Eleventh Edition

CHAPTER

13

VECTOR MECHANICS FOR ENGINEERS:

DYNAMICS

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Kinetics of Particles: Energy and Momentum Methods

HW Problems Week 6 (Due Mon 02/19): 13.163, 13.168, 13.175, 14.11, 14.18, 14.22

Lecture 10 02/12

Modified from Original

Impact

The coefficient of restitution is used to characterize the "bounciness" of different sports equipment. The U.S. Golf Association limits the COR of golf balls at 0.83

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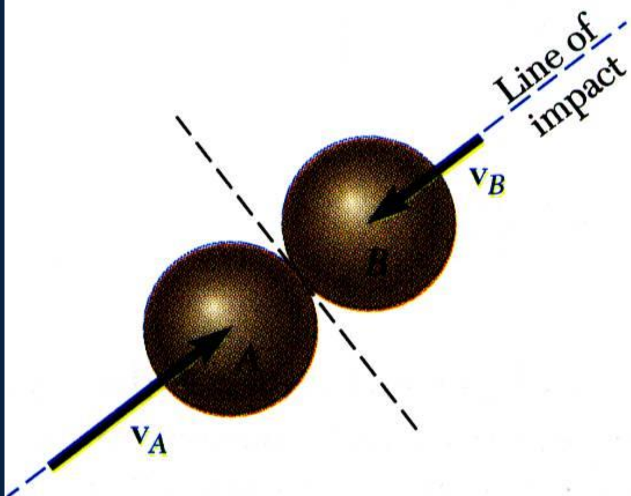
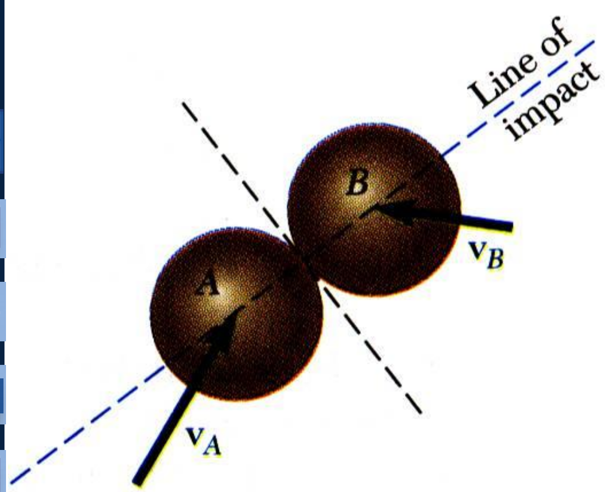
Impact



