# 9.S918: Statistical Inference in Brain and Cognitive Sciences

## Week 1 Day 2: Introduction to causal inference


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# A tiny bit of statistics

- On Monday we reviewed basics of **probability**: the logical calculus of uncertainty–a branch of mathematics
- The primary focus of this class is **statistics**: the mathematics, science, craft, and art of drawing inferences from data
- The two fields are fundamentally different
- But, probability is used extensively throughout statistics

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- Figuring out from observed data what the weighting is likely to be is **parameter estimation**
- In general, we will use $\mathbf{y}$ to refer to observed-outcome **data** and $\theta$ to refer to the model parameters to be estimated

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- Good estimators have favorable **bias–variance** tradeoff

# Maximum likelihood estimation

$$\mathrm{Lik}(\boldsymbol{\theta}; \boldsymbol{y}) \equiv P(\boldsymbol{y}|\boldsymbol{\theta}) \qquad \hat{\boldsymbol{\theta}}_{MLE} \stackrel{\text{def}}{=} \arg\max_{\boldsymbol{\theta}} \mathrm{Lik}(\boldsymbol{\theta}; \boldsymbol{y})$$

| ***i*** | $\boldsymbol{y_i}$ |
|---|---|
| 1 | T |
| 2 | T |
| 3 | H |
| 4 | T |

*(repeat slide from lecture 3)*

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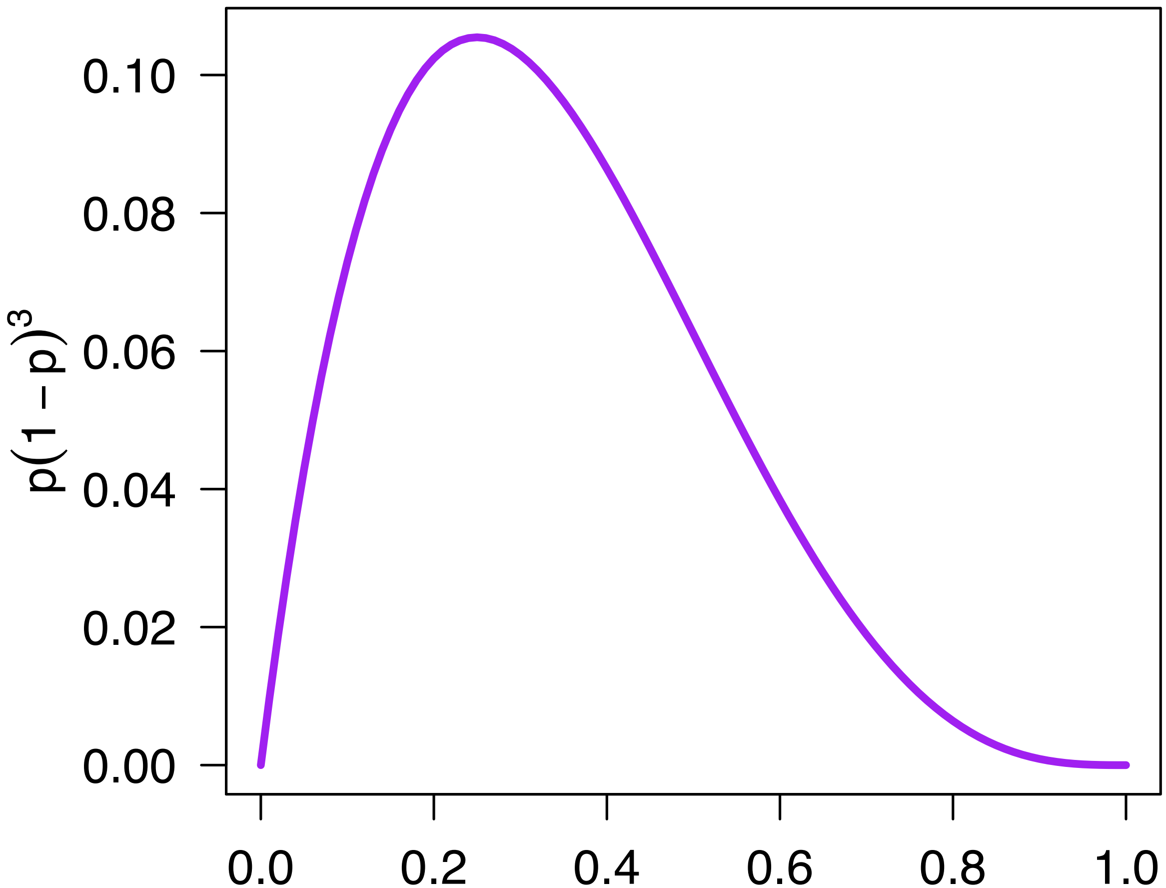


