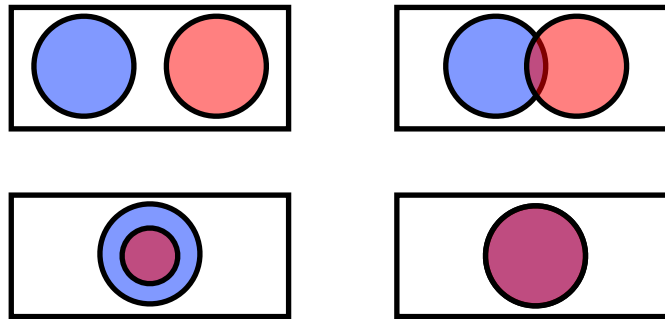


# Introductory Bayesian pragmatics



9.19: Computational Psycholinguistics

1 December 2021

Roger Levy

# Ad-hoc scalar inference

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A



B



C

*Bob can only say one word to communicate with you and he says: "**glasses**"*

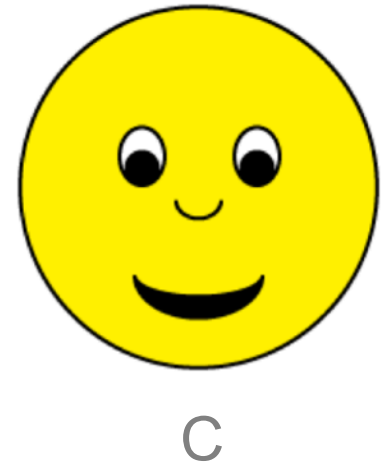
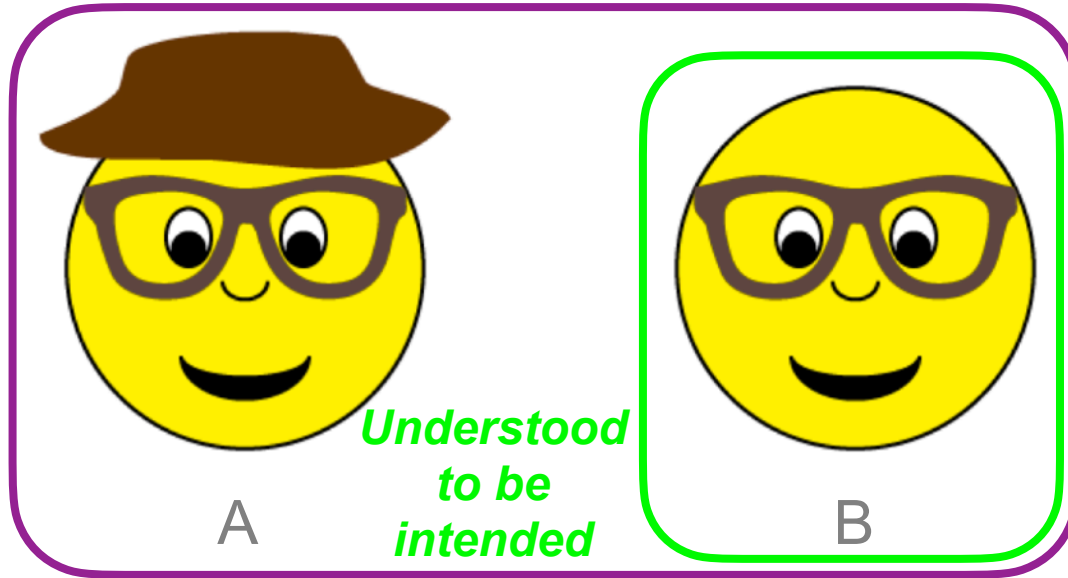
**Empirical finding:** >75% of experimental participants choose character **B**!

# What is *said* and what is *meant*

---

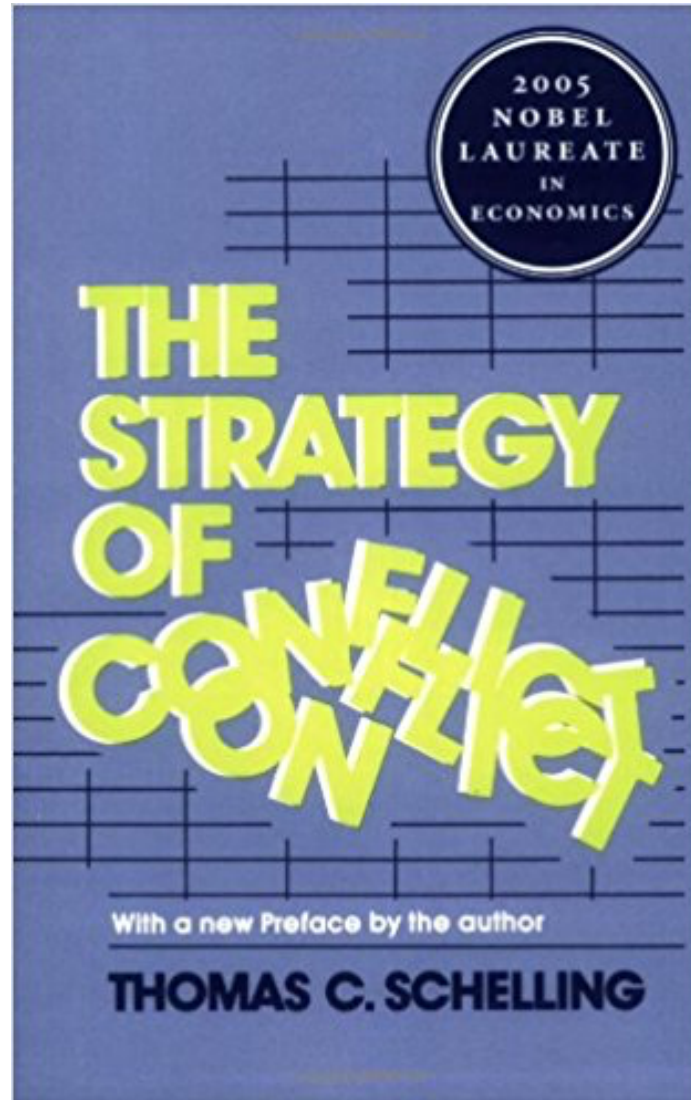
"glasses"

*Literally compatible*



# Coordination games

---



# Formalizing theories of semantics & pragmatics

---

- How does human language achieve its unbounded and highly context-dependent expressive capacity?
- **Semantics**: the “literal” meanings of words and the rules of composition by which words are combined
- **Pragmatics**: how a speaker’s communicative intent is inferred from literal meaning in context
  - A. *I could really use a cup of coffee.*
  - B. *There’s a good place called Area Four nearby.*
- Probabilistic models over rich logical structures finally allow us to formalize joint semantic/pragmatic models
- Allows us to connect insights about linguistic meaning from across cognitive science—linguistics, AI, cognitive psychology, social cognition, philosophy

# Semantics: principle of **compositionality**

---

*The meaning of a complex expression is determined by the rules by which the expression is formed as applied to the meaning of the expression's subparts.*

$$\left[ \begin{array}{c} S \\ \swarrow \quad \searrow \\ NP \quad VP \end{array} \right] = \llbracket VP \rrbracket(\llbracket NP \rrbracket)$$

$$\left[ \begin{array}{c} NP \\ \swarrow \quad \searrow \\ Adj \quad N \end{array} \right] = \llbracket Adj \rrbracket(\llbracket N \rrbracket)$$

...

# Pragmatics: Grice, 1975

---

Make your conversational contribution such as is required, at the stage at which it occurs, by the accepted purpose or direction of the talk exchange in which you are engaged. One might label this the COOPERATIVE PRINCIPLE.

# A simple communication game

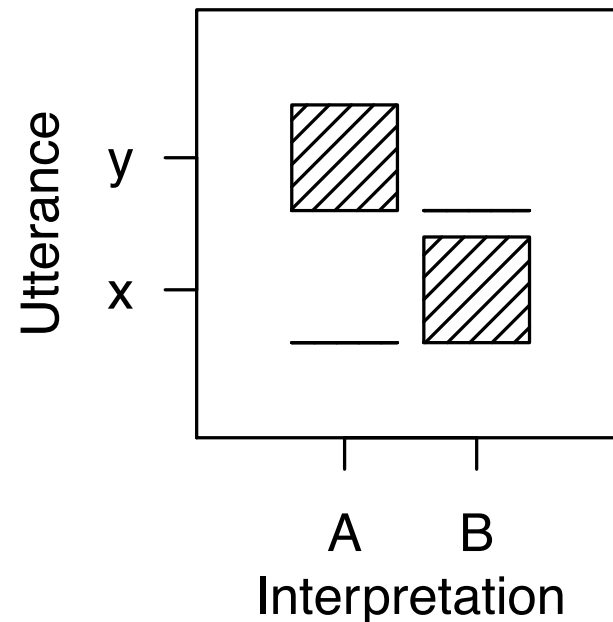
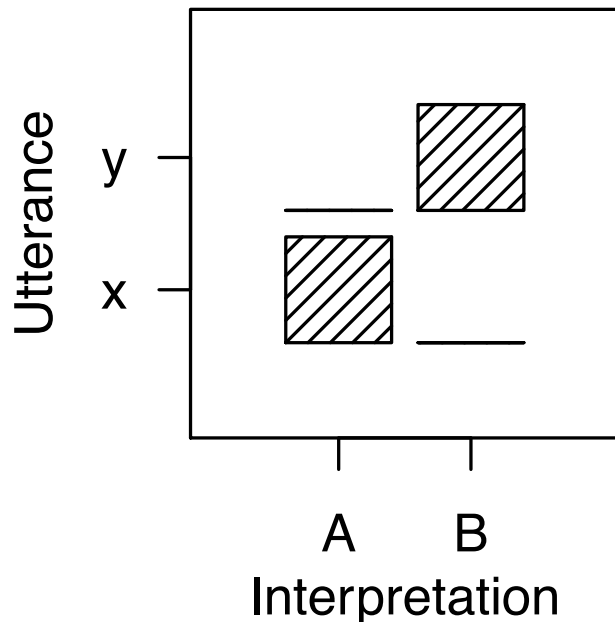
---

- The ***speaker*** knows which of two states  $\{\mathbf{A}, \mathbf{B}\}$  holds of the world
- She can transmit one of two messages  $\{x, y\}$  to the ***listener*** to signal which world state holds
- Speaker and listener have as common ground:
  - A ***prior distribution*** on world state  $P(\mathbf{A})$ ,  $P(\mathbf{B})$
  - Knowledge that messages  $x$  and  $y$  are equal in cost
  - That the game is purely cooperative



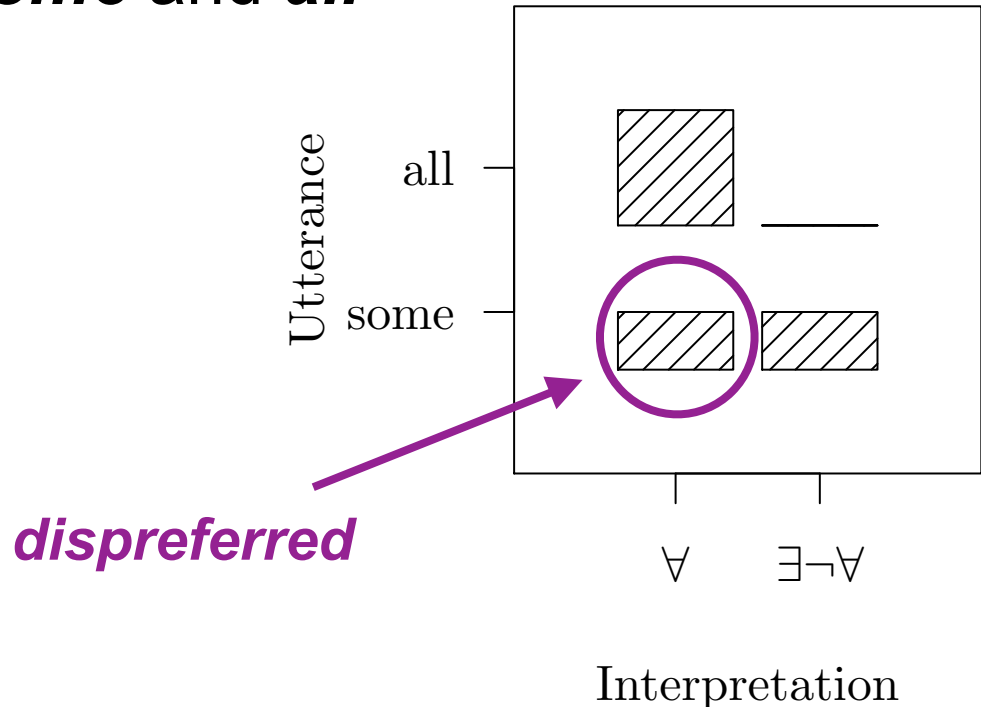
# A simple communication game

- A ***Pareto optimal strategy*** is one that delivers the highest possible reward to all players
  - A speaker who knows **A**
  - A speaker who knows **B**
  - A listener who hears **x**
  - A listener who hears **y**
- There are two Pareto optimal strategies in this game:



# Efficiency limits of literal meaning

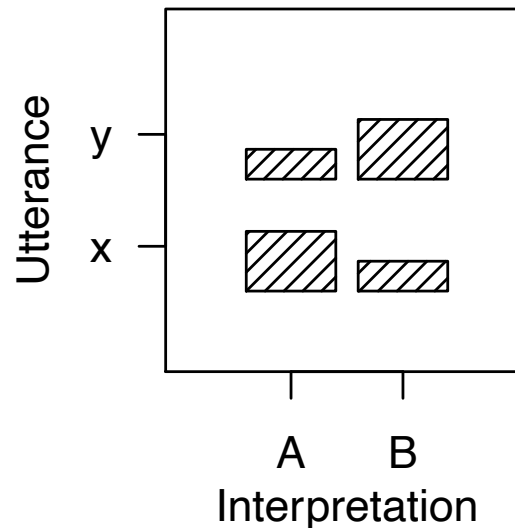
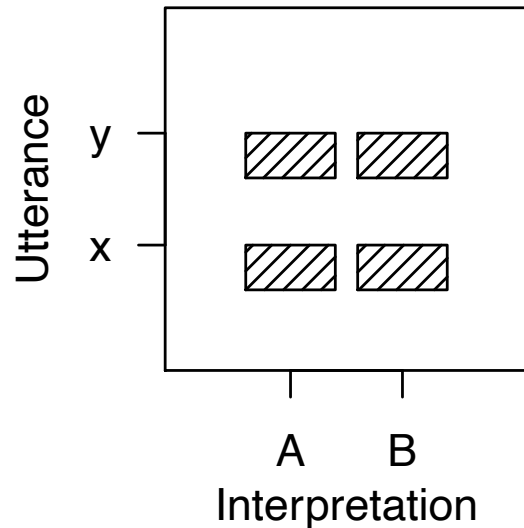
- But literal meanings don't hand us Pareto optimality
- A simple example: **some** and **all**



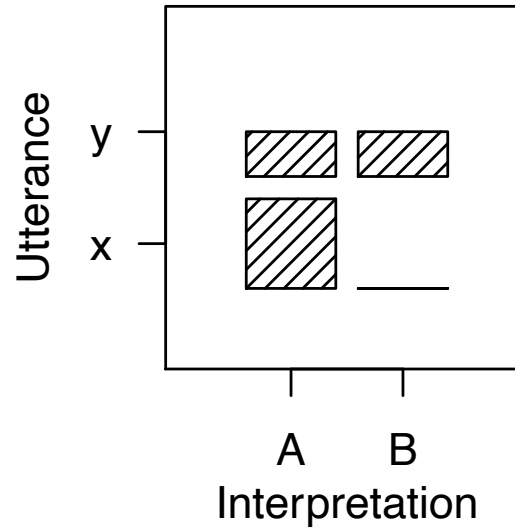
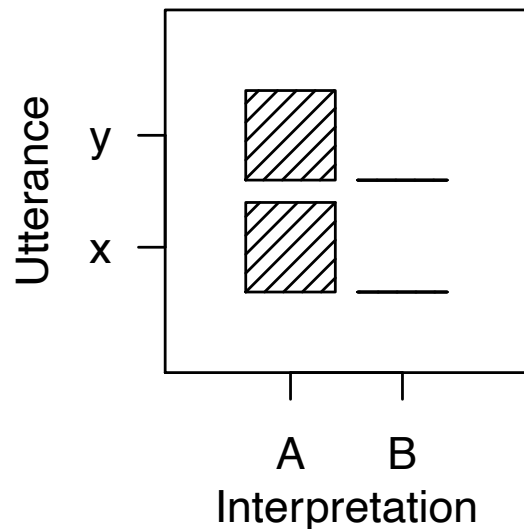
- “Remarkable” fact: the interpretation of **some** that is responsible for suboptimality is dispreferred!

