Seventh Edition

CHAPTER

4

# MECHANICS OF MATERIALS

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## Pure Bending

Lecture Notes:
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Lecture 9 02/07/2018
Modified from Original

HW Problems Week 5 (due Mon 02/12):

4.33, 4.39, 4.106, 4.107, 4.130, 4.141

# Unsymmetric Bending

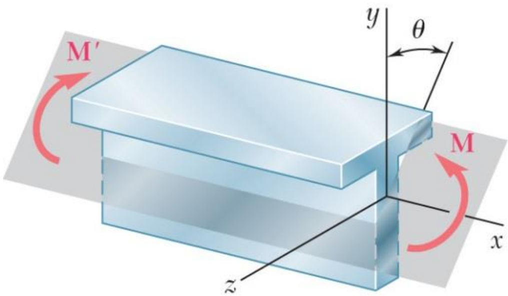



**Fig. 4.49** Unsymmetric bending, with bending moment not in a plane of symmetry.

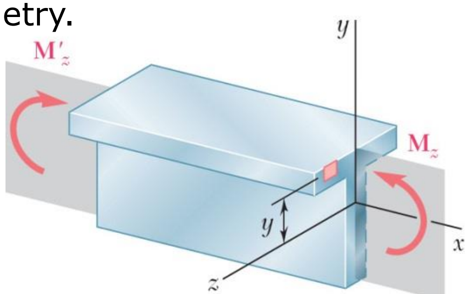



**Fig. 4.51** $\mathbf{M}_z$ acts in a plane that includes a principal centroidal axis, bending the member in the vertical plane.

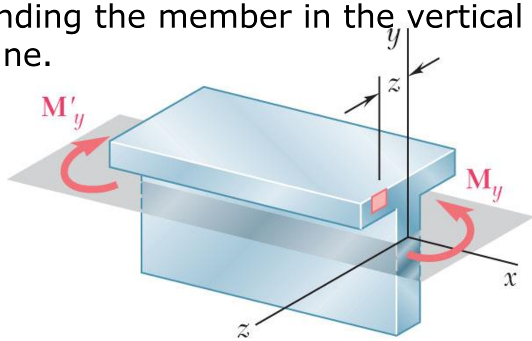



**Fig. 4.52** My acts in a plane that includes a principal centroidal axis, bending the member in the horizontal plane.

Superposition is applied to determine stresses in the most general case of unsymmetric bending.

- Resolve the couple vector into components along the principle centroidal axes.

$$M_z = M\cos\theta \qquad M_y = M\sin\theta$$

- Superpose the component stress distributions

$$\sigma_x = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

- Along the neutral axis,

$$\sigma_x = 0 = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y} = -\frac{(M\cos\theta)y}{I_z} + \frac{(M\sin\theta)z}{I_y}$$

$$\tan\phi = \frac{y}{z} = \frac{I_z}{I_y}\tan\theta$$

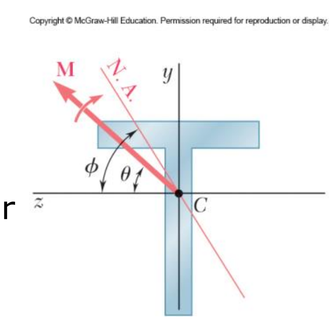
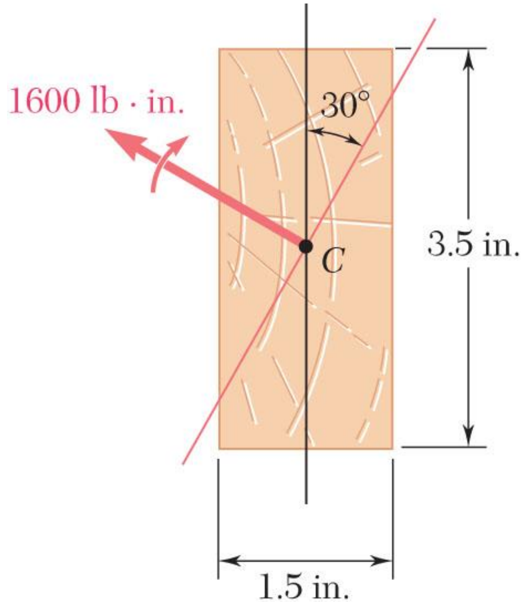



**Fig. 4.54** Neutral axis for unsymmetric bending.

# Concept Application 4.8




A 1600 lb-in couple is applied to a rectangular wooden beam in a plane forming an angle of 30° with the vertical. Determine (a) the maximum stress in the beam, (b) the angle that the neutral axis forms with the horizontal plane.