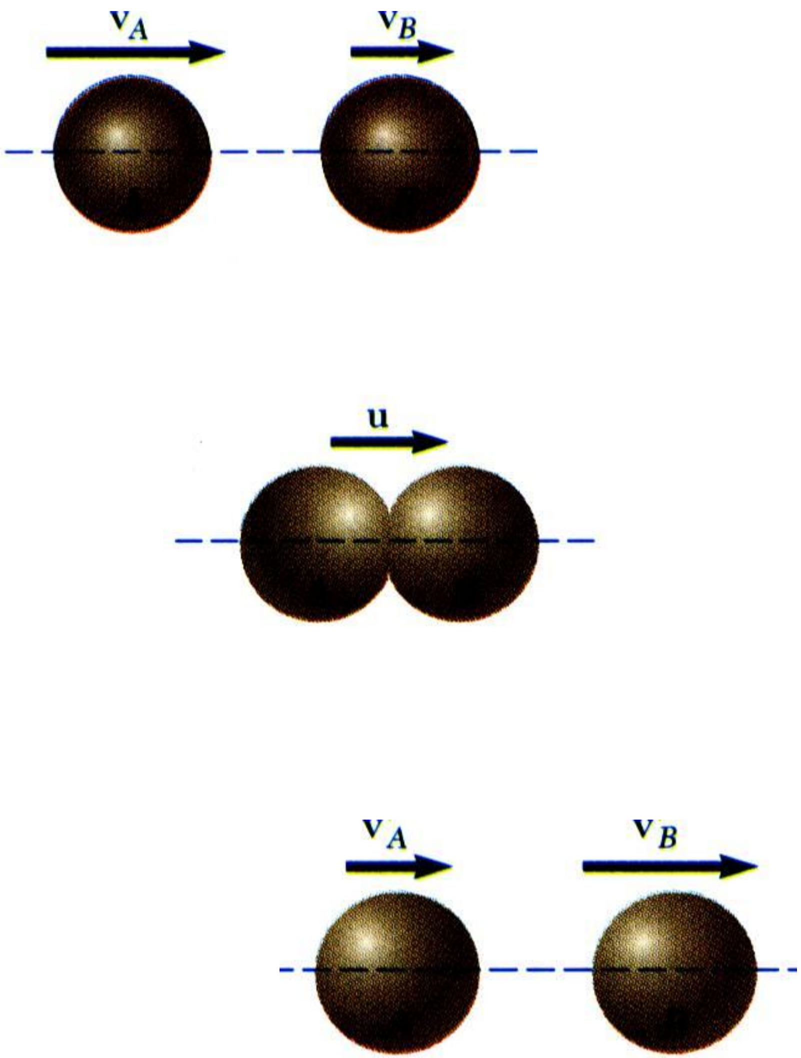
Direct Central Impact



Oblique Central Impact

- *Impact:* Collision between two bodies which occurs during a small time interval and during which the bodies exert large forces on each other.
- *Line of Impact:* Common normal to the surfaces in contact during impact.
- *Central Impact:* Impact for which the mass centers of the two bodies lie on the line of impact; otherwise, it is an *eccentric impact*..
- *Direct Impact:* Impact for which the velocities of the two bodies are directed along the line of impact.
- *Oblique Impact:* Impact for which one or both of the bodies move along a line other than the line of impact.

Direct Central Impact

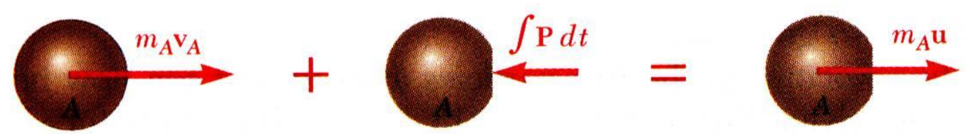


- Bodies moving in the same straight line, $v_A > v_B$.
- Upon impact the bodies undergo a *period of deformation*, at the end of which, they are in contact and moving at a common velocity.
- A *period of restitution* follows during which the bodies either regain their original shape or remain permanently deformed.
- Wish to determine the final velocities of the two bodies. The total momentum of the two body system is preserved,

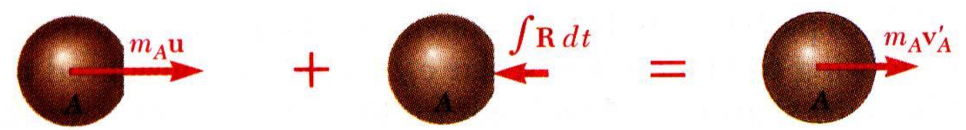
$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B$$

- A second relation between the final velocities is required.

Direct Central Impact



- Period of deformation: $m_A v_A - \int P dt = m_A u$



- Period of restitution: $m_A u - \int R dt = m_A v'_A$

$$e = \text{coefficient of restitution}$$

$$= \frac{\int R dt}{\int P dt} = \frac{u - v'_A}{v_A - u}$$

$$0 \le e \le 1$$

- A similar analysis of particle *B* yields

$$e = \frac{v'_B - u}{u - v_B}$$

- Combining the relations leads to the desired second relation between the final velocities.

$$v'_B - v'_A = e\left(v_A - v_B\right)$$

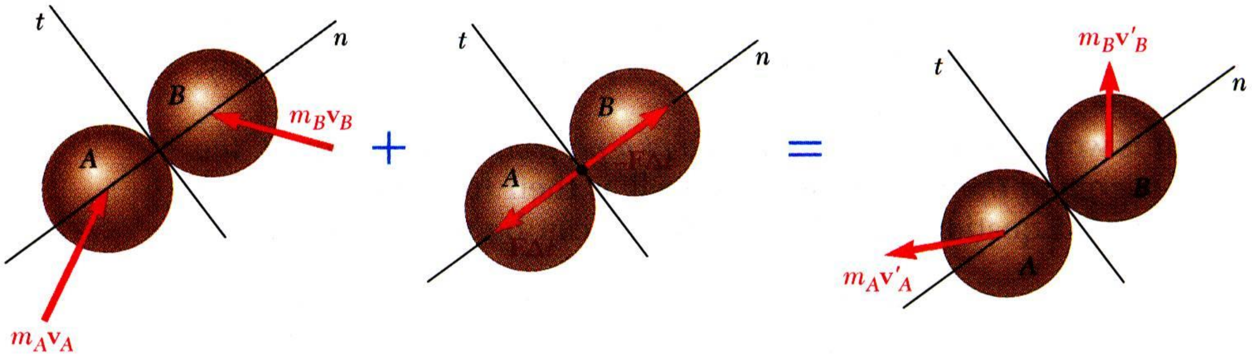
- *Perfectly plastic impact, e* = 0: $v'_B = v'_A = v'$

