COMP3121-Ass1-Q5

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Q5.

(a)

As n goes to infinity, f(n) and g(n) go to infinity. So, $\lim_{n\to\infty} f(x)/g(x) = \lim_{n\to\infty} f'(x)/g'(x)$ using L'Hopital's rule.
And also, we can find $\lim_{n\to\infty} f'(x)/g'(x) = \frac{10}{n^2 \log 2} = 0$. Hence, g(n) grows substantially faster than f(n) because the slope of g(n) is much steeper than the slope of f(n). Moreover, there exists positive constants c and n_0 such that $0 \le f(n) \le cg(n)$ for all n $\ge n_0$. So, we can say that g(n) is an asymptotic upper bound for f(n) (f(n) = O(g(n)).

(b)

g(n) = $2^{n \log n^2}$ is an asymptotic lower bound for f(n) = n^n because when n is greater than 2 we have $g(n) = 2^{n \log n^2} < n^n = f(n)$. And also, we can say that there exists positive constants and n_0 such that $0 \le cg(n) \le f(n)$ for all $n \ge n_0$ and here we can take c = 1 and n_0 = 3.

(c)

Firstly, in the case of f(n), when n is a natural number, $sin(\pi n)$ is always 0. So, we can say that f(n) = n for all natural number n. Then, $\lim_{n\to\infty} f(x)/g(x) =$

1 and it shows f(n) = O(g(n)) and $f(n) = \Omega(g(n))$. Hence, g(n) is both an asymptotical lower bound for f(n).