Pursuing Truth: A Guide to Critical Thinking

Randy Ridenour

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Preface

This is a textbook written primarily for my students in PHIL 1502: Critical Thinking, at Oklahoma Baptist University in Shawnee, Oklahoma.

There are many good textbooks for critical thinking on the market today, so why write another one? First, all of those books were written for someone else's course. None cover all of the topics that I would like to cover in class. Second, as I'm sure any student can attest to, textbooks are remarkably expensive, to the point that most of the world's population cannot afford access to good learning material. I'm beginning to think of access to information as a moral issue. Now, if I could become wealthy by publishing a book on critical thinking, I might be willing to put the ethical considerations aside.[1] Since all profit would likely go to the publisher, however, I might as well just give the book away.

The book is written in R Markdown with the R bookdown package, and published on R Studio Connect. There is a link at the top of the page to download a PDF version of the text. If you do that, keep in mind that there will be formatting errors.

Warning: if you are looking for the perfect book on critical thinking, this will not be it. For various reasons, this text is likely to always be a work in progress. I can promise, though, that it will be worth as least as much as you paid for it.

[1] As a wise man once said, every person has to decide for themselves what level of hypocrisy they're willing to live with.

Chapter 1

Introduction

1.1 Welcome

We human beings find it very difficult to completely clear our minds. That means you have been thinking of something nearly every waking moment since you began to think. If we assume eight hours of sleep every night, then that comes to just over 7,000,000 minutes of thinking in 20 years. Surely, after that much time spent doing something, you ought to have become pretty good at it. So, why should you consider reading a book or taking a course that claims to teach you how to do something you've been doing for years?

Well, as the old saying goes, I have some good news and some bad news. The bad news is that we're just not very good at thinking carefully. Some things are easy enough for us – you probably don't have a problem when it comes to deciding whether you should step out in front of a truck. On the other hand, when it comes to difficult, tricky subjects, we're often more likely to come up with wrong answers as right ones.

For example, consider this problem:

> *A ball and bat together cost $1.10, and the bat costs $1.00 more than the ball does. How much does the ball cost?*

I'll give you some time to think about your answer, although you shouldn't need much time. So, whenever you're ready. . . .

Was your answer ten cents? That's the most common answer, but it's also clearly wrong. If the ball costs ten cents, and the bat costs a dollar more than the ball, then the bat costs $1.10 and the total would be $1.20. The right answer has to be five cents: $1.05 + $.05 = $1.10. Even though it's not a difficult problem, most people get it wrong.

On the other hand, maybe we get it wrong *because* it's not a difficult problem. It looks so simple that we answer it without thinking about it. When we don't reason carefully about a problem, our minds provide us with an automatic answer. In some situations, the automatic answer provided by the mind is very likely to be true. In others, it is very likely to be false.

At this point, you might be asking yourself, "So what?" What's the worst that could happen, maybe getting a nickel extra in change when you buy the ball? This still doesn't justify taking a whole course to learn how to think better, does it?

Consider one more example:

> 1% of women at age forty who participate in routine screening have breast cancer. 80% of women with breast cancer will get positive mammographies. 9.6% of women without breast cancer will also get positive mammographies. A woman in this age group had a positive mammography in a routine screening. What is the probability that she actually has breast cancer?

Unlike the ball and the bat, being wrong in this case could have drastic consequences—if the doctor guessed too low, then the patient likely did not receive the treatment she needed. If the doctor guessed too high, then the patient may received radical treatments that she didn't need, unnecessary radiation, chemotherapy, or even a radical mastectomy.

Doctors have to make these diagnoses all the time, so surely they would be good at correctly estimating the patient's likelihood of having the disease. Most doctors estimated that, in this problem, the patient's chances of having breast cancer are somewhere between 70 and 80%. Only 15% of doctors surveyed were correct, however. Surprisingly, the right answer is 7.8%, a mere one-tenth of the estimates by the medical professionals.[1]

[1] That doesn't mean that the test should be ignored. It just means that the doctors should not immediately begin dangerous treatments. What is warranted is further testing

So, how does one avoid making such mistakes? The best way is to become a better critical thinker. You've taken the right initial steps by reading this book and taking this class.

1.2 What is Critical Thinking?

There are probably as many definitions of critical thinking as there books and articles on the subject. Here is a quick working definition that suit our purposes:

Critical Thinking Thinking clearly and carefully about what to believe or do in a way that is likely to produce a true belief or right action, if there is one.

There are a few things to note about this definition. First, critical thinking is practical. It is designed to produce a particular outcome, either a belief or an action. The goal is to gain true beliefs while avoiding false beliefs, or to do right actions and avoid doing wrong ones. At this point, we don't need to rehash old disputes about the nature of truth or morality, our ordinary understanding of the two concepts will be fine.

Second, there is nothing that we can do that guarantees a true belief—at best, we only get likelihood. Nevertheless, when we use the tools of critical thinking, we will be more likely to get to the truth than had we not used those tools.

Third, it is important to note that not every question has an answer that we can know just by thinking carefully about the problem. There are some questions that have right answers, but just cannot be known by us. How many life-supporting planets are there in the universe? We know there is at least one, but we don't yet have the ability to if there are any others. There are other questions for which there is no right answer, at least not in the objective sense. What kind of ice cream is best? You may have your preference, and I might have a different view. Is either of us wrong? Don't hold up the line in the ice cream shop telling yourself, "I know I like chocolate better than vanilla, but which one is *really* the best?" There is no best in this case, so order whatever you want.

to lessen the chances of a false positive.

This is a classic case of what philosophers call purely subjective. A subjective truth is one that is dependent on what a person prefers, thinks, believes, etc. Objective truths are true independently of what anyone thinks, believes, perceives, etc. Critical thinking won't help us answer the subjective questions, but we don't really have problems with those. In those cases, it's good enough just to report how we feel, since that is what makes those subjective beliefs true. Critical thinking, however, will help us decide if a question is objective or subjective, and if objective, if it can be answered.

1.3 The Tools of Critical Thinking

It's not enough to tell someone to think clearly and carefully—we have to know what clear and careful thinking, and clear and careful thinkers, look like. Critical thinking is a skill, and like many other skills, it involves the skilled use of tools. One set of tools will be no surprise; they are the tools of logic. Good critical thinkers can

- Identify arguments in propositional and categorical logic
- Evaluate arguments using truth tables and Venn diagrams
- Use the basic rules of probability, and
- Identify common logical fallacies.

Another set of tools has been provided by cognitive psychologists. Critical thinkers need to understand how the human mind works, especially the systematic ways that the mind is misled. So, critical thinkers must

- Understand common cognitive biases,
- Be aware of the ways that people try to mislead us,
- Know the situations in which we tend to reason badly.

Finally, I think that it's not enough that critical thinkers understand logic. It's not even enough to understand logic and cognitive psychology. I think there is a moral, or value component to critical thinking as well. To become a critical thinker is to become a certain kind of person, a person of intellectual virtue. So, we will discuss the importance and roles of such virtues as

- Open-mindedness
- Intellectual courage
- Intellectual humility

- Attentiveness
- Fairness
- Perseverance
- Firmness

So, by the end of this book, and by the end of your course, I hope that you are well on the road to acquiring these skills. Like any other skills, they cannot be acquired without practice. You will not become a perfect critical thinker in a semester, maybe not even over the course of a lifetime. You can, however, take some significant steps on a journey that leads to one of the most important destinations ever: the truth.