# A simple communication game

- There are also many *non-Pareto-optimal* strategies:

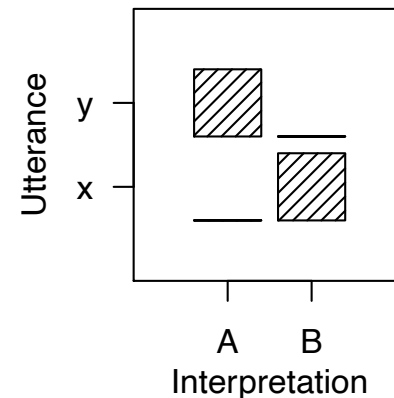
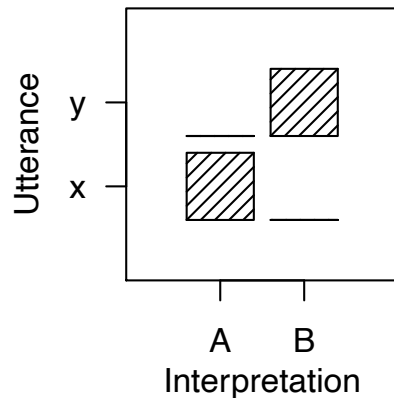


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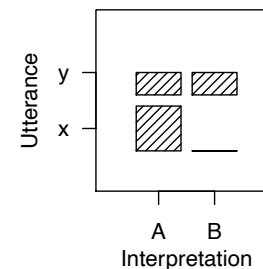
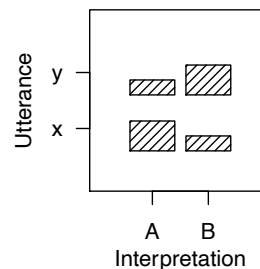
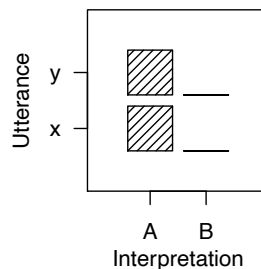
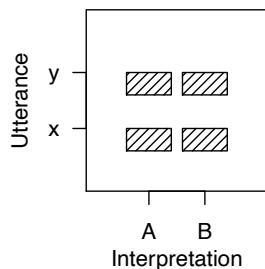


# A simple communication game

- *Efficient* communication would involve getting as close as possible to Pareto-optimal strategies...



- ...and away from the suboptimal strategies



...

- ...but without conventions, there's no way to do this reliably!

# Scalar implicature

---

- Consider the conventions offered us by **some** and **all**
- Two meanings:  $\forall$ ,  $\exists \neg \forall$
- Two signals:
  - **all** is compatible only with meaning  $\forall$
  - **some** is compatible with both meaning  $\forall$  and meaning  $\exists \neg \forall$
- For simplicity, assume prior  $P(\exists \neg \forall) = P(\forall) = 1/2$

# Bayesian theories of pragmatics

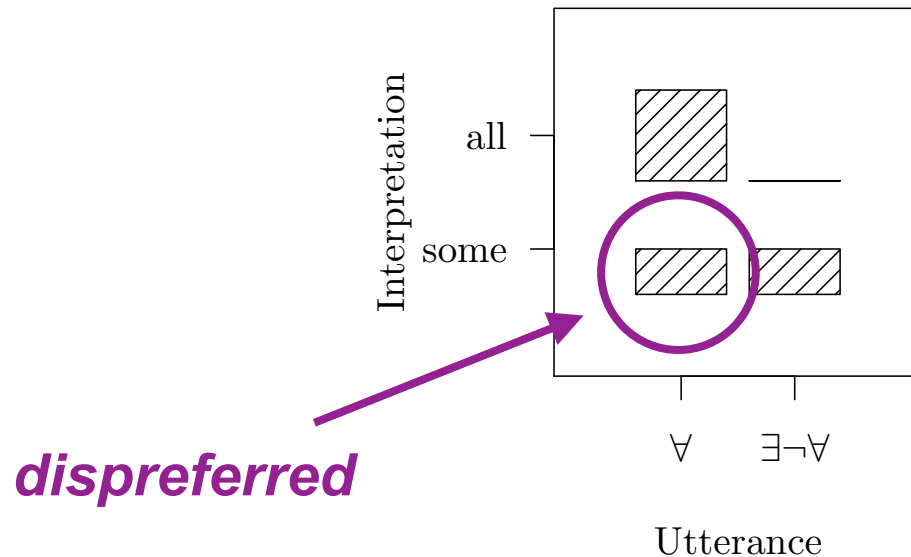
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## Assumptions:

- Speaker and listener beliefs represented as probability distributions over world states
- Joint communicative goal:
  - align the listener's beliefs with those of the speaker
  - but maintain brevity while doing so!
- Grammar and the literal meanings of words are common knowledge between speaker and listener
- Speaker and listener can recursively reason (probabilistically) about each other

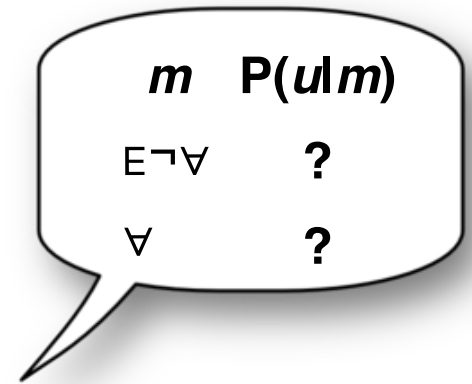
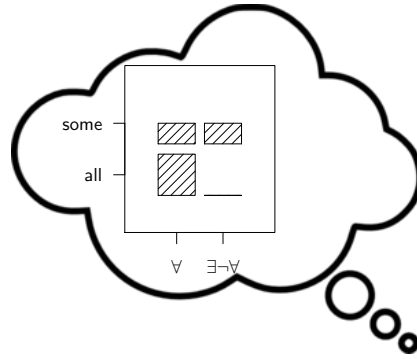
# Scalar implicature

- Simple model of *literal interpretation*:
  - Listener rules out meanings incompatible with message
  - Among meanings compatible with message, prefers those with higher prior probability
- Literal interpretation matrix for *some/all*:



- This is non-Pareto—!
- —and it fails to capture human preferences

# The Rational Speech-Act (RSA) model



*Modeling hypothesis: speakers are **softmax decision-theoretic agents** that prefer **high-utility utterances**:*

$$\text{Utility}^{(1)}(u; m) = \log P_{\text{Listener}}^{(0)}(m|u) - c(u)$$

*How **surprised** would  $u$  leave the listener about my intended meaning  $m$ ?*

*How intrinsically **costly** (time-consuming, effortful, taboo, ...) is the utterance  $u$ ?*

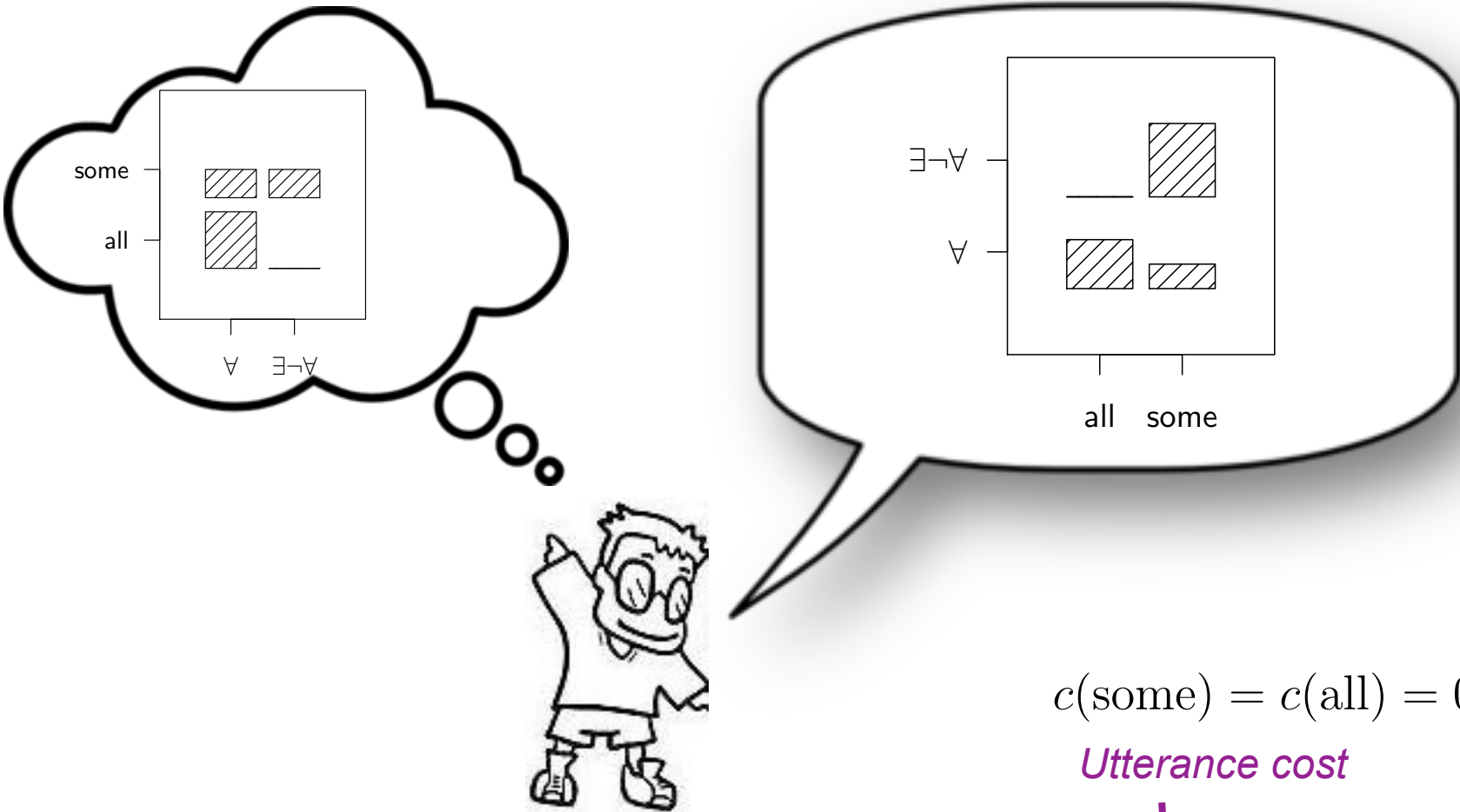
$$P_{\text{Speaker}}^{(1)} \propto \left[ \text{Utility}^{(1)}(u; m) \right]^{\lambda}$$

*Softmax optimality parameter*

*(Frank & Goodman, 2012;  
Goodman & Frank, 2016)*



# The Rational Speech-Act (RSA) model



$$c(\text{some}) = c(\text{all}) = 0$$

*Utterance cost*

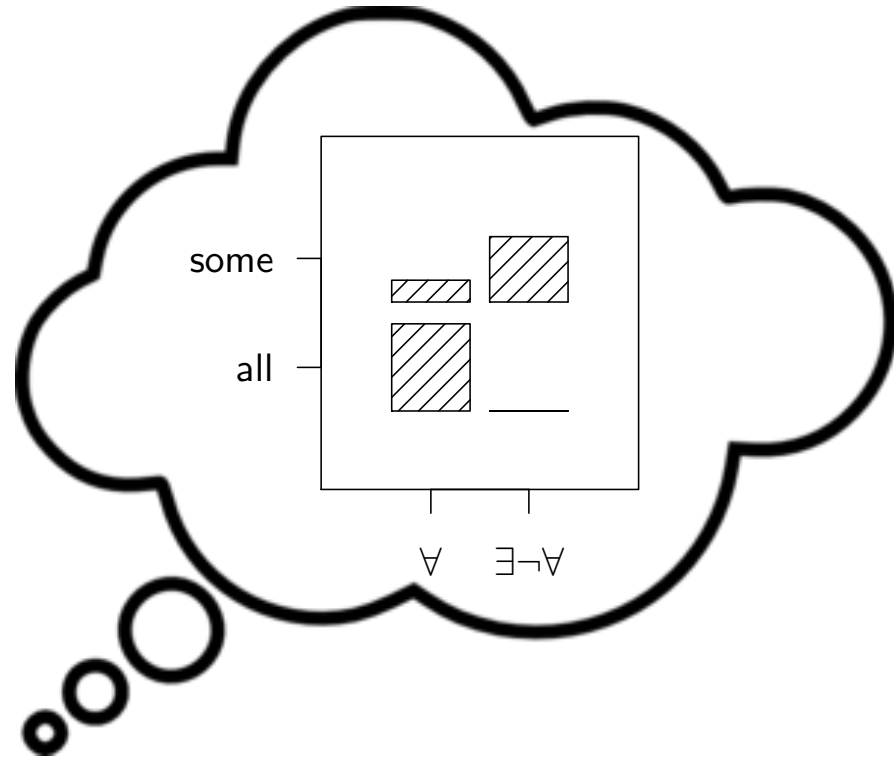
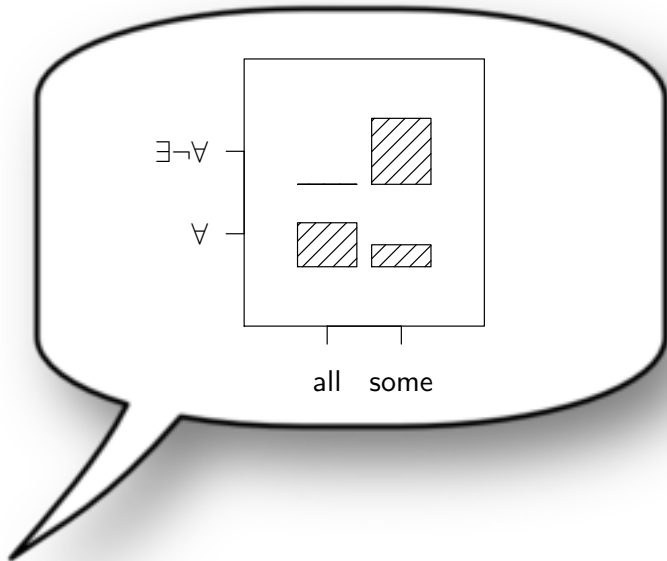
$$P_{\text{Speaker}}^{(1)}(u|m) \propto \left[ P_{\text{Listener}}^{(0)}(m|u) e^{-c(u)} \right]^{\lambda} \quad \lambda = 1$$

