Eleventh Edition

CHAPTER

13

VECTOR MECHANICS FOR ENGINEERS:
DYNAMICS

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Kinetics of Particles: Energy and Momentum Methods

HW Problems Week 4 (Due Wed 02/07):
12.74, 12.77, 12.82, 13.8, 13.24, 13.29

Lecture 7 01/31
Modified from Original

Introduction

- **Previously, problems dealing with the motion of particles were solved through the fundamental equation of motion,**

$$\Sigma\vec{F} = m\vec{a}.$$

- **The current chapter introduces two additional methods of analysis.**

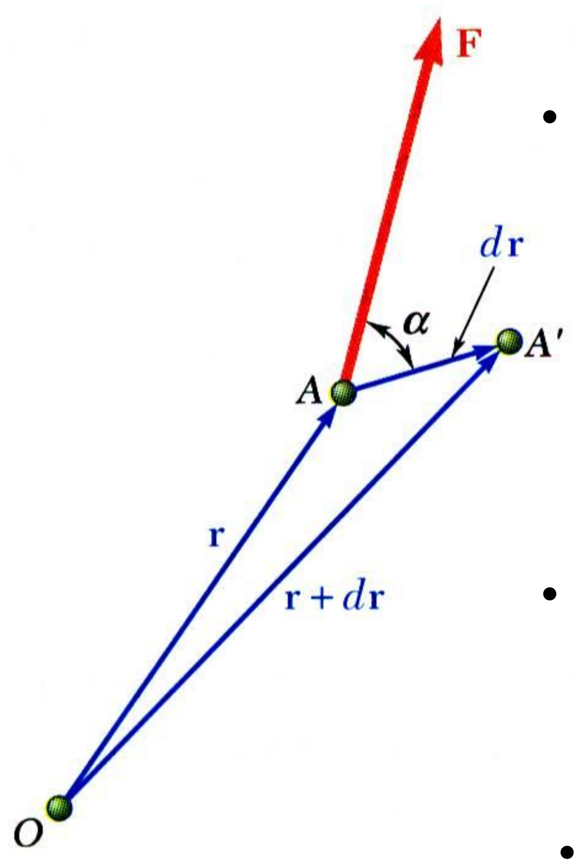
  - ***Method of work and energy*: directly relates force, mass, velocity and displacement.**

  - ***Method of impulse and momentum*: directly relates force, mass, velocity, and time.**

Approaches to Kinetics Problems

Forces and Accelerations	Velocities and Displacements	Velocities and Time
Newton's Second Law (last chapter)	Work-Energy	Impulse-Momentum
$\sum \vec{F} = m\vec{a}_G$	$T_1 + U_{1\to 2} = T_2$	$m\vec{v}_1 + \int_{t_1}^{t_2} \vec{F}\,dt = m\vec{v}_2$

Work of a Force



- Differential vector $d\vec{r}$ is the *particle displacement*.
- *Work of the force* is

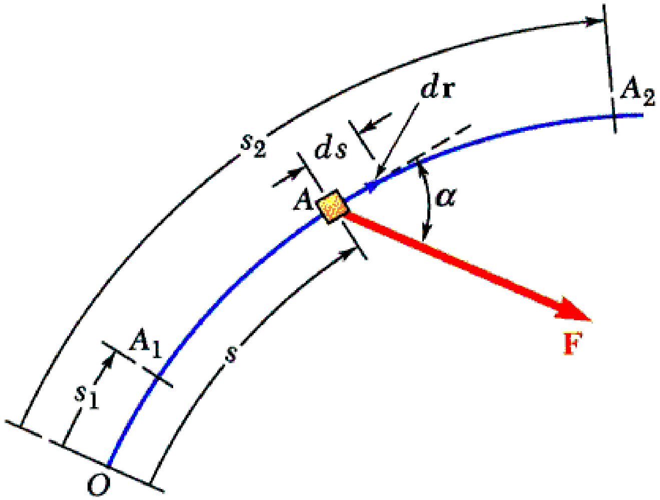
$$dU = \vec{F} \bullet d\vec{r} \quad \text{[scalar product of two vectors]}$$
$$= F\,ds\,\cos\alpha$$
$$= F_x dx + F_y dy + F_z dz$$

- Work is a ***scalar*** quantity, i.e., it has magnitude and sign but not direction.