Part I

Logic

Chapter 2

Arguments

The fundamental tool of the critical thinker is the argument. For a good example of what we are not talking about, consider a bit from a famous sketch by *Monty Python's Flying Circus*:[1]

Man: (Knock)
Mr. Vibrating: Come in.
Man: Ah, Is this the right room for an argument?
Mr. Vibrating: I told you once.
Man: No you haven't.
Mr. Vibrating: Yes I have.
Man: When?
Mr. Vibrating: Just now.
Man: No you didn't.
Mr. Vibrating: Yes I did.
Man: You didn't!
Mr. Vibrating: I did!
Man: You didn't!
Mr. Vibrating: I'm telling you I did!
Man: You did not!!
Mr. Vibrating: Oh, I'm sorry, just one moment. Is this a five minute argument or the full half hour?

[1] (**?**).

2.1 Identifying Arguments

People often use "argument" to refer to a dispute or quarrel between people. In critical thinking, an argument is defined as

Argument A set of statements, one of which is the conclusion and the others are the premises.

There are three important things to remember here:

1. Arguments contain statements.
2. They have a conclusion.
3. They have at least one premise

Arguments contain statements, or declarative sentences. Statements, unlike questions or commands, have a truth value. Statements assert that the world is a particular way; questions do not. For example, if someone asked you what you did after dinner yesterday evening, you wouldn't accuse them of lying. When the world is the way that the statement says that it is, we say that the statement is true. If the statement is not true, it is false.

One of the statements in the argument is called the conclusion. The conclusion is the statement that is intended to be proved. Consider the following argument:

> Calculus II will be no harder than Calculus I. Susan did well in Calculus I. So, Susan should do well in Calculus II.

Here the conclusion is that Susan should do well in Calculus II. The other two sentences are premises. Premises are the reasons offered for believing that the conclusion is true.

2.1.1 Standard Form

Now, to make the argument easier to evaluate, we will put it into what is called "standard form." To put an argument in standard form, write each premise on a separate, numbered line. Draw a line underneath the last premise, the write the conclusion underneath the line.

1. Calculus II will be no harder than Calculus I.
2. Susan did well in Calculus I.
3. Susan should do well in Calculus II.

Now that we have the argument in standard form, we can talk about premise 1, premise 2, and all clearly be referring to the same thing.

2.1.2 Indicator Words

Unfortunately, when people present arguments, they rarely put them in standard form. So, we have to decide which statement is intended to be the conclusion, and which are the premises. Don't make the mistake of assuming that the conclusion comes at the end. The conclusion is often at the beginning of the passage, but could even be in the middle. A better way to identify premises and conclusions is to look for indicator words. Indicator words are words that signal that statement following the indicator is a premise or conclusion. The example above used a common indicator word for a conclusion, 'so.' The other common conclusion indicator, as you can probably guess, is 'therefore.' Table

$$table : indwords$$

lists the indicator words you might encounter.

Conclusion	*Premise*
Therefore	Since
So	Because
Thus	For
Hence	Is implied by
Consequently	For the reason that
Implies that	
It follows that	

Each argument will likely use only one indicator word or phrase. When the conlusion is at the end, it will generally be preceded by a conclusion indicator. Everything else, then, is a premise. When the conclusion comes at the beginning, the next sentence will usually be introduced by a premise indicator. All of the following sentences will also be premises.

For example, here's our previous argument rewritten to use a premise indicator:

> Susan should do well in Calculus II, because Calculus II will be

no harder than Calculus I, and Susan did well in Calculus I.

Sometimes, an argument will contain no indicator words at all. In that case, the best thing to do is to determine which of the premises would logically follow from the others. If there is one, then it is the conclusion. Here is an example:

Spot is a mammal. All dogs are mammals, and Spot is a dog.

The first sentence logically follows from the others, so it is the conclusion. When using this method, we are forced to assume that the person giving the argument is rational and logical, which might not be true.

2.1.2.1 Non-Arguments

One thing that complicates our task of identifying arguments is that there are many passages that, although they look like arguments, are not arguments. The most common types are:

1. Explanations
2. Mere asssertions
3. Conditional statements
4. Loosely connected statements

Explanations can be tricky, because they often use one of our indicator words. Consider this passage:

Abraham Lincoln died because he was shot.

If this were an argument, then the conclusion would be that Abraham Lincoln died, since the other statement is introduced by a premise indicator. If this is an argument, though, it's a strange one. Do you really think that someone would be trying to prove that Abraham Lincoln died? Surely everyone knows that he is dead. On the other hand, there might be people who don't know how he died. This passage does not attempt to prove that something is true, but instead attempts to explain why it is true. To determine if a passage is an explanation or an argument, first find the statement that looks like the conclusion. Next, ask yourself if everyone likely already believes that statement to be true. If the answer to that question is yes, then the passage is an explanation.

Mere assertions are obviously not arguments. If a professor tells you simply that you will not get an A in her course this semester, she has not given you an argument. This is because she hasn't given you any reasons to believe that the statement is true. If there are no premises, then there is no argument.

Conditional statements are sentences that have the form "If. . . , then. . . ." A conditional statement asserts that *if* something is true, then something else would be true also. For example, imagine you are told, "If you have the winning lottery ticket, then you will win ten million dollars." What is being claimed to be true, that you have the winning lottery ticket, or that you will win ten million dollars? Neither. The only thing claimed is the entire conditional. Conditionals can be premises, and they can be conclusions. They can be parts of arguments, but that cannot, on their own, be arguments themselves.

Finally, consider this passage:

> I woke up this morning, then took a shower and got dressed. After breakfast, I worked on chapter 2 of the critical thinking text. I then took a break and drank some more coffee. . . .

This might be a description of my day, but it's not an argument. There's nothing in the passage that plays the role of a premise or a conclusion. The passage doesn't attempt to prove anything. Remember that arguments need a conclusion, there must be something that is the statement to be proved. Lacking that, it simply isn't an argument, no matter how much it looks like one.

2.1.3 Evaluating Arguments

The first step in evaluating an argument is to determine what kind of argument it is. We initially categorize arguments as either deductive or inductive, defined roughly in terms of their goals. In deductive arguments, the truth of the premises is intended to absolutely establish the truth of the conclusion. For inductive arguments, the truth of the premises is only intended to establish the probable truth of the conclusion. We'll focus on deductive arguments first, then examine inductive arguments in later chapters.

Once we have established that an argument is deductive, we then ask if it is valid. To say that an argument is valid is to claim that there is a very special logical relationship between the premises and the conclusion, such that if the premises are true, then the conclusion must also be true. Another way to state this is

Valid An argument is valid if and only if it is impossible for the premises to be true and the conclusion false.

Invalid An argument is invalid if and only if it is not valid.

Note that claiming that an argument is valid is not the same as claiming that it has a true conclusion, nor is it to claim that the argument has true premises. Claiming that an argument is valid is claiming nothing more that the premises, *if they were true*, would be enough to make the conclusion true. For example, is the following argument valid or not?

1. If pigs fly, then an increase in the minimum wage will be approved next term.
2. Pigs fly.
3. An increase in the minimum wage will be approved next term.

The argument is indeed valid. If the two premises were true, then the conclusion would have to be true also. What about this argument?

