Pursuing Truth

A Guide to Critical Thinking

Randy Ridenour

Contents

Preface

This is a textbook written primarily for my students in PHIL 1502: Critical Thinking, at Oklahoma Baptist University in Shawnee, Oklahoma.

There are many good textbooks for critical thinking on the market today, so why write another one? First, all of those books were written for someone else's course. None cover all of the topics that I would like to cover in class. Second, as I'm sure any student can attest to, textbooks are remarkably expensive, to the point that most of the world's population cannot afford access to good learning material. I'm beginning to think of access to information as a moral issue. Now, if I could become wealthy by publishing a book on critical thinking, I might be willing to put the ethical considerations aside.[1] Since all profit would likely go to the publisher, however, I might as well just give the book away.

Warning: if you are looking for the perfect book on critical thinking, this will not be it. For various reasons, this text is likely to always be a work in progress. I can promise, though, that it will be worth as least as much as you paid for it.

[1] As a wise man once said, every person has to decide for themselves what level of hypocrisy they're willing to live with.

Chapter 1

Introduction

1.1 Welcome

We human beings find it very difficult to completely clear our minds. That means you have been thinking of something nearly every waking moment since you began to think. If we assume eight hours of sleep every night, then that comes to just over 7,000,000 minutes of thinking in 20 years. Surely, after that much time spent doing something, you ought to have become pretty good at it. So, why should you consider reading a book or taking a course that claims to teach you how to do something you've been doing for years?

Well, as the old saying goes, I have some good news and some bad news. The bad news is that we're just not very good at thinking carefully. Some things are easy enough for us – you probably don't have a problem when it comes to deciding whether you should step out in front of a truck. On the other hand, when it comes to difficult, tricky subjects, we're often more likely to come up with wrong answers as right ones.

For example, consider this problem:

> A ball and bat together cost $1.10, and the bat costs $1.00 more than the ball does. How much does the ball cost?

I'll give you some time to think about your answer, although you shouldn't need much time. So, whenever you're ready, turn the page…

Was your answer ten cents? That's the most common answer, but it's also clearly wrong. If the ball costs ten cents, and the bat costs a dollar more than the ball, then the bat costs $1.10 and the total would be $1.20. The right answer has to be five cents: $1.05 + $.05 = $1.10. Even though it's not a difficult problem, most people get it wrong.

On the other hand, maybe we get it wrong *because* it's not a difficult problem. It looks so simple that we answer it without thinking about it. When we don't reason carefully about a problem, our minds provide us with an automatic answer. In some situations, the automatic answer provided by the mind is very likely to be true. In others, it is very likely to be false.

At this point, you might be asking yourself, "So what?" What's the worst that could happen, maybe getting a nickel extra in change when you buy the ball? This still doesn't justify taking a whole course to learn how to think better, does it?

Consider one more example:

> 1% of women at age forty who participate in routine screening have breast cancer. 80% of women with breast cancer will get positive mammographies. 9.6% of women without breast cancer will also get positive mammographies. A woman in this age group had a positive mammography in a routine screening. What is the probability that she actually has breast cancer?

Unlike the ball and the bat, being wrong in this case could have drastic consequences—if the doctor guessed too low, then the patient likely did not receive the treatment she needed. If the doctor guessed too high, then the patient may received radical treatments that she didn't need, unnecessary radiation, chemotherapy, or even a radical mastectomy.

Doctors have to make these diagnoses all the time, so surely they would be good at correctly estimating the patient's likelihood of having the disease. Most doctors estimated that, in this problem, the patient's chances of having breast cancer are somewhere between 70 and 80%. Only 15% of doctors surveyed were correct, however. Surprisingly, the right answer is 7.8%, a mere one-tenth of the estimates by the medical professionals.[1]

[1] That doesn't mean that the test should be ignored. It just means that the doctors should not immediately begin dangerous treatments. What is warranted is further testing to lessen the chances of a false positive.

So, how does one avoid making such mistakes? The best way is to become a better critical thinker. You've taken the right initial steps by reading this book and taking this class.

1.2 What is Critical Thinking?

There are probably as many definitions of critical thinking as there books and articles on the subject. Here is a quick working definition that will suit our purposes:

Critical Thinking Thinking clearly and carefully about what to believe or do in a way that is likely to produce a true belief or right action, if there is one.

There are a few things to note about this definition. First, critical thinking is practical. It is designed to produce a particular outcome, either a belief or an action. The goal is to gain true beliefs while avoiding false beliefs, or to do right actions and avoid doing wrong ones. At this point, we don't need to rehash old disputes about the nature of truth or morality, our ordinary understanding of the two concepts will be fine.

Second, there is nothing that we can do that guarantees a true belief—at best, we only get likelihood. Nevertheless, when we use the tools of critical thinking, we will be more likely to get to the truth than had we not used those tools.

Third, it is important to note that not every question has an answer that we can know just by thinking carefully about the problem. There are some questions that have right answers, but just cannot be known by us. How many life-supporting planets are there in the universe? We know there is at least one, but we don't yet have the ability to if there are any others. There are other questions for which there is no right answer, at least not in the objective sense. What kind of ice cream is best? You may have your preference, and I might have a different view. Is either of us wrong? Don't hold up the line in the ice cream shop telling yourself, "I know I like chocolate better than vanilla, but which one is *really* the best?" There is no best in this case, so order whatever you want.

This is a classic case of what philosophers call purely subjective. A subjective truth is one that is dependent on what a person prefers, thinks, believes, etc. Objective truths are true independently of what anyone thinks, believes, perceives, etc. Critical thinking won't help us answer the subjective questions, but we don't really have problems with those. In those cases, it's good enough just to report how we feel, since that is what makes those subjective beliefs true. Critical thinking, however, will help us decide if a question is objective or subjective, and if objective, if it can be answered.

1.3 The Tools of Critical Thinking

It's not enough to tell someone to think clearly and carefully—we have to know what clear and careful thinking, and clear and careful thinkers, look like. Critical thinking is a skill, and like many other skills, it involves the skilled use of tools. One set of tools will be no surprise; they are the tools of logic. Good critical thinkers can

1. Identify arguments in propositional and categorical logic
2. Evaluate arguments using truth tables and Venn diagrams
3. Use the basic rules of probability, and
4. Identify common logical fallacies.

Another set of tools has been provided by cognitive psychologists. Critical thinkers need to understand how the human mind works, especially the systematic ways that the mind is misled. So, critical thinkers must

1. Understand common cognitive biases,
2. Be aware of the ways that people try to mislead us,
3. Know the situations in which we tend to reason badly.

