1 Trig Functions

There are 6 trig functions in the curriculum,

$$\sin$$
$$\cos$$
$$\tan$$

and their reciprocols,

$$\csc$$
$$\sec$$
$$\cot$$

relating the ratios of the sides with the trig functions to the Pythagorean Theorum gives way to the Pythagorean identities

$$1 + \tan^2\theta = \sec^2\theta$$
$$1 + \cot^2\theta = \csc^2\theta$$

2 Inverse Trig Functions

Note that $\sin^-1$ is also known as *arcsin*, which is the inverse of sin. Plugging in the sin value will give you the angle. Same goes for *arccos* and *arctan* giving the angles for cos and tan.

In order for these arc functions to be functions, they have a range restriction. *arcsin* has a range of $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$, *arccos* has a range of $0 \leq y \leq \pi$, and *arctan* has a range of $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

3 Compound Angle Identities

3.1 Compound Identities

$$\sin(A \pm B) = \sin(A)\cos(B) \pm \cos(A)\sin(B)$$
$$\cos(A \pm B) = \cos(A)\cos(B) \mp \sin(A)\sin(B)$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan(A)\tan(B)}$$

3.2 Cofunction Identities

$$\sin(\frac{\pi}{2} - x) = \cos x$$
$$\cos(\frac{\pi}{2} - x) = \sin x$$
$$\csc(\frac{\pi}{2} - x) = \sec x$$
$$\sec(\frac{\pi}{2} - x) = \csc x$$
$$\tan(\frac{\pi}{2} - x) = \cot x$$
$$\cot(\frac{\pi}{2} - x) = \tan x$$

3.3 Example

Let $f(x) = \tan(x + \pi)\cos\left(x - \frac{\pi}{2}\right)$ where $0 < x < \frac{\pi}{2}$. Express $f(x)$ in terms of $\sin x$ and $\cos x$.

$$\begin{aligned} f(x) &= \tan(x+\pi)\cos\left(x - \frac{\pi}{2}\right) \\ &= \tan x \cdot \sin x \quad (\text{since } \tan(x+\pi) = \tan x, \text{ and } \cos(x - \frac{\pi}{2}) = \sin x) \\ &= \frac{\sin^2 x}{\cos x} \end{aligned}$$

Function	a (Amplitude)	b (Period)	c Shift	d Shift
$y = a\sin b(x-c) + d$	$\lvert a\rvert$	$\frac{2\pi}{b}$	Right if $c > 0$	Up if $d > 0$
$y = a\tan b(x-c) + d$	Stretch by a	$\frac{\pi}{b}$	Right if $c > 0$	Up if $d > 0$

Table 1: Transformations of sine and tangent functions

4 Complex Number

4.1 Roots of Complex Numbers

Finding the root, given $z^n = rcis(\theta)$

$$z = r^{\frac{1}{n}}(cis(\frac{\theta + 2\pi k}{n})$$

k = 0, 1, 2, n-1

5 Statistics

5.1 Binomial Expansion

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

5.2 Outlier

Outliers are calculated through seeing if they are 1.5 times the IQR above or below the Q1 or Q3 values of the data. This is not inclusive, so if the value is that exact number, it is not an outlier.

6 Proofs

6.1 Deduction

You need to prove that one side of equation is equal to the other side.

6.2 Contradiction

You assume the negation and you prvoe that this is always false, so the original is true.

7 Advanced Calc

7.1 L'hopital rule

If a limit gives you a indeterminate form, ie. $\frac{\text{inf}}{\text{inf}}$ or $\frac{0}{0}$, you can use the derivatve of the numerator and denominator to try to find the limit.

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

7.2 Seperable Differential Equations

A seperable differential equation is any equation that can be written in the form $y' = f(x)g(x)$. This means that you can seperate the RHS into two seperate functions.

First, check for values of y that make g(y) = 0, these lead to constant solutions. Then, you rewrite the euqation to the form of

$$\frac{dy}{g(y)} = f(x)dx$$

and you integrate both sides. Because both sides have c, there is only one. Solve the equation for y and if there is an intial condition sub in the value and solve for c.

7.3 Homogeneous Differential Equations

$$2xdy = (x + y)dx$$

Notice how all the terms have the same order. Sub in y as vx. That means that $dy = xdv + vdx$ through the power rule.

$$2x(xdv + vdx) = (x + vx)dx$$

You can factor out an x on the right side

$$2x(xdv + vdx) = x(1 + v)dx$$

Cancel out the x on both sides

$$2(xdv + vdx) = (1 + v)dx$$

$$2xdv + 2vdx = (1 + v)dx$$

Then, consolidating the vdx on both sides

$$2xdv + vdx = dx$$

Then, movign the vdx to the other side and factoring out dx again

$$2xdv = dx - vdx$$

$$2xdv = dx(1 - v)$$

Now, isolating x and v, we move some terms around

$$\frac{dv}{(1 - v)} = \frac{dx}{2x}$$

We can finally integrate both sides

$$\int \frac{dv}{(1 - v)} = \int \frac{dx}{2x}$$

$$-ln(1 - v) = 0.5ln(x) + c$$

You can use log rules and stuff to go beyond that, but when it is time you can sub v back out for $\frac{y}{x}$

7.4 Paper 3 Questions

7.4.1 May 22 Q1

This question asks you to explore properties of a family of curves of the type $y^2 = x^3 + ax + b$ for various values of a and b

1. On the same set of axes, sketch the following curves for $-2 \leq x \leq 2$ and $-2 \leq y \leq 2$, clearly indicating any points of intersection with the coordinate axes.