1. All dogs are mammals
2. Spot is a mammal.
3. Spot is a dog.

In this case, both of the premises are true and the conclusion is true. The question to ask, though, is whether the premises absolutely guarantee that the conclusion is true. The answer here is no. The two premises could be true and the conclusion false if Spot were a cat, whale, etc.

Neither of these arguments are good. The second fails because it is invalid. The two premises don't prove that the conclusion is true. The first argument is valid, however. So, the premises would prove that the conclusion is true, *if those premises were themselves true.* Unfortunately, (or fortunately, I guess, considering what would be dropping from the sky) pigs don't fly.

These examples give us two important ways that deductive arguments can fail. The can fail because they are invalid, or because they have at least one false premise. Of course, these are not mutually exclusive, an argument can be both invalid and have a false premise.

If the argument is valid, and has all true premises, then it is a sound argument. Sound arguments always have true conclusions.

Sound A deductively valid argument with all true premises.

Inductive arguments are never valid, since the premises only establish the probable truth of the conclusion. So, we evaluate inductive arguments according to their strength. A strong inductive argument is one in which the truth of the premises really do make the conclusion probably true. An argument is weak if the truth of the premises fail to establish the probable truth of the conclusion.

There is a significant difference between valid/invalid and strong/weak. If an argument is not valid, then it is invalid. The two categories are mutually exclusive and exhaustive. There can be no such thing as an argument being more valid than another valid argument. Validity is all or nothing. Inductive strength, however, is on a continuum. A strong inductive argument can be made stronger with the addition of another premise. More evidence can raise the probability of the conclusion. A valid argument cannot be made more valid with an additional premise. Why not? If the argument is valid, then the premises were enough to absolutely guarantee the truth of the conclusion. Adding another premise won't give any more guarantee of truth than was already there. If it could, then the guarantee wasn't absolute before, and the original argument wasn't valid in the first place.

2.1.4 Counterexamples

One way to prove an argument to be invalid is to use a counterexample. A counterexample is a consistent story in which the premises are true and the conclusion false. Consider the argument above:

1. All dogs are mammals
2. Spot is a mammal.
3. Spot is a dog.

By pointing out that Spot could have been a cat, I have told a story in which the premises are true, but the conclusion is false.

Here's another one:

1. If it is raining, then the sidewalks are wet.

2. The sidewalks are wet.
3. It is raining.

The sprinklers might have been on. If so, then the sidewalks would be wet, even if it weren't raining.

Counterexamples can be very useful for demonstrating invalidity. Keep in mind, though, that validity can never be proved with the counterexample method. If the argument is valid, then it will be impossible to give a counterexample to it. If you can't come up with a counterexample, however, that does not prove the argument to be valid. It may only mean that you're not creative enough.

2.2 Review

1. An **argument** is a set of statements; one is the conclusion, the rest are premises.
2. The **conclusion** is the statement that the argument is trying to prove.
3. The **premises** are the reasons offered for believing the conclusion to be true.
4. Explanations, conditional sentences, and mere assertions are not arguments.
5. **Deductive** reasoning attempts to absolutely guarantee the truth of the conclusion.
6. **Inductive** reasoning attempts to show that the conclusion is probably true.
7. In a **valid** argument, it is impossible for the premises to be true and the conclusion false.
8. In an **invalid** argument, it is possible for the premises to be true and the conclusion false.
9. A **sound** argument is valid and has all true premises.
10. An inductively **strong** argument is one in which the truth of the premises makes the the truth of the conclusion probable.
11. An inductively weak argument is one in which the truth of the premises do not make the conclusion probably true.
12. A **counterexample** is a consistent story in which the premises of an argument are true and the conclusion is false. Counterexamples can be used to prove that arguments are deductively invalid.

2.3 Exercises

Chapter 3

Categorical Logic

Now we turn to some structured logic systems. The first, categorical logic, is one of the oldest. It dates back at least to Aristotle (384–322 BCE). Categorical logic is a fairly simple logic of categories or classes. A class is a group of things that we designate with a common noun: students, teachers, dogs, politicians, etc. Each sentence will use two different classes. One is the subject class, and the other is the predicate class. In this logic, we can say something about all members of a class, called a universal sentence, or we can say something about some members of a class, called a particular sentence. We can also make a positive claim, called an affirmation, or we can make a negative claim, called a negation.

With these two distinctions, universal/particular and affirnation/negation, we can make four kinds of sentences. S and P stand for the subject class and the predicate class, respectively.

A: All S are P (universal affirmation)
E: No S are P (universal negation)
I: Some S are P (particular affirmation)
O: Some S are not P (particular negation)[1]

Here are some examples of categorical statements, some true and some false.

1. All dogs are mammals.

[1]The letters A, E, I, and O, are thought to come from the first two vowels of the Latin words *affirmo* and *nego*, meaning “I affirm” and “I deny.”

2. All mammals are dogs.
3. No reptiles are dogs.
4. No politicians are honest people.
5. Some politicians are honest people.
6. Some cats are amphibians.
7. Some dogs are not beagles.
8. Some beagles are not dogs.

Look at the sentences carefully. You should be able to tell that the odd-numbered ones are true and the even-numbered ones are false.

3.1 The Square of Opposition

We can visualize interesting logical relationships between these four types of sentences on something called "The Square of Opposition."

The first step is to place the sentence types in the corners of an imaginary square. A is at the upper left; E, the upper right; I, the lower left, and O, the lower right. Next, draw arrows on the diagonals, pointing to the sentences in the corners. Then, draw an arrow between the two at the top, and another one between the two at the bottom. Finally, draw an arrow on each side, going from top to bottom. When finished, you should have something like this:

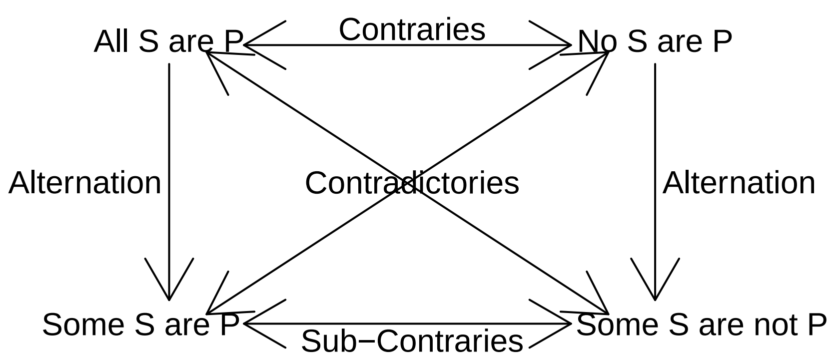

Figure 3.1: Square of Opposition

The next step is to note the relationship between the diagonals. The diagonals are contradictories, meaning they always have opposite truth values. They can't both be true, and they also can't both be false. If the A sentence is true, the O sentence must be false—if it is true that all dogs are mammals, it cannot be true that some dogs are not mammals. If the O sentence is true, then the A sentence must be false. It is the same for the E and the I.

Next, note the relationship between the A sentences and the E sentences, called contraries. Like the contradictories, they cannot both be true. Unlike the contradictories, they can both be false. If it's true that all critical thinking students are good students, then it must be false that no critical thinking students are good students. If it's false that all critical thinking students are good students, then it can be false that critical thinking students are good students. In fact, they are both false, because some critical thinking students are good and others are not.