Finally, I think that it's not enough that critical thinkers understand logic. It's not even enough to understand logic and cognitive psychology. I think there is a moral, or value component to critical thinking as well. To become a critical thinker is to become a certain kind of person, a person of intellectual virtue. So, we will discuss the importance and roles of such virtues as

1. Open mindedness
2. Intellectual courage

3. Intellectual humility
4. Attentiveness
5. Fairness
6. Perseverance
7. Firmness

So, by the end of this book, and by the end of your course, I hope that you are well on the road to acquiring these skills. Like any other skills, they cannot be acquired without practice. You will not become a perfect critical thinker in a semester, maybe not even over the course of a lifetime. You can, however, take some significant steps on a journey that leads to one of the most important destinations ever: the truth.

Part I

Logic

Chapter 2

Arguments

The fundamental tool of the critical thinker is the argument. For a good example of what we are not talking about, consider a bit from a famous sketch by *Monty Python's Flying Circus* (Cleese and Chapman 1980):

> Man: (Knock)
> Mr. Vibrating: Come in.
> Man: Ah, Is this the right room for an argument?
> Mr. Vibrating: I told you once.
> Man: No you haven't.
> Mr. Vibrating: Yes I have.
> Man: When?
> Mr. Vibrating: Just now.
> Man: No you didn't.
> Mr. Vibrating: Yes I did.
> Man: You didn't!
> Mr. Vibrating: I did!
> Man: You didn't!
> Mr. Vibrating: I'm telling you I did!
> Man: You did not!!
> Mr. Vibrating: Oh, I'm sorry, just one moment. Is this a five minute argument or the full half hour?

2.1 Identifying Arguments

People often use ”argument” to refer to a dispute or quarrel between people. In critical thinking, an argument is defined as

Argument A set of statements, one of which is the conclusion and the others are the premises.

There are three important things to remember here:

1. Arguments contain statements.
2. They have a conclusion.
3. They have at least one premise

Arguments contain statements, or declarative sentences. Statements, unlike questions or commands, have a truth value. Statements assert that the world is a particular way; questions do not. For example, if someone asked you what you did after dinner yesterday evening, you wouldn’t accuse them of lying. When the world is the way that the statement says that it is, we say that the statement is true. If the statement is not true, it is false.

One of the statements in the argument is called the conclusion. The conclusion is the statement that is intended to be proved. Consider the following argument:

> Calculus II will be no harder than Calculus I. Susan did well in Calculus I. So, Susan should do well in Calculus II.

Here the conclusion is that Susan should do well in Calculus II. The other two sentences are premises. Premises are the reasons offered for believing that the conclusion is true.

2.2 Standard Form

Now, to make the argument easier to evaluate, we will put it into what is called ”standard form.” To put an argument in standard form, write each premise on a separate, numbered line. Draw a line underneath the last premise, the write the conclusion underneath the line.

Conclusion	Premise
Therefore	Since
So	Because
Thus	For
Hence	Is implied by
Consequently	For the reason that
Implies that	
It follows that	

1. Calculus II will be no harder than Calculus I.

2. Susan did well in Calculus I.

∴ Susan will do well in Calculus II.

Now that we have the argument in standard form, we can talk about premise 1, premise 2, and all clearly be referring to the same thing.

2.3 Indicator Words

Unfortunately, when people present arguments, they rarely put them in standard form. So, we have to decide which statement is intended to be the conclusion, and which are the premises. Don't make the mistake of assuming that the conclusion comes at the end. The conclusion is often at the beginning of the passage, but could even be in the middle. A better way to identify premises and conclusions is to look for indicator words. Indicator words are words that signal that statement following the indicator is a premise or conclusion. The example above used a common indicator word for a conclusion, 'so.' The other common conclusion indicator, as you can probably guess, is 'therefore.' This table lists the indicator words you might encounter.

Each argument will likely use only one indicator word or phrase. When the conlusion is at the end, it will generally be preceded by a conclusion indicator. Everything else, then, is a premise. When the conclusion comes at the beginning, the next sentence will usually be introduced by a premise indicator. All of the following sentences will also be premises.

For example, here's our previous argument rewritten to use a premise indicator:

> Susan should do well in Calculus II, because Calculus II will be no harder than Calculus I, and Susan did well in Calculus I.

Sometimes, an argument will contain no indicator words at all. In that case, the best thing to do is to determine which of the premises would logically follow from the others. If there is one, then it is the conclusion. Here is an example:

> Spot is a mammal. All dogs are mammals, and Spot is a dog.

The first sentence logically follows from the others, so it is the conclusion. When using this method, we are forced to assume that the person giving the argument is rational and logical, which might not be true.

2.4 Non-Arguments

One thing that complicates our task of identifying arguments is that there are many passages that, although they look like arguments, are not arguments. The most common types are:

1. Explanations
2. Mere asssertions
3. Conditional statements
4. Loosely connected statements

Explanations can be tricky, because they often use one of our indicator words. Consider this passage:

> Abraham Lincoln died because he was shot.

If this were an argument, then the conclusion would be that Abraham Lincoln died, since the other statement is introduced by a premise indicator. If this is an argument, though, it's a strange one. Do you really think that someone would be trying to prove that Abraham Lincoln died? Surely everyone knows that he is dead. On the other hand, there might be people who don't know how he died. This passage does not attempt to prove that something is true, but instead attempts to explain why it is true. To determine if a passage is an explanation or an argument, first find the statement that looks like the conclusion. Next, ask yourself if everyone likely already believes that statement to be true. If the answer to that question is yes, then the passage is an explanation.

Mere assertions are obviously not arguments. If a professor tells you simply that you will not get an A in her course this semester, she has not given you an argument. This is because she hasn't given you any reasons to believe that the statement is true. If there are no premises, then there is no argument.

Conditional statements are sentences that have the form "If…, then…" A conditional statement asserts that *if* something is true, then something else would be true also. For example, imagine you are told, "If you have the winning lottery ticket, then you will win ten million dollars." What is being claimed to be true, that you have the winning lottery ticket, or that you will win ten million dollars? Neither. The only thing claimed is the entire conditional. Conditionals can be premises, and they can be conclusions. They can be parts of arguments, but that cannot, on their own, be arguments themselves.

Finally, consider this passage:

> I woke up this morning, then took a shower and got dressed. After breakfast, I worked on chapter 2 of the critical thinking text. I then took a break and drank some more coffee….

