MATH1853 Tutorial 1

Who am I

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Question 1: Matrix Addition

1. *(Matrix Addition)* Compute the following matrix additions, if possible:

a. $\begin{bmatrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$ **b.** $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 3 \end{bmatrix}$

c. $\begin{bmatrix} 2 & 0 & 4 \\ 3 & 1 & 3 \\ 2 & 5 & 9 \end{bmatrix} - 3 \cdot \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 3 \end{bmatrix}$

Key points:

- The ***<u>dimensions</u>*** of addends must match.
- Perform ***<u>element-wise addition.</u>***
- When multiply with a scalar, do multiplication ***<u>with each element.</u>***

Question 1: Matrix Addition

a. $\begin{bmatrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 3 \end{bmatrix}$

c. $\begin{bmatrix} 2 & 0 & 4 \\ 3 & 1 & 3 \\ 2 & 5 & 9 \end{bmatrix} - 3 \cdot \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 3 \end{bmatrix}$

- (a) cannot add because unmatched dimension

- (b) Element-wise addition

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1+2 & 2+1 & 3+0 \\ 4+0 & 5+1 & 6+1 \\ 7+1 & 8+0 & 9+3 \end{bmatrix} = \begin{bmatrix} 3 & 3 & 3 \\ 4 & 6 & 7 \\ 8 & 8 & 12 \end{bmatrix}$$

- (c) Multiply with scalar first, then element-wise addition

$$\begin{bmatrix} 2 & 0 & 4 \\ 3 & 1 & 3 \\ 2 & 5 & 9 \end{bmatrix} - 3 \cdot \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 4 \\ 3 & 1 & 3 \\ 2 & 5 & 9 \end{bmatrix} - \begin{bmatrix} 3\times 2 & 3\times 1 & 3\times 0 \\ 3\times 0 & 3\times 1 & 3\times 1 \\ 3\times 1 & 3\times 0 & 3\times 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2-6 & 0-3 & 4-0 \\ 3-0 & 1-3 & 3-3 \\ 2-3 & 5-0 & 9-9 \end{bmatrix} = \begin{bmatrix} -4 & -3 & 4 \\ 3 & -2 & 0 \\ -1 & 5 & 0 \end{bmatrix}$$

Question 2: Matrix Multiplication

2. *(Matrix Multiplication)* Compute the following matrix products, if possible:

a. $\begin{bmatrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 3 \end{bmatrix}$

c. $\begin{bmatrix} 0 \\ 3 \\ 2 \\ 4 \end{bmatrix} \begin{bmatrix} 0 & 1 & 3 & 0 \end{bmatrix}$

d. $\begin{bmatrix} 0 & 1 & 3 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \\ 2 \\ 4 \end{bmatrix}$

e. $\begin{bmatrix} 2 & 7 & 3 \\ 5 & 4 & 4 \end{bmatrix} \begin{bmatrix} 6 & 3 & 2 \\ 0 & 4 & 3 \\ 6 & 5 & 3 \end{bmatrix}$

f. $3 \cdot \begin{bmatrix} 6 & 3 & 2 \\ 0 & 4 & 3 \\ 6 & 5 & 3 \end{bmatrix}$

Key points:

- The dimensions of operands ***must fit*** ($n \times m$ with $m \times p$)
- Product would have shape ($n \times p$)
- Result elements: dot product of ***corresponding row and column***.

Question 2: Matrix Multiplication

a. $\begin{bmatrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 3 \end{bmatrix}$

c. $\begin{bmatrix} 0 \\ 3 \\ 2 \\ 4 \end{bmatrix} \begin{bmatrix} 0 & 1 & 3 & 0 \end{bmatrix}$

d. $\begin{bmatrix} 0 & 1 & 3 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \\ 2 \\ 4 \end{bmatrix}$

- (a) not possible, dimensions not fit (3×2 with 3×3)
- (b) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 3 & 11 \\ 14 & 9 & 23 \\ 23 & 15 & 35 \end{bmatrix}$
- (c) $\begin{bmatrix} 0 \\ 3 \\ 2 \\ 4 \end{bmatrix} \begin{bmatrix} 0 & 1 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 3 & 9 & 0 \\ 0 & 2 & 6 & 0 \\ 0 & 4 & 12 & 0 \end{bmatrix}$
- (d) $\begin{bmatrix} 0 & 1 & 3 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \\ 2 \\ 4 \end{bmatrix} = 9$