- Dimensions of work are length × force. Units are

$$1\,\mathrm{J}\,(joule) = (1\,\mathrm{N})(1\,\mathrm{m}) \qquad 1\,\mathrm{ft}\cdot\mathrm{lb} = 1.356\,\mathrm{J}$$

Work of a Force

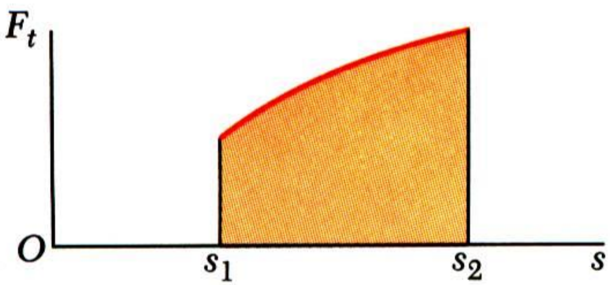


- Work of a force during a finite displacement,

$$U_{1\to 2}=\int_{A_1}^{A_2}\vec{F}\bullet d\vec{r}$$

$$=\int_{s_1}^{s_2}(F\cos\alpha)ds=\int_{s_1}^{s_2}F_t\ ds$$

$$=\int_{A_1}^{A_2}\left(F_x dx+F_y dy+F_z dz\right)$$



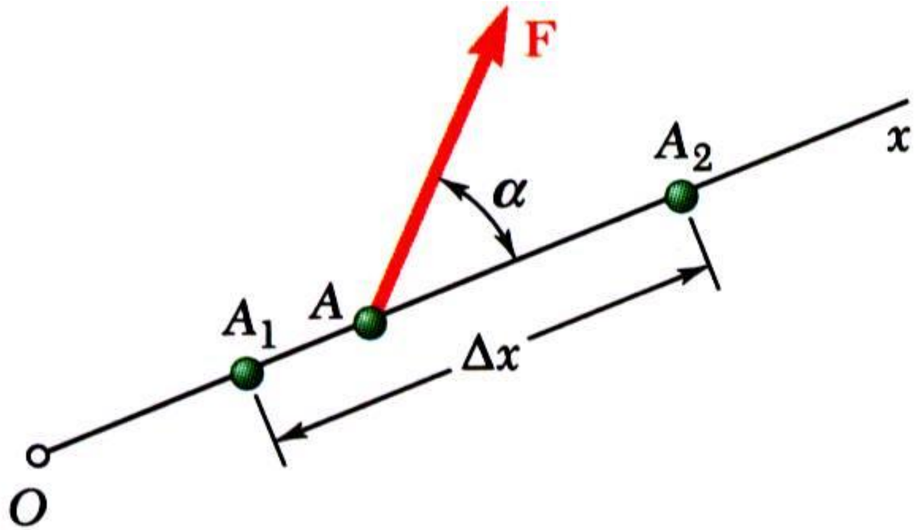
- Work is represented by the area under the curve of F_t plotted against s.

- F_t is the force in the direction of the displacement ds

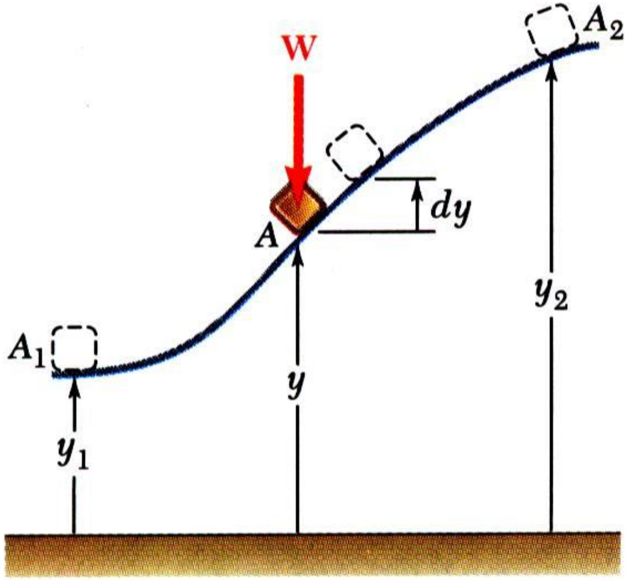
Work of a Force

Work of a constant force in rectilinear motion?

