9.S918: Quantitative Inference in Brain and Cognitive Sciences

Week 2 Day 12: Causal inference continued

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Introductory causal inference

- You have probably had previous exposure to both probability and statistics
- You are less likely to have had exposure to **causal inference**
- Causal inference uses probability and statistics, but it is something separate from the traditional construal of those two fields
- You can think of causal inference as being a framework extending more traditional statistics by:
  - Adding new probability-based mathematical constructs; and,
  - Developing a set of practice for statistical inference based on those constructs
- Two causal inference frameworks:
  - The **potential outcomes** framework
  - The **causal graphical models** framework

The potential-outcomes framework

- In epidemiology and many other areas of statistics, causal inference was developed out of the idea of **potential outcomes** (Neyman 1923, Rubin 1974)
- Consider an outcome, Y, and a potential **treatment** A
- **Example:**

Y: an individual survives to the end of the year (0: no, 1: yes)

A: an individual with heart disease receives a heart transplant (0: no, 1: yes)

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- So we are interested in (and likewise for $Y^{a=1}$):

$$E[Y^{a=0}] = \sum_y yP(Y^{a=0} = y) = 0 \times P(Y^{a=0} = 0) + 1 \times P(Y^{a=0} = 1) = P(Y^{a=0} = 1)$$

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Counterfactual data and causal effects

(Hernan & Robins, 2020, Table 1.1)

	$Y^{a=0}$	$Y^{a=1}$
Rheia	0	1
Kronos	1	0
Demeter	0	0
Hades	0	0
Hestia	0	0
Poseidon	1	0
Hera	0	0
Zeus	0	1
Artemis	1	1
Apollo	1	0
Leto	0	1
Ares	1	1
Athena	1	1
Hephaestus	0	1
Aphrodite	0	1
Cyclope	0	1
Persephone	1	1
Hermes	1	0
Hebe	1	0
Dionysus	1	0
$P(Y^{a=*}) = 1$	0.5	0.5

Counterfactual data and causal effects

- Suppose we knew **what would happen** for each individual in the population under each value of the treatment

(Hernan & Robins, 2020, Table 1.1)

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- The **average causal effect** of treatment A is defined as the difference of counterfactual risks:

$$E[Y^{a=1}] - E[Y^{a=0}] = 0$$

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Counterfactual data and causal effects

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- Then we could compute the counterfactual risks:

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- The **average causal effect** of treatment A is defined as the difference of counterfactual risks:

$$E[Y^{a=1}] - E[Y^{a=0}] = 0$$

- Here, treatment is **ineffective**

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$P(Y^{a=*}) = 1$	0.5	0.5

Estimating causal effects

Remember, $E[Y^{a=i}] = P(Y^{a=i} = 1)$

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Rheia	0	0	0	0	?
Kronos	0	0	1	1	?
Demeter	0	0	0	0	?
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Crucial step; make sure you understand it!

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Consistency: when $A = i,\, Y = Y^{a=i}$

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$$= \hat{P}_{MLE}(Y^{a=i} = 1 \,|\, A = i)$$

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Crucial step; make sure you understand it!

$$= \hat{P}_{MLE}(Y^{a=i} = 1 \,|\, A = i)$$

- So, the following condition suffices:

$$P(Y^{a=i} = 1 \,|\, A = i) = P(Y^{a=i} = 1)$$

Estimating causal effects

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Polyphemus	1	1	1	?	1
Persephone	1	1	1	?	1
Hermes	1	1	0	?	0
Hebe	1	1	0	?	0
Dionysus	1	1	0	?	0

- Naively, we might estimate the counterfactual risks $P(Y^{a=i} = 1)$ directly from observed A and Y:

$$\hat{P}_{MLE}(Y = 1 \,|\, A = 0) = \frac{3}{7} \quad \hat{P}_{MLE}(Y = 1 \,|\, A = 1) = \frac{7}{13}$$

- But under what circumstances $\hat{P}_{MLE}(Y \,|\, A = i) = \hat{P}_{MLE}(Y^{a=i} = 1)$?
- The following is certainly true:

$$\hat{P}_{MLE}(Y = 1 \,|\, A = i) = \frac{\text{Count}(Y = 1 \wedge A = i)}{\text{Count}(A = i)}$$