(Frank & Goodman, 2012;  
Goodman & Stühlmüller, 2013)

*“Greedy optimality” parameter*

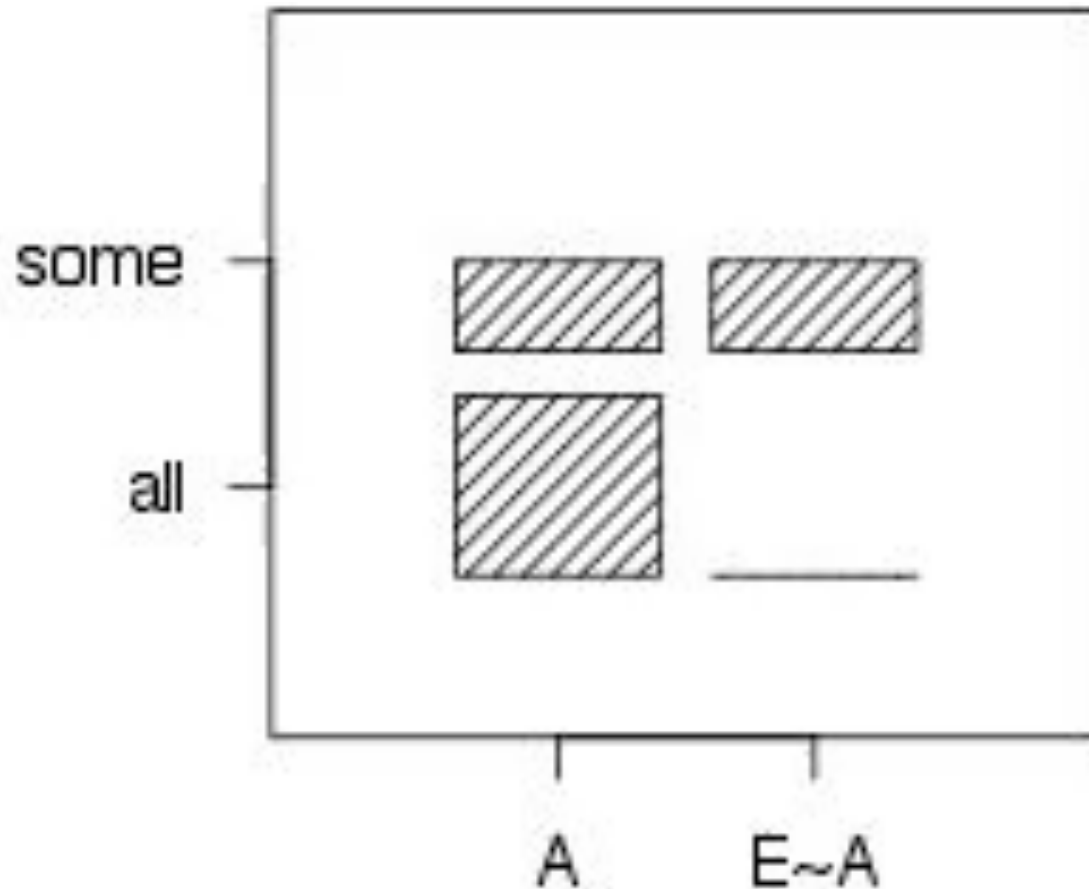
# Scalar implicature in RSA: listening

$$P_{Listener}^{(1)}(m|u) \propto P_{Speaker}^{(1)}(u|m)P(m)$$



# Speaker—listener recursion in RSA

- The process of recursion strengthens the implicature



# Conceptual framing

---

- Speaker and listener got (close) to a Pareto-optimal strategy by combining two ingredients:
  - Language knowledge (lexicon/grammar) as the *raw materials* for initial solutions to the communication game
  - General principles of socio-cognitive reasoning to craft these raw materials into more efficient solutions
- These two ingredients together allow discourse participants to do *so much more* than either one alone

# Levinson's (2000) typology of implicature

---

## Q-implicature

(Horn's Q)

*What isn't said isn't meant*

Pat has three children



Pat has **exactly three** children

## M-implicature

(Horn's "division of pragmatic labor")

*Align utterance simplicity  
with situation stereotypicality*

I started the car



...by **just** turning the key

## I-implicature

(Horn's R, sort of)

*Interpret utterances as  
the prototypical case*

The cup is on the table



It's **in contact with** the table

### *Q/I Tradeoff*

I injured a child



I didn't injure **my** child

I got the car to start



...I needed to do **more than just** turn the key

I injured a finger



I injured **my own** finger

*Can we explain this typology from basic principles in a probabilistic pragmatic framework, respecting linguistic form, semantic composition, and world knowledge?*

# Q/I tradeoff in rational speech-act theory

*I injured a child* → *it WASN'T my child*    *I burned my finger* → *it WASN'T my finger*

$$c(\text{my}) = c(\text{a}) = 0$$

$$c(\text{someone else's}) = 1$$

$$P(\text{MINE}) = \frac{1}{2}$$

*I injured **my** child*

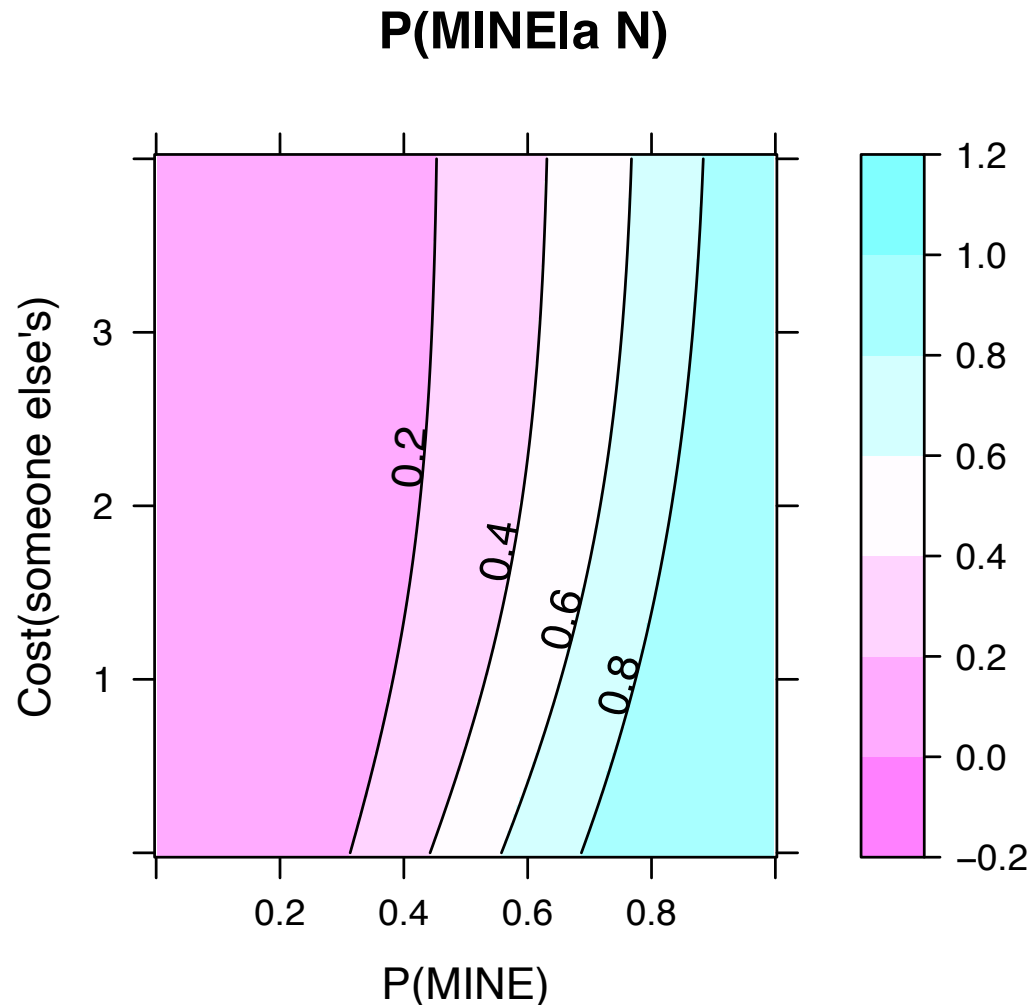
*I injured **a** child*     $c(\text{my}) = 0, c(\text{a}) = 0$

*I injured **someone else's** child*     $c(\text{someone else's}) = 1$

$$P(\text{MINE}) = \frac{5}{6}$$

# Q/I tradeoff in rational speech-act theory

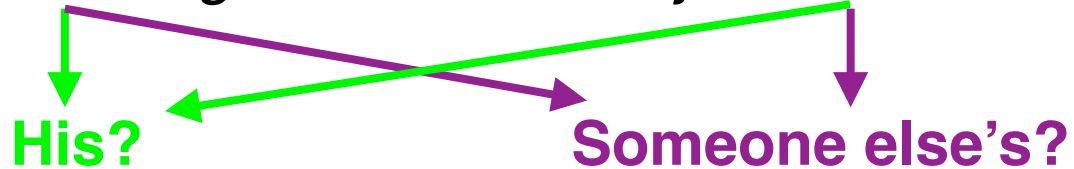
- Prior probability and simplicity trade off against one another
- But they aren't symmetric!



# A rich testbed for exploring Q/I tradeoff

---

*The man injured a finger*      *The man injured a child*

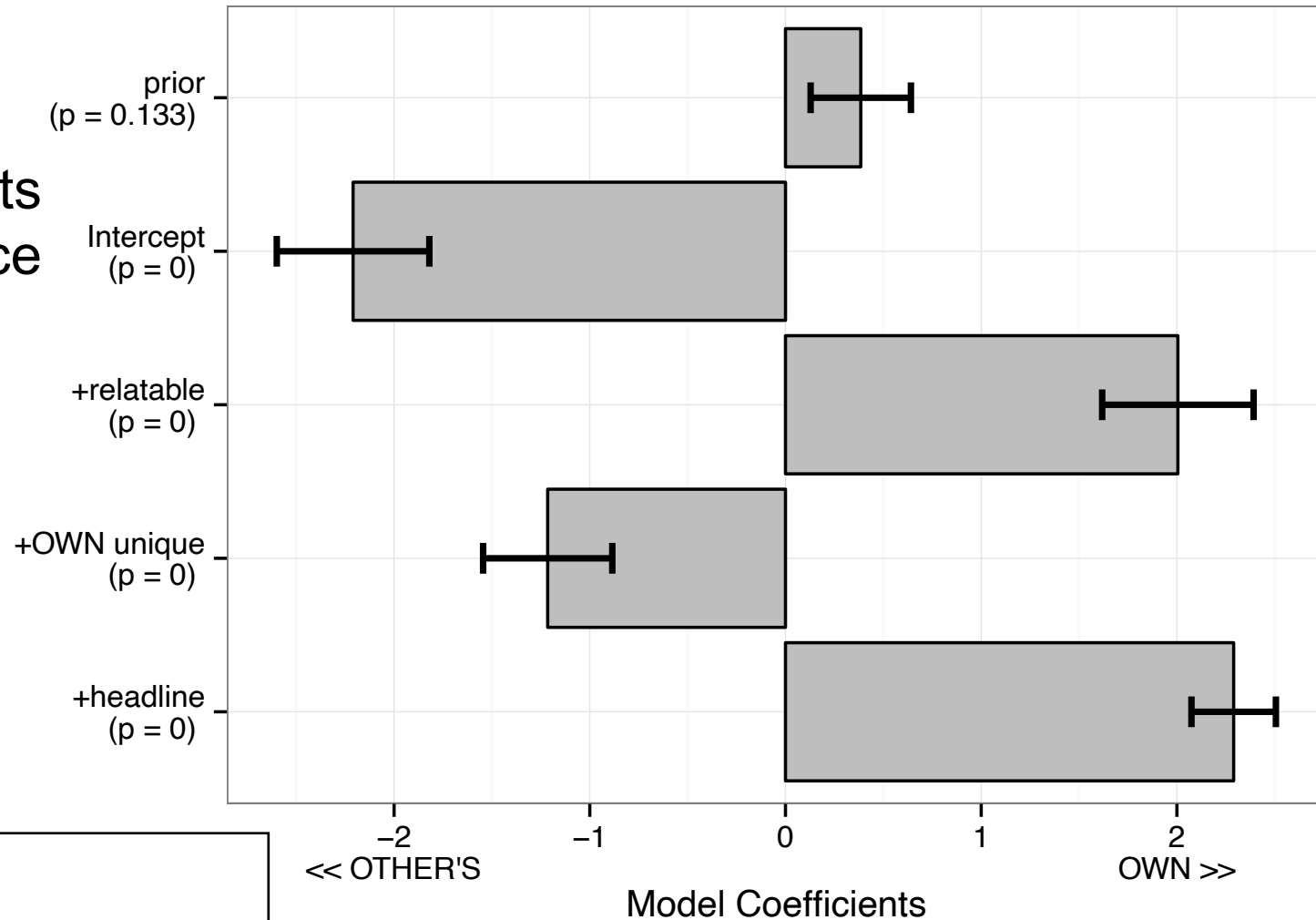


- Five predictions from the rational speech-act model:
  1. Judgments should track prior event probabilities  
*The man broke a nose*      *The python broke a nose*
  2. Judgments should be *other* skewed relative to prior
  3. Relational nouns should favor *own* judgments  
*The man injured a child*      *The father injured a child*
  4. “Only-one-of” nouns should favor *other* judgments  
*The man broke a finger*      *The man broke a nose*
  5. Allowing null determiners should favor *own* judgments  
*The man injured a child*      *Man injured child*



