Eleventh Edition

VECTOR MECHANICS FOR ENGINEERS:

DYNAMICS

CHAPTER

13

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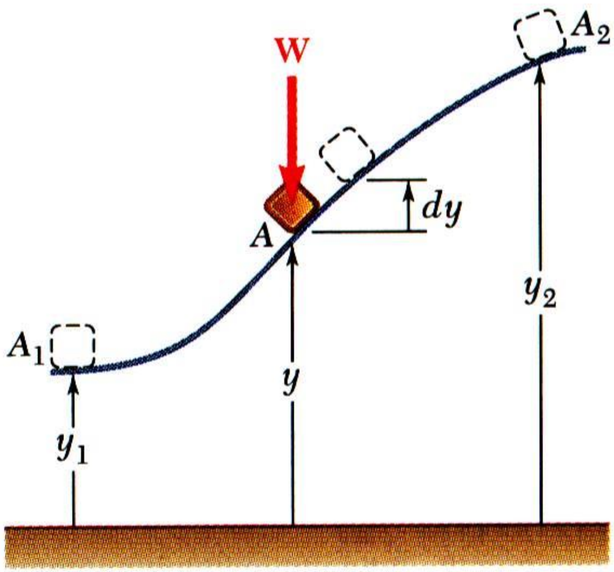
Kinetics of Particles: Energy and Momentum Methods

HW Problems Week 5 (Due Wed 02/14): 13.60, 13.63, 13.68, 13.122, 13.146, 13.151

Lecture 8 02/05
Modified from Original

Potential Energy

If the work of a force only depends on differences in position, we can express this work as potential energy. Eg. Weight, friction



- Work of the force of gravity $\vec{W}$,

$$U_{1\to 2} = W y_1 - W y_2$$

- Work is independent of path followed; depends only on the initial and final values of Wy.

$$V_g = Wy$$

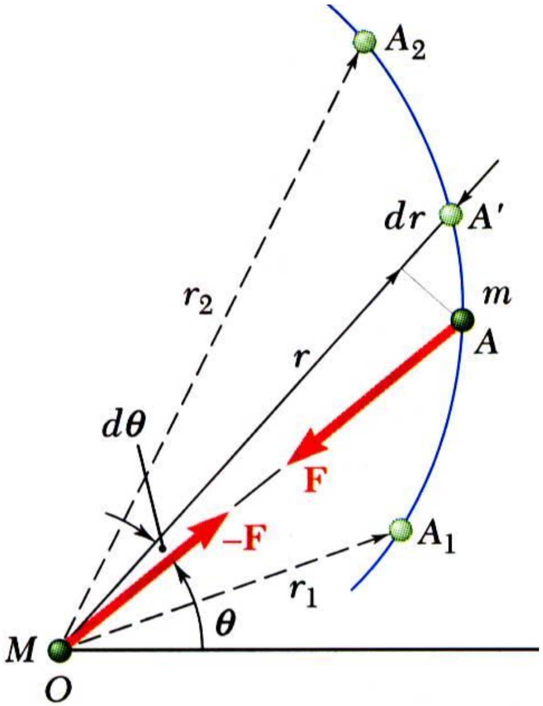
$= \textit{potential energy}$ of the body with respect to *force of gravity*.

$$U_{1\to 2} = \left(V_g\right)_1 - \left(V_g\right)_2$$

- Choice of datum from which the elevation y is measured is arbitrary.
- Units of work and potential energy are the same:

$$V_g = Wy = \mathrm{N}\cdot\mathrm{m} = \mathrm{J}$$

Potential Energy



- Previous expression for potential energy of a body with respect to gravity is only valid when the weight of the body can be assumed constant.

- For a space vehicle, the variation of the force of gravity with distance from the center of the earth should be considered.