Consistency: when $A = i,\, Y = Y^{a=i}$

$$= \frac{\text{Count}(Y^{a=1} = 1 \wedge A = i)}{\text{Count}(A = i)}$$

Crucial step; make sure you understand it!

$$= \hat{P}_{MLE}(Y^{a=i} = 1 \,|\, A = i)$$

- So, the following condition suffices:

$$P(Y^{a=i} = 1 \,|\, A = i) = P(Y^{a=i} = 1)$$

- This is called **Exchangeability**:

$$Y^a \perp A \,|\, \{\}$$

Exchangeability and randomization

$\boxed{\textbf{Goal: } \hat{P}(Y^a = 1)}$

	$Y^{a=0}$	$Y^{a=1}$
Rheia	0	1
Kronos	1	0
Demeter	0	0
Hades	0	0
Hestia	0	0
Poseidon	1	0
Hera	0	0
Zeus	0	1
Artemis	1	1
Apollo	1	0
Leto	0	1
Ares	1	1
Athena	1	1
Hephaestus	0	1
Aphrodite	0	1
Polyphemus	0	1
Persephone	1	1
Hermes	1	0
Hebe	1	0
Dionysus	1	0

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- Why is a randomized experiment so powerful?

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Demeter	0	0
Hades	0	0
Hestia	0	0
Poseidon	1	0
Hera	0	0
Zeus	0	1
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Apollo	1	0
Leto	0	1
Ares	1	1
Athena	1	1
Hephaestus	0	1
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Polyphemus	0	1
Persephone	1	1
Hermes	1	0
Hebe	1	0
Dionysus	1	0

Exchangeability and randomization

Goal: $\hat{P}(Y^a = 1)$

- Why is a randomized experiment so powerful?
- Recap of exchangeability criterion:

$$Y^a \perp A \mid \{\}$$

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Kronos	1	0
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Hades	0	0
Hestia	0	0
Poseidon	1	0
Hera	0	0
Zeus	0	1
Artemis	1	1
Apollo	1	0
Leto	0	1
Ares	1	1
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Demeter	0	0
Hades	0	0
Hestia	0	0
Poseidon	1	0
Hera	0	0
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	A
Rheia	0
Kronos	0
Demeter	0
Hades	0
Hestia	1
Poseidon	1
Hera	1
Zeus	1
Artemis	0
Apollo	0
Leto	0
Ares	1
Athena	1
Hephaestus	1
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	A	Y
Rheia	0	0
Kronos	0	1
Demeter	0	0
Hades	0	0
Hestia	1	0
Poseidon	1	0
Hera	1	0
Zeus	1	1
Artemis	0	1
Apollo	0	1
Leto	0	0
Ares	1	1
Athena	1	1
Hephaestus	1	1
Aphrodite	1	1
Polyphemus	1	1
Persephone	1	1
Hermes	1	0
Hebe	1	0
Dionysus	1	0

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	$Y^{a=0}$	$Y^{a=1}$	A	Y
Rheia	0	1	0	0
Kronos	1	0	0	1
Demeter	0	0	0	0
Hades	0	0	0	0
Hestia	0	0	1	0
Poseidon	1	0	1	0
Hera	0	0	1	0
Zeus	0	1	1	1
Artemis	1	1	0	1
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Ares	1	1	1	1
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- We can now go ahead and estimate

$$\hat{P}(Y^{a=i} = 1) = \hat{P}(Y = 1 \,|\, A = i)$$

	$Y^{a=0}$	$Y^{a=1}$	A	Y
Rheia	0	1	0	0
Kronos	1	0	0	1
Demeter	0	0	0	0
Hades	0	0	0	0
Hestia	0	0	1	0
Poseidon	1	0	1	0
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Zeus	0	1	1	1
Artemis	1	1	0	1
Apollo	1	0	0	1
Leto	0	1	0	0
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$$\hat{P}(Y^{a=i} = 1) = \hat{P}(Y = 1 \,|\, A = i)$$

- Hooray!!!

