Seventh Edition

CHAPTER

4

# MECHANICS OF MATERIALS


**Ferdinand P. Beer**

**E. Russell Johnston, Jr.**

**John T. DeWolf**

**David F. Mazurek**


## Pure Bending


**Lecture Notes:**

**Brock E. Barry**

**U.S. Military Academy**


Lecture 8 02/05/2018
Modified from Original

HW Problems Week 5 (due Mon 02/12):

4.33, 4.39, 4.106, 4.107, 4.130, 4.141

# Bending of Members Made of Several Materials

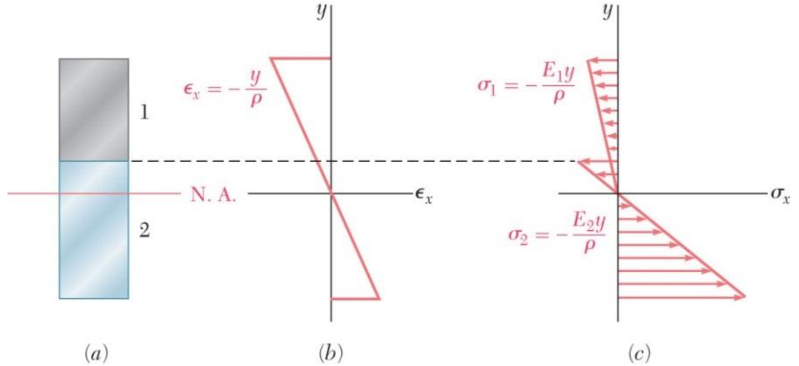



**Fig. 4.19** Stress and strain distributions in bar Made of two materials. (a) Neutral axis shifted from centroid. (b) Strain distribution. (c) Corresponding stress distribution.

- Consider a bar consisting of two different materials with $E_1$ and $E_2$.
- Normal strain varies linearly with distance y.

$$\varepsilon_x = -\frac{y}{\rho}$$

- Piecewise linear normal stress variation.

$$\sigma_1 = E_1 \varepsilon_x = -\frac{E_1 y}{\rho} \qquad \sigma_2 = E_2 \varepsilon_x = -\frac{E_2 y}{\rho}$$

Neutral axis does not pass through section centroid of composite section.

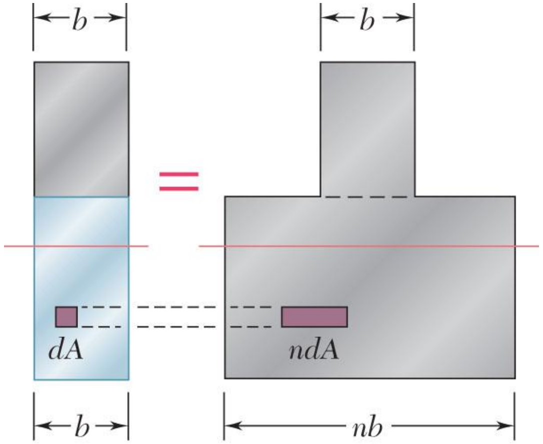



**Fig. 4.20** Transformed section based on stiffness is used to locate neutral axis.

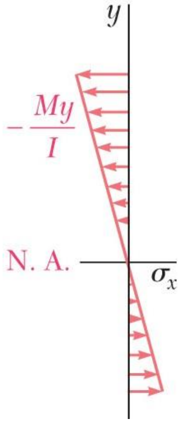



**Fig. 4.21** Distribution of stresses in transformed section.

- Elemental forces on the section are

$$dF_1 = \sigma_1 dA = -\frac{E_1 y}{\rho} dA \qquad dF_2 = \sigma_2 dA = -\frac{E_2 y}{\rho} dA$$

- Define a transformed section such that

$$dF_2 = -\frac{(nE_1)y}{\rho} dA = -\frac{E_1 y}{\rho}(n\,dA) \qquad n = \frac{E_2}{E_1}$$

