Seventh Edition

CHAPTER

5

MECHANICS OF MATERIALS

Ferdinand P. Beer

E. Russell Johnston, Jr.

John T. DeWolf

David F. Mazurek

Lecture Notes:

Brock E. Barry

U.S. Military Academy

Analysis and Design of Beams for Bending

Lecture 10 02/12/2018

Modified from Original

HW Problems Week 6 (due Mon 02/26):

5.11, 5.15, 5.16, 5.53, 5.56, 5.57

Introduction

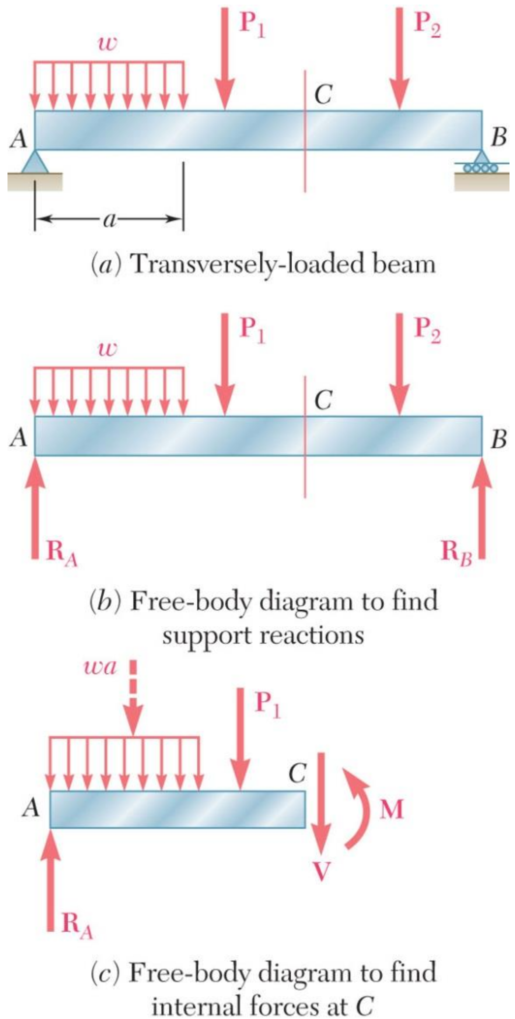


Fig. 5.4 Analysis of a simply supported beam.

- Goal - Analysis and design of beams
- *Beams* - structural members supporting loads at various points along the member
- Transverse loadings of beams are classified as *concentrated loads* or *distributed loads*
- Applied loads result in internal forces consisting of a shear force (from the shear stress distribution) and a bending couple (from the normal stress distribution)
- Normal stress is often the critical design criteria