	$Y^{a=0}$	$Y^{a=1}$	A	Y
Rheia	0	1	0	0
Kronos	1	0	0	1
Demeter	0	0	0	0
Hades	0	0	0	0
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	L
Rheia	0
Kronos	0
Demeter	0
Hades	0
Hestia	0
Poseidon	0
Hera	0
Zeus	0
Artemis	1
Apollo	1
Leto	1
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	L	A
Rheia	0	0
Kronos	0	0
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  - E.g., in this example, patients in critical condition are surely more likely to die overall!

	L	A	$Y^{a=0}$	$Y^{a=1}$	Y
Rheia	0	0	0	1	0
Kronos	0	0	1	0	1
Demeter	0	0	0	0	0
Hades	0	0	0	0	0
Hestia	0	1	0	0	0
Poseidon	0	1	1	0	0
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Zeus	0	1	0	1	1
Artemis	1	0	1	1	1
Apollo	1	0	1	0	1
Leto	1	0	0	1	0
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$$A \perp Y^a \mid \{\}$$

	L	A	$Y^{a=0}$	$Y^{a=1}$	Y
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Kronos	0	0	1	0	1
Demeter	0	0	0	0	0
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Poseidon	0	1	1	0	0
Hera	0	1	0	0	0
Zeus	0	1	0	1	1
Artemis	1	0	1	1	1
Apollo	1	0	1	0	1
Leto	1	0	0	1	0
Ares	1	1	1	1	1
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$$\cancel{A \perp\!\!\!\perp Y^a \mid \{\}}$$

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Demeter	0	0	0	0	0
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Persephone	1	1	1	1	1
Hermes	1	1	1	0	0
Hebe	1	1	1	0	0
Dionysus	1	1	1	0	0

Conditional exchangeability

	L	A	$Y^{a=0}$	$Y^{a=1}$	Y
Rheia	0	0	0	1	0
Kronos	0	0	1	0	1
Demeter	0	0	0	0	0
Hades	0	0	0	0	0
Hestia	0	1	0	0	0
Poseidon	0	1	1	0	0
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Zeus	0	1	0	1	1
Artemis	1	0	1	1	1
Apollo	1	0	1	0	1
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Ares	1	1	1	1	1
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Persephone	1	1	1	1	1
Hermes	1	1	1	0	0
Hebe	1	1	1	0	0
Dionysus	1	1	1	0	0

Conditional exchangeability

- But now suppose we have observed (i.e., it's in our dataset) the factor L that affected whether the treatment A was applied

	L	A	$Y^{a=0}$	$Y^{a=1}$	Y
Rheia	0	0	0	1	0
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Demeter	0	0	0	0	0
Hades	0	0	0	0	0
Hestia	0	1	0	0	0
Poseidon	0	1	1	0	0
Hera	0	1	0	0	0
Zeus	0	1	0	1	1
Artemis	1	0	1	1	1
Apollo	1	0	1	0	1
Leto	1	0	0	1	0
Ares	1	1	1	1	1
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Hebe	1	1	1	0	0
Dionysus	1	1	1	0	0

Conditional exchangeability

- But now suppose we have observed (i.e., it's in our dataset) the factor L that affected whether the treatment A was applied
- If the following condition holds, it can help us estimate the counterfactual risks $P(Y^a)$:

$$A \perp Y^a \,|\, L$$

	L	A	$Y^{a=0}$	$Y^{a=1}$	Y
Rheia	0	0	0	1	0
Kronos	0	0	1	0	1
Demeter	0	0	0	0	0
Hades	0	0	0	0	0
Hestia	0	1	0	0	0
Poseidon	0	1	1	0	0
Hera	0	1	0	0	0
Zeus	0	1	0	1	1
Artemis	1	0	1	1	1
Apollo	1	0	1	0	1
Leto	1	0	0	1	0
Ares	1	1	1	1	1
Athena	1	1	1	1	1
Hephaestus	1	1	0	1	1
Aphrodite	1	1	0	1	1
Polyphemus	1	1	0	1	1
Persephone	1	1	1	1	1
Hermes	1	1	1	0	0
Hebe	1	1	1	0	0
Dionysus	1	1	1	0	0

