Eleventh Edition

CHAPTER

12

VECTOR MECHANICS FOR ENGINEERS:

DYNAMICS

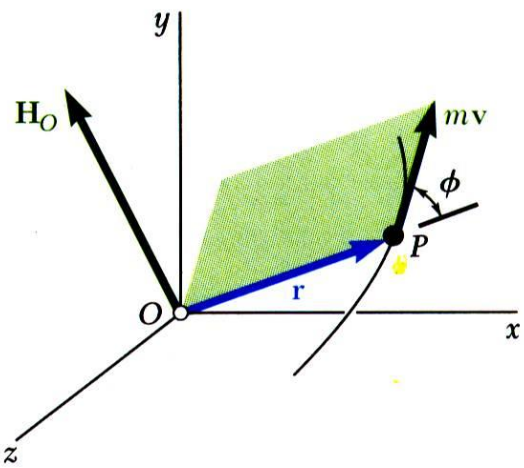
Ferdinand P. Beer
E. Russell Johnston, Jr.
Phillip J. Cornwell
Brian P. Self

Kinetics of Particles: Newton's Second Law

HW Problems Week 4 (Due Wed 02/07):
12.74, 12.77, 12.82, 13.8, 13.24, 13.29

Lecture 6 01/29
Modified from Original

Angular Momentum of a Particle



- $\vec{H}_O = \vec{r} \times m\vec{V} =$ *moment of momentum* or the *angular momentum* of the particle about *O*.
- $\vec{H}_O$ is perpendicular to plane containing $\vec{r}$ and $m\vec{V}$

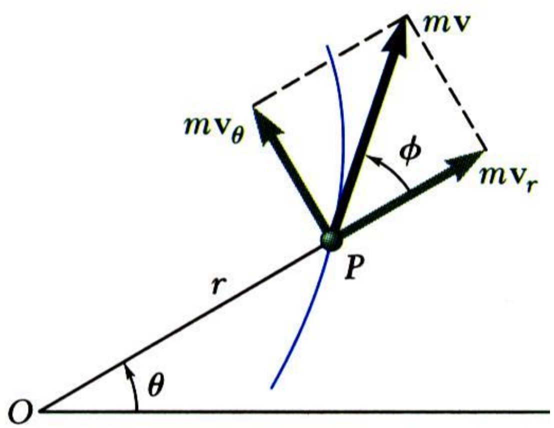
Magnitude: $H_O = rmV\sin\phi$

$= rmv_\theta$

$= mr^2\dot{\theta}$

Note: $v_\theta = r\dot{\theta}$

$$\vec{H}_O = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ mv_x & mv_y & mv_z \end{vmatrix}$$



- Derivative of angular momentum with respect to time,

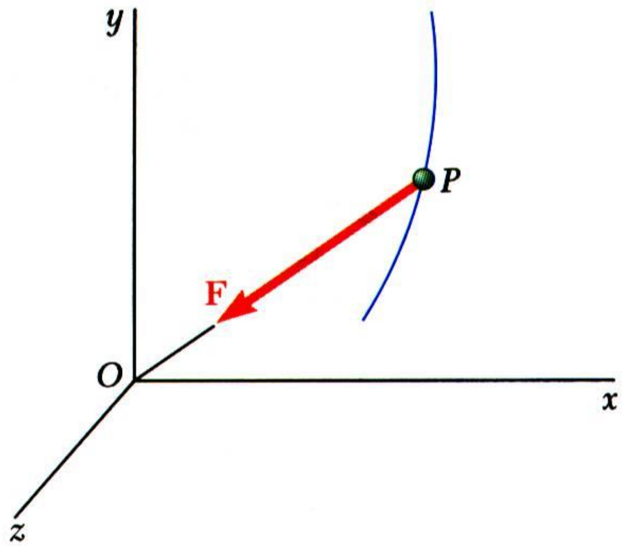
$$\dot{\vec{H}}_O = \dot{\vec{r}} \times m\vec{V} + \vec{r} \times m\dot{\vec{V}} = \vec{V} \times m\vec{V} + \vec{r} \times m\vec{a}$$