At the bottom, we have sub-contraries. They can both be false, but cannot both be true.

Finally, we have the relationship between the top level sentences and the bottom level sentences on the same side. This is called alternation. The universal is called the superaltern and the particular is called the subaltern. If the superaltern is true, then the subaltern must also be true. If the superaltern is false, then the subaltern can be either true or false. If the subaltern is false, then the superaltern must be false. If the subaltern is true, then the superaltern can be either true or false. It is easy to remember this way: truth goes down, falsity goes up.

3.2 Diagramming Sentences

We diagram sentences and arguments in categorical logic using Venn diagrams. You've probably used these in a math class at some time. Before we can use these to evaluate arguments in categorical logic, we first have to learn how to diagram individual sentences.

The first step is to draw two interlocking circles. Label the left circle with an "S" and the right circle with "P"—standing for the subject term and predicate term, respectively.

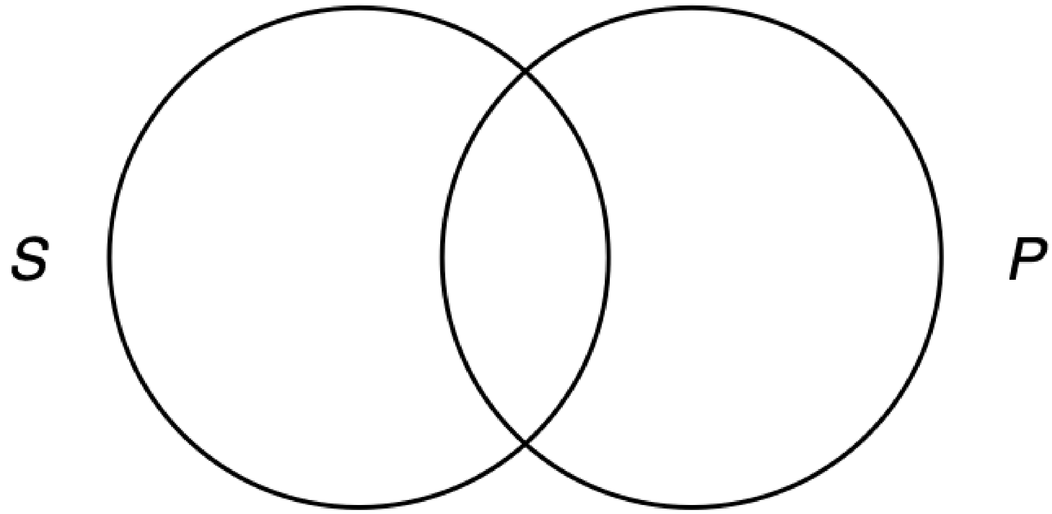

Figure 3.2: Starting a venn diagram

3.2.1 A-Sentences

Remember that the A-sentence has the form All S are P. That means that everything that is in the S circle must also be in the P circle. To diagram this, we shade the region of the S circle that is not contained in the P circle. If a region is shaded, that means that nothing is in that region.

3.2.2 E-Sentences

To shade the universal negation, we shade the region that is shared by both S and P:

3.2.3 I-Sentences

To diagram a particular affirmation, we place an x in the region shared by S and P:

3.2.4 O-Sentences

Finally, to diagram an O-sentence, we place an x in S, but not in P:

