Seventh Edition

CHAPTER

4

MECHANICS OF MATERIALS

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Lecture Notes:

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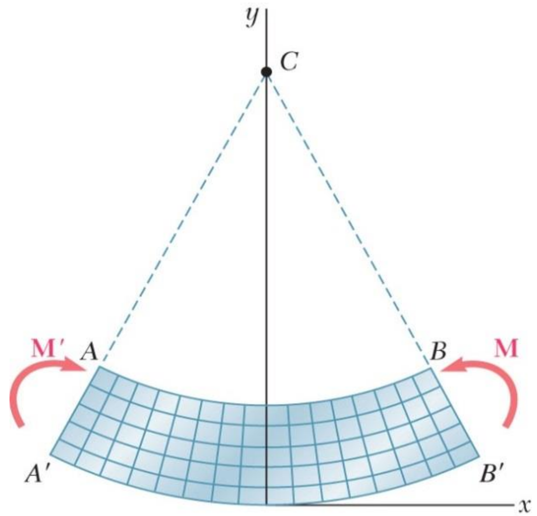
Pure Bending

Lecture 7 01/31/2018
Modified from Original

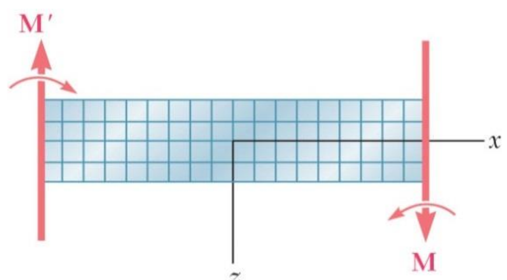
HW Problems Week 4 (due Mon 02/05):

3.11, 3.15, 4.9, 4.18

Bending Deformations



(a) Longitudinal, vertical section (plane of symmetry)



(b) Longitudinal, horizontal section

Fig. 4.9 Member subject to pure bending shown in two views. (a) Longitudinal, vertical view (plane of symmetry) and (b) Longitudinal, horizontal view.

Beam with a plane of symmetry in pure bending:

- member remains symmetric
- bends uniformly to form a circular arc
- cross-sectional plane passes through arc center and remains planar
- length of top decreases and length of bottom increases
- a *neutral surface* must exist that is parallel to the upper and lower surfaces and for which the length does not change
- stresses and strains are negative (compressive) above the neutral plane and positive (tension) below it

Strain Due to Bending

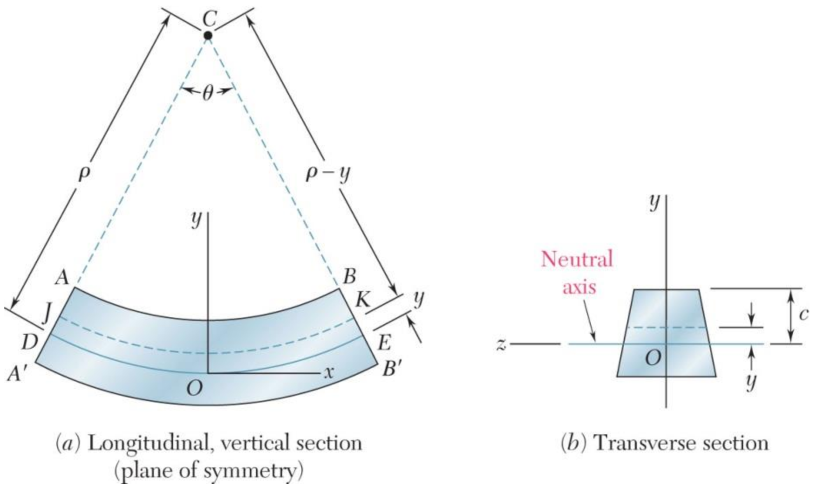


Fig. 4.10 Kinematic definitions for pure bending. (a) Longitudinal-vertical view and (b) Transverse section at origin.

Consider a beam segment of length L.