# Concept Application 4.3

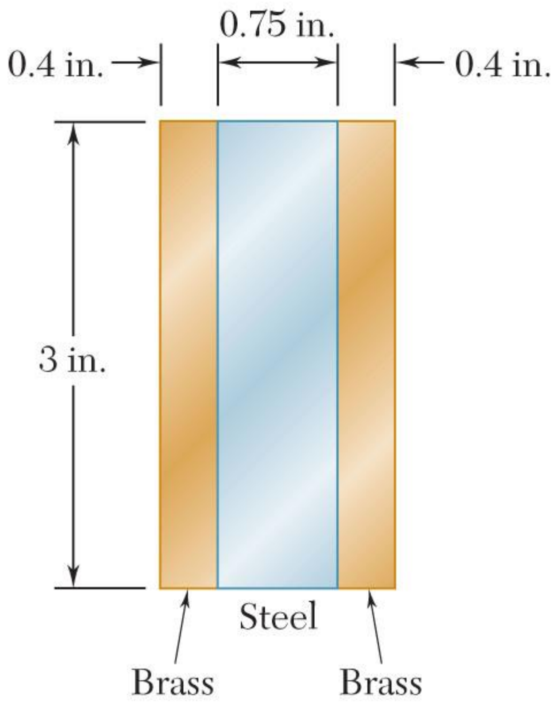



**Fig. 4.22a** Composite, sandwich structure cross section.

Bar is made from bonded pieces of steel ($E_s$ = 29x10$^6$ psi) and brass ($E_b$ = 15x10$^6$ psi). Determine the maximum stress in the steel and brass when a moment of 40 kip*in is applied.

SOLUTION:

- Transform the bar to an equivalent cross section made entirely of brass
- Evaluate the cross sectional properties of the transformed section
- Calculate the maximum stress in the transformed section. This is the correct maximum stress for the brass pieces of the bar.
- Determine the maximum stress in the steel portion of the bar by multiplying the maximum stress for the transformed section by the ratio of the moduli of elasticity.

# Example 4.03

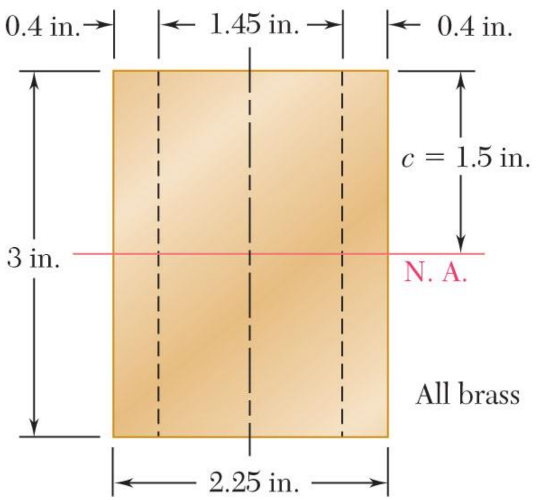



**Fig. 4.22b** Bar length and height dimensions.

SOLUTION:

- Transform the bar to an equivalent cross section made entirely of brass.

$$n = \frac{E_s}{E_b} = \frac{29\times10^6\,\text{psi}}{15\times10^6\,\text{psi}} = 1.933$$

$$b_T = 0.4\,\text{in} + 1.933\times0.75\,\text{in} + 0.4\,\text{in} = 2.25\,\text{in}$$

- Evaluate the transformed cross sectional properties

$$I = \tfrac{1}{12} b_T h^3 = \tfrac{1}{12}(2.25\,\text{in.})(3\,\text{in.})^3$$
$$= 5.063\,\text{in.}^4$$

- Calculate the maximum stresses

$$\sigma_m = \frac{Mc}{I} = \frac{(40\,\text{kip}\cdot\text{in.})(1.5\,\text{in.})}{5.063\,\text{in.}^4} = 11.85\,\text{ksi}$$

$$(\sigma_b)_{max} = \sigma_m$$
$$(\sigma_s)_{max} = n\sigma_m = 1.933\times11.85\,\text{ksi}$$

$$\boxed{\begin{aligned}(\sigma_b)_{max} &= 11.85\,\text{ksi}\\ (\sigma_s)_{max} &= 22.9\,\text{ksi}\end{aligned}}$$

