Calculus 3

Rishi Lokesh

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1 Parametric Equations and Polar Coordinates

1.1 Parametric Equations

If x and y are continuous functions, then

$$x = x(t) \text{ and } y = y(t)$$

are parametric equations, and t is the parameter. The points that are obtained when t is varied over an interval is the graph, or is called the parametric curve.

To understand parametric functions further, you can eliminate the parameter by relating t to x and y. For the function $x(t) = 4\cos t$ and $y(t) = 3\sin t$, divide both sides by 4 and 3 respectivly and plug the cosine and sin functions into the Pythagoreus identity to create the equation of a circle.

1.2 Calculus on Parametric Curves

1.2.1 Derivatives of Parametric Equations

Given a curve defined by $x = x(t)$ and $y = y(t)$, given that their derivatives exist, the derivative is given by

$$\frac{y'(t)}{x'(t)}$$

The second derivative is given by

$$\frac{\frac{d}{dt}(\frac{dy}{dx})}{\frac{dx}{dt}}$$

by using the quotient rule on the first derivative

1.2.2 Integrals of Parametric Equations

The area under a parametric curve is

$$\int_a^b y(t)x'(t)dt$$

1.2.3 Arc Length of a Parametric Curve

The length of a arc on a parametric curve is

$$s = \int_{t_1}^{t_2} \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2} \quad dt$$

1.2.4 Surface Area of a Parametric Curve

When a parametric curve is rotated about the x axis, the area is,

$$S = 2\pi \int_a^b f(x)\sqrt{1 + (f'(x))^2} \quad dx$$