$$m_A v_A + m_B v_B = \left(m_A + m_B\right) v'$$

- *Perfectly elastic impact, e* = 1:
  Total energy and total momentum conserved.

$$v'_B - v'_A = v_A - v_B$$

Oblique Central Impact



- Final velocities are unknown in magnitude and direction. Four equations are required.

- No tangential impulse component; tangential component of momentum for each particle is conserved.

$$(v_A)_t = (v'_A)_t \qquad (v_B)_t = (v'_B)_t$$

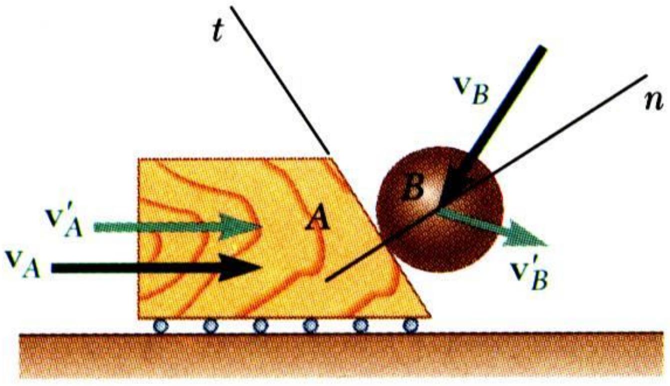
- Normal component of total momentum of the two particles is conserved.

$$m_A(v_A)_n + m_B(v_B)_n = m_A(v'_A)_n + m_B(v'_B)_n$$

- Normal components of relative velocities before and after impact are related by the coefficient of restitution.

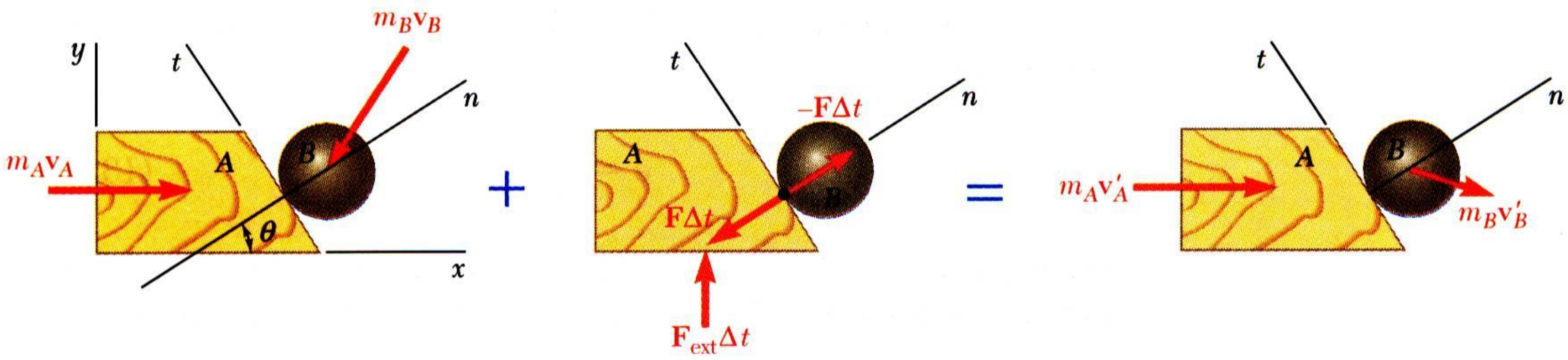
$$(v'_B)_n - (v'_A)_n = e\left[(v_A)_n - (v_B)_n\right]$$

Oblique Central Impact



- Block constrained to move along horizontal surface.
- Impulses from internal forces $\vec{F}$ and $-\vec{F}$ along the n axis and from external force $\vec{F}_{ext}$ exerted by horizontal surface and directed along the vertical to the surface.
- Final velocity of ball unknown in direction and magnitude and unknown final block velocity magnitude. Three equations required.