   (a) $y^2 = x^3, x \geq 0$

   The move here is to graph out $y = \sqrt{x^3}$, then have it symmetric over the x axis because including a square root makes it plus or minus

   (b) $y^2 = x^3 + 1, x \geq -1$

   The move is the same here

2. (a) Write down the coordinates for two points of inflection on the curve $y^3 = x^3 + 1$

   This is a write down question, so it cannot be taking the derivitive. Instead, you can clearly tell from the graph that is switches at (0, 1) and (0, -1)

   (b) By considering each curve from part(a), identify two key features which would differentiate the graph.

   Given that it asked us for points of inflection, we can talk about how they are different. You could also reference the shart point, the domain, the x and y intercepts

   Now consider curves in the form $y^2 = x^3 + b$ where $x \geq -\sqrt[3]{b}$

3. By varying the value of b, suggest two key features common to these curves

   There is a huge list of things you can say here. The easiest is their limits are the same, they have the same number of intercepts and points of inflection, and there is no sharp point

   Now consider the curve, $y^2 = x^3 + x$, for $x \geq 0$

4. (a) Show that $\frac{dy}{dx} = \pm\frac{3x^2+1}{2\sqrt{x^3+x}}$

$$(y^2)' = (x^3 + x)'$$
$$2y\frac{dy}{dx} = 3x^2 + 1$$
$$\frac{dy}{dx} = \frac{3x^2 + 1}{2y}$$

We can solve for y as

$$y = \pm\sqrt{x^3 + x}$$

And plug it in

$$\frac{dy}{dx} = \pm\frac{3x^2 + 1}{\sqrt{x^3 + x}}$$

(b) Hence deduce that the curve has no local minimum or maximum points

$$\frac{dy}{dx} = 0 = \pm\frac{3x^2 + 1}{\sqrt{x^3 + x}}$$

$$0 = 3x^2 + 1$$
$$-\frac{1}{3} = x^2$$

And x^2 cannot be negative

5. The curve $y^2 = x^3 + x$ has two points of inflexion. Due to the symmetry of the curve these points have the same x-coordinate. Find the value of the coordinate in the form $x = \sqrt{\frac{p\sqrt{3}+q}{r}}$

$$\frac{dy}{dx} = \pm\frac{3x^2 + 1}{\sqrt{x^3 + x}}$$
$$\frac{d^2y}{dx^2} = (\pm\frac{3x^2 + 1}{\sqrt{x^3 + x}})'$$
$$\frac{d^2y}{dx^2} = \pm\frac{(\sqrt{x^3 + x})(3x^2 + 1)' + (\sqrt{x^3 + x})'(3x^2 + 1)}{x^3 + x}$$
$$\frac{d^2y}{dx^2} = \pm\frac{(\sqrt{x^3 + x})(6x) + 0.5((x^3 + x)^{-1/2}(3x^2 + 1))(3x^2 + 1)}{x^3 + x}$$
$$0 = \pm(\sqrt{x^3 + x})(6x) + 0.5((x^3 + x)^{-1/2}(3x^2 + 1))(3x^2 + 1)$$
$$(\sqrt{x^3 + x})(6x) = 0.5(x^3 + x)^{-1/2}(3x^2 + 1)(3x^2 + 1)$$
$$(x^3 + x)(6x) = 0.5(3x^2 + 1)^2$$

$$6x^4 + 6x^2 = 0.5(9x^4 + 6x^2 + 1)$$
$$1.5x^4 + 3x^2 - 0.5 = 0$$

Then, recognize how you can turn this into a quadratic using u substitution of x^2

$$3u^2 + 6u - 1 = 0$$

$$u = \frac{-6 \pm \sqrt{6^3 - 4 * 3 * -3}}{6}$$
$$u = \frac{-6 \pm \sqrt{48}}{6}$$
$$x^2 = \frac{-6 \pm \sqrt{48}}{6}$$
$$x = \sqrt{\frac{-6 \pm 4\sqrt{3}}{6}}$$

Bing bong badabang thats the answer

6. P(x, y) is defined to be a rational point if x and y are rational numbers.

   The tangent to the curve $y^2 = x^3 + 2$, for $x \geq -\sqrt[3]{2}$. The rational point P(-1, -1) Lies on C.

   (a) Find the equation of the tangent to C at P

   You can quickly implicitly differentiate to find the equation.

   $$2yy' = 3x^2$$

   And the slope is $-\frac{3}{2}$
   Then plug into the point slope for to get

   $$y + 1 = -\frac{3}{2}(x + 1)$$

   (b) Hence, find the coordinate of the rational point Q where this tangent intersects C, expressing each coordinate as a fraction.

   Reorder the tangent equation in terms of y.

   $$y = -\frac{3}{2}x - \frac{1}{2}$$

   Then plug into the calculator, using the math to frac function to get $\frac{17}{4}$ and $-\frac{71}{8}$

7. The point S(-1, 1) also lies on C. The line [QS] intersects C at a further point. Determine the coordinates of the point.

   First find the equation of QS by finding the slope as -1.88 then plugging that into point slope form. Then solve simultaneously against C