Robert Y. Lewis

Bessemerstraat 9A – 1097AM Amsterdam – The Netherlands

+1 617 872 3556 • +31 6 837 77159 • r.y.lewis@vu.nl • http://robertylewis.com

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My research interests fall broadly into the category of formal methods, more specifically in interactive theorem proving and automated reasoning. *Proof assistants* are tools that provide languages for defining objects, stating properties of these objects, and proving that these properties hold; they also offer engines for verifying the correctness of these proofs. *Automated reasoning* tools, including SAT solvers, SMT solvers, and first- and higher-order provers, are by contrast push-button tools that check the correctness of a statement. While they are sometimes used in the context of a proof assistant to close proof obligations automatically, they are seen more often in unverified settings.

Formal methods are widespread in program verification and analysis, and the use of proof assistants in programming language research is growing more common. In mathematics, however, the picture is very different. Outside of a few landmark results, these tools remain largely unapplied to mathematical problems. These applications are not different in kind to those seen in computer science, but have a significantly different flavor to them: the depth and layers of abstraction limit the applicability of general proof search techniques. My goal as a researcher is to adapt the logics, tools, and paradigms of formal methods in computer science to attack mathematical problems. I envision a future where trustworthy computer assistance in mathematical research is commonplace. The routine use of proof assistants will increase the reliability of proofs; large libraries of formal proofs will be used for reference and training by students and AI programs; advances in automated reasoning will do away with tedious parts of proving.

Early in the course of my PhD studies, I became one of the first users of Lean, a proof assistant based on dependent type theory. I have contributed to the Lean source code, its standard library, and its automated proof tools. I am an official maintainer of Lean's standard library and an active member of its community, which includes Kevin Buzzard, professor of mathematics at Imperial College London, and a number of Imperial undergraduates. My work has also been recognized by the broader interactive theorem proving community: I organized a session on formal libraries at the International Congress on Mathematical Software; I was an invited speaker at a conference in honor of Thomas Hales, a prominent figure in formal proof; alongside leading researchers, I have been asked to contribute a chapter to a Springer book on proof assistants and their applications, which is expected to become the standard reference in that area.

I have a number of ongoing research projects related to logic and formal methods. One component of my dissertation, a method for automatically proving nonlinear inequalities, remains in active development. These inequalities appear frequently as goals in interactive proofs and are frustrating to prove by hand. While classical decision procedures like cylindrical algebraic decomposition (CAD) can sometimes be applied, they tend to be slow and limited in scope. Drawing on theory combination methods like SMT, my method combines separate solvers for linear arithmetic, multiplicative arithmetic, and other theories in a heuristic proof search; successful searches produce certificates that can be verified in a proof assistant. While this search is not complete (and can be applied to undecidable theories), it is efficient and often successful on problems that appear naturally in mathematical formalizations, which is not the case for CAD. Prototyping has shown that the approach also works on industrial problems, particularly those arising in the verification of hybrid systems. I am investigating how to extend the techniques used in this system to reason about special functions, convex optimization, and other topics of theoretical and applied interest. I have found this kind of domain-specific automation to be more successful than general tools, at the current time, in speeding up mathematical formalizations.

Another project of mine integrates Lean with the computer algebra system (CAS) Mathematica. The link I have developed is generic, extensible, and can be used for communication in both directions. There are many benefits of accessing CAS tools from proof assistants: many CAS computations are certifiable, and other computations and visualizations can be used to guide automatic and interactive proof search. Communication in the opposite direction also has interesting applications. By introducing the library of a proof assistant to the computation and visualization capabilities of a CAS, one can inspect and analyze proofs, investigate the proof graph as a whole, and test queries from the CAS against the proof assistant library. These applications connect to research programs in the mathematical knowledge management community. There are many educational applications of such a link; working with the developers of CoCalc, an online IDE and course management tool that supports Lean and Sage, I plan to incorporate my link into their framework. I also maintain ties to traditional computer algebra research. During my PhD, I spent two months interning at Wolfram Research, and have continued to consult for their Mathematica R&D group.

Lean's `mathlib` library forms the basis for much of my current research. As a medium-term project, I plan to formalize Maryna Viazovska's recent groundbreaking results on sphere packings. The topic of sphere packing is familiar to the interactive theorem proving community through the formalization of the Kepler Conjecture, and I expect to collaborate with researchers involved in that effort. This project will lead to formal libraries for Fourier analysis and modular forms; it will also require the development of optimization tools, related to my work on nonlinear inequalities, which will be useful in many other applications. I approach formalizations aiming to notice pain points and to think of tools and design paradigms that can make the process easier.

To spread formal methods more widely, mathematicians and computer scientists must collaborate. Pushing this collaboration is an essential part of my research project. I am a member of the Formal Abstracts project, launched recently by Thomas Hales, which aims to introduce mathematicians to proof assistants by formalizing the statements of contemporary results. My latest publication describes a formal proof of Ellenberg and Gijswijt's solution to the cap set problem, a recent and celebrated result in combinatorics; the paper was written with Sander Dahmen (a mathematician) and Johannes Hölzl (a computer scientist). It is exceedingly rare for modern mathematics to be formalized, and by formalizing a result from 2017 that received popular press, we hope to bring more attention to the field. Mathematicians have so far hesitated to try proof assistants, and even heavily promoted events like the 2017 Big Proof program at the Isaac Newton Institute have attracted mainly computer scientists. Lean Together, a workshop that I organized in January 2019, was attended by 29 mathematicians (including two IMO gold medalists) and 29 computer scientists (including a Spinoza Prize winner). I have since given a series of invited lectures to mathematical audiences in Australia and Vietnam, and am organizing a follow-up workshop to Lean Together in 2020.

My lines of research connect well with various groups at Imperial, particularly those in Theory and Algorithms and Analysis and Verification. I expect to find fruitful collaborations with researchers working on logic and type theory, optimization, trusted computing, and software verification. I already have close ties to the Imperial mathematics department, where Kevin Buzzard's innovative use of proof assistants aligns perfectly with my own goals. My aim is to spread the use of formal methods from computer science in mathematics research and education, and Imperial's strengths on both sides make it a uniquely suited place to pursue this plan.

I understand the importance of external grant support to a modern academic institution. At the VU, I contributed to the successful €800,000 proposal Lean Forward, and I will use this experience as I apply for grants of my own, for instance, to support formalizing the new sphere packing results. The EPSRC lists research at the interface of mathematics and other disciplines as a priority area for their early career fellowships. Kevin Buzzard and I will also apply jointly to the EPSRC. With a collaborator from the University of Western Ontario, I plan to apply for a newly open New Frontiers in Research Fund grant, meant to fund collaboration between Canadian and non-Canadian researchers.