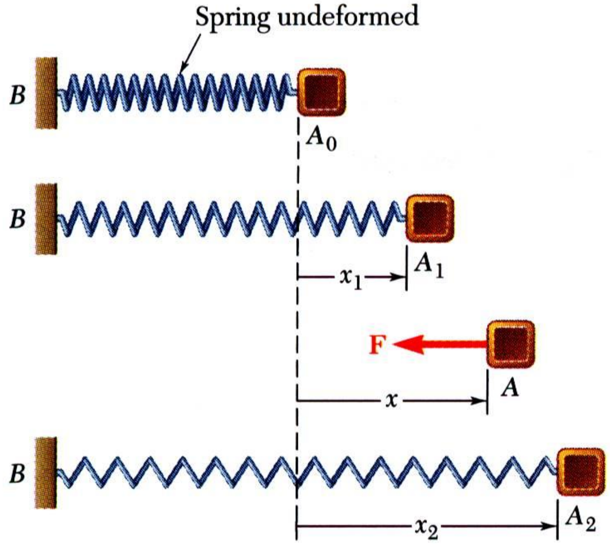
- Work of a gravitational force,

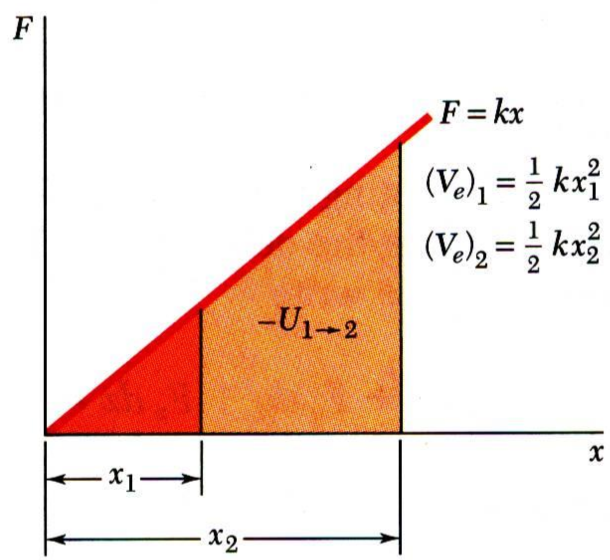
$$U_{1\rightarrow 2} = \frac{GMm}{r_2} - \frac{GMm}{r_1}$$

- Potential energy V_g when the variation in the force of gravity can not be neglected,

$$V_g = -\frac{GMm}{r} = -\frac{WR^2}{r}$$

Potential Energy





- Work of the force exerted by a spring depends only on the initial and final deflections of the spring,

$$U_{1\to 2} = \tfrac{1}{2}kx_1^2 - \tfrac{1}{2}kx_2^2$$

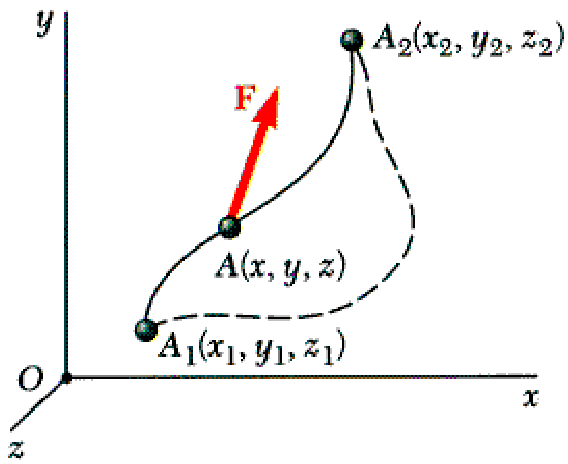
- The potential energy of the body with respect to the elastic force,