This might be a description of my day, but it's not an argument. There's nothing in the passage that plays the role of a premise or a conclusion. The passage doesn't attempt to prove anything. Remember that arguments need a conclusion, there must be something that is the statement to be proved. Lacking that, it simply isn't an argument, no matter how much it looks like one.

2.5 Evaluating Arguments

The first step in evaluating an argument is to determine what kind of argument it is. We initially categorize arguments as either deductive or inductive, defined roughly in terms of their goals. In deductive arguments, the truth of the premises is intended to absolutely establish the truth of the conclusion. For inductive arguments, the truth of the premises is only intended to establish the probable truth of the conclusion. We'll focus on deductive arguments first, then examine inductive arguments in later chapters.

Once we have established that an argument is deductive, we then ask if it is valid. To say that an argument is valid is to claim that there is a very special logical relationship between the premises and the conclusion, such that if the premises are true, then the conclusion must also be true. Another way to state this is

Valid An argument is valid if and only if it is impossible for the premises to be true and the conclusion false.

Invalid An argument is invalid if and only if it is not valid.

Note that claiming that an argument is valid is not the same as claiming that it has a true conclusion, nor is it to claim that the argument has true premises. Claiming that an argument is valid is claiming nothing more that the premises, *if they were true*, would be enough to make the conclusion true. For example, is the following argument valid or not?

1. If pigs fly, then an increase in the minimum wage will be approved next term.
2. Pigs fly.

∴ An increase in the minimum wage will be approved next term.

The argument is indeed valid. If the two premises were true, then the conclusion would have to be true also. What about this argument?

1. All dogs are mammals
2. Spot is a mammal.

∴ Spot is a dog.

In this case, both of the premises are true and the conclusion is true. The question to ask, though, is whether the premises absolutely guarantee that the conclusion is true. The answer here is no. The two premises could be true and the conclusion false if Spot were a cat, whale, etc.

Neither of these arguments are good. The second fails because it is invalid. The two premises don't prove that the conclusion is true. The first argument is valid, however. So, the premises would prove that the conclusion is true, *if those premises were themselves true.* Unfortunately, (or fortunately, I guess, considering what would be dropping from the sky) pigs don't fly.

These examples give us two important ways that deductive arguments can fail. The can fail because they are invalid, or because they have at least one false premise. Of course, these are not mutually exclusive, an argument can be both invalid and have a false premise.

If the argument is valid, and has all true premises, then it is a sound argument. Sound arguments always have true conclusions.

Sound A deductively valid argument with all true premises.

Inductive arguments are never valid, since the premises only establish the probable truth of the conclusion. So, we evaluate inductive arguments according to their strength. A strong inductive argument is one in which the truth of the premises really do make the conclusion probably true. An argument is weak if the truth of the premises fail to establish the probable truth of the conclusion.

There is a significant difference between valid/invalid and strong/weak. If an argument is not valid, then it is invalid. The two categories are mutually exclusive and exhaustive. There can be no such thing as an argument being more valid than another valid argument. Validity is all or nothing. Inductive strength, however, is on a continuum. A strong inductive argument can be made stronger with the addition of another premise. More evidence can raise the probability of the conclusion. A valid argument cannot be made more valid with an additional premise. Why not? If the argument is valid, then the premises were enough to absolutely guarantee the truth of the conclusion. Adding another premise won't give any more guarantee of truth than was already there. If it could, then the guarantee wasn't absolute before, and the original argument wasn't valid in the first place.

2.6 Counterexamples

One way to prove an argument to be invalid is to use a counterexample. A counterexample is a consistent story in which the premises are true and the conclusion false. Consider the argument above:

1. All dogs are mammals
2. Spot is a mammal.

∴ Spot is a dog.

By pointing out that Spot could have been a cat, I have told a story in which the premises are true, but the conclusion is false.

Here's another one:

1. If it is raining, then the sidewalks are wet.
2. The sidewalks are wet.

∴ It is raining.

The sprinklers might have been on. If so, then the sidewalks would be wet, even if it weren't raining.

Counterexamples can be very useful for demonstrating invalidity. Keep in mind, though, that validity can never be proved with the counterexample method. If the argument is valid, then it will be impossible to give a counterexample to it. If you can't come up with a counterexample, however, that does not prove the argument to be valid. It may only mean that you're not creative enough.

2.7 Review

1. An **argument** is a set of statements; one is the conclusion, the rest are premises.
2. The **conclusion** is the statement that the argument is trying to prove.
3. The **premises** are the reasons offered for believing the conclusion to be true.
4. Explanations, conditional sentences, and mere assertions are not arguments.
5. **Deductive** reasoning attempts to absolutely guarantee the truth of the conclusion.
6. **Inductive** reasoning attempts to show that the conclusion is probably true.
7. In a **valid** argument, it is impossible for the premises to be true and the conclusion false.

8. In an **invalid** argument, it is possible for the premises to be true and the conclusion false.

9. A **sound** argument is valid and has all true premises.

10. An inductively **strong** argument is one in which the truth of the premises makes the the truth of the conclusion probable.

11. An inductively **weak** argument is one in which the truth of the premises do not make the conclusion probably true.

12. A **counterexample** is a consistent story in which the premises of an argument are true and the conclusion is false. Counterexamples can be used to prove that arguments are deductively invalid

Chapter 3

Categorical Logic

Now we turn to some structured logic systems. The first, categorical logic, is one of the oldest. It dates back at least to Aristotle (384–322 BCE). Categorical logic is a fairly simple logic of categories or classes. A class is a group of things that we designate with a common noun: students, teachers, dogs, politicians, etc. Each sentence will use two different classes. One is the subject class, and the other is the predicate class. In this logic, we can say something about all members of a class, called a universal sentence, or we can say something about some members of a class, called a particular sentence. We can also make a positive claim, called an affirmation, or we can make a negative claim, called a negation.

With these two distinctions, universal/particular and affirnation/negation, we can make four kinds of sentences. S and P stand for the subject class and the predicate class, respectively.

A : All S are P (universal affirmation)

E : No S are P (universal negation)

I : Some S are P (particular affirmation)

O : Some S are not P (particular negation)[1]

Here are some examples of categorical statements, some true and some false.

[1] The letters A, E, I, and O, are thought to come from the first two vowels of the Latin words *affirmo* and *nego*, meaning "I affirm" and "I deny."

1. All dogs are mammals.
2. All mammals are dogs.
3. No reptiles are dogs.
4. No politicians are honest people.
5. Some politicians are honest people.
6. Some cats are amphibians.
7. Some dogs are not beagles.
8. Some beagles are not dogs.

