: 5)
Comparison: LIBSVM with same C and default values otherwise gives accuracy 84.22%



Accuracy when number of outer iterations is increased (number of inner iterations: 1)