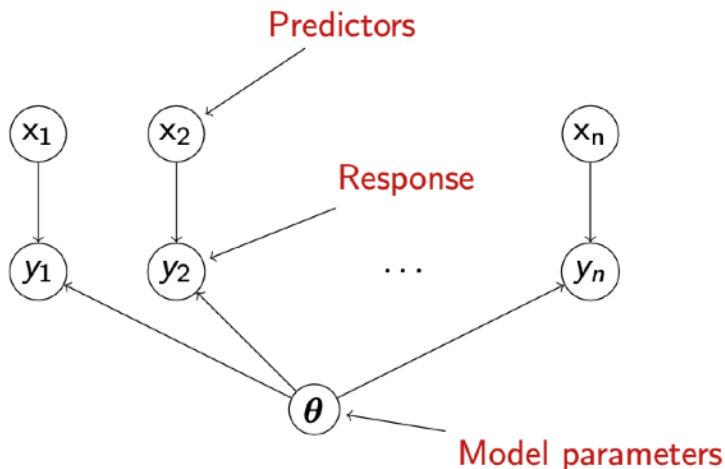


Bayes Net plate notation

- Here is our simple (single-level, non-hierarchical) Bayes net for regression:



- Here is a version of it using **plate notation**:

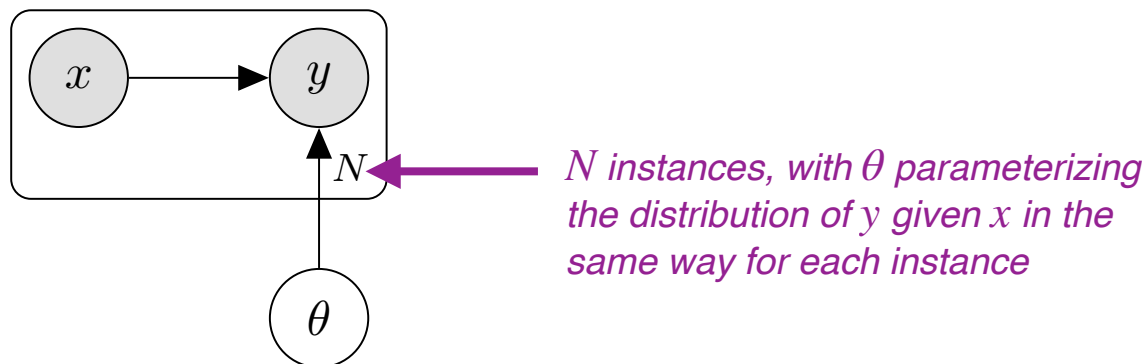
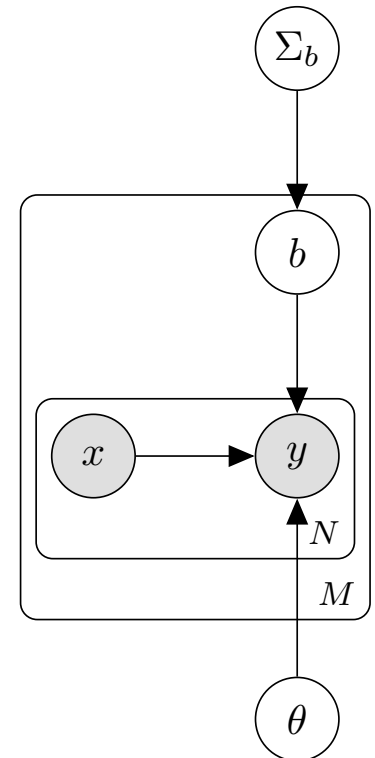
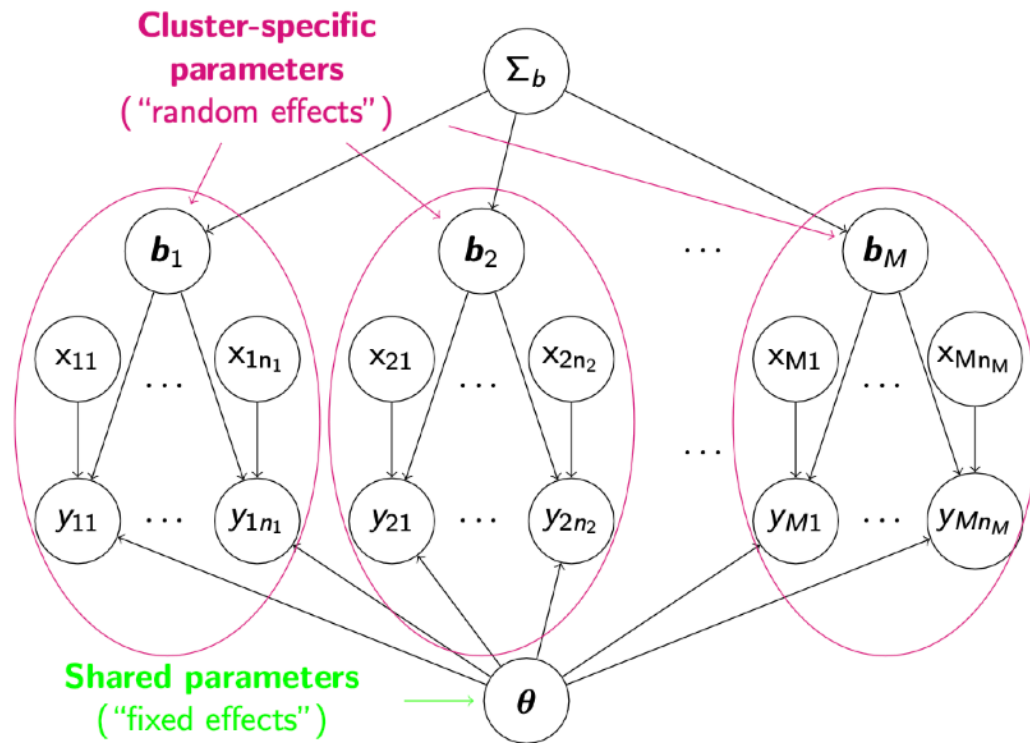


