Eleventh Edition

CHAPTER

14

VECTOR MECHANICS FOR ENGINEERS:

DYNAMICS

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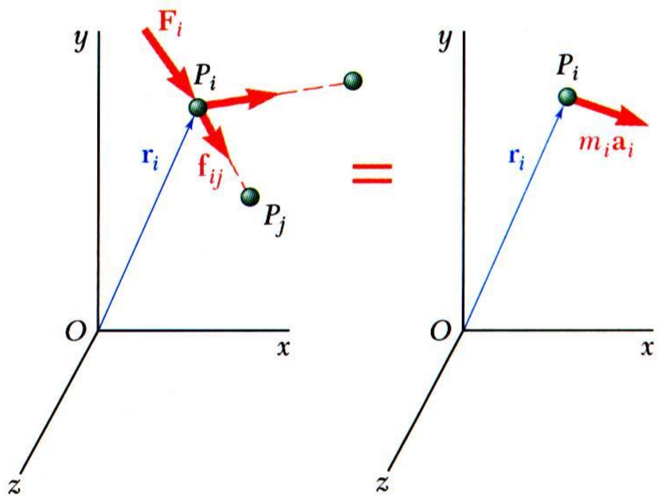
Systems of Particles

HW Problems Week 6 (Due Mon 02/19):
13.163, 13.168, 13.175, 14.11, 14.18, 14.22

Lecture 11 02/14
Modified from Original

Applying Newton's Law and Momentum Principles



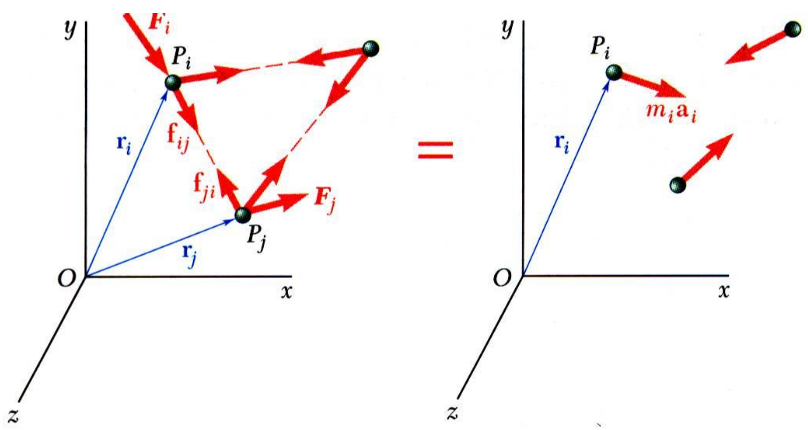
- Newton's second law for each particle P_i in a system of n particles,

$$\vec{F}_i + \sum_{j=1}^{n} \vec{f}_{ij} = m_i \vec{a}_i$$

$$\vec{r}_i \times \vec{F}_i + \sum_{j=1}^{n} \left(\vec{r}_i \times \vec{f}_{ij} \right) = \vec{r}_i \times m_i \vec{a}_i$$