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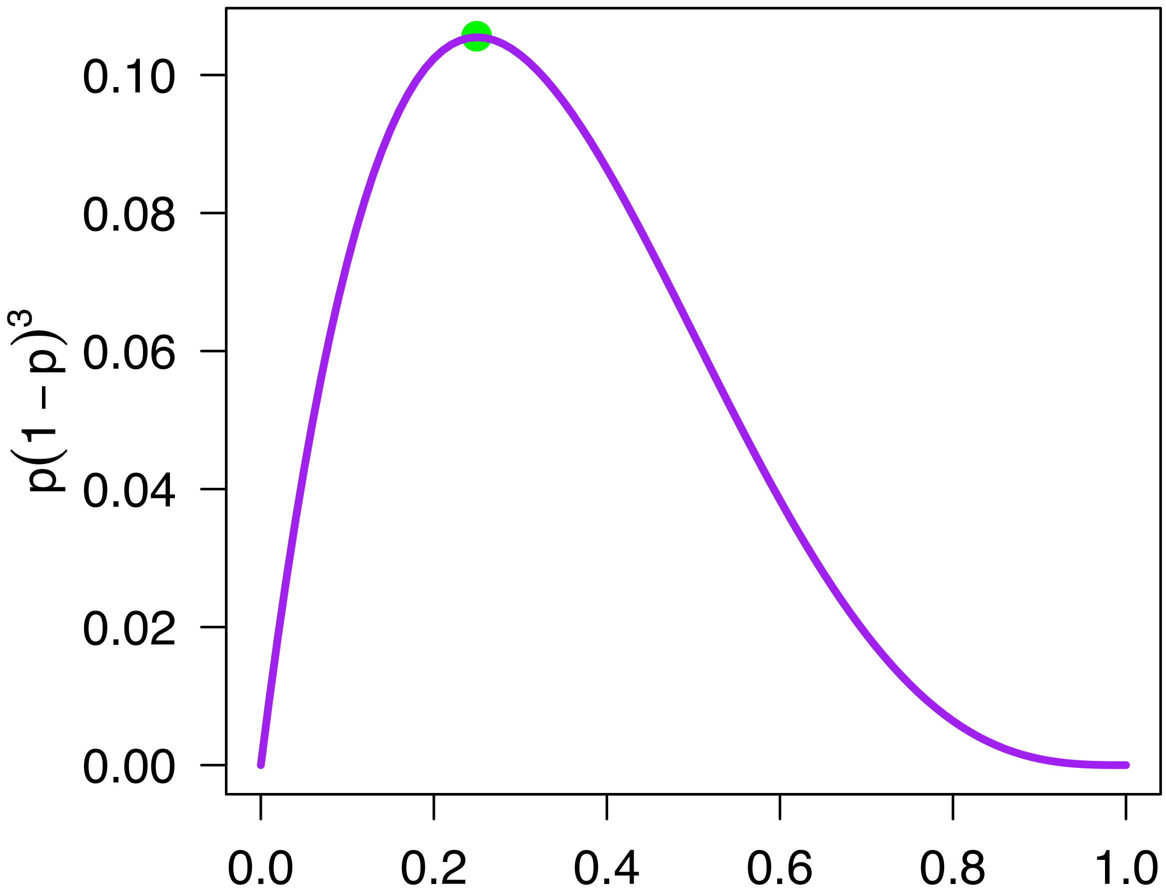


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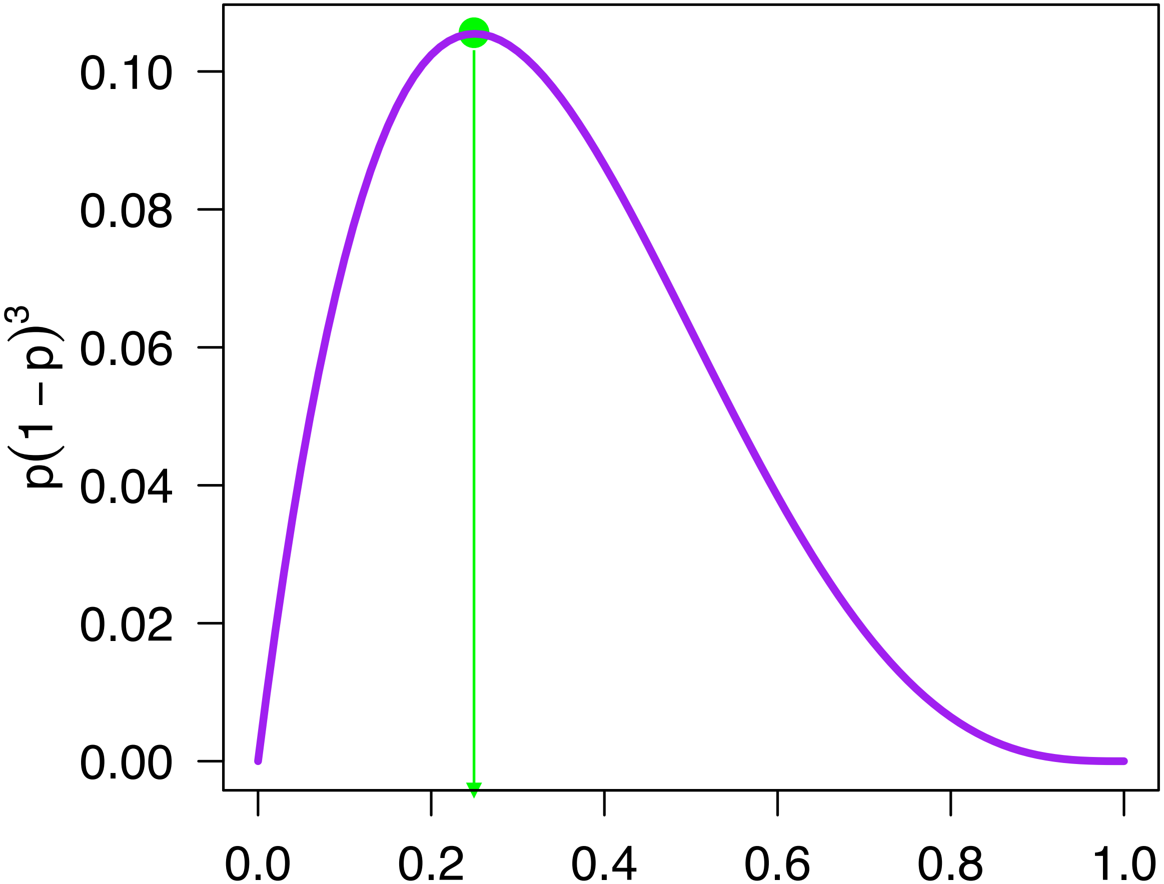