Look at the sentences carefully. You should be able to tell that the odd-numbered ones are true and the even-numbered ones are false.

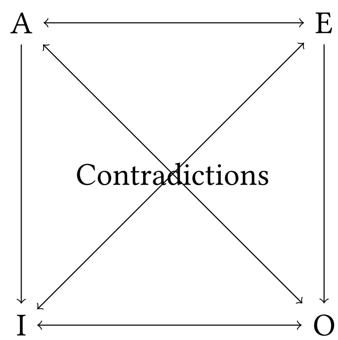
3.1 The Square of Opposition

We can visualize interesting logical relationships between these four types of sentences with something called "The Square of Opposition."

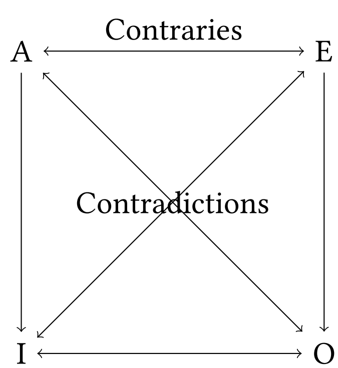
The first step is to place the sentence types in the corners of an imaginary square. A is at the upper left; E, the upper right; I, the lower left, and O, the lower right. Next, draw arrows on the diagonals, pointing to the sentences in the corners. Then, draw an arrow between the two at the top, and another one between the two at the bottom. Finally, draw an arrow on each side, going from top to bottom. When finished, you should have something like this:

$$
\begin{tikzcd}
A \arrow[r, leftrightarrow] \arrow[d] \arrow[dr, leftrightarrow] & E \arrow[d] \arrow[dl, leftrightarrow] \\
I \arrow[r, leftrightarrow] & O
\end{tikzcd}
$$

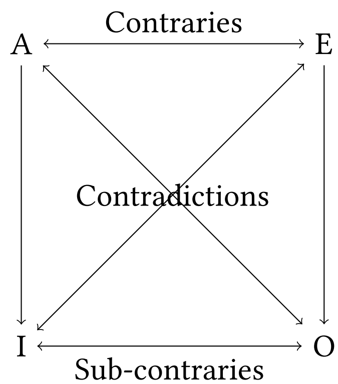
The next step is to note the relationship between the diagonals. The diagonals are contradictories, meaning they always have opposite truth values. They can't both be true, and they also can't both be false. If the A sentence is true, the O sentence must be false—if it is true that all dogs are mammals, it cannot be true that some dogs are not mammals. If the O sentence is true, then the A sentence must be false. It is the same for the E and the I.



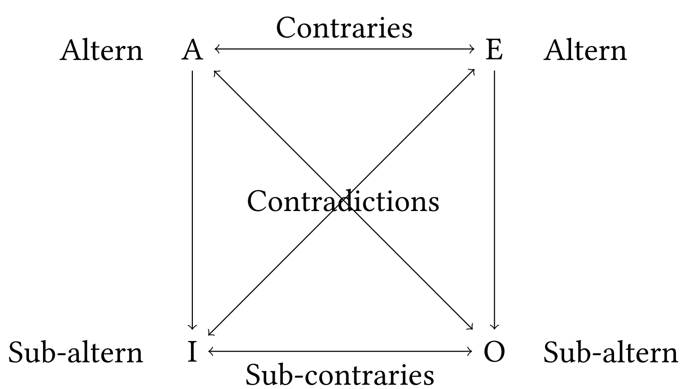
Next, note the relationship between the A sentences and the E sentences, called contraries. Like the contradictories, they cannot both be true. Unlike the contradictories, they can both be false. If it's true that all critical thinking students are good students, then it must be false that no critical thinking students are good students. If it's false that all critical thinking students are good students, then it can be false that critical thinking students are good students. In fact, they are both false, because some critical thinking students are good and others are not.



At the bottom, we have sub-contraries. They can both be false, but cannot both be true.



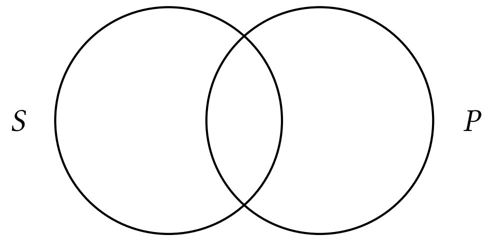
Finally, we have the relationship between the top level sentences and the bottom level sentences on the same side. This is called alternation. The universal is called the superaltern and the particular is called the subaltern. If the superaltern is true, then the subaltern must also be true. If the superaltern is false, then the subaltern can be either true or false. If the subaltern is false, then the superaltern must be false. If the subaltern is true, then the superaltern can be either true or false. It is easy to remember this way: truth goes down, falsity goes up.



3.2 Diagramming Sentences

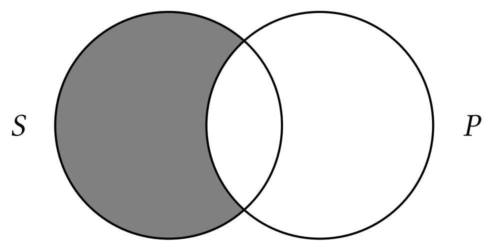
We diagram sentences and arguments in categorical logic using Venn diagrams. You've probably used these in a math class at some time. Before we can use these to evaluate arguments in categorical logic, we first have to learn how to diagram individual sentences.

The first step is to draw two interlocking circles. Label the left circle with an “S” and the right circle with “P”—standing for the subject term and predicate term, respectively.



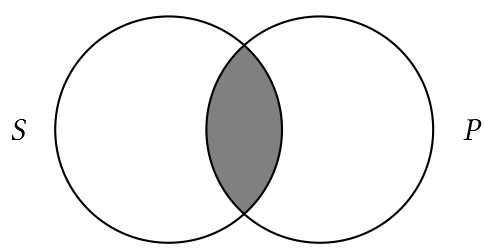
A-Sentences

Remember that the A-sentence has the form All S are P. That means that everything that is in the S circle must also be in the P circle. To diagram this, we shade the region of the S circle that is not contained in the P circle. If a region is shaded, that means that nothing is in that region.



