Convergent Temporal-Difference Learning with Arbitrary Smooth Function Approximation

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24 mars 2017

Motivations

TD algorithms are great but potentially **unstable** in the **off-policy** case, or when the function approximation is **non-linear**.

Gradient methods (TDC, GTD...) help in the off-policy case.

The problem in the non-linear case remains $\Rightarrow$ **non-linear extensions of GTD and TDC**

Convergent as long as the function approximation is **smooth** enough.

The objective function

Linear TDC, GTD :

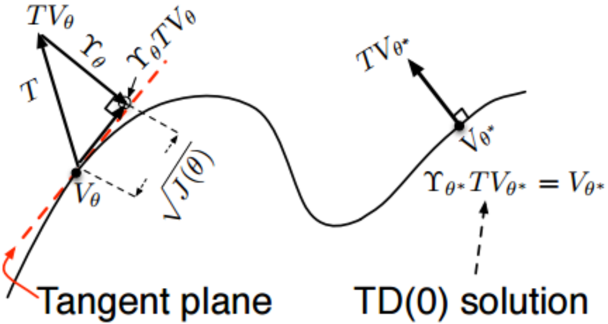
$$J(\theta) = MSPBE(\theta) = \|\Pi(TV_\theta - V_\theta)\|_D$$

where Π projects onto $\mathcal{M} = \{V_\theta | \theta \in \mathbb{R}^n\} = \{\Phi\theta | \theta \in \mathbb{R}^n\}$ (linear).

Non-linear TDC, GTD : $\mathcal{M}$ is a non-linear manifold... $\Rightarrow$ projection onto the tangent space of $\mathcal{M}$ at θ : $T\mathcal{M}_\theta = \{\Phi_\theta\theta | \theta \in \mathbb{R}^n\}$ where $(\Phi_\theta)_{s,i} = \frac{\partial}{\partial\theta_i} V_\theta(s)$:

$$J(\theta) = \|\Pi_\theta(TV_\theta - V_\theta)\|$$

Visually...



Υ_θ : projection on the tangent plane
$\Pi_\theta(TV_\theta - V_\theta) = \Upsilon_\theta TV_\theta - V_\theta$

Derivations

Let $\Phi_\theta \equiv \nabla V_\theta(s)$

$$J(\theta) = \mathbb{E}[\delta\Phi_\theta]^T \mathbb{E}[\Phi_\theta\Phi_\theta^T]\mathbb{E}[\delta\Phi_\theta]$$

Gradient of $J(\theta)$

$$\begin{aligned} -\frac{1}{2}\nabla J(\theta) &= -\mathbb{E}[(\gamma\Phi'_\theta - \Phi_\theta)\Phi_\theta^T\omega] + h(\theta,\omega) && \text{(GTD)} \\ &= -\mathbb{E}[\delta\Phi_\theta] - \gamma\mathbb{E}[\Phi'_\theta\Phi_\theta^T\omega] + h(\theta,\omega) && \text{(TDC)} \end{aligned}$$

where $\omega = \mathbb{E}[\Phi_\theta\Phi_\theta^T]^{-1}\mathbb{E}[\delta\Phi_\theta]$ and $h(\theta,\omega) = -\mathbb{E}[\nabla^2 V_\theta(s)\omega]$

In comparison with the linear case, the only difference is the presence of a second order term $h(\theta,\omega)$.

Non-linear GTD/TDC updates

Non-linear GTD

$$\theta_{k+1} = \theta_k + \alpha_k \left[(\Phi_k - \gamma \Phi'_k)(\Phi_k^T w_k) - h_k \right]$$

Non-linear TDC

$$\theta_{k+1} = \theta_k + \alpha_k \left[\delta_k \Phi_k - \gamma \Phi'_k (\Phi_k^T w_k) - h_k \right]$$

Same approximation trick for ω (and h) as in the linear case :

$$\omega_{k+1} = \omega_k + \beta_k (\delta_k - \Phi_k^T \omega_k) \Phi_k$$
$$h_k = (\delta_k - \Phi_k^T w_k) \nabla^2 V_{\theta_k}(s_k) \omega_k$$

The spiral counterexample

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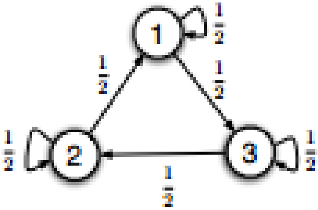
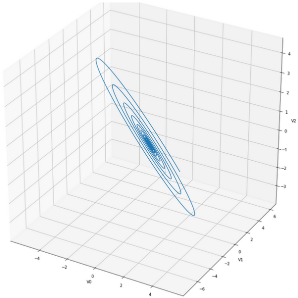
$$V_\theta(s) = (a[s]\cos(\lambda\theta - b[s]\sin(\lambda\theta)))\exp(\epsilon\theta)$$





The spiral counterexample