Question 3: Matrix Multiplication

3. *(Matrix Multiplication)* Compute the following matrix products:

a. $\begin{bmatrix} 6 & 2 & 7 \\ 0 & 1 & 5 \\ 8 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 6 & 2 & 7 \\ 0 & 1 & 5 \\ 8 & 0 & 3 \end{bmatrix}$

c. $\begin{bmatrix} 2 & 1 & 0 \\ 4 & 3 & 2 \\ -1 & 1 & 4 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

d. $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 4 & 3 & 2 \\ -1 & 1 & 4 \end{bmatrix}$

e. $\begin{bmatrix} 6 & 2 & 7 \\ 0 & 1 & 5 \\ 8 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Key point:

- Multiplication with the ***identity matrix.***

$$\forall \boldsymbol{A} \in \mathbb{R}^{m \times n} : \boldsymbol{I}_m \boldsymbol{A} = \boldsymbol{A} \boldsymbol{I}_n = \boldsymbol{A} \qquad \boldsymbol{I}_m \neq \boldsymbol{I}_n \text{ for } m \neq n.$$

Question 3: Matrix Multiplication

a. $\begin{bmatrix} 6 & 2 & 7 \\ 0 & 1 & 5 \\ 8 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 6 & 2 & 7 \\ 0 & 1 & 5 \\ 8 & 0 & 3 \end{bmatrix}$

- (a) $\begin{bmatrix} 6 & 2 & 7 \\ 0 & 1 & 5 \\ 8 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 2 & 7 \\ 0 & 1 & 5 \\ 8 & 0 & 3 \end{bmatrix}$
- (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 6 & 2 & 7 \\ 0 & 1 & 5 \\ 8 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 6 & 2 & 7 \\ 0 & 1 & 5 \\ 8 & 0 & 3 \end{bmatrix}$

Question 3: Matrix Multiplication

c. $\begin{bmatrix} 2 & 1 & 0 \\ 4 & 3 & 2 \\ -1 & 1 & 4 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ d. $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 4 & 3 & 2 \\ -1 & 1 & 4 \end{bmatrix}$

- (c) $\begin{bmatrix} 2 & 1 & 0 \\ 4 & 3 & 2 \\ -1 & 1 & 4 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 1 \\ 2 & 4 & 3 \\ 4 & -1 & 1 \end{bmatrix}$

- (d) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 4 & 3 & 2 \\ -1 & 1 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 3 & 2 \\ -1 & 1 & 4 \\ 2 & 1 & 0 \end{bmatrix}$

Key point:

- Get a feeling of ***Matrix transformation !***

Question 4: Matrix Multiplication Properties

a. $\left(\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}\right) \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$

b. $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \left(\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}\right)$

Key point:

- ***Matrix Multiplication Associativity***

$$\forall \boldsymbol{A} \in \mathbb{R}^{m \times n}, \boldsymbol{B} \in \mathbb{R}^{n \times p}, \boldsymbol{C} \in \mathbb{R}^{p \times q} : (\boldsymbol{AB})\boldsymbol{C} = \boldsymbol{A}(\boldsymbol{BC})$$

Question 4: Matrix Multiplication Properties

- (a) $\left(\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}\right)\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$

$= \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$

$= \begin{bmatrix} a_{11}b_{11}c_{11} + a_{12}b_{21}c_{11} + a_{11}b_{12}c_{21} + a_{12}b_{22}c_{21} & a_{11}b_{11}c_{12} + a_{12}b_{21}c_{12} + a_{11}b_{12}c_{22} + a_{12}b_{22}c_{22} \\ a_{21}b_{11}c_{11} + a_{22}b_{21}c_{11} + a_{21}b_{12}c_{21} + a_{22}b_{22}c_{21} & a_{21}b_{11}c_{12} + a_{22}b_{21}c_{12} + a_{21}b_{12}c_{22} + a_{22}b_{22}c_{22} \end{bmatrix}$

- (b) $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\left(\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}\right)$

$= \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} b_{11}c_{11} + b_{12}c_{21} & b_{11}c_{12} + b_{12}c_{22} \\ b_{21}c_{11} + b_{22}c_{21} & b_{21}c_{12} + b_{22}c_{22} \end{bmatrix}$

