ments, such as those we find in Euclid's geometry. A central philosophical problem for Aristotle was therefore to give an account of the inferences that make for a valid deduction.

ARISTOTLE'S LOGIC

Aristotle's logic concerns sentences that have a simple structure consisting of a *quantifier* such as "all" or "some" or "no" (as in "none"), a subject term such as "humans" or "Socrates," and a predicate term such as "are animals" or "is not snub-nosed" or "are mortal." For example, "All humans are mortal" or "Socrates is not snub-nosed" are the kind of sentences whose logic Aristotle described.

The characteristic form of inference in Aristotle's logic is the *syllogism*, which consists of a pair of sentences that serve as *premises* and a sentence that serves as the *conclusion*. You have seen an example of syllogistic argument in the previous section. Here is another:

Syllogism 1
All humans are animals.
All animals are mortal.
Therefore, all humans are mortal.

This is a *valid syllogism. What makes it valid is that if the premises are true, then it follows necessarily that the conclusion is also true.* If the premises happen to be false in a valid syllogism, then the conclusion may be either true or false. What matters is that in every conceivable case in which the premises could be true, the conclusion would also be true.

You can see why this syllogism counts as valid by drawing some circles. (This is not a device that Aristotle used. It was first developed during the Renaissance). Suppose you introduce a circle *H* to represent the set of all humans, another circle *A* to represent the set of all animals, and a third circle *M* to represent the set of all mortal things. The first premise says that the set of all men is contained in the set of all animals. So put circle *H inside circle A* to represent the state of affairs required for the first premise to be true (figure 2.1). The second premise says that the set of all animals is contained in the set of all mortal things. So put circle *M* around circle *A* to represent the state of affairs required for the second premise to be true (figure 2.2). Now consider the figure drawn (2.2). To represent the state of affairs required to make both premises true, you *had* to put *H* inside *A* and *A* inside *M*. So necessarily *H* is inside *M*, which is what the conclusion asserts. What makes a syllogism valid is that in any way you represent

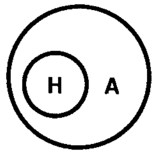

Figure 2.1
Premise 1 of syllogism 1

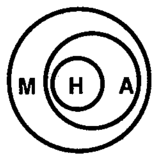

Figure 2.2
Syllogism 1

circumstances so that both of the premises are true, the conclusion is true as well.

Here is another valid syllogism:

Syllogism 2
All humans are animals.
Some humans are quiet.
Therefore, some quiet things are animals.

Represent the class of all humans by the circle H, and the class all animals by the circle A, and the class of all quiet things by the circle Q. The first premise, as before, says that H is contained in A. The second premise is different. It says that there are things that are both human and quiet. This can only be represented by having circle Q, representing the set of all quiet things, *intersect* circle H, representing the set of all humans. So every representation that makes the first two premises of the syllogism both true has Q intersecting H and H contained in A (figure 2.3). But then Q must necessarily intersect A, which is what the conclusion asserts.

By contrast the following syllogism is *not valid*, even though all its premises and its conclusion are true:

Syllogism 3
All humans are animals.
Some animals are mortal.
Therefore, all humans are mortal.