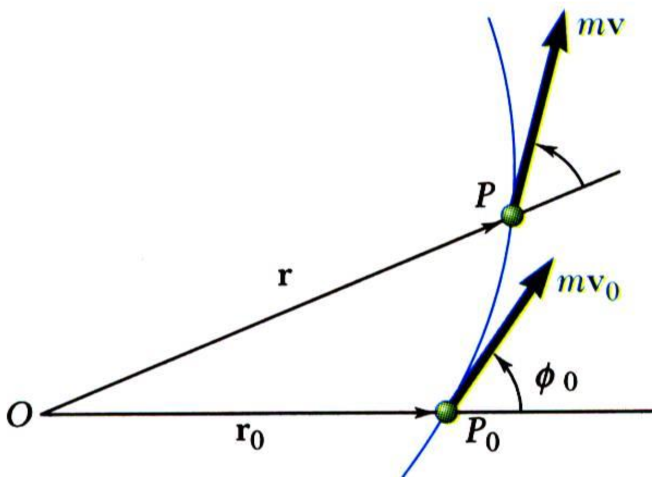
$$= r \times \sum \vec{F}$$

$$= \sum \vec{M}_O$$

- It follows from Newton’s second law that the sum of the moments about *O* of the forces acting on the particle is equal to the rate of change of the angular momentum of the particle about *O*.

Conservation of Angular Momentum





- When only force acting on particle is directed toward or away from a fixed point *O*, the particle is said to be *moving under a central force*.
- Since the line of action of the central force passes through *O*, $\sum \vec{M}_O = \dot{\vec{H}}_O = 0$ and

$$\vec{r} \times m\vec{V} = \vec{H}_O = \text{constant}$$

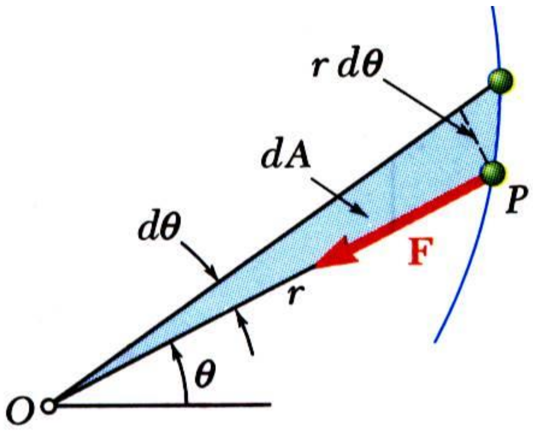
- Position vector and motion of particle are in a plane perpendicular to $\vec{H}_O$.
- Magnitude of angular momentum,

$$H_O = rmV \sin\phi = \text{constant}$$
$$= r_0 m V_0 \sin\phi_0$$

or $$H_O = mr^2\dot{\theta} = \text{constant}$$

$$\frac{H_O}{m} = r^2\dot{\theta} = h = \frac{\text{angular momentum}}{\text{unit mass}}$$

Conservation of Angular Momentum



- Radius vector *OP* sweeps infinitesimal area

$$dA = \tfrac{1}{2} r^2 d\theta$$

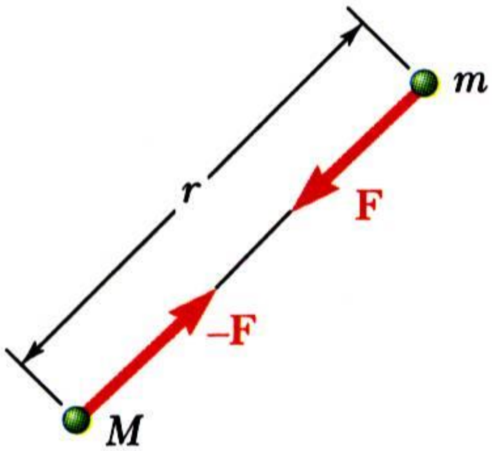
- Define *areal velocity* $= \dfrac{dA}{dt} = \tfrac{1}{2} r^2 \dfrac{d\theta}{dt} = \tfrac{1}{2} r^2 \dot{\theta}$

- Recall, for a body moving under a central force,

$$h = r^2 \dot{\theta} = \text{constant}$$

- When a particle moves under a central force, its areal velocity is constant.

