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Hash Functions, Bit Commitment, and Zero-Knowledge Proofs

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Overview

1. Cryptography Introduction
2. 1-way Functions
3. Bit Commitment
4. Zero-Knowledge Proofs

Cryptography Introduction

Cryptography Introduction

Short Introduction to Cryptography

Study of secrecy and communication across insecure channels to potentially dishonest users.

Basic assumptions:

- All users have at least equal computational power to us (usually assumed to be equal)
- All methods and processes are well-known to all users

Cryptography Introduction

Background: P vs. NP

Class **P**: problems which can be solved with a polynomial-time algorithm **(quickly solvable)**

Class **NP**: problems whose solutions can be verified in polynomial time **(quickly checkable)**

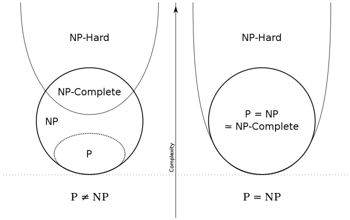
- Clearly, $P \subseteq NP$
- Converse is unknown (widely believed to be false)

Cryptography Introduction
NP-hard and NP-complete

Any NP problem can be easily reduced to an NP-hard problem, and all NP-hard problems in NP are called NP-complete.
Note: If there exists an NP-complete problem in P, all NP problems must be in P ($P = NP$).

Examples of NP-complete problems:

- Boolean satisfiability problem (**SAT**)
- Graph coloring problem
- Subgraph isomorphism problem
- Traveling salesman problem



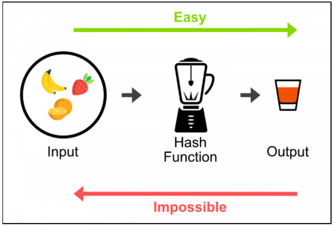
Complexity Venn diagram

1-way Functions

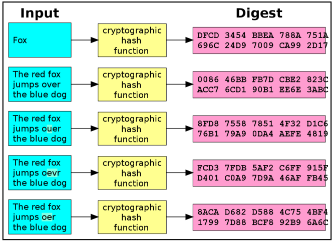
1-way Functions

Definition and Motivation

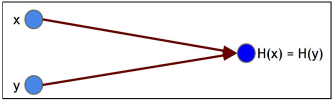
A 1-way function (or cryptographic hash function) is a deterministic function that is easy to compute and hard to invert. The most useful ones have a possibly infinite input length and fixed output length.



1-way-ness



Hides information about input



Collision resistance

1-way Functions

Defining Hardness-to-Invert

1-way functions' strength can be defined by strong and weak:

- Strong: any inversion algorithm succeeds with negligible probability
- Weak: any inversion algorithm fails with noticeable probability

Theorem: **Weak 1-way functions exist if and only if strong 1-way functions exist.**[1]

1-way Functions

Hardcore Predicates and Functions

Hardcore predicate of a 1-way function: **secure bit** based on a 1-way function's output.

Hardcore function is a generalization: secure sequence of bits.

Examples of hardcore predicates:

- The least significant bit of an RSA output
- XOR of a random subset of bits of a 1-way function

No noticeable advantage

Bit Commitment

Bit Commitment

Commitment scheme: protocol for a sender to commit to a bit **unambiguously** and **secretly**.

2 phases:

- Commit phase: sender commits to value publicly
- Reveal phase: sender reveals committed value and commitment process

Such a commitment scheme can be designed based on hardcore predicates or pseudorandom number generators.

Zero-Knowledge Proofs

Zero-Knowledge Proofs
Introduction

Zero-knowledge proof (ZK-pf): protocol for a sender to **unambiguously** demonstrate knowledge to a verifier while keeping the knowledge **secret**.

Simulation paradigm: **zero-knowledge** means that anything the verifier was able to compute after the ZK-pf, it could compute beforehand.

Zero-Knowledge Proofs

Creating ZK-Pfs

A ZK-pf for a solution to an NP-complete problem is a ZK-pf for a solution to any NP problem, since NP-complete problems can be reduced to any other NP problem.

Theorem: **Polynomially many repetitions of a ZK-pf remains zero-knowledge.**[1]

So we can repeat a proof that provides non-zero evidence over and over to create strong ZK-pfs.

Zero-Knowledge Proofs

Using SAT (E3SAT)

Prover must first:

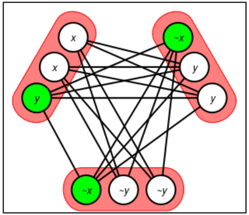
- Permute variable order and labels, clause order
- Invert each variable/assignment with probability 0.5
- Commit to new formula and assignment

Verifier can ask for either:

- New formula and the variable permutations
- One clause of the new formula and those assignments

(x OR y OR z) AND (x OR $\bar{y}$ OR z) AND
(x OR y OR $\bar{z}$) AND (x OR $\bar{y}$ OR $\bar{z}$) AND
($\bar{x}$ OR y OR z) AND ($\bar{x}$ OR $\bar{y}$ OR $\bar{z}$)

Formulaic Form



Tripartite Graph Form

References

O. Goldreich.
Foundations of Cryptography.
Cambridge University Press, Cambridge, United Kingdom, 2004.