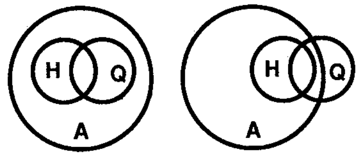

Figure 2.3
Two possibilities for syllogism 2

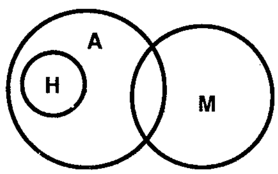

Figure 2.4
A counterexample to syllogism 3

To see that the syllogism is not valid, remember that for validity there must be *no possible way* of arranging the circles representing the sets of things that are human, *H*, animals, *A*, and mortal, *M*, so that in that representation of possible circumstances the premises are both true but the conclusion of the syllogism is false. The first premise says, as before, that *H* is included in *A*. The second premise says that circles *A* and *M* intersect. One way in which the two premises *could imaginably* be true is given in figure 2.4. In this figure *M* intersects *A*, and *H* is included in *A*, but *M* does not include any of *H*. The figure represents an imaginable circumstance in which all humans are animals, some animals are mortal, but some humans (in fact, all humans) are *immortal*. The circumstances represented are not those that obtain in our world, where in fact all humans are mortal, but they are consistently imaginable circumstances, and they show that *the truth of the premises of the syllogism do not by themselves necessitate the truth of the conclusion of the syllogism.*

That a syllogism is *valid* does not imply that its premises are true or that its conclusion is true. A valid syllogism may have false premises and a true conclusion, false premises and a false conclusion, or true premises and a true conclusion. What it may *not* have is true premises and a false conclusion. What it means for a syllogism to be valid is that if its premises were true, its conclusion would of necessity be true. So if the premises are actually true and the syllogism is valid, then the conclusion must actually be true.

Here is an example of a valid syllogism in which the premises are in fact false but the conclusion is true:

All humans are apes.
All apes have opposing thumbs.
Therefore, all humans have opposing thumbs.

Here is an example of a valid syllogism in which the premises are false and the conclusion is false:

All humans are apes.
All apes are stockbrokers.
Therefore, all humans are stockbrokers.

Aristotle realized that the validity of a syllogism has nothing to do with what the predicate terms and the subject terms *mean*, but has everything to do with what quantifiers occur in the premises and the conclusion and with where one and the same term occurs in both the premises and the conclusion. The first syllogism we considered has the following form:

All A are B.
All B are C.
Therefore, all A are C.

Any syllogism of this form will be valid, no matter what classes A, B, and C denote. A could be stars, B olives, C dragons. The following syllogism is silly, but valid.

All stars are dragons.
All dragons are olives.
Therefore, all stars are olives.

By contrast, the following form is not valid.

All A are B.
Some B are C.
Therefore, some A are C.

It is easy to see that this form of syllogistic argument is not valid by considering an example of that form in which the premises are true but the conclusion is false:

All men are mammals.
Some mammals are female.
Therefore, some men are female.

Study Questions

1. Give new examples of *valid* syllogisms with the following properties: (a) The premises are false and the conclusion is true. (b) The premises are false and the conclusion is false. (c) One premise is false, one premise is true, and the conclusion is false.

2. Give examples of *invalid* syllogisms with the following properties: (a) The premises are true and the conclusion is true. (b) The premises are false and the conclusion is true.

THE THEORY OF THE SYLLOGISM

Aristotle described fourteen valid forms of syllogistic argument. Medieval logicians gave each of them names, such as *Barbara* and *Celerant*. In Aristotle's logical theory there are four expressions, now called *quantifiers*, that can be prefixed to a subject-predicate phrase. The quantifiers are "all," "no," "some," and "not all." The traditional abbreviations for these quantifiers are respectively A, E, I, and O. By prefixing one of the quantifiers to a subject-predicate phrase, we obtain a sentence. An Aristotelian syllogism consists of three such sentences: two premises and a conclusion. (The names of the syllogisms contain a code for the quantifiers in the sentences in syllogisms of that form. The vowels in the names indicate the kind of quantifier in the second premise, the first premise, and the conclusion. Thus Darapti is a syllogism with two premises having "all" as their quantifier and a conclusion having "some" as its quantifier.)

These syllogisms are written so that the conclusion is always "(Quantifier) *A* are *C*." The term that occurs in the subject place in the conclusion (*A* in the examples below) is called the *minor* term. The term that occurs in the predicate place in the conclusion (*C* in the examples below) is called the *major* term. The term that occurs in the premises but not in the conclusion (*B* in the examples) is called the *middle term*.