$$\sigma_x = -\frac{My}{I} \qquad \sigma_m = \frac{|M|c}{I} = \frac{|M|}{S}$$

Requires determination of the location and magnitude of maximum bending moment

Introduction

Classification of Beam Supports

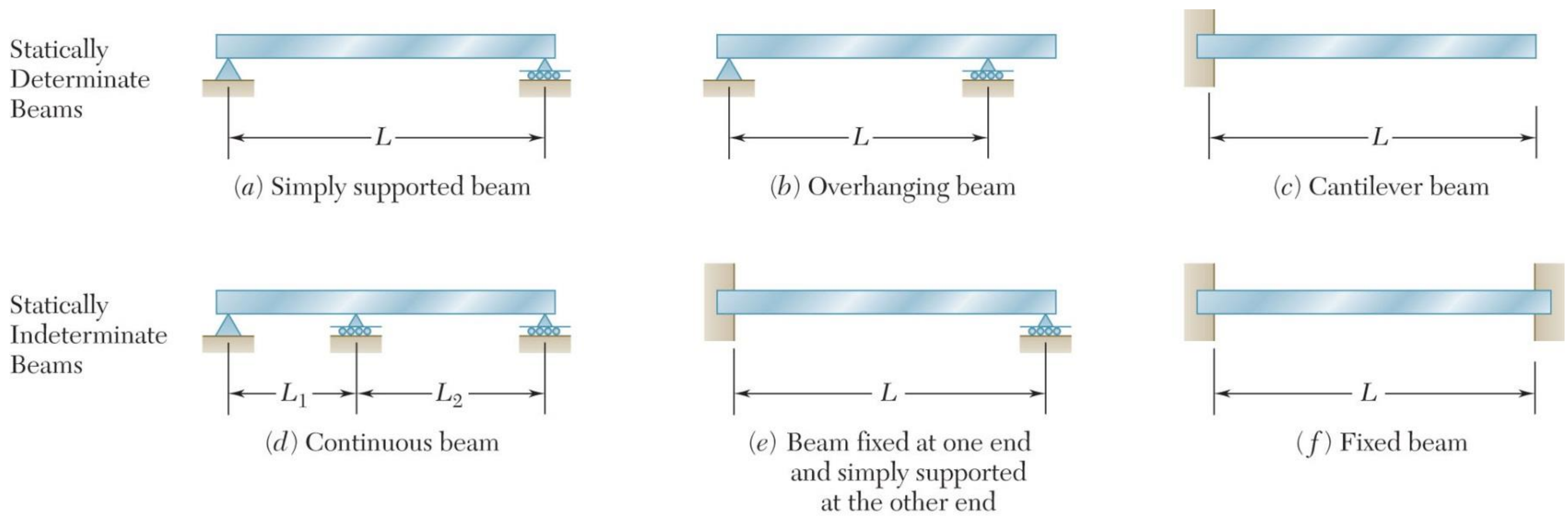


Fig. 5.2 Common beam support configurations.

Shear and Bending Moment Diagrams

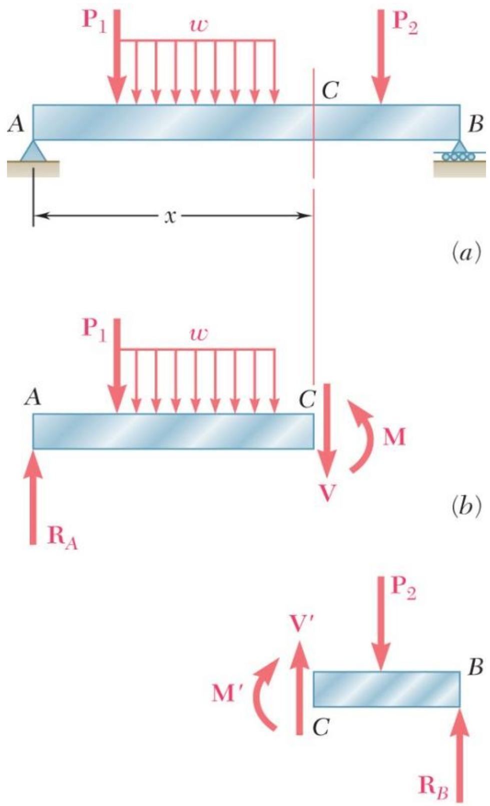


Fig. 5.5 Determination of shear force, V and bending moment, M, at a given section. (a) Loaded beam with section indicated at arbitrary positions S. (b) Free-body diagrams of left and right sections at C.

- Determination of maximum normal and shearing stresses requires identification of maximum internal shear force and bending couple.
- Shear force and bending couple at a point are determined by passing a section through the beam and applying an equilibrium analysis on the beam portions on either side of the section.