SOLUTION:

- Resolve the couple vector into components along the principle centroidal axes and calculate the corresponding maximum stresses.

$$M_z = M\cos\theta \qquad M_y = M\sin\theta$$

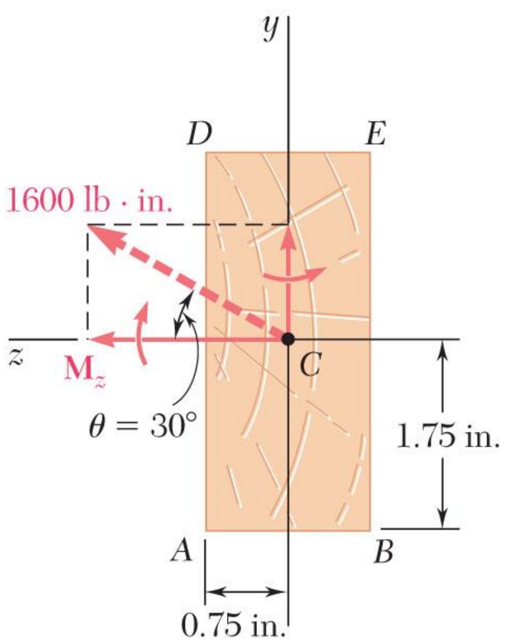
- Combine the stresses from the component stress distributions.

$$\sigma_x = \frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$

- Determine the angle of the neutral axis.

$$\tan\phi = \frac{y}{z} = \frac{I_z}{I_y}\tan\theta$$

# Concept Application 4.8




- Resolve the couple vector into components and calculate the corresponding maximum stresses.

$$M_z = (1600\,\text{lb}\cdot\text{in})\cos 30 = 1386\,\text{lb}\cdot\text{in}$$
$$M_y = (1600\,\text{lb}\cdot\text{in})\sin 30 = 800\,\text{lb}\cdot\text{in}$$
$$I_z = \tfrac{1}{12}(1.5\,\text{in})(3.5\,\text{in})^3 = 5.359\,\text{in}^4$$
$$I_y = \tfrac{1}{12}(3.5\,\text{in})(1.5\,\text{in})^3 = 0.9844\,\text{in}^4$$

The largest tensile stress due to $M_z$ occurs along $AB$

$$\sigma_1 = \frac{M_z y}{I_z} = \frac{(1386\,\text{lb}\cdot\text{in})(1.75\,\text{in})}{5.359\,\text{in}^4} = 452.6\,\text{psi}$$

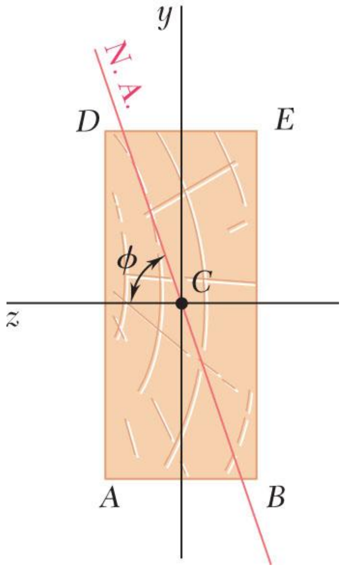
The largest tensile stress due to $M_y$ occurs along $AD$

