9.S918: Statistical Inference in Brain and Cognitive Sciences

Week 1 Day 2: Introduction to causal inference

Roger Levy
Dept. of Brain & Cognitive Sciences
Massachusetts Institute of Technology

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A tiny bit of statistics

- On Tuesday we reviewed basics of **probability**: the logical calculus of uncertainty–a branch of mathematics
- The primary focus of this class is **statistics**: the mathematics, science, craft, and art of drawing inferences from data
- The two fields are fundamentally different
- But, probability is used extensively throughout statistics

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- In general, we will use $\mathbf{y}$ to refer to observed-outcome **data** and θ to refer to the model parameters to be estimated

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(see reading materials)

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- Good estimators have favorable **bias–variance** tradeoff

Maximum likelihood estimation

$$\mathrm{Lik}(\boldsymbol{\theta}; \boldsymbol{y}) \equiv P(\boldsymbol{y}|\boldsymbol{\theta}) \qquad \hat{\boldsymbol{\theta}}_{MLE} \stackrel{\mathrm{def}}{=} \arg\max_{\boldsymbol{\theta}} \mathrm{Lik}(\boldsymbol{\theta}; \boldsymbol{y})$$

i	y_i
1	T
2	T
3	H
4	T

(repeat slide from lecture 3)

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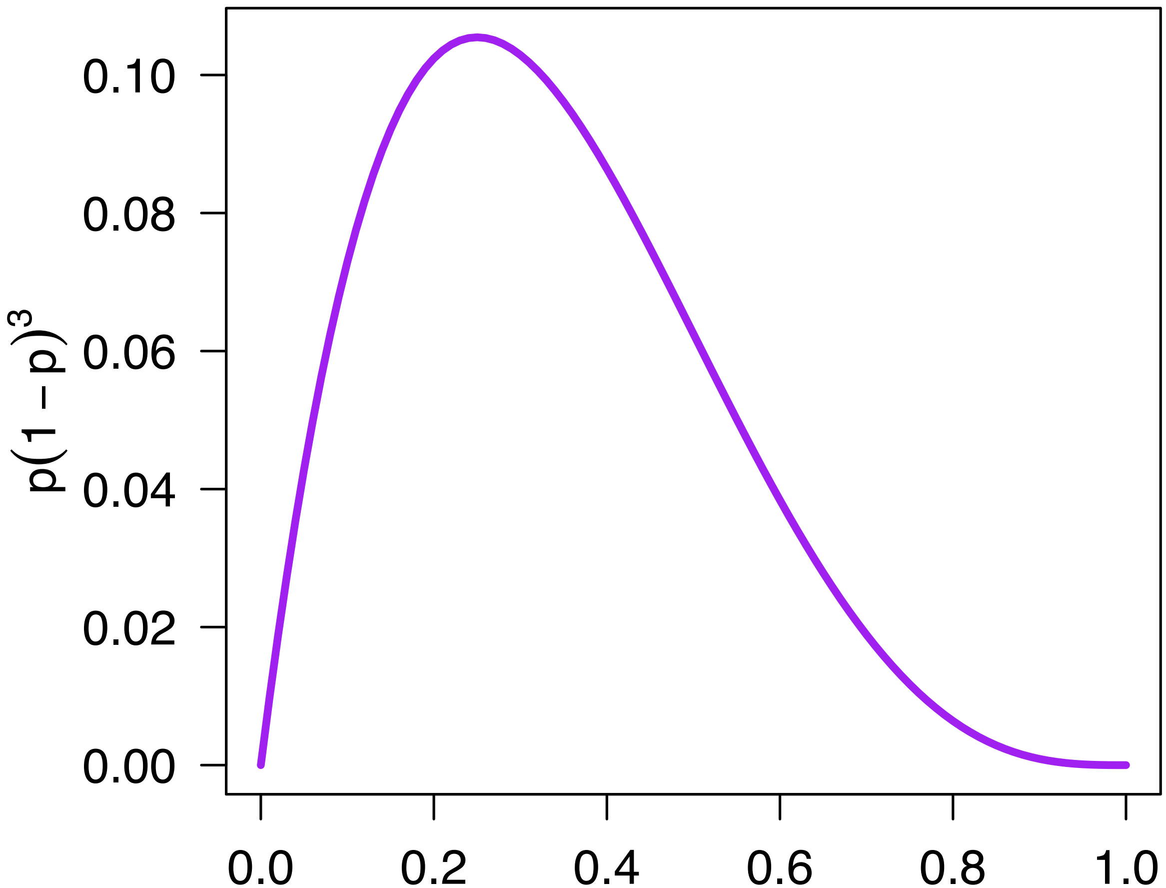