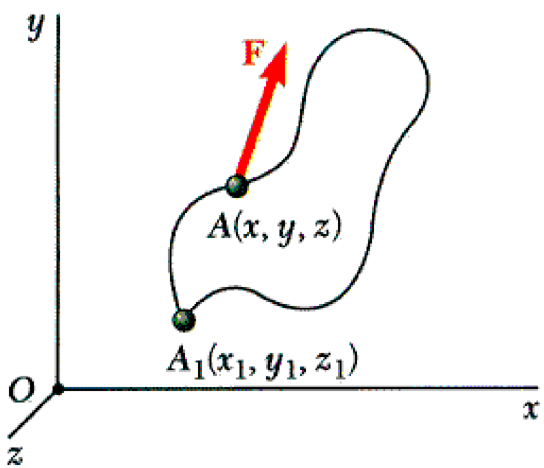
$$V_e = \tfrac{1}{2}kx^2$$

$$U_{1\to 2} = (V_e)_1 - (V_e)_2$$

- Note that the preceding expression for V_e is valid only if the deflection of the spring is measured from its undeformed position.

Conservative Forces





- Concept of potential energy can be applied if the work of the force is independent of the path followed by its point of application.

  $$U_{1\to 2} = V(x_1, y_1, z_1) - V(x_2, y_2, z_2)$$

  Such forces are described as *conservative forces*.

- For any conservative force applied on a closed path,

  $$\oint \vec{F} \bullet d\vec{r} = 0$$

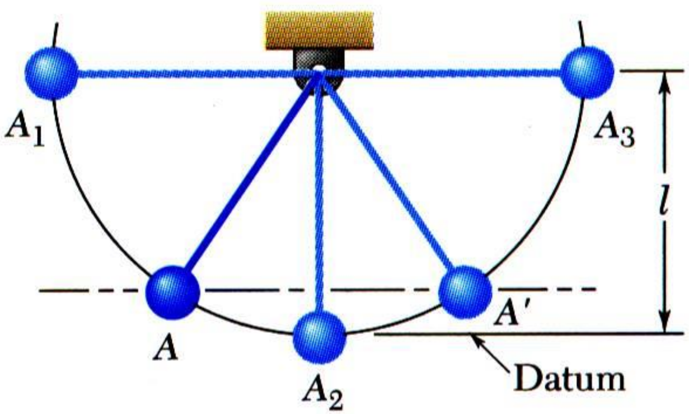
- Elementary work corresponding to displacement between two neighboring points,

  $$dU = V(x, y, z) - V(x+dx, y+dy, z+dz) = -dV(x, y, z)$$

  $$F_x dx + F_y dy + F_z dz = -\left(\frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz\right)$$

  $$\vec{F} = -\left(\frac{\partial V}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial V}{\partial z}\right) = -\mathbf{grad}\, V$$

Conservation of Energy

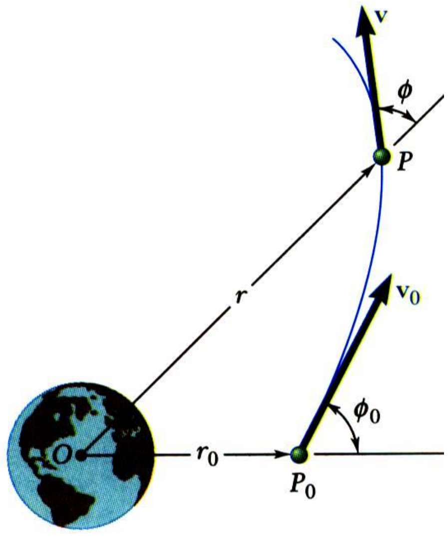


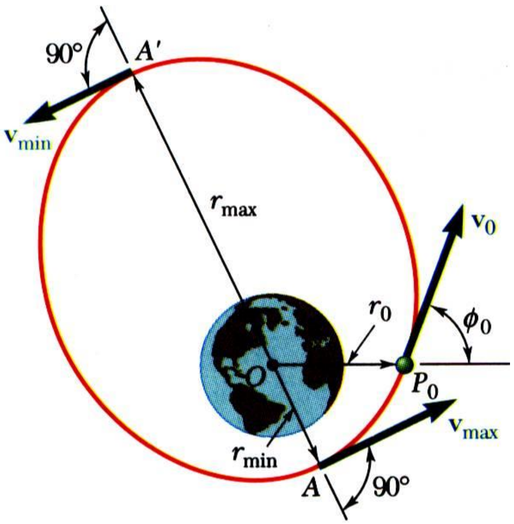
$$T_1 = 0 \quad V_1 = W\ell$$
$$T_1 + V_1 = W\ell$$

$$T_2 = \tfrac{1}{2} m v_2^2 = \frac{1}{2}\frac{W}{g}(2g\ell) = W\ell \quad V_2 = 0$$
$$T_2 + V_2 = W\ell$$

