In [1]:
```python
import math
import numpy as np
import pandas as pd
from matplotlib import pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
from sklearn.model_selection import train_test_split
from sklearn.model_selection import cross_val_score
from sklearn.model_selection import RepeatedKFold
from sklearn.model_selection import GridSearchCV
from sklearn.linear_model import LinearRegression
from sklearn.linear_model import Ridge
from sklearn.linear_model import Lasso
from sklearn.linear_model import ElasticNet
from sklearn.metrics import mean_squared_error
```

Reading in the data about wine preferences

In [2]:
```python
wine_quality_data = pd.read_csv("winequality-red.csv", sep=';')
```

In [3]:
```python
wine_quality_data.shape
```

Out[3]: (1599, 12)

In [4]:
```python
wine_quality_data.head(5)
```

Out[4]:

	fixed acidity	volatile acidity	citric acid	residual sugar	chlorides	free sulfur dioxide	total sulfur dioxide	density	pH	sulphates	alcohol	qu
0	7.4	0.70	0.00	1.9	0.076	11.0	34.0	0.9978	3.51	0.56	9.4	
1	7.8	0.88	0.00	2.6	0.098	25.0	67.0	0.9968	3.20	0.68	9.8	
2	7.8	0.76	0.04	2.3	0.092	15.0	54.0	0.9970	3.26	0.65	9.8	
3	11.2	0.28	0.56	1.9	0.075	17.0	60.0	0.9980	3.16	0.58	9.8	
4	7.4	0.70	0.00	1.9	0.076	11.0	34.0	0.9978	3.51	0.56	9.4	

In [5]:
```python
wine_quality_data.tail(5)
```

Out[5]:

	fixed acidity	volatile acidity	citric acid	residual sugar	chlorides	free sulfur dioxide	total sulfur dioxide	density	pH	sulphates	alcohol
1594	6.2	0.600	0.08	2.0	0.090	32.0	44.0	0.99490	3.45	0.58	10.5
1595	5.9	0.550	0.10	2.2	0.062	39.0	51.0	0.99512	3.52	0.76	11.2
1596	6.3	0.510	0.13	2.3	0.076	29.0	40.0	0.99574	3.42	0.75	11.0
1597	5.9	0.645	0.12	2.0	0.075	32.0	44.0	0.99547	3.57	0.71	10.2
1598	6.0	0.310	0.47	3.6	0.067	18.0	42.0	0.99549	3.39	0.66	11.0

Train/Test Split

In [6]:
```
X = wine_quality_data.iloc[:,:-1]
```

In [7]:
```
X.head(5)
```

Out[7]:

	fixed acidity	volatile acidity	citric acid	residual sugar	chlorides	free sulfur dioxide	total sulfur dioxide	density	pH	sulphates	alcohol
0	7.4	0.70	0.00	1.9	0.076	11.0	34.0	0.9978	3.51	0.56	9.4
1	7.8	0.88	0.00	2.6	0.098	25.0	67.0	0.9968	3.20	0.68	9.8
2	7.8	0.76	0.04	2.3	0.092	15.0	54.0	0.9970	3.26	0.65	9.8
3	11.2	0.28	0.56	1.9	0.075	17.0	60.0	0.9980	3.16	0.58	9.8
4	7.4	0.70	0.00	1.9	0.076	11.0	34.0	0.9978	3.51	0.56	9.4

In [8]:
```
y = wine_quality_data.iloc[:,-1:]
```

In [9]:
```
y.head(5)
```

Out[9]:

	quality
0	5
1	5
2	5
3	6
4	5

In [10]:
```
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2, ra
```

Model Development

Ordinary Least Squares

In [11]:
```
lr_model = LinearRegression()
lr_model.fit(X_train, y_train)
```

Out[11]:
```
LinearRegression(copy_X=True, fit_intercept=True, n_jobs=None,
         normalize=False)
```