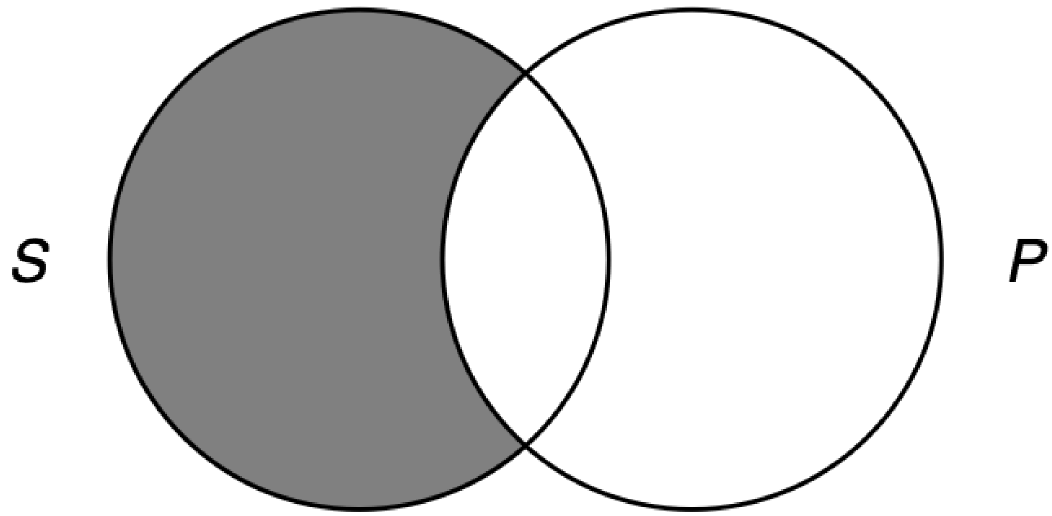

Figure 3.3: Diagramming an A-Sentence

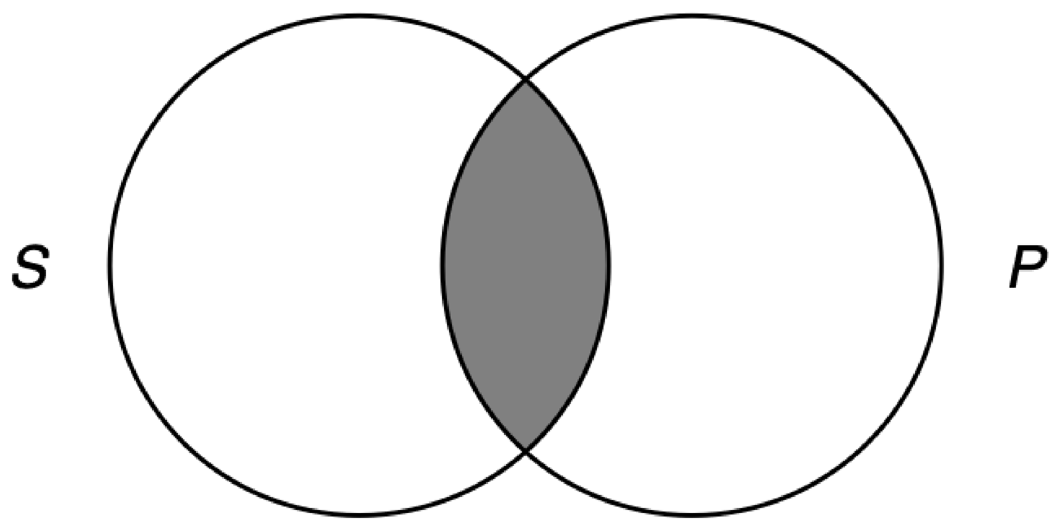

Figure 3.4: Diagramming an E-Sentence

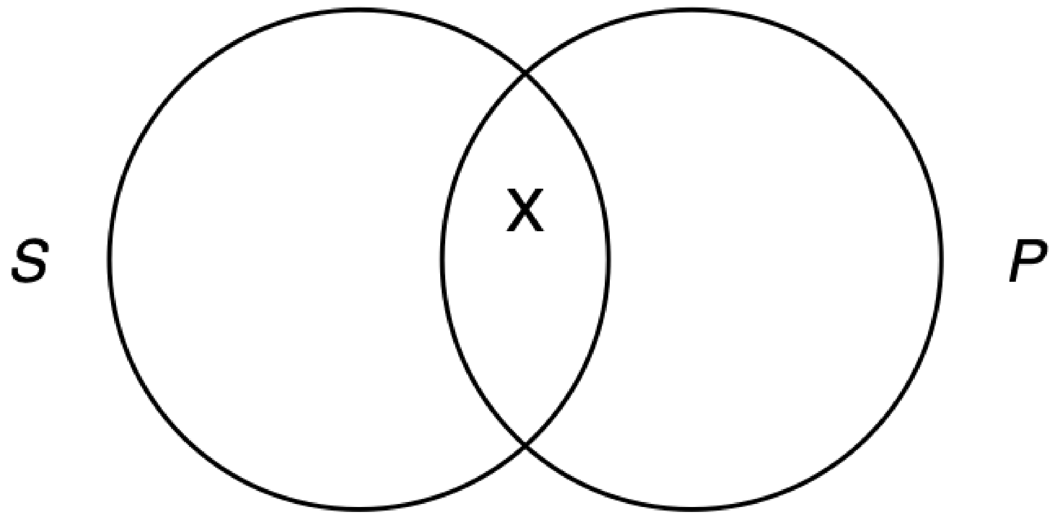

Figure 3.5: Diagramming an I-Sentence

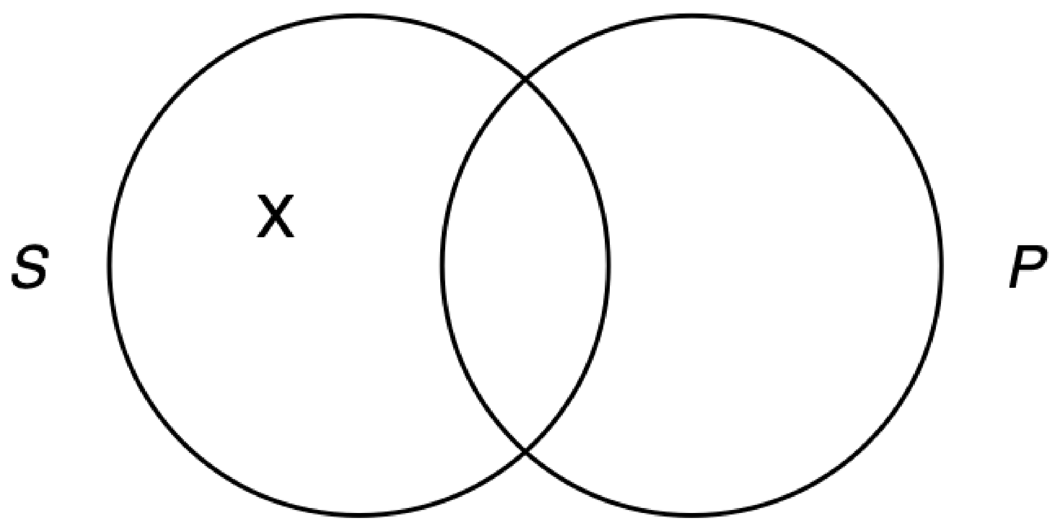

Figure 3.6: Diagramming an O-Sentence

3.3 Relations of Equivalence

There are three operations that can be applied to categorical sentences: conversion, obversion, and contraposition. It is important to know how to apply them and in what cases does operation result in an equivalent sentence. We're particularly interested in the conditions those different operations are *truth-preserving*. The operation is truth preserving when, applied to a true sentence, always results in a true sentence.

3.3.1 Conversion

Conversion is the simplest of the three. The converse of a sentence simply exchanges the subject and predicate terms of the original sentence. Conversion applied to A-sentences is *not* truth-preserving. "All dogs are mammals" is true, but "All mammals are dogs" is not. Conversion is truth-preserving for E-sentences and I-sentences. If it is true that no dogs are reptiles, it must be true that no reptiles are dots. Likewise, if it is true that some dogs are brown things, it must be true that some brown things are dogs.

3.3.2 Obversion

Take another look at the square of opposition in figure 4.1. Note that the A and the E are straight across from each other, as are the I and the O. The first step in forming the obverse is to first change the sentence into the type that is straight across the square of opposition. That is, if you started with an A-sentence, then make it into and E. The O becomes and I, and so on.

Once you've changed the sentence type, the next step is to change predicate into its complement. The complement of a class C is the class of everything that is not in C. The easiest way to form a complement is to prefix the class with 'non'. For example, the complement of the class of students is the class of non-students.

So, the obverse of all dogs are mammals is no dogs are non-mammals. The obverse of no OBU students are martians is all OBU students are non-martians. Obversion is truth-preserving in all cases.

3.3.3 Contraposition

The last of our three relations is contraposition. To form the contrapositive of a sentence, first form the converse, then exchange both terms for their complements.

The contrapositive of all dogs are mammals is all non-mammals are non-dogs. Contraposition is truth-preserving for A-sentences and O-sentences only.

Here's a table to help keep this straight (operations that are truth-preserving are in bold type):

Original	Converse	Obverse	Contrapositive
All S are P	All P are S	**No S are non-P**	**All non-P are non-S**
No S are P	**No P are S**	**All S are non-P**	No non-P are non-S
Some S are P	**Some P are S**	**Some S are not non-P**	Some non-P are non-S
Some S are not P	Some P are not S	**Some S are non-P**	**Some non-P are not no**

3.4 Evaluating Categorical Syllogisms

A syllogism is an argument that has two premises and a conclusion. A categorical syllogism is a syllogism that contains only categorical sentences. Here is an example:

1. All Dogs are mammals.
2. All mammals are animals.
3. All dogs are animals

Both premises and the conclusion are A-sentences. Notice that we have three terms in the argument: dogs, mammals, and animals. Every categorical syllogism, in proper form, has three terms. Each term occurs in two sentences. Two of those terms will be found in the conclusion, and one term is only in the premises. The predicate term of the conclusion is called the major term. The subject of the conclusion is called the minor term. The term that is not in the conclusion is called the middle term.

There are two ways to determine if a categorical syllogism is valid. One way uses Venn diagrams, and the other involves applying some simple rules.

3.4.1 Diagram Method

Since we have three terms in the argument, we'll need three intersecting circles. We'll start by drawing two circles for the conclusion, just as we did before. Then, in the middle and below, we'll draw another circle for the middle term. For labels, use letters that correspond to the classes in the argument. Here, we'll use D for dogs, M for mammals, and A for animals.