# Reinforced Concrete Beams

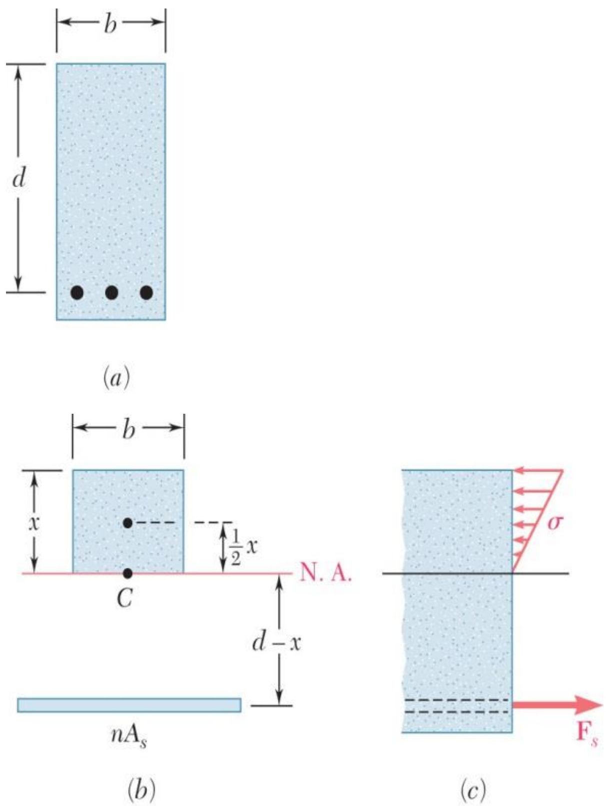



**Fig. 4.23** Reinforced concrete beam: (a) Cross section showing location of reinforcing steel. (b) Transformed section of all concrete. (c) Concrete stresses and resulting steel force.

- Concrete beams subjected to bending moments are reinforced by steel rods.
- The steel rods carry the entire tensile load below the neutral surface. The upper part of the concrete beam carries the compressive load.
- In the transformed section, the cross sectional area of the steel, $A_s$, is replaced by the equivalent area $nA_s$ where $n = E_s/E_c$.
  - To determine the location of the neutral axis,

$$(bx)\frac{x}{2} - n A_s (d - x) = 0$$

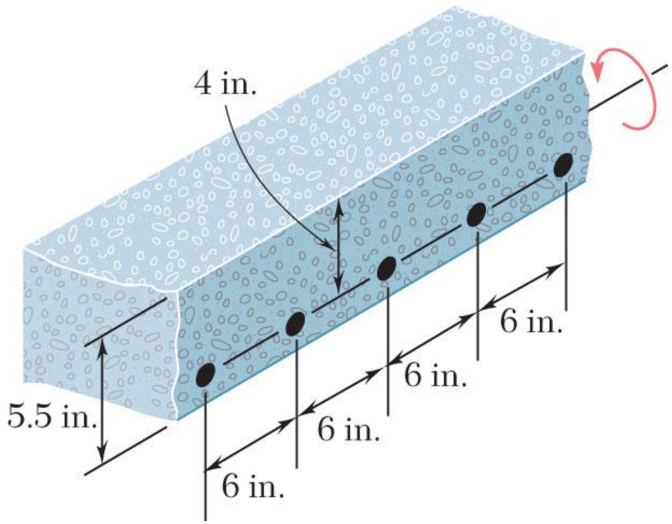
$$\frac{1}{2} b x^2 + n A_s x - n A_s d = 0$$

- The normal stress in the concrete and steel

$$\sigma_x = -\frac{My}{I}$$

$$\sigma_c = \sigma_x \qquad \sigma_s = n\sigma_x$$

# Sample Problem 4.4




A concrete floor slab is reinforced with 5/8-in-diameter steel rods placed 1.5 in. above the lower face of the slab and spaced 6 in. on centers. The modulus of elasticity is $29 \times 10^6$ psi for steel and $3.6 \times 10^6$ psi for concrete. With an applied bending moment of 40 kip*in for 1-ft width of the slab, determine the maximum stress in the concrete and steel.

SOLUTION:

- Transform to a section made entirely of concrete.
- Evaluate geometric properties of transformed section.
- Calculate the maximum stresses in the concrete and steel.