$\vec{F}_i$ = external force $\quad$ $\vec{f}_{ij}$ = internal forces

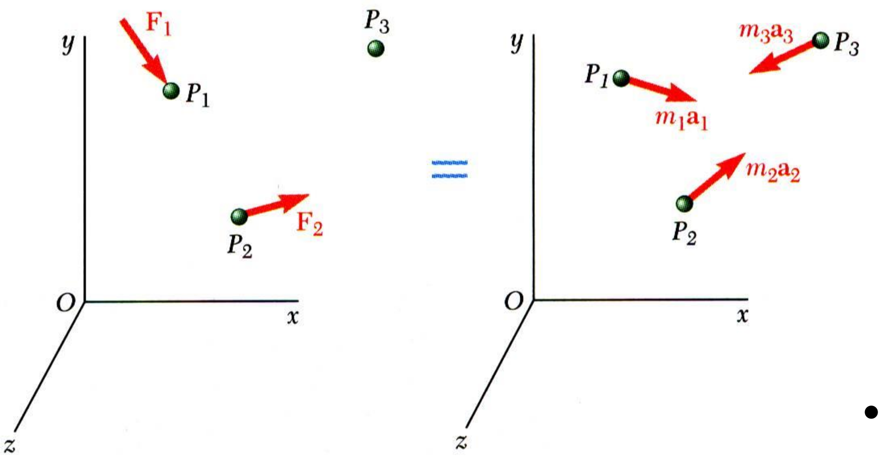
$m_i \vec{a}_i$ = effective force

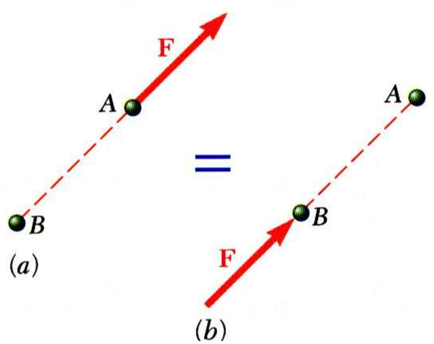


- The system of external and internal forces on a particle is *equivalent* to the effective force of the particle.

- The system of external and internal forces acting on the entire system of particles is *equivalent* to the system of effective forces.

Applying Newton's Law and Momentum Principles





- Summing over all the elements,

$$\sum_{i=1}^{n}\vec{F}_i+\sum_{i=1}^{n}\sum_{j=1}^{n}\vec{f}_{ij}=\sum_{i=1}^{n}m_i\vec{a}_i$$

$$\sum_{i=1}^{n}\left(\vec{r}_i\times\vec{F}_i\right)+\sum_{i=1}^{n}\sum_{j=1}^{n}\left(\vec{r}_i\times\vec{f}_{ij}\right)=\sum_{i=1}^{n}\left(\vec{r}_i\times m_i\vec{a}_i\right)$$

- Since the internal forces occur in equal and opposite collinear pairs, the resultant force and couple due to the internal forces are zero,

$$\sum\vec{F}_i=\sum m_i\vec{a}_i$$
$$\sum\left(\vec{r}_i\times\vec{F}_i\right)=\sum\left(\vec{r}_i\times m_i\vec{a}_i\right)$$

- The system of external forces and the system of effective forces are *equipollent* but not *equivalent*.

Linear & Angular Momentum

- Linear momentum of the system of particles,

$$\vec{L} = \sum_{i=1}^{n} m_i \vec{v}_i$$

$$\dot{\vec{L}} = \sum_{i=1}^{n} m_i \dot{\vec{v}}_i = \sum_{i=1}^{n} m_i \vec{a}_i$$

- Resultant of the external forces is equal to rate of change of linear momentum of the system of particles,

$$\sum \vec{F} = \dot{\vec{L}}$$

- Angular momentum about fixed point *O* of system of particles,

$$\vec{H}_O = \sum_{i=1}^{n} \left(\vec{r}_i \times m_i \vec{v}_i \right)$$

$$\dot{\vec{H}}_O = \sum_{i=1}^{n} \left(\dot{\vec{r}}_i \times m_i \vec{v}_i \right) + \sum_{i=1}^{n} \left(\vec{r}_i \times m_i \dot{\vec{v}}_i \right)$$

$$= \sum_{i=1}^{n} \left(\vec{r}_i \times m_i \vec{a}_i \right)$$

- Moment resultant about fixed point *O* of the external forces is equal to the rate of change of angular momentum of the system of particles,

$$\sum \vec{M}_O = \dot{\vec{H}}_O$$

Motion of the Mass Center of a System of Particles

- Mass center *G* of system of particles is defined by position vector $\vec{r}_G$ which satisfies

$$m\vec{r}_G = \sum_{i=1}^{n} m_i \vec{r}_i$$

- Differentiating twice,

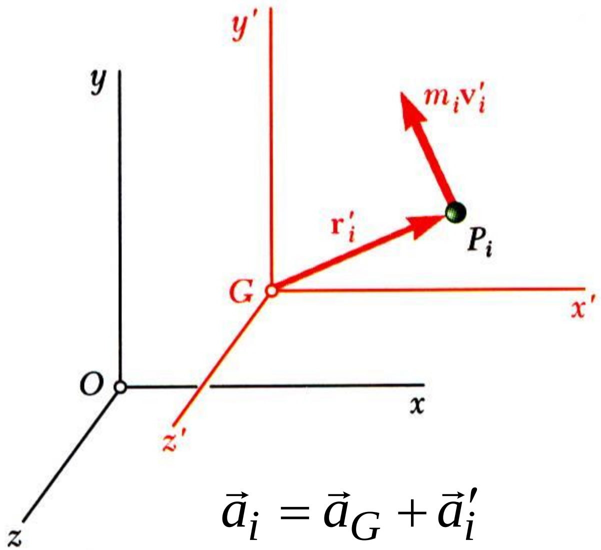
$$m\dot{\vec{r}}_G = \sum_{i=1}^{n} m_i \dot{\vec{r}}_i$$

$$m\vec{v}_G = \sum_{i=1}^{n} m_i \vec{v}_i = \vec{L}$$

$$m\vec{a}_G = \dot{\vec{L}} = \sum \vec{F}$$

- The mass center moves as if the entire mass and all of the external forces were concentrated at that point.

