Eleventh Edition

VECTOR MECHANICS FOR ENGINEERS:
DYNAMICS

CHAPTER
13

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Kinetics of Particles: Energy and Momentum Methods

HW Problems Week 5 (Due Wed 02/14): 13.60, 13.63, 13.68, 13.122, 13.146, 13.151

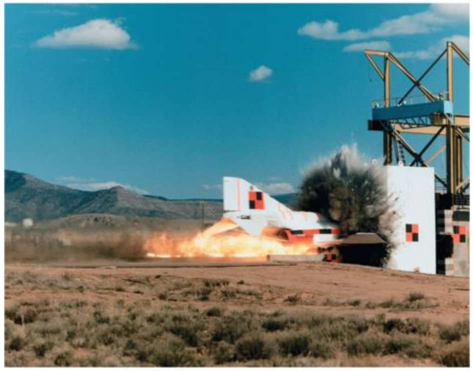
Lecture 9 02/07
Modified from Original

Impulsive Motion

The thrust of a rocket acts over a specific time period to give the rocket linear momentum.

Crash tests are often performed to help improve safety in different vehicles.

NASA.



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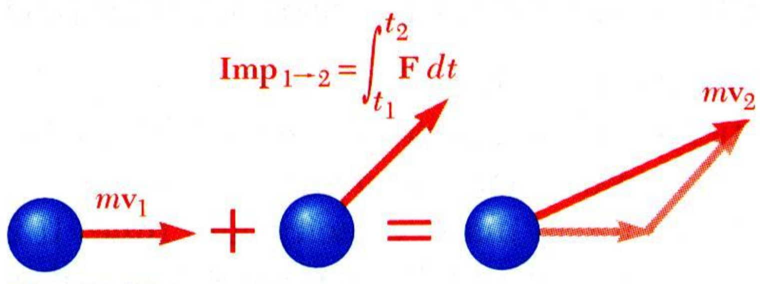
Principle of Impulse and Momentum



- Dimensions of the impulse of a force are

  *force*time.*

- Units for the impulse of a force are

$$\mathrm{N}\cdot\mathrm{s} = \left(\mathrm{kg}\cdot\mathrm{m/s^2}\right)\cdot\mathrm{s} = \mathrm{kg}\cdot\mathrm{m/s}$$

- From Newton's second law,

$$\vec{F} = \frac{d}{dt}\left(m\vec{v}\right) \qquad m\vec{v} = \text{linear momentum}$$

$$\vec{F}dt = d\left(m\vec{v}\right)$$

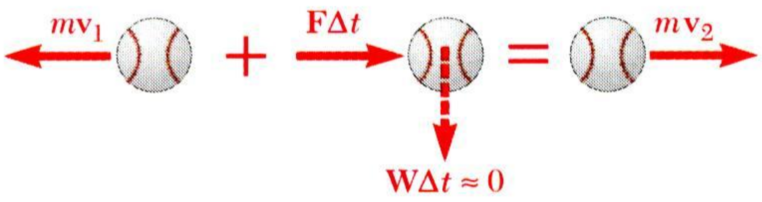
$$\int_{t_1}^{t_2} \vec{F}dt = m\vec{v}_2 - m\vec{v}_1$$

$$\int_{t_1}^{t_2} \vec{F}dt = \mathbf{Imp}_{1\to2} = \text{impulse of the force } \vec{F}$$

$$m\vec{v}_1 + \mathbf{Imp}_{1\to2} = m\vec{v}_2$$

- The final momentum of the particle can be obtained by adding vectorially its initial momentum and the impulse of the force during the time interval.

Impulsive Motion



- Force acting on a particle during a very short time interval that is large enough to cause a significant change in momentum is called an *impulsive force*.
- When impulsive forces act on a particle,

$$m\vec{v}_1 + \sum \vec{F}\,\Delta t = m\vec{v}_2$$

- When a baseball is struck by a bat, contact occurs over a short time interval but force is large enough to change sense of ball motion.
- *Nonimpulsive forces* are forces for which $\vec{F}\Delta t$ is small and therefore, may be neglected – an example of this is the weight of the baseball.

Sample Problem 13.13

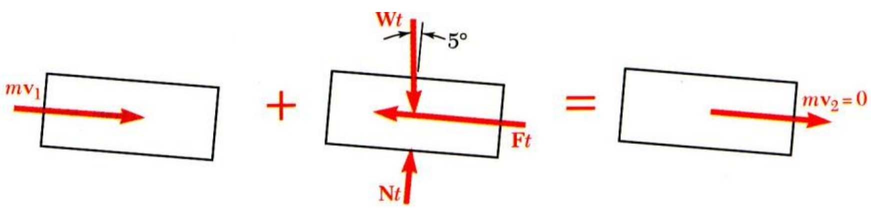
An automobile weighing 4000 lb is driven down a 5° incline at a speed of 60 mi/h when the brakes are applied, causing a constant total braking force of 1500 lb.