Oblique Central Impact



- Tangential momentum of ball is conserved.

$$(v_B)_t = (v'_B)_t$$

- Total horizontal momentum of block and ball is conserved.

$$m_A(v_A) + m_B(v_B)_x = m_A(v'_A) + m_B(v'_B)_x$$

- Normal component of relative velocities of block and ball are related by coefficient of restitution.

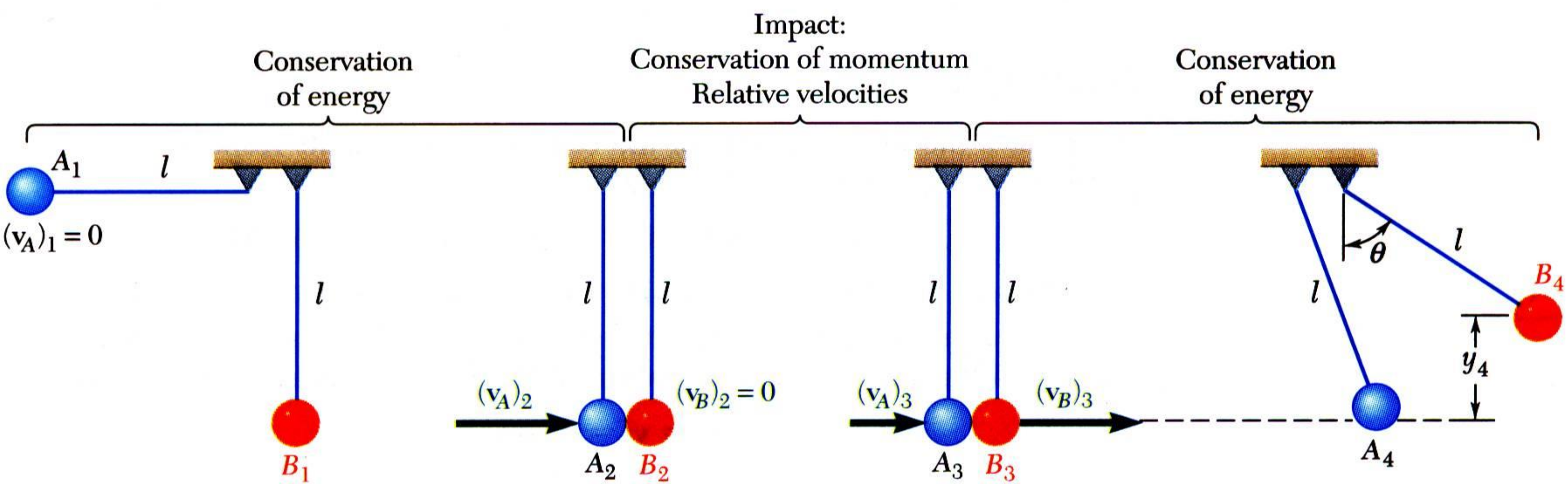
$$(v'_B)_n - (v'_A)_n = e\left[(v_A)_n - (v_B)_n\right]$$

- Note: Validity of last expression does not follow from previous relation for the coefficient of restitution. A similar but separate derivation is required.

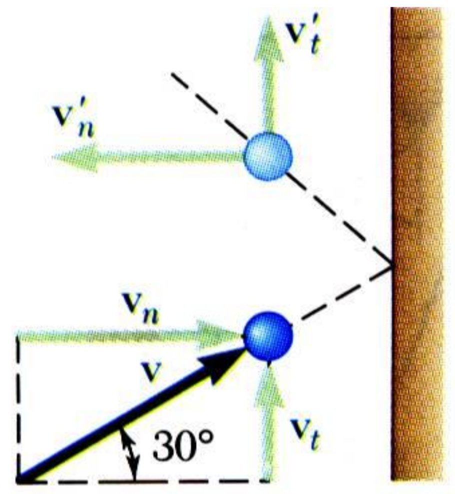
Problems Involving Multiple Principles

- Three methods for the analysis of kinetics problems:
  - Direct application of Newton's second law
  - Method of work and energy
  - Method of impulse and momentum

- Select the method best suited for the problem or part of a problem under consideration.