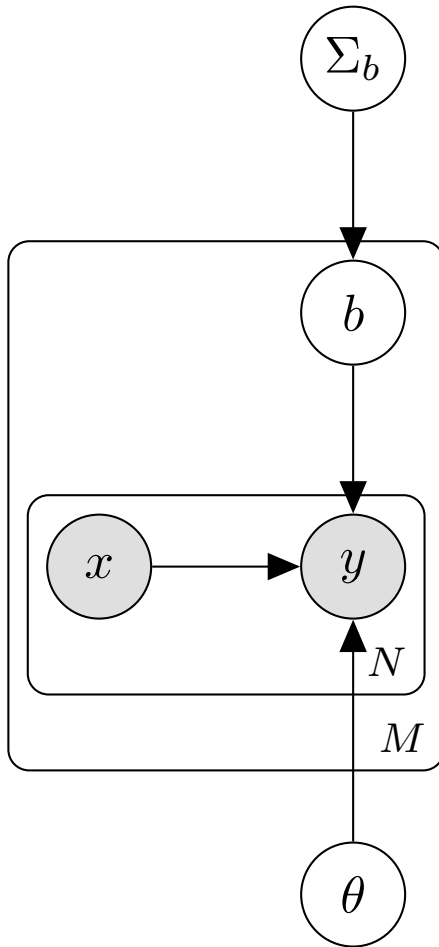
Plate notation for hierarchical models

- Our multi-level/mixed-effects/hierarchical model (with one level of hierarchy):
- The same model, using plate notation:



Compact mixed-effects model notation

- Multi-level GLM assumptions:



$$\theta = \langle \beta, \sigma^2 \rangle$$

$$b \sim N(0, \Sigma_b)$$

$$\eta = (\beta + b) x$$

$$\hat{y} \mid x = l^{-1}(\eta)$$

$P(Y \mid \hat{y})$ follows an *exponential* family

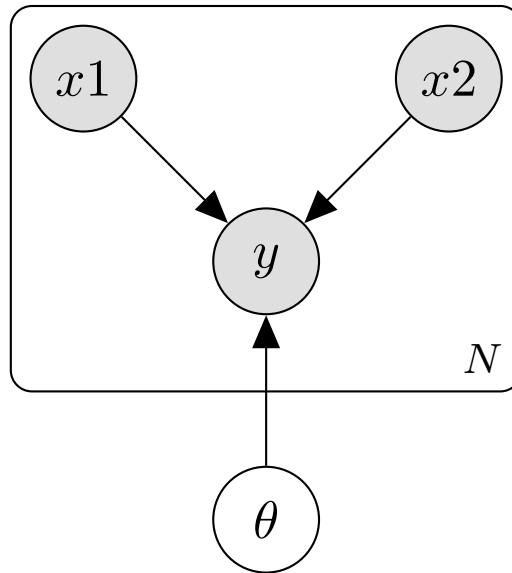
And for **multi-level linear regression**,

$$l^{-1}(\eta) = \eta \quad \text{(the link function is the **identity** function)}$$

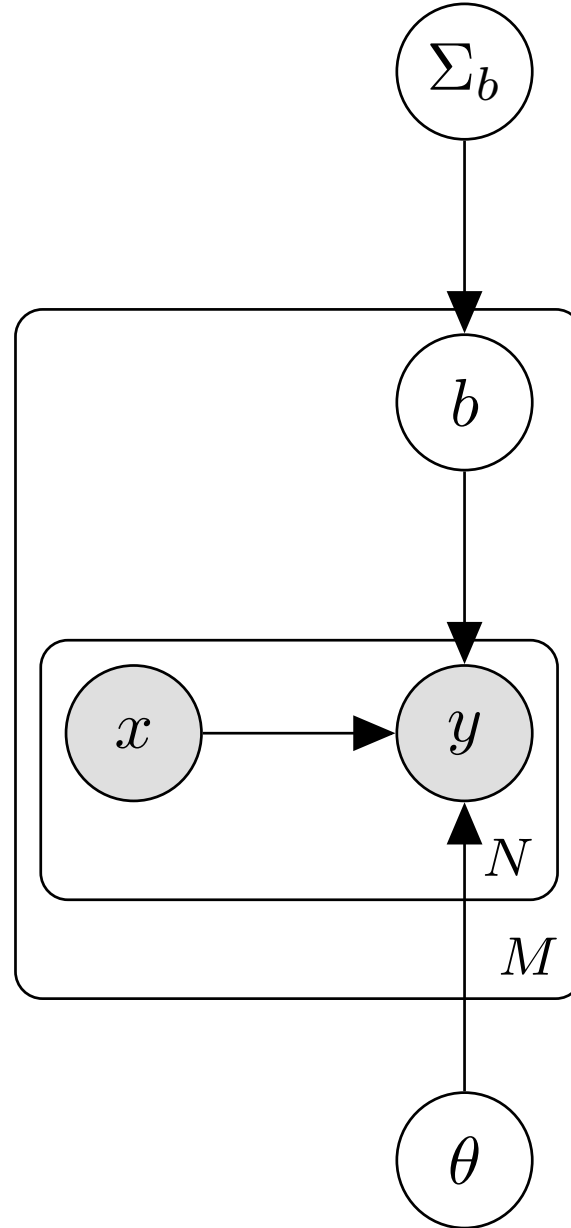
$$y \mid \hat{y} \sim N(\hat{y}, \sigma^2)$$

Credit-assignment problems and regression

- Example:
 - x_1 : how long a student studies for an exam
 - x_2 : a student's proclivity toward the subject matter
 - y : the student's score on the exam