- Work of a conservative force,
  $$U_{1\to 2} = V_1 - V_2$$
- Concept of work and energy,
  $$U_{1\to 2} = T_2 - T_1$$
- Follows that
  $$T_1 + V_1 = T_2 + V_2$$
  $$E = T + V = \text{constant}$$
- When a particle moves under the action of conservative forces, the total mechanical energy is constant.
- Friction forces are not conservative. Total mechanical energy of a system involving friction decreases.
- Mechanical energy is dissipated by friction into thermal energy. Total energy is constant.

Motion Under a Conservative Central Force





- When a particle moves under a conservative central force, both the principle of conservation of angular momentum

$$r_0 m v_0 \sin\phi_0 = r m v \sin\phi$$

and the principle of conservation of energy

$$T_0 + V_0 = T + V$$

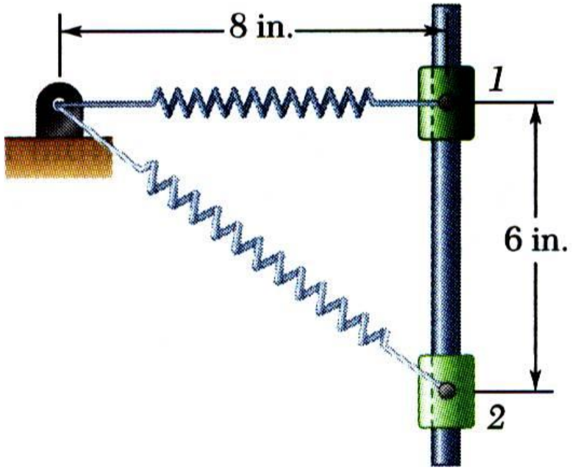
$$\tfrac{1}{2} m v_0^2 - \frac{GMm}{r_0} = \tfrac{1}{2} m v^2 - \frac{GMm}{r}$$

may be applied.

- Given r, the equations may be solved for v and φ.

- At minimum and maximum r, $\varphi = 90^{\circ}$. Given the launch conditions, the equations may be solved for r_{min}, r_{max}, v_{min}, and v_{max}.

Sample Problem 13.8



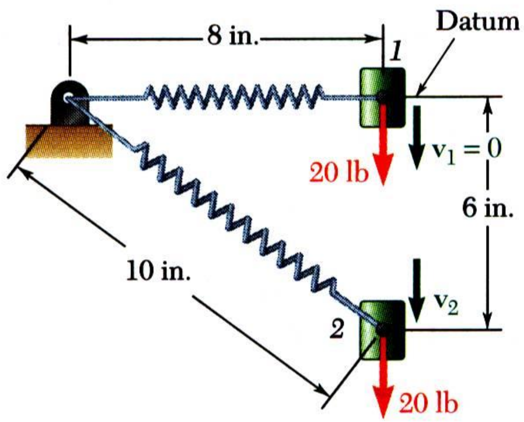
A 20 lb collar slides without friction along a vertical rod as shown. The spring attached to the collar has an undeflected length of 4 in. and a constant of 3 lb/in.

