Seventh Edition

CHAPTER

5

MECHANICS OF MATERIALS

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Analysis and Design of Beams for Bending

Lecture Notes:

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Lecture 11 02/14/2018
Modified from Original

HW Problems Week 6 (due Mon 02/26):

5.11, 5.15, 5.16, 5.53, 5.56, 5.57

Relations Among Load, Shear, and Bending Moment

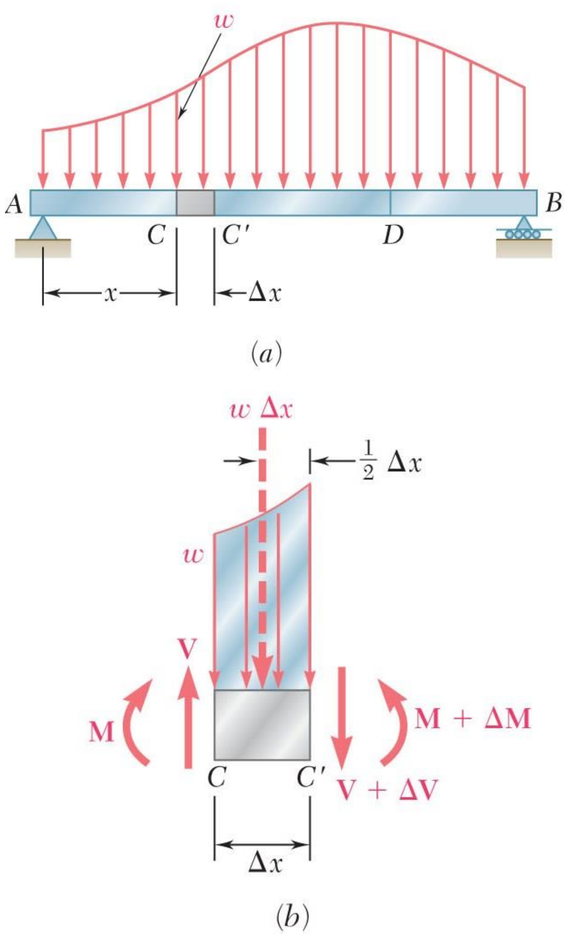


Fig. 5.9 (a) Simply supported beam subjected to a distributed load, with a small element between C and C′, (b) Free-body diagram of the element.

- Relationship between load and shear:

$$\sum F_y = 0: \quad V - (V + \Delta V) - w\Delta x = 0$$

$$\Delta V = -w\Delta x$$

$$\frac{dV}{dx} = -w$$

$$V_D - V_C = -\int_{x_C}^{x_D} w\, dx$$

$$= -(\text{area under load curve between } C \text{ and } D)$$

- Relationship between shear and bending moment:

$$\sum M_{C'} = 0: \quad (M + \Delta M) - M - V\,\Delta x + w\Delta x \frac{\Delta x}{2} = 0$$

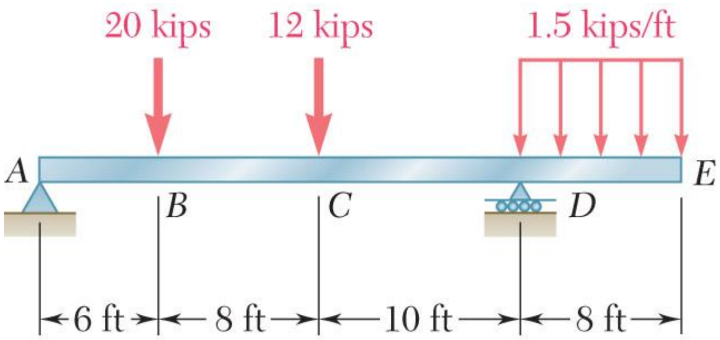
$$\Delta M = V\,\Delta x - \tfrac{1}{2} w (\Delta x)^2$$

$$\frac{dM}{dx} = V$$

$$M_D - M_C = \int_{x_C}^{x_D} V\, dx$$

$$= \text{area under shear curve between } C \text{ and } D$$

Sample Problem 5.3



Draw the shear and bending moment diagrams for the beam and loading shown.