Determine the time required for the automobile to come to a stop.

STRATEGY:

- Apply the principle of impulse and momentum. The impulse is equal to the product of the constant forces and the time interval.

Sample Problem 13.13



MODELING and ANALYSIS:

- Apply the principle of impulse and momentum.

$$m\vec{v}_1 + \sum \mathbf{Imp}_{1\to 2} = m\vec{v}_2$$

Taking components parallel to the incline,

$$mv_1 + (W\sin 5°)t - Ft = 0$$

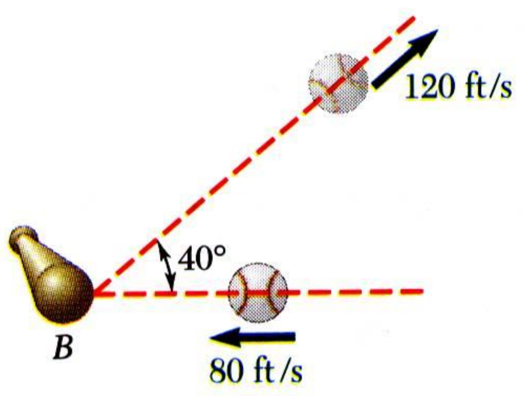
$$\left(\frac{4000}{32.2}\right)(88\,\mathrm{ft/s}) + (4000\sin 5°)t - 1500t = 0$$

$$\boxed{t = 9.49\,\mathrm{s}}$$

REFLECT and THINK

- You could use Newton’s second law to solve this problem. First, you would determine the car’s deceleration, separate variables, and then integrate a = dv/dt to relate the velocity, deceleration, and time. You could not use conservation of energy to solve this problem, because this principle does not involve time.

Sample Problem 13.16

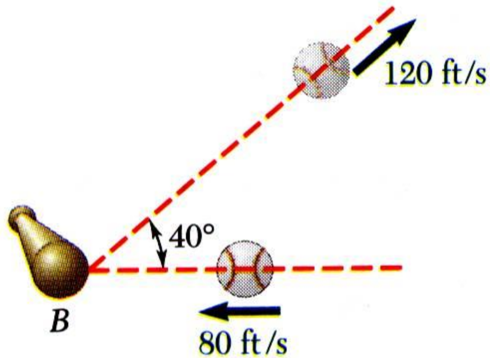


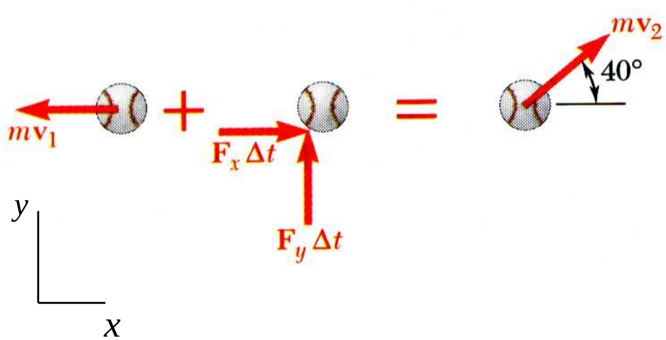
A 4 oz baseball is pitched with a velocity of 80 ft/s. After the ball is hit by the bat, it has a velocity of 120 ft/s in the direction shown. If the bat and ball are in contact for 0.015 s, determine the average impulsive force exerted on the ball during the impact.

STRATEGY:

- Apply the principle of impulse and momentum in terms of horizontal and vertical component equations.

Sample Problem 13.16





MODELING and ANALYSIS:

- Apply the principle of impulse and momentum in terms of horizontal and vertical component equations.

$$m\vec{v}_1 + \mathbf{Imp}_{1\to 2} = m\vec{v}_2$$

x component equation:

$$-mv_1 + F_x \Delta t = mv_2 \cos 40°$$

$$-\frac{4/16}{32.2}(80) + F_x(0.15) = \frac{4/16}{32.2}(120 \cos 40°)$$

$$F_x = 89 \text{ lb}$$

y component equation:

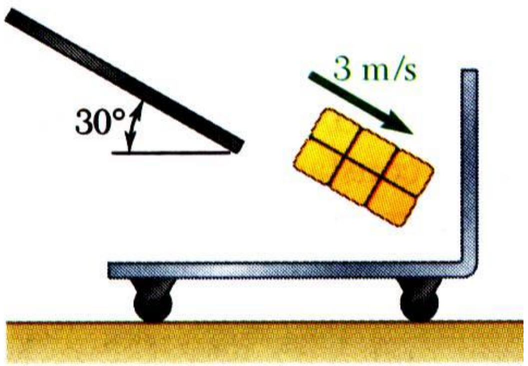
$$0 + F_y \Delta t = mv_2 \sin 40°$$

$$F_y(0.015) = \frac{4/16}{32.2}(120 \sin 40°)$$

$$F_y = 39.9 \text{ lb}$$

$$\boxed{\vec{F} = (89 \text{ lb})\vec{i} + (39.9 \text{ lb})\vec{j}, \quad F = 97.5 \text{ lb}}$$

Sample Problem 13.17



A 10 kg package drops from a chute into a 24 kg cart with a velocity of 3 m/s. Knowing that the cart is initially at rest and can roll freely, determine *(a)* the final velocity of the cart, *(b)* the impulse exerted by the cart on the package, and *(c)* the fraction of the initial energy lost in the impact.

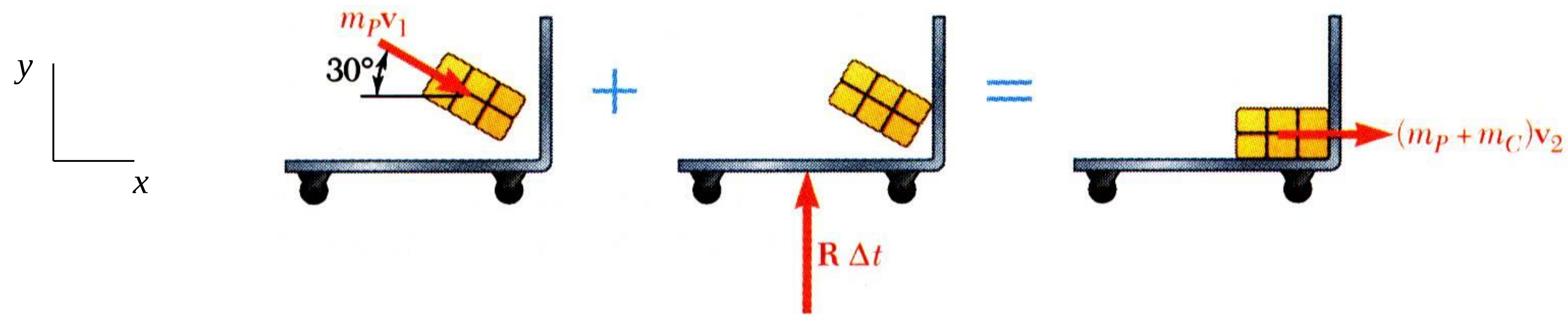
STRATEGY:

- Apply the principle of impulse and momentum to the package-cart system to determine the final velocity.
- Apply the same principle to the package alone to determine the impulse exerted on it from the change in its momentum.

Sample Problem 13.17

MODELING and ANALYSIS

- Apply the principle of impulse and momentum to the package-cart system to determine the final velocity.



$$m_p\vec{v}_1 + \sum \mathbf{Imp}_{1\to 2} = (m_p + m_c)\vec{v}_2$$

x components:

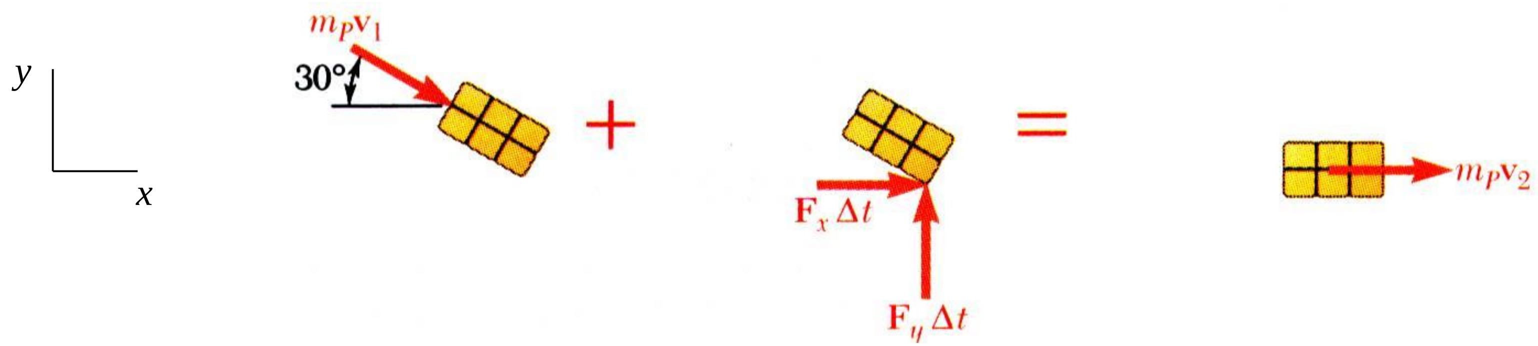
$$m_p v_1 \cos 30° + 0 = (m_p + m_c)v_2$$