After deformation, the length of the neutral surface remains L. At other sections,

$$L' = (\rho - y)\theta$$
$$\delta = L' - L = (\rho - y)\theta - \rho\theta = -y\theta$$
$$\varepsilon_x = \frac{\delta}{L} = -\frac{y\theta}{\rho\theta} = -\frac{y}{\rho} \quad \text{(strain varies linearly)}$$
$$\varepsilon_m = \frac{c}{\rho} \quad \text{or} \quad \rho = \frac{c}{\varepsilon_m} \quad \left(\varepsilon_m \text{ is the maximum absolute value of strain}\right)$$
$$\varepsilon_x = -\frac{y}{c}\varepsilon_m$$

The minus sign is due to the fact that it is assumed the bending moment is positive and thus the beam is concave upward.

Stress Due to Bending

- For a linearly elastic and homogeneous material,

$$\sigma_x = E\varepsilon_x = -\frac{y}{c}E\varepsilon_m$$

$$= -\frac{y}{c}\sigma_m \quad \text{(stress varies linearly)}$$

- For static equilibrium,

$$F_x = 0 = \int \sigma_x \, dA = \int -\frac{y}{c}\sigma_m \, dA$$

$$0 = -\frac{\sigma_m}{c}\int y \, dA$$

First moment with respect to neutral axis is zero. Therefore, the neutral axis must pass through the section centroid.

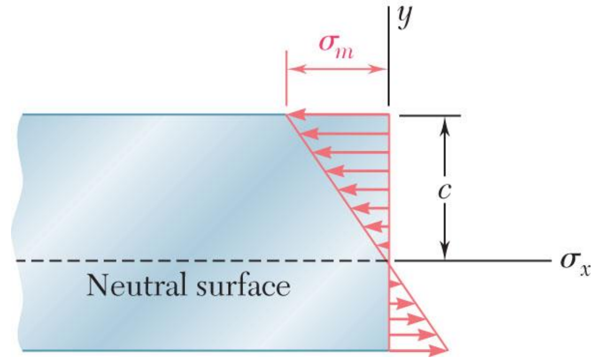


Fig. 4.11 Bending stresses vary linearly with distance from the neutral axis.

- For static equilibrium,

$$M = \int(-y\sigma_x \, dA) = \int(-y)\left(-\frac{y}{c}\sigma_m\right)dA$$

$$M = \frac{\sigma_m}{c}\int y^2 \, dA = \frac{\sigma_m I}{c}$$

$$\sigma_m = \frac{Mc}{I} = \frac{M}{S}$$

Substituting $\sigma_x = -\frac{y}{c}\sigma_m$

$$\sigma_x = -\frac{My}{I}$$

Beam Section Properties

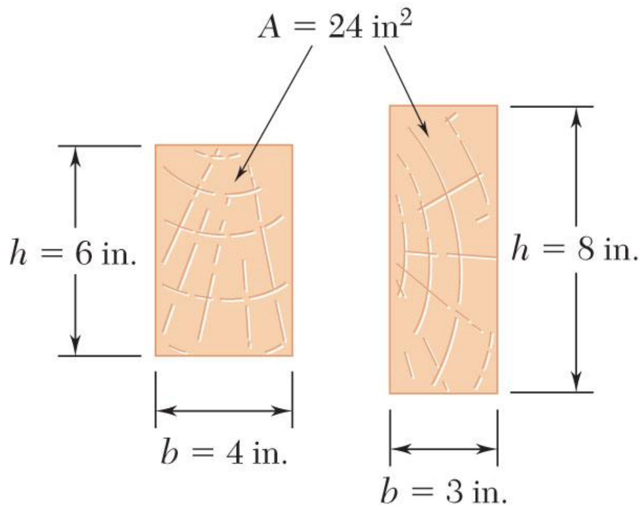


Fig. 4.12 Wood beam cross sections.