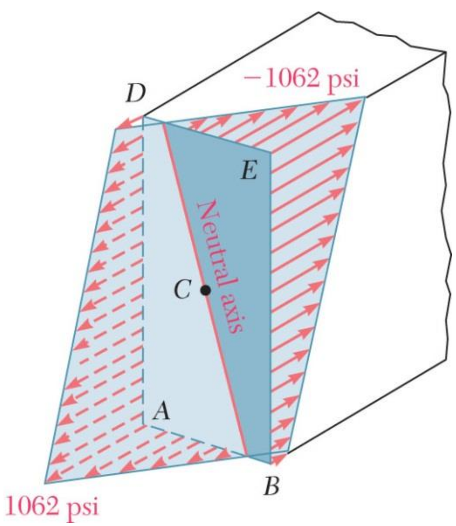
$$\sigma_2 = \frac{M_y z}{I_y} = \frac{(800\,\text{lb}\cdot\text{in})(0.75\,\text{in})}{0.9844\,\text{in}^4} = 609.5\,\text{psi}$$

- The largest tensile stress due to the combined loading occurs at $A$.

$$\sigma_{\max} = \sigma_1 + \sigma_2 = 452.6 + 609.5 \qquad \boxed{\sigma_{\max} = 1062\text{psi}}$$

# Concept Application 4.8




- Determine the angle of the neutral axis.

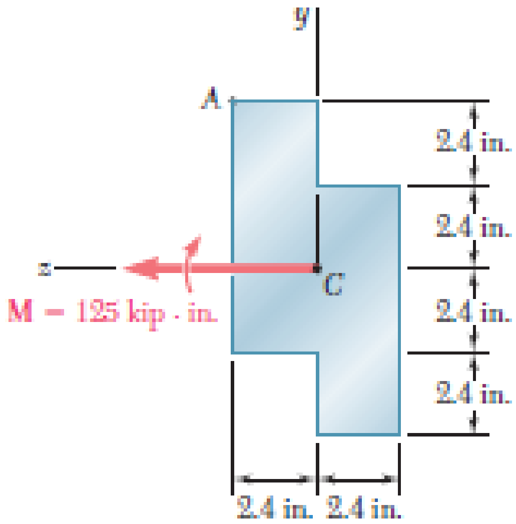
$$\tan\phi = \frac{I_z}{I_y}\tan\theta = \frac{5.359\text{in}^4}{0.9844\text{in}^4}\tan 30$$

$$= 3.143$$

$$\boxed{\phi = 72.4^o}$$

**Fig. 4.55** Cross section with neutral axis and stress distribution.

# In-Class Problem




The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine the stress at point *A*.

# In-Class Problem Solution

**SOLUTION**

$$I_y = 2\left\{\frac{1}{3}(7.2)(2.4)^3\right\} = 66.355 \text{ in}^4$$

$$I_z = 2\left\{\frac{1}{12}(2.4)(7.2)^3 + (2.4)(7.2)(1.2)^2\right\} = 199.066 \text{ in}^4$$

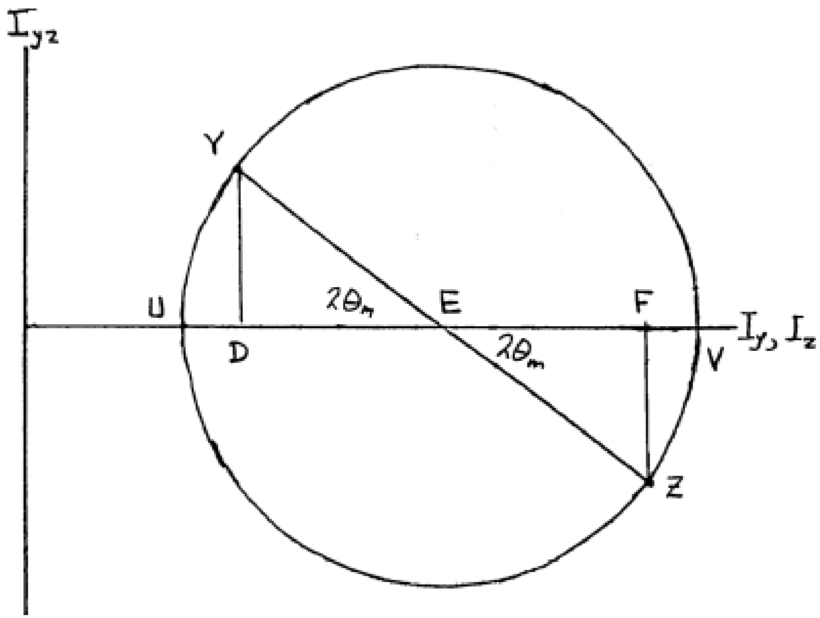
$$I_{yz} = 2\{(2.4)(7.2)(1.2)(1.2)\} = 49.766 \text{ in}^4$$

