Categorical predictors, interactions, logistic regression, and hierarchical models

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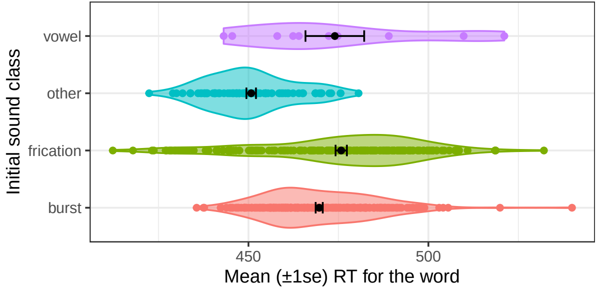
  - Is the vowel lax or tense?
- Example in word naming: the category of the initial sound of the word

`burst`	*bait*
`frication`	*chess*
`other consonant`	*wrist*
`vowel`	*inch*

Categorical predictors

- Initial evidence suggests that there may well be some real effects of initial phone class on word naming RT:



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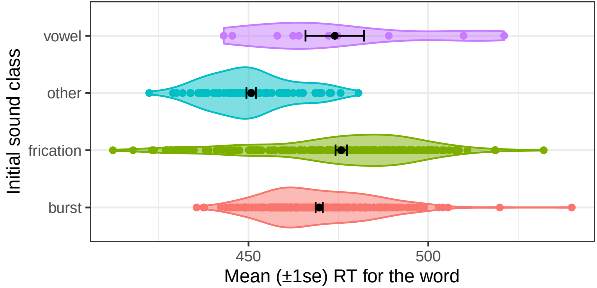

- **Note:** within-class variance seems to differ across classes—this is sometimes called heteroskedasticity. We won't worry about this for now, but it's something to keep in mind and we can eventually incorporate ways to deal with it

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- Many different coding schemes available (I'll cover three); critically, these models are equivalent in the following key sense: they can express the same set of probability distributions $P(Y|X)$!
- Hence, your choice of coding scheme should generally be guided by how you want to interpret model parameters

Dummy coding

With dummy coding, you drop the intercept term and create one 0/1 “dummy” predictor per value of your categorical predictor:

Level of `Init`	X_1	X_2	X_3	X_4
`burst`	1	0	0	0
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(Note: you cannot use dummy coding for more than one additively combined categorical predictor, since you can only sacrifice the intercept once)

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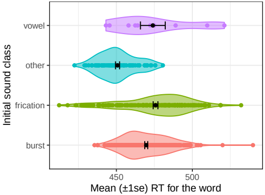

Predictor	$\widehat{\beta}$	$SE(\widehat{\beta})$	$p(>\|t\|)$
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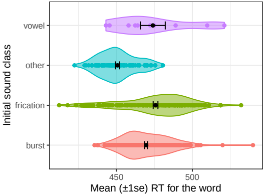

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- The t statistics reflect the null hypothesis that mean RT for that initial phone type is 0 (not a very useful null hypothesis!)

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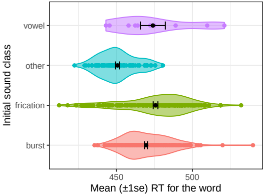

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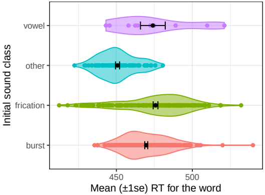

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- The *t*-statistic-based *p*-value indicates that fricative-initial words are slower than burst-initial words, and that words starting with other consonants are faster, but that we have insufficient information to conclude that vowel-initial words are either faster or slower

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subect to the constraint that

$$\beta_1 + \beta_2 + \beta_3 + \beta_4 = 0$$

Or equivalently

$$\beta_4 = -\beta_1 - \beta_2 - \beta_3$$

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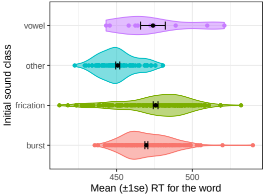

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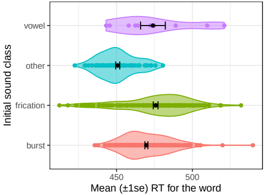

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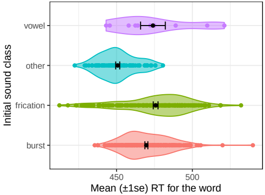

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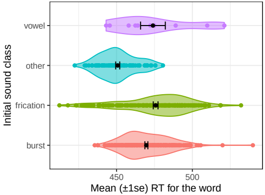

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This gives us $\widehat{\beta}_4 = -6.45$; using the formula for variance of the sum of random variables we can recover that $SE(\widehat{\beta}_4) = 4.33$ and hence the p=value for that offset is 0.136

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  - Consulting the cumulative distribution function for $F_{3,535}$, we find that this is highly significant—$p \ll 0.001$
  - Rather, your choice of contrasts should reflect how you want to *interpret* the parameters in your model!