Conditional exchangeability

- But now suppose we have observed (i.e., it's in our dataset) the factor L that affected whether the treatment A was applied
- If the following condition holds, it can help us estimate the counterfactual risks $P(Y^a)$:

$$A \perp Y^a \mid L$$

- That is, L captures all the information available in A that is relevant to all Y^a

	L	A	$Y^{a=0}$	$Y^{a=1}$	Y
Rheia	0	0	0	1	0
Kronos	0	0	1	0	1
Demeter	0	0	0	0	0
Hades	0	0	0	0	0
Hestia	0	1	0	0	0
Poseidon	0	1	1	0	0
Hera	0	1	0	0	0
Zeus	0	1	0	1	1
Artemis	1	0	1	1	1
Apollo	1	0	1	0	1
Leto	1	0	0	1	0
Ares	1	1	1	1	1
Athena	1	1	1	1	1
Hephaestus	1	1	0	1	1
Aphrodite	1	1	0	1	1
Polyphemus	1	1	0	1	1
Persephone	1	1	1	1	1
Hermes	1	1	1	0	0
Hebe	1	1	1	0	0
Dionysus	1	1	1	0	0

Conditional exchangeability

- But now suppose we have observed (i.e., it's in our dataset) the factor L that affected whether the treatment A was applied
- If the following condition holds, it can help us estimate the counterfactual risks $P(Y^a)$:

$$A \perp Y^a \,|\, L$$

- That is, L captures all the information available in A that is relevant to all Y^a
- This is called CONDITIONAL EXCHANGEABILITY

	L	A	$Y^{a=0}$	$Y^{a=1}$	Y
Rheia	0	0	0	1	0
Kronos	0	0	1	0	1
Demeter	0	0	0	0	0
Hades	0	0	0	0	0
Hestia	0	1	0	0	0
Poseidon	0	1	1	0	0
Hera	0	1	0	0	0
Zeus	0	1	0	1	1
Artemis	1	0	1	1	1
Apollo	1	0	1	0	1
Leto	1	0	0	1	0
Ares	1	1	1	1	1
Athena	1	1	1	1	1
Hephaestus	1	1	0	1	1
Aphrodite	1	1	0	1	1
Polyphemus	1	1	0	1	1
Persephone	1	1	1	1	1
Hermes	1	1	1	0	0
Hebe	1	1	1	0	0
Dionysus	1	1	1	0	0

Using conditional exchangeability

	L	Y
Rheia	0	0
Kronos	0	1
Demeter	0	0
Hades	0	0
Hestia	0	0
Poseidon	0	0
Hera	0	0
Zeus	0	1
Artemis	1	1
Apollo	1	1
Leto	1	0
Ares	1	1
Athena	1	1
Hephaestus	1	1
Aphrodite	1	1
Polyphemus	1	1
Persephone	1	1
Hermes	1	0
Hebe	1	0
Dionysus	1	0

Using conditional exchangeability

- Can we estimate $P(Y^{a=i} = 1 \,|\, L)$?

	L	Y
Rheia	0	0
Kronos	0	1
Demeter	0	0
Hades	0	0
Hestia	0	0
Poseidon	0	0
Hera	0	0
Zeus	0	1
Artemis	1	1
Apollo	1	1
Leto	1	0
Ares	1	1
Athena	1	1
Hephaestus	1	1
Aphrodite	1	1
Polyphemus	1	1
Persephone	1	1
Hermes	1	0
Hebe	1	0
Dionysus	1	0

Using conditional exchangeability

- Can we estimate $P(Y^{a=i} = 1 \mid L)$?
- It turns out we can!