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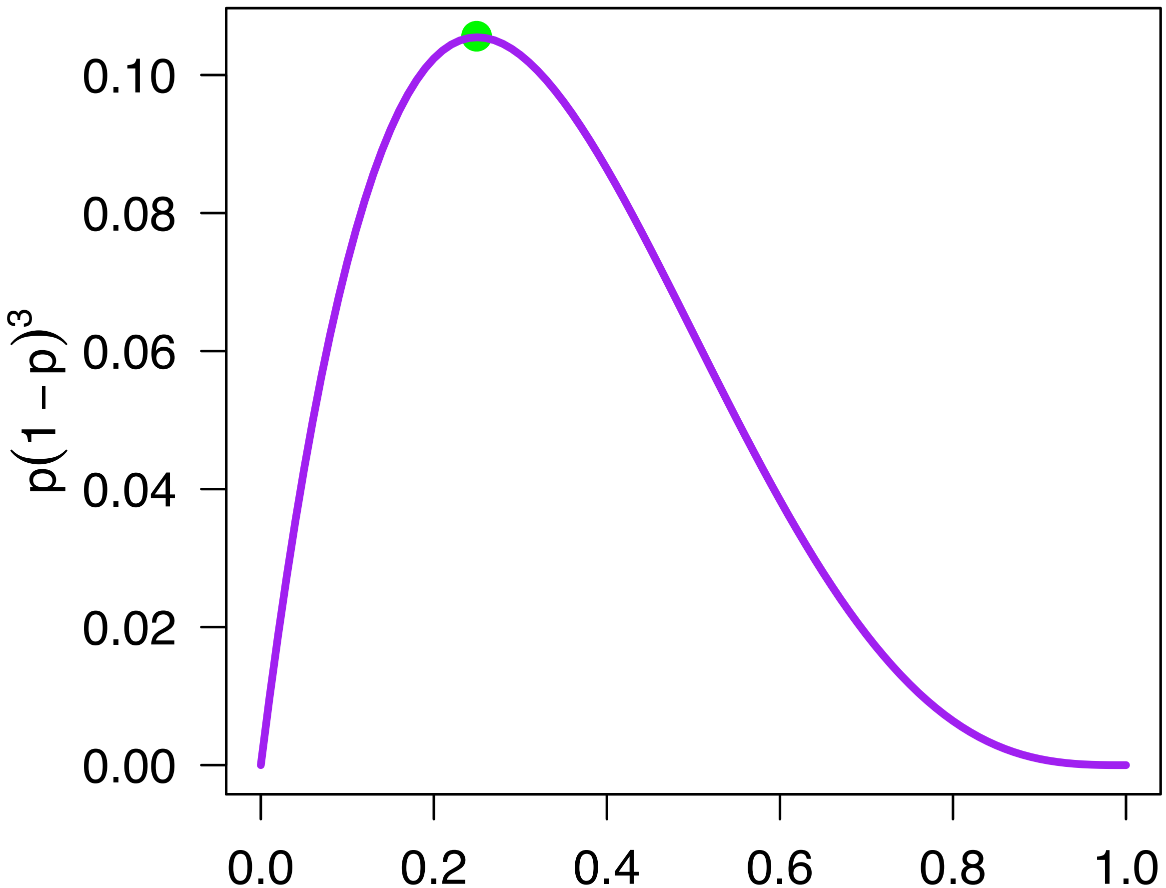