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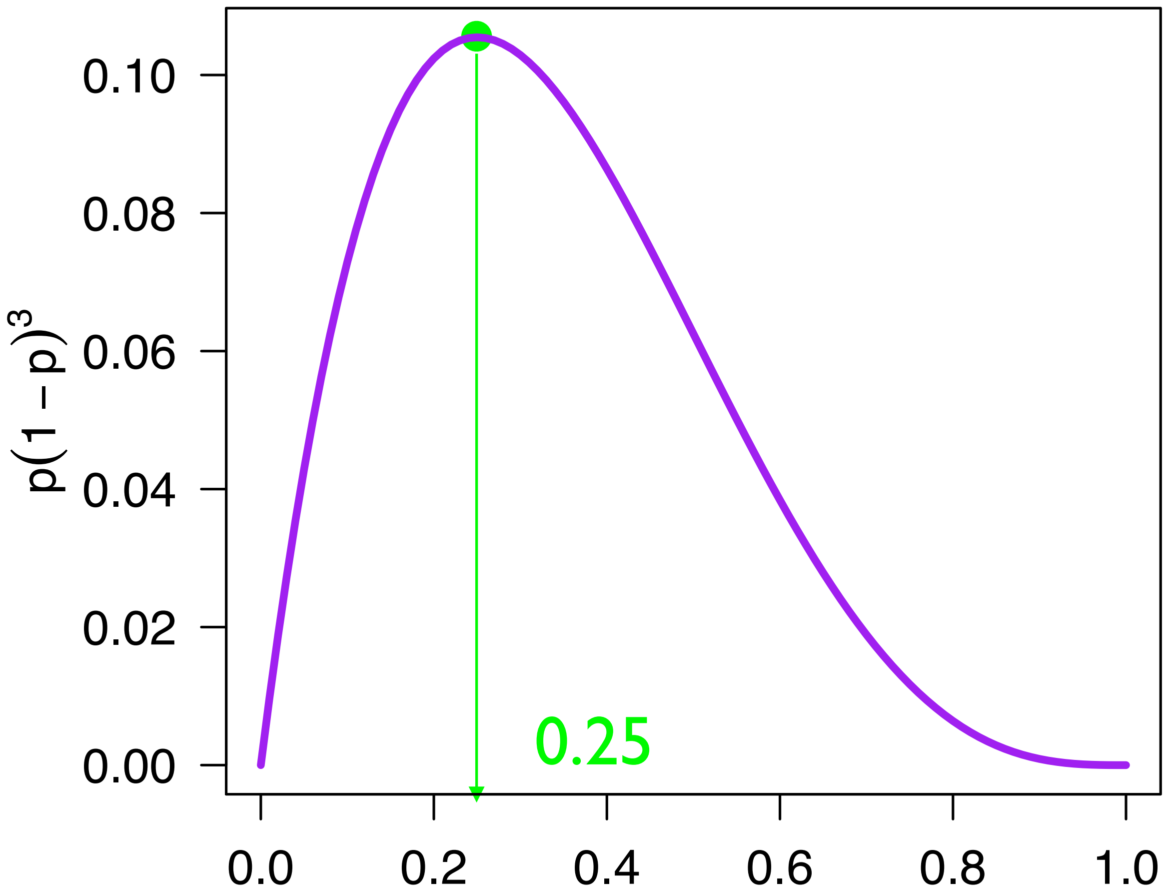


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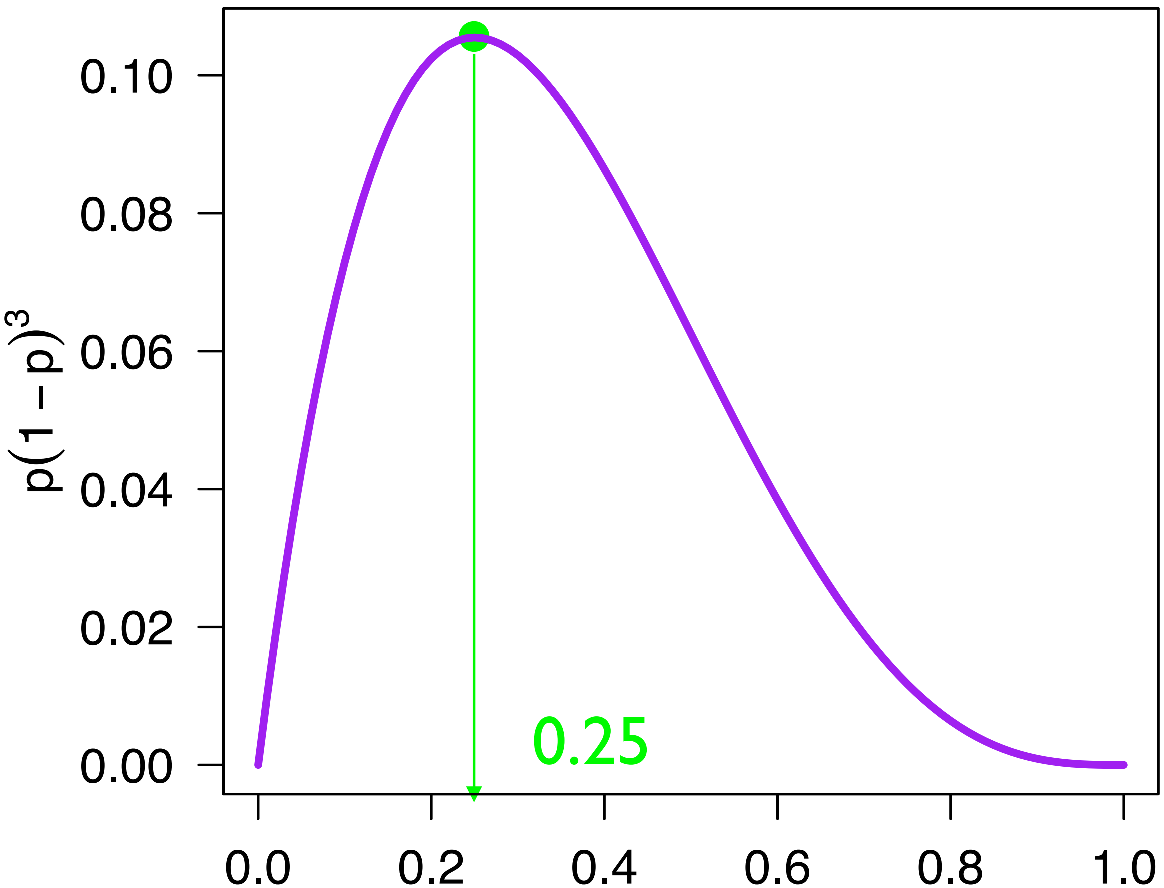