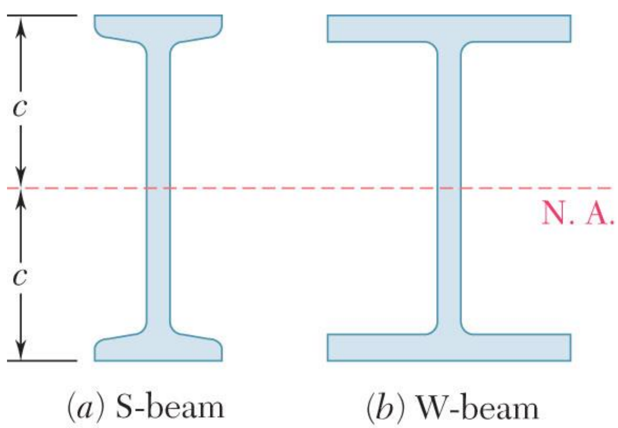


Fig. 4.13 Two type of steel beam cross sections. (a) S-beam and (b) W-beam

- The maximum normal stress due to bending,

$$\sigma_m = \frac{Mc}{I} = \frac{M}{S}$$

$$I = \text{section moment of inertia}$$

$$S = \frac{I}{c} = \text{section modulus}$$

A beam section with a larger section modulus will have a lower maximum stress

- Consider a rectangular beam cross section,

$$S = \frac{I}{c} = \frac{\frac{1}{12}bh^3}{h/2} = \frac{1}{6}bh^3 = \frac{1}{6}Ah$$

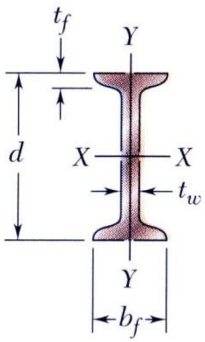
Between two beams with the same cross sectional area, the beam with the larger depth h will be more effective in resisting bending.

- Structural steel beams are designed to have a large section modulus.

Properties of American Standard Shapes

Appendix C. Properties of Rolled-Steel Shapes (SI Units)

S Shapes
(American Standard Shapes)



Designation†	Area A, mm^2	Depth d, mm	Flange Width b_f, mm	Flange Thickness t_f, mm	Web Thickness t_w, mm	Axis X-X I_x 10^6 mm^4	Axis X-X S_x 10^3 mm^3	Axis X-X r_x mm	Axis Y-Y I_y 10^6 mm^4	Axis Y-Y S_y 10^3 mm^3	Axis Y-Y r_y mm
S610 × 180	22900	622	204	27.7	20.3	1320	4240	240	34.9	341	39.0
158	20100	622	200	27.7	15.7	1230	3950	247	32.5	321	39.9
149	19000	610	184	22.1	18.9	995	3260	229	20.2	215	32.3
134	17100	610	181	22.1	15.9	938	3080	234	19.0	206	33.0
119	15200	610	178	22.1	12.7	878	2880	240	17.9	198	34.0
S510 × 143	18200	516	183	23.4	20.3	700	2710	196	21.3	228	33.9
128	16400	516	179	23.4	16.8	658	2550	200	19.7	216	34.4
112	14200	508	162	20.2	16.1	530	2090	193	12.6	152	29.5
98.3	12500	508	159	20.2	12.8	495	1950	199	11.8	145	30.4
S460 × 104	13300	457	159	17.6	18.1	385	1685	170	10.4	127	27.5
81.4	10400	457	152	17.6	11.7	333	1460	179	8.83	113	28.8
S380 × 74	9500	381	143	15.6	14.0	201	1060	145	6.65	90.8	26.1
64	8150	381	140	15.8	10.4	185	971	151	6.15	85.7	27.1

