

COMP3121-Ass4-Q1

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1. Solution

Firstly, we construct a flow network as a directed graph where all computers are vertices $1, 2, 3, \dots, N$ and the M one-directional links are edges. And also, we set the computer 1 as a source and the computer N as a sink. After that, we set each weight of edges as the cost of removing.

If there are connection from computer 1 to computer N , we must eliminate them to prevent sending a virus. To find the minimum total cost of removing edges, we run the Edmonds-Karp algorithm to find the maximal flow through such a network. After the algorithm has converged, we construct the last residual network flow and look at all the vertices to which there is a path from the source S and this process takes $O(NM^2)$ time.

Therefore, this will define a minimal cut and we can find all the edges crossing such a minimal cut. Hence, we need to remove the edges found on minimal cut to prevent sending a virus and the sum of weight of edges will be the minimum total cost of removing edges. At the worst case, we need to remove all edges from vertices $1, 2, \dots, N-1$ to the vertex N and it takes $O(M)$ time. So, the total time complexity is $O(M) + O(NM^2) = O(NM^2)$.