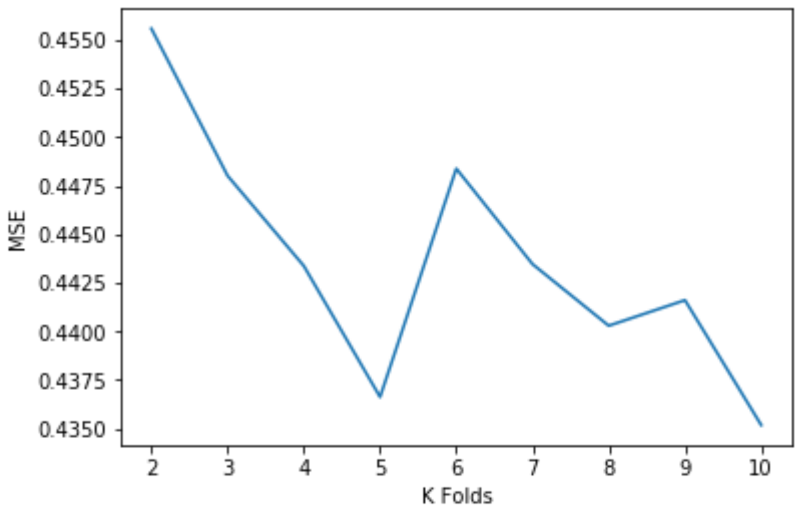
Perform Cross Validation

In [12]:
```
cv_values = [i for i in range(2,11)]
accuracy_values_mse = []
```

In [13]:
```
for i in cv_values:
    accuracy_mse = -cross_val_score(lr_model, X, y, scoring='neg_mean_squar
    accuracy_values_mse.append(accuracy_mse)
```

Show plot of k number of folds and accuracy values

In [14]:
```
plt.plot(cv_values, accuracy_values_mse)
plt.xlabel("K Folds")
plt.ylabel("MSE")
plt.show()
```



Print out the coefficients of the linear regression model corresponding to each of the features, along with the intercept term.

In [15]:
```
for id_value, column_name in enumerate(X_train.columns):
    print("The coefficient for {} is {}".format(column_name, lr_model.coef_

lr_intercept = lr_model.intercept_[0]
print("The intercept term for our model is {}".format(lr_intercept))
```

```
The coefficient for fixed acidity is 0.023085333909279103
The coefficient for volatile acidity is -1.0013044340678174
The coefficient for citric acid is -0.14082146122412922
The coefficient for residual sugar is 0.0065643110414779346
The coefficient for chlorides is -1.8065031490473662
The coefficient for free sulfur dioxide is 0.005627334387083055
The coefficient for total sulfur dioxide is -0.0036444489338687603
The coefficient for density is -10.351593588833333
The coefficient for pH is -0.3936877323398524
The coefficient for sulphates is 0.8411716226093248
The coefficient for alcohol is 0.2818895674091875
The intercept term for our model is 14.355105195764843
```

In [16]:
```
print("The minimum mse value for ols is: " + str(min(accuracy_values_mse)))
```

```
The minimum mse value for ols is: 0.43518490403009247
```

Ridge Regression

Perform repeated k-fold cross validation

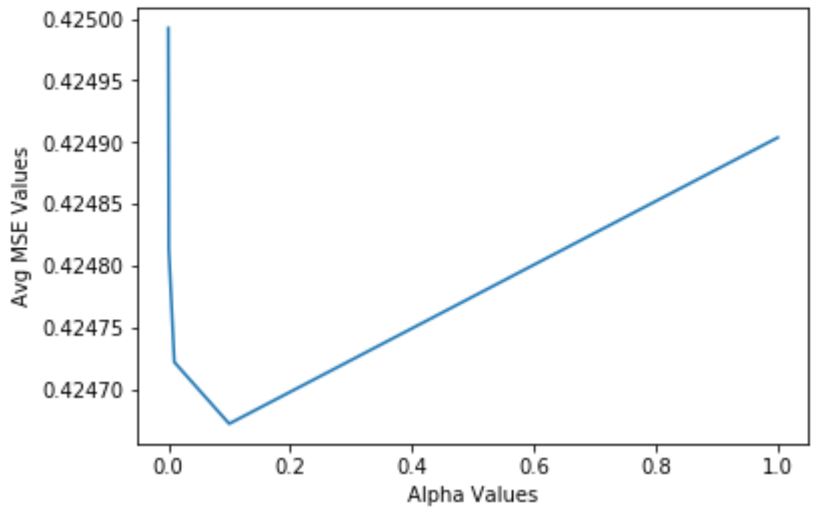
In [17]:
```
mse_values = [0, 0, 0, 0, 0]
alphas = [0.0001, 0.001, 0.01, 0.1, 1.0]
num_iterations = 0
```