Angular Momentum About the Mass Center



$$\vec{a}_i = \vec{a}_G + \vec{a}'_i$$

- Consider the centroidal frame of reference *Gx'y'z'*, which translates with respect to the Newtonian frame *Oxyz*.

- The centroidal frame is not, in general, a Newtonian frame.

- The angular momentum of the system of particles about the mass center,

$$\vec{H}'_G = \sum_{i=1}^{n}\left(\vec{r}'_i \times m_i\vec{v}'_i\right)$$

$$\dot{\vec{H}}'_G = \sum_{i=1}^{n}\left(\vec{r}'_i \times m_i\vec{a}'_i\right) = \sum_{i=1}^{n}\left(\vec{r}'_i \times m_i\left(\vec{a}_i - \vec{a}_G\right)\right)$$

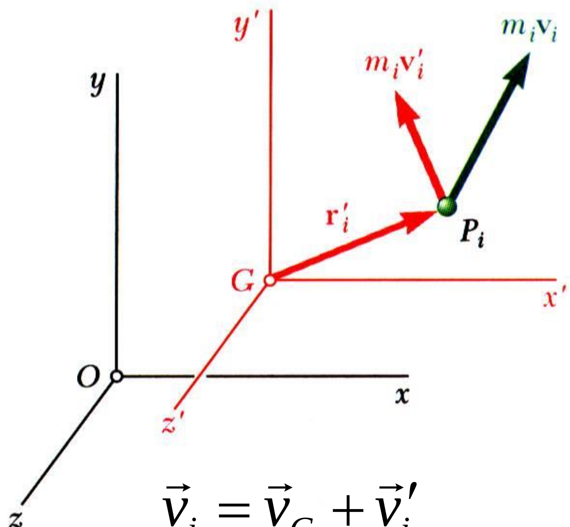
$$= \sum_{i=1}^{n}\left(\vec{r}'_i \times m_i\vec{a}_i\right) - \left(\sum_{i=1}^{n} m_i\vec{r}'_i\right)\times\vec{a}_G$$

$$= \sum_{i=1}^{n}\left(\vec{r}'_i \times m_i\vec{a}_i\right) = \sum_{i=1}^{n}\left(\vec{r}'_i \times \vec{F}_i\right)$$

$$= \sum \vec{M}_G$$

- The moment resultant about *G* of the external forces is equal to the rate of change of angular momentum about *G* of the system of particles.

Angular Momentum About the Mass Center



$$\vec{v}_i = \vec{v}_G + \vec{v}'_i$$

- Angular momentum about *G* of the particles in their motion relative to the centroidal *Gx'y'z'* frame of reference,

$$\vec{H}'_G = \sum_{i=1}^{n} \left(\vec{r}'_i \times m_i \vec{v}'_i\right)$$

- Angular momentum about *G* of particles in their absolute motion relative to the Newtonian *Oxyz* frame of reference.

$$\vec{H}_G = \sum_{i=1}^{n} \left(\vec{r}'_i \times m_i \vec{v}_i\right)$$
$$= \sum_{i=1}^{n} \left(\vec{r}'_i \times m_i \left(\vec{v}_G + \vec{v}'_i\right)\right)$$
$$= \left(\sum_{i=1}^{n} m_i \vec{r}'_i\right) \times \vec{v}_G + \sum_{i=1}^{n} \left(\vec{r}'_i \times m_i \vec{v}'_i\right)$$
$$\vec{H}_G = \vec{H}'_G = \sum \vec{M}_G$$

- Angular momentum about *G* of the particle momenta can be calculated with respect to either the Newtonian or centroidal frames of reference.