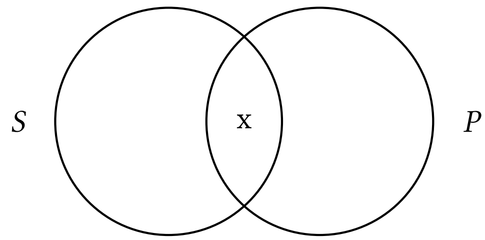
E-Sentences

To shade the universal negation, we shade the region that is shared by both S and P:



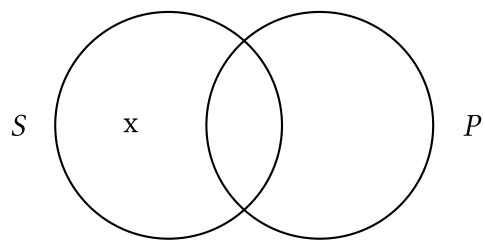
I-Sentences

To diagram a particular affirmation, we place an x in the region shared by S and P:



O-Sentences

Finally, to diagram an O-sentence, we place an x in S, but not in P:



Evaluating Categorical Syllogisms

A syllogism is an argument that has two premises and a conclusion. A categorical syllogism is a syllogism that contains only categorical sentences. Here is an example:

1. All Dogs are mammals.
2. <u>All mammals are animals.</u>

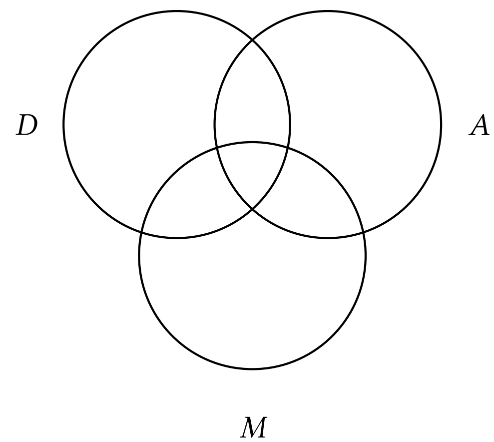
∴ All dogs are animals

Both premises and the conclusion are A-sentences. Notice that we have three terms in the argument: dogs, mammals, and animals. Every categorical syllogism, in proper form, has three terms. Each term occurs in two sentences. Two of those terms will be found in the conclusion, and one term is only in the premises. The predicate term of the conclusion is called the major term. The subject of the conclusion is called the minor term. The term that is not in the conclusion is called the middle term.

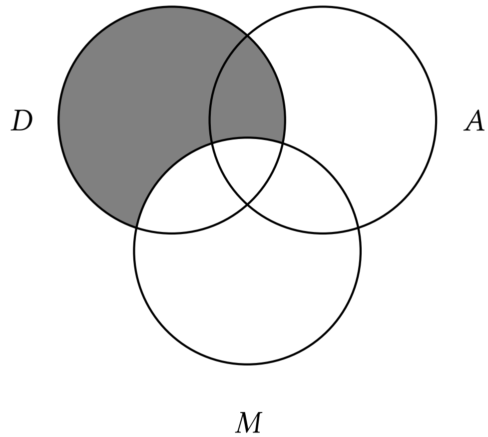
There are two ways to determine if a categorical syllogism is valid. One way uses Venn diagrams, and the other involves applying some simple rules.

Diagram Method

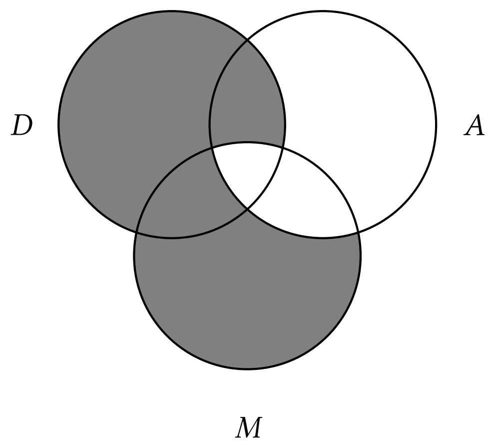
Since we have three terms in the argument, we'll need three intersecting circles. We'll start by drawing two circles for the conclusion, just as we did before. Then, in the middle and below, we'll draw another circle for the middle term. For labels, use letters that correspond to the classes in the argument. Here, we'll use D for dogs, M for mammals, and A for animals.



Next, we finish diagramming the premises by shading or placing an x. Since our first premise is "All dogs are mammals", we need to shade everything in the D circle that is not in the M circle.



Next, we diagram the second premise by shading everything that is in the M circle but not in the A circle.



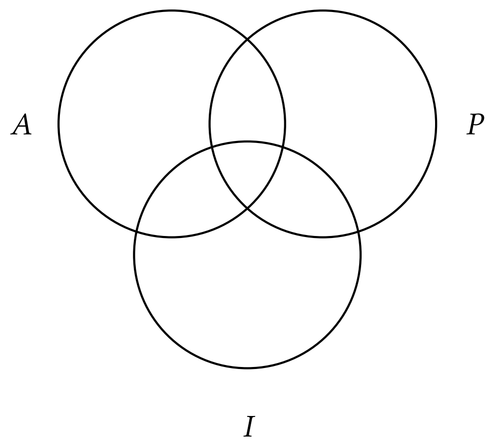
If there is any circle that has only one region left unshaded, you can place an ‘X’ in that region. This is because categorical logic assumes that there are no empty categories, meaning that every category has at least one thing in it. This is really only important for arguments that have an I or an O-sentence for a conclusion. In this case, we won’t worry about it. Now that the premises are diagrammed, check to see if the conclusion has also been diagrammed, which in this case means that everything in the D circle that is not also in the A circle is shaded out. If so, then the argument is valid. This shows that making the premises true was enough to make the conclusion true also.

Let’s try to diagram this argument:

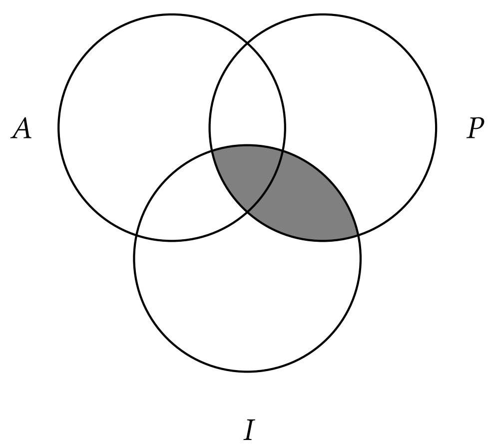
1. No introverts are politicians

2. <u>All artists are introverts</u>
3. No artists are politicians

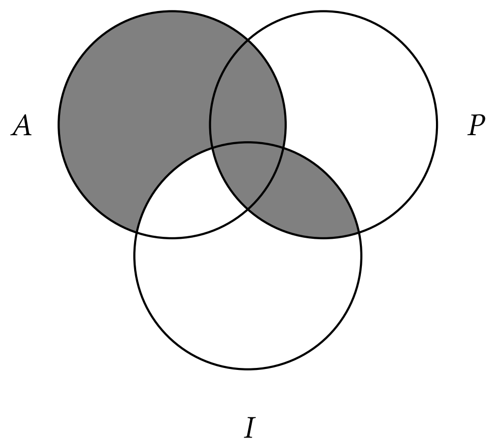
First, we draw and label the circles:



Then we diagram the premises, always doing the universals before any particulars. In this case, we have two universal premises, so we will just begin with the first premise:



Now, we'll diagram the second premise:

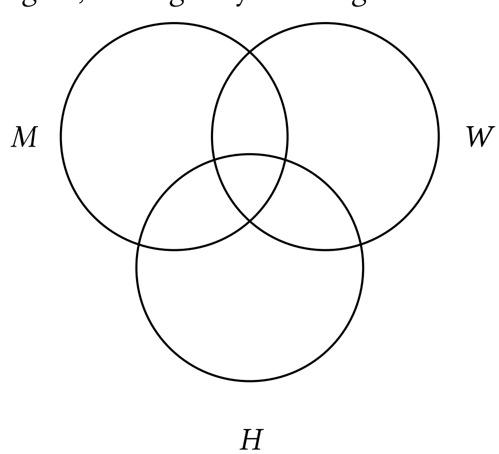


Diagramming the conclusion would require the intersection of A and P to be shaded. Notice, though, that the region between A and P has already been shaded by just diagramming the premises. That means that making the premises true was enough to guarantee that the conclusion would also be true, and the argument is valid.

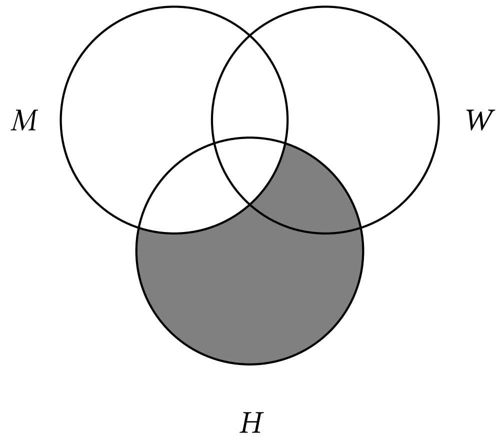
Let's try one more argument.

1. Some horses are things that weigh over 2,000 pounds.
2. <u>All horses are mammals.</u>
3. Some mammals are things that weigh over 2,000 pounds.

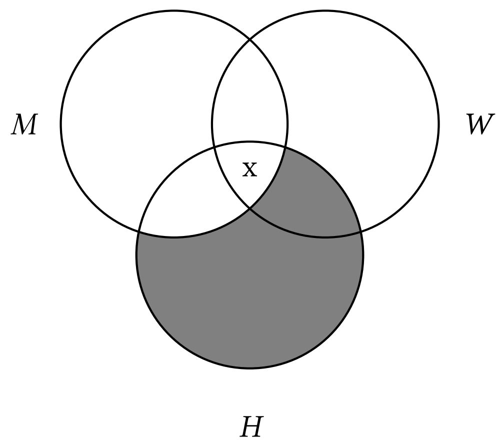
Again, we begin by drawing and labeling the circles.



The we diagram any universal premises, which, in this case, is the second premise.



Then, we diagram any particular premises.



Finally, we check to see if diagramming the premises was enough to make the conclusion also diagrammed. In this case, it was, so the argument is valid.

Hints for Diagramming Categorical Syllogisms

1. Diagram universals before particulars (shade before making an x.)
2. If it is not clear where the x goes, then put it on the line.

3.3 Rules for Categorical Syllogisms

There is another way to determine validity for categorical syllogisms. Every valid syllogism must meet three conditions:

1. There must be the same number of negations in the conclusion as in the premises.
2. The middle term must be distributed at least once.
3. Any term distributed in the conclusion must be distributed in the premises.

Before these rules can be applied, we'll have to explain what distribution is. Every categorical statement says something about a category or class. A statement distributes a term just in case what it says about that class is true of every subset of the class. For example, it is true that all dogs are mammals. It's also true that all members of any subset of the set of dogs are mammals—all dogs in Oklahoma are mammals, and all dogs in Greece are mammals, and so on. All dogs are not necessarily members of every subset of the class of mammals, however. The class of cats is a subset of the class of mammals, and no dog is a cat. So, the subject of an A-sentence is distributed, but the predicate is not. To remember when something is distributed, keep this in mind:

1. Universals distribute subjects, and
2. Negations distribute predicates.

So, A-sentences distribute the subject, E-sentences distribute both terms, I-sentences don't distribute anything, and O sentences distribute the predicate.

The rules are easy to apply. First, put the argument in standard form:

1. All A are B.
2. All B are C.

∴ All A are C.

Then, circle all of the distributed terms.

1. All A are B.
2. All B are C.

∴ All A are C.

Now, just check to see if there are any violations of the rules:

1. Are there the same number of negations in the conclusion as in the premises? Yes, since there are no negations at all.
2. Is the middle term distributed at least once? Yes, the middle term is B and it is distributed in the second premise.
3. Is any term that distributed in the conclusion also distributed in the premises? Yes, A is distributed in the conclusion, but it is also distributed in the first premise.

So, since the argument breaks none of the rules, it is valid.

3.4 Relations of Equivalence

Properly formed categorical syllogisms have only three terms. Unfortunately, some arguments that you will encounter won't always be in proper form. One common way this happens is for a person to use a term like "Americans" in one premise, but use "non-Americans" in another. This can result in a syllogism with four or more terms, making it impossible to evaluate using either of our two methods. What we then need to do is to convert the sentence using one of the terms into a logically equivalent sentence that uses the other term.

There are three operations that can be applied to categorical sentences: conversion, obversion, and contraposition. It is important to know both how to apply them and in what cases does an operation result in an equivalent sentence. We're particularly interested in the conditions that those different operations are *truth-preserving*. An operation is truth preserving when, applied to a true sentence, it always results in a true sentence.

Conversion

Conversion is the simplest of the three. The converse of a sentence simply exchanges the subject and predicate terms of the original sentence. Conversion applied to A-sentences is *not* truth-preserving. "All dogs are mammals" is true, but "All mammals are dogs" is not. Conversion is truth-preserving for E-sentences and I-sentences. If it is true that no dogs are reptiles, it must be true that no reptiles are dots. Likewise, if it is true that some dogs are brown things, it must be true that some brown things are dogs.