Sample Problem 13.19

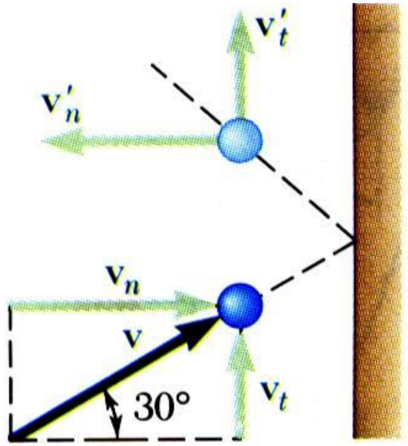


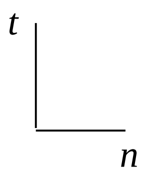
A ball is thrown against a frictionless, vertical wall. Immediately before the ball strikes the wall, its velocity has a magnitude *v* and forms angle of 30° with the horizontal. Knowing that $e = 0.90$, determine the magnitude and direction of the velocity of the ball as it rebounds from the wall.

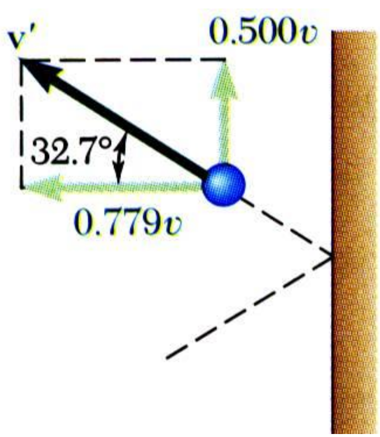
STRATEGY:

- Resolve ball velocity into components normal and tangential to wall.
- Impulse exerted by the wall is normal to the wall. Component of ball momentum tangential to wall is conserved.
- Assume that the wall has infinite mass so that wall velocity before and after impact is zero. Apply coefficient of restitution relation to find change in normal relative velocity between wall and ball, i.e., the normal ball velocity.

Sample Problem 13.19







MODELING and ANALYSIS:

- Resolve ball velocity into components parallel and perpendicular to wall.

$$v_n = v\cos 30° = 0.866v \qquad v_t = v\sin 30° = 0.500v$$

- Component of ball momentum tangential to wall is conserved.

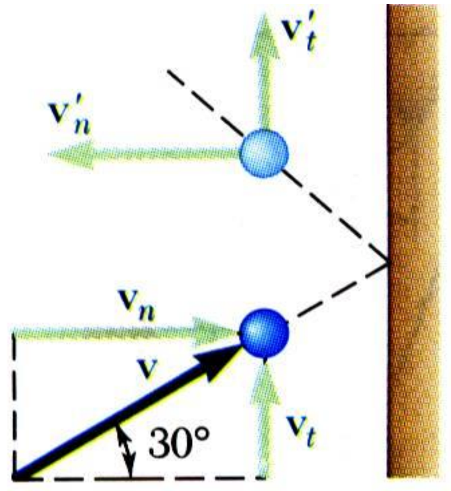
$$v'_t = v_t = 0.500v$$

- Apply coefficient of restitution relation with zero wall velocity.

$$0 - v'_n = e\left(v_n - 0\right)$$
$$v'_n = -0.9\left(0.866v\right) = -0.779v$$

$$\boxed{\begin{array}{l} \vec{v}' = -0.779v\,\vec{\lambda}_n + 0.500v\,\vec{\lambda}_t \\ v' = 0.926v \quad \tan^{-1}\left(\dfrac{0.5}{0.779}\right) = 32.7° \end{array}}$$

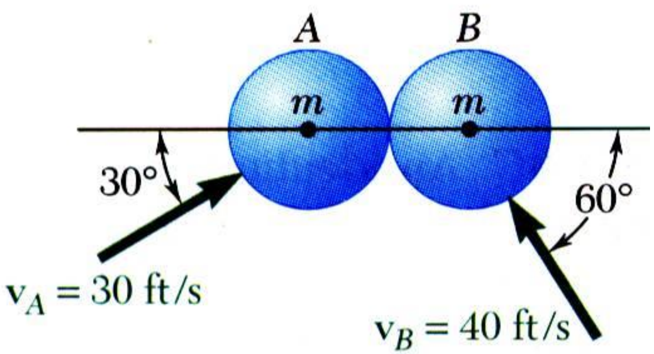
Sample Problem 13.19



REFLECT and THINK:

Tests similar to this are done to make sure that sporting equipment—such as tennis balls, golf balls, and basketballs—are consistent and fall within certain specifications. Testing modern golf balls and clubs shows that the coefficient of restitution actually decreases with increasing club speed (from about 0.84 at a speed of 90 mph to about 0.80 at club speeds of 130 mph).