Conservation of Momentum

- If no external forces act on the particles of a system, then the linear momentum and angular momentum about the fixed point *O* are conserved.

$$\dot{\vec{L}} = \sum \vec{F} = 0 \qquad \dot{\vec{H}}_O = \sum \vec{M}_O = 0$$
$$\vec{L} = \text{constant} \qquad \vec{H}_O = \text{constant}$$

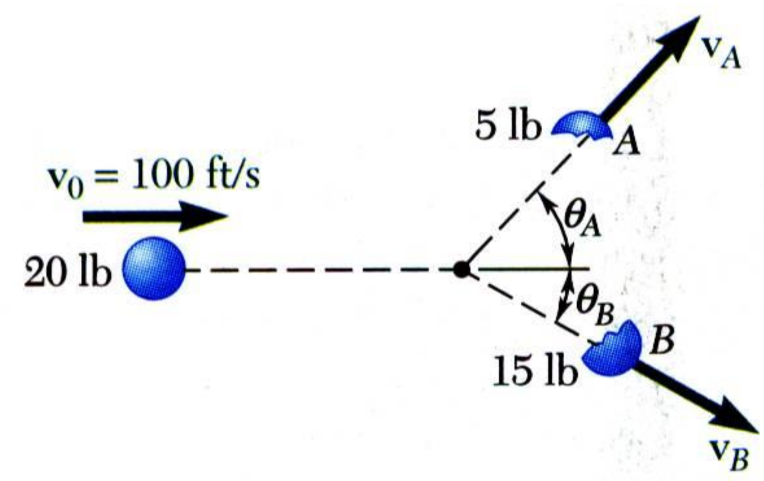
- In some applications, such as problems involving central forces,

$$\dot{\vec{L}} = \sum \vec{F} \neq 0 \qquad \dot{\vec{H}}_O = \sum \vec{M}_O = 0$$
$$\vec{L} \neq \text{constant} \qquad \vec{H}_O = \text{constant}$$

- Concept of conservation of momentum also applies to the analysis of the mass center motion,

$$\dot{\vec{L}} = \sum \vec{F} = 0 \qquad \dot{\vec{H}}_G = \sum \vec{M}_G = 0$$
$$\vec{L} = m\vec{v}_G = \text{constant}$$
$$\vec{v}_G = \text{constant} \qquad \vec{H}_G = \text{constant}$$

Sample Problem 14.2



A 20-lb projectile is moving with a velocity of 100 ft/s when it explodes into 5 and 15-lb fragments. Immediately after the explosion, the fragments travel in the directions $\theta_A = 45^o$ and $\theta_B = 30^o$.

Determine the velocity of each fragment.

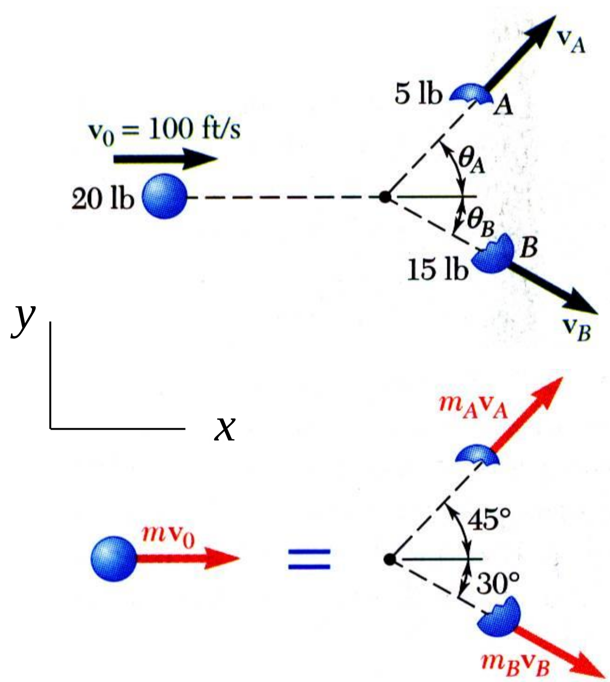
STRATEGY:

- Since there are no external forces, the linear momentum of the system is conserved.
- Write separate component equations for the conservation of linear momentum.
- Solve the equations simultaneously for the fragment velocities.

Sample Problem 14.2

MODELING and ANALYSIS:

- Since there are no external forces, the linear momentum of the system is conserved.