Interactions between predictors

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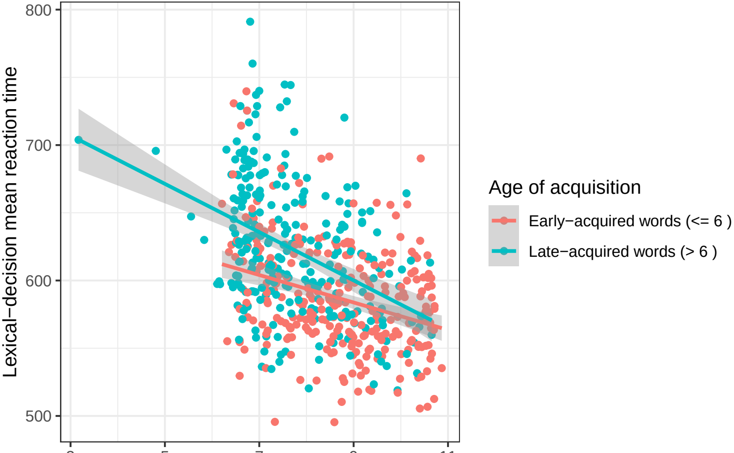
- Example: back to age of acquisition and word frequency: let's make a median split on AoA and examine frequency effects on lexical decision time:



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- Effect on the model matrix for our problem:

$$\mathrm{X} = \begin{bmatrix} 1 & \mathrm{Freq}_1 & \mathrm{AoA}_1 & \mathrm{Freq}_1\mathrm{AoA}_1 \\ 1 & \mathrm{Freq}_2 & \mathrm{AoA}_2 & \mathrm{Freq}_2\mathrm{AoA}_2 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & \mathrm{Freq}_n & \mathrm{AoA}_n & \mathrm{Freq}_n\mathrm{AoA}_n \end{bmatrix}$$

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Resulting model fit:

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$\widehat{\beta}_{\text{AoA}}$	33.47	7.35	$\ll 0.001$
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- Interpretation of the interaction parameter: as AoA increases, the effect of frequency becomes “less positive”
- Critically important issue: when there is a higher-order interaction in the model, care must be taken with the interpretation of lower-order terms involved in the interaction
- Here, the “main” effect of frequency ($\widehat{\beta}_{\text{Freq}}$) indicates the effect on RT of increasing word frequency for a hypothetical word with age of acquisition at 0 years!

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- Interpretation of the interaction parameter: as AoA increases, the effect of frequency becomes "less positive"
- Critically important issue: when there is a higher-order interaction in the model, care must be taken with the interpretation of lower-order terms involved in the interaction
- Here, the "main" effect of frequency ($\widehat{\beta}_{\mathrm{Freq}}$) indicates the effect on RT of increasing word frequency for a hypothetical word with age of acquisition at 0 years!
- But such words can't exist!

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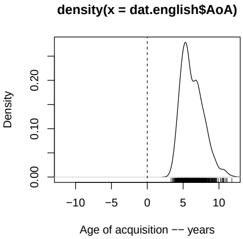
- We can achieve this by centering the predictor(s)

Interpreting interactions—centering predictors



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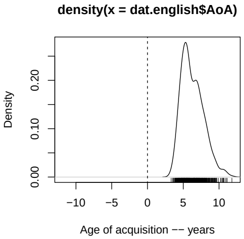

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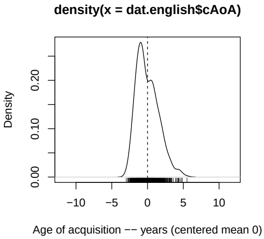