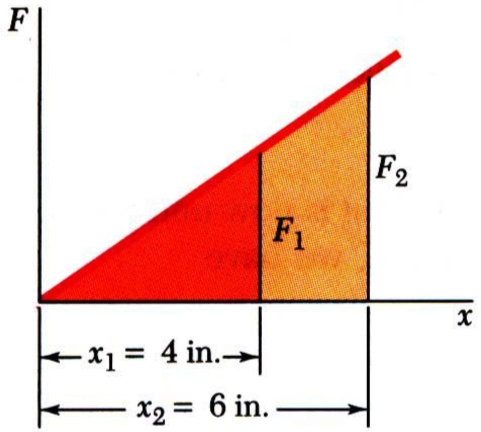
If the collar is released from rest at position 1, determine its velocity after it has moved 6 in. to position 2.

STRATEGY:

- Apply the principle of conservation of energy between positions 1 and 2.
- The elastic and gravitational potential energies at 1 and 2 are evaluated from the given information. The initial kinetic energy is zero.
- Solve for the kinetic energy and velocity at 2.

Sample Problem 13.8





MODELING and ANALYSIS:

- Apply the principle of conservation of energy between positions 1 and 2.

Position 1: $V_e = \frac{1}{2}kx_1^2 = \frac{1}{2}(3\,\text{lb/in.})(8\text{ in.} - 4\text{ in.})^2 = 24\text{ in.}\cdot\text{lb}$

$$V_1 = V_e + V_g = 24\text{ in.}\cdot\text{lb} + 0 = 2\text{ ft}\cdot\text{lb}$$

$$T_1 = 0$$

Position 2: $V_e = \frac{1}{2}kx_2^2 = \frac{1}{2}(3\,\text{lb/in.})(10\text{ in.} - 4\text{ in.})^2 = 54\text{ in.}\cdot\text{lb}$

$$V_g = Wy = (20\text{ lb})(-6\text{ in.}) = -120\text{ in.}\cdot\text{lb}$$

$$V_2 = V_e + V_g = 54 - 120 = -66\text{ in.}\cdot\text{lb} = -5.5\text{ ft}\cdot\text{lb}$$

$$T_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}\frac{20}{32.2}v_2^2 = 0.311v_2^2$$

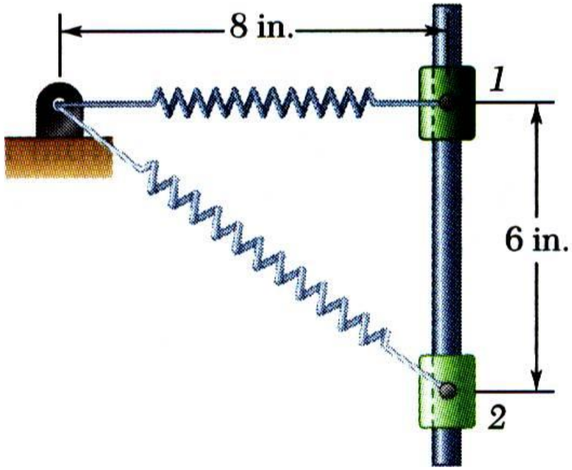
Conservation of Energy:

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 2\text{ ft}\cdot\text{lb} = 0.311v_2^2 - 5.5\text{ ft}\cdot\text{lb}$$

$$\boxed{v_2 = 4.91\text{ ft/s}\downarrow}$$

Sample Problem 13.8

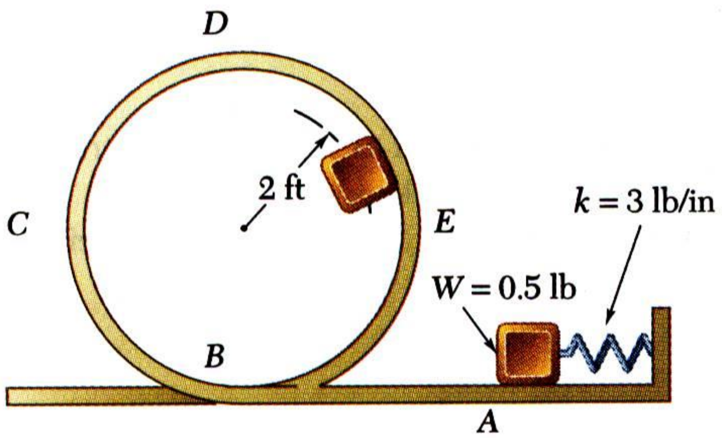


REFLECT and THINK

If you had not included the spring in your system, you would have needed to treat it as an external force; therefore, you would have needed to determine the work.