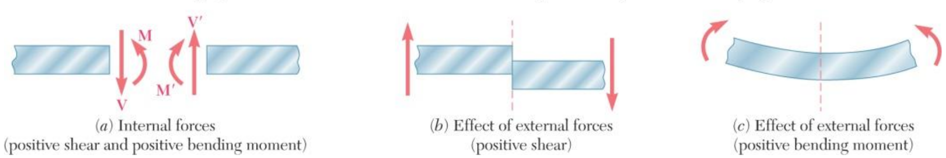
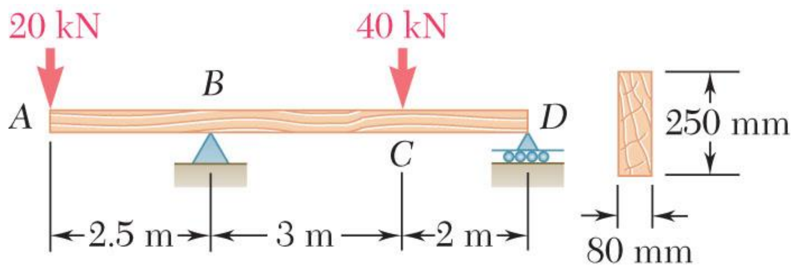


Fig. 5.6 Sign convention for shear and bending moment.

Sample Problem 5.1



For the timber beam and loading shown, draw the shear and bend-moment diagrams and determine the maximum normal stress due to bending.

SOLUTION:

- Treating the entire beam as a rigid body, determine the reaction forces
- Section the beam at points near supports and load application points. Apply equilibrium analyses on resulting free-bodies to determine internal shear forces and bending couples
- Identify the maximum shear and bending-moment from plots of their distributions.
- Apply the elastic flexure formulas to determine the corresponding maximum normal stress.

Sample Problem 5.1

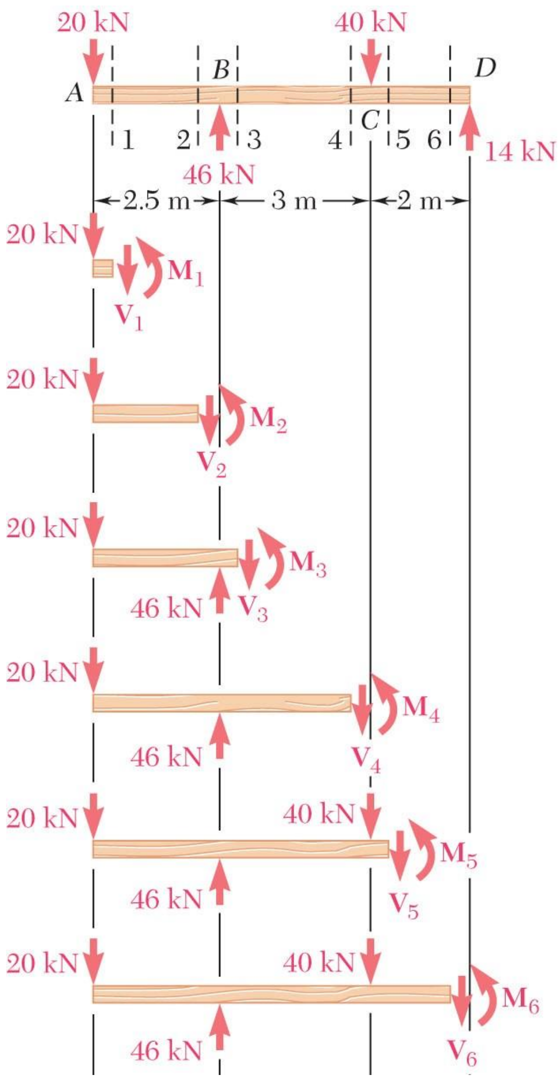


Fig. 1 Six sections identified for analysis, and the free body diagram for each section.