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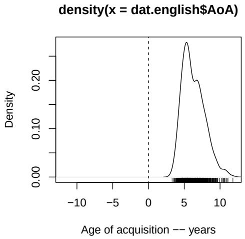

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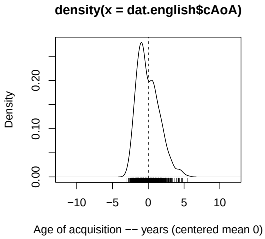

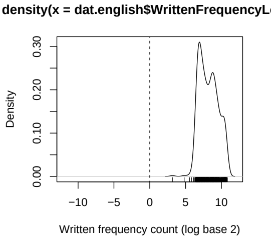

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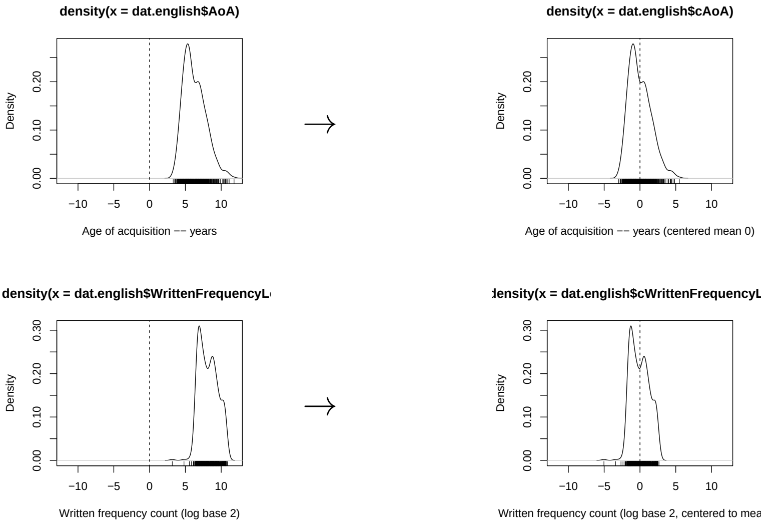

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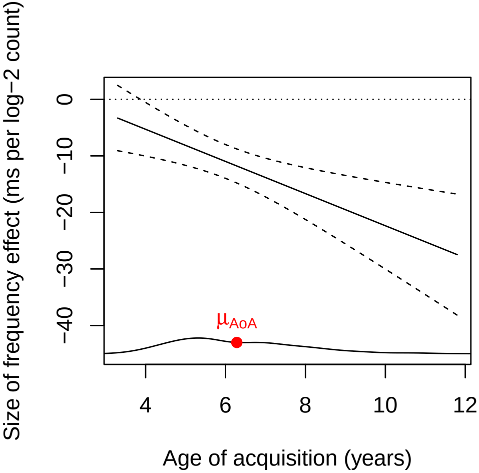
- Let us explore the effect of a word's frequency as a function of its AoA in this model



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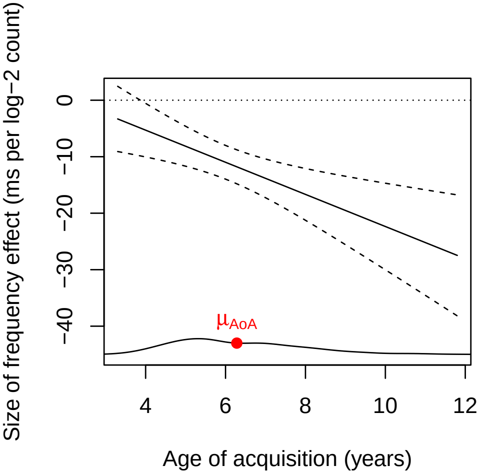

- What are some possible theoretical interpretations of this

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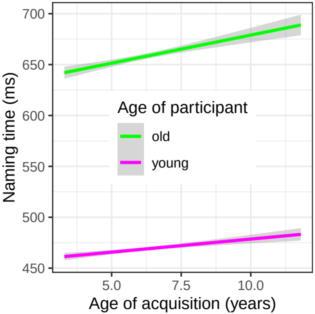

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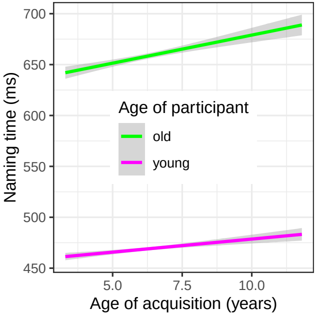

