MECHANICS OF MATERIALS

CHAPTER

3

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Torsion

Lecture Notes:

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Lecture 6 01/29/2018
Modified from Original

HW Problems Week 4 (due Mon 02/05):

3.11, 3.15, 4.9, 4.18

Mc Graw Hill Education

Statically Indeterminate Shafts

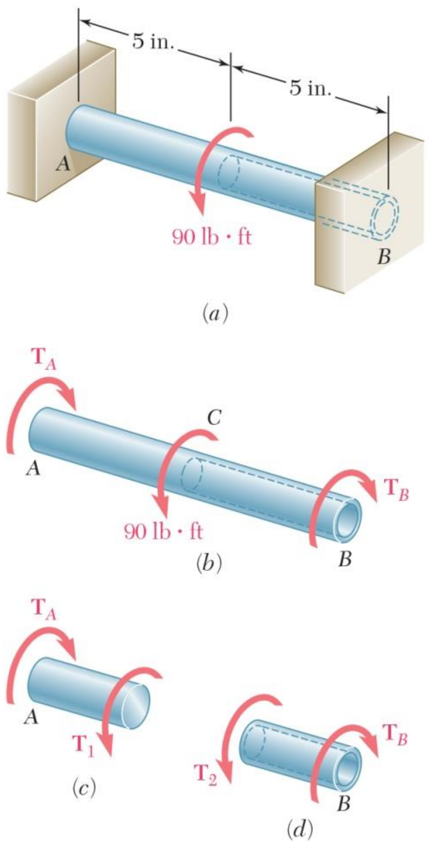


Fig. 3.25 (a) Shaft with central applied torque and fixed ends. (b) free-body diagram of shaft AB. (c) Free-body diagrams for solid and hollow segments.

- Given the shaft dimensions and the applied torque, we would like to find the torque reactions at *A* and *B*.
- From a free-body analysis of the shaft,

$$T_A + T_B = 90\,\mathrm{lb \cdot ft}$$

which is not sufficient to find the end torques. The problem is *statically indeterminate*.

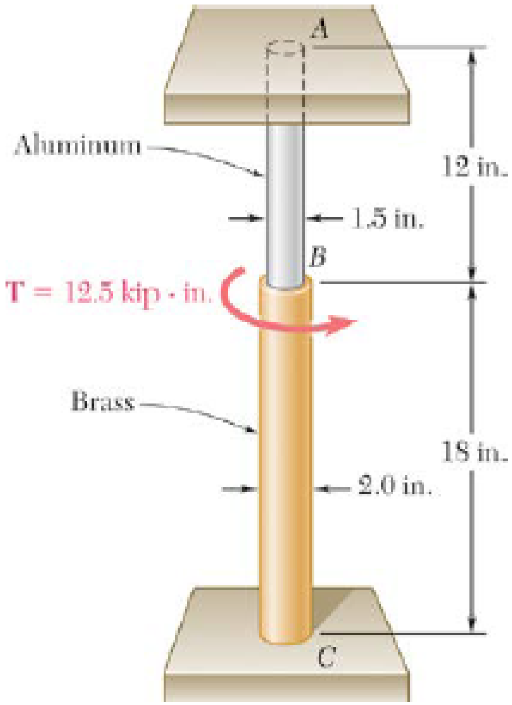
- Divide the shaft into two components which must have compatible deformations,

$$\phi = \phi_1 + \phi_2 = \frac{T_A L_1}{J_1 G} - \frac{T_B L_2}{J_2 G} = 0 \qquad T_B = \frac{L_1 J_2}{L_2 J_1} T_A$$

