Finite State Machines

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- **Recommendation:** compare this inductive definition with syntax & semantics of regular expressions, as they're closely related

Constructing an FSA from a regex r (slightly informal)

- Base cases:

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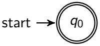

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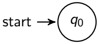
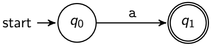


$r =$**a**



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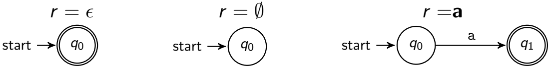

- Additional operators we need for inductive construction:

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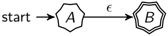
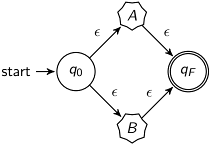
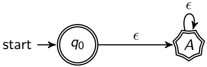


Disjunction



Kleene closure



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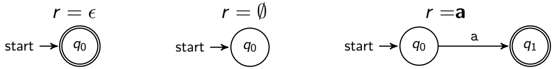

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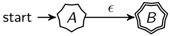

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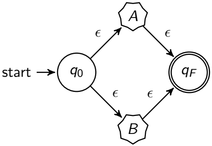

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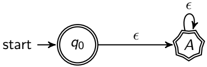
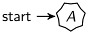
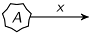


- Semantics regarding the resulting automaton A':

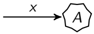


The start state of A is the start state of A'

Every final state of A is final in A'



A' has an x-labeled transition from every final state in A to wherever the arrow points to



A' has an x-labeled transition from wherever the arrow originates to the start state of A

Closure properties of regular languages

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- Closure under intersection plays an important role in what's coming up!

Finite state transducers, briefly

- A **finite-state transducer** consists of:
  - A finite set of N **states** $Q = \{q_0, q_1, \ldots, q_{N-1}\}$, with q_0 the **start state**
  - A finite **input alphabet** Σ of symbols
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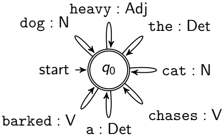

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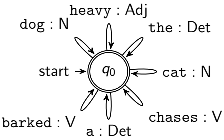

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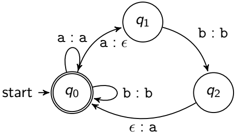

Optional exchange of ab

Input	Possible outputs
aabbab	aabbab, ababab
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- **Suggested exercise:** before going on, try to express the following generalizations in an FSA for this fragment of English syntax, over the alphabet $\Sigma = \{\texttt{they},\texttt{are},\texttt{Vgnd},\texttt{Adj},\texttt{N}\}$

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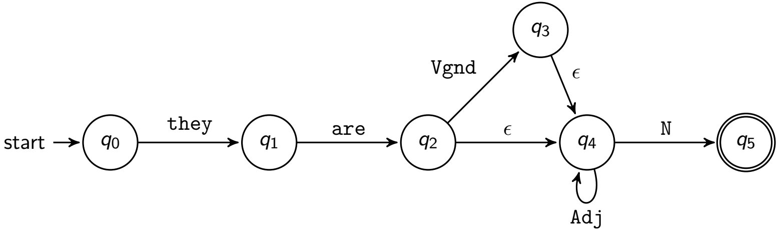
- An FSA that expresses these generalization:



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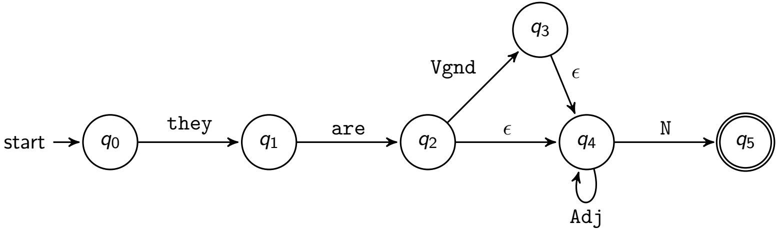