Deformations in a Transverse Cross Section

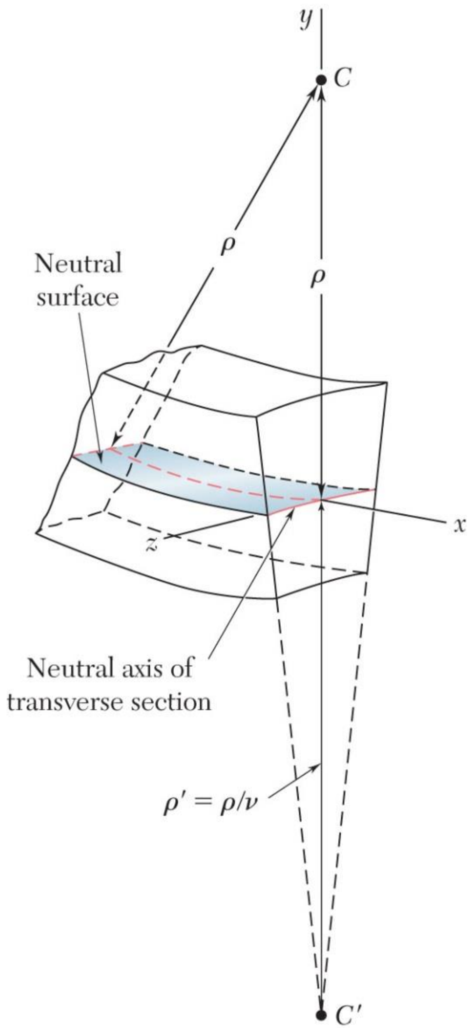


Fig. 4.16 Deformation of a transverse cross section.

- Deformation due to bending moment M is quantified by the curvature of the neutral surface

$$\frac{1}{\rho} = \frac{\varepsilon_m}{c} = \frac{\sigma_m}{Ec} = \frac{1}{Ec}\frac{Mc}{I}$$

$$= \frac{M}{EI}$$

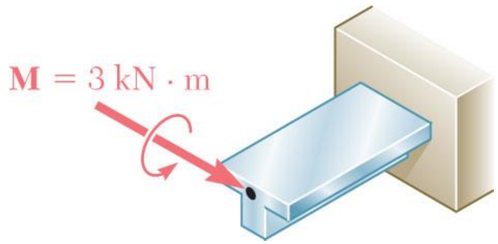
- Although transverse cross sectional planes remain planar when subjected to bending moments, in-plane deformations are nonzero,

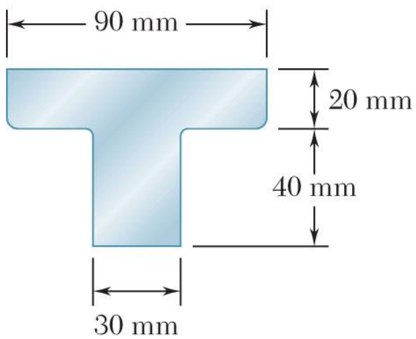
$$\varepsilon_y = -\nu\varepsilon_x = \frac{\nu y}{\rho} \qquad \varepsilon_z = -\nu\varepsilon_x = \frac{\nu y}{\rho}$$

- Expansion above the neutral surface and contraction below it cause an in-plane curvature,

$$\frac{1}{\rho'} = \frac{\nu}{\rho} = \text{anticlastic curvature}$$

Sample Problem 4.2





A cast-iron machine part is acted upon by a 3 kN-m couple. Knowing E = 165 GPa and neglecting the effects of fillets, determine (a) the maximum tensile and compressive stresses, (b) the radius of curvature.

SOLUTION:

- Based on the cross section geometry, calculate the location of the section centroid and moment of inertia.

$$\bar{Y} = \frac{\sum \bar{y}A}{\sum A} \qquad I_{x'} = \sum\left(\bar{I} + Ad^2\right)$$

- Apply the elastic flexural formula to find the maximum tensile and compressive stresses.

$$\sigma_m = \frac{Mc}{I}$$

- Calculate the curvature

$$\frac{1}{\rho} = \frac{M}{EI}$$

Sample Problem 4.2

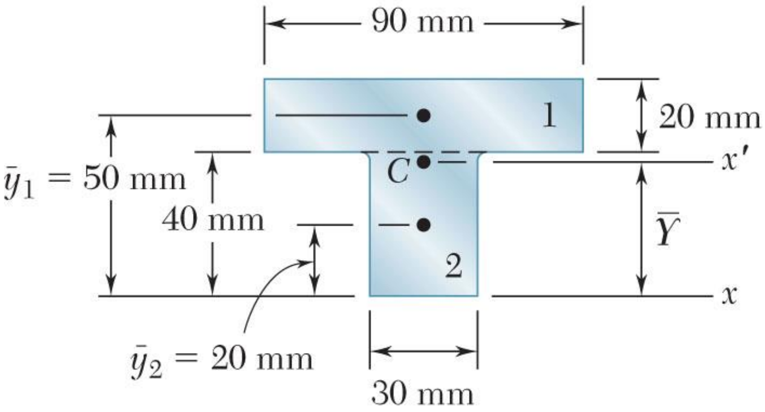


Fig. 1 Composite areas for calculating centroid.