Similarly, if there was friction acting on the collar, you would have needed to use the more general work–energy principle to solve this problem. It turns out that the work done by friction is not very easy to calculate because the normal force depends on the spring force.

Sample Problem 13.10

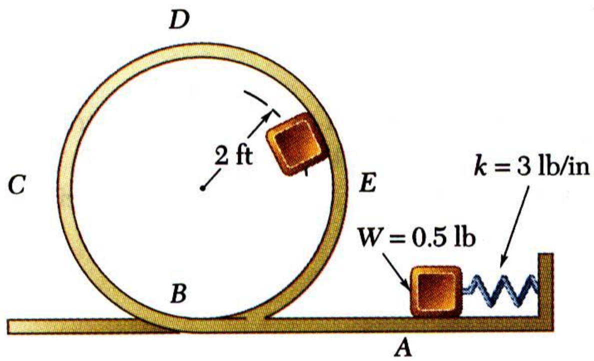


The 0.5 lb pellet is pushed against the spring and released from rest at *A*. Neglecting friction, determine the smallest deflection of the spring for which the pellet will travel around the loop and remain in contact with the loop at all times.

STRATEGY:

- Since the pellet must remain in contact with the loop, the force exerted on the pellet must be greater than or equal to zero. Setting the force exerted by the loop to zero, solve for the minimum velocity at *D*.
- Apply the principle of conservation of energy between points *A* and *D*. Solve for the spring deflection required to produce the required velocity and kinetic energy at *D*.

Sample Problem 13.10

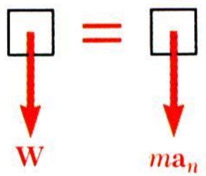


MODELING and ANALYSIS:

- Setting the force exerted by the loop to zero, solve for the minimum velocity at *D*.

$$+\downarrow \sum F_n = ma_n : \quad W = ma_n \qquad mg = mv_D^2/r$$

$$v_D^2 = rg = (2\text{ ft})(32.2\text{ ft/s}) = 64.4\text{ ft}^2/\text{s}^2$$



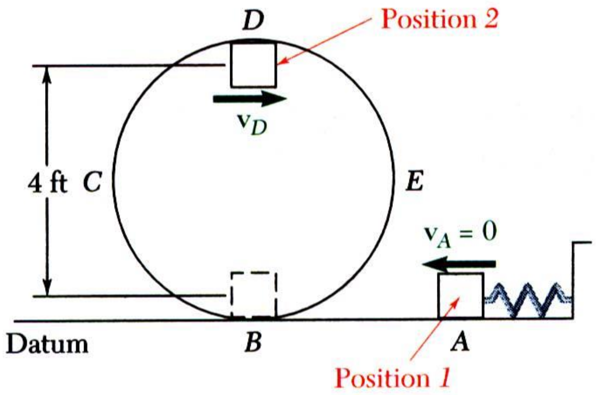
- Apply the principle of conservation of energy between points *A* and *D*.

$$V_1 = V_e + V_g = \tfrac{1}{2}kx^2 + 0 = \tfrac{1}{2}(36\text{ lb/ft})x^2 = 18x^2$$

$$T_1 = 0$$

$$V_2 = V_e + V_g = 0 + Wy = (0.5\text{ lb})(4\text{ ft}) = 2\text{ ft}\cdot\text{lb}$$