# A rich testbed for exploring Q/I tradeoff

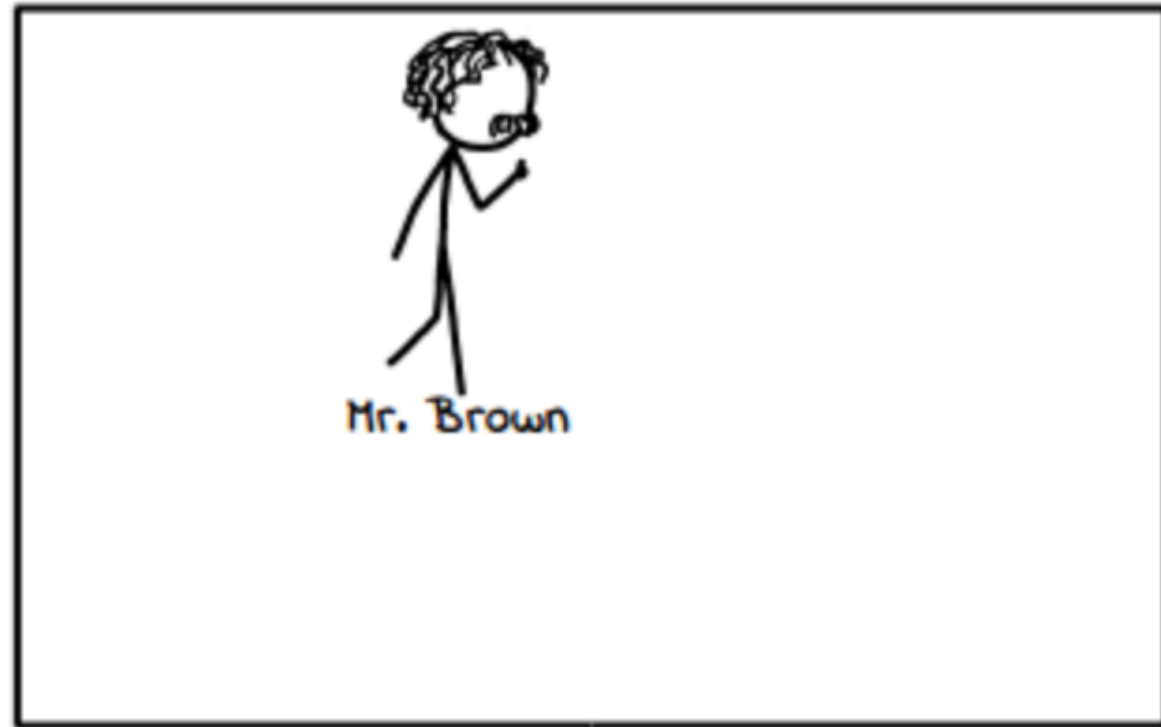
- 1348 judgments of 108 sentence prompts
- Multivariate mixed-effects logistic regression analysis



1. Effect of prior
2. Overall *other* skew
3. Relational nouns favor *own*
4. “Only-one-of” nouns favor *other*
5. Null determiners favor *own*

# A controlled Q/I experiment

---

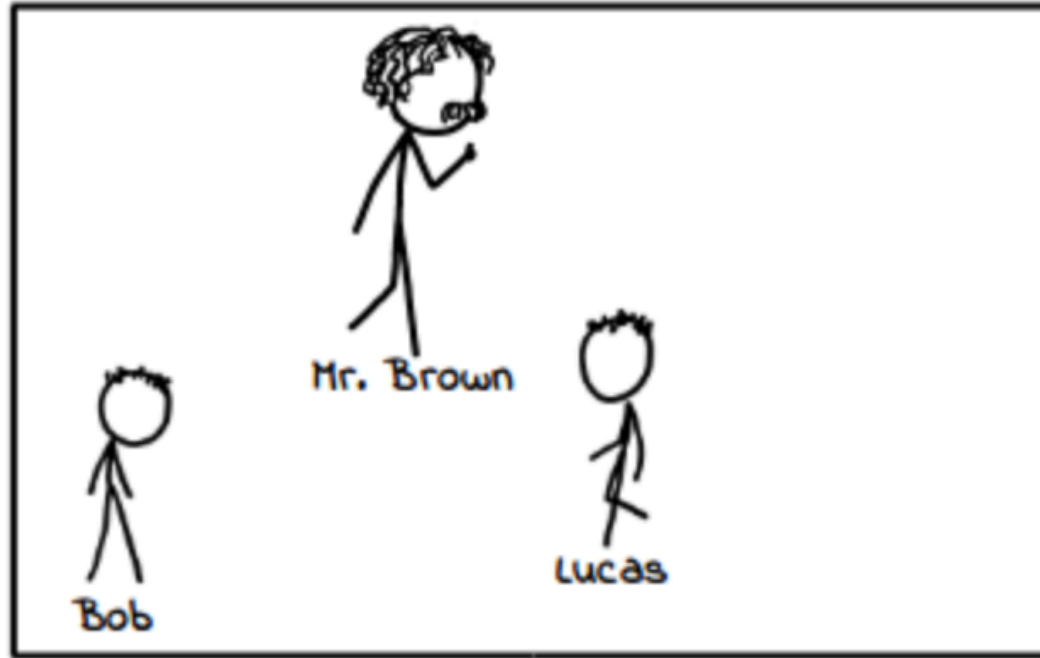


**"Mr. Brown is the principal at an element##### school."**

('#####' indicates phone static.)

# A controlled Q/I experiment

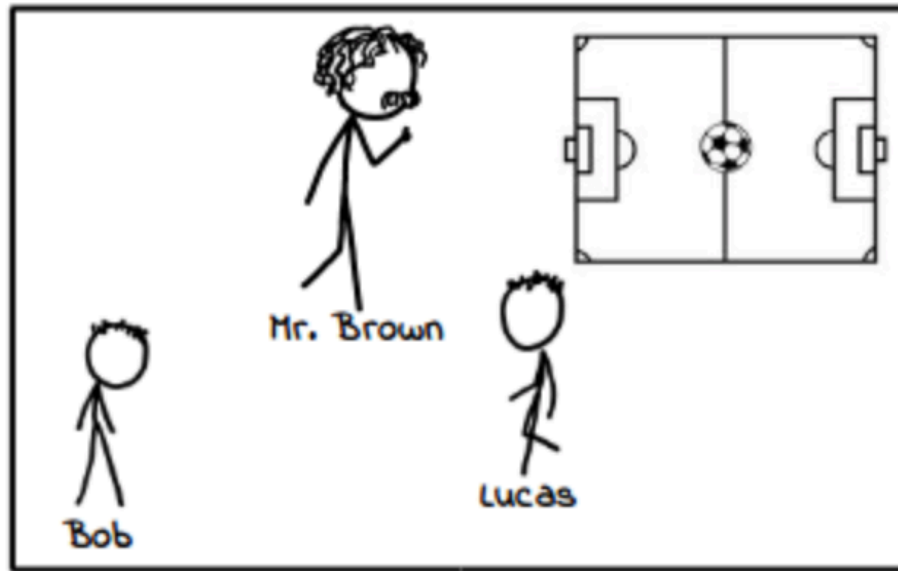
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**"He has two sons called Bob and Lucas who are 8 and 9 ye#####old."**

# A controlled Q/I experiment

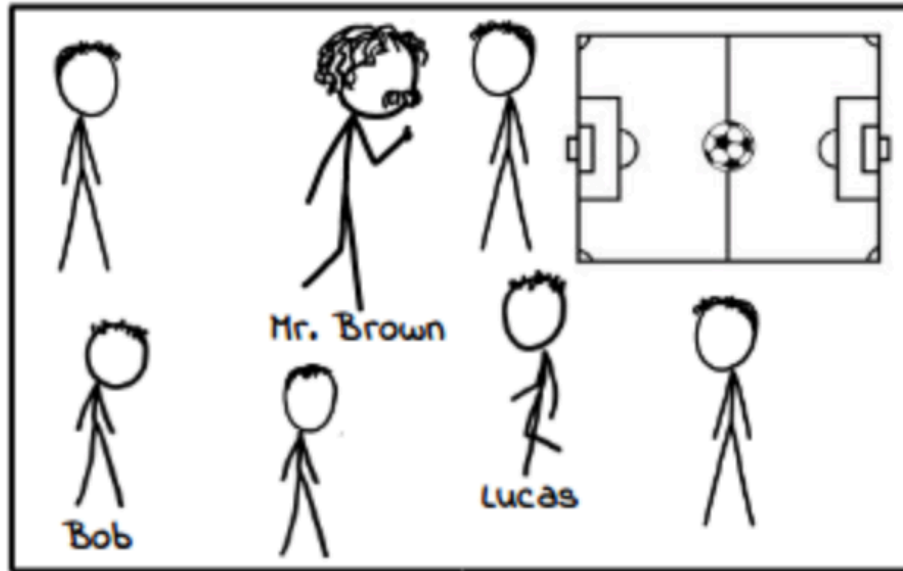
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**"Bob and Lucas are Mr. Brown's only children and they participated in the school's soccer tourna##### last weekend."**

# A controlled Q/I experiment

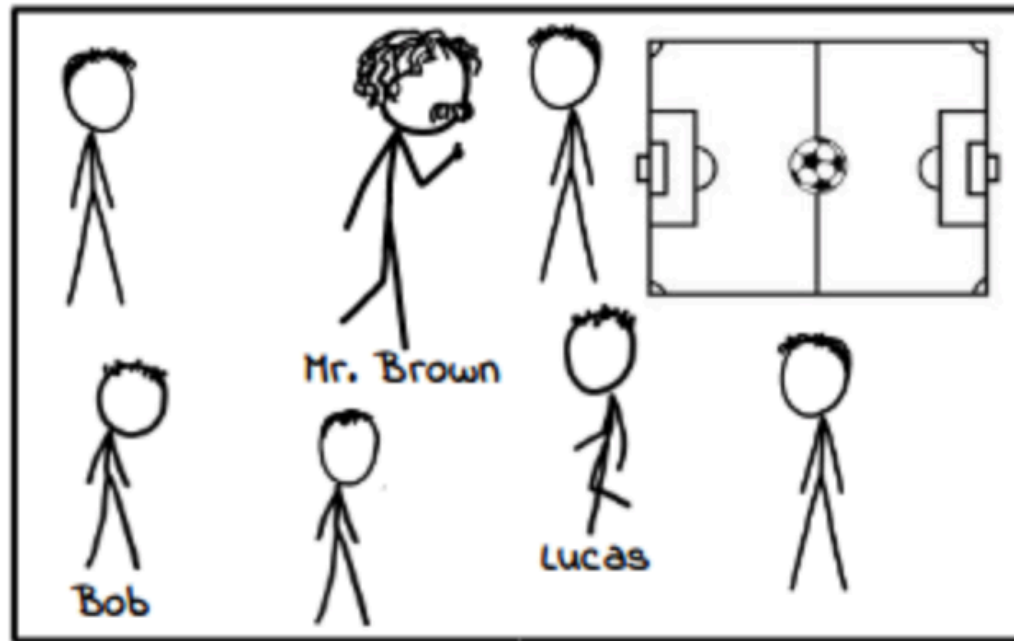
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**"After the #####nament, Mr. Brown played tag with Bob, Lucas, and 4 other boys when the accident happened:"**

# A controlled Q/I experiment

---



**"Brown knocks over#####son!"**

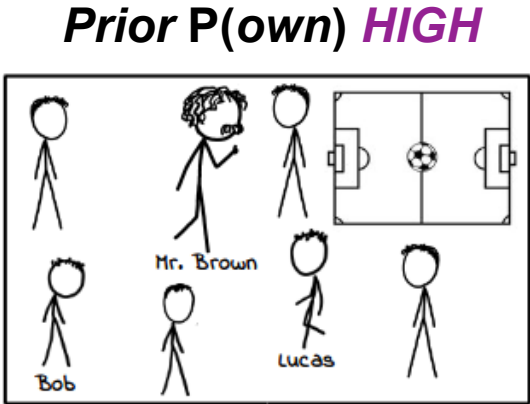
What do you think the speaker is trying to convey with this sentence?

- That Mr. Brown knocked over **someone else's child**.
- That Mr. Brown knocked over **his own child**.

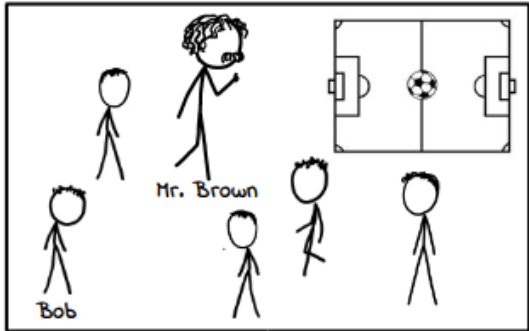
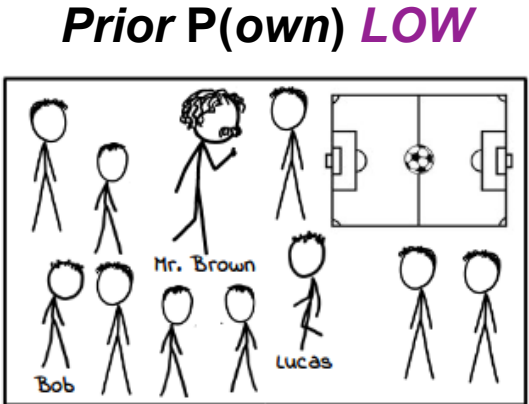
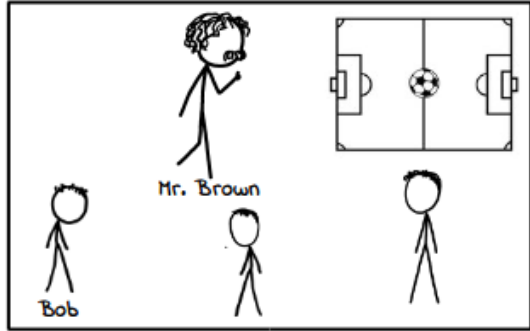
What do you think the speaker is trying to convey with this sentence?