Sample Problem 13.20

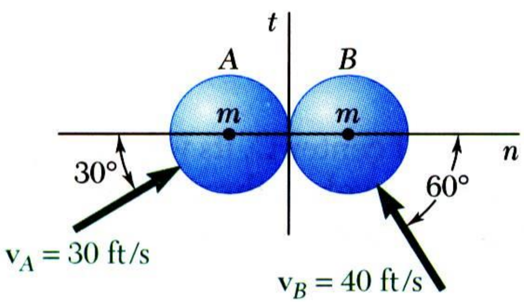


The magnitude and direction of the velocities of two identical frictionless balls before they strike each other are as shown. Assuming $e = 0.9$, determine the magnitude and direction of the velocity of each ball after the impact.

STRATEGY:

- Resolve the ball velocities into components normal and tangential to the contact plane.
- Tangential component of momentum for each ball is conserved.
- Total normal component of the momentum of the two ball system is conserved.
- The normal relative velocities of the balls are related by the coefficient of restitution.
- Solve the last two equations simultaneously for the normal velocities of the balls after the impact.

Sample Problem 13.20

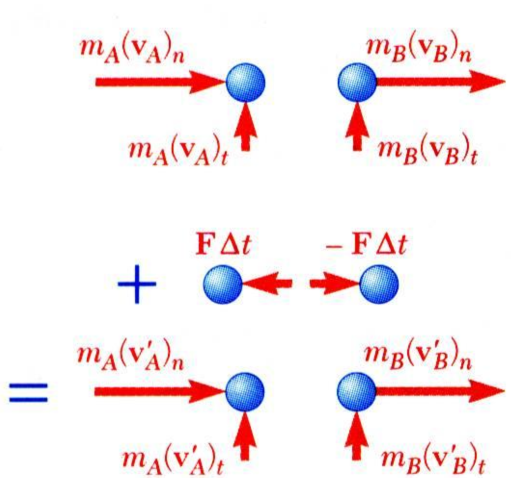


MODELING and ANALYSIS:

- Resolve the ball velocities into components normal and tangential to the contact plane.

$$(v_A)_n = v_A \cos 30° = 26.0\,\text{ft/s} \qquad (v_A)_t = v_A \sin 30° = 15.0\,\text{ft/s}$$

$$(v_B)_n = -v_B \cos 60° = -20.0\,\text{ft/s} \qquad (v_B)_t = v_B \sin 60° = 34.6\,\text{ft/s}$$



- Tangential component of momentum for each ball is conserved.

$$(v'_A)_t = (v_A)_t = 15.0\,\text{ft/s} \qquad (v'_B)_t = (v_B)_t = 34.6\,\text{ft/s}$$

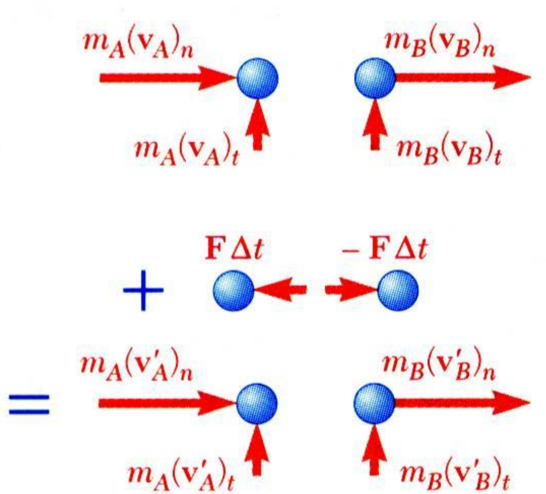
- Total normal component of the momentum of the two ball system is conserved.