In [18]:
```
kf = RepeatedKFold(n_splits = 10, n_repeats = 10)

for train_index, test_index in kf.split(X):
    X_train_indexed, X_test_indexed = X.iloc[train_index, :], X.iloc[test_i
    y_train_indexed, y_test_indexed = y.iloc[train_index, :], y.iloc[test_i

    for i in range(5):
        alpha = alphas[i]
        ridge_model = Ridge(alpha=alpha)
        ridge_model.fit(X_train_indexed, y_train_indexed)
        y_predicted = ridge_model.predict(X_test_indexed)
        mse = mean_squared_error(y_test_indexed, y_predicted)
        mse_values[i] += mse

    num_iterations += 1
```

In [19]:
```
avg_mse_values = [mse_value / num_iterations for mse_value in mse_values]
```

Create a plot of the alpha (tuning parameter) and average mean squared error values

In [20]:
```
plt.plot(alphas, avg_mse_values)
plt.xlabel("Alpha Values")
plt.ylabel("Avg MSE Values")
plt.show()
```



Find the optimal alpha (tuning parameter) value and use it to build a ridge regression model.

In [21]:
```
min_avg_mse_value = min(avg_mse_values)
min_error_index = avg_mse_values.index(min_avg_mse_value)
alpha = alphas[min_error_index]
```

In [22]:
```
ridge_model = Ridge(alpha=alpha)
ridge_model.fit(X_train, y_train)
y_predicted = ridge_model.predict(X_test)
mse = mean_squared_error(y_test, y_predicted)
```

Specify the ridge regression model coefficients and intercept

In [23]:
```
coefficients = pd.DataFrame({"Features":X.columns,"Coefficients":np.transpo
coefficients.head(11)
```

Out[23]:

	Features	Coefficients
0	fixed acidity	0.014517
1	volatile acidity	-1.012339
2	citric acid	-0.146866
3	residual sugar	0.002027
4	chlorides	-1.735658
5	free sulfur dioxide	0.005721
6	total sulfur dioxide	-0.003657
7	density	-0.079639
8	pH	-0.436087
9	sulphates	0.816781
10	alcohol	0.292164

In [24]:
```
intercept = pd.DataFrame({"Intercept": ridge_model.intercept_})
intercept.head()
```

Out[24]:

	Intercept
0	4.249021

Specify the minimum cross validation error

In [25]:
```
print("The minimum average cross validation mse is: " + str(min_avg_mse_val
```

```
The minimum average cross validation mse is: 0.4246719328182429
```

Lasso Regression

Perform repeated k-fold cross validation

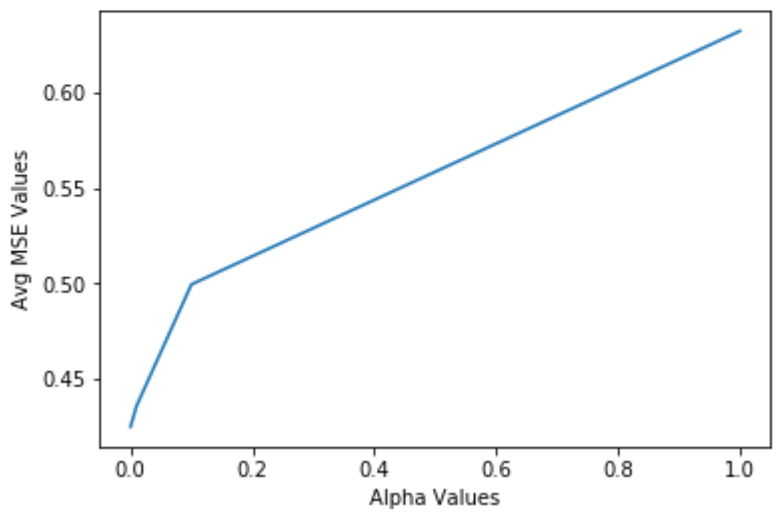
In [26]:
```
mse_values = [0, 0, 0, 0, 0]
alphas = [0.0001, 0.001, 0.01, 0.1, 1.0]
num_iterations = 0
```