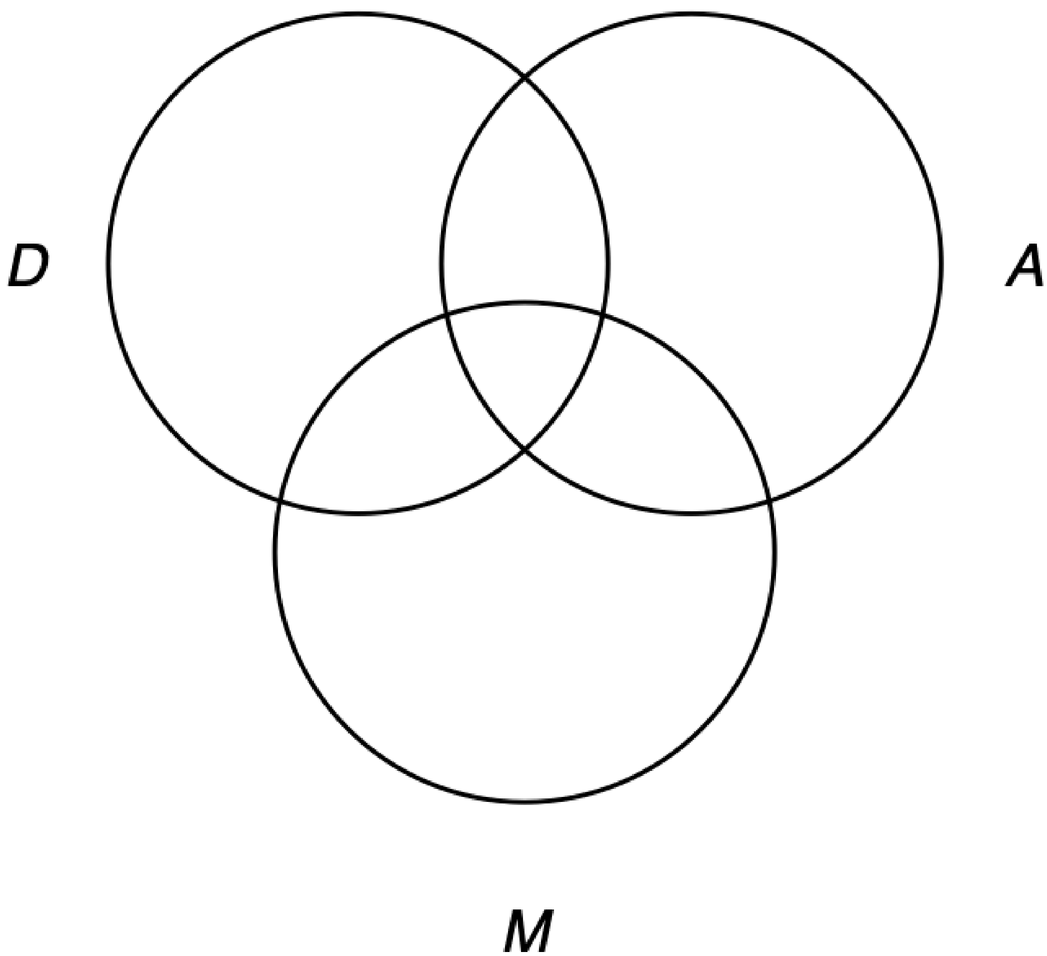

Figure 3.7: Diagramming a categorical syllogism, step 1

Next, we finish diagramming the premises by shading or placing an x. Since our first premise is "All dogs are mammals,'' we need to shade everything in the D circle that is not in the M circle.

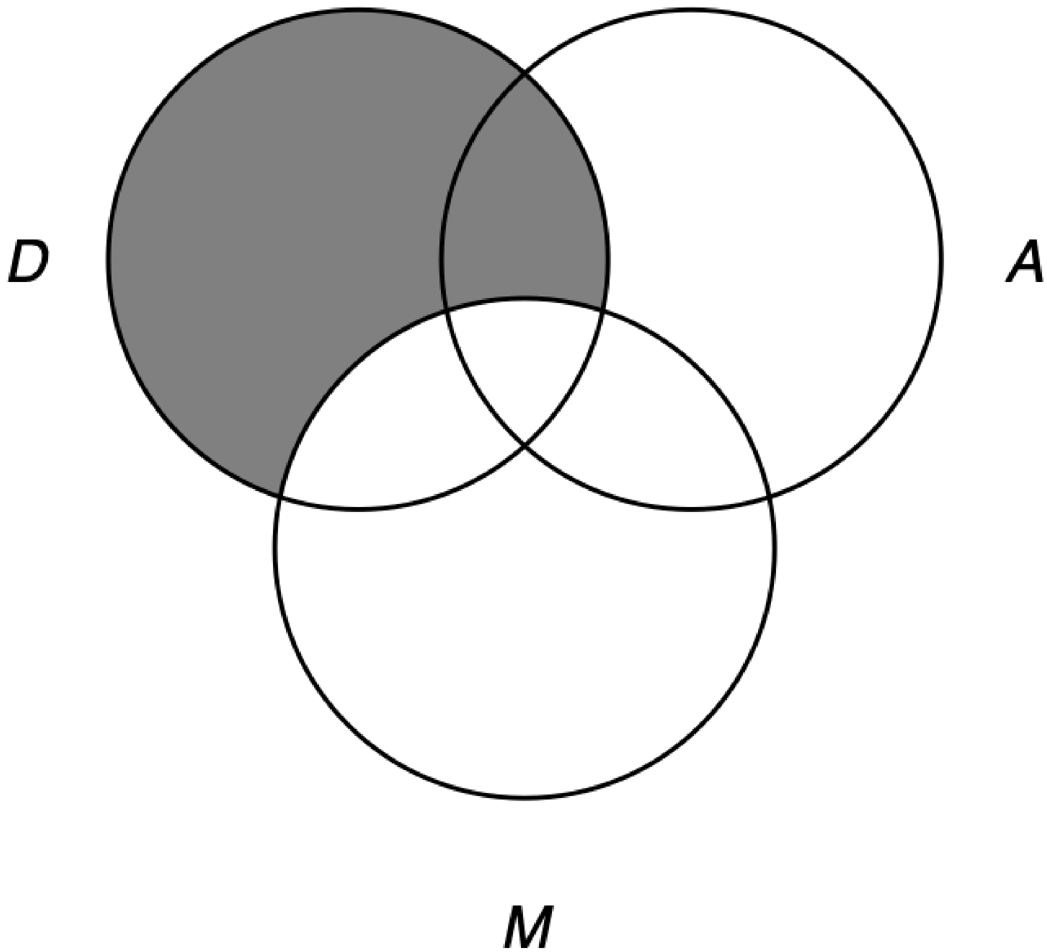

Figure 3.8: Diagramming a categorical syllogism, step 2

Next, we diagram the second premise by shading everything that is in the M circle but not in the A circle.

