Finite State Machines

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- The empty string is also a valid regular expression
- Semantics of what each part of a regular expression **matches**:

The empty string	A zero-character sequence
Any character from Σ	That character
Concatenation: XY	what X matches followed by what Y matches
$X*$	0 or more repetitions of X ($*$ is the "Kleene star")
$X \mid Y$	X or Y
()	determine operator precedence

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`^` and `$`	**Anchors** requiring certain in-string position for partial matches
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`(`X`)`Y`\1`	X then Y then a repetition of the string matching X, e.g.:
`([ab]*)b\1`	Matches `b`, `aba`, `bbb`, `abbba`, ...

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- In this discussion going forward, we cover regexes *without* backreferences.

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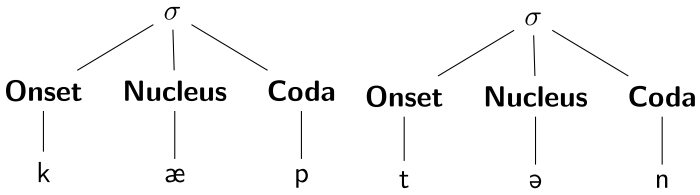
- Can you think of similar examples involving English and another language you know?

The grammar of English onsets

- Basic syllable structure of English, e.g. for *captain* /kæptən/:



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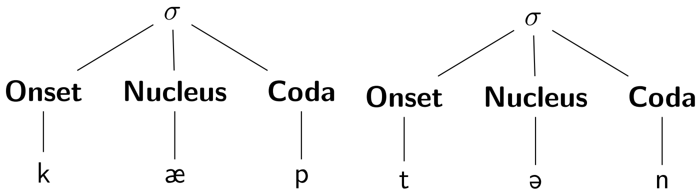

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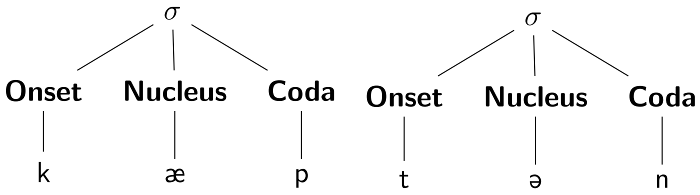

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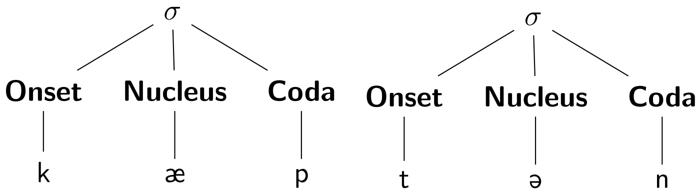

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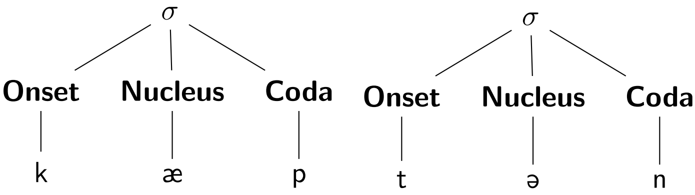

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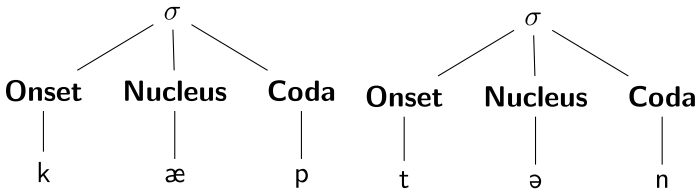

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- Note that these prohibitions vary in generality!

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Feature	$+$ phonemes	$-$ phonemes
Voice	b, d, ð (first sound of *the*), g, v, z, j, ʒ (second consonant of *measure*)	tʃ ("ch"), f, k, p, s, ʃ ("sh"), t, θ ("th"), h
Labial	b, p, f, v, m, w	none
Sonorant	l, m, n, ŋ ("ng"), r, w, y	all others
Strident	tʃ, j, s, ʃ, z, ʒ	d, ð, t, θ, n, l, r
Continuant	ð, f, s, ʃ, θ, v, z, ʒ, h	b, tʃ, d, g, j, k, p, t

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- We can now write phonotactic prohibitions as little regular expressions:

Constraint	Regex	Explanation
$*\begin{bmatrix}+\text{Son}\end{bmatrix}\begin{bmatrix}\quad\end{bmatrix}$	`[mnŋlrwy].`	Sonorants may only be onset-final
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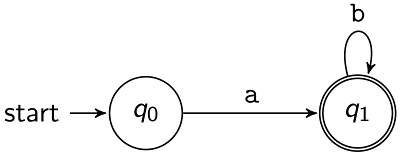
- A natural way to do this turns out to be through the formalism of **finite-state machines**

Finite-state automata: an example

- Example **finite-state automaton** (FSA):



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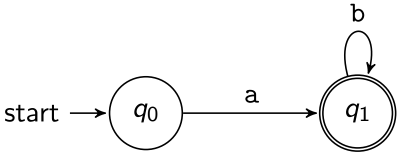

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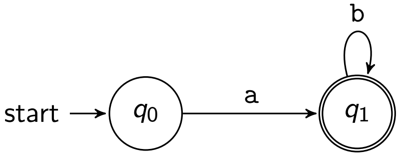

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- More precisely, it accepts all and only the strings accepted by the regular expression `ab*`

Finite-state automata, formally defined

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- A finite set of N **states** $Q = \{q_0, q_1, \ldots, q_{N-1}\}$, with q_0 the **start state**;

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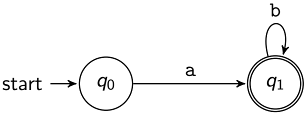
Equivalent specifications of the same FSA



$$Q = \{q_0, q_1\}$$
$$\Sigma = \{\mathtt{a}, \mathtt{b}\}$$
$$F = \{q_1\}$$
$$\Delta = \{q_0 \overset{\mathtt{a}}{\rightsquigarrow} q_1,\ q_1 \overset{\mathtt{b}}{\rightsquigarrow} q_1\}$$

Acceptance criterion in FSAs (slightly informal)

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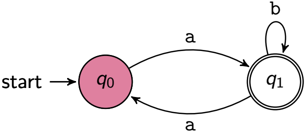

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b
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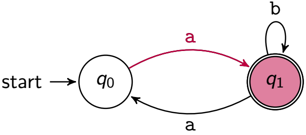

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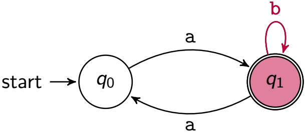

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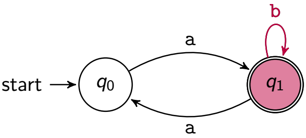

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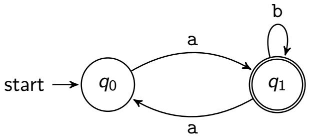

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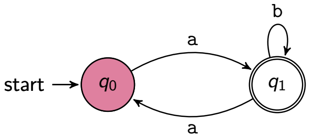

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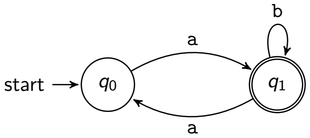

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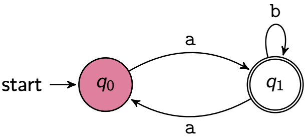

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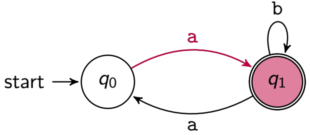

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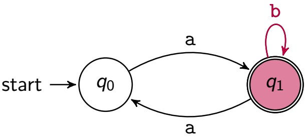

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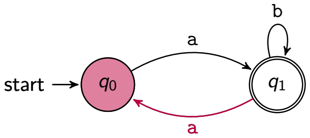

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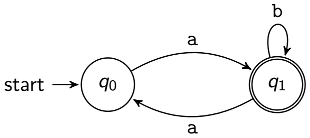

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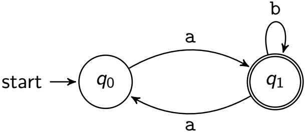

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- **For every regex there is an FSA that accepts all and only the strings the regex matches, and vice versa!**