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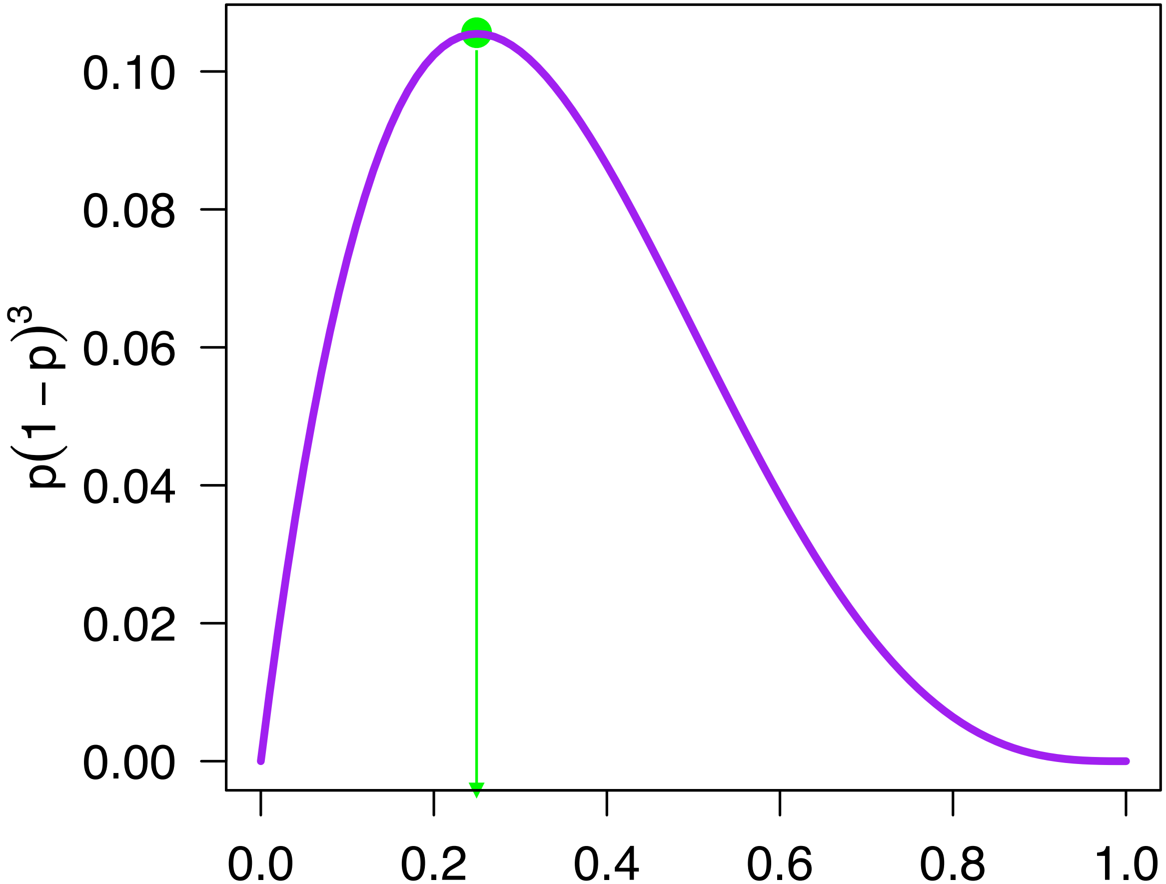