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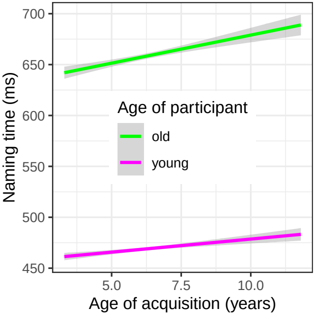

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- We can use the F-test to do this instead—comparing with a 3-parameter model (sans interaction) gives us $F_{1,1074} = 7.84$, for a p-value of 0.00519

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- However, we have not yet addressed the case when the *response* variable is categorical
- Let's consider the case of a dichotomous response variable
- Example: the dative alternation (Bresnan et al., 2007)

Sally sent [the children]$_{\text{Recip}}$ [toys]$_{\text{Theme}}$	**Double Object**
Sally sent [toys]$_{\text{Theme}}$ to [the children]$_{\text{Recip}}$	**Prepositional Object**

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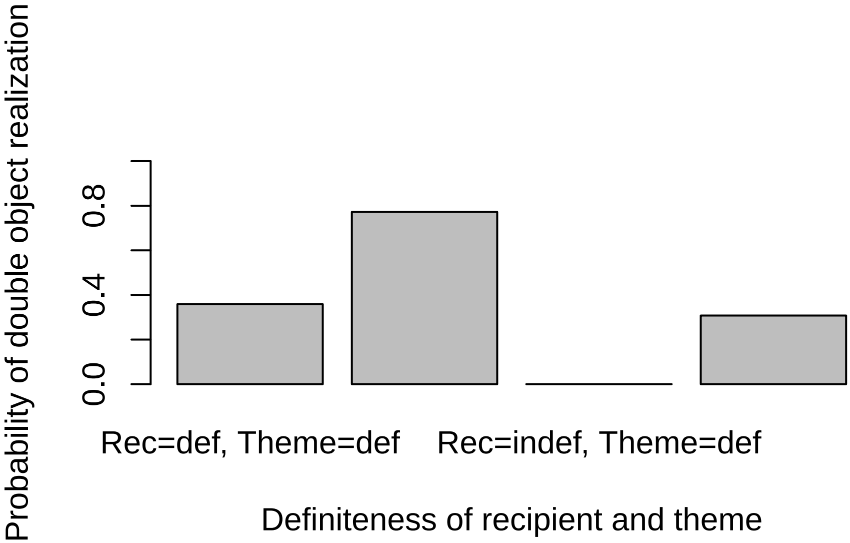
- We looked briefly before at the effects of definiteness of the theme (*toys/the toys*) and recipient (*children/the children*)

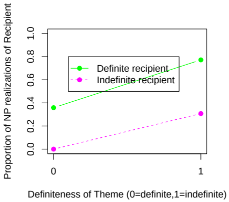
Dichotomous categorical responses



- We could learn these four separate means, but we would fail to capture the systematicity of the effects seen here

Dichotomous categorical responses

Another way of representing the four means:



- This looks like what we called an *additive pattern* for linear regression

$$Y = \alpha + \beta_1 X_1 + \beta_2 X_2 + \epsilon$$

where X_1 is 1 iff the theme is indefinite, and X_2 is 1 iff the recipient is indefinite (both 0 otherwise)

Problems for linear models with categorical response

$Y = \alpha + \beta_1 X_1 + \beta_2 X_2 + \epsilon$ $X_1 = 1$ iff theme indefinite, $X_2 = 1$ iff recipient indefinite (both 0 otherwise)

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   - Remember that our observed "means" are averages of many 0 and 1 observations!