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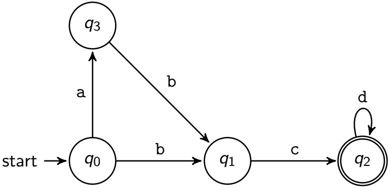
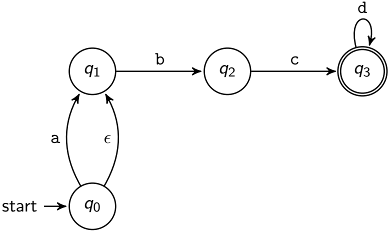
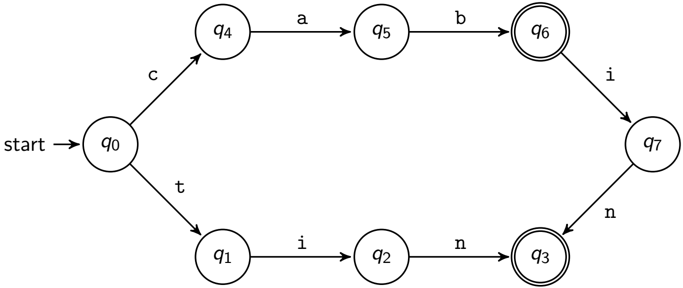
DFSA

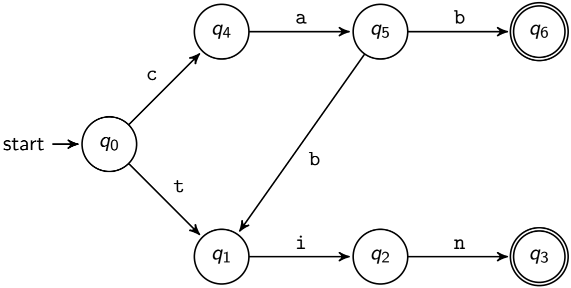


Equivalent NFSA

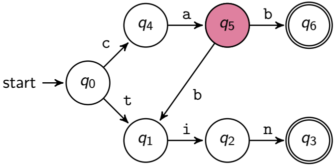


Another equivalent DFSA/NFSA pair





Checking string acceptance/rejection in an NFSA



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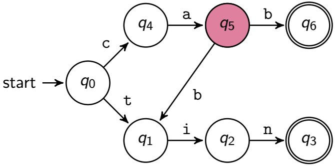

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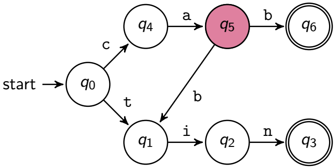

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- Above, there are two possible outward transitions in q_5 for input symbol b.
- Algorithmic options:
  - **Backup:** whenever we encounter a choicepoint, generate a list of transition options and mark our position. Try one. If we fail, go back to the last choicepoint and try the next option on the list. If we run out of options, then the string is rejected.
  - **Lookahead:** look forward in the string to guide choice.
  - **Parallelism:** Instead of maintaining and updating a single state, build a *set* of possible states for each string position.

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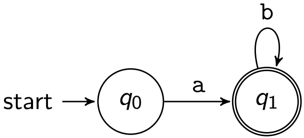
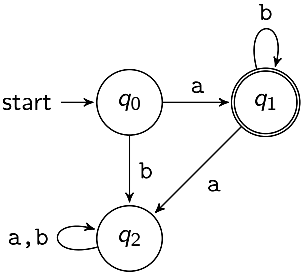


Equivalent total FSA



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- Broad goal: as we accumulate grammar fragments for a language, we should obtain an increasingly close approximation of the *true* characterization of the language and its structure

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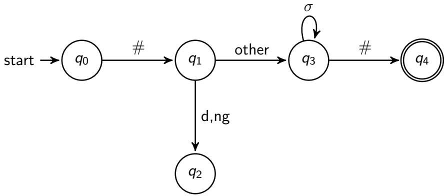

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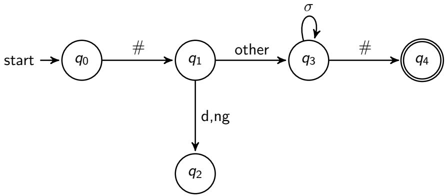

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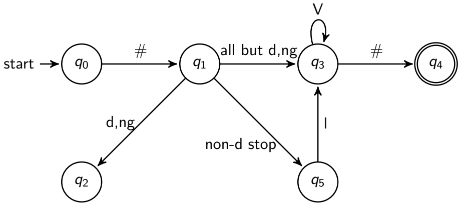
- Add constraint 2: Word-initially, CC sequences must be stop+liquid(=l).



