# (Conditional) Independence

Events A and B are said to be Conditionally Independent given information C if

$$P(A, B|C) = P(A|C)P(B|C)$$

Conditional independence of A and B given C is often expressed as

$$A \perp B \mid C$$

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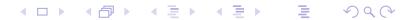
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  - ▶ Remember that probability densities have units (the inverse of the unit of the continuum), and the densities can exceed 1 per unit!



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- Unless I mention otherwise, things I say will hold for both discrete and continuous random variables, and I will freely use sums or integrals with the implicit understanding that what I say applies to both cases

## Mean, variance, and standard deviation

(Population) mean, or expected value:

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Population) **standard deviation**, often notated  $\sigma$ , is just the (positive) square root of the variance:

$$(\sigma_X =) SD[X] = \sqrt{\operatorname{Var}[X]}$$

## Linearity of the expectation

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Note that this requires no further assumptions (e.g., the random variables don't need to be independent)

#### Covariance and correlation

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The **correlation** between two random variables, sometimes notated  $\rho_{XY}$  for random variables X and Y, is their covariance "standardized" by their standard deviations:

$$(\rho_{XY} =) \operatorname{Cor}(X, Y) = \frac{\operatorname{Cov}(X, Y)}{\sigma_X \sigma_Y}$$