# Sample Problem 4.4

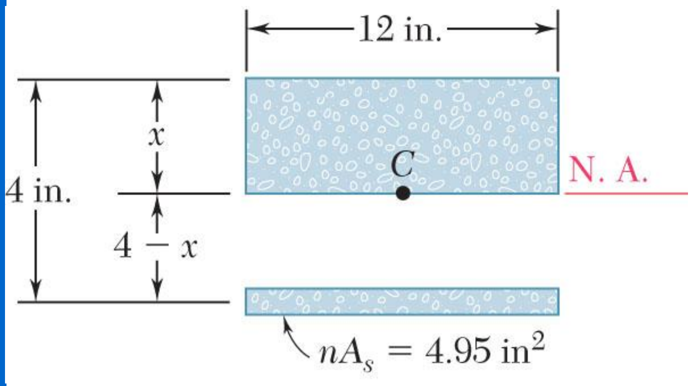



**Fig. 1** Transformed section to calculate neutral axis.

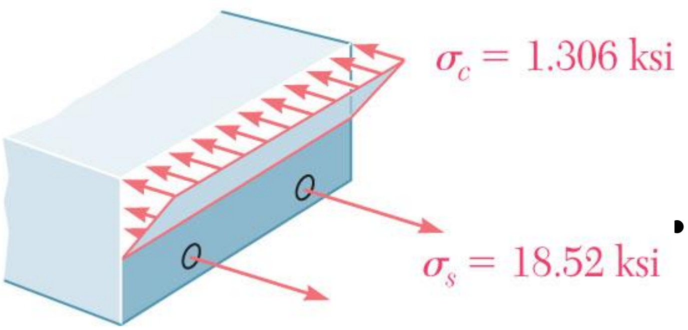



**Fig. 3** Force diagram at a cross section to calculate stresses.

SOLUTION:

- Transform to a section made entirely of concrete.

$$n = \frac{E_s}{E_c} = \frac{29\times10^6\ \text{psi}}{3.6\times10^6\ \text{psi}} = 8.06$$

$$nA_s = 8.06\times 2\left[\frac{\pi}{4}\left(\frac{5}{8}\text{in}\right)^2\right] = 4.95\text{in}^2$$

- Evaluate the geometric properties of the transformed section.

$$12x\left(\frac{x}{2}\right) - 4.95(4-x) = 0 \qquad x = 1.450\text{in}$$

$$I = \tfrac{1}{3}(12\text{in})(1.45\text{in})^3 + \left(4.95\text{in}^2\right)(2.55\text{in})^2 = 44.4\text{in}^4$$

- Calculate the maximum stresses.

$$\sigma_c = \frac{Mc_1}{I} = \frac{40\text{kip}\cdot\text{in}\times1.45\text{in}}{44.4\text{in}^4} \qquad \boxed{\sigma_c = 1.306\text{ksi}}$$

$$\sigma_s = n\frac{Mc_2}{I} = 8.06\frac{40\text{kip}\cdot\text{in}\times2.55\text{in}}{44.4\text{in}^4} \qquad \boxed{\sigma_s = 18.52\text{ksi}}$$

# Eccentric Axial Loading in a Plane of Symmetry

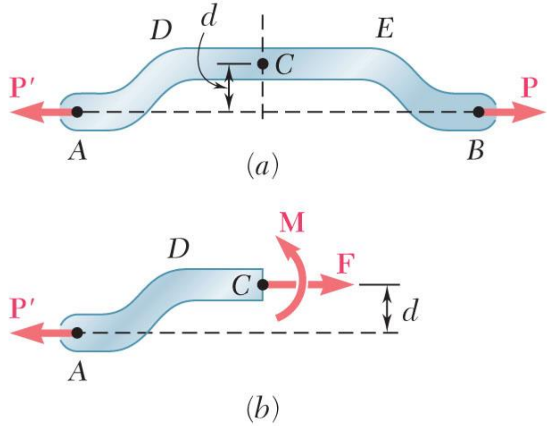



**Fig. 4.39** (a) Member with eccentric loading. (b) Free-body diagram of a member with internal loads at section C.

- Eccentric loading

$$F = P$$
$$M = Pd$$

- Stress due to eccentric loading found by superposing the uniform stress due to a centric load and linear stress distribution due to a pure bending moment

$$\sigma_x = (\sigma_x)_{\text{centric}} + (\sigma_x)_{\text{bending}}$$
$$= \frac{P}{A} - \frac{My}{I}$$

- Result are valid if stresses do not exceed the proportional limit, deformations have negligible effect on geometry, and stresses are not evaluated near points of load application.