$$(10\text{ kg})(3\text{ m/s})\cos 30° = (10\text{ kg} + 25\text{ kg})v_2$$

$$\boxed{v_2 = 0.742\text{ m/s}}$$

Sample Problem 13.17

- Apply the same principle to the package alone to determine the impulse exerted on it from the change in its momentum.



$$m_p\vec{v}_1 + \sum \mathbf{Imp}_{1\to 2} = m_p\vec{v}_2$$

x components:

$$m_p v_1 \cos 30° + F_x\Delta t = m_p v_2$$

$$(10\,\text{kg})(3\,\text{m/s})\cos 30° + F_x\Delta t = (10\,\text{kg})v_2 \qquad F_x\Delta t = -18.56\,\text{N}\cdot\text{s}$$

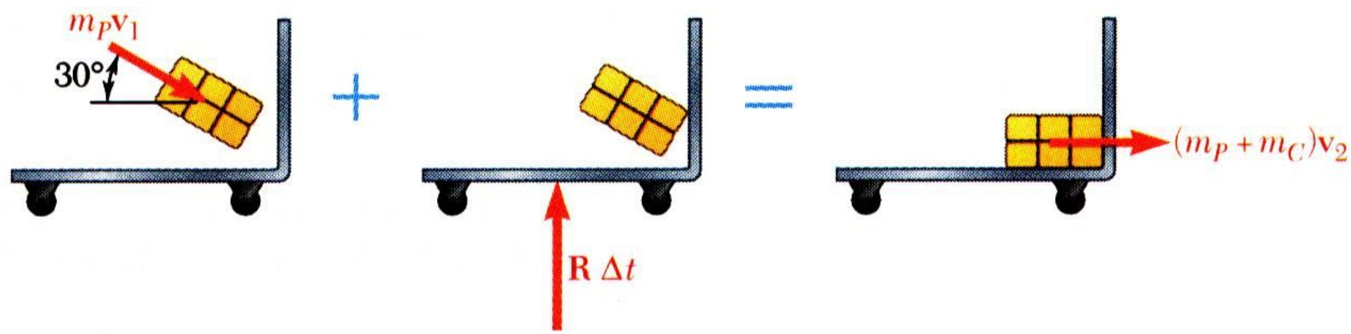
y components:

$$-m_p v_1 \sin 30° + F_y\Delta t = 0$$

$$-(10\,\text{kg})(3\,\text{m/s})\sin 30° + F_y\Delta t = 0 \qquad F_y\Delta t = 15\,\text{N}\cdot\text{s}$$

$$\boxed{\sum \mathbf{Imp}_{1\to 2} = \vec{F}\Delta t = (-18.56\,\text{N}\cdot\text{s})\vec{i} + (15\,\text{N}\cdot\text{s})\vec{j} \qquad F\Delta t = 23.9\,\text{N}\cdot\text{s}}$$

Sample Problem 13.17



To determine the fraction of energy lost,

$$T_1 = \tfrac{1}{2} m_p v_1^2 = \tfrac{1}{2}(10\text{ kg})(3\text{ m/s})^2 = 45\text{ J}$$

$$T_2 = \tfrac{1}{2}(m_p + m_c)v_2^2 = \tfrac{1}{2}(10\text{ kg} + 25\text{ kg})(0.742\text{ m/s})^2 = 9.63\text{ J}$$

$$\boxed{\frac{T_1 - T_2}{T_1} = \frac{45\text{ J} - 9.63\text{ J}}{45\text{ J}} = 0.786}$$

REFLECT and THINK:

Except in the purely theoretical case of a “perfectly elastic” collision, mechanical energy is never conserved in a collision between two objects, even though linear momentum may be conserved. Note that, in this problem, momentum was conserved in the x direction but was not conserved in the y direction because of the vertical impulse on the wheels of the cart. Whenever you deal with an impact, you need to use impulse-momentum methods.