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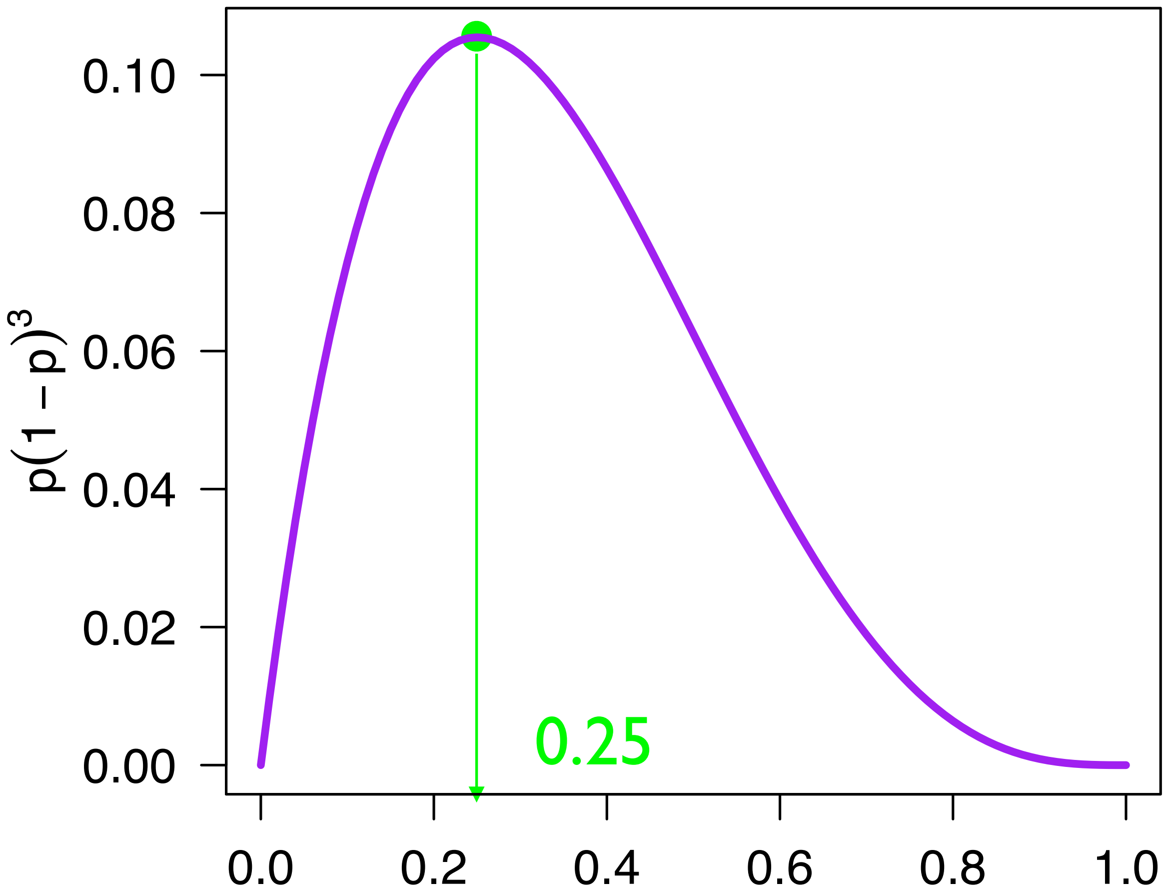