Random effects as a "credit-assignment" confound

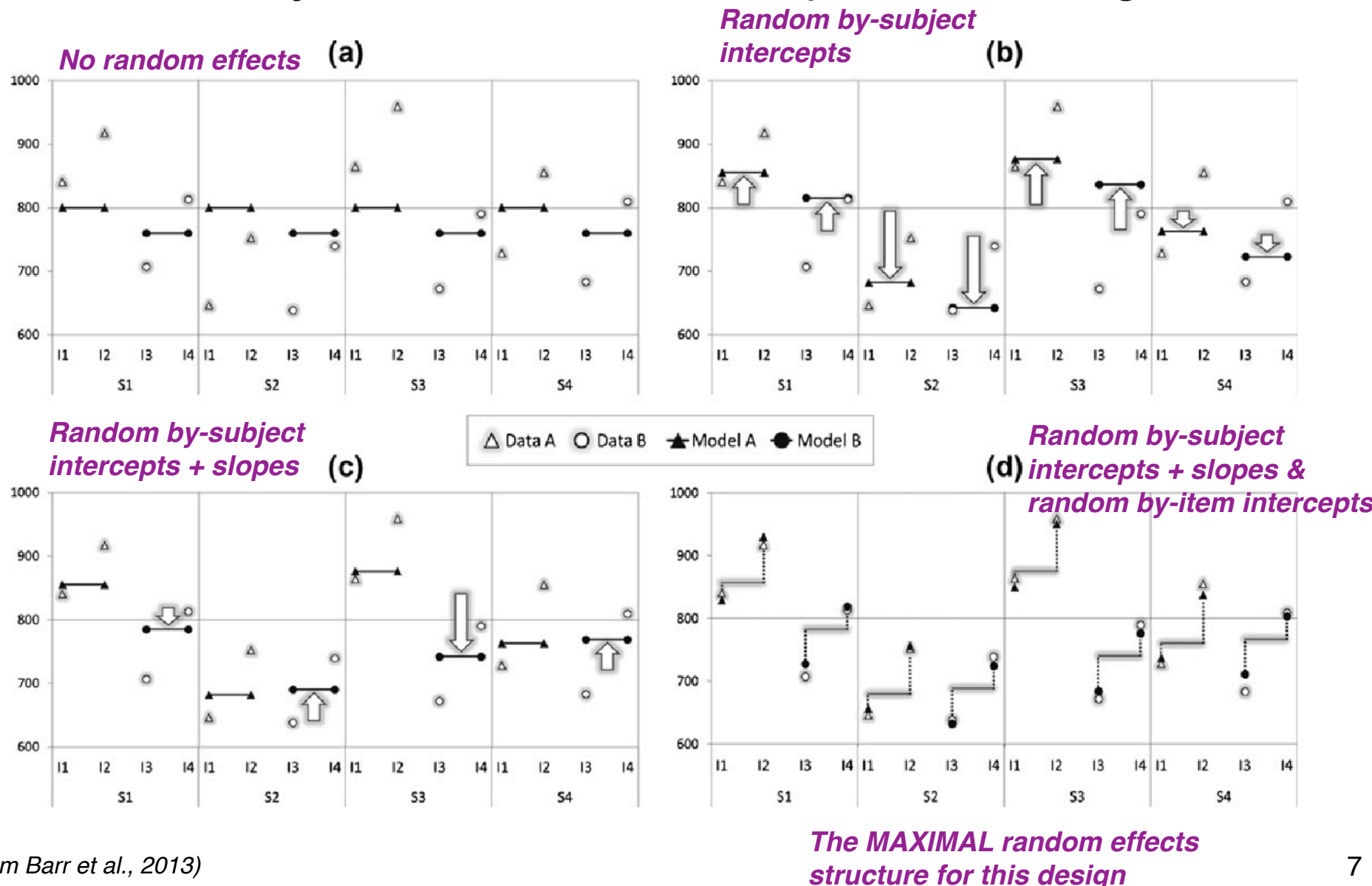


The rule for random effects for a predictor

- If the average value of a predictor x varies **between** groups in a random-effects factor F , include a **random intercept** for F
- If the value of a predictor x varies **within** groups in a random-effects predictor F , include a **random slope of x** for F

Illustration of random intercepts & slopes

- Within-subject, between-items experiment design



Barr et al. analysis of different models

Table 1

Summary of models considered and associated lmer syntax.