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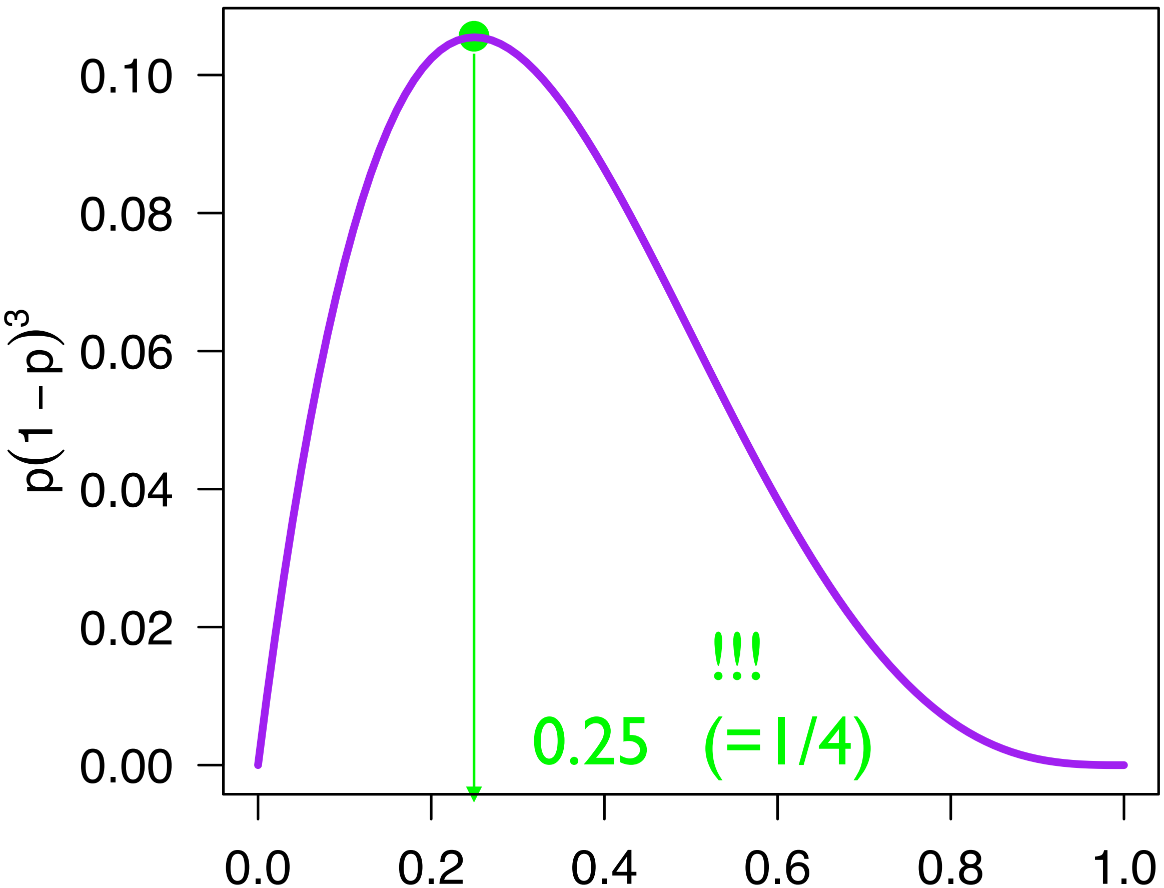


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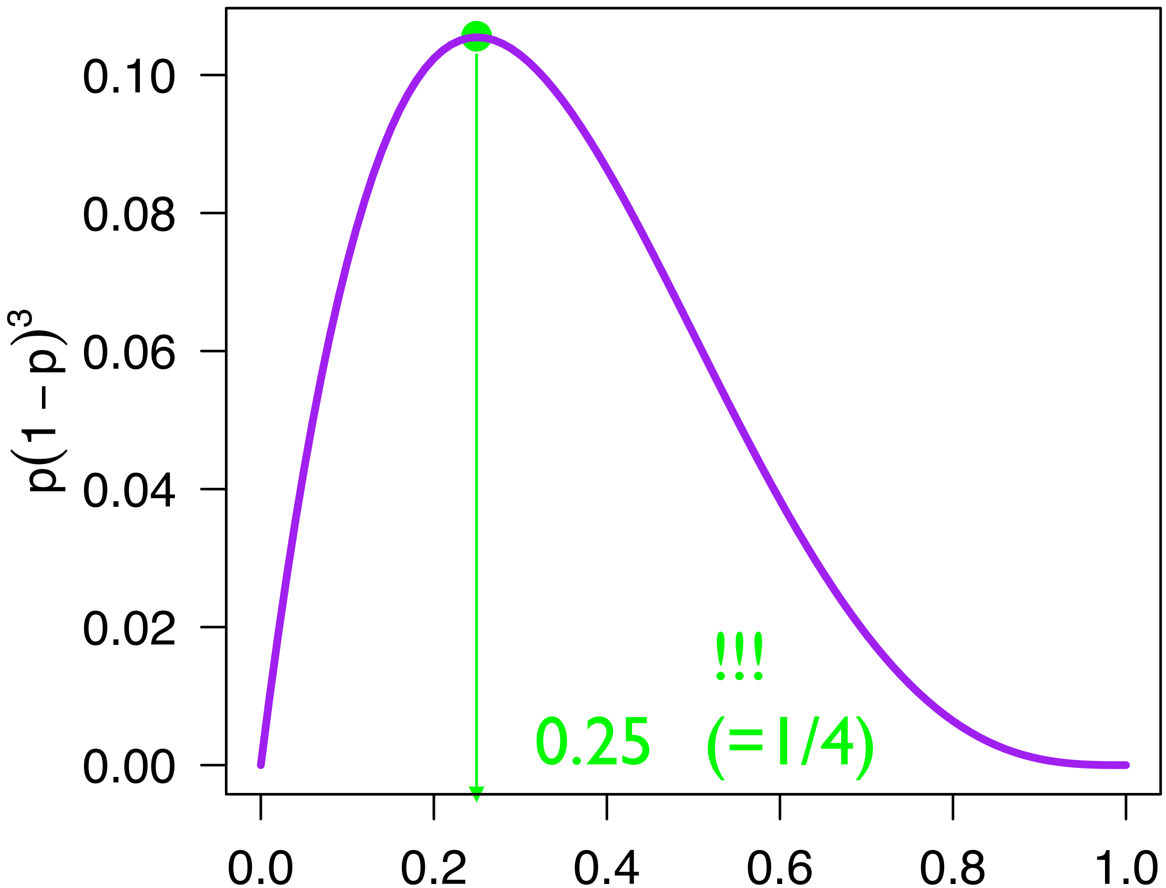