Another way to think about this is to consider what the diagrams would like before the change and after the change. Before the change, the diagram looks like figure below, with the intersection of the S and P circles shaded.

After the change, the diagram looks like figure , with the intersection of the S and P circles shaded. Essentially, there's been no change. Imagine what it would like to view the first diagram from behind, or upside-down. In either case, what you would see is the same as the first diagram.

Obversion

Take another look at the square of opposition in figure 4.1. Note that the A and the E are straight across from each other, as are the I and the O. The first step in forming the obverse is to first change the sentence into the type that is straight across the square of opposition. That is, if you started with an A-sentence, then make it into and E. The O becomes and I, and so on.

Once you've changed the sentence type, the next step is to change predicate into its complement. The complement of a class C is the class of everything that is not in C. The easiest way to form a complement is to prefix the class with 'non'. For example, the complement of the class of students is the class of non-students.

So, the obverse of all dogs are mammals is no dogs are non-mammals. The obverse of no OBU students are martians is all OBU students are non-martians. Obversion is truth-preserving in all cases.

Contraposition

The last of our three relations is contraposition. To form the contrapositive of a sentence, first form the converse, then exchange both terms for their complements.

The contrapositive of all dogs are mammals is all non-mammals are non-dogs. Contraposition is truth-preserving for A-sentences and O-sentences only.

Original	Converse	Obverse	Contrapositive
All S are P	All P are S	**No S are non-P**	**All non-P are non-S**
No S are P	**No P are S**	**All S are non-P**	No non-P are non-S
Some S are P	**Some P are S**	**Some S are not non-P**	Some non-P are non-S
Some S are not P	Some P are not S	**Some S are non-P**	**Some non-P are not non-S**

Here's a table to help keep this straight (operations that are truth-preserving are in bold type):

Example

Look at the following argument:

1. All Catholics are non-Protestants.
2. <u>All Lutherans are Protestants.</u>
3. No Catholics are Lutherans.

Note that this argument has four terms:

1. Catholics
2. Non-Protestants
3. Lutherans
4. Protestants

To evaluate the argument, we will first have to either change "non-Protestants" to "Protestants" in the first premise, or "Protestants" to "non-Protestants" in the second premise and conclusion. To minimize errors, we should probably try the option requiring the fewest changes. The only two truth-preserving operations on A-sentences are obversion and contraposition. The contrapositive of "All Catholics are non-Protestants" is "All non-non-Protestants are non-Catholics." The double-non will cancel out, which will fix our original problem, but it will leave us with a new term, "non-Catholic." So, let's try the obverse. The obverse of "All Catholics are non-Protestants" is "No Catholics are Protestants." So, using that for our first premise, the argument becomes:

1. No Catholics are Protestants.
2. <u>All Lutherans are Protestants.</u>
3. No Catholics are Lutherans.

Now, we can check for validity — I'll leave that for you.

Chapter 4

Propositional Logic

Categorical logic is a great way to analyze arguments, but only certain kinds of arguments. It is limited to arguments that have only two premises and the four kinds of categorical sentences. This means that certain common arguments that are obviously valid will not even be well-formed arguments in categorical logic. Here is an example:

1. I will either go out for dinner tonight or go out for breakfast tomorrow.
2. I won't go out for dinner tonight.
3. I will go out for breakfast tomorrow.

None of these sentences fit any of the four categorical schemes. So, we need a new logic, called propositional logic. The good news is that it is fairly simple.

4.1 Simple and Complex Sentences

The fundamental logical unit in categorical logic was a category, or class of things. The fundamental logical unit in propositional logic is a statement, or proposition[1] Simple statements are statements that contain no other statement as a part. Here are some examples:

[1]Informally, we use 'proposition' and 'statement' interchangeably. Strictly speaking, the proposition is the content, or meaning, that the statement expresses. When different sentences in different languages mean the same thing, it is because they express the same proposition.

- Oklahoma Baptist University is in Shawnee, Oklahoma.
- Barack Obama was succeeded as President of the US by Donald Trump.
- It is 33 degrees outside.

Simple sentences are symbolized by uppercase letters. Just pick a letter that makes sense, given the sentence to be symbolized, that way you can more easily remember which letter means which sentence.

Complex sentences have at least one sentence as a component. There are five types in propositional logic:

- Negations
- Conjunctions
- Disjunctions
- Conditionals
- Biconditionals

Negations

Negations are "not" sentences. They assert that something is not the case. For example, the negation of the simple sentence "Oklahoma Baptist University is in Shawnee, Oklahoma" is "Oklahoma Baptist University is not in Shawnee, Oklahoma." In general, a simple way to form a negation is to just place the phrase "It is not the case that" before the sentence to be negated.

A negation is symbolized by placing this symbol '¬' before the sentence-letter. The symbol looks like a dash with a little tail on its right side. If D = 'It is 33 degrees outside,' then ¬D = 'It is not 33 degrees outside.' The negation symbol is used to translate these English phrases:

- not
- it is not the case that
- it is not true that
- it is false that

A negation is true whenever the negated sentence is false. If it is true that it is not 33 degrees outside, then it must be false that it is 33 degrees outside. if it is false that Tulsa is the capital of Oklahoma, then it is true that Tulsa is not the capital of Oklahoma.

When translating, try to keep the simple sentences positive in meaning. Note the warning on page 24, about the example of affirming and denying. Denying is not simply the negation of affirming.

4.2 Conjunction

Negations are “and” sentences. They put two sentences, called conjuncts, together and claim that they are both true. We’ll use the ampersand (&) to signify a negation. Other common symbols are a dot and an upside down wedge. The English words that are translated with the ampersand include:

- and
- but
- also
- however
- yet
- still
- moreover
- although
- nevertheless
- both

For example, we would translate the sentence ‘It is raining today and my sunroof is open’ as ‘R&O’.

4.3 Disjunction

A disjunction is an "or" sentence. It claims that at least one of two sentences, called disjuncts, is true. For example, if I say that either I will go to the movies this weekend or I will stay home and grade critical thinking homework, then I have told the truth provided that I do one or both of those things. If I do neither, though, then my claim was false.

We use this symbol, called a "vel," for disjunctions: $\vee$. The vel is used to translate - or - either...or - unless

4.4 Conditional

The conditional is a common type of sentence. It claims that something is true, if something else is also. Examples of conditionals are

- "If Sarah makes an A on the final, then she will get an A for the course."
- "Your car will last many years, provided you perform the required maintenance."
- "You can light that match only if it is not wet."