SOLUTION:

- Taking the entire beam as a free body, determine the reactions at *A* and *D*.
- Apply the relationship between shear and load to develop the shear diagram.
- Apply the relationship between bending moment and shear to develop the bending moment diagram.

Sample Problem 5.3

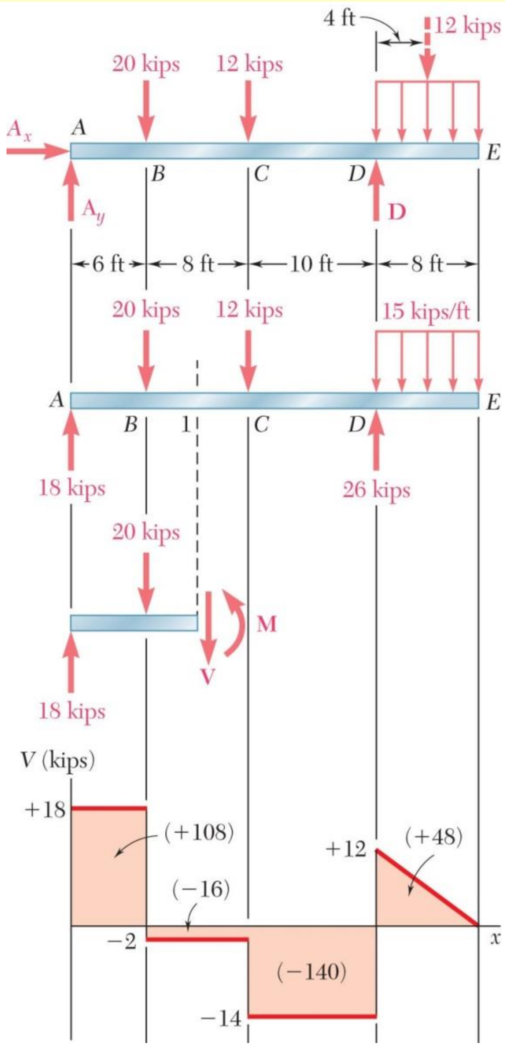


Fig. 1 Free-body diagrams for finding the reactions as well as the shear diagram.

SOLUTION:

- Taking the entire beam as a free body, determine the reactions at *A* and *D*.

$$\sum M_A = 0$$

$$0 = D(24\,\text{ft}) - (20\,\text{kips})(6\,\text{ft}) - (12\,\text{kips})(14\,\text{ft}) - (12\,\text{kips})(28\,\text{ft})$$

$$D = 26\,\text{kips}\uparrow$$

$$\sum F_y = 0$$

$$0 = A_y - 20\,\text{kips} - 12\,\text{kips} + 26\,\text{kips} - 12\,\text{kips}$$

$$A_y = 18\,\text{kips}\uparrow$$

- Apply the relationship between shear and load to develop the shear diagram.

$$\frac{dV}{dx} = -w \qquad dV = -w\,dx$$

- zero slope between concentrated loads
- linear variation over uniform load segment