Newton's Law of Gravitation



- Gravitational force exerted by the sun on a planet or by the earth on a satellite is an important example of gravitational force.

- *Newton's law of universal gravitation* - two particles of mass M and m attract each other with equal and opposite force directed along the line connecting the particles,

$$F = G\frac{Mm}{r^2}$$

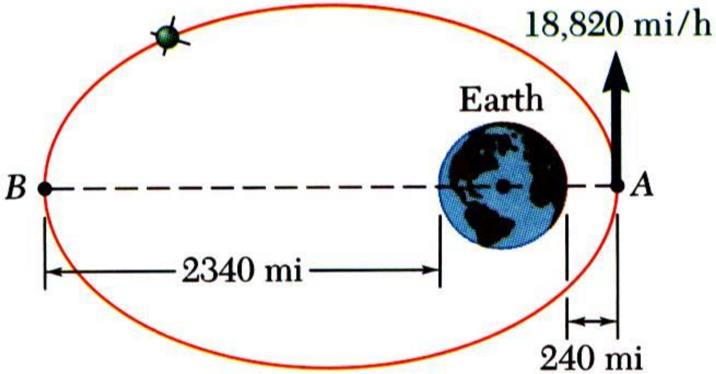
$$G = \text{constant of gravitation}$$

$$= 66.73\times10^{-12}\frac{\mathrm{m}^3}{\mathrm{kg}\cdot\mathrm{s}^2} = 34.4\times10^{-9}\frac{\mathrm{ft}^4}{\mathrm{lb}\cdot\mathrm{s}^4}$$

- For particle of mass m on the earth's surface,

$$W = m\frac{MG}{R^2} = mg \qquad g = 9.81\frac{\mathrm{m}}{\mathrm{s}^2} = 32.2\frac{\mathrm{ft}}{\mathrm{s}^2}$$

Sample Problem 12.12

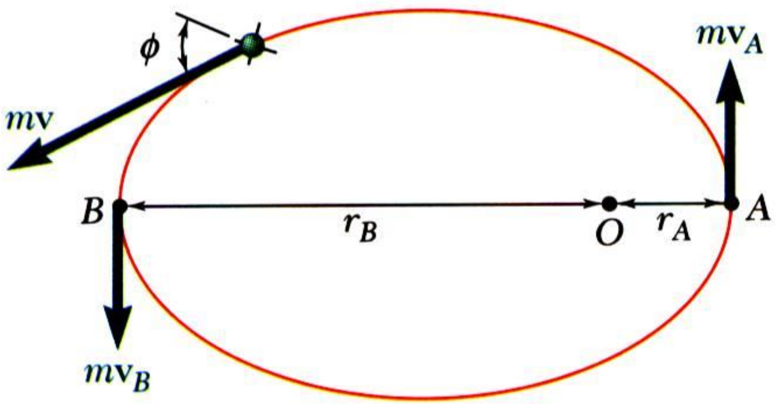


A satellite is launched in a direction parallel to the surface of the earth with a velocity of 18820 mi/h from an altitude of 240 mi. Determine the velocity of the satellite as it reaches it maximum altitude of 2340 mi. The radius of the earth is 3960 mi.

STRATEGY:

- Since the satellite is moving under a central force, its angular momentum is constant. Equate the angular momentum at *A* and *B* and solve for the velocity at *B*.

Sample Problem 12.12



MODELING and ANALYSIS:

- Since the satellite is moving under a central force, its angular momentum is constant. Equate the angular momentum at *A* and *B* and solve for the velocity at *B*.

$$r m v \sin\phi = H_O = \text{constant}$$

$$r_A m v_A = r_B m v_B$$

$$v_B = v_A \frac{r_A}{r_B}$$

$$= (18820\,\text{mi/h}) \frac{(3960+240)\text{mi}}{(3960+2340)\text{mi}}$$

$$\boxed{v_B = 12550\,\text{mi/h}}$$

REFLECT and THINK:

- Note that in order to increase velocity, a spacecraft often applies thrusters to push it closer to the earth. This central force means the spacecraft’s angular momentum remains constant, its radial distance r decreases, and its velocity v increases.