SOLUTION:

- Treating the entire beam as a rigid body, determine the reaction forces

$$\sum F_y = 0 = \sum M_B : \quad R_B = 46\,\text{kN} \quad R_D = 14\,\text{kN}$$

- Section the beam and apply equilibrium analyses on resulting free-bodies

$$\sum F_y = 0 \quad -20\,\text{kN} - V_1 = 0 \qquad V_1 = -20\,\text{kN}$$

$$\sum M_1 = 0 \quad (20\,\text{kN})(0\,\text{m}) + M_1 = 0 \quad M_1 = 0$$

$$\sum F_y = 0 \quad -20\,\text{kN} - V_2 = 0 \qquad V_2 = -20\,\text{kN}$$

$$\sum M_2 = 0 \quad (20\,\text{kN})(2.5\,\text{m}) + M_2 = 0 \quad M_2 = -50\,\text{kN}\cdot\text{m}$$

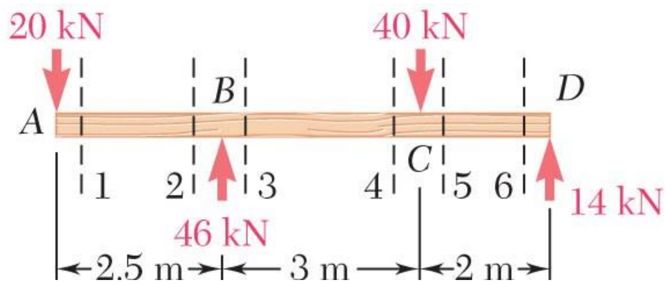
$$V_3 = +26\,\text{kN} \quad M_3 = -50\,\text{kN}\cdot\text{m}$$

$$V_4 = +26\,\text{kN} \quad M_4 = +28\,\text{kN}\cdot\text{m}$$

$$V_5 = -14\,\text{kN} \quad M_5 = +28\,\text{kN}\cdot\text{m}$$

$$V_6 = -14\,\text{kN} \quad M_6 = 0$$

Sample Problem 5.1



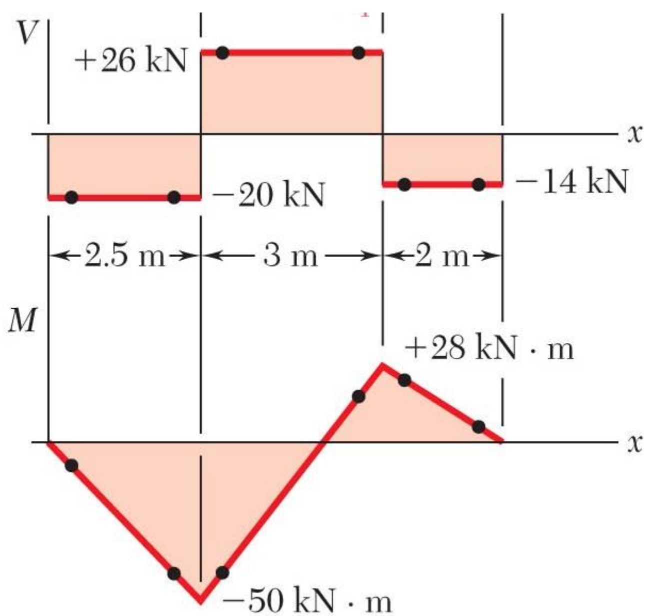


Fig. 1 Shear and bending moment diagrams result from the analysis of each section.

- Identify the maximum shear and bending-moment from plots of their distributions.

$$V_m = 26\,\text{kN} \qquad M_m = |M_B| = 50\,\text{kN}\cdot\text{m}$$

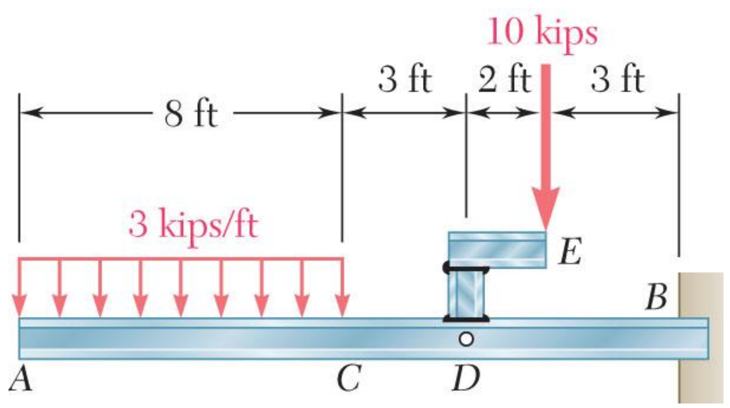
- Apply the elastic flexure formulas to determine the corresponding maximum normal stress.