Sample Problem 5.3

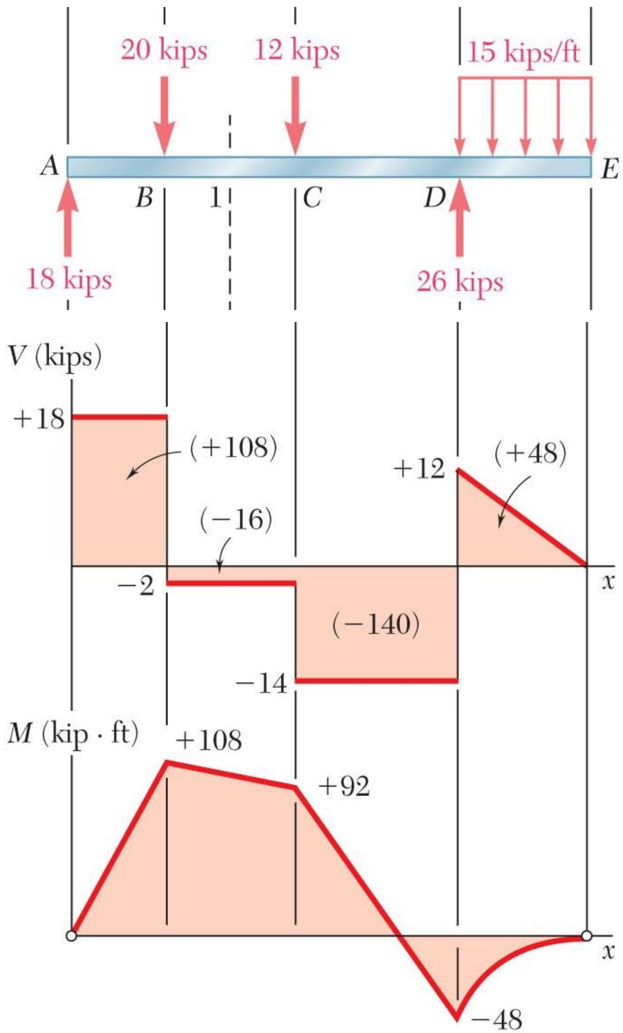


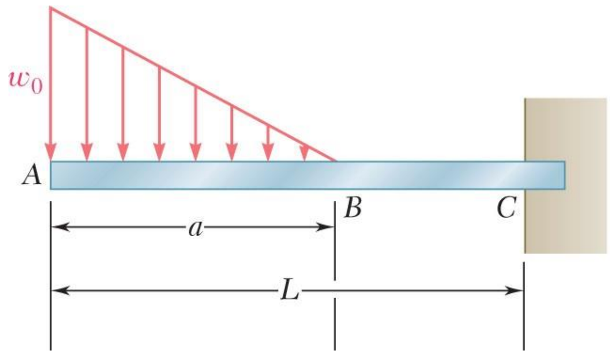
Fig. 1 Free-body diagram for finding the reactions as well as the shear and bending-moment diagrams.

- Apply the relationship between bending moment and shear to develop the bending moment diagram.

$$\frac{dM}{dx} = V \qquad dM = V\,dx$$

- bending moment at *A* and *E* is zero
- bending moment variation between *A*, *B*, *C* and *D* is linear
- bending moment variation between D and *E* is quadratic
- net change in bending moment is equal to areas under shear distribution segments
- total of all bending moment changes across the beam should be zero

Sample Problem 5.5



Draw the shear and bending moment diagrams for the beam and loading shown.

SOLUTION:

- Taking the entire beam as a free body, determine the reactions at *C*.
- Apply the relationship between shear and load to develop the shear diagram.
- Apply the relationship between bending moment and shear to develop the bending moment diagram.

Sample Problem 5.5

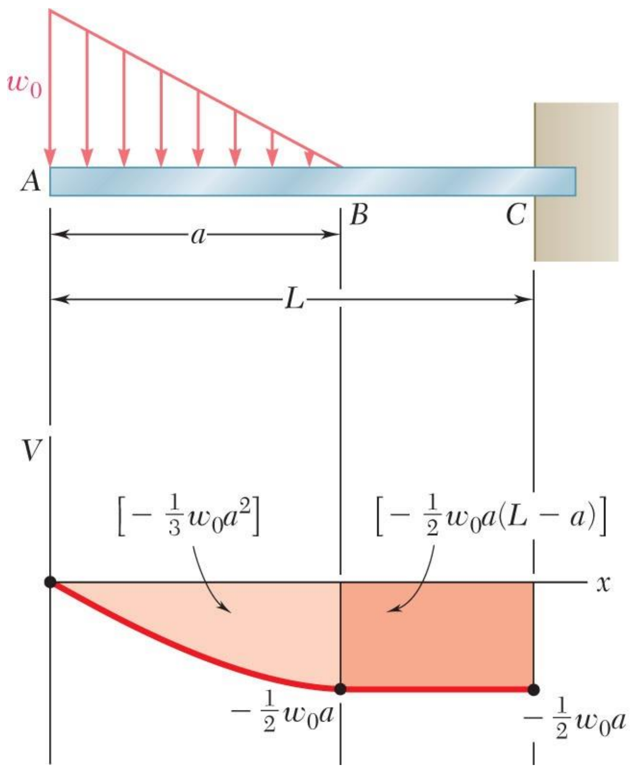


Fig. 1 Cantilevered beam having distributed loading and the resulting the shear diagram.