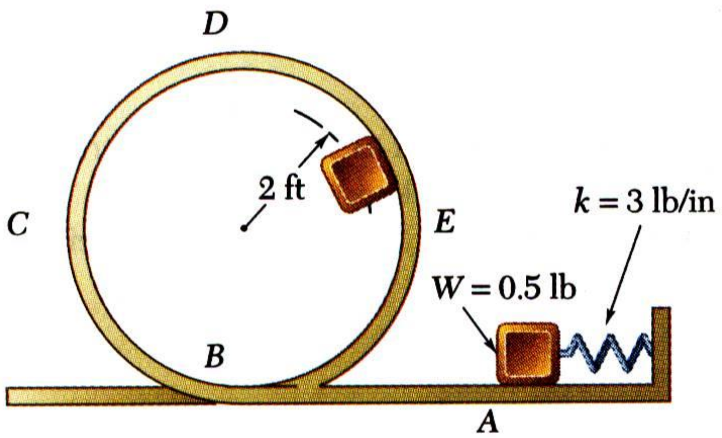
$$T_2 = \tfrac{1}{2}mv_D^2 = \frac{1}{2}\frac{0.5\text{ lb}}{32.2\text{ ft/s}^2}\left(64.4\text{ ft}^2/\text{s}^2\right) = 0.5\text{ ft}\cdot\text{lb}$$



$$T_1 + V_1 = T_2 + V_2$$

$$0 + 18x^2 = 0.5 + 2 \qquad \boxed{x = 0.3727\text{ ft} = 4.47\text{ in.}}$$

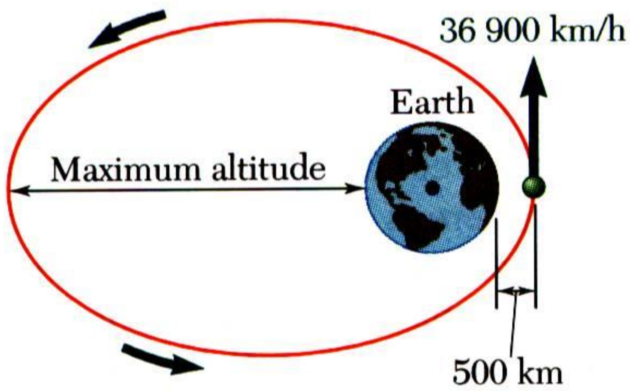
Sample Problem 13.10



REFLECT and THINK

A common misconception in problems like this is assuming that the speed of the particle is zero at the top of the loop, rather than that the normal force is equal to or greater than zero. If the pellet had a speed of zero at the top, it would clearly fall straight down, which is impossible.

Sample Problem 13.12



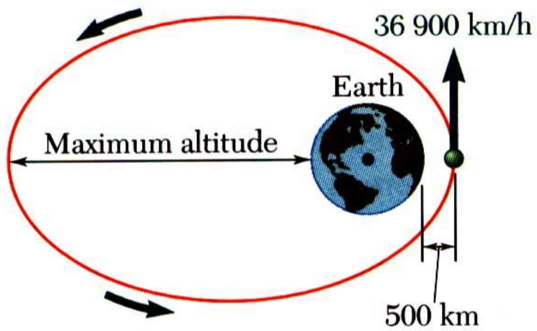
A satellite is launched in a direction parallel to the surface of the earth with a velocity of 36900 km/h from an altitude of 500 km.

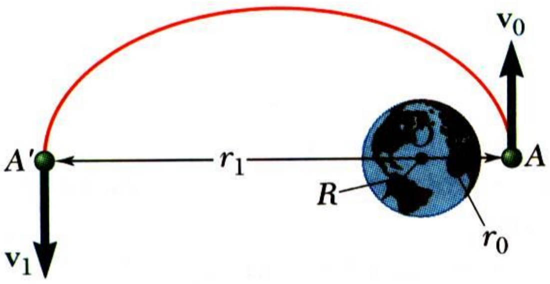
Determine *(a)* the maximum altitude reached by the satellite, and *(b)* the maximum allowable error in the direction of launching if the satellite is to come no closer than 200 km to the surface of the earth

STRATEGY:

- For motion under a conservative central force, the principles of conservation of energy and conservation of angular momentum may be applied simultaneously.
- Apply the principles to the points of minimum and maximum altitude to determine the maximum altitude.
- Apply the principles to the orbit insertion point and the point of minimum altitude to determine maximum allowable orbit insertion angle error.