$= \begin{bmatrix} a_{11}b_{11}c_{11} + a_{12}b_{21}c_{11} + a_{11}b_{12}c_{21} + a_{12}b_{22}c_{21} & a_{11}b_{11}c_{12} + a_{12}b_{21}c_{12} + a_{11}b_{12}c_{22} + a_{12}b_{22}c_{22} \\ a_{21}b_{11}c_{11} + a_{22}b_{21}c_{11} + a_{21}b_{12}c_{21} + a_{22}b_{22}c_{21} & a_{21}b_{11}c_{12} + a_{22}b_{21}c_{12} + a_{21}b_{12}c_{22} + a_{22}b_{22}c_{22} \end{bmatrix}$

Question 4: Matrix Multiplication Properties

$$\textbf{c.}\ \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \left(\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} + \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \right)$$

$$\textbf{d.}\ \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$$

Key point:

- ***Matrix Multiplication Distributivity***

$$\begin{aligned} \forall \boldsymbol{A}, \boldsymbol{B} \in \mathbb{R}^{m \times n}, \boldsymbol{C}, \boldsymbol{D} \in \mathbb{R}^{n \times p} : (\boldsymbol{A} + \boldsymbol{B})\boldsymbol{C} &= \boldsymbol{AC} + \boldsymbol{BC} \\ \boldsymbol{A}(\boldsymbol{C} + \boldsymbol{D}) &= \boldsymbol{AC} + \boldsymbol{AD} \end{aligned}$$

Question 4: Matrix Multiplication Properties

- (c) $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \left(\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} + \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \right)$

$= \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} + c_{11} & b_{12} + c_{12} \\ b_{21} + c_{21} & b_{22} + c_{22} \end{bmatrix}$

$= \begin{bmatrix} a_{11}b_{11} + a_{11}c_{11} + a_{12}b_{21} + a_{12}c_{21} & a_{11}b_{12} + a_{11}c_{12} + a_{12}b_{22} + a_{12}c_{22} \\ a_{21}b_{11} + a_{21}c_{11} + a_{22}b_{21} + a_{22}c_{21} & a_{21}b_{12} + a_{21}c_{12} + a_{22}b_{22} + a_{22}c_{22} \end{bmatrix}$

- (d) $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$

$= \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix} + \begin{bmatrix} a_{11}c_{11} + a_{12}c_{21} & a_{11}c_{12} + a_{12}c_{22} \\ a_{21}c_{11} + a_{22}c_{21} & a_{21}c_{12} + a_{22}c_{22} \end{bmatrix}$

$= \begin{bmatrix} a_{11}b_{11} + a_{11}c_{11} + a_{12}b_{21} + a_{12}c_{21} & a_{11}b_{12} + a_{11}c_{12} + a_{12}b_{22} + a_{12}c_{22} \\ a_{21}b_{11} + a_{21}c_{11} + a_{22}b_{21} + a_{22}c_{21} & a_{21}b_{12} + a_{21}c_{12} + a_{22}b_{22} + a_{22}c_{22} \end{bmatrix}$

Question 5: Multiplication by a Scalar

5. *(Multiplication by a Scalar)* Compute the following questions

a. $(3 \times 2) \cdot \begin{bmatrix} 4 & 3 \\ 5 & 2 \end{bmatrix}$ **b.** $2 \cdot \left(3 \cdot \begin{bmatrix} 4 & 3 \\ 5 & 2 \end{bmatrix}\right)$

MatMul Associativity

c. $(3 + 2) \cdot \begin{bmatrix} 4 & 3 \\ 5 & 2 \end{bmatrix}$ **d.** $3 \cdot \begin{bmatrix} 4 & 3 \\ 5 & 2 \end{bmatrix} + 2 \cdot \begin{bmatrix} 4 & 3 \\ 5 & 2 \end{bmatrix}$

e. $2 \cdot \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix} + 2 \cdot \begin{bmatrix} 3 & 2 & 0 \\ 1 & 5 & 4 \end{bmatrix}$ **f.** $2 \cdot \left(\begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix} + \begin{bmatrix} 3 & 2 & 0 \\ 1 & 5 & 4 \end{bmatrix}\right)$

MatMul Distributivity

Key point:

- ***Matrix-Scalar Multiplication Distributivity***

$$(\lambda\psi)\boldsymbol{C} = \lambda(\psi\boldsymbol{C}), \quad \boldsymbol{C} \in \mathbb{R}^{m \times n}$$

Key point:

- ***Matrix-Scalar Multiplication Distributivity***

$$(\lambda + \psi)\boldsymbol{C} = \lambda\boldsymbol{C} + \psi\boldsymbol{C}, \quad \boldsymbol{C} \in \mathbb{R}^{m \times n}$$

$$\lambda(\boldsymbol{B} + \boldsymbol{C}) = \lambda\boldsymbol{B} + \lambda\boldsymbol{C}, \quad \boldsymbol{B}, \boldsymbol{C} \in \mathbb{R}^{m \times n}$$