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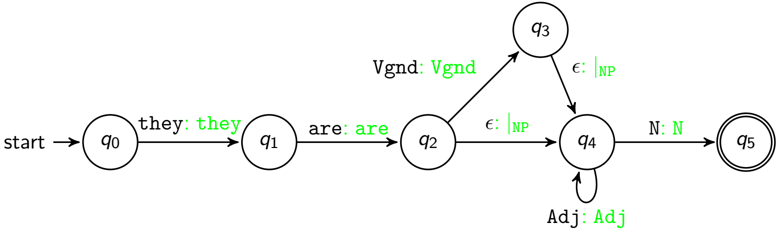

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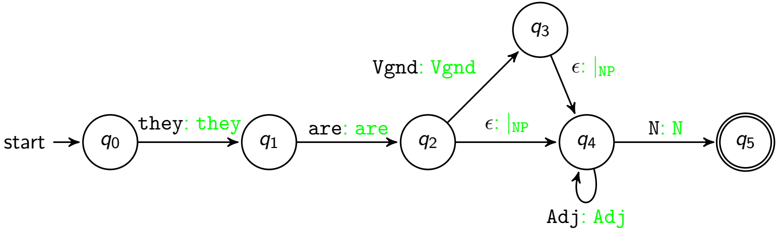

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- The multiple paths through the automaton offer the possibility for different **structural descriptions** of strings

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Phenomenon	Finite-state machine (FSM) **weak**?	FSM **strong**?
Gerund/adjective ambiguity	✓	✓

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- Example NPs that these generalizations license:

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a joke about the woman with an umbrella
a joke about the woman with an umbrella on the street
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- **Recommended exercise:** try writing an FSA for this before going on!
- **Observe:** as PPs accumulate, the meanings multiply!

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Multiple prepositional phrases in English

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- \# meanings grows as the Catalan numbers, $C_k = \binom{2k}{k} - \binom{2k}{k-1}$ (Church & Patil, 1982)

Multiple prepositional phrases in English II

- An NP can consist of a determiner and a noun, optionally followed by one or more PREPOSITIONAL PHRASEs.
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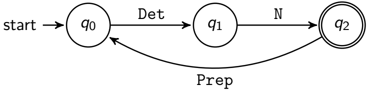

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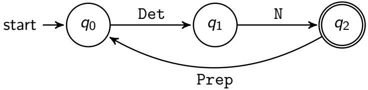

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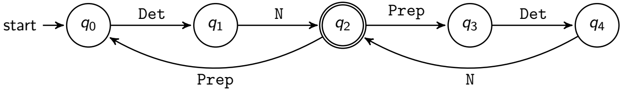

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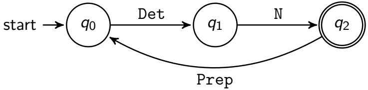

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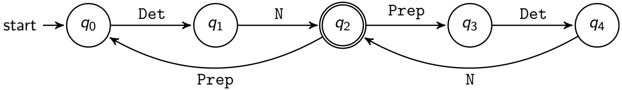

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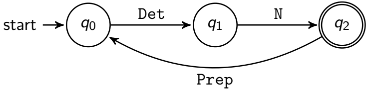

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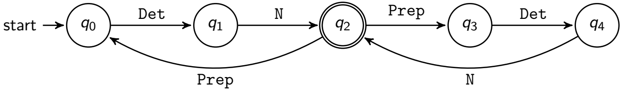

- ...but we would have to add states for every additional level of PP stacking.
- Since PP stacking is unbounded, a finite-state machine won't be able to generate enough structural descriptions for an unbounded number of PPs.