$$U_{1\to 2} = \left(F \cos\alpha\right) \Delta x$$



Work of a Force

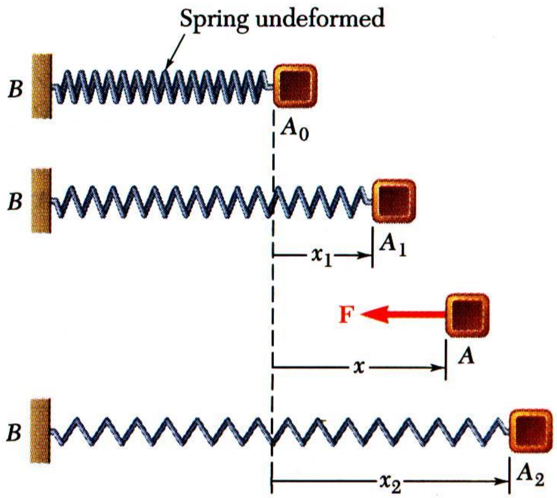


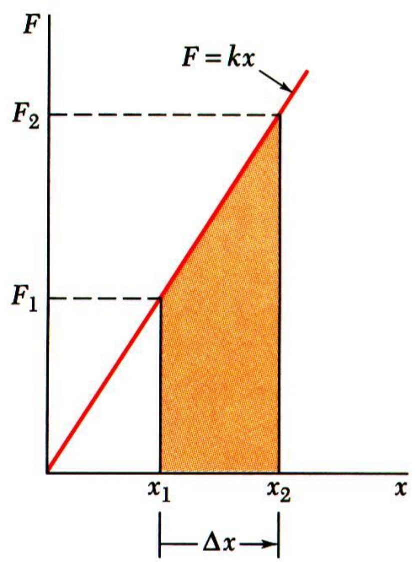
- Work of the force of gravity,

$$
\begin{aligned}
dU &= F_x dx + F_y dy + F_z dz \\
&= -W\ dy \\
U_{1\to 2} &= -\int_{y_1}^{y_2} W\ dy \\
&= -W(y_2 - y_1) = -W\ \Delta y
\end{aligned}
$$

- Work *of the weight* is equal to product of weight *W* and vertical displacement Δy.

Work of a Force





- Magnitude of the force exerted by a spring is proportional to deflection,

$$F = kx$$

$$k = \text{spring constant } (\text{N/m or lb/in.})$$

- Work *of the force exerted by spring*,

$$dU = -F\,dx = -kx\,dx$$

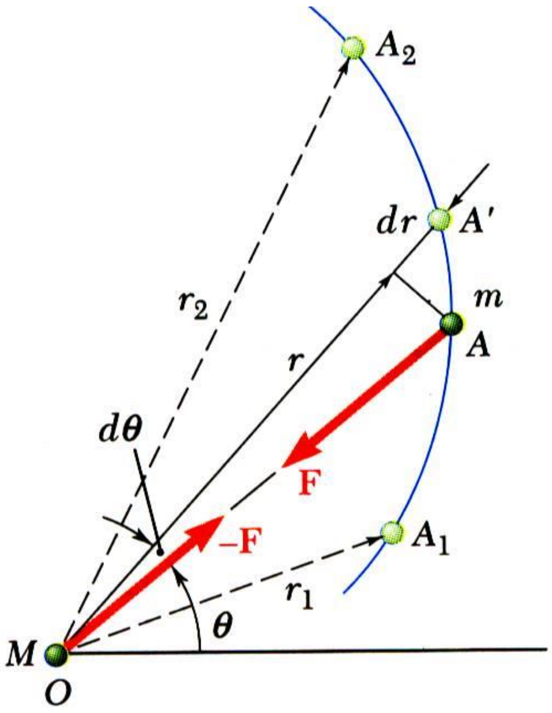
$$U_{1\to2} = -\int_{x_1}^{x_2} kx\,dx = \tfrac{1}{2}kx_1^2 - \tfrac{1}{2}kx_2^2$$

- Work *of the force exerted by spring* is positive when $x_2 < x_1$, i.e., when the spring is returning to its undeformed position.

- Work of the force exerted by the spring is equal to negative of area under curve of F plotted against x,

$$U_{1\to2} = -\tfrac{1}{2}(F_1 + F_2)\,\Delta x$$

Work of a Force

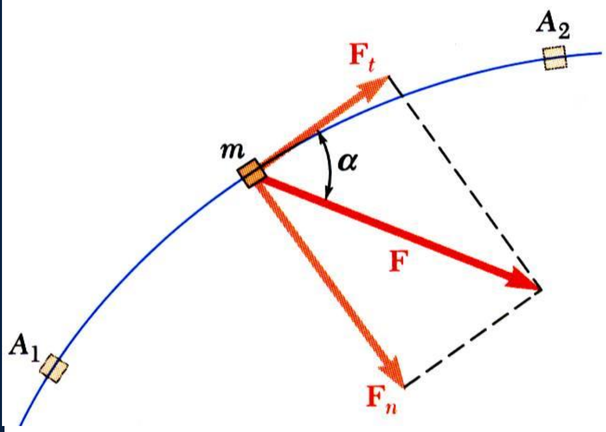


Work of a gravitational force (assume particle M occupies fixed position O while particle m follows path shown),

$$dU = -F dr = -G\frac{Mm}{r^2}dr$$

$$U_{1\to 2} = -\int_{r_1}^{r_2} G\frac{Mm}{r^2}dr = G\frac{Mm}{r_2} - G\frac{Mm}{r_1}$$