No. Model	lmer model syntax
(1) $Y_{si} = \beta_0 + \beta_1 X_i + e_{si}$	n/a (Not a mixed-effects model)
(2) $Y_{si} = \beta_0 + S_{0s} + \beta_1 X_i + e_{si}$	$Y \sim X + (1 \text{Subject})$
(3) $Y_{si} = \beta_0 + S_{0s} + (\beta_1 + S_{1s})X_i + e_{si}$	$Y \sim X + (1 + X \text{Subject})$
(4) $Y_{si} = \beta_0 + S_{0s} + I_{0i} + (\beta_1 + S_{1s})X_i + e_{si}$	$Y \sim X + (1 + X \text{Subject}) + (1 \text{Item})$
(5) $Y_{si} = \beta_0 + S_{0s} + I_{0i} + \beta_1 X_i + e_{si}$	$Y \sim X + (1 \text{Subject}) + (1 \text{Item})$
(6) ^a As (4), but S_{0s} , S_{1s} independent	$Y \sim X + (1 \text{Subject}) + (0 + X \text{Subject}) + (1 \text{Item})$
(7) ^a $Y_{si} = \beta_0 + I_{0i} + (\beta_1 + S_{1s})X_i + e_{si}$	$Y \sim X + (0 + X \text{Subject}) + (1 \text{Item})$

^a Performance is sensitive to the coding scheme for variable X (see [Online Appendix](#)).

Performance for **between-items** designs

- Type 1 error, power, and corrected power

N_{items}	Type I		Power		Power'	
	12	24	12	24	12	24
<i>Type I: Error at or near $\alpha = .05$</i>						
min- F'	.044	.045	.210	.328	.328	.328
LMEM, maximal, χ^2_{LR}	.070	.058	.267	.364	.223	.342
LMEM, no random correlations, χ^2_{LR} ^a	.069	.057	.267	.363	.223	.343
LMEM, no within-unit intercepts, χ^2_{LR} ^a	.081	.065	.288	.380	.223	.342
LMEM, maximal, t	.086	.065	.300	.382	.222	.343
LMEM, no random correlations, t	.086	.064	.300	.382	.223	.343
LMEM, no within-unit intercepts, t ^a	.100	.073	.323	.401	.222	.342
$F_1 \times F_2$.063	.077	.252	.403	.224	.337
<i>Type I: Error far exceeding $\alpha = .05$</i>						
LMEM, random intercepts only, χ^2_{LR}	.102	.111	.319	.449	.216	.314
LMEM, random intercepts only, t	.128	.124	.360	.472	.472	.314
LMEM, no random correlations, MCMC ^a	.172	.192	.426	.582		
LMEM, random intercepts only, MCMC	.173	.211	.428	.601		
F_1	.421	.339	.671	.706	.134	.212

^a Performance is sensitive to coding of the predictor (see the [Online Appendix](#)); simulations use deviation coding.

Performance for **within-items** designs

N_{items}	Type I		Power		Power'	
	12	24	12	24	12	24
<i>Type I: Error at or near $\alpha = .05$</i>						
min- F'	.027	.031	.327	.512	.327	.512
LMEM, maximal, χ^2_{LR}	.059	.056	.460	.610	.433	.591
LMEM, no random correlations, χ^2_{LR} ^a	.059	.056	.461	.610	.432	.593
LMEM, no within-unit intercepts, χ^2_{LR} ^a	.056	.055	.437	.596	.416	.579
LMEM, maximal, t	.072	.063	.496	.629	.434	.592
LMEM, no random correlations, t	.072	.062	.497	.629	.432	.593
LMEM, no within-unit intercepts, t ^a	.070	.064	.477	.620	.416	.580
$F_1 \times F_2$.057	.072	.440	.643	.416	.578
<i>Type I: Error far exceeding $\alpha = .05$</i>						
F_1	.176	.139	.640	.724	.345	.506
LMEM, no random correlations, MCMC ^a	.187	.198	.682	.812		
LMEM, random intercepts only, MCMC	.415	.483	.844	.933		
LMEM, random intercepts only, χ^2_{LR}	.440	.498	.853	.935	.379	.531
LMEM, random intercepts only, t	.441	.499	.854	.935	.379	.531

^a Performance is sensitive to coding of the predictor (see the Online Appendix); simulations use deviation coding.

Anti-conservativity of model selection

