9.S918: Statistical Inference for Brain and Cognitive Sciences, Pset 3
due 16 April 2025

9 April 2025

1 Causal estimation and graphical models

We discussed the BACK-DOOR CRITERION for identifying causal effects in causal graphical models using observational data. We start with the definition of BLOCKING (or *d*-SEPARATING) a path:

A set of Z blocks, or d-separates, a path iff:

- there is some node N on the path whose arrows do not converge and which *is* in Z; or,
- there is some node N on the path with converging arrows, and neither N nor any of its descendants is in Z.

With that in hand: a set of variables Z satisfies the back-door criterion relative to an ordered pair of variables $\langle X, Y \rangle$ if:

- No node in Z is a descendant of X; and
- Z blocks every path between X and Y that starts with an arrow into X.

If Z fulfills the back-door criterion relative to $\langle X, Y \rangle$, then the causal effect of X on Y, i.e., $P(Y = y|\text{do}(X = x))$, is identifiable:

$$P(Y = y|\text{do}(X = x)) = \sum_z P(Y = y|X = x, Z = z)P(Z = z). \tag{1}$$

With that in hand, consider the following experiment design. An experimenter designs logic puzzles parametrically varying in difficulty D according to some candidate cognitive theory. The experimenter is interested in the effect of the puzzle's difficulty on the time T participants take to work on it, under the assumption that the causal effect of D is linear on T. However, the theory only predicts the difficulty of *correctly* solving puzzles, so the experimenter uses the correctness C (where $C = 1$ means a correct solution and $C = 0$ means an incorrect solution) of the participant's solution as part of the analysis, either discarding

trials where $C = 0$ or including C as a covariate alongside D in a multiple regression where T is the dependent variable.

Questions:

1. What are plausible causal graphical models of the relationship among D, T, and C?
2. Explain the theoretical issue with the experimenter's data analysis plan in terms of the back-door criterion.
3. Demonstrate (e.g., in simulations; simple linear regression is fine) how the experimenter's data analysis plan can lead to systematically incorrect estimates of the causal effect of D on T.
4. Suggest some ideas for addressing the issue in practice, so that the experimenter can test the theory in some meaningful way.