$$S = \tfrac{1}{6} b h^2 = \tfrac{1}{6}(0.080\,\text{m})(0.250\,\text{m})^2$$
$$= 833.33\times10^{-6}\,\text{m}^3$$

$$\sigma_m = \frac{|M_B|}{S} = \frac{50\times10^3\,\text{N}\cdot\text{m}}{833.33\times10^{-6}\,\text{m}^3}$$

$$\boxed{\sigma_m = 60.0\times10^6\,\text{Pa}}$$

Sample Problem 5.2



The structure shown is constructed of a W10x112 rolled-steel beam. (*a*) Draw the shear and bending-moment diagrams for the beam and the given loading. (*b*) determine maximum normal stress in sections just to the right and left of point *D*.

SOLUTION:

- Replace the 10 kip load with an equivalent force-couple system at *D*. Find the reactions at *B* by considering the beam as a rigid body.
- Section the beam at points near the support and load application points. Apply equilibrium analyses on resulting free-bodies to determine internal shear forces and bending couples.
- Apply the elastic flexure formulas to determine the maximum normal stress to the left and right of point *D*.

Sample Problem 5.2

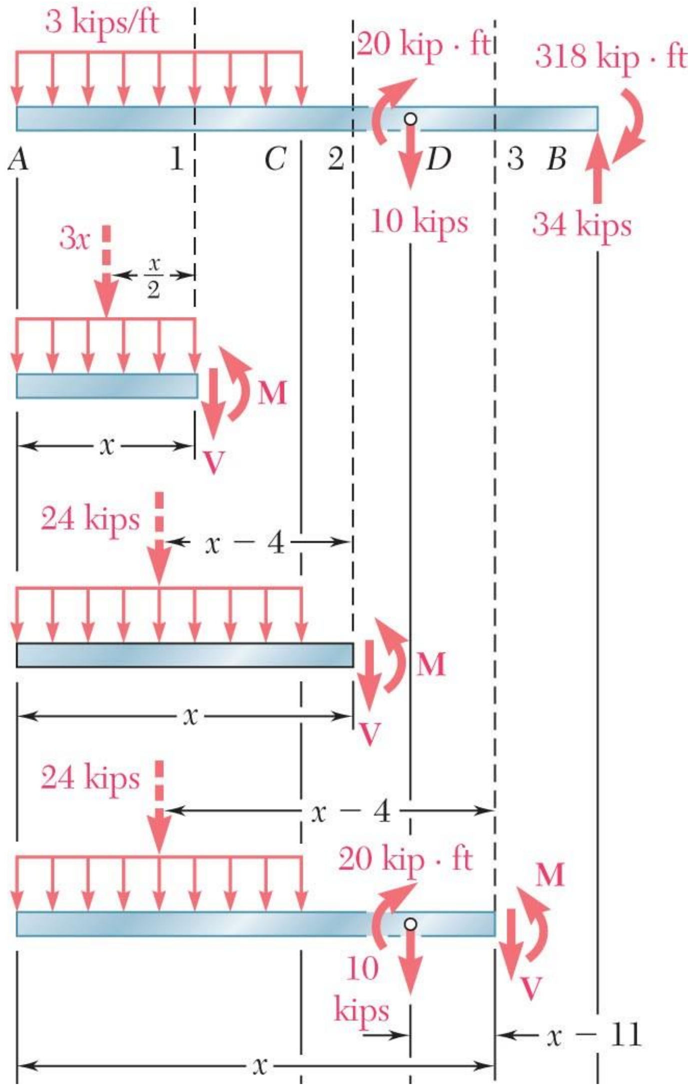


Fig. 1 Three sections identified for analysis, and the free-body diagram for each section.