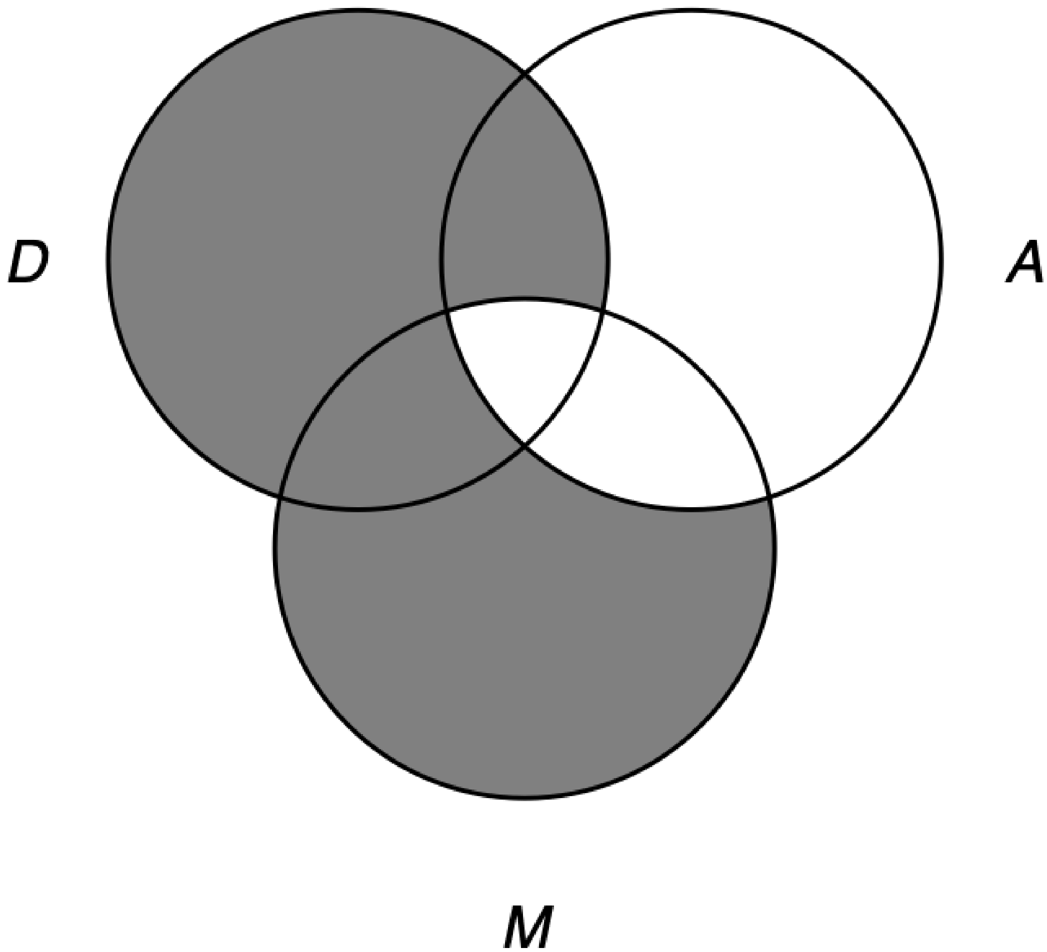

Figure 3.9: Diagramming a categorical syllogism, step 3

If there is any circle that has only one region left unshaded, you can place an 'X' in that region. This is because categorical logic assumes that there are no empty categories, meaning that every category has at least one thing in it. This is really only important for arguments that have an I or an O-sentence for a conclusion. In this case, we won't worry about it. Now that the premises are diagrammed, check to see if the conclusion has also been diagrammed. If so, then the argument is valid. This shows that making the premises true was enough to make the conclusion true also.

Let's try to diagram this argument:

1. No introverts are politicians
2. All artists are introverts
3. No artists are politicians

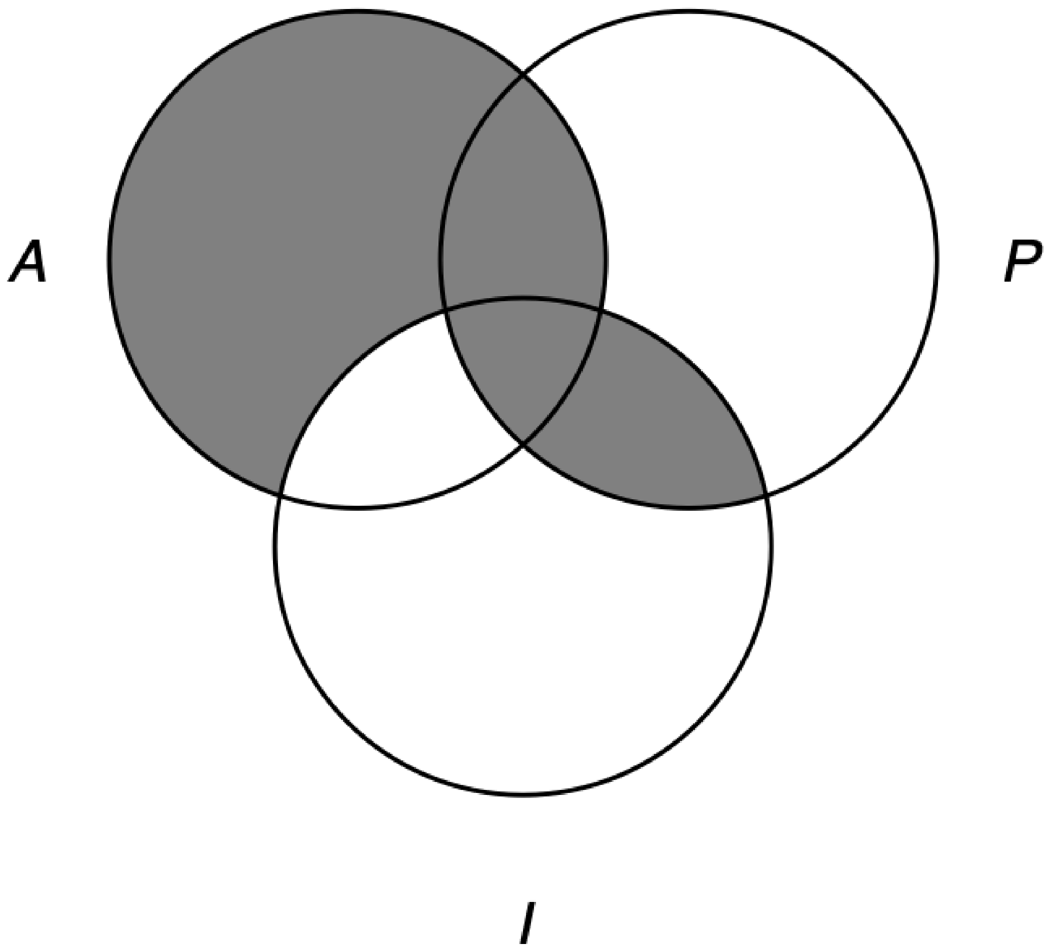

Figure 3.10: Celarent

1. Some horses are things that weigh over 2,000 pounds.
2. All horses are mammals.
3. Some mammals are things that weigh over 2,000 pounds.