Weak vs. strong generative capacity

Phenomenon	FSM **weak**?	FSM **strong**?
Gerund/adjective ambiguity	✓	✓
NPs with stacked PP postmodifiers	✓	✗

Case study 3: object-extracted relative clauses

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	rock	that the squirrel likes ~~the rock~~	
the	rock		can be found in the garden
the	rock	that the squirrel likes	can be found in the garden

Multiple center-embedding with relative clauses

	N	that NP V ~~NP~~	
the	N		⟨rest_of_clause⟩
the	N	that NP V	⟨rest_of_clause⟩

- Subject-modifying object-extracted relative clauses can be nested:

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  the rock that the squirrel that the dog that the woman owns chases likes can be found in the garden

  ⋮

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- That is, they may tax human linguistic **performance**, but they should be part of a theory of human grammatical **competence**

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- **Recommended exercise:** before going on, think of how you would try to capture this pattern (at least in part) with a finite-state model.

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- Showing this rigorously requires additional technical apparatus that we'll cover next

Pumping strings

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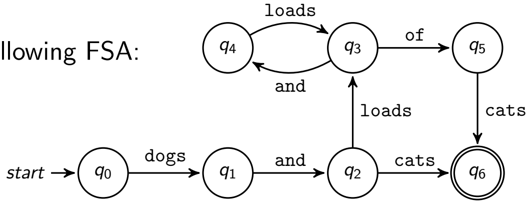

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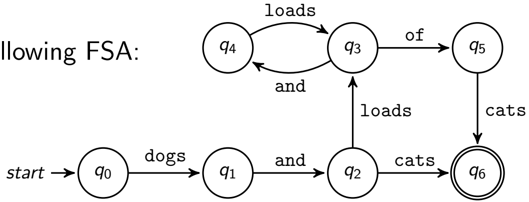

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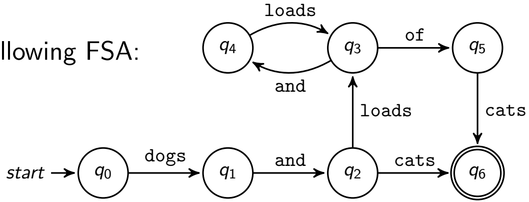

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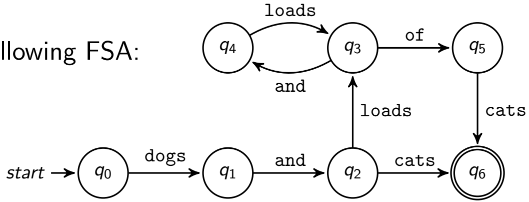

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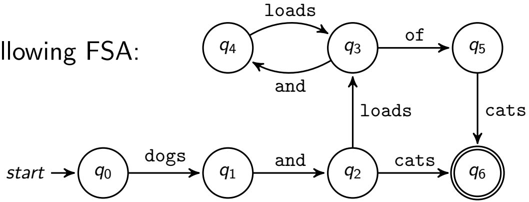

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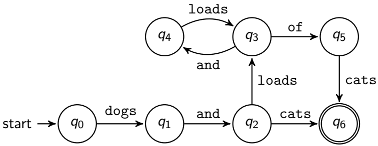
- This is called **pumping** s_3 with the substring `and loads`.

The Pumping Lemma for regular languages

- Informally: if L is regular, then every string that is “long enough” contains some non-empty “intermediate” section that can be arbitrarily pumped without leaving L.



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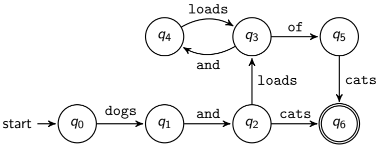

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- Formally: if $L \subseteq \Sigma^*$ is regular, then there is some integer k such that for every string $s \in L$ such that $|s| > k$ can be written as $s = xyz$ for $x, y, z \in \Sigma^*$, with:
  - $|y| \geq 1$ (y is non-empty);
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Example of the pumping lemma's application

if $L \subseteq \Sigma^*$ is regular, then there is some integer k such that for every string $s \in L$ such that $|s| > k$ can be written as $s = xyz$ for $x, y, z \in \Sigma^*$, with:

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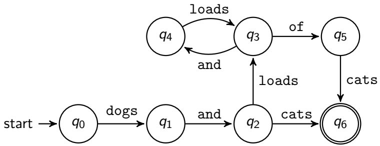

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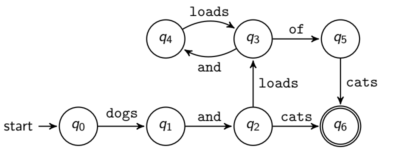

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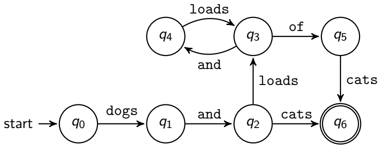

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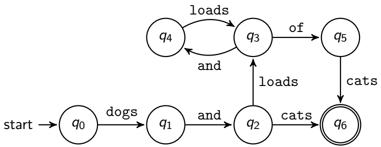

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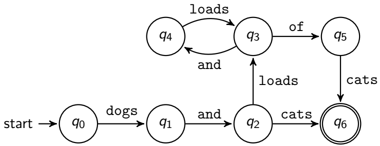

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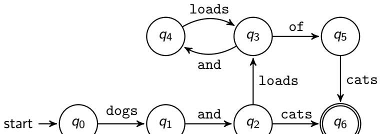

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- y is non-empty;
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- We could do a similar decomposition for every other string in the language of length > 6. Thus, the pumping lemma is satisfied, and the language is regular!

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- We use the pumping lemma for regular languages to show that $L^{\dagger}$ is *not* regular.
- Therefore, $\textsc{English}$ is not regular.

Evaluating multiple center-embedding with the Pumping Lemma

```
the rock can be found in the garden
the rock that the N V can be found in the garden
the rock that the N that the N V V can be found in the garden
the rock that the N that the N that the N V V V can be found in the
garden
⋮
```

- We will call this infinite set $L^{\dagger}$ and summarize it as:

  $$\texttt{the rock (that the N)}^{i}\ \texttt{V}^{i}\ \texttt{can be found in the garden}$$

  for $i \geq 0$, with the multiple appearances of ^{i} indicating that `(that the N)` and `V` must appear in place the same number of times as each other.

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  `the rock (that the N)`^{i} `V`^{i} `can be found in the garden`

  for $i \geq 0$, with the multiple appearances of ^{i} indicating that `(that the N)` and `V` must appear in place the same number of times as each other.
- Note that strings of the following form are not OK English when $i \neq j$:

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  e.g., **the rock that the squirrel can be found in the garden* is not in English.

Evaluating multiple center-embedding with the Pumping Lemma

```
the rock can be found in the garden
the rock that the N V can be found in the garden
the rock that the N that the N V V can be found in the garden
the rock that the N that the N that the N V V V can be found in the the
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⋮
```

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  e.g., **the rock that the squirrel can be found in the garden* is not in English.
- We assume that $L^\dagger$ is part (formally, a subset) of ENGLISH.

Intersecting $\textsc{English}$ with a regular language

$L^{\dagger} = \texttt{the rock (that the N)}^{i}\ \texttt{V}^{i}\ \texttt{can be found in the garden}$

- Call the regular language to the regex below $L^{\ddagger}$:

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- We will be able to use the Pumping Lemma for regular languages to show that $L^{\dagger}$ is *not* regular

Proving by contradiction that $L^\dagger$ is not regular

if $L \subseteq \Sigma^*$ is regular, then there is some integer k such that for every string $s \in L$ such that $|s| > k$ can be written as $s = xyz$ for $x, y, z \in \Sigma^*$, with:

- $|y| \geq 1$ (y is non-empty);
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- Thus English is **not regular**

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Phenomenon	FSM **weak**?	FSM **strong**?
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Multiply nested object relative clauses	✗	✗

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- Finite-state models can capture some features of English syntactic structure, but have neither the strong nor the weak generative capacity for other features
- **Looking ahead:** these classic results motivate more expressive grammatical formalisms that have been central to the cognitive science of language for decades