$$m_A(v_A)_n + m_B(v_B)_n = m_A(v'_A)_n + m_B(v'_B)_n$$

$$m(26.0) + m(-20.0) = m(v'_A)_n + m(v'_B)_n$$

$$(v'_A)_n + (v'_B)_n = 6.0$$

Sample Problem 13.20

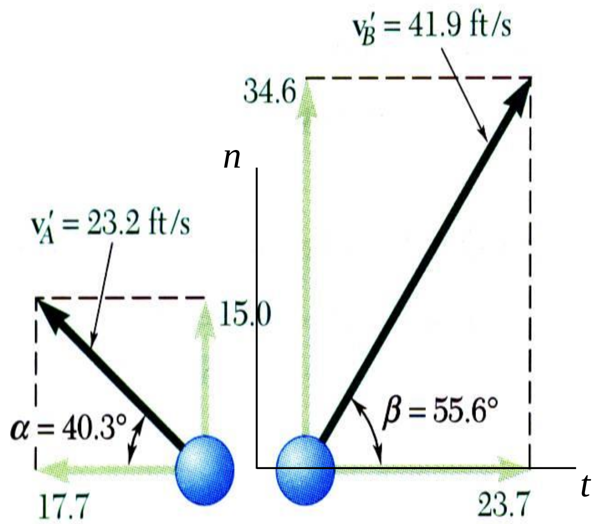


- The normal relative velocities of the balls are related by the coefficient of restitution.

$$(v'_A)_n - (v'_B)_n = e\left[(v_A)_n - (v_B)_n\right]$$
$$= 0.90\left[26.0 - (-20.0)\right] = 41.4$$

- Solve the last two equations simultaneously for the normal velocities of the balls after the impact.

$$(v'_A)_n = -17.7\,\text{ft/s} \qquad (v'_B)_n = 23.7\,\text{ft/s}$$



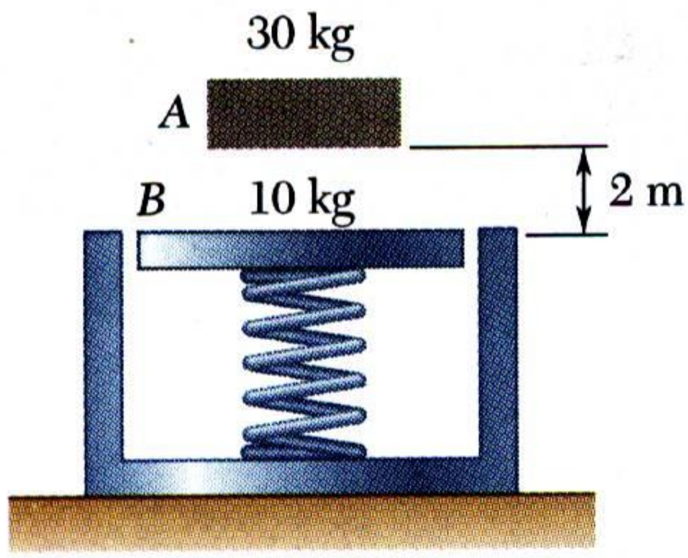
$$\vec{v}'_A = -17.7\vec{\lambda}n + 15.0\vec{\lambda}t$$

$$v'_A = 23.2\,\text{ft/s} \qquad \tan^{-1}\left(\frac{15.0}{17.7}\right) = 40.3°$$

$$\vec{v}'_B = 23.7\vec{\lambda}_n + 34.6\vec{\lambda}_t$$

$$v'_B = 41.9\,\text{ft/s} \qquad \tan^{-1}\left(\frac{34.6}{23.7}\right) = 55.6°$$

Sample Problem 13.22

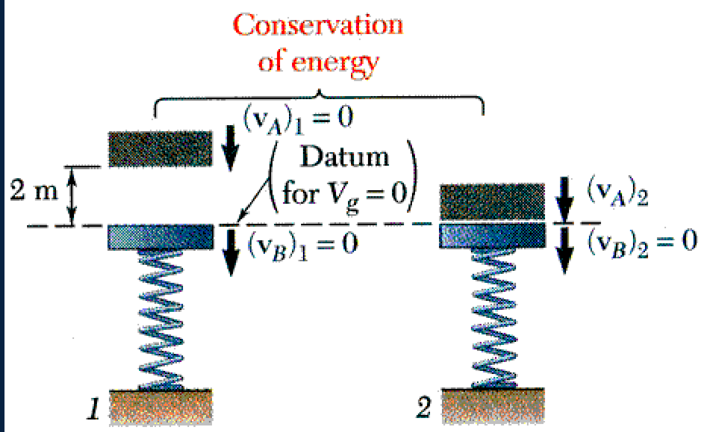


A 30 kg block is dropped from a height of 2 m onto the 10 kg pan of a spring scale. Assuming the impact to be perfectly plastic, determine the maximum deflection of the pan. The constant of the spring is k = 20 kN/m.

STRATEGY:

- Apply the principle of conservation of energy to determine the velocity of the block at the instant of impact.
- Since the impact is perfectly plastic, the block and pan move together at the same velocity after impact. Determine that velocity from the requirement that the total momentum of the block and pan is conserved.
- Apply the principle of conservation of energy to determine the maximum deflection of the spring.