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The **MLE** also turns out to be the *relative frequency estimate* (RFE)

*(repeat slide from lecture 3)*

# Introductory causal inference

- You have probably had previous exposure to both probability and statistics
- You are less likely to have had exposure to **causal inference**
- Causal inference uses probability and statistics, but it is something separate from the traditional construal of those two fields
- You can think of causal inference as being a framework extending more traditional statistics by:
  - Adding new probability-based mathematical constructs; and,
  - Developing a set of practice for statistical inference based on those constructs
- Two causal inference frameworks:
  - The **potential outcomes** framework
  - The **causal graphical models** framework

# The potential-outcomes framework

- In epidemiology and many other areas of statistics, causal inference was developed out of the idea of **potential outcomes** (Neyman 1923, Rubin 1974)
- Consider an outcome, $Y$, and a potential **treatment** $A$
- **Example:**

$Y$: an individual survives to the end of the year (0: no, 1: yes)

$A$: an individual with heart disease receives a heart transplant (0: no, 1: yes)

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- So we are interested in (and likewise for $Y^{a=1}$):

$$E[Y^{a=0}] = \sum_y yP(Y^{a=0} = y) = 0 \times P(Y^{a=0} = 0) + 1 \times P(Y^{a=0} = 1) = P(Y^{a=0} = 1)$$

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# Counterfactual data and causal effects

(Hernan & Robins, 2020, Table 1.1)

| | $Y^{a=0}$ | $Y^{a=1}$ |
|---|---|---|
| Rheia | 0 | 1 |
| Kronos | 1 | 0 |
| Demeter | 0 | 0 |
| Hades | 0 | 0 |
| Hestia | 0 | 0 |
| Poseidon | 1 | 0 |
| Hera | 0 | 0 |
| Zeus | 0 | 1 |
| Artemis | 1 | 1 |
| Apollo | 1 | 0 |
| Leto | 0 | 1 |
| Ares | 1 | 1 |
| Athena | 1 | 1 |
| Hephaestus | 0 | 1 |
| Aphrodite | 0 | 1 |
| Cyclope | 0 | 1 |
| Persephone | 1 | 1 |
| Hermes | 1 | 0 |
| Hebe | 1 | 0 |
| Dionysus | 1 | 0 |
| $P(Y^{a=*}) = 1$ | 0.5 | 0.5 |

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- Suppose we knew **what would happen** for each individual in the population under each value of the treatment

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- The **average causal effect** of treatment $A$ is defined as the difference of counterfactual risks:

$$E[Y^{a=1}] - E[Y^{a=0}] = 0$$

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- The **average causal effect** of treatment $A$ is defined as the difference of counterfactual risks:

$$E[Y^{a=1}] - E[Y^{a=0}] = 0$$

- Here, treatment is **ineffective**

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# Estimating causal effects

Remember, $E[Y^{a=i}] = P(Y^{a=i} = 1)$

| | $L$ | $A$ | $Y$ | $Y^0$ | $Y^1$ |
|---|---|---|---|---|---|
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| Hera | 0 | 1 | 0 | ? | 0 |
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| Artemis | 1 | 0 | 1 | 1 | ? |
| Apollo | 1 | 0 | 1 | 1 | ? |
| Leto | 1 | 0 | 0 | 0 | ? |
| Ares | 1 | 1 | 1 | ? | 1 |
| Athena | 1 | 1 | 1 | ? | 1 |
| Hephaestus | 1 | 1 | 1 | ? | 1 |
| Aphrodite | 1 | 1 | 1 | ? | 1 |
| Polyphemus | 1 | 1 | 1 | ? | 1 |
| Persephone | 1 | 1 | 1 | ? | 1 |
| Hermes | 1 | 1 | 0 | ? | 0 |
| Hebe | 1 | 1 | 0 | ? | 0 |
| Dionysus | 1 | 1 | 0 | ? | 0 |

# Estimating causal effects

Remember, $E[Y^{a=i}] = P(Y^{a=i} = 1)$

| | $L$ | $A$ | $Y$ | $Y^0$ | $Y^1$ |
|---|---|---|---|---|---|
| Rheia | 0 | 0 | 0 | 0 | ? |
| Kronos | 0 | 0 | 1 | 1 | ? |
| Demeter | 0 | 0 | 0 | 0 | ? |
| Hades | 0 | 0 | 0 | 0 | ? |
| Hestia | 0 | 1 | 0 | ? | 0 |
| Poseidon | 0 | 1 | 0 | ? | 0 |
| Hera | 0 | 1 | 0 | ? | 0 |
| Zeus | 0 | 1 | 1 | ? | 1 |
| Artemis | 1 | 0 | 1 | 1 | ? |
| Apollo | 1 | 0 | 1 | 1 | ? |
| Leto | 1 | 0 | 0 | 0 | ? |
| Ares | 1 | 1 | 1 | ? | 1 |
| Athena | 1 | 1 | 1 | ? | 1 |
| Hephaestus | 1 | 1 | 1 | ? | 1 |
| Aphrodite | 1 | 1 | 1 | ? | 1 |
| Polyphemus | 1 | 1 | 1 | ? | 1 |
| Persephone | 1 | 1 | 1 | ? | 1 |
| Hermes | 1 | 1 | 0 | ? | 0 |
| Hebe | 1 | 1 | 0 | ? | 0 |
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|---|---|---|---|---|---|
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| Kronos | 0 | 0 | 1 | 1 | ? |
| Demeter | 0 | 0 | 0 | 0 | ? |
| Hades | 0 | 0 | 0 | 0 | ? |
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| Poseidon | 0 | 1 | 0 | ? | 0 |
| Hera | 0 | 1 | 0 | ? | 0 |
| Zeus | 0 | 1 | 1 | ? | 1 |
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# Estimating causal effects