	L	Y
Rheia	0	0
Kronos	0	1
Demeter	0	0
Hades	0	0
Hestia	0	0
Poseidon	0	0
Hera	0	0
Zeus	0	1
Artemis	1	1
Apollo	1	1
Leto	1	0
Ares	1	1
Athena	1	1
Hephaestus	1	1
Aphrodite	1	1
Polyphemus	1	1
Persephone	1	1
Hermes	1	0
Hebe	1	0
Dionysus	1	0

Using conditional exchangeability

- Can we estimate $P(Y^{a=i} = 1 \mid L)$?
- It turns out we can!

$$P(Y^{a=i} = 1 \mid L) = P(Y^{a=i} = 1 \mid L, A) \quad \boxed{\text{CONDITIONAL EXCHANGEABILITY}}$$

	L	Y
Rheia	0	0
Kronos	0	1
Demeter	0	0
Hades	0	0
Hestia	0	0
Poseidon	0	0
Hera	0	0
Zeus	0	1
Artemis	1	1
Apollo	1	1
Leto	1	0
Ares	1	1
Athena	1	1
Hephaestus	1	1
Aphrodite	1	1
Polyphemus	1	1
Persephone	1	1
Hermes	1	0
Hebe	1	0
Dionysus	1	0

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	L	A	Y
Rheia	0	0	0
Kronos	0	0	1
Demeter	0	0	0
Hades	0	0	0
Hestia	0	1	0
Poseidon	0	1	0
Hera	0	1	0
Zeus	0	1	1
Artemis	1	0	1
Apollo	1	0	1
Leto	1	0	0
Ares	1	1	1
Athena	1	1	1
Hephaestus	1	1	1
Aphrodite	1	1	1
Polyphemus	1	1	1
Persephone	1	1	1
Hermes	1	1	0
Hebe	1	1	0
Dionysus	1	1	0

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$$P(Y^{a=i} = 1 \mid L) = P(Y^{a=i} = 1 \mid L, A)$$ CONDITIONAL EXCHANGEABILITY

	L	A	Y	Y^0	Y^1
Rheia	0	0	0	0	?
Kronos	0	0	1	1	?
Demeter	0	0	0	0	?
Hades	0	0	0	0	?
Hestia	0	1	0	?	0
Poseidon	0	1	0	?	0
Hera	0	1	0	?	0
Zeus	0	1	1	?	1
Artemis	1	0	1	1	?
Apollo	1	0	1	1	?
Leto	1	0	0	0	?
Ares	1	1	1	?	1
Athena	1	1	1	?	1
Hephaestus	1	1	1	?	1
Aphrodite	1	1	1	?	1
Polyphemus	1	1	1	?	1
Persephone	1	1	1	?	1
Hermes	1	1	0	?	0
Hebe	1	1	0	?	0
Dionysus	1	1	0	?	0

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CONDITIONAL EXCHANGEABILITY

$$\hat{P}_{\text{MLE}}(Y^{a=i} = 1 \mid L = j, A = k) = \frac{\text{Count}(Y^{a=i} = 1, L = j, A = k)}{\text{Count}(L = j, A = k)}$$

	L	A	Y	Y^0	Y^1
Rheia	0	0	0	0	?
Kronos	0	0	1	1	?
Demeter	0	0	0	0	?
Hades	0	0	0	0	?
Hestia	0	1	0	?	0
Poseidon	0	1	0	?	0
Hera	0	1	0	?	0
Zeus	0	1	1	?	1
Artemis	1	0	1	1	?
Apollo	1	0	1	1	?
Leto	1	0	0	0	?
Ares	1	1	1	?	1
Athena	1	1	1	?	1
Hephaestus	1	1	1	?	1
Aphrodite	1	1	1	?	1
Polyphemus	1	1	1	?	1
Persephone	1	1	1	?	1
Hermes	1	1	0	?	0
Hebe	1	1	0	?	0
Dionysus	1	1	0	?	0

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	L	A	Y	Y^0	Y^1
Rheia	0	0	0	0	?
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Hestia	0	1	0	?	0
Poseidon	0	1	0	?	0
Hera	0	1	0	?	0
Zeus	0	1	1	?	1
Artemis	1	0	1	1	?
Apollo	1	0	1	1	?
Leto	1	0	0	0	?
Ares	1	1	1	?	1
Athena	1	1	1	?	1
Hephaestus	1	1	1	?	1
Aphrodite	1	1	1	?	1
Polyphemus	1	1	1	?	1
Persephone	1	1	1	?	1
Hermes	1	1	0	?	0
Hebe	1	1	0	?	0
Dionysus	1	1	0	?	0