- That Mr. Brown knocked over **someone else's child**.
- That Mr. Brown knocked over **his own child**.

|sons of Brown|=2



|sons of Brown|=1

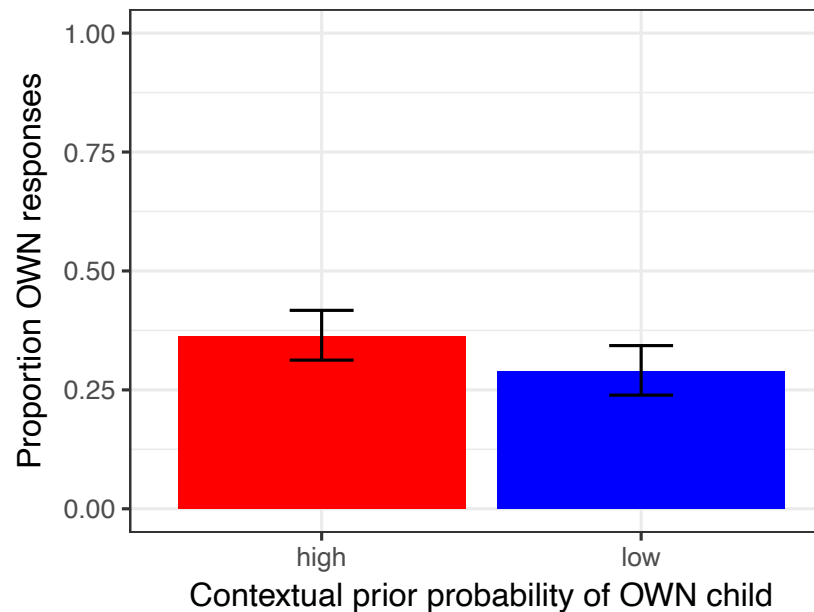
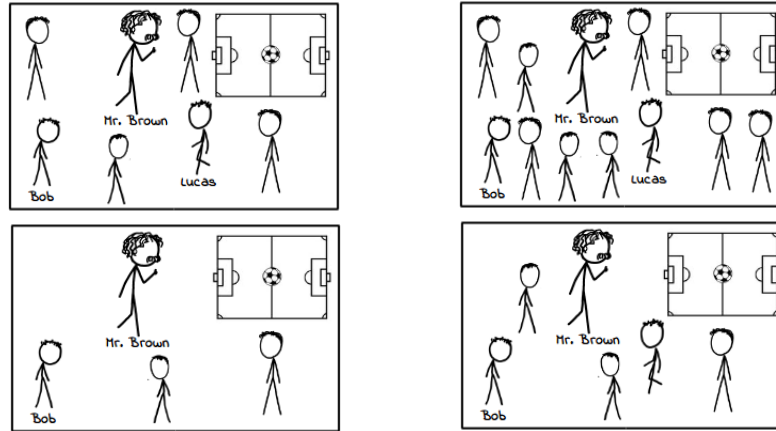


Utterance condition	unrelatable noun (boy)	relatable noun (son)
(prior elicitation)	Brown knocks over#####boy!	Brown knocks over#####son!
Posterior elicitation	Brown knocks over a boy!	Brown knocks over a son!
“Headline” interpretation	Brown knocks over boy!	Brown knocks over son!

# Checking predicted effects

- Effect of contextual prior probability of OWN child

*Prior P(own) **HIGH** Prior P(own) **LOW***



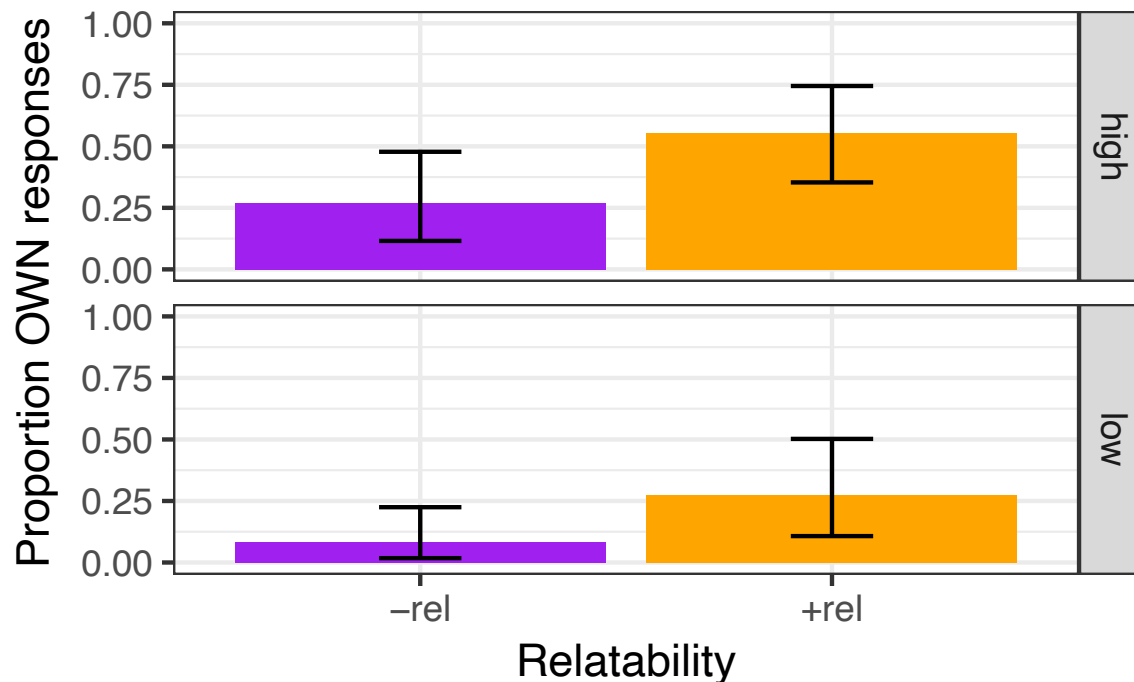


# Checking predicted effects

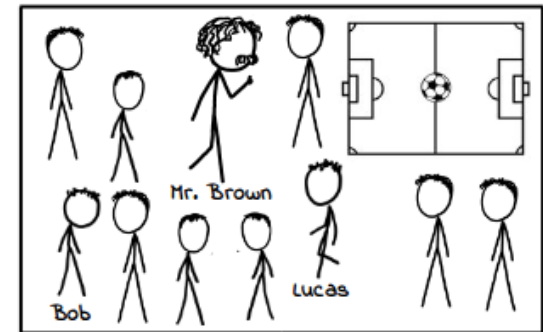
- The relatability effect

unrelatable noun ( <i>boy</i> )	relatable noun ( <i>son</i> )
<i>Brown knocks over a boy!</i>	<i>Brown knocks over a son!</i>

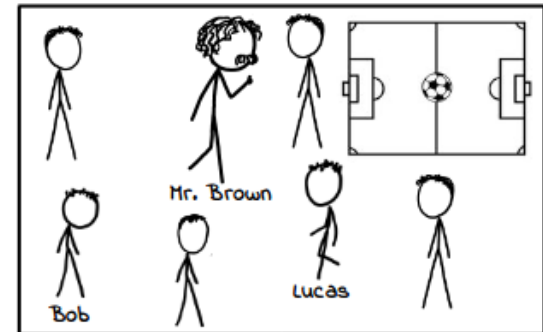
Relatability: ■ -rel ■ +rel



**Prior P(own) HIGH**



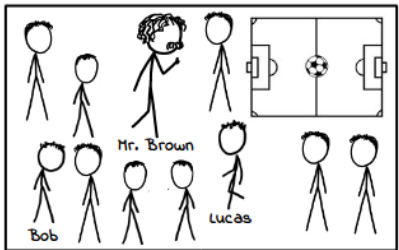
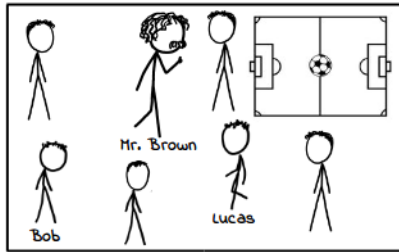
**Prior P(own) LOW**



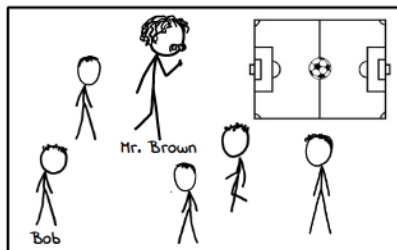
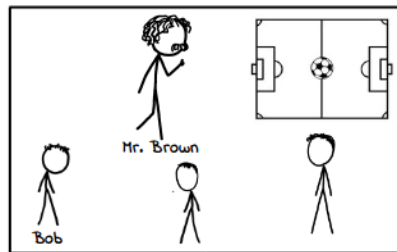
# Checking predicted effects

- The effect of contextual uniqueness

$|\text{sons of Brown}|=2$

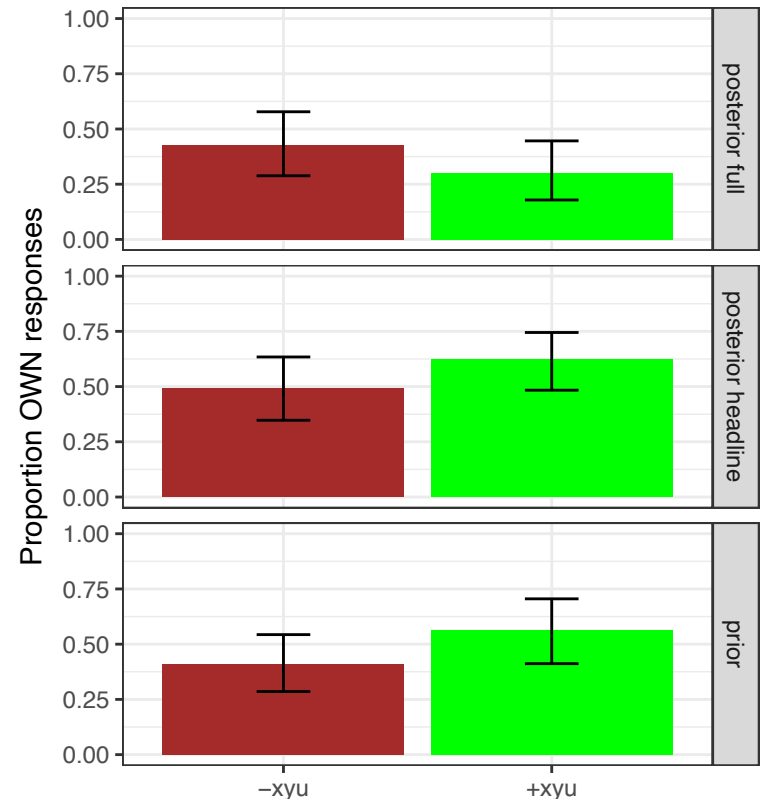


$|\text{sons of Brown}|=1$



Posterior	<i>Brown knocks over a son!</i>
“Headline”	<i>Brown knocks over son!</i>
Prior	<i>Brown knocks over#####son!</i>

X–Y relationship contextually unique? ■ –xyu ■ +xyu



# A little more about logical semantics

---

- **Logical** approaches to semantics influential in linguistics, psychology & AI
  - Linguistic expressions have **semantic types**
    - **Nouns**: sets (equiv: functions from entities to truth values, or  $e \rightarrow t$ )
    - **NPs** (oversimplified): individuals  $e$
    - **Intransitive verbs**: sets, or  $e \rightarrow t$  functions
  - Syntactic rules have corresponding semantic rules too
    - e.g., set membership check for  $S \rightarrow NP VP$

# Super-brief introduction to logical semantics

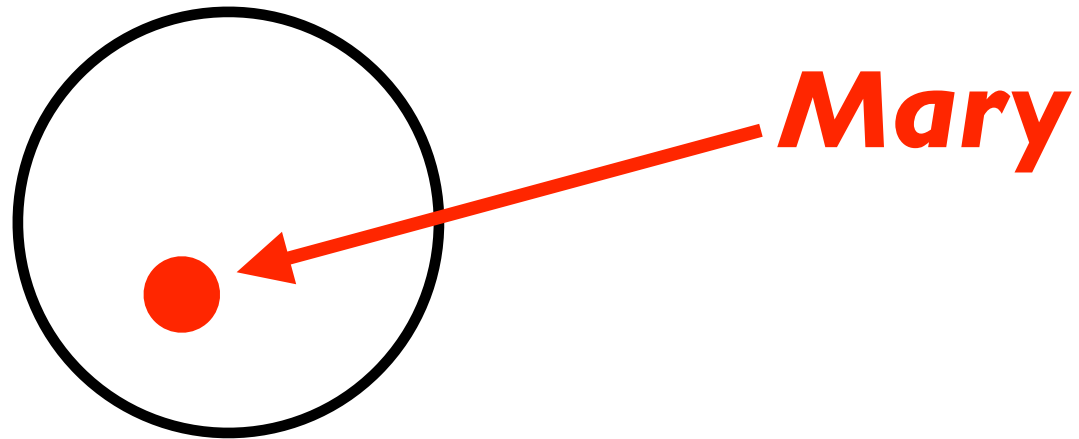
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*denotation  
function*

$$\left[ \begin{array}{c} S \\ \swarrow \quad \searrow \\ NP \quad VP \end{array} \right] = [VP]([NP])$$

$$[Mary \text{ sings}] = [sings]([Mary])$$

set of entities  
that sing



# Background: logical semantics

---

## **Warning!**

- The previous slide is a serious oversimplification of NP semantics & composition with VP into S, e.g.:
  - *Dogs growl* (dogs is a generic)
  - *Every student studied hard* (quantifiers every need different treatment)
- But, this example hopefully gives a sense of the context in which set-based semantics is put to use

# Adjectives: a range of semantic types

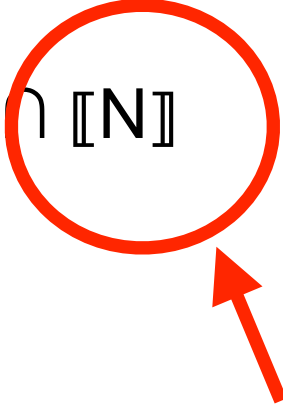
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- **Intersective:** *living, blue*
- **Scalar:**
  - **Relative:** *short, expensive*
  - **Absolute:** *dangerous, full*
- **Non-intersective:** *possible, alleged*
- **Anti-intersective:** *former, counterfeit*

# Intersective adjectives

---

- Examples: living, blue
- The criterion imposed by the adjective is independent of the noun
- Formally,  $[[\text{Adj N}]] = [[\text{Adj}]] \cap [[\text{N}]]$



$[[X]]$ : “*meaning of X*”

# Scalar adjectives

---

- Examples: *short, expensive, dangerous, full*
- Constrain referent's value on a **scale**
- **Relative adjs** (*short, expensive*): scale constraint based on comparison class
  - Highly context-sensitive; compare:
    - Bill is a big man!*
    - Bill is a big mouse!*
    - Bill is a big elephant!*
    - Bill is a big basketball player!*
- **Absolute adjs** (*dangerous, full, empty*): constraint is tied to scale boundary
  - Less context sensitive?
    - The glass is empty.*
    - The gas tank is empty.*
    - The auditorium is empty.*



# Non-intersective adjectives

---

- Examples: *alleged*, *possible*
- Adj's meaning contribution depends fundamentally on modified noun
- Adj “releases” referent from constraints on the noun