- Write separate component equations for the conservation of linear momentum.

$$m_A\vec{v}_A + m_B\vec{v}_B = m\vec{v}_0$$
$$(5/g)\vec{v}_A + (15/g)\vec{v}_B = (20/g)\vec{v}_0$$

x components:

$$5v_A\cos 45° + 15v_B\cos 30° = 20(100)$$

y components:

$$5v_A\sin 45° - 15v_B\sin 30° = 0$$

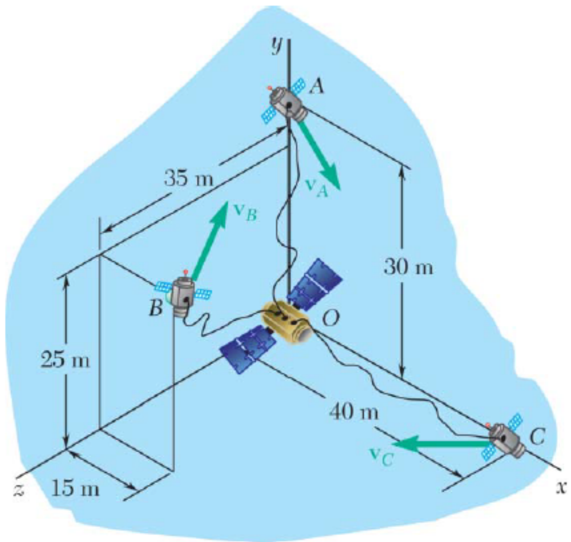
- Solve the equations simultaneously for the fragment velocities.

$$\boxed{v_A = 207\,\text{ft/s} \qquad v_B = 97.6\,\text{ft/s}}$$

REFELCT and THINK:

As you might have predicted, the less massive fragment winds up with a larger magnitude of velocity and departs the original trajectory at a larger angle.

In-Class Problem



PROBLEM 14.9

A 20-kg base satellite deploys three sub-satellites, each which has its own thrust capabilities, to perform research on tether propulsion. The weights of sub-satellite *A*, *B*, and *C* are 4 kg, 6 kg, and 8 kg, respectively, and their velocities expressed in m/s are given by $\mathbf{v_A} = 4\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$, $\mathbf{v_B} = \mathbf{i} + 4\mathbf{j}$, $\mathbf{v_C} = 2\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$. At the instant shown, what is the angular momentum $\mathbf{H_O}$ of the system about the base satellite?

In-Class Problem Solution

SOLUTION

Given: $m_A = 4\text{ kg},\ m_B = 6\text{ kg},\ m_C = 8\text{ kg}$

$\mathbf{v}_A = 4\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}\text{ m/s},\ \mathbf{v}_B = \mathbf{i} + 4\mathbf{j}\text{ m/s},\ \mathbf{v}_C = 2\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}\text{ m/s}$

Linear momentum of each particle expressed in kg·m/s :

$$m_A\mathbf{v}_A = 16.0\mathbf{i} - 8.0\mathbf{j} + 8.0\mathbf{k}$$
$$m_B\mathbf{v}_B = 6.0\mathbf{i} + 24.0\mathbf{j} + 0\mathbf{k}$$
$$m_C\mathbf{v}_C = 16.0\mathbf{i} + 16.0\mathbf{j} + 32.0\mathbf{k}$$

Position Vectors from point O to each satellite in meters:

$$\mathbf{r}_A = 30\mathbf{j}$$
$$\mathbf{r}_B = 15\mathbf{i} + 25\mathbf{j} + 35\mathbf{k}$$
$$\mathbf{r}_C = 40\mathbf{i}$$

Angular Momentum about O, kg·m^2/s :

$$\mathbf{H}_O = \mathbf{r}_A \times (m_A\mathbf{v}_A) + \mathbf{r}_B \times (m_B\mathbf{v}_B) + \mathbf{r}_C \times (m_C\mathbf{v}_C)$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 30 & 0 \\ 16.0 & -8.0 & 8.0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 15 & 25 & 35 \\ 6.0 & 24.0 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 40 & 0 & 0 \\ 16.0 & 16.0 & 32.0 \end{vmatrix}$$

$$= (240.0\mathbf{i} - 480.0\mathbf{k}) + (-840.0\mathbf{i} + 210.0\mathbf{j} + 210.0\mathbf{k}) + (-1280.0\mathbf{j} + 640.0\mathbf{k})$$

$$= -600.0\mathbf{i} - 1070.0\mathbf{j} + 370.0\mathbf{k}$$

$$\mathbf{H}_O = -(600\text{ kg}\cdot\text{m}^2/\text{s})\mathbf{i} - (1070.0\text{ kg}\cdot\text{m}^2/\text{s})\mathbf{j} + (370.0\text{ kg}\cdot\text{m}^2/\text{s})\mathbf{k} \blacktriangleleft$$