In [27]:
```
kf = RepeatedKFold(n_splits = 10, n_repeats = 10)

for train_index, test_index in kf.split(X):
    X_train_indexed, X_test_indexed = X.iloc[train_index, :], X.iloc[test_i
    y_train_indexed, y_test_indexed = y.iloc[train_index, :], y.iloc[test_i

    for i in range(5):
        alpha = alphas[i]
        lasso_model = Lasso(alpha=alpha)
        lasso_model.fit(X_train_indexed, y_train_indexed)
        y_predicted = lasso_model.predict(X_test_indexed)
        mse = mean_squared_error(y_test_indexed, y_predicted)
        mse_values[i] += mse

    num_iterations += 1
```

In [28]:
```
avg_mse_values = [mse_value / num_iterations for mse_value in mse_values]
```

Create a plot of the alpha (tuning parameter) and average mean squared error values

In [29]:
```
plt.plot(alphas, avg_mse_values)
plt.xlabel("Alpha Values")
plt.ylabel("Avg MSE Values")
plt.show()
```



Find the optimal alpha (tuning parameter) value and use it to build a lasso regression model.

In [30]:
```
min_avg_mse_value = min(avg_mse_values)
min_error_index = avg_mse_values.index(min_avg_mse_value)
alpha = alphas[min_error_index]
```

In [31]:
```
lasso_model = Lasso(alpha=alpha)
lasso_model.fit(X_train, y_train)
y_predicted = lasso_model.predict(X_test)
mse = mean_squared_error(y_test, y_predicted)
```

Specify the lasso regression model coefficients and intercept

In [32]:
```
coefficients = pd.DataFrame({"Features":X.columns,"Coefficients":np.transpo
coefficients.head(11)
```

Out[32]:

	Features	Coefficients
0	fixed acidity	0.013953
1	volatile acidity	0.013953
2	citric acid	0.013953
3	residual sugar	0.013953
4	chlorides	0.013953
5	free sulfur dioxide	0.013953
6	total sulfur dioxide	0.013953
7	density	0.013953
8	pH	0.013953
9	sulphates	0.013953
10	alcohol	0.013953

In [33]:
```
intercept = pd.DataFrame({"Intercept": lasso_model.intercept_})
intercept.head()
```

Out[33]:

	Intercept
0	4.163343

Specify the minimum cross validation error

In [34]:
```
print("The minimum average cross validation mse is: " + str(min_avg_mse_val
```

```
The minimum average cross validation mse is: 0.42491066451983206
```

Elastic Net

Perform repeated k-fold cross validation

In [35]:
```
mse_values = [[0, 0, 0, 0, 0], [0, 0, 0, 0, 0], [0, 0, 0, 0, 0], [0, 0, 0,
alphas = [0.0001, 0.001, 0.01, 0.1, 1.0]
ratios = [0.2, 0.4, 0.6, 0.8, 1.0]
num_iterations = 0
```

In [36]:
```
kf = RepeatedKFold(n_splits = 10, n_repeats = 10)

for train_index, test_index in kf.split(X):
    X_train_indexed, X_test_indexed = X.iloc[train_index, :], X.iloc[test_i
    y_train_indexed, y_test_indexed = y.iloc[train_index, :], y.iloc[test_i

    for i in range(5):
        alpha = alphas[i]
        for j in range(5):
            ratio = ratios[j]
            elastic_net_model = ElasticNet(alpha=alpha, l1_ratio=ratio)
            elastic_net_model.fit(X_train_indexed, y_train_indexed)
            y_predicted = elastic_net_model.predict(X_test_indexed)
            mse = mean_squared_error(y_test_indexed, y_predicted)
            mse_values[i][j] += mse
            num_iterations += 1
```