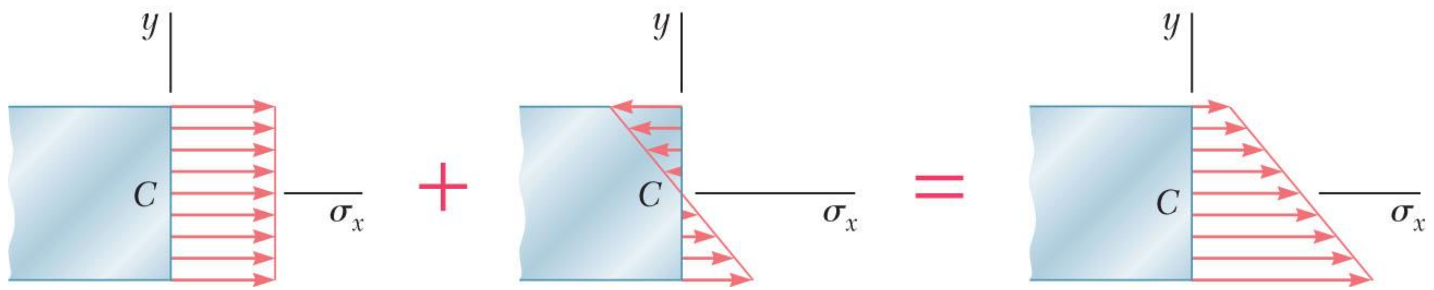



**Fig. 4.41** Stress distribution for eccentric loading is obtained by superposing the axial and pure bending distributions.

# Concept Application 4.7

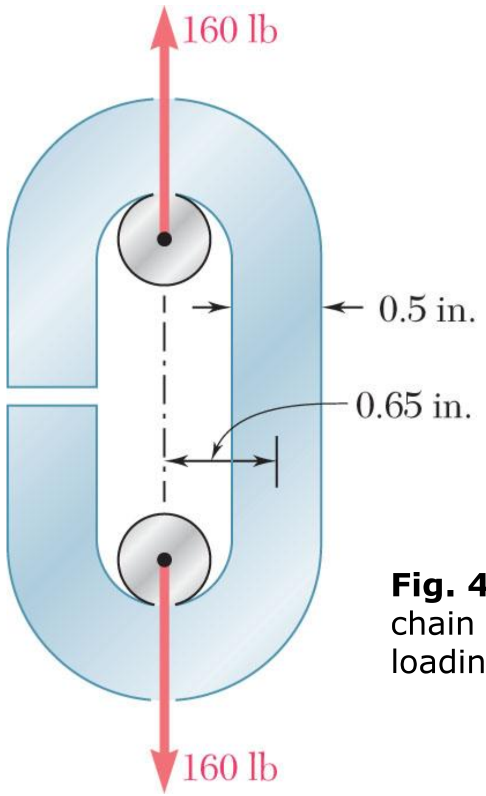


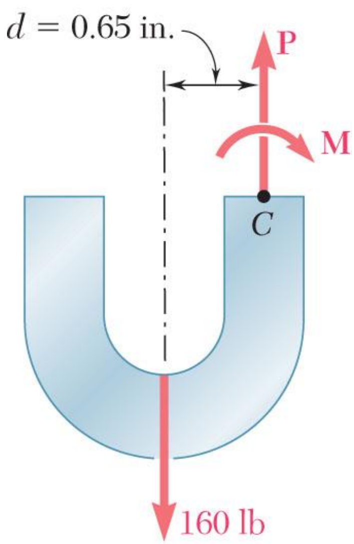

**Fig. 4.43** Open chain link under loading.

An open-link chain is obtained by bending low-carbon steel rods into the shape shown. For 160 lb load, determine (a) maximum tensile and compressive stresses, (b) distance between section centroid and neutral axis

SOLUTION:

- Find the equivalent centric load and bending moment
- Superpose the uniform stress due to the centric load and the linear stress due to the bending moment.
- Evaluate the maximum tensile and compressive stresses at the inner and outer edges, respectively, of the superposed stress distribution.
- Find the neutral axis by determining the location where the normal stress is zero.