# Anti-intersective adjectives

---

- Examples: *counterfeit*, *former*
- Adjective's meaning contribution depends fundamentally on modified noun
- Adjective adds commitment that the referent is NOT in noun's denotation

# Adjectives: a range of semantic types

---

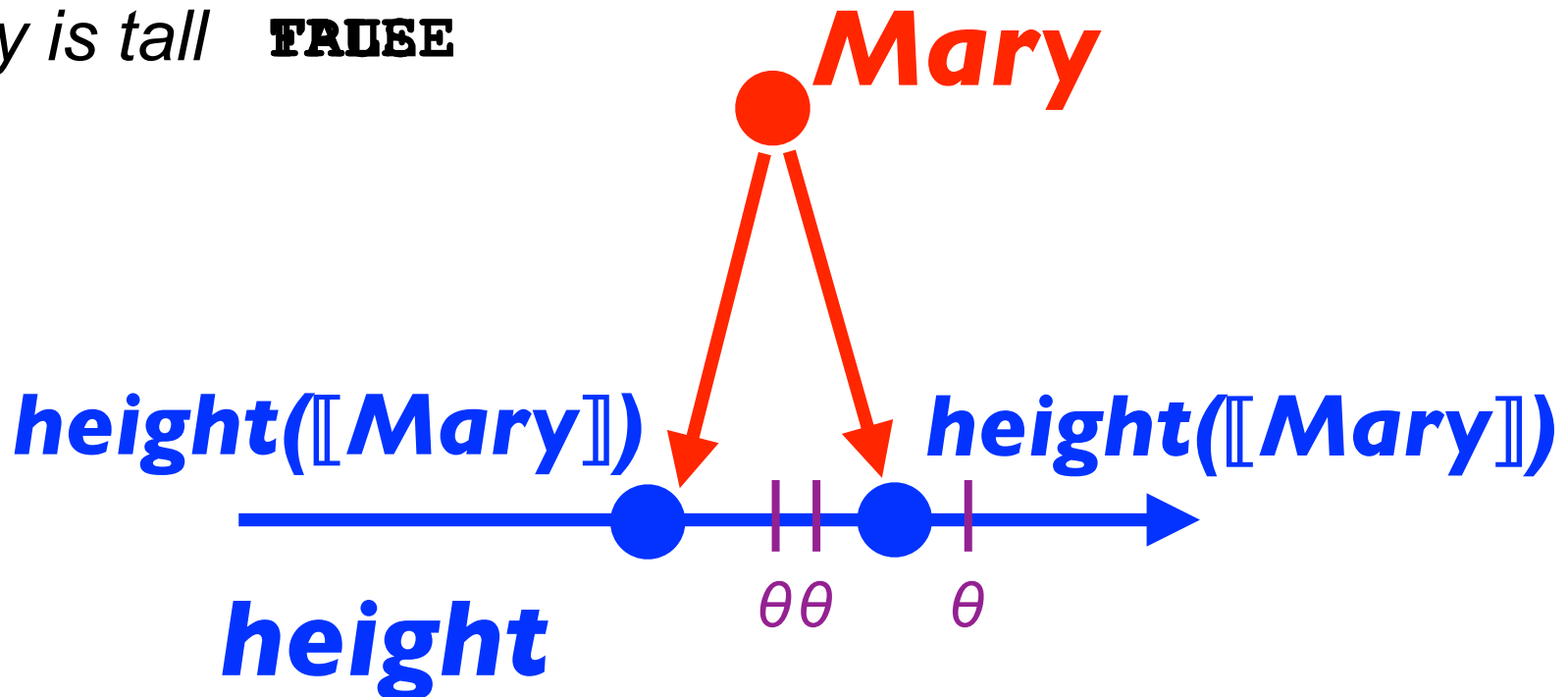
- **Intersective:** *living, blue*
- **Scalar:**
  - **Relative:** *short, expensive*
  - **Absolute:** *dangerous, full*
- **Non-intersective:** *possible, alleged*
- **Anti-intersective:** *former, counterfeit*

**Today**

# Degree semantics for scalar adjectives

- The meaning of a scalar adjective like *tall* does two things
  1. Projects a referent onto some **value** on a *scale*
  2. Predicates that that **value** is greater than some **threshold  $\theta$**

*Mary is tall* **FALSE**



# Observations regarding degree semantics

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- Differences in scale structure can predict validity of compositions



***fullness***

✓ *The glass is perfectly full.*

✓ *The glass is perfectly empty.*

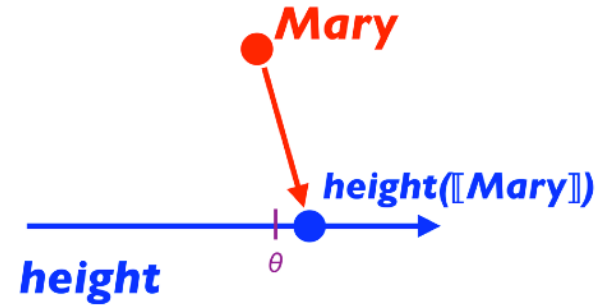


***danger***

✓ *The neighborhood is perfectly safe.*

\* *The neighborhood is perfectly dangerous.*

# What the degree semantics doesn't say



- This is a very elegant model
  - The abstractness of the model allows for context-sensitivity
- But it doesn't say *how* this context-sensitivity is achieved!
  - How does *tall elephant* turn out to mean something different from *tall mouse*?
  - How can the same *individual* be evaluated as either tall or not tall in different contexts?

*Stephen Curry is tall.*

*Stephen Curry is a tall basketball player.*

(Stephen Curry is 6'2"; this is the 12th percentile of NBA player heights)



# Towards a pragmatic model for scalar adjectives

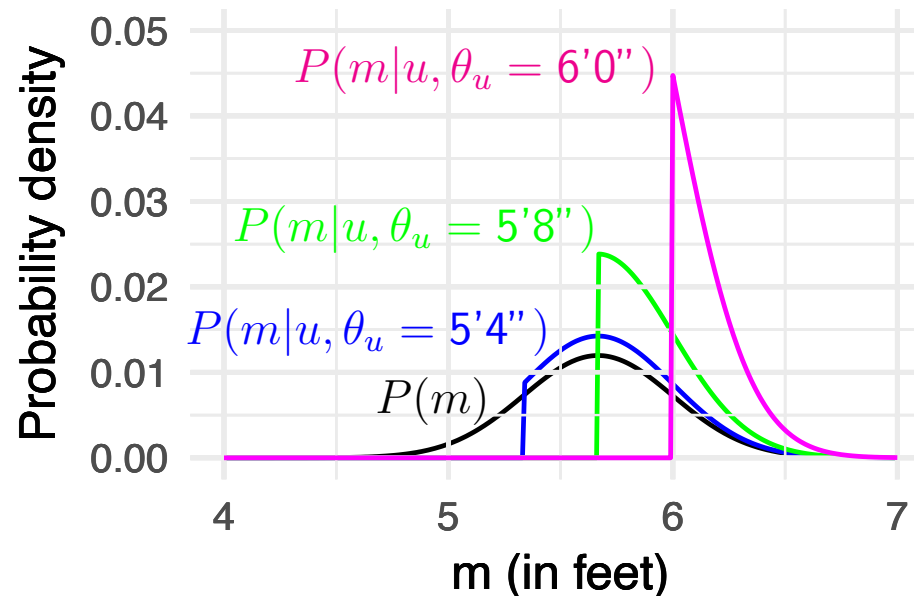
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- Desiderata
  - Inference on a continuum of possible scalar values
  - A threshold representation

# The Lassiter & Goodman model

- The literal-listener model of interpretation:

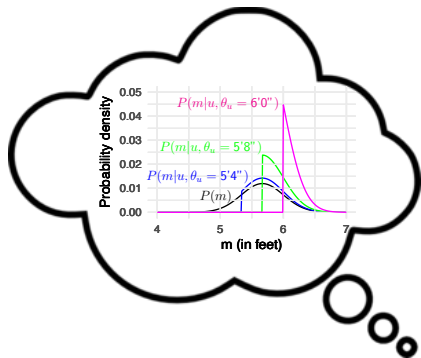
$$L_0(m|u, \theta) \propto \begin{cases} P(m) & m \geq \theta \\ 0 & \text{otherwise} \end{cases}$$





# A speaker model

- Assume a set of *alternative utterances* available to speaker
  - For “Pat ate some of the cookies”, alternatives were *some/all*
  - For “I injured a finger”, alternatives were *a/my/someone else’s*
- Here, we assume alternatives (to start) *tall* and **silence** ( $\emptyset$ )



*tall?*  
 $\emptyset$ ?

**Lassiter & Goodman's  
cost assumption:**

$$\text{cost}(\text{tall}) = \text{cost}(\emptyset) + 2$$

$$\text{Utility}(u|m, \theta_u) = \log L_0(m|u, \theta_u) - \text{cost}(u)$$

$$S_1(u|m, \theta_u) \propto e^{\text{Utility}(u|m, \theta_u)}$$

$$S_1(u|m, \theta_u) \propto \frac{L_0(m|u, \theta_u)}{e^{\text{cost}(u)}}$$

# A pragmatic listener

*Pragmatic listener is a standard Bayesian comprehender:*

$$L_1(m, \theta_u | u) \propto S_1(u | m, \theta_u) \underbrace{P(m, \theta_u)}$$

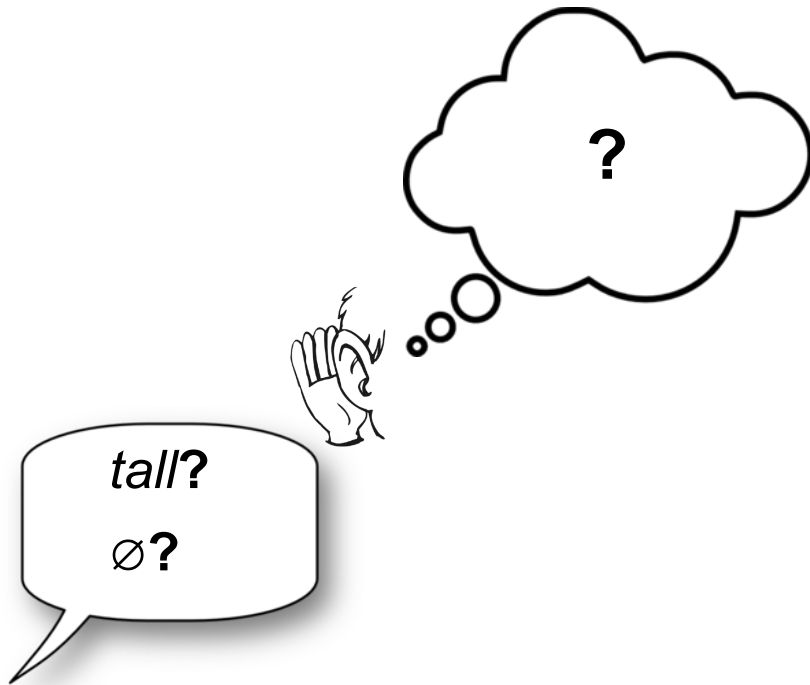
*What do we do with this joint distribution?*

*Proposal: they are conditionally independent...*

$$L_1(m, \theta_u | u) \propto S_1(u | m, \theta_u) P(m) P(\theta_u)$$

*...and  $\theta_u$  has a **uniform** prior:*

$$L_1(m, \theta_u | u) \propto S_1(u | m, \theta_u) P(m)$$



*This is a proposal of non-trivial theoretical depth and interest; let's discuss!*

# Visualizing pragmatic listener inferences

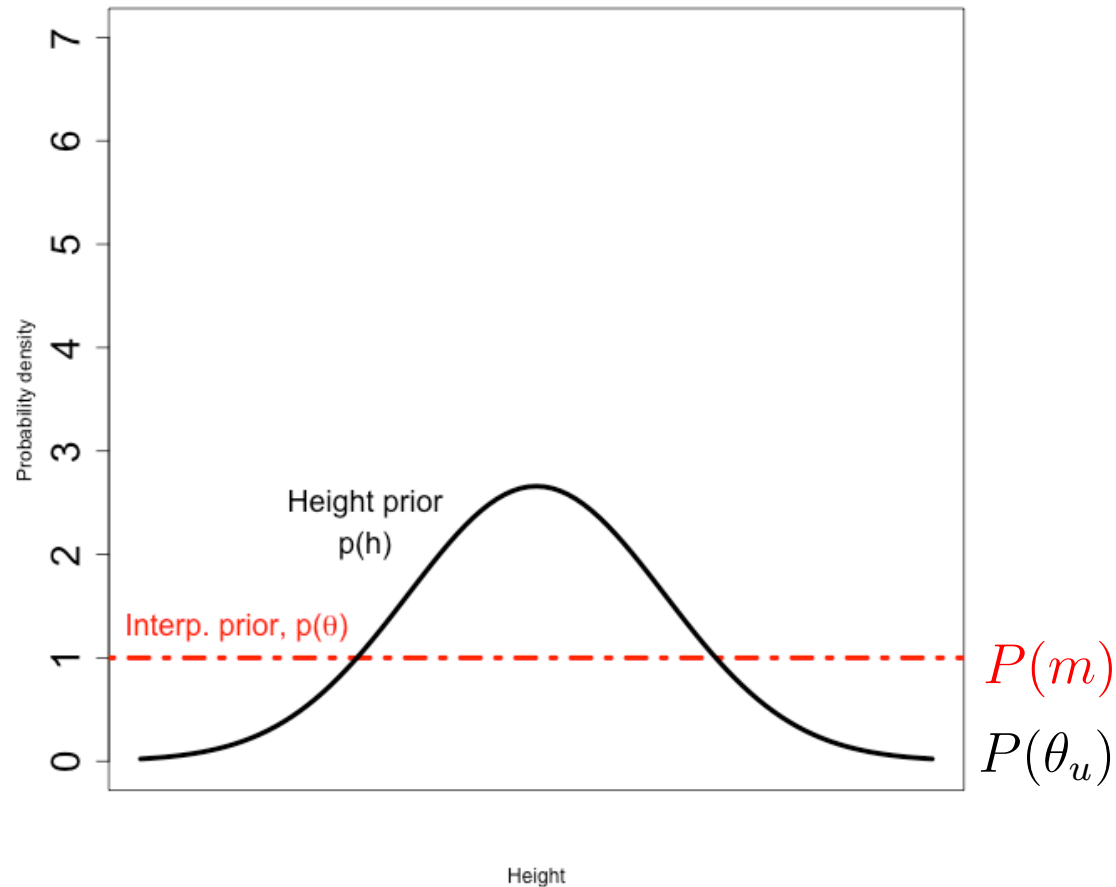
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$$L_1(m, \theta_u | u) \propto S_1(u | m, \theta_u) P(m) \underline{P(\theta_u)}$$