Principle of Work & Energy



- Consider a particle of mass m acted upon by force $\vec{F}$

$$F_t = ma_t = m\frac{dv}{dt}$$

$$= m\frac{dv}{ds}\frac{ds}{dt} = mv\frac{dv}{ds}$$

$$F_t\, ds = mv\, dv$$

- Integrating from A_1 to A_2 ,

$$\int_{s_1}^{s_2} F_t ds = m\int_{v_1}^{v_2} v\, dv = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2$$

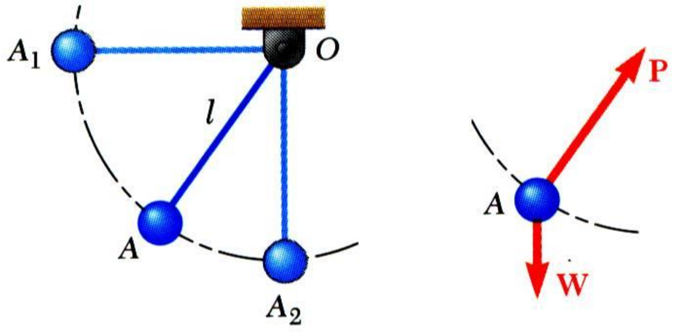
$$U_{1\to 2} = T_2 - T_1 \qquad T = \tfrac{1}{2}mv^2 = kinetic\ energy$$

- *The work of the force* $\vec{F}$ *is equal to the change in kinetic energy of the particle.*

- Units of work and kinetic energy are the same:

$$T = \tfrac{1}{2}mv^2 = \mathrm{kg}\left(\frac{\mathrm{m}}{\mathrm{s}}\right)^2 = \left(\mathrm{kg}\frac{\mathrm{m}}{\mathrm{s}^2}\right)\mathrm{m} = \mathrm{N}\cdot\mathrm{m} = \mathrm{J}$$

Applications of the Principle of Work and Energy



- The bob is released from rest at position A_1. Determine the velocity of the pendulum bob at A_2 using work & kinetic energy.

- Force $\vec{P}$ acts normal to path and does no work.

$$T_1 + U_{1\to 2} = T_2$$

$$0 + Wl = \frac{1}{2}\frac{W}{g}v_2^2$$

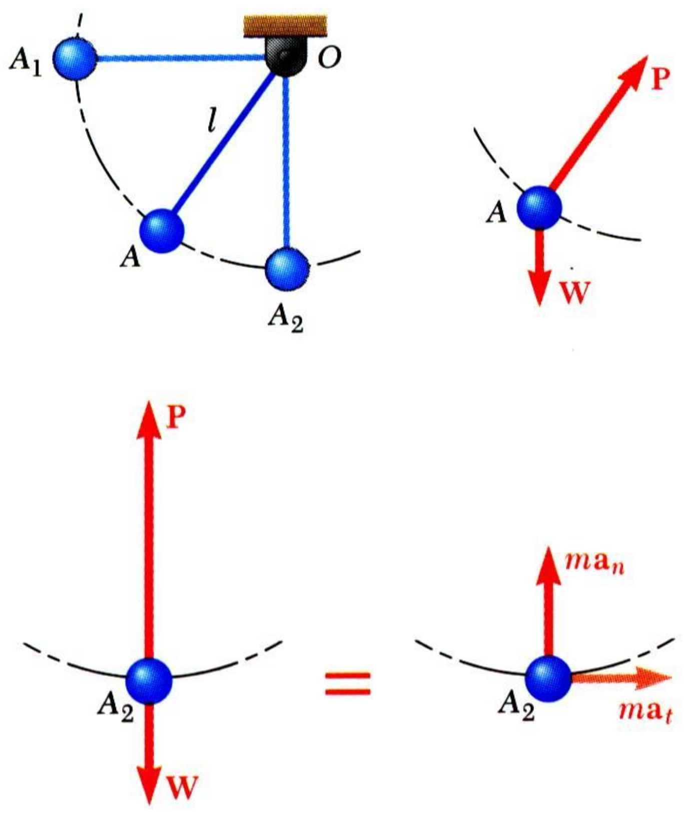
$$v_2 = \sqrt{2gl}$$

- Velocity is found without determining expression for acceleration and integrating.