# Concept Application 4.7





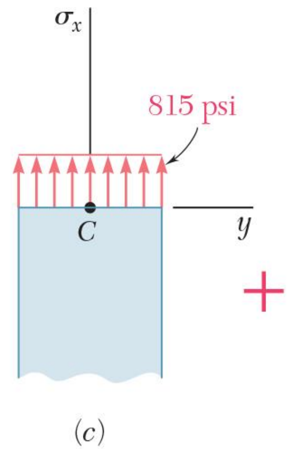
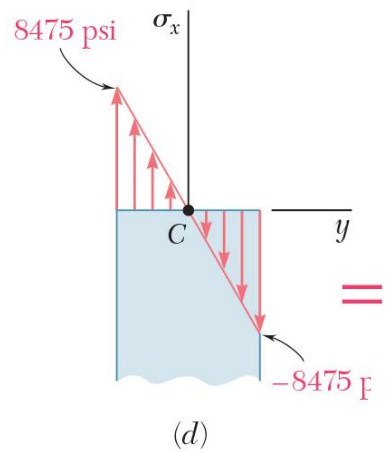




**Fig. 4.43** Free-body diagram for section at C to find axial force and moment. Stress at section C is superposed axial and bending stresses.




- Equivalent centric load and bending moment

$$P = 160\,\text{lb}$$

$$M = Pd = (160\,\text{lb})(0.65\,\text{in})$$

$$= 104\,\text{lb}\cdot\text{in}$$

- Normal stress due to a centric load

$$A = \pi c^2 = \pi(0.25\text{in})^2$$

$$= 0.1963\text{in}^2$$

$$\sigma_0 = \frac{P}{A} = \frac{160\text{lb}}{0.1963\text{in}^2}$$

$$= 815\text{psi}$$

- Normal stress due to bending moment

$$I = \tfrac{1}{4}\pi c^4 = \tfrac{1}{4}\pi(0.25)^4$$

$$= 3.068\times10^{-3}\,\text{in}^4$$

$$\sigma_m = \frac{Mc}{I} = \frac{(104\,\text{lb}\cdot\text{in})(0.25\,\text{in})}{3.068\times10^{-3}\,\text{in}^4}$$

$$= 8475\,\text{psi}$$

# Concept Application 4.7

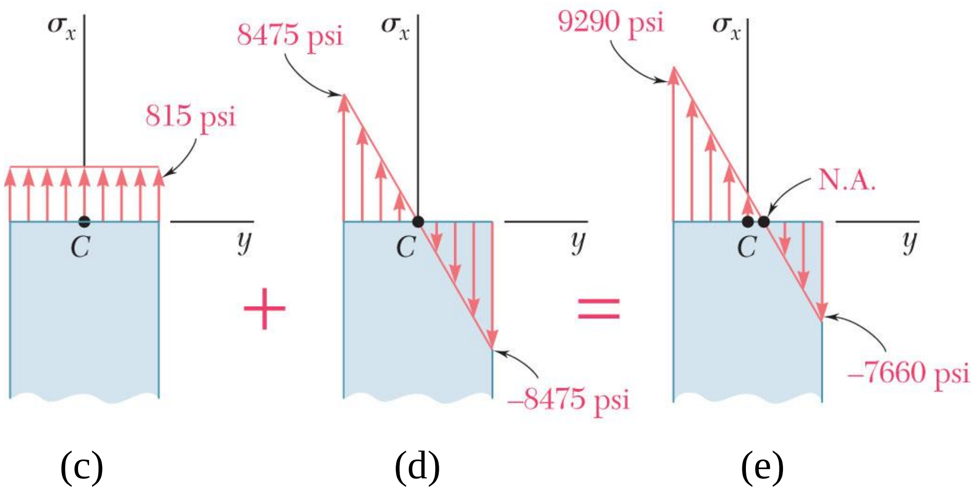



**Fig. 4.43** (c) Axial stress at section C. (d) Bending stress at C. (e) Superposition of stresses.

- Maximum tensile and compressive stresses

$$\sigma_t = \sigma_0 + \sigma_m = 815 + 8475$$

$$\boxed{\sigma_t = 9260\,\text{psi}}$$

$$\sigma_c = \sigma_0 - \sigma_m = 815 - 8475$$

$$\boxed{\sigma_c = -7660\,\text{psi}}$$

- Neutral axis location

$$0 = \frac{P}{A} - \frac{M y_0}{I}$$

$$y_0 = \frac{P}{A}\frac{I}{M} = (815\,\text{psi})\frac{3.068\times10^{-3}\,\text{in}^4}{105\,\text{lb}\cdot\text{in}}$$