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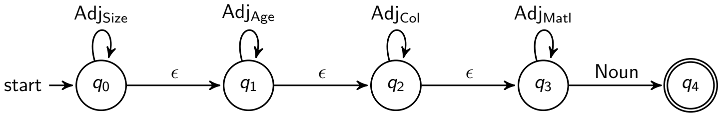

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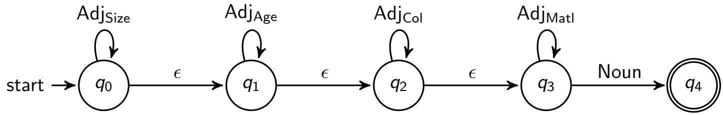

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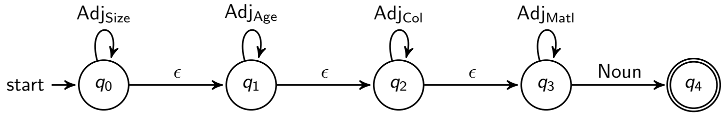

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- **Note:** the fuller picture is more complicated! For example, contrastive stress can help bring an adjective leftward (e.g., "the **WOODEN** old door, not the **STONE** one"). But this simple description captures some major trends.

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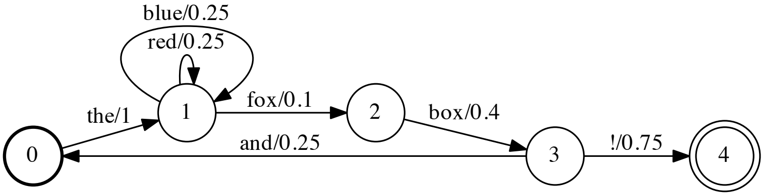
  - What parts of natural language structure can and cannot be captured by these formalisms?

Weighted Finite-State Automata (WFSAs)

- An example of a WFSA:



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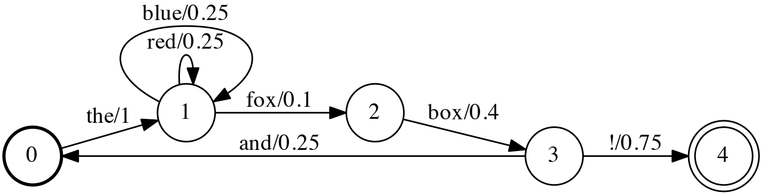

- A useful exercise: write regular expressions that are equivalent to each of the automata on this page, and on the previous page

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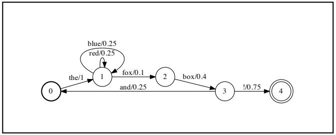

A WEIGHTED FINITE-STATE AUTOMATON (WFSA) consists of a tuple (Q, V, S, R) such that:

- Q is a finite set of STATES $q_0 q_1 \ldots q_N$, with q_0 the designated START STATE;
- Σ is a finite set of terminal symbols;
- $F \subseteq Q$ is the set of FINAL STATES;
- Δ is a finite set of TRANSITIONS each of the form $q \stackrel{i}{\rightsquigarrow} q'$, meaning that “if you are in state q and see symbol i you can consume it and move to state q'”;
- λ is a function mapping transitions to real numbers (weights);
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- $w_{1 \dots N} \in \Sigma^N$ is ACCEPTED or RECOGNIZED by an automaton iff there is a PATH of transitions $\underset{1 \dots N}{\rightsquigarrow}$ to a final state $q^* \in F$ such that

$$q_0 \underset{1}{\overset{w_1}{\rightsquigarrow}} \underset{2}{\overset{w_2}{\rightsquigarrow}} \dots \underset{N-1}{\overset{w_{N-1}}{\rightsquigarrow}} \underset{N}{\overset{w_N}{\rightsquigarrow}} q^*$$

- The WEIGHT of such a path $\underset{1 \dots N}{\rightsquigarrow}$ is the product of the weights of each of the transitions, together with the weight of the final state:

$$P(q_0 \underset{1}{\overset{w_1}{\rightsquigarrow}} \underset{2}{\overset{w_2}{\rightsquigarrow}} \dots \underset{N-1}{\overset{w_{N-1}}{\rightsquigarrow}} \underset{N}{\overset{w_N}{\rightsquigarrow}} q^*) = \rho(q^*) \prod_{i=1}^{N} \lambda(\underset{i}{\rightsquigarrow}) \quad (1)$$