The *form of a syllogistic argument* is determined entirely by the quantifiers attached to each sentence and by the positions of the terms in the premises. If we ignore the quantifiers for the moment, it is easy to see that there are four different patterns or *figures* (as they are called) in which the major, middle, and minor terms can be distributed (table 2.1). The valid Aristotelian syllogisms, with their medieval names, are listed in table 2.2. You may notice that the table of valid syllogisms contains no syllogisms having the pattern of figure 4. Aristotle did not include a study of syllogisms of this figure.

Table 2.1
The four figures of syllogistic arguments

Figure 1	Figure 2	Figure 3	Figure 4
A are *B*	*A* are *B*	*B* are *A*	*B* are *A*
B are *C*	*C* are *B*	*B* are *C*	*C* are *B*
A are *C*	*A* are *C*	*A* are *C*	*A* are *C*

Table 2.2
Valid Aristotelian syllogisms in the first three figures

1st figure	Barbara	Celarent	Darii	Ferio
A are *B* *B* are *C*	All *A* are *B* All *B* are *C*	All *A* are *B* No *B* are *C*	Some *A* are *B* All *B* are *C*	Some *A* are *B* No *B* are *C*
A are *C*	All *A* are *C*	No *A* are *C*	Some *A* are *C*	Not all *A* are *C*
2nd figure	Cesare	Camestres	Festino	Baroco
A are *B* *C* are *B*	All *A* are *B* No *C* are *B*	No *A* are *B* All *C* are *B*	Some *A* are *B* No *C* are *B*	Not all *A* are *B* All *C* are *B*
A are *C*	No *A* are *C*	No *A* are *C*	Not all *A* are *C*	Not all *A* are *C*
3rd figure	Darapti	Felapton	Disamis	Datisi
B are *A* *B* are *C*	All *B* are *A* All *B* are *C*	All *B* are *A* No *B* are *C*	All *B* are *A* Some *B* are *C*	Some *B* are *A* All *B* are *C*
A are *C*	Some *A* are *C*	Not all *A* are *C*	Some *A* are *C*	Some *A* are *C*
	Bocardo	Ferison		
	All *B* are *A* Not all *B* are *C*	Some *B* are *A* No *B* are *C*		
	Not all *A* are *C*	Not all *A* are *C*		

There are four possible quantifiers, any of which can attach to any sentence in a syllogism of any figure. Each syllogism has three sentences, and there are four choices of quantifier for the first sentence, four choices for the second sentence, and four choices for the third sentence, and thus there are $4 \times 4 \times 4 = 64$ distinct syllogistic forms in each figure. And since there are four figures, there are 256 distinct forms of syllogistic arguments altogether. Of the 192 syllogistic forms in the first three figures, Aristotle held that only the 14 illustrated are valid. All others are invalid. How did Aristotle come to this conclusion?

Aristotle held that the valid syllogisms of the first figure are *perfect*, by which he meant that their validity is obvious and self-evident and requires no proof. Assuming this is true, it remains to show that the other syllogistic forms he gives are also valid and that other syllogistic forms in the first three figures are invalid. To show the first, Aristotle assumed certain *rules of conversion*, which are really logical rules for inferring one sentence from another. Aristotle's rules of conversion include the following:

Rule 1 From "No *X* are *Y*," infer "No *Y* are *X*."

Rule 2 From "All *X* are *Y*," infer "Some *Y* are *X*."

Rule 3 From "Some *X* are *Y*," infer "Some *Y* are *X*."

With these three rules, some of the valid syllogisms of the second and third figures can be derived from the valid syllogisms of the first figure. Aristotle's strategy is to start with the premises of a second- or third-figure syllogism and to use the rules of conversion to derive the *premises* of a first-figure perfect syllogism. If the perfect syllogism shares its conclusion with the original second- or third-figure syllogism, it follows that the original syllogism is valid (assuming the first-figure perfect syllogisms are valid and that the rules of conversion preserve truth).

For example, Cesare can be transformed into Celarent by using the first rule of conversion on the second premise. That is, from "No *C* are *B*" we infer "No *B* are *C*" by rule 1 to obtain Celarent.