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$$\hat{P}_{\text{MLE}}(Y^{a=i} = 1 \mid L = j, A = k) = \frac{\text{Count}(Y^{a=i} = 1, L = j, A = k)}{\text{Count}(L = j, A = k)}$$

- In estimating this condprob:
  - when $i = k$ we use CONSISTENCY

CONSISTENCY: when $A = i$, $Y = Y^{a=i}$

	L	A	Y	Y^0	Y^1
Rheia	0	0	0	0	?
Kronos	0	0	1	1	?
Demeter	0	0	0	0	?
Hades	0	0	0	0	?
Hestia	0	1	0	?	0
Poseidon	0	1	0	?	0
Hera	0	1	0	?	0
Zeus	0	1	1	?	1
Artemis	1	0	1	1	?
Apollo	1	0	1	1	?
Leto	1	0	0	0	?
Ares	1	1	1	?	1
Athena	1	1	1	?	1
Hephaestus	1	1	1	?	1
Aphrodite	1	1	1	?	1
Polyphemus	1	1	1	?	1
Persephone	1	1	1	?	1
Hermes	1	1	0	?	0
Hebe	1	1	0	?	0
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Using conditional exchangeability

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CONDITIONAL EXCHANGEABILITY

$$\hat{P}_{\text{MLE}}(Y^{a=i} = 1 \mid L = j, A = k) = \frac{\text{Count}(Y^{a=i} = 1, L = j, A = k)}{\text{Count}(L = j, A = k)}$$

- In estimating this condprob:
  - when $i = k$ we use CONSISTENCY
  - WHEN $i \neq k$ we have "missing data", so ignore those instances

CONSISTENCY: when $A = i, Y = Y^{a=i}$

L	A	Count(L, A)

	L	A	Y	Y^0	Y^1
Rheia	0	0	0	0	?
Kronos	0	0	1	1	?
Demeter	0	0	0	0	?
Hades	0	0	0	0	?
Hestia	0	1	0	?	0
Poseidon	0	1	0	?	0
Hera	0	1	0	?	0
Zeus	0	1	1	?	1
Artemis	1	0	1	1	?
Apollo	1	0	1	1	?
Leto	1	0	0	0	?
Ares	1	1	1	?	1
Athena	1	1	1	?	1
Hephaestus	1	1	1	?	1
Aphrodite	1	1	1	?	1
Polyphemus	1	1	1	?	1
Persephone	1	1	1	?	1
Hermes	1	1	0	?	0
Hebe	1	1	0	?	0
Dionysus	1	1	0	?	0

Using conditional exchangeability

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Artemis	1	0	1	1	?
Apollo	1	0	1	1	?
Leto	1	0	0	0	?
Ares	1	1	1	?	1
Athena	1	1	1	?	1
Hephaestus	1	1	1	?	1
Aphrodite	1	1	1	?	1
Polyphemus	1	1	1	?	1
Persephone	1	1	1	?	1
Hermes	1	1	0	?	0
Hebe	1	1	0	?	0
Dionysus	1	1	0	?	0

L	A	Count(L, A)
0	0	
0	1	
1	0	
1	1	

Using conditional exchangeability

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	L	A	Y	Y^0	Y^1
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Demeter	0	0	0	0	?
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Hestia	0	1	0	?	0
Poseidon	0	1	0	?	0
Hera	0	1	0	?	0
Zeus	0	1	1	?	1
Artemis	1	0	1	1	?
Apollo	1	0	1	1	?
Leto	1	0	0	0	?
Ares	1	1	1	?	1
Athena	1	1	1	?	1
Hephaestus	1	1	1	?	1
Aphrodite	1	1	1	?	1
Polyphemus	1	1	1	?	1
Persephone	1	1	1	?	1
Hermes	1	1	0	?	0
Hebe	1	1	0	?	0
Dionysus	1	1	0	?	0

L	A	Count(L, A)
0	0	4
0	1	4
1	0	3
1	1	9

Using conditional exchangeability

- Can we estimate $P(Y^{a=i} = 1 \mid L)$?
- It turns out we can!