$$\boxed{y_0 = 0.0240\,\text{in}}$$

# Sample Problem 4.8

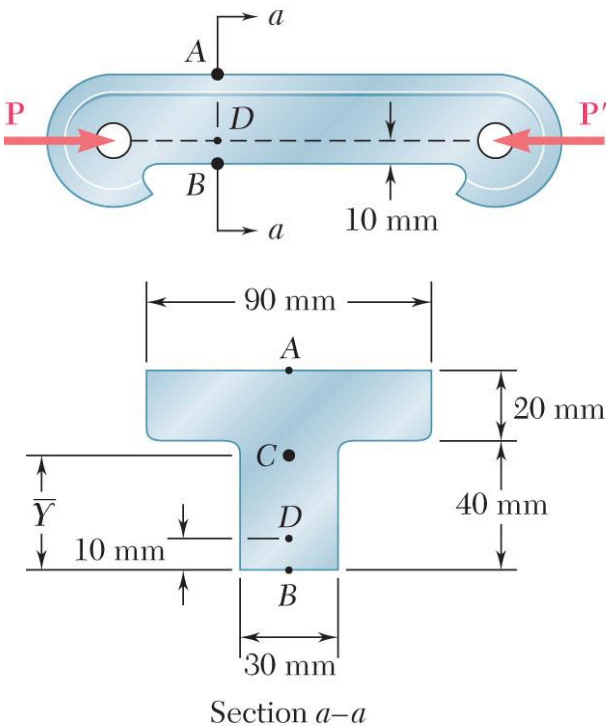



**Fig. 1** Section geometry to find centroid location.

From Sample Problem 4.2,

$$A = 3\times10^{-3}\,\mathrm{m}^2$$
$$\bar{Y} = 0.038\,\mathrm{m}$$
$$I = 868\times10^{-9}\,\mathrm{m}^4$$

The largest allowable stresses for the cast iron link are 30 MPa in tension and 120 MPa in compression. Determine the largest force *P* which can be applied to the link.

SOLUTION:

- Determine equivalent centric load and bending moment.
- Superpose the stress due to a centric load and the stress due to bending.
- Evaluate the critical loads for the allowable tensile and compressive stresses.
- The largest allowable load is the smallest of the two critical loads.

# Sample Problem 4.8

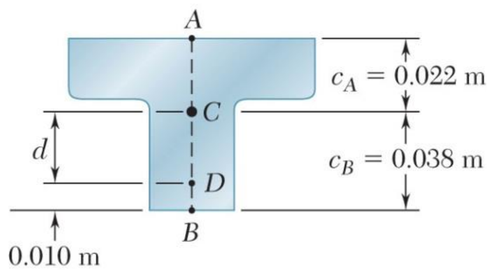
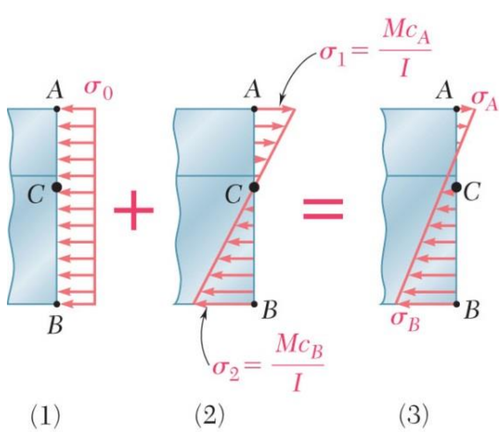



**Fig. 2** Section dimensions for finding location of point D.




**Figs. 4** Stress distribution at section C is superposition of axial and bending distributions acting at centroid.

- Determine equivalent centric and bending loads.

$$d = 0.038 - 0.010 = 0.028\text{m}$$
$$P = \text{centric load}$$
$$M = Pd = 0.028P = \text{bending moment}$$

- Superpose stresses due to centric and bending loads

$$\sigma_A = -\frac{P}{A} + \frac{Mc_A}{I} = -\frac{P}{3\times10^{-3}} + \frac{(0.028P)(0.022)}{868\times10^{-9}} = +377P$$
$$\sigma_B = -\frac{P}{A} - \frac{Mc_A}{I} = -\frac{P}{3\times10^{-3}} - \frac{(0.028P)(0.038)}{868\times10^{-9}} = -1559P$$

- Evaluate critical loads for allowable stresses.

$$\sigma_A = +377P = 30\,\text{MPa} \qquad P = 79.6\,\text{kN}$$
$$\sigma_B = -1559P = -120\,\text{MPa} \qquad P = 77.0\,\text{kN}$$

- The largest allowable load $\boxed{P = 77.0\text{ kN}}$