Sample Problem 13.12





MODELING and ANALYSIS:

- Apply the principles of conservation of energy and conservation of angular momentum to the points of minimum and maximum altitude to determine the maximum altitude.

Conservation of energy:

$$T_A + V_A = T_{A'} + V_{A'} \qquad \tfrac{1}{2} m v_0^2 - \frac{GMm}{r_0} = \tfrac{1}{2} m v_1^2 - \frac{GMm}{r_1}$$

Conservation of angular momentum:

$$r_0 m v_0 = r_1 m v_1 \qquad v_1 = v_0 \frac{r_0}{r_1}$$

Combining,

$$\tfrac{1}{2} v_0^2 \left(1 - \frac{r_0^2}{r_1^2}\right) = \frac{GM}{r_0}\left(1 - \frac{r_0}{r_1}\right) \qquad 1 + \frac{r_0}{r_1} = \frac{2GM}{r_0 v_0^2}$$

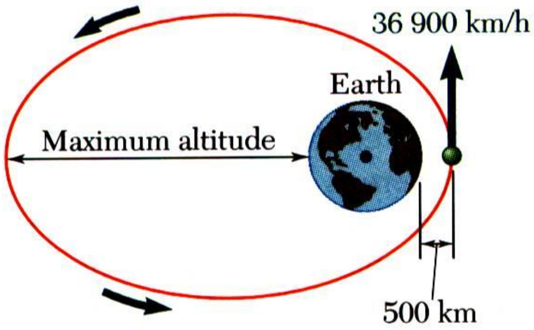
$$r_0 = 6370\,\text{km} + 500\,\text{km} = 6870\,\text{km}$$

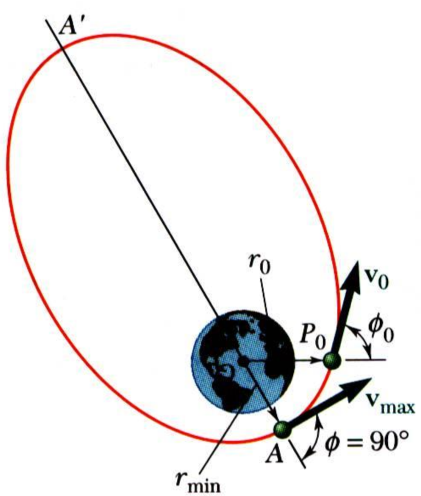
$$v_0 = 36900\,\text{km/h} = 10.25 \times 10^6\,\text{m/s}$$

$$GM = gR^2 = \left(9.81\,\text{m/s}^2\right)\left(6.37 \times 10^6\,\text{m}\right)^2 = 398 \times 10^{12}\,\text{m}^3/\text{s}^2$$

$$\boxed{r_1 = 60.4 \times 10^6\,\text{m} = 60400\,\text{km}}$$

Sample Problem 13.12





- Apply the principles to the orbit insertion point and the point of minimum altitude to determine maximum allowable orbit insertion angle error.

Conservation of energy:

$$T_0 + V_0 = T_A + V_A \qquad \tfrac{1}{2}mv_0^2 - \frac{GMm}{r_0} = \tfrac{1}{2}mv_{max}^2 - \frac{GMm}{r_{min}}$$

Conservation of angular momentum:

$$r_0 m v_0 \sin\phi_0 = r_{min} m v_{max} \qquad v_{max} = v_0 \sin\phi_0 \frac{r_0}{r_{min}}$$

Combining and solving for sin φ_0,

$$\sin\phi_0 = 0.9801$$

$$\varphi_0 = 90° \pm 11.5° \qquad \boxed{\text{allowable error} = \pm 11.5°}$$

REFLECT and THINK:

- Space probes and other long-distance vehicles are designed with small rockets to allow for mid-course corrections. Satellites launched from the Space Station usually do not need this kind of fine-tuning.