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| | $L$ | $A$ | $Y$ | $Y^0$ | $Y^1$ |
|---|---|---|---|---|---|
| Rheia | 0 | 0 | 0 | 0 | ? |
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| Demeter | 0 | 0 | 0 | 0 | ? |
| Hades | 0 | 0 | 0 | 0 | ? |
| Hestia | 0 | 1 | 0 | ? | 0 |
| Poseidon | 0 | 1 | 0 | ? | 0 |
| Hera | 0 | 1 | 0 | ? | 0 |
| Zeus | 0 | 1 | 1 | ? | 1 |
| Artemis | 1 | 0 | 1 | 1 | ? |
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| Hermes | 1 | 1 | 0 | ? | 0 |
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| Hestia | 0 | 1 | 0 | ? | 0 |
| Poseidon | 0 | 1 | 0 | ? | 0 |
| Hera | 0 | 1 | 0 | ? | 0 |
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|---|---|---|---|---|---|
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| Hades | 0 | 0 | 0 | 0 | ? |
| Hestia | 0 | 1 | 0 | ? | 0 |
| Poseidon | 0 | 1 | 0 | ? | 0 |
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|---|---|---|---|---|---|
| Rheia | 0 | 0 | 0 | 0 | ? |
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| | $L$ | $A$ | $Y$ | $Y^0$ | $Y^1$ |
|---|---|---|---|---|---|
| Rheia | 0 | 0 | 0 | 0 | ? |
| Kronos | 0 | 0 | 1 | 1 | ? |
| Demeter | 0 | 0 | 0 | 0 | ? |
| Hades | 0 | 0 | 0 | 0 | ? |
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| Poseidon | 0 | 1 | 0 | ? | 0 |
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|---|---|---|---|---|---|
| Rheia | 0 | 0 | 0 | 0 | ? |
| Kronos | 0 | 0 | 1 | 1 | ? |
| Demeter | 0 | 0 | 0 | 0 | ? |
| Hades | 0 | 0 | 0 | 0 | ? |
| Hestia | 0 | 1 | 0 | ? | 0 |
| Poseidon | 0 | 1 | 0 | ? | 0 |
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# Estimating causal effects

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| | $L$ | $A$ | $Y$ | $Y^0$ | $Y^1$ |
|---|---|---|---|---|---|
| Rheia | 0 | 0 | 0 | 0 | ? |
| Kronos | 0 | 0 | 1 | 1 | ? |
| Demeter | 0 | 0 | 0 | 0 | ? |
| Hades | 0 | 0 | 0 | 0 | ? |
| Hestia | 0 | 1 | 0 | ? | 0 |
| Poseidon | 0 | 1 | 0 | ? | 0 |
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- So, the following condition suffices:

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# Estimating causal effects

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| | $L$ | $A$ | $Y$ | $Y^0$ | $Y^1$ |
|---|---|---|---|---|---|
| Rheia | 0 | 0 | 0 | 0 | ? |
| Kronos | 0 | 0 | 1 | 1 | ? |
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| Hades | 0 | 0 | 0 | 0 | ? |
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| Ares | 1 | 1 | 1 | ? | 1 |
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| Hephaestus | 1 | 1 | 1 | ? | 1 |
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| Polyphemus | 1 | 1 | 1 | ? | 1 |
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| Hermes | 1 | 1 | 0 | ? | 0 |
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- So, the following condition suffices:

$$P(Y^{a=i} = 1 \,|\, A = i) = P(Y^{a=i} = 1)$$

- This is called **Exchangeability**:

$$Y^a \perp A \,|\, \{\}$$

# Exchangeability and randomization

**Goal:** $\hat{P}(Y^a = 1)$

| | $Y^{a=0}$ | $Y^{a=1}$ |
|---|---|---|
| Rheia | 0 | 1 |
| Kronos | 1 | 0 |
| Demeter | 0 | 0 |
| Hades | 0 | 0 |
| Hestia | 0 | 0 |
| Poseidon | 1 | 0 |
| Hera | 0 | 0 |
| Zeus | 0 | 1 |
| Artemis | 1 | 1 |
| Apollo | 1 | 0 |
| Leto | 0 | 1 |
| Ares | 1 | 1 |
| Athena | 1 | 1 |
| Hephaestus | 0 | 1 |
| Aphrodite | 0 | 1 |
| Polyphemus | 0 | 1 |
| Persephone | 1 | 1 |
| Hermes | 1 | 0 |
| Hebe | 1 | 0 |
| Dionysus | 1 | 0 |

# Exchangeability and randomization

**Goal:** $\hat{P}(Y^a = 1)$

- Why is a randomized experiment so powerful?

| | $Y^{a=0}$ | $Y^{a=1}$ |
|---|---|---|
| Rheia | 0 | 1 |
| Kronos | 1 | 0 |
| Demeter | 0 | 0 |
| Hades | 0 | 0 |
| Hestia | 0 | 0 |
| Poseidon | 1 | 0 |
| Hera | 0 | 0 |
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# Exchangeability and randomization

**Goal:** $\hat{P}(Y^a = 1)$

- Why is a randomized experiment so powerful?
- Recap of exchangeability criterion:

$$Y^a \perp A \mid \{\}$$

| | $Y^{a=0}$ | $Y^{a=1}$ |
|---|---|---|
| Rheia | 0 | 1 |
| Kronos | 1 | 0 |
| Demeter | 0 | 0 |
| Hades | 0 | 0 |
| Hestia | 0 | 0 |
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# Exchangeability and randomization

**Goal:** $\hat{P}(Y^a = 1)$

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- If we ourselves determine $A$ in a way that is *truly blind to* $Y^a$, it **imposes** exchangeability!

| | $Y^{a=0}$ | $Y^{a=1}$ |
|---|---|---|
| Rheia | 0 | 1 |
| Kronos | 1 | 0 |
| Demeter | 0 | 0 |
| Hades | 0 | 0 |
| Hestia | 0 | 0 |
| Poseidon | 1 | 0 |
| Hera | 0 | 0 |
| Zeus | 0 | 1 |
| Artemis | 1 | 1 |
| Apollo | 1 | 0 |
| Leto | 0 | 1 |
| Ares | 1 | 1 |
| Athena | 1 | 1 |
| Hephaestus | 0 | 1 |
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| |
|---|
| Rheia |
| Kronos |
| Demeter |
| Hades |
| Hestia |
| Poseidon |
| Hera |
| Zeus |
| Artemis |
| Apollo |
| Leto |
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| | $A$ |
|---|---|
| Rheia | 0 |
| Kronos | 0 |
| Demeter | 0 |
| Hades | 0 |
| Hestia | 1 |
| Poseidon | 1 |
| Hera | 1 |
| Zeus | 1 |
| Artemis | 0 |
| Apollo | 0 |
| Leto | 0 |
| Ares | 1 |
| Athena | 1 |
| Hephaestus | 1 |
| Aphrodite | 1 |
| Polyphemus | 1 |
| Persephone | 1 |
| Hermes | 1 |
| Hebe | 1 |
| Dionysus | 1 |