- All quantities are scalars and can be added directly.

- Forces which do no work are eliminated from the problem.

Applications of the Principle of Work and Energy



$$v_2 = \sqrt{2gl}$$

- Principle of work and energy cannot be applied to directly determine the acceleration of the pendulum bob.

- Calculating the tension in the cord requires supplementing the method of work and energy with an application of Newton's second law.

- As the bob passes through A_2 ,

$$\sum F_n = m\,a_n$$

$$P - W = \frac{W}{g}\frac{v_2^2}{l}$$

$$P = W + \frac{W}{g}\frac{2gl}{l} = 3W$$

Power and Efficiency

- *Power* = rate at which work is done.

$$= \frac{dU}{dt} = \frac{\vec{F} \bullet d\vec{r}}{dt}$$

$$= \vec{F} \bullet \vec{v}$$

- Dimensions of power are work/time or force*velocity. Units for power are

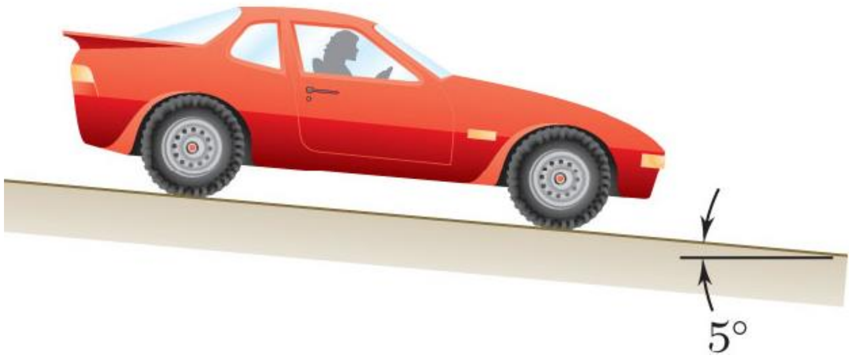
$$1\,\mathrm{W}\ (\mathrm{watt}) = 1\frac{\mathrm{J}}{\mathrm{s}} = 1\,\mathrm{N}\cdot\frac{\mathrm{m}}{\mathrm{s}} \quad \text{or} \quad 1\,\mathrm{hp} = 550\frac{\mathrm{ft}\cdot\mathrm{lb}}{\mathrm{s}} = 746\,\mathrm{W}$$

- η = efficiency

$$= \frac{\text{output work}}{\text{input work}}$$

$$= \frac{\text{power output}}{\text{power input}}$$

Sample Problem 13.1



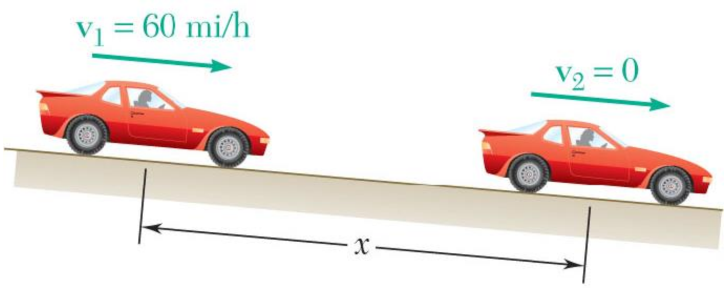
An automobile weighing 4000 lb is driven down a 5° incline at a speed of 60 mi/h when the brakes are applied causing a constant total breaking force of 1500 lb.

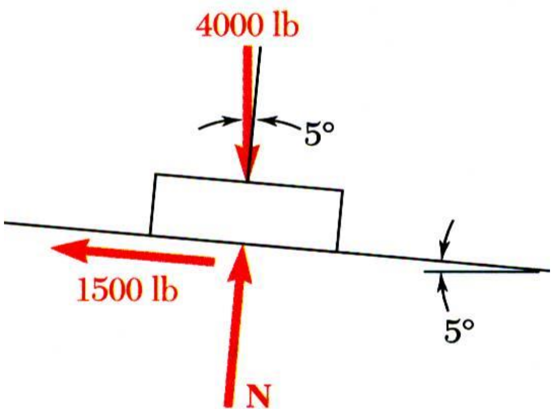
Determine the distance traveled by the automobile as it comes to a stop.

STRATEGY:

- Evaluate the change in kinetic energy.
- Determine the distance required for the work to equal the kinetic energy change.