- Substitute into the original equilibrium equation,

$$T_A + \frac{L_1 J_2}{L_2 J_1} T_A = 90\,\mathrm{lb \cdot ft}$$

In-Class Problem

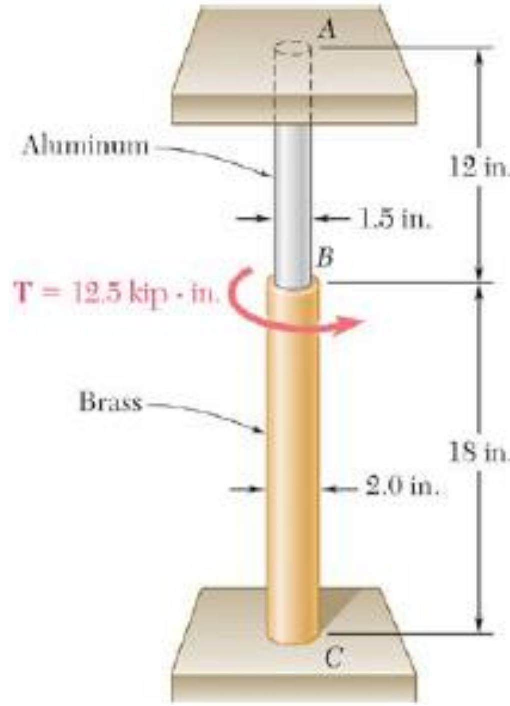


PROBLEM 3.51

The solid cylinders *AB* and *BC* are bonded together at *B* and are attached to fixed supports at *A* and *C*. Knowing that the modulus of rigidity is 3.7×10^6 psi for aluminum and 5.6×10^6 psi for brass, determine the maximum shearing stress (*a*) in cylinder *AB*, (*b*) in cylinder *BC*.

Solution

The torques in cylinders *AB* and *BC* are statically indeterminate. Match the rotation φ_B for each cylinder.



Cylinder *AB*:

$$c = \frac{1}{2}d = 0.75 \text{ in.} \qquad L = 12 \text{ in.}$$

$$J = \frac{\pi}{2}c^4 = 0.49701 \text{ in}^4$$

$$\varphi_B = \frac{T_{AB}L}{GJ} = \frac{T_{AB}(12)}{(3.7\times 10^6)(0.49701)} = 6.5255\times 10^{-6}T_{AB}$$

Cylinder *BC*:

$$c = \frac{1}{2}d = 1.0 \text{ in.} \qquad L = 18 \text{ in.}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(1.0)^4 = 1.5708 \text{ in}^4$$

$$\varphi_B = \frac{T_{BC}L}{GJ} = \frac{T_{BC}(18)}{(5.6\times 10^6)(1.5708)} = 2.0463\times 10^{-6}T_{BC}$$

Matching expressions for φ_B:

$$6.5255\times 10^{-6}T_{AB} = 2.0463\times 10^{-6}T_{BC}$$

$$T_{BC} = 3.1889\, T_{AB}$$

Equilibrium of connection at *B*:

$$T_{AB} + T_{BC} - T = 0 \qquad T = 12.5\times 10^3 \text{ lb}\cdot\text{in.}$$

$$T_{AB} + T_{BC} = 12.5\times 10^3$$

Solution

Substituting (1) into (2), $4.1889\, T_{AB} = 12.5 \times 10^3$

$$T_{AB} = 2.9841 \times 10^3 \text{ lb} \cdot \text{in.} \qquad T_{BC} = 9.5159 \times 10^3 \text{ lb} \cdot \text{in.}$$

(*a*) Maximum stress in cylinder *AB*.

$$\tau_{AB} = \frac{T_{AB}c}{J} = \frac{(2.9841 \times 10^3)(0.75)}{0.49701} = 4.50 \times 10^3 \text{ psi} \qquad \tau_{AB} = 4.50 \text{ ksi} \blacktriangleleft$$

(*b*) Maximum stress in cylinder *BC*.

$$\tau_{BC} = \frac{T_{BC}c}{J} = \frac{(9.5159 \times 10^3)(1.0)}{1.5708} = 6.06 \times 10^3 \text{ psi} \qquad \tau_{BC} = 6.06 \text{ ksi} \blacktriangleleft$$

Design of Transmission Shafts

- Principal transmission shaft performance specifications are:
  - *power*
  - *Speed of rotation*

- Designer must select shaft material and dimensions of the cross-section to meet performance specifications without exceeding allowable shearing stress.

Self Review from Text:
Concept Application Problems 3.6 and 3.7 (pages 186 and 187)

- Determine torque applied to shaft at specified power and speed,

$$P = T\omega = 2\pi f T$$

$$T = \frac{P}{\omega} = \frac{P}{2\pi f}$$

- Find shaft cross-section which will not exceed the maximum allowable shearing stress,

$$\tau_{\max} = \frac{Tc}{J}$$

$$\frac{J}{c} = \frac{\pi}{2}c^3 = \frac{T}{\tau_{\max}} \quad \text{(solid shafts)}$$

$$\frac{J}{c_2} = \frac{\pi}{2c_2}\left(c_2^4 - c_1^4\right) = \frac{T}{\tau_{\max}} \quad \text{(hollow shafts)}$$