SOLUTION:

Based on the cross section geometry, calculate the location of the section centroid and moment of inertia.

	Area, mm^2	$\bar{y}$, mm	$\bar{y}A$, mm^3
1	$20\times 90 = 1800$	50	90×10^3
2	$40\times 30 = 1200$	20	24×10^3
	$\sum A = 3000$		$\sum \bar{y}A = 114\times 10^3$

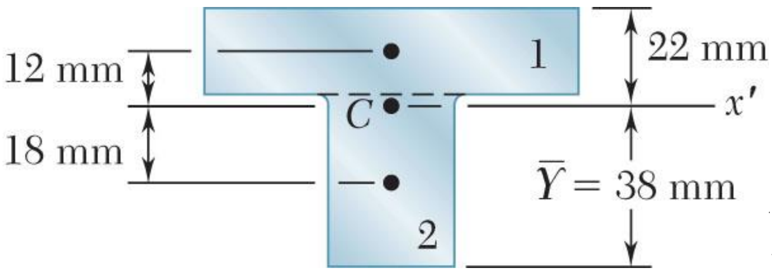


Fig. 2 Composite sections for calculating moment of inertia.

$$\bar{Y} = \frac{\sum \bar{y}A}{\sum A} = \frac{114\times 10^3}{3000} = 38\,\text{mm}$$

$$I_{x'} = \sum\left(\bar{I} + Ad^2\right) = \sum\left(\tfrac{1}{12}bh^3 + Ad^2\right)$$

$$= \left(\tfrac{1}{12}90\times 20^3 + 1800\times 12^2\right) + \left(\tfrac{1}{12}30\times 40^3 + 1200\times 18^2\right)$$

$$I = 868\times 10^3\,\text{mm}^4 = 868\times 10^{-9}\,\text{m}^4$$

Sample Problem 4.2

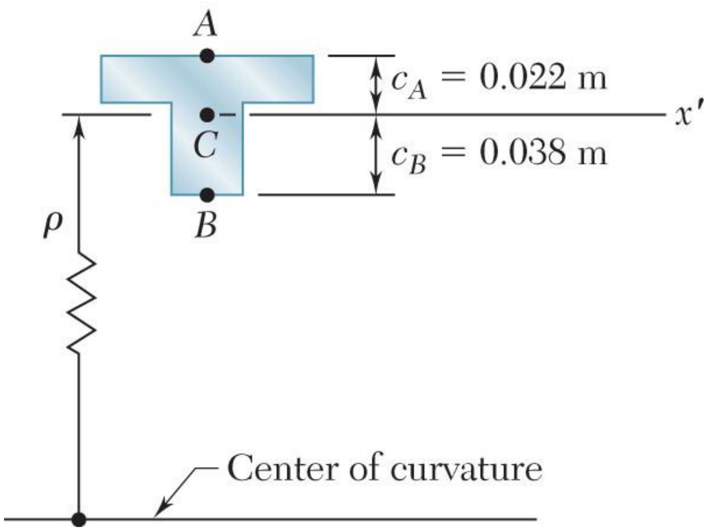


Fig. 3 Deformed radius of curvature is measured to the centroid of the cross sections.