# Exchangeability and randomization

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| | $A$ | $Y$ |
|---|---|---|
| Rheia | 0 | 0 |
| Kronos | 0 | 1 |
| Demeter | 0 | 0 |
| Hades | 0 | 0 |
| Hestia | 1 | 0 |
| Poseidon | 1 | 0 |
| Hera | 1 | 0 |
| Zeus | 1 | 1 |
| Artemis | 0 | 1 |
| Apollo | 0 | 1 |
| Leto | 0 | 0 |
| Ares | 1 | 1 |
| Athena | 1 | 1 |
| Hephaestus | 1 | 1 |
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| Persephone | 1 | 1 |
| Hermes | 1 | 0 |
| Hebe | 1 | 0 |
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# Exchangeability and randomization

**Goal: $\hat{P}(Y^a = 1)$**

- Why is a randomized experiment so powerful?
- Recap of exchangeability criterion:

$$Y^a \perp A \,|\, \{\}$$

- If we ourselves determine $A$ in a way that is *truly blind to* $Y^a$, it **imposes** exchangeability!

| | $Y^{a=0}$ | $Y^{a=1}$ | $A$ | $Y$ |
|---|---|---|---|---|
| Rheia | 0 | 1 | 0 | 0 |
| Kronos | 1 | 0 | 0 | 1 |
| Demeter | 0 | 0 | 0 | 0 |
| Hades | 0 | 0 | 0 | 0 |
| Hestia | 0 | 0 | 1 | 0 |
| Poseidon | 1 | 0 | 1 | 0 |
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| Apollo | 1 | 0 | 0 | 1 |
| Leto | 0 | 1 | 0 | 0 |
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- We can now go ahead and estimate

$$\hat{P}(Y^{a=i} = 1) = \hat{P}(Y = 1 \,|\, A = i)$$

| | $Y^{a=0}$ | $Y^{a=1}$ | $A$ | $Y$ |
|---|---|---|---|---|
| Rheia | 0 | 1 | 0 | 0 |
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- Hooray!!!

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| Hestia | 0 |
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| Hera | 0 |
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  - E.g., in this example, patients in critical condition are surely more likely to die overall!

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|---|---|---|---|---|---|
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| Ares | 1 | 1 | 1 | 1 | 1 |
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# Conditional exchangeability

| | $L$ | $A$ | $Y^{a=0}$ | $Y^{a=1}$ | $Y$ |
|---|---|---|---|---|---|
| Rheia | 0 | 0 | 0 | 1 | 0 |
| Kronos | 0 | 0 | 1 | 0 | 1 |
| Demeter | 0 | 0 | 0 | 0 | 0 |
| Hades | 0 | 0 | 0 | 0 | 0 |
| Hestia | 0 | 1 | 0 | 0 | 0 |
| Poseidon | 0 | 1 | 1 | 0 | 0 |
| Hera | 0 | 1 | 0 | 0 | 0 |
| Zeus | 0 | 1 | 0 | 1 | 1 |
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| Hermes | 1 | 1 | 1 | 0 | 0 |
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- But now suppose we have observed (i.e., it's in our dataset) the factor $L$ that affected whether the treatment $A$ was applied

| | $L$ | $A$ | $Y^{a=0}$ | $Y^{a=1}$ | $Y$ |
|---|---|---|---|---|---|
| Rheia | 0 | 0 | 0 | 1 | 0 |
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- But now suppose we have observed (i.e., it's in our dataset) the factor $L$ that affected whether the treatment $A$ was applied
- If the following condition holds, it can help us estimate the counterfactual risks $P(Y^a)$:

$$A \perp Y^a \,|\, L$$

| | $L$ | $A$ | $Y^{a=0}$ | $Y^{a=1}$ | $Y$ |
|---|---|---|---|---|---|
| Rheia | 0 | 0 | 0 | 1 | 0 |
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$$A \perp Y^a \mid L$$

- That is, $L$ captures all the information available in $A$ that is relevant to all $Y^a$

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- That is, $L$ captures all the information available in $A$ that is relevant to all $Y^a$
- This is called CONDITIONAL EXCHANGEABILITY

| | $L$ | $A$ | $Y^{a=0}$ | $Y^{a=1}$ | $Y$ |
|---|---|---|---|---|---|
| Rheia | 0 | 0 | 0 | 1 | 0 |
| Kronos | 0 | 0 | 1 | 0 | 1 |
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| Hephaestus | 1 | 1 | 0 | 1 | 1 |
| Aphrodite | 1 | 1 | 0 | 1 | 1 |
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| Persephone | 1 | 1 | 1 | 1 | 1 |
| Hermes | 1 | 1 | 1 | 0 | 0 |
| Hebe | 1 | 1 | 1 | 0 | 0 |
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# Conditional exchangeability

| | $L$ | $A$ | $Y$ |
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- Suppose we observe some other potentially relevant factor in our data, such as $L =$ whether the patient was in critical condition (1=yes, 0=no)
- CONDITIONAL EXCHANGEABILITY holds iff the counterfactual outcomes are conditionally independent of treatment, GIVEN this other relevant factor $L$:

$$A \perp Y^a \mid L$$

- Conditional exchangeability intuitively seems to give us an inroad into estimating the average causal effect of $A$ on $Y$

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***Not an issue of sample size—no amount of data would help!***

# The three criteria for identifiability

- **Consistency**: $Y = Y^{a=i}$ whenever $A = i$
  - Consequence: different individuals' outcomes don't affect each other
  - Consequence: there can be no "multiple versions" of the same treatment $A$ in terms of their influence on $Y$
- **Exchangeability**: for all $i$, $Y^{a=i} \perp A \,|\, Z$ for some set of observed variables $Z$
  - Consequence: there can be no "hidden common causes" or "hidden mediators" of $A$ and $Y^{a=i}$
- **Positivity**: for all $i$ and all values of $Z$, $P(A = i \,|\, Z) > 0$
  - e.g., in our example, it can't be the case that individuals with heart disease are *always* given transplants