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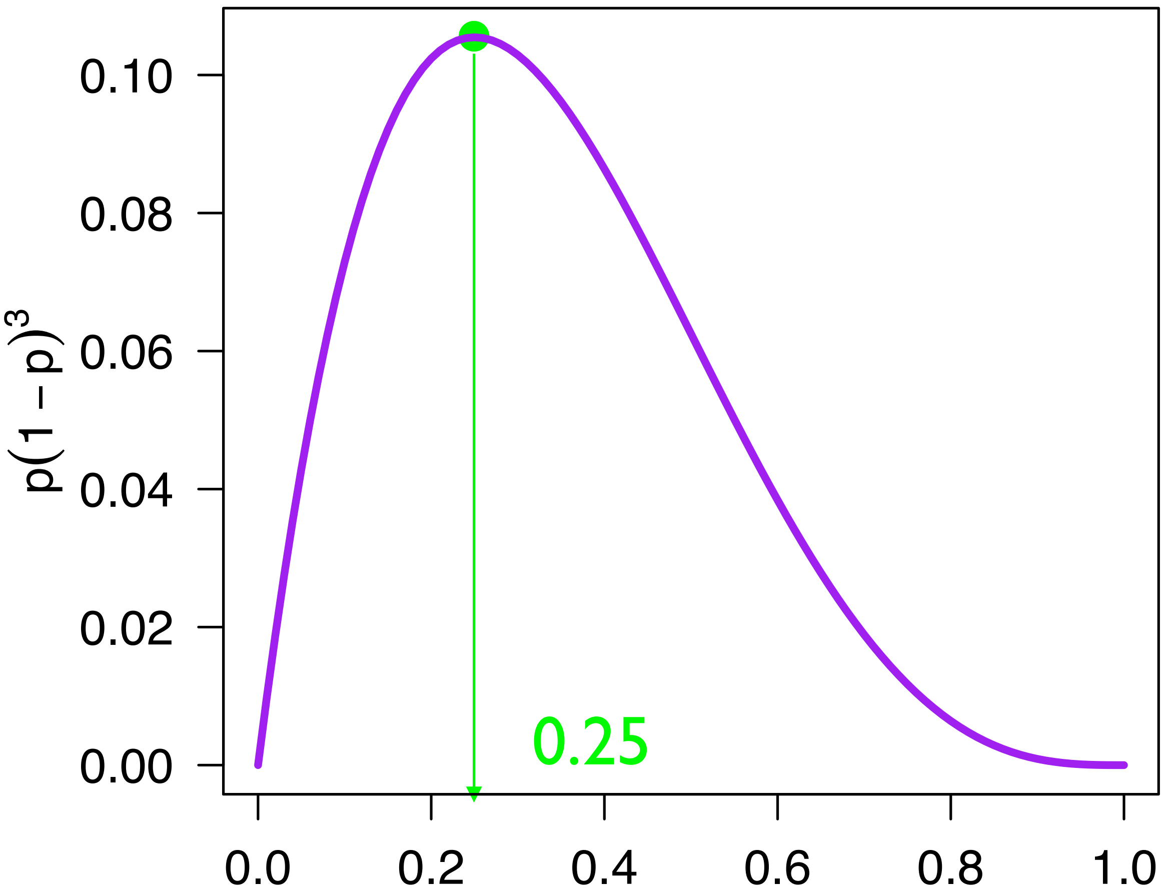

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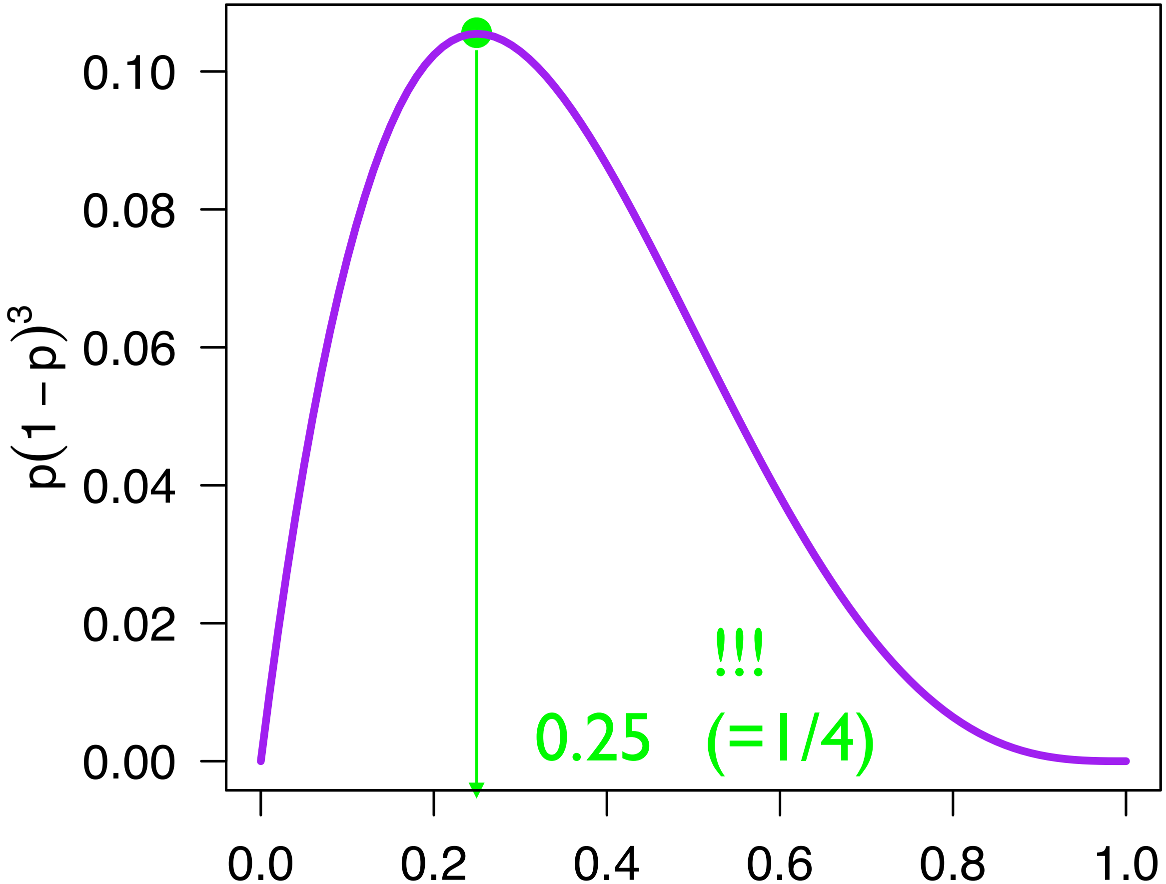