- Apply the elastic flexural formula to find the maximum tensile and compressive stresses.

$$\sigma_m = \frac{Mc}{I}$$

$$\sigma_A = \frac{M\,c_A}{I} = \frac{3\,\text{kN}\cdot\text{m}\times 0.022\,\text{m}}{868\times 10^{-9}\,\text{m}^4} \qquad \boxed{\sigma_A = +76.0\,\text{MPa}}$$

$$\sigma_B = -\frac{M\,c_B}{I} = -\frac{3\,\text{kN}\cdot\text{m}\times 0.038\,\text{m}}{868\times 10^{-9}\,\text{m}^4} \qquad \boxed{\sigma_B = -131.3\,\text{MPa}}$$

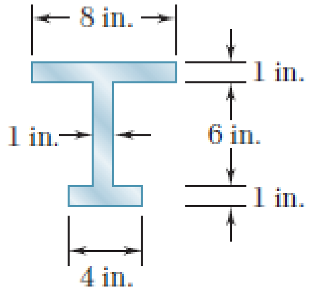
- Calculate the curvature

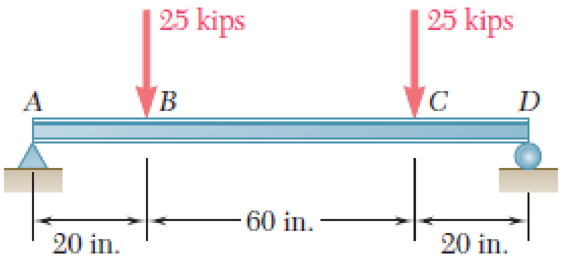
$$\frac{1}{\rho} = \frac{M}{EI}$$

$$= \frac{3\,\text{kN}\cdot\text{m}}{(165\,\text{GPa})\left(868\times 10^{-9}\,\text{m}^4\right)}$$

$$\boxed{\begin{aligned}\frac{1}{\rho} &= 20.95\times 10^{-3}\,\text{m}^{-1}\\ \rho &= 47.7\,\text{m}\end{aligned}}$$

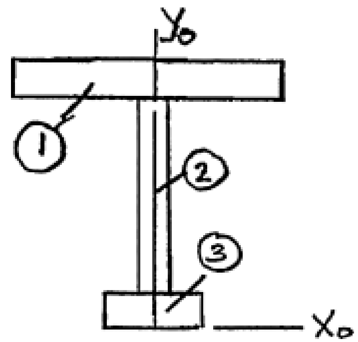
In-Class Problem





Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.

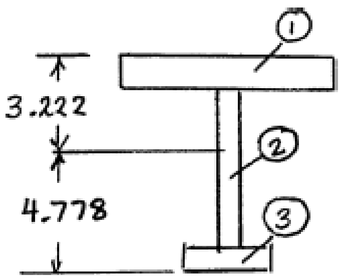
Solution



	A	$\bar{y}_0$	$A\bar{y}_0$
①	8	7.5	60
②	6	4	24
③	4	0.5	2
Σ	18		86

$$\bar{Y}_o = \frac{86}{18} = 4.778 \text{ in.}$$

Neutral axis lies 4.778 in. above the base.



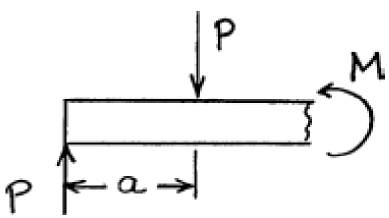
$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(8)(1)^3 + (8)(2.772)^2 = 59.94 \text{ in}^4$$

$$I_2 = \frac{1}{12}b_2h_2^3 + A_2d_2^2 = \frac{1}{12}(1)(6)^3 + (6)(0.778)^2 = 21.63 \text{ in}^4$$

$$I_3 = \frac{1}{12}b_3h_3^3 + A_3d_3^2 = \frac{1}{12}(4)(1)^3 + (4)(4.278)^2 = 73.54 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 59.94 + 21.63 + 73.57 = 155.16 \text{ in}^4$$

$y_{top} = 3.222$ in. $\quad y_{bot} = -4.778$ in.



$$M - Pa = 0$$

$$M = Pa = (25)(20) = 500 \text{ kip}\cdot\text{in.}$$

$$\sigma_{top} = -\frac{My_{top}}{I} = -\frac{(500)(3.222)}{155.16} \qquad \sigma_{top} = -10.38 \text{ ksi (compression)} \blacktriangleleft$$

$$\sigma_{bot} = -\frac{My_{bot}}{I} = -\frac{(500)(-4.778)}{155.16} \qquad \sigma_{bot} = 15.40 \text{ ksi (tension)} \blacktriangleleft$$