# What to do with conditional exchangeability

- One approach is to

| | $L$ | $A$ | $Y$ |
|---|---|---|---|
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# Why causal inference is challenging

(Hernan & Robins, 2020, Table 1.1)

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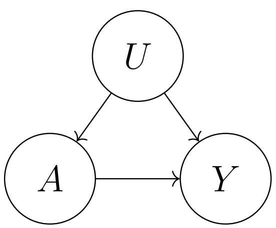


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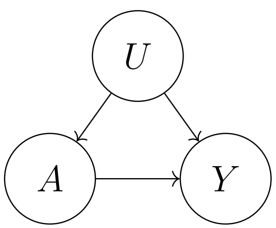


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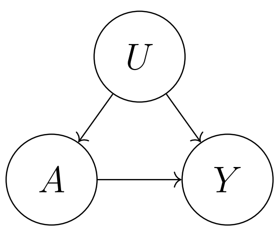


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- This is why randomized experiments are so powerful!

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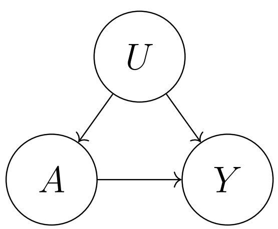


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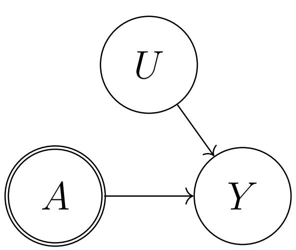


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- where $\mathrm{pa}_i$ is the subset of $V$ that are parents of $X_i$

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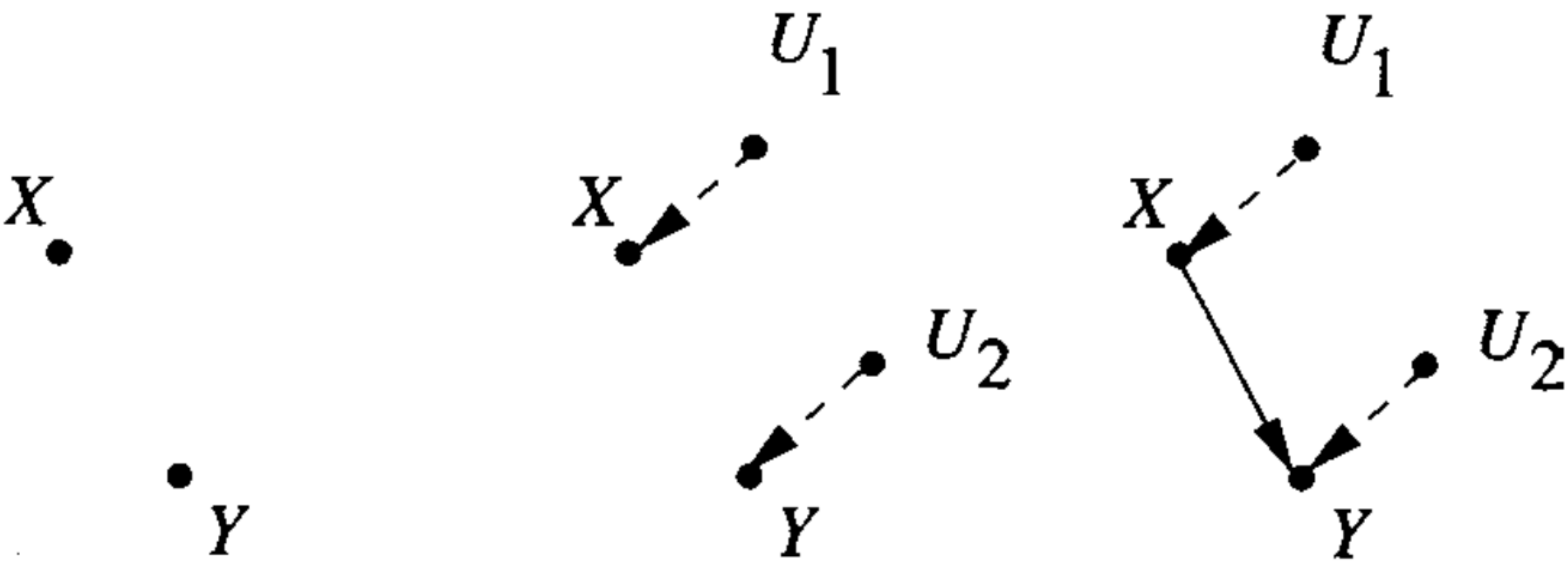
# Bayes Nets & functional causal models

**Bayes Net**

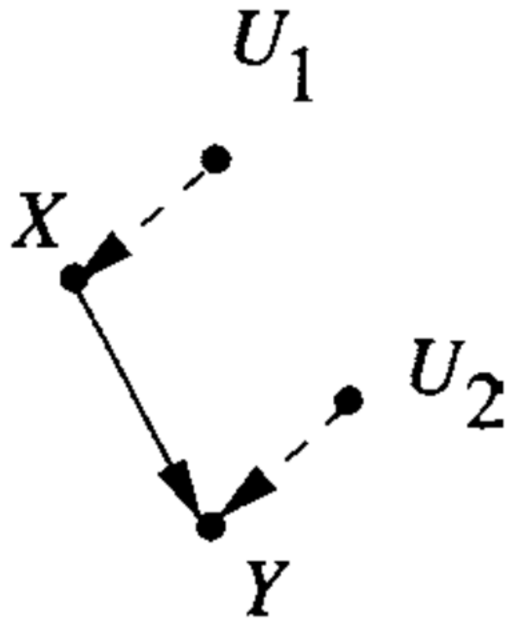
**Corresponding functional causal model**

**Functional causal model with $X$ causally upstream of $Y$**




(Pearl 2009, Figure 1.6)

# Example model & structural equations




$x = f_1(u_1)$ $\qquad\qquad$ $y = f_2(x, u_2)$

- An example structural equation:

$$y = a + bx + u_2$$

**(Could be linear regression!)**

(Pearl 2009, Figure 1.6)