In [37]:
```
avg_mse_values = [mse_value / num_iterations for mse_value_array in mse_val
```

Create a plot of the alpha and l1_ratio (tuning parameters) and average mean squared error values

In [38]:
```
print(avg_mse_values)
```

```
[0.016997942416315717, 0.016998189361100223, 0.016998450227699847, 0.0169
9873982050371, 0.016999056299432342, 0.017024256119224184, 0.017026600759
85282, 0.017029355454885278, 0.01703249650813154, 0.017035778082767013,
0.017325910060408744, 0.017373185322344442, 0.017387701929380456, 0.01740
949731946985, 0.017440247655287455, 0.01900360166096478, 0.01958639602768
0045, 0.019688180480933883, 0.01981779864210686, 0.0199795805993513, 0.02
2735024044205916, 0.02520520482272836, 0.025266918431463843, 0.0252752831
60936517, 0.025288100441754766]
```

In [39]:
```
print(alphas)
```

```
[0.0001, 0.001, 0.01, 0.1, 1.0]
```

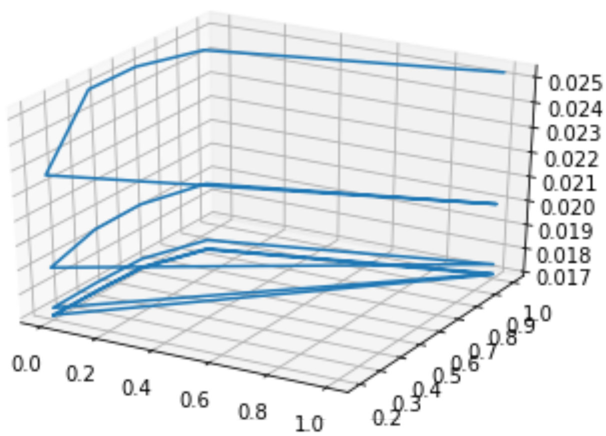
In [40]:
```
print(ratios)
```

```
[0.2, 0.4, 0.6, 0.8, 1.0]
```

In [41]:
```
x_values = alphas * 5
y_values = ratios * 5
z_values = avg_mse_values
```

In [42]:
```
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
ax.plot(xs = x_values, ys = y_values, zs = z_values)
```

Out[42]:
```
[<mpl_toolkits.mplot3d.art3d.Line3D at 0x1a20cd6cc0>]
```



Find the optimal alpha and alpha and l1_ratio (tuning parameters) values and use it to build a elastic net model.

In [43]:
```
min_avg_mse_value = min(avg_mse_values)
min_error_index = avg_mse_values.index(min_avg_mse_value)
alpha = alphas[min_error_index]
l1_ratio = ratios[min_error_index]
```

In [44]:
```
elastic_net_model = ElasticNet(alpha=alpha, l1_ratio=l1_ratio)
elastic_net_model.fit(X_train_indexed, y_train_indexed)
y_predicted = elastic_net_model.predict(X_test_indexed)
mse = mean_squared_error(y_test_indexed, y_predicted)
```

Specify the elastic net model coefficients and intercept

In [45]:
```
coefficients = pd.DataFrame({"Features":X.columns,"Coefficients":np.transpo
coefficients.head(11)
```

Out[45]:

	Features	Coefficients
0	fixed acidity	0.015212
1	volatile acidity	0.015212
2	citric acid	0.015212
3	residual sugar	0.015212
4	chlorides	0.015212
5	free sulfur dioxide	0.015212
6	total sulfur dioxide	0.015212
7	density	0.015212
8	pH	0.015212
9	sulphates	0.015212
10	alcohol	0.015212

In [46]:
```
intercept = pd.DataFrame({"Intercept": elastic_net_model.intercept_})
intercept.head()
```

Out[46]:

	Intercept
0	4.241787