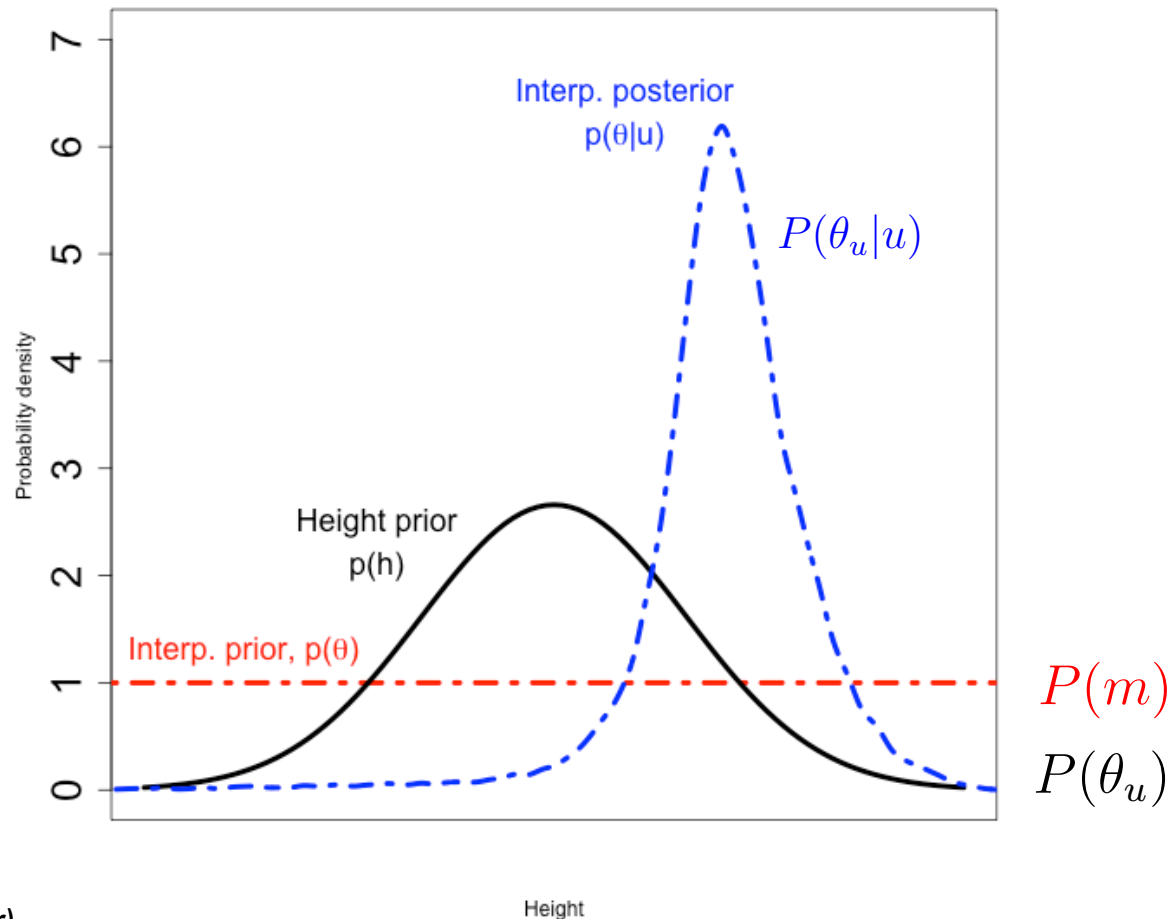
# Visualizing pragmatic listener inferences

$$L_1(m, \theta_u | u) \propto S_1(u | m, \theta_u) \underline{P(m)} P(\theta_u)$$



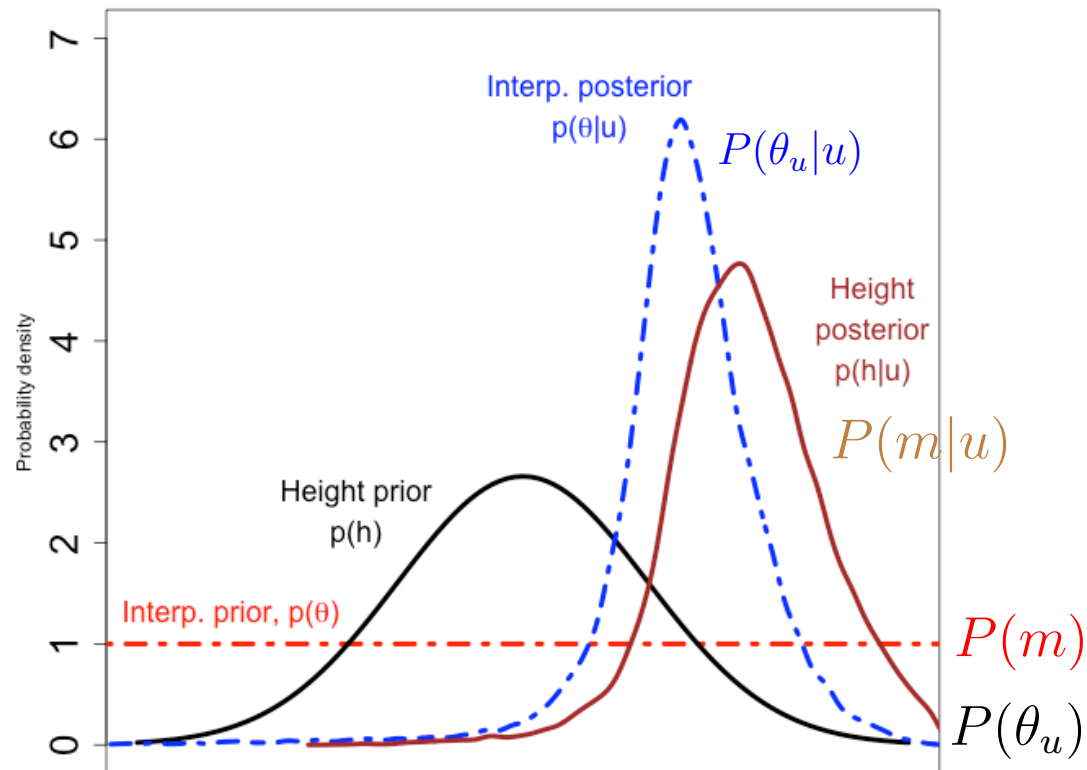
# Visualizing pragmatic listener inferences

$$L_1(m, \theta_u | u) \propto S_1(u | m, \theta_u) P(m) P(\theta_u)$$

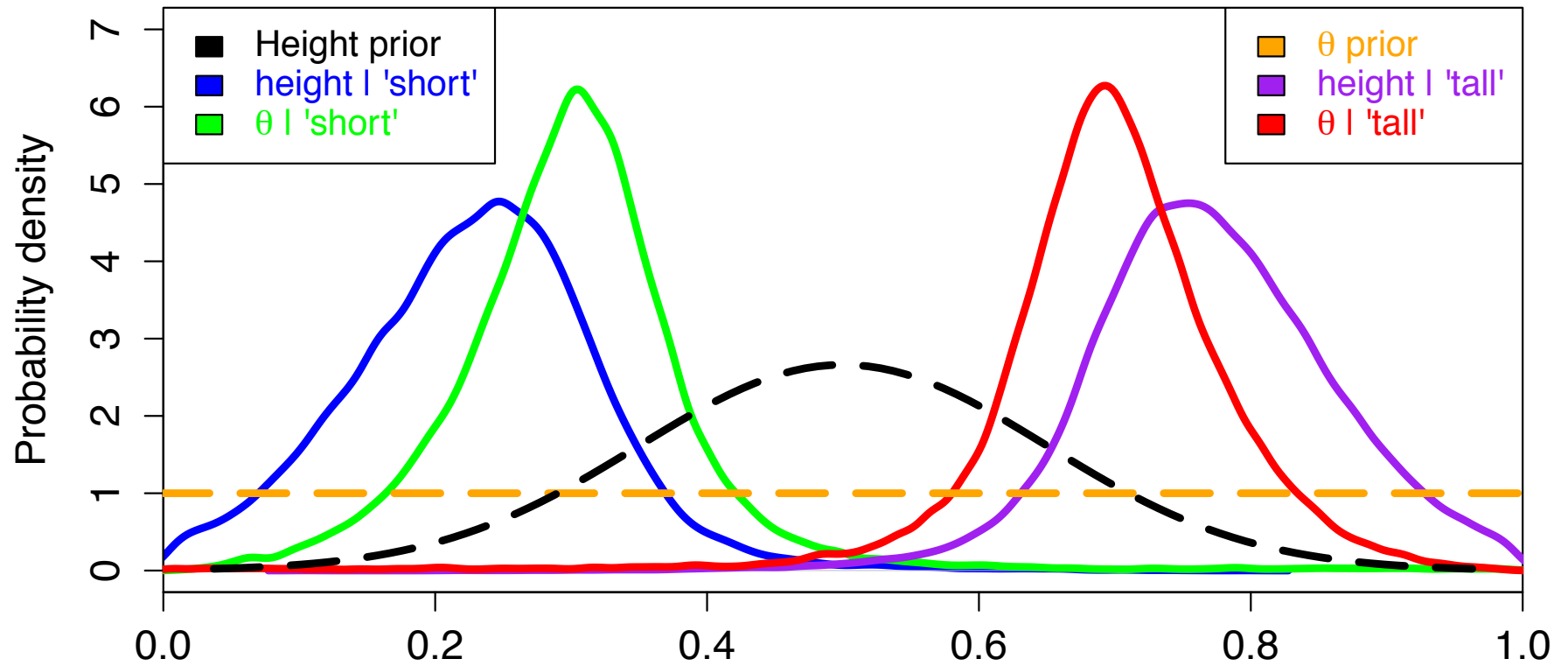


# Visualizing pragmatic listener inferences

$$L_1(\underline{m}, \underline{\theta}_u | u) \propto S_1(u | m, \theta_u) P(m) P(\theta_u)$$



# Antonyms



(Due to Dan Lassiter)

# Absolute adjectives

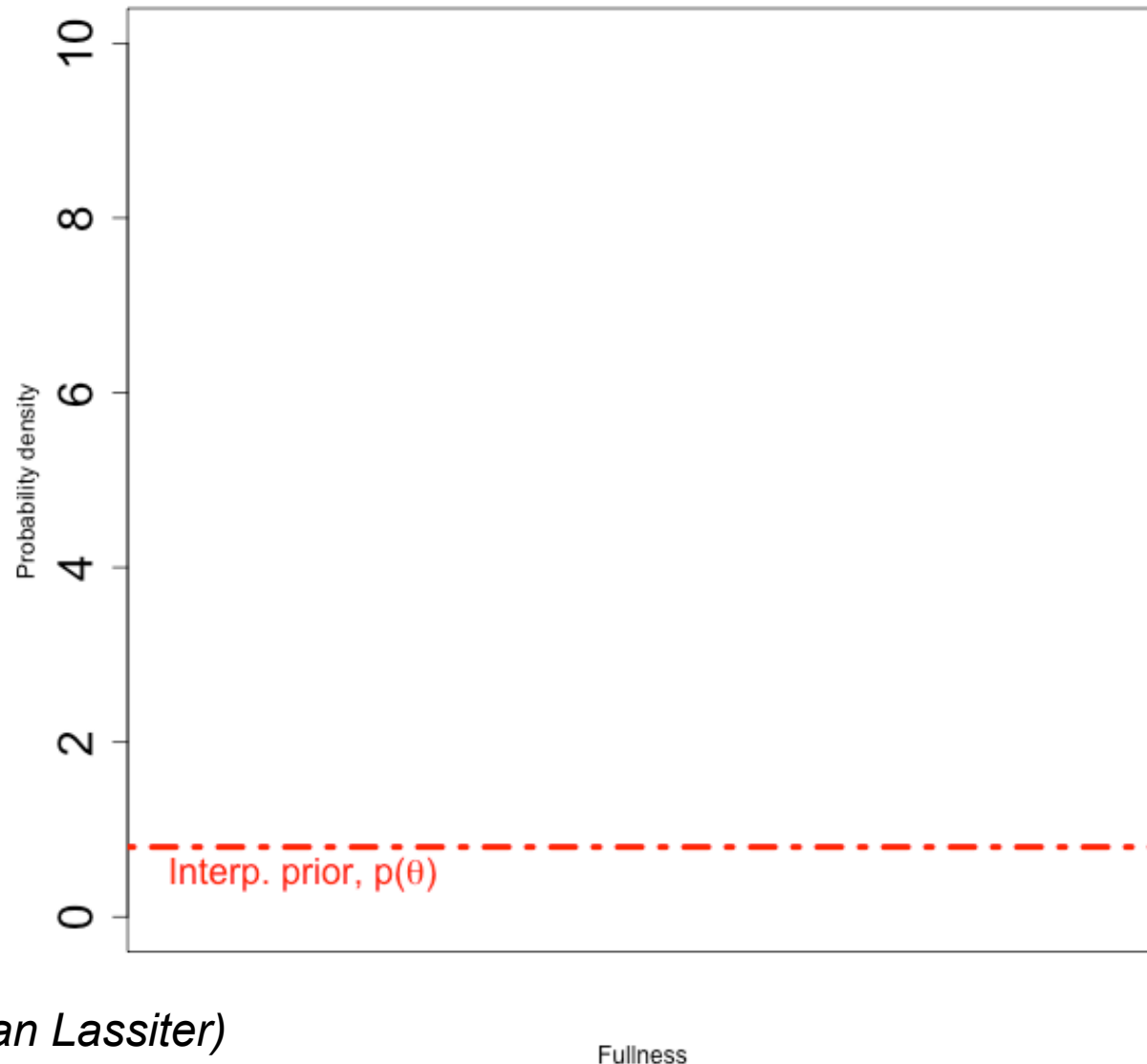
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- *full/empty, wet/dry, safe/dangerous, ...*
  - meanings are less (not?) context-dependent
  - meanings are sharp(er)
  - reference classes apparently not relevant to interpretation