SOLUTION:

- Taking the entire beam as a free body, determine the reactions at C.

$$\sum F_y = 0 = -\tfrac{1}{2}w_0 a + R_C \qquad R_C = \tfrac{1}{2}w_0 a$$

$$\sum M_C = 0 = \tfrac{1}{2}w_0 a\left(L - \frac{a}{3}\right) + M_C \qquad M_C = -\tfrac{1}{2}w_0 a\left(L - \frac{a}{3}\right)$$

Results from integration of the load and shear distributions should be equivalent.

- Apply the relationship between shear and load to develop the shear diagram.

$$V_B - V_A = -\int_0^a w_0\left(1 - \frac{x}{a}\right)dx = -\left[w_0\left(x - \frac{x^2}{2a}\right)\right]_0^a$$

$$V_B = -\tfrac{1}{2}w_0 a = -\left(area\ under\ load\ curve\right)$$

- No change in shear between B and C.
- Compatible with free body analysis

Sample Problem 5.5

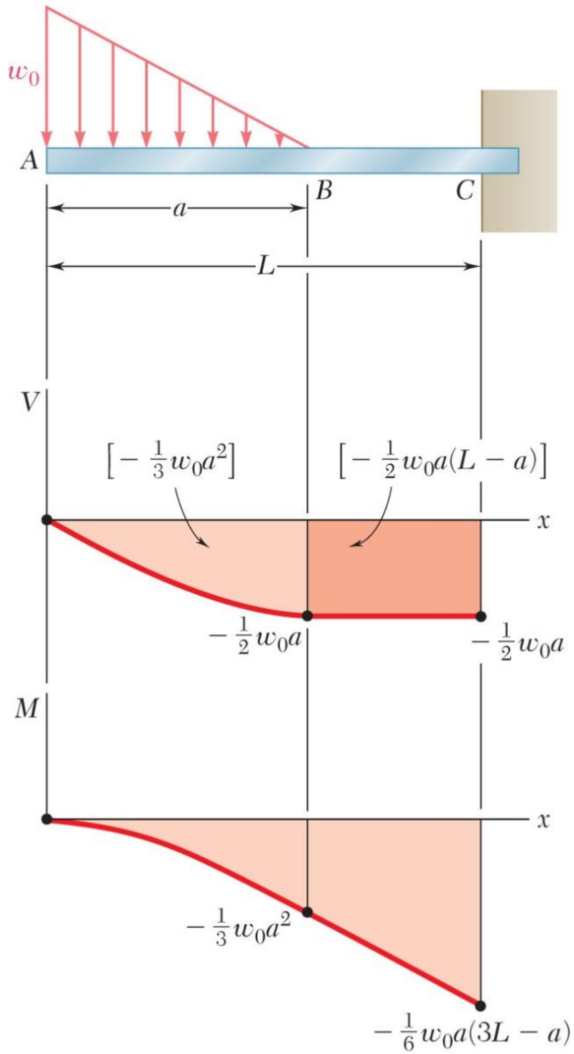


Fig. 1 Cantilevered beam having distributed loading and the resulting the shear and bending-moment diagrams.

- Apply the relationship between bending moment and shear to develop the bending moment diagram.

$$M_B - M_A = \int_0^a \left(-w_0 \left(x - \frac{x^2}{2a} \right) \right) dx = \left[-w_0 \left(\frac{x^2}{2} - \frac{x^3}{6a} \right) \right]_0^a$$

$$M_B = -\tfrac{1}{3} w_0 a^2$$

$$M_B - M_C = \int_a^L \left(-\tfrac{1}{2} w_0 a \right) dx = -\tfrac{1}{2} w_0 a (L-a)$$

$$M_C = -\tfrac{1}{6} w_0 a (3L - a) = \frac{a\, w_0}{2} \left(L - \frac{a}{3} \right)$$

Results at C are compatible with free-body analysis