Sample Problem 13.22



MODELING and ANALYSIS:

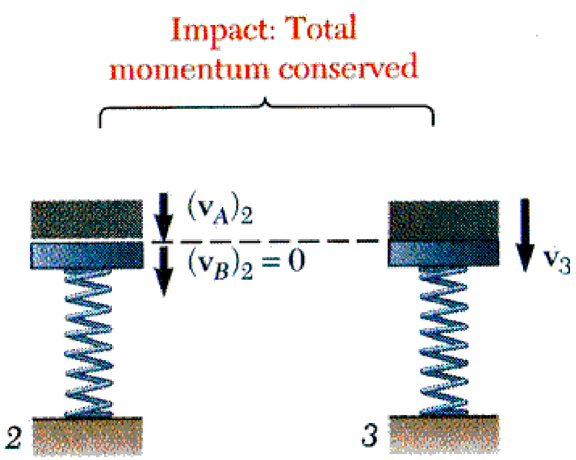
- Apply principle of conservation of energy to determine velocity of the block at instant of impact.

$$T_1 = 0 \qquad V_1 = W_A y = (30)(9.81)(2) = 588\ \mathrm{J}$$

$$T_2 = \tfrac{1}{2} m_A (v_A)_2^2 = \tfrac{1}{2}(30)(v_A)_2^2 \qquad V_2 = 0$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 + 588\ \mathrm{J} = \tfrac{1}{2}(30)(v_A)_2^2 + 0 \qquad (v_A)_2 = 6.26\ \mathrm{m/s}$$

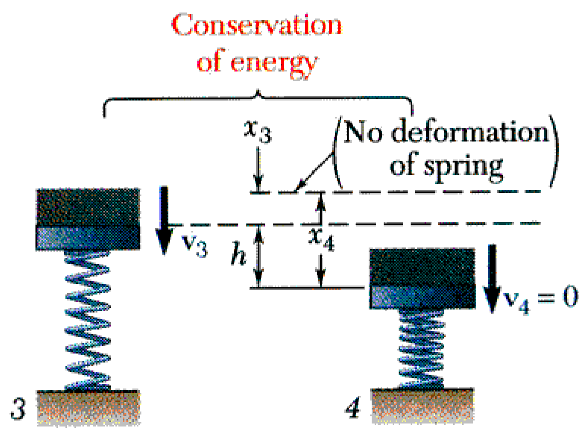


- Determine velocity after impact from requirement that total momentum of the block and pan is conserved.

$$m_A (v_A)_2 + m_B (v_B)_2 = (m_A + m_B) v_3$$

$$(30)(6.26) + 0 = (30 + 10) v_3 \qquad v_3 = 4.70\ \mathrm{m/s}$$

Sample Problem 13.22



Initial spring deflection due to pan weight:

$$x_3 = \frac{W_B}{k} = \frac{(10)(9.81)}{20\times10^3} = 4.91\times10^{-3}\,\text{m}$$

- Apply the principle of conservation of energy to determine the maximum deflection of the spring.

$$T_3 = \tfrac{1}{2}(m_A + m_B)v_3^2 = \tfrac{1}{2}(30+10)(4.7)^2 = 442\ \text{J}$$

$$V_3 = V_g + V_e$$

$$= 0 + \tfrac{1}{2}kx_3^2 = \tfrac{1}{2}\left(20\times10^3\right)\left(4.91\times10^{-3}\right)^2 = 0.241\,\text{J}$$

$$T_4 = 0$$

$$V_4 = V_g + V_e = (W_A + W_B)(-h) + \tfrac{1}{2}kx_4^2$$

$$= -392(x_4 - x_3) + \tfrac{1}{2}\left(20\times10^3\right)x_4^2$$

$$= -392\left(x_4 - 4.91\times10^{-3}\right) + \tfrac{1}{2}\left(20\times10^3\right)x_4^2$$

$$T_3 + V_3 = T_4 + V_4$$

$$442 + 0.241 = 0 - 392\left(x_4 - 4.91\times10^{-3}\right) + \tfrac{1}{2}\left(20\times10^3\right)x_4^2$$

$$x_4 = 0.230\,\text{m}$$

$$h = x_4 - x_3 = 0.230\,\text{m} - 4.91\times10^{-3}\,\text{m} \qquad \boxed{h = 0.225\,\text{m}}$$