# The pragmatic model on *full*

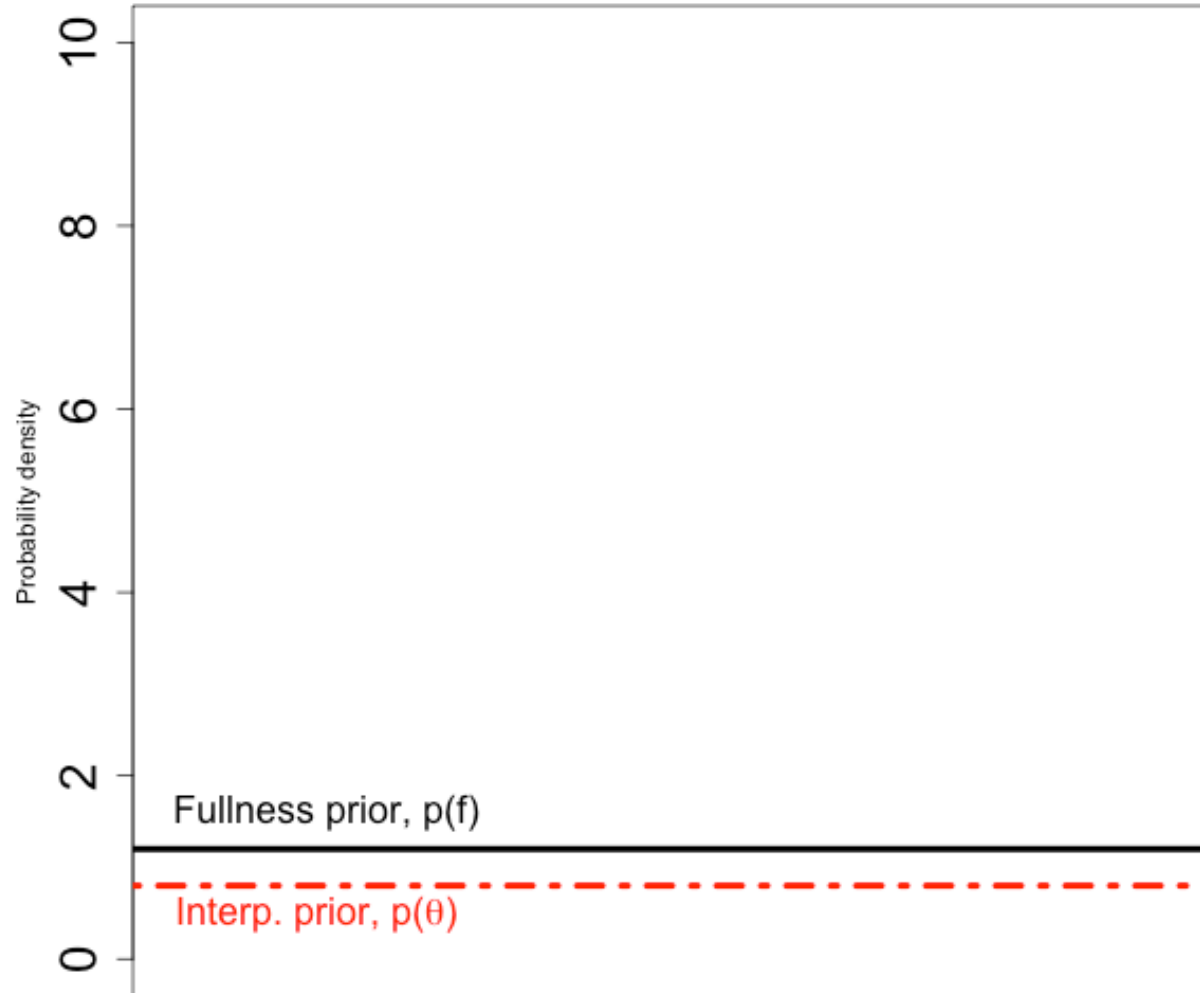
- Crucially, fullness is a ***bounded*** scale!



(Graph due to Dan Lassiter)

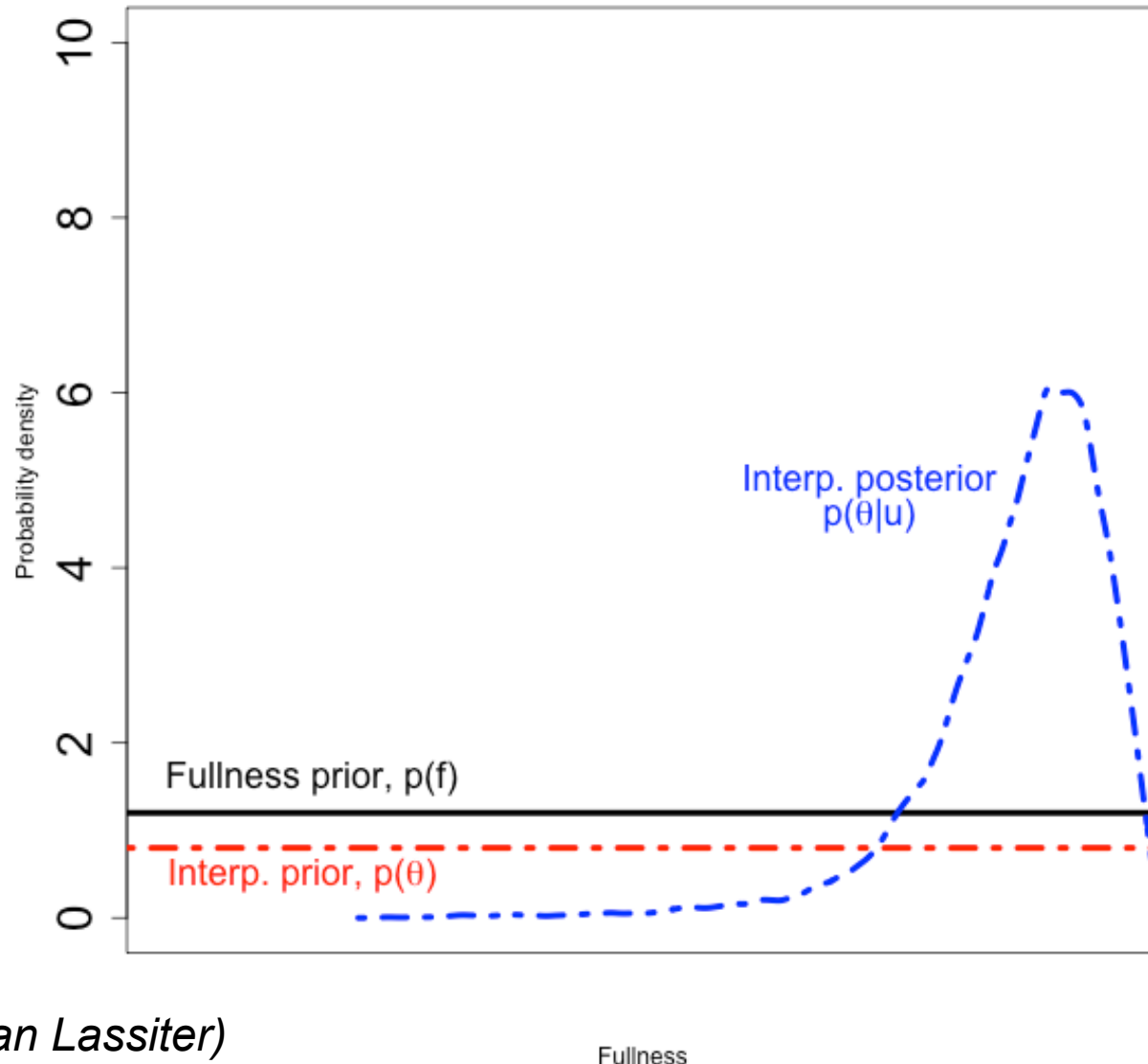
# The pragmatic model on *full*

- Crucially, fullness is a ***bounded*** scale!



# The pragmatic model on *full*

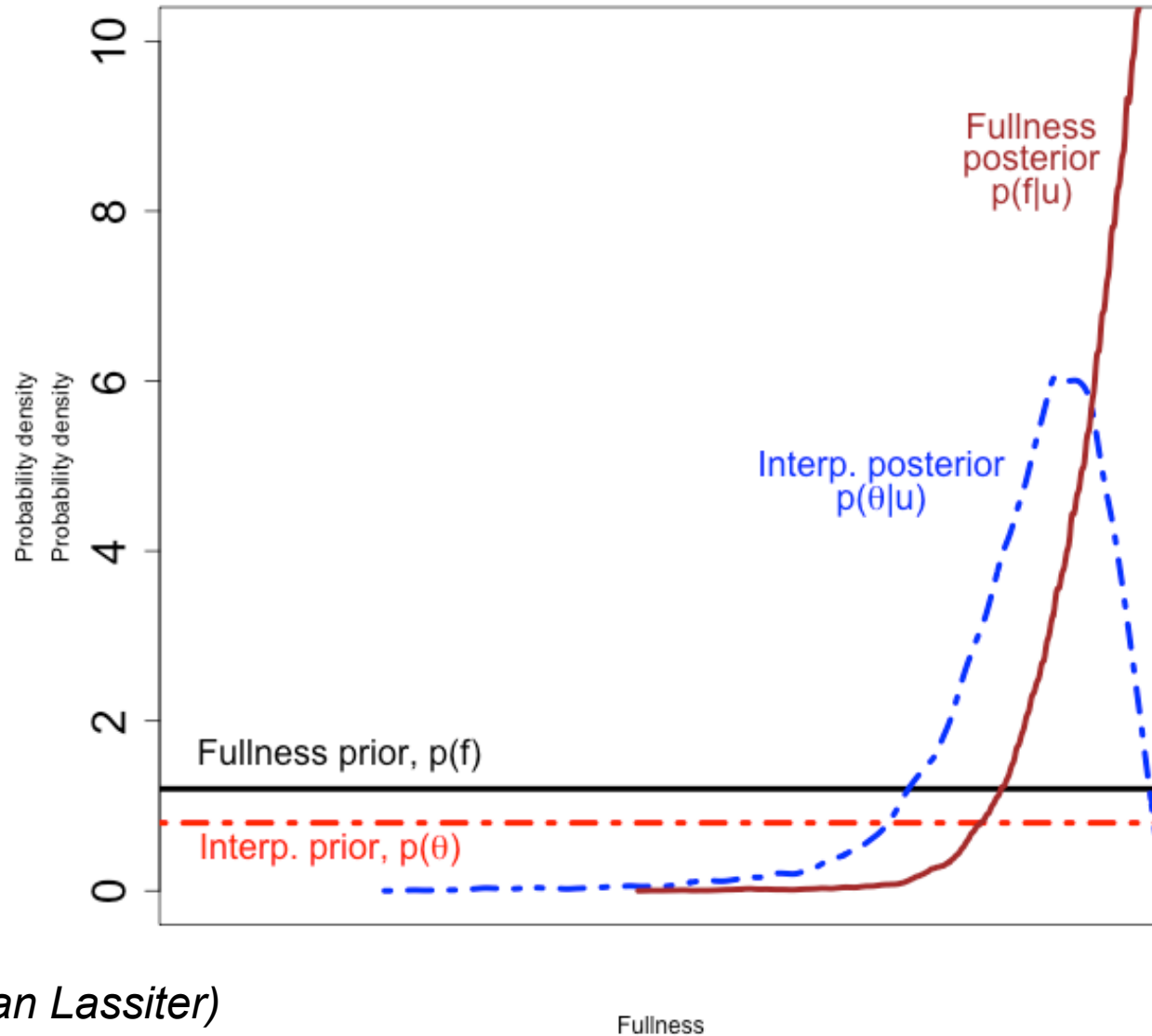
- Crucially, fullness is a ***bounded*** scale!



(Graph due to Dan Lassiter)

# The pragmatic model on *full*

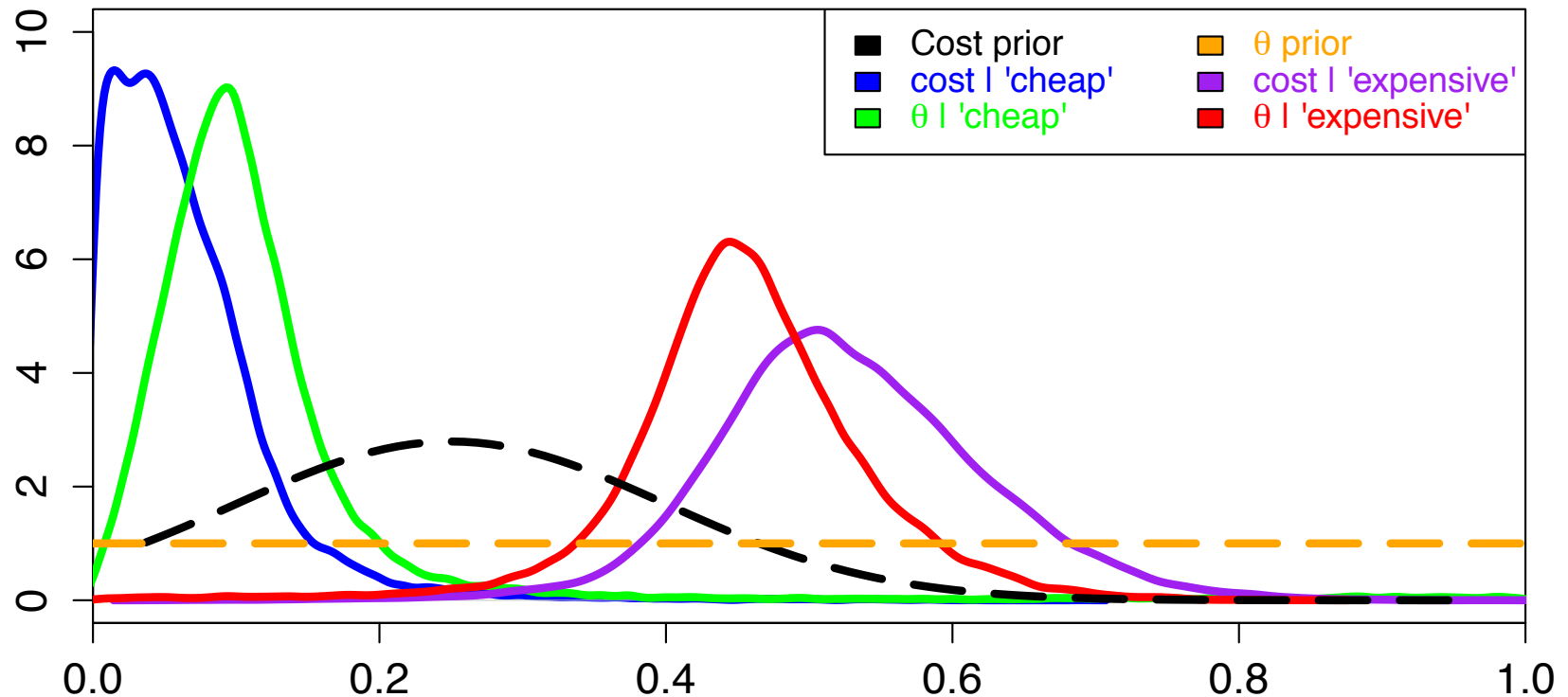
- Crucially, fullness is a ***bounded*** scale!



(Graph due to Dan Lassiter)

# Bounds on scales

- On the Lassiter & Goodman model, asymmetries in the interpretations of adjectives arise naturally as a consequence of the prior



# Summary

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- Scalar adjectives are a simple example, but pose an additional challenge for pragmatics models
- Some part of the *literal meaning of an utterance* must get contextually determined
- This is one of the simplest examples of interleaving of semantic representation and probabilistic pragmatic inference
- Pieces of the puzzle:
  - Logical semantic representations
  - Latent-variable treatment of pieces of these representations
  - Prior probabilities on likely speaker meanings
  - Joint, utility-driven posterior inference on latent semantic variables and speaker meaning