We can translate those sentences with an arrow like this:

- $F \longrightarrow C$
- $M \longrightarrow L$
- $L \longrightarrow \neg W$

The arrow translates many English words and phrases, including

- if
- if... then
- only if
- whenever
- when

- only when
- implies
- provided that
- means
- entails
- is a sufficient condition for
- is a necessary condition for
- given that
- on the condition that
- in case

One big difference between conditionals and other sentences, like conjunctions and disjunctions, is that order matters. Notice that there is no logical difference between the following two sentences:

- Albany is the capital of New York and Austin is the capital of Texas.
- Austin is the capital of Texas and Albany is the capital of New York.

They essentially assert exactly the same thing, that both of those conjuncts are true. So, changing order of the conjuncts or disjuncts does not change the meaning of the sentence, and if meaning doesn't change, then true value doesn't change.

That's not true of conditionals. Note the difference between these two sentences:

- If you drew a diamond, then you drew a red card.
- If you drew a red card, then you drew a diamond.

The first sentence *must* be true. if you drew a diamond, then that guarantees that it's a red card. The second sentence, though, could be false. Your drawing a red card doesn't guarantee that you drew a diamond, you could have drawn a heart instead. So, we need to be able to specify which sentence goes before the arrow and which sentence goes after. The sentence before the arrow is called the antecedent, and the sentence after the arrow is called the consequent.

Look at those three examples again:

1. "If Sarah makes an A on the final, then she will get an A for the course."

2. "Your car will last many years, provided you perform the required maintenance."

3. "You can light that match only if it is not wet."

The antecedent for the first sentence is "Sarah makes an A on the final." The consequent is "She will get an A for the course." Note that the `if` and the `then` are not parts of the antecedent and consequent.

In the second sentence, the antecdent is "You perform the required maintenance." The consequent is "Your car will last many years." This tells us that the antecedent won't always come first in the English sentence.

The third sentence is tricky. The antecedent is "You can light that match." Why? The explanation involves something called necessary and sufficient conditions.

Necessary and Sufficient Conditions

A sufficient condition is something that is enough to guarantee the truth of something else. For example, getting a 95 on an exam is sufficient for making an A, assuming that exam is worth 100 points. A necessary condition is something that must be true in order for something else to be true. Making a 95 on an exam is not necessary for making an A—a 94 would have still been an A. Taking the exam is necessary for making an A, though. You can't make an A if you don't take the exam, or, in other words, you can make an a only if you enroll in the course.

Here are some important rules to keep in mind:

- 'If' introduces antecedents, but `Only if` introduces consequents.

- If A is a sufficient condition for B, then $A \longrightarrow B$.
- If A is a necessary condition for B, then $B \longrightarrow A$.

4.5 Biconditional

We won't spend much time on biconditionals. There are times when something is both a necessary and a sufficient condition for something else. For example, making at least a 90 and getting an A (assuming a standard scale, no curve, and no rounding up). If you make at least a 90, then you will get an A. If you got an A, then you made at least a 90. We can use a double arrow to translate a biconditional, like this:

- $N \longrightarrow A$

For biconditionals, as for conjunctions and disjunctions, order doesn't matter. Here are some English phrases that signify biconditionals:

- it and only if
- when and only when
- just in case
- is a necessary and sufficient condition for

4.6 Translations

Propositional logic is language. Like other languages, it has a syntax and a semantics. The syntax of a language includes the basic symbols of the language plus rules for putting together proper statements in the language. To use propositional logic, we need to know how to translate English sentences into the language of propositional logic. We start with our sentence letters, which represent simple English sentences. Let's use three borrowed from an elementary school reader:

T: Tom hit the ball.
J: Jane caught the ball.
S: Spot chased the ball.

We then build complex sentences using the sentence letters and our five logical operators, like this:

English Sentence	PL Translation
Tom did not hit the ball.	$\neg T$
Either Tom hit the ball or Jane caught the ball.	$T \vee J$
Spot chased the ball, but Jane caught it.	$S \,\&\, J$
If Jane caught the ball, then Spot did not chase it.	$J \longrightarrow \neg S$
Spot chased the ball if and only if Tom hit the ball.	$S \leftrightarrow T$

We can make even more complex sentences, but we will soon run into a problem. Consider this example:

$$T \,\&\, J \longrightarrow S$$

We don't know what this means. It could be either one of the following:

1. Tom hit the ball, and if Jane caught the ball, then Spot chased it.
2. If Tom hit the ball and Jane caught it, then Spot chased it.

The first sentence is a conjunction, T is the first conjunct and $M \longrightarrow S$ is the second conjunct. The second sentence, though, is a conditional, $T \,\&\, M$ is the antecdent, and S is the consequent. Our two interpretations are not equivalent, so we need a way to clear up the ambiguity. We can do this with parentheses. Our first sentence becomes:

$$T \,\&\, (J \longrightarrow S)$$

The second sentence is:

$$(T \,\&\, J) \longrightarrow S$$

If we need higher level parentheses, we can use brackets and braces. For instance, this is a perfectly good formula in propositional logic:[2]

$$[(P \,\&\, Q) \vee R] \longrightarrow \{[(\neg P \leftrightarrow Q) \,\&\, S] \vee \neg P\}$$

Every sentence in propositional logic is one of six types:

[2]It may be a good formula in propositional logic, but that doesn't mean it would be a good English sentence.

1. Simple
2. Negation
3. Conjunction
4. Disjunction
5. Conditional
6. Biconditional

What type of sentence it is will be determined by its main logical operator. Sentences can have several logical operators, but they will always have one, and only one, main operator. Here are some general rules for finding the main operator in a symbolized formula of propositional logic:

1. If a sentence has only one logical operator, then that is the main operator.
2. If a sentence has more than one logical operator, then the main operator is the one outside the parentheses.
3. If a sentence has two logical operators outside the parentheses, then the main operator is not the negation.

Here are some examples:

Formula	Main Operator	Sentence Type
P	None	Simple
$P \,\&\, Q$	&	Conjunction
$\neg P \,\&\, Q$	$\neg$	Negation
$P \vee (Q \longrightarrow R)$	$\vee$	Disjunction
$[(P \,\&\, \neg Q) \leftrightarrow R] \longrightarrow P$	$\longrightarrow$	Conditional

Bibliography

Cleese, John, and Graham Chapman. 1980. "The Argument Clinic." In *Monty Python Live at the Hollywood Bowl.* Columbia Pictures.