Cesare	**Celarent**
All *A* are *B*	All *A* are *B*
No *C* are *B*	No *B* are *C*
No *A* are *C*	No *A* are *C*

In the same way, other valid syllogisms of the second and third figure can be converted into a syllogism of the first figure (with the same conclusion) with the rules of conversion.

How did Aristotle show that the many syllogistic forms of the second and third figures that do not occur in the table above are not valid? To answer that, we have to be clearer about what it means for an argument *form* to be valid. The syllogistic forms in the table above are not sentences, they are abstract schemes that would become sentences if genuine terms were substituted for *A*, *B*, and *C*. An argument is valid if and only if it is not possible for its premises to be true and its conclusion false. A *syllogistic argument form is valid provided that, however we substitute real terms for the abstract A, B, and C in the syllogistic form, if the result is a syllogism with true premises, then the resulting conclusion is also true*. So in order to show that a syllogistic form is not valid, Aristotle needed only to find examples of syllogisms of that form in which the premises are both true and the conclusion is false.

Consider the following form of syllogistic argument:

No *A* are *B*
All *B* are *C*
____
No *A* are *C*

Aristotle shows that this form is not valid by considering the following example:

No horse is a man.
All men are animals.
____
No horses are animals.

In this case it is obvious that both of the premises are true but the conclusion is false. Hence the syllogistic form is not valid.

Study Questions

1. By providing an example in which the premises are clearly true and the conclusion is clearly false, show that each of the following syllogistic forms is *invalid*:

No *A* are *B*	No *A* are *B*	No *A* are *B*
All *B* are *C*	All *B* are *C*	All *B* are *C*
No *A* are *C*	Some *A* are *C*	Not all *A* are *C*

2. Use the valid syllogistic forms of the first figure and the rules of conversion to show the validity of the form Camestres and the form Felapton.

3. Do the rules of conversion given in the text suffice to show the validity of the forms Baroco and Bocardo? Why or why not?

4. Find the valid syllogistic forms in the fourth figure.*

LIMITATIONS OF ARISTOTLE'S SYLLOGISTIC THEORY OF DEDUCTIVE ARGUMENT

Although the theory of the syllogism is an interesting and impressive theory of deductive inference, it is not comprehensive. It does not include arguments that we and Aristotle's contemporaries recognize as valid. In other respects it is *too* comprehensive: Aristotle counts as valid some arguments that we would not count as valid.

Aristotle developed his theory of the syllogism as part of a theory of scientific demonstration. One of the great ironies of intellectual history is that while geometry was the paradigmatic Greek science and Euclid lived only a generation after Aristotle, the theory of the syllogism cannot account for even the simplest demonstrations in Euclid's *Elements*. There are several reasons why.

First, the propositions of geometry are not all of a simple subject-predicate form. In fact, rather few of them are. Instead, geometrical propositions deal with *relations* among objects. Second, the propositions of geometry do not all have *just* one quantifier; they may essentially involve repeated uses of "all" and "there exists." Third, proofs require devices for referring to the same object in different ways within the same sentence. Recall from chapter 1 the content of Euclid's first proposition:

Proposition 1 For every straight line segment, there exists an equilateral triangle having that line segment as one side.

To treat this claim as the conclusion of a syllogism, Aristotle would have to treat this sentence as having a single quantifier, "all"; a subject, "straight line segment"; and a predicate, "thing for which there exists an equilateral triangle having that thing as one side." Aristotle would therefore have to interpret the conclusion of Euclid's first proof as of the form

All *A* are *C*.

That is,

All straight line segments are things for which there exists an equilateral triangle having that thing as one side.

If we look at the table of valid syllogistic forms, we see that a conclusion of this form can only be obtained from a syllogism of the form Barbara. So for Aristotle's theory of deductive argument to apply, Euclid's proof would have to provide some middle term *B* and axioms or subconclusions of the following forms:

All *A* are *B*

All *B* are *C*

Or more concretely,

All straight line segments are *B*.