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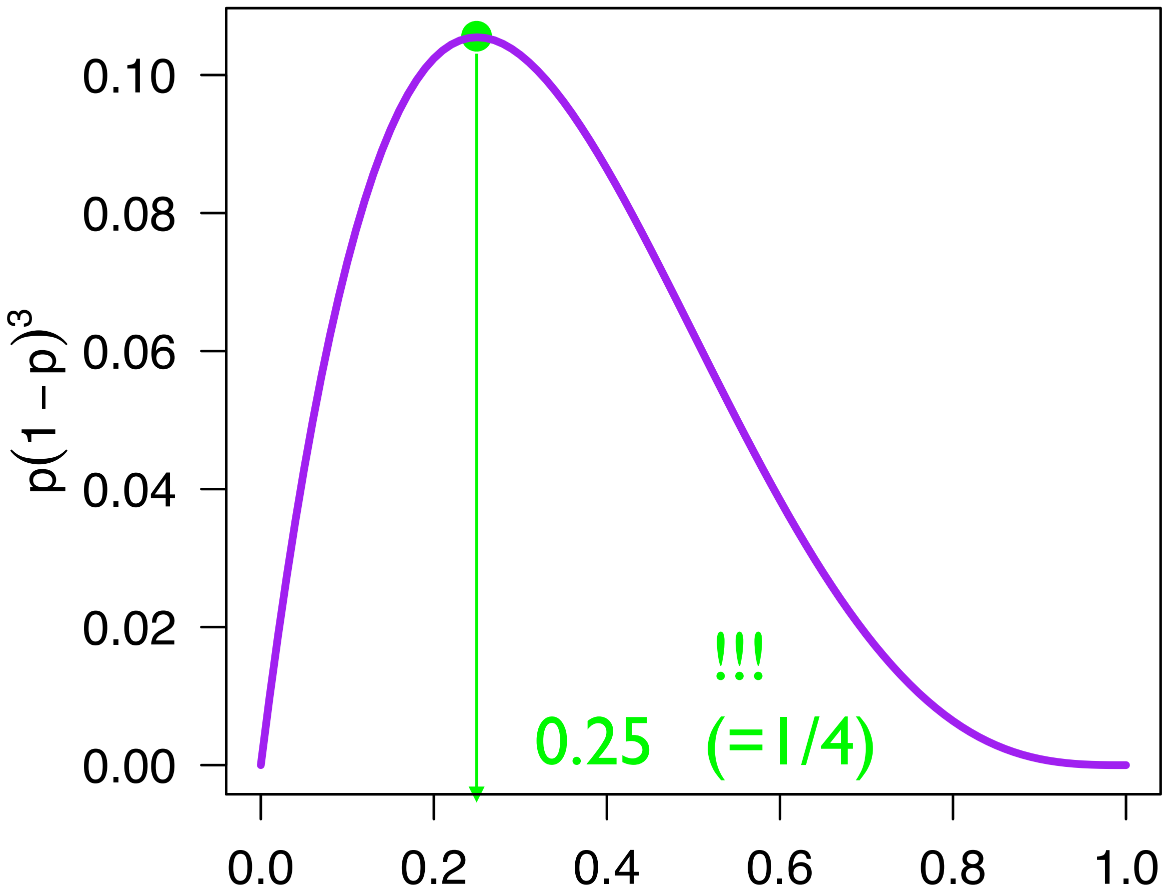