Question 5: Multiplication by a Scalar

$$\mathbf{a.}(3 \times 2) \cdot \begin{bmatrix} 4 & 3 \\ 5 & 2 \end{bmatrix}$$

$$\mathbf{c.}(3 + 2) \cdot \begin{bmatrix} 4 & 3 \\ 5 & 2 \end{bmatrix}$$

$$\mathbf{e.} \quad 2 \cdot \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix} + 2 \cdot \begin{bmatrix} 3 & 2 & 0 \\ 1 & 5 & 4 \end{bmatrix}$$

(a) $(3 \times 2) \cdot \begin{bmatrix} 4 & 3 \\ 5 & 2 \end{bmatrix} = 6 \cdot \begin{bmatrix} 4 & 3 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 24 & 18 \\ 30 & 12 \end{bmatrix}$

(c) $(3 + 2) \begin{bmatrix} 4 & 3 \\ 5 & 2 \end{bmatrix} = 5 \begin{bmatrix} 4 & 3 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 20 & 15 \\ 25 & 10 \end{bmatrix}$

(e) $2 \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix} + 2 \begin{bmatrix} 3 & 2 & 0 \\ 1 & 5 & 4 \end{bmatrix}$
$= \begin{bmatrix} 2 & 6 & 10 \\ 4 & 8 & 12 \end{bmatrix} + \begin{bmatrix} 6 & 4 & 0 \\ 2 & 10 & 8 \end{bmatrix} = \begin{bmatrix} 8 & 10 & 10 \\ 6 & 18 & 20 \end{bmatrix}$

Question 6: Systems of Linear Equations

6. *(Systems of Linear Equations)* Solve the System of Linear Equation

$$
\begin{aligned}
2x_1 + 7x_2 + 8x_3 &= -9 \\
6x_1 - 3x_2 - 7x_3 &= 14 \\
4x_1 + 5x_2 - 3x_3 &= -10
\end{aligned}
$$

Then Compute the product

$$
\begin{bmatrix} 2 & 7 & 8 \\ 6 & -3 & -7 \\ 4 & 5 & -3 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}
$$

Key point:

- Get a feeling of how to ***use matrix to represent linear equations!***

Question 6: Systems of Linear Equations

Q6.1

$$2x_1 + 7x_2 + 8x_3 = -9$$
$$6x_1 - 3x_2 - 7x_3 = 14$$
$$4x_1 + 5x_2 - 3x_3 = -10$$

Q6.2

$$\begin{bmatrix} 2 & 7 & 8 \\ 6 & -3 & -7 \\ 4 & 5 & -3 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$$

$$2x_1 + 7x_2 + 8x_3 = -9 \qquad (1)$$
$$6x_1 - 3x_2 - 7x_3 = 14 \qquad (2)$$
$$4x_1 + 5x_2 - 3x_3 = -10 \qquad (3)$$

$Eq.(2) - 3 \times Eq.(1)$:
$$(6x_1 - 3x_2 - 7x_3) - 3(2x_1 + 7x_2 + 8x_3) = -24x_2 - 31x_3 = 14 - 3 \times (-9) = 41$$

$Eq.(3) - 2 \times Eq.(1)$:
$$(4x_1 + 5x_2 - 3x_3) - 2(2x_1 + 7x_2 + 8x_3) = -9x_2 - 19x_3 = -10 - 2 \times (-9) = 8$$

$$3(-24x_2 - 31x_3) - 8(-9x_2 - 19x_3) = 59\mathrm{x}_3 = 3 \times 41 - 8 \times 8 = 59$$

$x_3 = 1$
$x_2 = -3$
$x_1 = 2$

$$\begin{bmatrix} 2 & 7 & 8 \\ 6 & -3 & -7 \\ 4 & 5 & -3 \end{bmatrix} \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix} = \begin{bmatrix} -9 \\ 14 \\ -10 \end{bmatrix}$$

Summary

- ***Matrix addition***
- ***Matrix multiplication***
  - Matrix multiplication with the identity matrix
  - Get a feeling of matrix transformation
- ***Matrix(-Scalar) multiplication properties***
  - Associativity
  - Distributivity
- ***Systems of Linear Equations***
  - Get a feeling of how to use matrix to represent linear equations.