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##                            Definiteness of recipient and theme
## Realization     Rec=def, Theme=def Rec=def, Theme=indef
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2. Bad predicted means: in linear regression, there is no guarantee that the predicted mean response $\hat{y}$ will fall between 0 and 1, even if all individual observations fall within this range

Bad predicted means with linear regression for categorical response

- Consider a case where a predictor is continuous and the response is categorical:

Recipient is NP	Recipient is PP
Mary sent John a shiny toy	Mary sent a shiny toy to John
Mary sent her friend a shiny toy	Mary sent a shiny toy to her friend
Mary sent every kid in the room a shiny toy	Mary sent a shiny toy to every kid in the room

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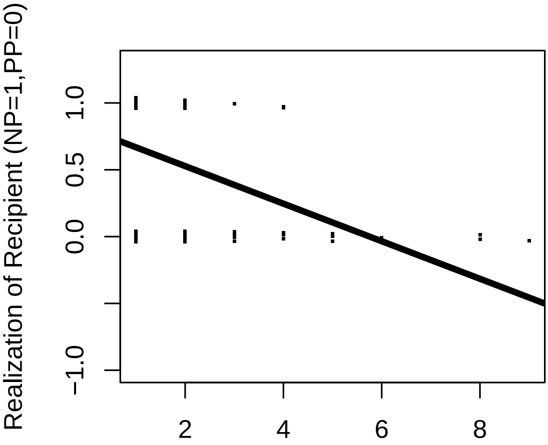
- We could quantify the *size* of the recipient in any number of ways (here we'll use length in # of words)

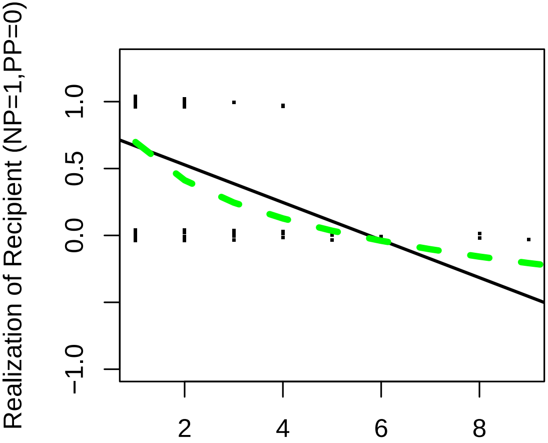
Dichotomous categorical response variables

Here's what happens when we learn a linear regression model on recipient length:



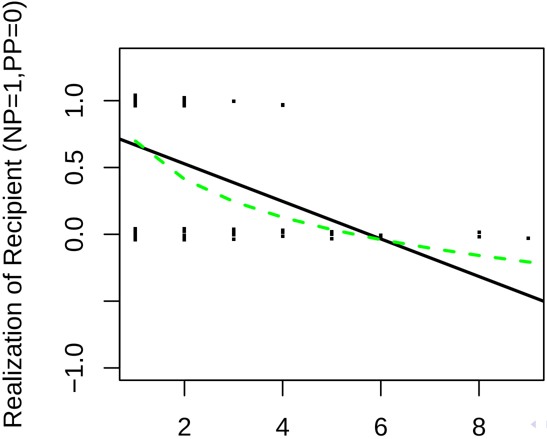
Bad predicted means with linear regression for categorical response

Same problem if we use *log* of recipient length as a predictor:



Bad predicted means with linear regression for categorical response

Even spline-based methods (which we haven't covered yet) give us the same problem, too, at the far end of the range of lengths:



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- So linear regression is bad for categorical response variables in:
  1. The noise distribution it assumes around the predicted mean
  2. The range of the predicted mean allowed
- Fortunately, the framework of generalized linear models (GLMs) gives us the flexibility to deal with these problems!

Reviewing GLMs

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$$P(Y = y; \mu)$$

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- Using logit GLMs to fit data with dichotomous response variables is called logistic regression

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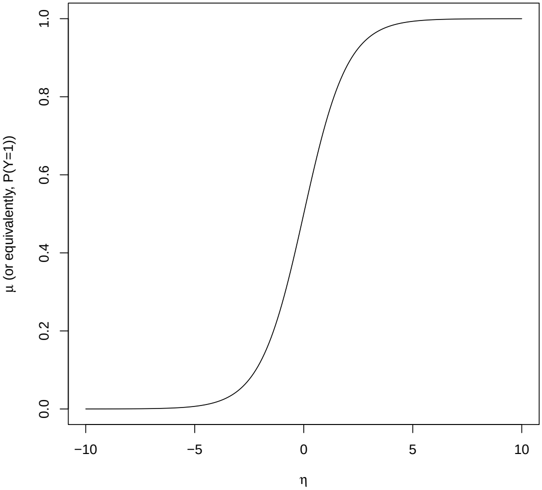