$$P(Y^{a=i} = 1 \mid L) = P(Y^{a=i} = 1 \mid L, A)$$

CONDITIONAL EXCHANGEABILITY

$$\hat{P}_{\text{MLE}}(Y^{a=i} = 1 \mid L = j, A = k) = \frac{\text{Count}(Y^{a=i} = 1, L = j, A = k)}{\text{Count}(L = j, A = k)}$$

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Zeus	0	1	1	?	1
Artemis	1	0	1	1	?
Apollo	1	0	1	1	?
Leto	1	0	0	0	?
Ares	1	1	1	?	1
Athena	1	1	1	?	1
Hephaestus	1	1	1	?	1
Aphrodite	1	1	1	?	1
Polyphemus	1	1	1	?	1
Persephone	1	1	1	?	1
Hermes	1	1	0	?	0
Hebe	1	1	0	?	0
Dionysus	1	1	0	?	0

L	A	Count(L, A)	Count$(Y^{a=0} = 1, L, A)$	Count$(Y^{a=1} = 1, L, A)$
0	0	4		
0	1	4		
1	0	3		
1	1	9		

Using conditional exchangeability

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$$P(Y^{a=i} = 1 \mid L) = P(Y^{a=i} = 1 \mid L, A)$$ CONDITIONAL EXCHANGEABILITY

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Hades	0	0	0	0	?
Hestia	0	1	0	?	0
Poseidon	0	1	0	?	0
Hera	0	1	0	?	0
Zeus	0	1	1	?	1
Artemis	1	0	1	1	?
Apollo	1	0	1	1	?
Leto	1	0	0	0	?
Ares	1	1	1	?	1
Athena	1	1	1	?	1
Hephaestus	1	1	1	?	1
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Polyphemus	1	1	1	?	1
Persephone	1	1	1	?	1
Hermes	1	1	0	?	0
Hebe	1	1	0	?	0
Dionysus	1	1	0	?	0

L	A	$\text{Count}(L, A)$	$\text{Count}(Y^{a=0} = 1, L, A)$	$\text{Count}(Y^{a=1} = 1, L, A)$
0	0	4	1	?
0	1	4		
1	0	3		
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L	A	Count(L, A)	Count$(Y^{a=0} = 1, L, A)$	Count$(Y^{a=1} = 1, L, A)$
0	0	4	1	?
0	1	4	?	1
1	0	3		
1	1	9		

	L	A	Y	Y^0	Y^1
Rheia	0	0	0	0	?
Kronos	0	0	1	1	?
Demeter	0	0	0	0	?
Hades	0	0	0	0	?
Hestia	0	1	0	?	0
Poseidon	0	1	0	?	0
Hera	0	1	0	?	0
Zeus	0	1	1	?	1
Artemis	1	0	1	1	?
Apollo	1	0	1	1	?
Leto	1	0	0	0	?
Ares	1	1	1	?	1
Athena	1	1	1	?	1
Hephaestus	1	1	1	?	1
Aphrodite	1	1	1	?	1
Polyphemus	1	1	1	?	1
Persephone	1	1	1	?	1
Hermes	1	1	0	?	0
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	L	A	Y	Y^0	Y^1
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Hades	0	0	0	0	?
Hestia	0	1	0	?	0
Poseidon	0	1	0	?	0
Hera	0	1	0	?	0
Zeus	0	1	1	?	1
Artemis	1	0	1	1	?
Apollo	1	0	1	1	?
Leto	1	0	0	0	?
Ares	1	1	1	?	1
Athena	1	1	1	?	1
Hephaestus	1	1	1	?	1
Aphrodite	1	1	1	?	1
Polyphemus	1	1	1	?	1
Persephone	1	1	1	?	1
Hermes	1	1	0	?	0
Hebe	1	1	0	?	0
Dionysus	1	1	0	?	0

L	A	Count(L, A)	Count$(Y^{a=0} = 1, L, A)$	Count$(Y^{a=1} = 1, L, A)$
0	0	4	1	?
0	1	4	?	1
1	0	3	2	?
1	1	9		

Using conditional exchangeability

- Can we estimate $P(Y^{a=i} = 1 \mid L)$?
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Hera	0	1	0	?	0
Zeus	0	1	1	?	1
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This is just like estimating $P(Y \mid L, A)$!