SOLUTION:

- Replace the 10 kip load with equivalent force-couple system at *D*. Find reactions at *B*.
- Section the beam and apply equilibrium analyses on resulting free-bodies.

From A to C:

$$\sum F_y = 0 \quad -3x - V = 0 \qquad V = -3x \text{ kips}$$

$$\sum M_1 = 0 \quad (3x)\left(\tfrac{1}{2}x\right) + M = 0 \qquad M = -1.5x^2 \text{ kip} \cdot \text{ft}$$

From C to D:

$$\sum F_y = 0 \quad -24 - V = 0 \qquad V = -24 \text{ kips}$$

$$\sum M_2 = 0 \quad 24(x-4) + M = 0 \qquad M = (96 - 24x) \text{ kip} \cdot \text{ft}$$

From D to B:

$$V = -34 \text{ kips} \qquad M = (226 - 34x) \text{ kip} \cdot \text{ft}$$

Sample Problem 5.2

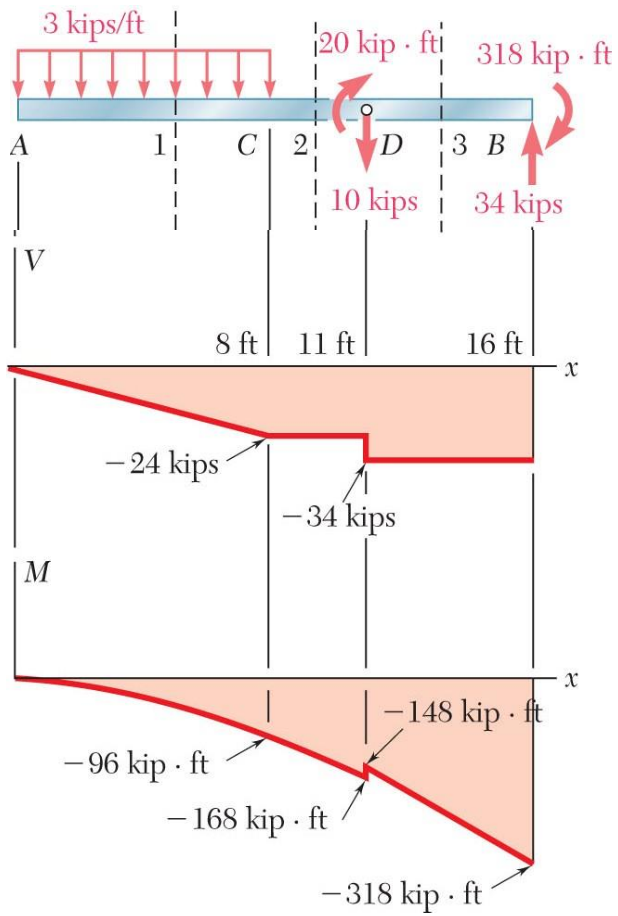


Fig. 1 Shear and bending-moment diagrams result from the analysis of each section

- Apply the elastic flexure formulas to determine the maximum normal stress to the left and right of point *D*.

From Appendix C for a W10x112 rolled steel shape, $S = 126\ \text{in}^3$ about the *X*-*X* axis.

To the left of D:

$$\sigma_m = \frac{|M|}{S} = \frac{2016\,\text{kip}\cdot\text{in}}{126\,\text{in}^3} \qquad \boxed{\sigma_m = 16.0\,\text{ksi}}$$

To the right of D:

$$\sigma_m = \frac{|M|}{S} = \frac{1776\,\text{kip}\cdot\text{in}}{126\,\text{in}^3} \qquad \boxed{\sigma_m = 14.1\,\text{ksi}}$$