Sample Problem 13.1





MODELING and ANALYSIS:

- Evaluate the change in kinetic energy.

$$v_1 = \left(60\frac{\text{mi}}{\text{h}}\right)\left(\frac{5280\text{ ft}}{\text{mi}}\right)\left(\frac{\text{h}}{3600\text{ s}}\right) = 88\text{ ft/s}$$

$$T_1 = \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}(4000/32.2)(88)^2 = 481000\text{ ft}\cdot\text{lb}$$

$$v_2 = 0 \qquad T_2 = 0$$

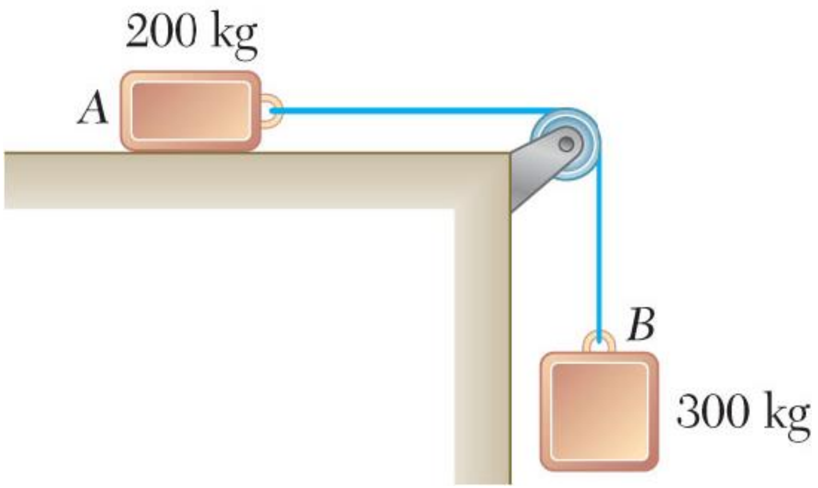
- Determine the distance required for the work to equal the kinetic energy change.

$$U_{1\to 2} = (-1500\text{ lb})x + (4000\text{ lb})(\sin 5°)x$$
$$= -(1151\text{ lb})x$$

$$T_1 + U_{1\to 2} = T_2$$
$$481000\text{ ft}\cdot\text{lb} - (1151\text{ lb})x = 0$$

$$\boxed{x = 418\text{ ft}}$$

Sample Problem 13.2

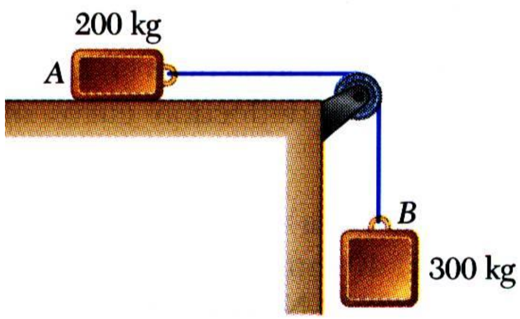


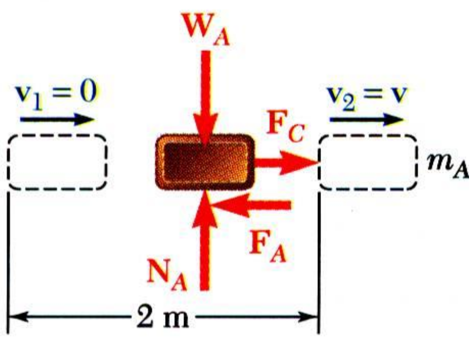
Two blocks are joined by an inextensible cable as shown. If the system is released from rest, determine the velocity of block *A* after it has moved 2 m. Assume that the coefficient of friction between block *A* and the plane is $\mu_k = 0.25$ and that the pulley is weightless and frictionless.

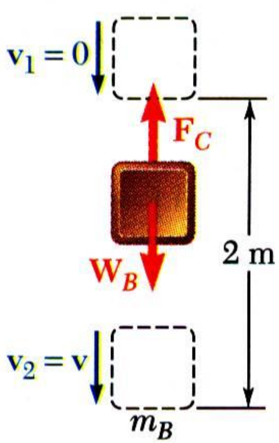
STRATEGY:

- Apply the principle of work and energy separately to blocks *A* and *B*.
- When the two relations are combined, the work of the cable forces cancel. Solve for the velocity.