All *B* are things for which there exists an equilateral triangle having that thing as one side.

But that is not how Euclid's proof works. Recall that if the line segment has endpoints *P* and *Q*, Euclid constructs a circle centered on *P* and another circle centered on Q, each having the line segment as a radius. One of his postulates says that for every point and every length, a circle centered on that point having that length as radius exists (or can be constructed). Then Euclid assumes that there is a point at which the circle centered on Q and the circle centered on P intersect one another. This point, call it S, must be the same distance from P as P is from Q, and also the same distance from Q as Q is from P. By the construction and the definition of circle, the distance from Q to P is the same as the distance from P to Q, so point S must be the same distance from Q as P is from Q. Then Euclid uses the axiom that things equal to the same thing are equal to one another to infer that the distance from S to P is the distance from P to Q. So the distances PQ, PS, and QS are all equal. Another axiom guarantees that for all pairs of points there is a line segment connecting the points, and the definition of a triangle shows that the figure thus shown to exist is a triangle.

Aristotle might let B stand for "thing with endpoints that are the centers of circles with radii equal to the distance between the points." Then Aristotle would need to show that Euclid's proof contains a syllogistic demonstration of each of the following:

All straight line segments are things with endpoints that are centers of circles with radii equal to the distance between the points.

All things for which there exists an equilateral triangle having that thing as one side are things with endpoints that are the centers of circles with radii equal to the distance between the points.

Each of these will again have to be established by means of a syllogism of the form Barbara. But however many times we compound syllogisms of the Barbara form, we will never obtain a proof that looks at all like the argument that Euclid provided.

Aristotle's theory also fails to cover several other types of arguments. Recall that Aristotle *proves* that the syllogistic forms of the second and third figures shown in table 2.2 are valid forms. What is the form of those proofs? The proof I illustrated has the following form:

If Celarent is valid, then Cesare is valid.
Celarent is valid.
___
Therefore, Cesare is valid.

This is a perfectly valid deductive argument. It has the following form:

If P then Q
P
Therefore Q

Here P and Q stand for any complete sentences that are either true or false. *This argument is not one of Aristotle's valid syllogistic forms.* So Aristotle's own proof of the properties of his logical system uses logical principles that his system can neither represent nor account for. The argument just sketched depends on the logical properties of "If . . . then ____," where the ellipsis and the blank are filled by *sentences*. This form of argument is sometimes called a "hypothetical syllogism."

There is a third difficulty with Aristotle's theory of the syllogism. Look at the first four valid syllogisms of the third figure: Darapti, Felapton, Disamis, and Datisi. Each of them has an existential conclusion; that is, in each case the conclusion says that something exists having specified properties. So, for example, in Darapti we have the following inference:

All B are A
All B are C
Some A are C

Aristotle meant "Some A are C" to be read as "There exist some things that are A and C." So understood, it is not clear that Darapti is a valid form of inference. Consider the following example:

All unicorns are animals with hoofs.
All unicorns are horses with one horn.
Therefore, some animals with hoofs are horses with one horn.

This looks like an argument in which the premises are true but the conclusion is false. The problem is with the second rule of conversion:

From "All X are Y," infer "Some Y are X."

We don't think it is legitimate to infer "Some little people are leprechauns" from "All leprechauns are little people." We don't think it is legitimate to infer "Some numbers that are divisible by two are both even and odd" from "All numbers that are both even and odd are divisible by two." We reason all the time (both in fairy tales and in mathematics) about *all* things of a certain kind, even when we don't believe or mean to imply that things of that kind exist. In fact, in mathematics we often reason about such things just to prove that they don't exist! Aristotle would have agreed with our practice, but his *theory* seems not to agree.