Estimating parameters in logistic regression

$$\begin{aligned} \eta &= \alpha + \beta_1 X_1 + \cdots + \beta_m X_m \\ \mu &= \frac{e^\eta}{1 + e^\eta} \\ P(Y = y|\mu) &= \begin{cases} \mu & y = 1 \\ 1 - \mu & y = 0 \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

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- As with linear regression, the regression weights $\langle \alpha, \beta_1, \ldots, \beta_m \rangle$ must be learned
- Unlike linear regression, there is no additional noise parameter to be learned (σ^2 in linear regression)
- Once again, we can use the method of maximum likelihood (which we use in the example here) or Bayesian inference to estimate parameters

Interpreting an additive logistic regression model

- Here's a logistic regression model for additive effects of theme and recipient definiteness:

$$\eta = \alpha + \beta_{\text{ThemeDef}} X_{\text{ThemeDef}} + \beta_{\text{RecDef}} X_{\text{RecDef}}$$

$$\mu = \frac{e^{\eta}}{1 + e^{\eta}}$$

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- The maximum likelihood estimate for the three regression parameters is

$\widehat{\alpha}$	-0.61
$\widehat{\beta}_{\text{ThemeDef}}$	1.8
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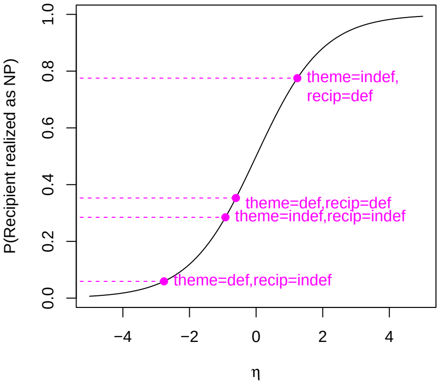
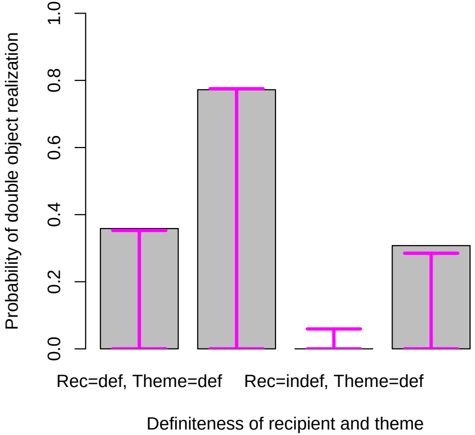


Interpreting an additive logistic regression model

This additive model does a decent job of modeling the true means!



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- In logistic regression and other GLMs, we make confidence regions and model comparisons based on constructs whose *asymptotic* (=approximately true, and increasingly accurate as sample sizes increase) form we can state
- For confidence regions: asymptotically, the covariance matrix of $\widehat{\boldsymbol{\beta}}$ is

$$\mathrm{Cov}(\widehat{\boldsymbol{\beta}}) = \begin{bmatrix} \frac{\partial^2 L(\beta_1)}{\partial \beta_1^2} & \frac{\partial^2 L(\beta_2)}{\partial \beta_1 \beta_2} & \cdots & \frac{\partial^2 L(\beta_m)}{\partial \beta_1 \beta_m} \\ \frac{\partial^2 L(\beta_1)}{\partial \beta_1 \beta_2} & \frac{\partial^2 L(\beta_2)}{\partial \beta_2^2} & \cdots & \frac{\partial^2 L(\beta_m)}{\partial \beta_2 \beta_m} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 L(\beta_1)}{\partial \beta_1 \beta_m} & \frac{\partial^2 L(\beta_2)}{\partial \beta_2 \beta_m} & \cdots & \frac{\partial^2 L(\beta_m)}{\partial \beta_m^2} \end{bmatrix}$$

(when certain regularity conditions hold). This is known as the Fisher information matrix.