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The **MLE** also turns out to be the *relative frequency estimate* (RFE)

(repeat slide from lecture 3)

Introductory causal inference

- You have probably had previous exposure to both probability and statistics
- You are less likely to have had exposure to **causal inference**
- Causal inference uses probability and statistics, but it is something separate from the traditional construal of those two fields
- You can think of causal inference as being a framework extending more traditional statistics by:
    - Adding new probability-based mathematical constructs; and,
    - Developing a set of practice for statistical inference based on those constructs
- Two causal inference frameworks:
    - The **potential outcomes** framework
    - The **causal graphical models** framework

The potential-outcomes framework

- In epidemiology and many other areas of statistics, causal inference was developed out of the idea of **potential outcomes** (Neyman 1923, Rubin 1974)
- Consider an outcome, Y, and a potential **treatment** A
- **Example:**

Y: an individual survives to the end of the year (0: no, 1: yes)

A: an individual with heart disease receives a heart transplant (0: no, 1: yes)

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- So we are interested in (and likewise for $Y^{a=1}$):

$$E[Y^{a=0}] = \sum_y yP(Y^{a=0} = y) = 0 \times P(Y^{a=0} = 0) + 1 \times P(Y^{a=0} = 1) = P(Y^{a=0} = 1)$$

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Counterfactual data and causal effects

(Hernan & Robins, 2020, Table 1.1)

	$Y^{a=0}$	$Y^{a=1}$
Rheia	0	1
Kronos	1	0
Demeter	0	0
Hades	0	0
Hestia	0	0
Poseidon	1	0
Hera	0	0
Zeus	0	1
Artemis	1	1
Apollo	1	0
Leto	0	1
Ares	1	1
Athena	1	1
Hephaestus	0	1
Aphrodite	0	1
Cyclope	0	1
Persephone	1	1
Hermes	1	0
Hebe	1	0
Dionysus	1	0
$P(Y^{a=*}) = 1$	0.5	0.5

Counterfactual data and causal effects

- Suppose we knew **what would happen** for each individual in the population under each value of the treatment

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- The **average causal effect** of treatment A is defined as the difference of counterfactual risks:

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- The **average causal effect** of treatment A is defined as the difference of counterfactual risks:

$$E[Y^{a=1}] - E[Y^{a=0}] = 0$$

- Here, treatment is **ineffective**

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Estimating causal effects

Remember, $E[Y^{a=i}] = P(Y^{a=i} = 1)$

	L	A	Y	Y^0	Y^1
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Kronos	0	0	1	1	?
Demeter	0	0	0	0	?
Hades	0	0	0	0	?
Hestia	0	1	0	?	0
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Dionysus	1	1	0	?	0

- Naively, we might estimate the counterfactual risks $P(Y^{a=i} = 1)$ directly from observed A and Y:

Estimating causal effects

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Zeus	0	1	1	?	1
Artemis	1	0	1	1	?
Apollo	1	0	1	1	?
Leto	1	0	0	0	?
Ares	1	1	1	?	1
Athena	1	1	1	?	1
Hephaestus	1	1	1	?	1
Aphrodite	1	1	1	?	1
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Hebe	1	1	0	?	0
Dionysus	1	1	0	?	0

