COMP3121-Ass4-Q4

z5302513, Kihwan Baek

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1. Solution

Before we solve the problem, we make a bipartite graph to change the problem to a maximum matching in a bipartite graph. Firstly, we set all n columns as vertices and put those on the left hand side. And also, we set all n rows as vertices and put on the right hand side and we will draw an edge between ith row and ith column if a rook at cell (a_i,b_i) is not under the attack of any of the k bishops.

Then, we turn the maximum matching problem into a max flow problem by adding a super source and a super sink, and by giving all edges a capacity of 1. Hence, we use Edmonds-Karp algorithm to increase the total throughput. If we get m flows ($0 \leq m \leq k$) using the algorithm, we can say we can place up to m black rooks on the board. If we get n flows ($n > k$) using the algorithm, we can say we can place up to k black rooks on the board.

What we need to note at this stage is that we already drew the edges between vertices by considering only the attack of k bishops and not considering that no two rooks are in the same row..(1). However, because all weights of edges are 1, by changing this problem as max flow problem, we can satisfy this condition (1) using residual network.

2. Time Complexity

The total time complexity includes drawing edges, using Edmonds-Karp algorithm. Firstly, in the case of drawing edges, if there is a cell at (a_i,b_i) in which we are considering to put a rook, we need to calculate the slope between the cell and cells in which k bishops are located. If the slope is 1 or -1 or 0, we cannot put rook at (a_i,b_i). So, it takes O(kn^2) time.

And also, if we define the number of edges as $|E|$ after we draw all edges before using Edmonds-Karp algorithm, it takes min(O($n|E|^2$), O($|E|f$)) = O($|E|f$) = O($n|E|$) time because the maximum time complexity of f(=flow) is O(n). Therefore, the total time complexity is O($kn^2 + n|E|$) = O($kn^2 + n|E|$).