Lecture 21 - Inverse Trigonometric Derivatives

Propiedad 1. Las derivadas de las funciones trigonométricas inversas son

$$\frac{\mathrm{d}}{\mathrm{d}x}(\arcsin(x)) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{\mathrm{d}}{\mathrm{d}x}(\arccos(x)) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(\arctan(x)) = \frac{1}{\sqrt{1+x^2}} \qquad \frac{\mathrm{d}}{\mathrm{d}x}(\arccos(x)) = -\frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(\mathrm{arcsec}(x)) = \frac{1}{|x|\sqrt{x^2-1}} \qquad \frac{\mathrm{d}}{\mathrm{d}x}(\mathrm{arccot}(x)) = -\frac{1}{1+x^2}$$

Ejemplo 1. Muestre que

1.
$$\frac{\mathrm{d}}{\mathrm{d}x}(\arcsin(x)) = \frac{1}{\sqrt{1-x^2}}$$

2.
$$\frac{\mathrm{d}}{\mathrm{d}x}(\mathrm{arccot}(x)) = -\frac{1}{1+x^2}$$

Propiedad 2. En general, $\frac{\mathrm{d}}{\mathrm{d}x}f^{-1}(x) = \frac{1}{f'(y)}$.

Ejemplo 2. Encuentre

1.
$$\frac{\mathrm{d}}{\mathrm{d}x}(\arcsin(7x))$$

2.
$$\frac{\mathrm{d}}{\mathrm{d}x}(\arcsin(x^2))$$

3.
$$\frac{\mathrm{d}}{\mathrm{d}x}(\arctan(\sqrt{x}))$$

4.
$$\frac{\mathrm{d}}{\mathrm{d}x}(x \operatorname{arcsec}(3x^2))$$

5.
$$\frac{\mathrm{d}}{\mathrm{d}x}(x\arccos(x) - \sqrt{1-x^2})$$

6.
$$\frac{\mathrm{d}}{\mathrm{d}x}(\operatorname{arccsc}(4x-1))$$

7.
$$\frac{\mathrm{d}}{\mathrm{d}x}(\arcsin(7x))$$

8.
$$\frac{\mathrm{d}}{\mathrm{d}x}(\arccos(\frac{1}{x}))$$

Ejemplo 3. Muestre que $\frac{\mathrm{d}}{\mathrm{d}x}(\mathrm{arccsc}(x)) = -\frac{1}{|x|\sqrt{x^2-1}}$.