Sample Problem 13.2







MODELING and ANALYSIS

- Apply the principle of work and energy separately to blocks *A* and *B*.

$$W_A = (200\,\text{kg})\left(9.81\,\text{m/s}^2\right) = 1962\,\text{N}$$

$$F_A = \mu_k N_A = \mu_k W_A = 0.25(1962\,\text{N}) = 490\,\text{N}$$

$$T_1 + U_{1\to 2} = T_2 :$$

$$0 + F_C(2\,\text{m}) - F_A(2\,\text{m}) = \tfrac{1}{2} m_A v^2$$

$$F_C(2\,\text{m}) - (490\,\text{N})(2\,\text{m}) = \tfrac{1}{2}(200\,\text{kg})v^2$$

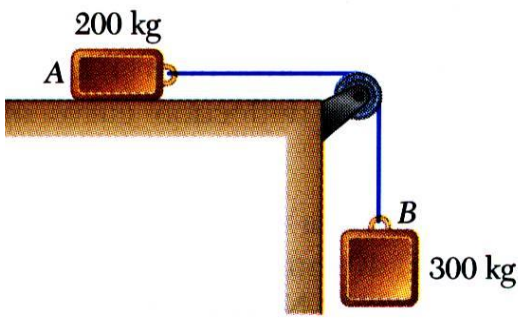
$$W_B = (300\,\text{kg})\left(9.81\,\text{m/s}^2\right) = 2940\,\text{N}$$

$$T_1 + U_{1\to 2} = T_2 :$$

$$0 - F_c(2\,\text{m}) + W_B(2\,\text{m}) = \tfrac{1}{2} m_B v^2$$

$$-F_c(2\,\text{m}) + (2940\,\text{N})(2\,\text{m}) = \tfrac{1}{2}(300\,\text{kg})v^2$$

Sample Problem 13.2



- When the two relations are combined, the work of the cable forces cancel. Solve for the velocity.

$$F_C(2\,\mathrm{m})-(490\,\mathrm{N})(2\,\mathrm{m})=\tfrac{1}{2}(200\,\mathrm{kg})v^2$$

$$-F_c(2\,\mathrm{m})+(2940\,\mathrm{N})(2\,\mathrm{m})=\tfrac{1}{2}(300\,\mathrm{kg})v^2$$

$$(2940\,\mathrm{N})(2\,\mathrm{m})-(490\,\mathrm{N})(2\,\mathrm{m})=\tfrac{1}{2}(200\,\mathrm{kg}+300\,\mathrm{kg})v^2$$

$$4900\,\mathrm{J}=\tfrac{1}{2}(500\,\mathrm{kg})v^2$$

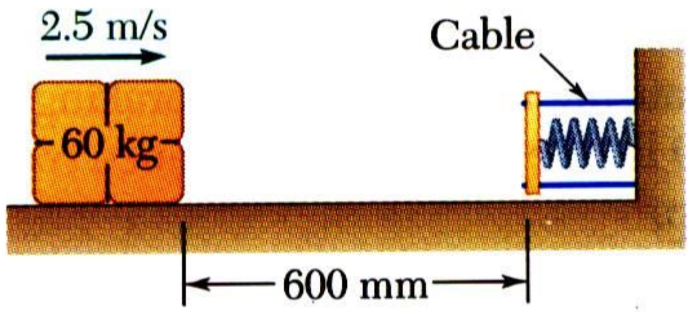
$$\boxed{v=4.43\,\mathrm{m/s}}$$

REFLECT and THINK:

This problem can also be solved by applying the principle of work and energy to the combined system of blocks.

When using the principle of work and energy, it usually saves time to choose your system to be everything that moves.

Sample Problem 13.3



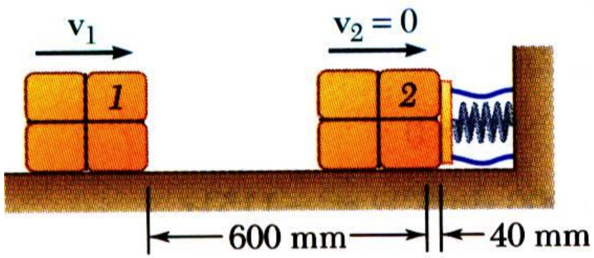
A spring is used to stop a 60 kg package which is sliding on a horizontal surface. The spring has a constant k = 20 kN/m and is held by cables so that it is initially compressed 120 mm. The package has a velocity of 2.5 m/s in the position shown and the maximum deflection of the spring is 40 mm.