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- But there are situations where the basic counterfactual risk $E[Y^{a=i}]$ may be of interest
  - e.g., "how many lives would it save if everyone who came to the hospital with heart disease received a heart transplant?
- But we can recover the basic counterfactual risks through **standardization** (or the mathematically equivalent **inverse probability weighting**)

Standardization

L = whether the patient was in critical condition (1=yes, 0=no)

L	A	$\hat{P}_{\text{MLE}}(Y^{a=i} \mid L)$
0	0	1/4
0	1	1/4
1	0	2/3
1	1	2/3

	L	A	Y	Y^0	Y^1
Rheia	0	0	0	0	?
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$$P(Y^{a=i}) = \sum_j P(Y^{a=i} \mid L)\, P(L)$$

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$$= \frac{1}{4} \times \frac{2}{5} + \frac{2}{3} \times \frac{3}{5}$$

$$= \frac{1}{10} + \frac{2}{5}$$

Standardization

L = whether the patient was in critical condition (1=yes, 0=no)

L	A	$\hat{P}_{\text{MLE}}(Y^{a=i} \mid L)$
0	0	1/4
0	1	1/4
1	0	2/3
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- By the law of total probability,

We have just estimated this

We can estimate this from the data, too

$$P(Y^{a=i}) = \sum_j P(Y^{a=i} \mid L) P(L)$$

- Expanding the sum and plugging in our estimates we get:

$$\begin{aligned} P(Y^{a=i} = 1) &= P(Y^{a=i} = 1 \mid L = 0) P(L = 0) + P(Y^{a=i} = 1 \mid L = 1) P(L = 1) \\ &= \frac{1}{4} \times \frac{2}{5} + \frac{2}{3} \times \frac{3}{5} \\ &= \frac{1}{10} + \frac{2}{5} \\ &= \frac{5}{10} = \frac{1}{2} \end{aligned}$$

	L	A	Y	Y^0	Y^1
Rheia	0	0	0	0	?
Kronos	0	0	1	1	?
Demeter	0	0	0	0	?
Hades	0	0	0	0	?
Hestia	0	1	0	?	0
Poseidon	0	1	0	?	0
Hera	0	1	0	?	0
Zeus	0	1	1	?	1
Artemis	1	0	1	1	?
Apollo	1	0	1	1	?
Leto	1	0	0	0	?
Ares	1	1	1	?	1
Athena	1	1	1	?	1
Hephaestus	1	1	1	?	1
Aphrodite	1	1	1	?	1
Polyphemus	1	1	1	?	1
Persephone	1	1	1	?	1
Hermes	1	1	0	?	0
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$$\hat{P}_{MLE}(L = 1) = \frac{12}{20} = \frac{3}{5}$$

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(Because $\hat{P}_{\text{MLE}}(Y^{a=i} \mid L)$ are the same for $a=0$ and $a=1$, this work gives us the result for both counterfactual treatments, and the risk ratio is 1)

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Not an issue of sample size—no amount of data would help!

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- If all three criteria hold, we can estimate causal effects

Summary of intro to potential outcomes

- The potential outcomes framework formalizes causal effects (or risks) through **counterfactual outcome** (also called **potential outcome**) **variables**
- At most one counterfactual outcome is observable in each datum, so causal effects cannot in general be naively estimated from data

	L	A	Y	Y^0	Y^1
Rheia	0	0	0	0	?
Kronos	0	0	1	1	?
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- However, if the three following conditions hold, the data can be viewed as a **conditionally randomized experiment** and causal effects can be estimated

Consistency	**Conditional Exchangeability**	**Positivity**
$Y = Y^{a=i}$ whenever $A = i$	$\exists Z . \forall i . Z$ is observed $\wedge\, Y^{a=i} \perp A \mid Z$	$\forall i . \forall Z . P(A = i \mid Z) > 0$

- This analysis also sheds light on the power of randomized experiments: they offer **unconditional exchangeability**