Using Mohr's circle, determine the principal axes and principal moments of inertia.

$$Y: (66.355 \text{ in}^4,\ 49.766 \text{ in}^4)$$

$$Z: (199.066 \text{ in}^4,\ -49.766 \text{ in}^4)$$

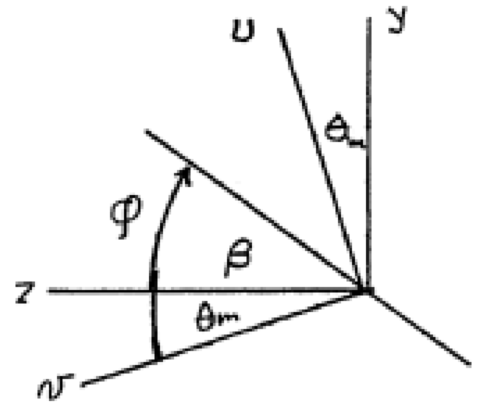
$$E: (132.710 \text{ in}^4,\ 0)$$

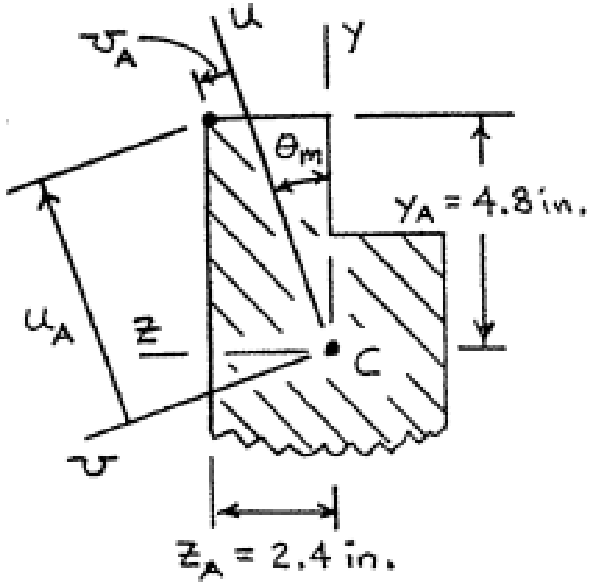


# In-Class Problem Solution

$$\tan 2\theta_m = \frac{DY}{DE} = \frac{49.766}{66.355}$$
$$2\theta_m = 36.87° \quad \theta_m = 18.435°$$

$$R = \sqrt{\overline{DE}^2 + \overline{DY}^2} = 82.944\ \text{in}^4$$
$$I_u = 132.710 - 82.944 = 49.766\ \text{in}^4$$
$$I_v = 132.710 + 82.944 = 215.654\ \text{in}^4$$




$$M_u = 125 \sin 18.435° = 39.529\ \text{kip}\cdot\text{in.}$$
$$M_v = 125 \cos 18.435° = 118.585\ \text{kip}\cdot\text{in.}$$

$$u_A = 4.8 \cos 18.435° + 2.4 \sin 18.435° = 5.3126\ \text{in.}$$
$$v_A = -4.8 \sin 18.435° + 2.4 \cos 18.435° = 0.7589\ \text{in.}$$

$$\sigma_A = -\frac{M_v u_A}{I_v} + \frac{M_u v_A}{I_u}$$
$$= -\frac{(118.585)(5.3126)}{215.654} + \frac{(39.529)(0.7589)}{49.766}$$
$$= -2.32\ \text{ksi} \qquad \sigma_A = -2.32\ \text{ksi} \blacktriangleleft$$