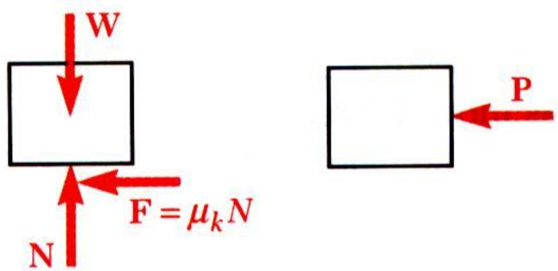
Determine *(a)* the coefficient of kinetic friction between the package and surface and *(b)* the velocity of the package as it passes again through the position shown.

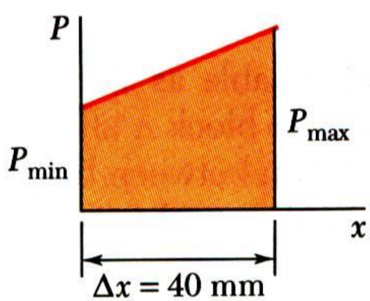
STRATEGY:

- Apply the principle of work and energy between the initial position and the point at which the spring is fully compressed and the velocity is zero. The only unknown in the relation is the friction coefficient.
- Apply the principle of work and energy for the rebound of the package. The only unknown in the relation is the velocity at the final position.

Sample Problem 13.3







MODELING and ANALYSIS:

- Apply principle of work and energy between initial position and the point at which spring is fully compressed.

$$T_1 = \tfrac{1}{2} m v_1^2 = \tfrac{1}{2}(60\,\text{kg})(2.5\,\text{m/s})^2 = 187.5\,\text{J} \qquad T_2 = 0$$

$$\begin{aligned}(U_{1\to 2})_f &= -\mu_k W x \\ &= -\mu_k (60\,\text{kg})(9.81\,\text{m/s}^2)(0.640\,\text{m}) = -(377\,\text{J})\mu_k\end{aligned}$$

$$P_{\min} = k x_0 = (20\,\text{kN/m})(0.120\,\text{m}) = 2400\,\text{N}$$

$$P_{\max} = k(x_0 + \Delta x) = (20\,\text{kN/m})(0.160\,\text{m}) = 3200\,\text{N}$$

$$\begin{aligned}(U_{1\to 2})_e &= -\tfrac{1}{2}(P_{\min} + P_{\max})\Delta x \\ &= -\tfrac{1}{2}(2400\,\text{N} + 3200\,\text{N})(0.040\,\text{m}) = -112.0\,\text{J}\end{aligned}$$

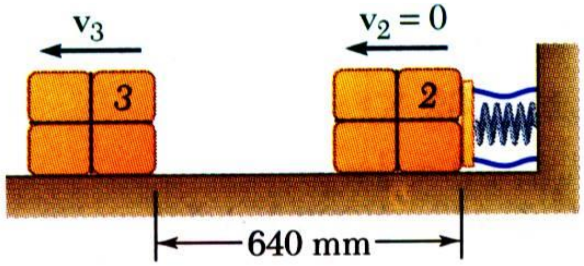
$$U_{1\to 2} = (U_{1\to 2})_f + (U_{1\to 2})_e = -(377\,\text{J})\mu_k - 112\,\text{J}$$

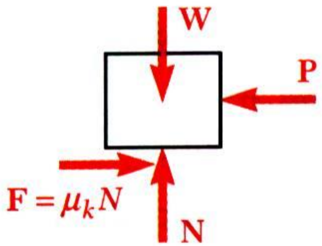
$$T_1 + U_{1\to 2} = T_2 :$$

$$187.5\,\text{J} - (377\,\text{J})\mu_k - 112\,\text{J} = 0$$

$$\boxed{\mu_k = 0.20}$$

Sample Problem 13.3





- Apply the principle of work and energy for the rebound of the package.

$$T_2 = 0 \qquad T_3 = \tfrac{1}{2} m v_3^2 = \tfrac{1}{2}(60\,\text{kg}) v_3^2$$

$$U_{2\to3} = (U_{2\to3})_f + (U_{2\to3})_e = -(377\,\text{J})\mu_k + 112\,\text{J}$$
$$= +36.5\,\text{J}$$

$$T_2 + U_{2\to3} = T_3:$$
$$0 + 36.5\,\text{J} = \tfrac{1}{2}(60\,\text{kg}) v_3^2$$

$$\boxed{v_3 = 1.103\,\text{m/s}}$$

REFLECT and THINK:

You needed to break this problem into two segments. From the first segment you were able to determine the coefficient of friction. Then you could use the principle of work and energy to determine the velocity of the package at any other location. Note that the system does not lose any energy due to the spring; it returns all of its energy back to the package. You would need to design something that could absorb the kinetic energy of the package in order to bring it to rest.