Confidence regions for logistic regression

- A confidence region for predictors of a model estimated under maximum likelihood can be constructed similarly to the case in linear regression: for any size-k subset of predictors $\boldsymbol{\beta}'$, the quantity

$$(\widehat{\boldsymbol{\beta}}' - \boldsymbol{\beta})^T \left(\mathrm{Cov}(\widehat{\boldsymbol{\beta}}')\right)^{-1} (\widehat{\boldsymbol{\beta}}' - \boldsymbol{\beta})^T$$

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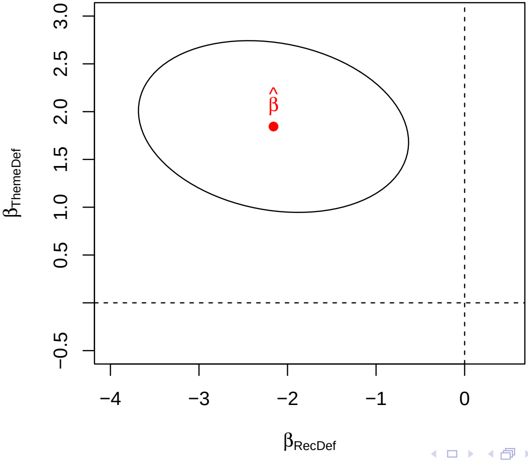
- Caution! These approximations break down when the estimates $\widehat{\beta}$ are large—most notably, when a single predictor allows *perfect* prediction of an outcome (always 0, or always 1)

Confidence regions in logistic regression

For example, a confidence region for the effects of recipient and theme definiteness:



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- Crucial to remember the coding scheme for these categorical predictors—here we'll stay with $X_{\text{ThemeDef}} = 1$ iff theme indefinite, $X_{\text{RecDef}} = 1$ iff recipient indefinite (both 0 otherwise)

Interactions in logistic regression

- MLE fit of the with-interactions model for the *send* data:

$\widehat{\alpha}$	-0.5819215
$\widehat{\beta}_{\text{RecDef}}$	-16
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- Remember, in these situations you cannot trust the Wald z-statistic $\left(\frac{\widehat{\beta}}{SE(\widehat{\beta})}\right)$!

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- Instead, the more general method for hypothesis testing is the likelihood ratio test
- We saw this before, in the end of the chapter on frequentist hypothesis testing

The likelihood ratio test

- For *nested* models $M_0 \subset M_A$ with k_0 and k_A free parameters respectively, the statistic

$$-2 \log \frac{\max \mathrm{Lik}_{M_0}(\mathrm{y})}{\max \mathrm{Lik}_{M_A}(\mathrm{y})}$$

is (asymptotically, in the limit of large data) distributed as $\chi^2_{k_A - k_0}$ if M_0 is true

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- In our case, the additive model for recipient and theme animacy had log-likelihood of -97.1, whereas the interactive model had log-likelihood of -96.8
- They differed in 1 parameter, and the cumulative distribution function of χ^2_1 for 0.66 is 0.582, so we conclude that the interaction is statistically

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  is (asymptotically, in the limit of large data) distributed as $\chi^2_{k_A - k_0}$ if M_0 is true
- This statistic doesn't have the same problems that the Wald z statistic has, so it can be used very generally to compare nested models
- In our case, the additive model for recipient and theme animacy had log-likelihood of -97.1, whereas the interactive model had log-likelihood of -96.8
- They differed in 1 parameter, and the cumulative distribution function of χ^2_1 for 0.66 is 0.582, so we conclude that the interaction is statistically

The likelihood ratio test

- For *nested* models $M_0 \subset M_A$ with k_0 and k_A free parameters respectively, the statistic

$$-2 \log \frac{\max \mathrm{Lik}_{M_0}(\mathrm{y})}{\max \mathrm{Lik}_{M_A}(\mathrm{y})}$$

  is (asymptotically, in the limit of large data) distributed as $\chi^2_{k_A - k_0}$ if M_0 is true
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- They differed in 1 parameter, and the cumulative distribution function of χ^2_1 for 0.66 is 0.582, so we conclude that the interaction is statistically insignificant

References I

Bresnan, J., Cueni, A., Nikitina, T., and Baayen, H. (2007). Predicting the dative alternation. In Boume, G., Kraemer, I., and Zwarts, J., editors, *Cognitive Foundations of Interpretation*, pages 69–95. Amsterdam: Royal Netherlands Academy of Science.

Grodner, D. and Gibson, E. (2005). Some consequences of the serial nature of linguistic input. *Cognitive Science*, 29(2):261–290.

Traxler, M. J., Morris, R. K., and Seely, R. E. (2002). Processing subject and object relative clauses: Evidence from eye movements. *Journal of Memory and Language*, 47:69–90.