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Estimating causal effects

Remember, $E[Y^{a=i}] = P(Y^{a=i} = 1)$

	L	A	Y	Y^0	Y^1
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Demeter	0	0	0	0	?
Hades	0	0	0	0	?
Hestia	0	1	0	?	0
Poseidon	0	1	0	?	0
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CONSISTENCY: when $A = i$, $Y = Y^{a=i}$

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$$= \hat{P}_{MLE}(Y^{a=i} = 1 \,|\, A = i)$$

- So, the following condition suffices:

$$P(Y^{a=i} = 1 \,|\, A = i) = P(Y^{a=i} = 1)$$

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Remember, $E[Y^{a=i}] = P(Y^{a=i} = 1)$

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$$= \frac{\text{Count}(Y^{a=1} = 1 \wedge A = i)}{\text{Count}(A = i)}$$

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- So, the following condition suffices:

$$P(Y^{a=i} = 1 \,|\, A = i) = P(Y^{a=i} = 1)$$

- This is called **Exchangeability**:

$$Y^a \perp A \,|\, \{\}$$

Exchangeability and randomization

Goal: $\hat{P}(Y^a = 1)$

	$Y^{a=0}$	$Y^{a=1}$
Rheia	0	1
Kronos	1	0
Demeter	0	0
Hades	0	0
Hestia	0	0
Poseidon	1	0
Hera	0	0
Zeus	0	1
Artemis	1	1
Apollo	1	0
Leto	0	1
Ares	1	1
Athena	1	1
Hephaestus	0	1
Aphrodite	0	1
Polyphemus	0	1
Persephone	1	1
Hermes	1	0
Hebe	1	0
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Exchangeability and randomization

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- Why is a randomized experiment so powerful?

	$Y^{a=0}$	$Y^{a=1}$
Rheia	0	1
Kronos	1	0
Demeter	0	0
Hades	0	0
Hestia	0	0
Poseidon	1	0
Hera	0	0
Zeus	0	1
Artemis	1	1
Apollo	1	0
Leto	0	1
Ares	1	1
Athena	1	1
Hephaestus	0	1
Aphrodite	0	1
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Persephone	1	1
Hermes	1	0
Hebe	1	0
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Exchangeability and randomization

$\boxed{\textbf{Goal: } \hat{P}(Y^a = 1)}$

- Why is a randomized experiment so powerful?
- Recap of exchangeability criterion:

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Kronos	1	0
Demeter	0	0
Hades	0	0
Hestia	0	0
Poseidon	1	0
Hera	0	0
Zeus	0	1
Artemis	1	1
Apollo	1	0
Leto	0	1
Ares	1	1
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Rheia	0	1
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Demeter	0	0
Hades	0	0
Hestia	0	0
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Rheia
Kronos
Demeter
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	A
Rheia	0
Kronos	0
Demeter	0
Hades	0
Hestia	1
Poseidon	1
Hera	1
Zeus	1
Artemis	0
Apollo	0
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	A	Y
Rheia	0	0
Kronos	0	1
Demeter	0	0
Hades	0	0
Hestia	1	0
Poseidon	1	0
Hera	1	0
Zeus	1	1
Artemis	0	1
Apollo	0	1
Leto	0	0
Ares	1	1
Athena	1	1
Hephaestus	1	1
Aphrodite	1	1
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Persephone	1	1
Hermes	1	0
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Rheia	0	1	0	0
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$$\hat{P}(Y^{a=i} = 1) = \hat{P}(Y = 1 \mid A = i)$$

- Hooray!!!

	$Y^{a=0}$	$Y^{a=1}$	A	Y
Rheia	0	1	0	0
Kronos	1	0	0	1
Demeter	0	0	0	0
Hades	0	0	0	0
Hestia	0	0	1	0
Poseidon	1	0	1	0
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	L
Rheia	0
Kronos	0
Demeter	0
Hades	0
Hestia	0
Poseidon	0
Hera	0
Zeus	0
Artemis	1
Apollo	1
Leto	1
Ares	1
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	L	A
Rheia	0	0
Kronos	0	0
Demeter	0	0
Hades	0	0
Hestia	0	1
Poseidon	0	1
Hera	0	1
Zeus	0	1
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Apollo	1	0
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