AFTER ARISTOTLE

Aristotle's theory of deductive reasoning may have had many flaws. Yet despite minor improvements in the theory of syllogistic reasoning and some other developments in logical theory, no fundamental advances appeared for the next 2,400 years. Aristotle's successors at the Lyceum and after them the Stoic philosophers developed some of the principles of the logic of propositions. Their principles were understood by medieval logicians. For example, it was recognized that for any propositions P and Q, one could infer Q from premises consisting of the assertion of P and the assertion of "If P then Q." Medieval logicians even gave this form of inference a name, *modus ponens*:

Modus ponens From "P" and "If P then Q," infer "Q."

Other related logical principles were also understood, for example, the principle *modus tollens*:

Modus tollens From "Not Q" and "If P then Q," infer "Not P."

Theophrastus, who succeeded Aristotle as the head of the Lyceum, gave conditions for the truth of sentences compounded of simpler sentences. He proposed that any sentence of the form "If P then Q" is false only when P is true and Q is false. In any other circumstance, "If P then Q" is true. So in Theophrastus' view, "If P then Q" is true if P and Q are both false, if P is false and Q is true, and if both P and Q are true. In Theophrastus' conception, therefore, the truth or falsity of "If P then Q" is a function of the truth values (true or false) of P and Q. In other words, the truth value (true or false) of "If P then Q" is uniquely determined by the truth values of P and Q, just as the numerical value of the sum $X + Y$ is uniquely determined by the numerical values of X and Y. Sentences of the form "If ... then ____" are now known as *conditional sentences* or simply *conditionals*. The account of conditionals as truth functions of the simpler sentences from which they are composed was not widely accepted by logicians of the Middle Ages. They held instead that "If P then Q" is true only if the truth of P necessitates the truth of Q. With that understanding, the truth value of "If P then Q" is not a function of the truth values of P and Q. It isn't the truth or falsity of P and Q alone that determines the truth or falsity of "If P then Q," but whether the truth of P necessitates the truth of Q.

Further principles about inference with quantifiers were also recognized by Aristotle's successors. For example, they recognized the principle that from a universal claim one may infer any instance of it. From "Everything is such that if it is human, then it is mortal" one may infer "If Socrates is human, then Socrates is mortal."

Logic was extensively studied in the late Middle Ages from the twelfth through the fourteenth centuries. The theory of the syllogism was understood and extended in minor ways, and tracts were written on various sorts of quantifiers. Medieval logicians were especially interested in what we call *modal logic*, which is the study of deductive inferences that involve notions of necessity, possibility, and ability. Aristotle himself had written on the subject. Aristotle had maintained the following logical principles (which he did not clearly distinguish):

For any proposition *P*, "Necessarily *P*" is true if and only if "Not possibly not *P*" is true.

"*A* is necessarily *B*" is true if and only if "*A* is not possibly not *B*" is true.

Modal reasoning was of special concern to logicians of the Middle Ages because the motivation for their studies of logic was as much religious as it was scientific. They were concerned with features of God and with humanity's relations with God. These subjects involved complicated uses of claims about necessity and possibility. For example, Saint Anselm's proof of God's existence seems to turn on the idea that God is an entity that could not possibly not exist, an entity that necessarily exists. Notions of possibility and necessity can easily lead to paradoxes, which require a logical theory to untangle.

These and other logical investigations amounted to some limited progress in understanding valid reasoning. But at the end of the fourteenth century, Western civilization was not substantially closer to understanding deductive inference than it had been in the fourth century B.C. It was still not possible, for example, to give a systematic theory of proof that would include the proofs of geometry and exclude fallacies. Although additional logical principles had been developed after Aristotle, they had not been formed into a powerful systematic theory. The three central questions posed in chapter 1 were not much closer to being answered.

ARISTOTELIAN REASONING IN ARTIFICIAL INTELLIGENCE*

Although Aristotle's theory of demonstrative reasoning is inadequate to represent most proofs in mathematics and the sciences, a lot of simple reasoning can be represented as syllogistic.