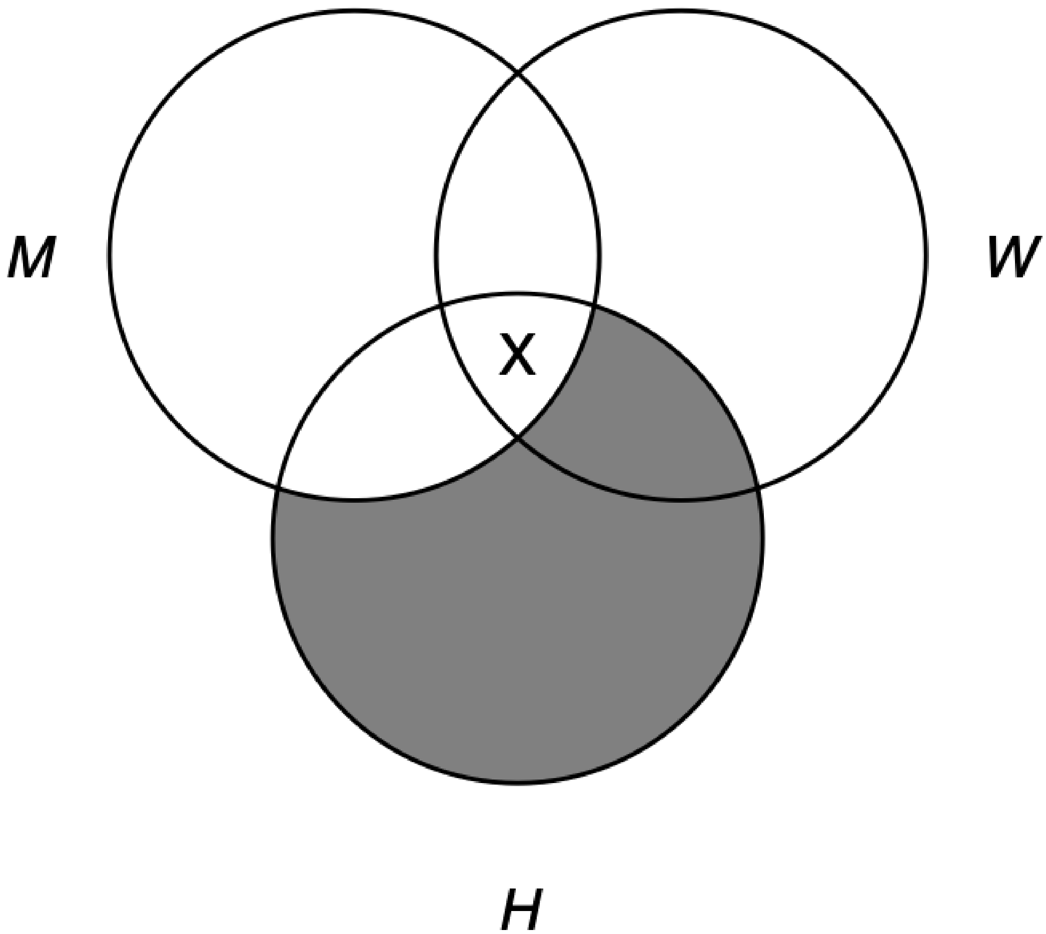

Figure 3.11: Disamis

3.4.2 Hints for Diagramming Categorical Syllogisms

1. Diagram universals before particulars (shade before making an x.)
2. If it is not clear where the x goes, then put it on the line.

3.5 Rules for Categorical Syllogisms

There is another way to determine validity for categorical syllogisms. Every valid syllogism must meet three conditions:

1. There must be the same number of negations in the conclusion as in the premises.
2. The middle term must be distributed at least once.
3. Any term distributed in the conclusion must be distributed in the premises.

Before these rules can be applied, we'll have to explain what distribution is. Every categorical statement says something about a category or class. A statement distributes a term just in case what it says about that class is true of every subset of the class. To remember when something is distributed, keep this in mind:

1. Universals distribute subjects, and
2. Negations distribute predicates.

So, A-sentences distribute the subject, E-sentences distribute both terms, I-sentences don't distribute anything, and O sentences distribute the predicate.

Chapter 4

Propositional Logic

Categorical logic is a great way to analyze arguments, but only certain kinds of arguments. It is limited to arguments that have only two premises and the four kinds of categorical sentences. This means that certain common arguments that are obviously valid will not even be well-formed arguments in categorical logic. Here is an example:

1. I will either go out for dinner tonight or go out for breakfast tomorrow.
2. I won't go out for dinner tonight.
3. I will go out for breakfast tomorrow.

None of these sentences fit any of the four categorical schemes. So, we need a new logic, called propositional logic. The good news is that it is fairly simple.

4.1 Simple and Complex Sentences

The fundamental logical unit in categorical logic was a category, or class of things. The fundamental logical unit in propositional logic is a statement, or proposition[1] Simple statements are statements that contain no other statement as a part. Here are some examples:

[1]Informally, we use 'proposition' and 'statement' interchangeably. Strictly speaking, the proposition is the content, or meaning, that the statement expresses. When different sentences in different languages mean the same thing, it is because they express the same proposition.

- Oklahoma Baptist University is in Shawnee, Oklahoma.
- Barack Obama was succeeded as President of the US by Donald Trump.
- It is 33 degrees outside.

Simple sentences are symbolized by uppercase letters. Just pick a letter that makes sense, given the sentence to be symbolized, that way you can more easily remember which letter means which sentence.

Complex sentences have at least one sentence as a component. There are five types in propositional logic:

- Negations
- Conjunctions
- Disjunctions
- Conditionals
- Biconditionals

4.1.1 Negations

Negations are "not" sentences. They assert that something is not the case. For example, the negation of the simple sentence "Oklahoma Baptist University is in Shawnee, Oklahoma" is "Oklahoma Baptist University is not in Shawnee, Oklahoma." In general, a simple way to form a negation is to just place the phrase "It is not the case that" before the sentence to be negated.

A negation is symbolized by placing this symbol '$\neg$' before the sentence-letter. The symbol looks like a dash with a little tail on its right side. If D = 'It is 33 degrees outside,' then $\neg$D = 'It is not 33 degrees outside.' The negation symbol is used to translate these English phrases:

- not
- it is not the case that
- it is not true that
- it is false that

A negation is true whenever the negated sentence is false. If it is true that it is not 33 degrees outside, then it must be false that it is 33 degrees outside. if it is false that Tulsa is the capital of Oklahoma, then it is true that Tulsa is not the capital of Oklahoma.

When translating, try to keep the simple sentences positive in meaning. Note the warning on page 24, about the example of affirming and denying. Denying is not simply the negation of affirming.

4.2 Conjunction

Negations are "and" sentences. They put two sentences, called conjuncts, together and claim that they are both true. We'll use the ampersand (&) to signify a negation. Other common symbols are a dot and an upside down wedge. The English words that are translated with the ampersand include:

- and
- but
- also
- however
- yet
- still
- moreover
- although
- nevertheless
- both

For example, we would translate the sentence 'It is raining today and my sunroof is open' as 'R&O'.

4.3 Disjunction

A disjunction is an "or" sentence. It claims that at least one of two sentences, called disjuncts, is true. For example, if I say that either I will go to the movies this weekend or I will stay home and grade critical thinking homework, then I have told the truth provided that I do one or both of those things. If I do neither, though, then my claim was false.

We use this symbol, called a "vel," for disjunctions: $\vee$. The vel is used to translate - or - either. . . or - unless

4.4 Conditional

The conditional is a common type of sentence. It claims that something is true, if something else is also. Examples of conditionals are

- "If Sarah makes an A on the final, then she will get an A for the course."
- "Your car will last many years, provided you perform the required maintenance."
- "You can light that match only if it is not wet."

We can translate those sentences with an arrow like this:

- $F \to C$
- $M \to L$
- $L \to \neg W$

The arrow translates many English words and phrases, including

- if
- if… then
- only if
- whenever
- when
- only when
- implies
- provided that
- means
- entails
- is a sufficient condition for
- is a necessary condition for
- given that
- on the condition that
- in case

One big difference between conditionals and other sentences, like conjunctions and disjunctions, is that order matters. Notice that there is no logical difference between the following two sentences:

- Albany is the capital of New York and Austin is the capital of Texas.
- Austin is the capital of Texas and Albany is the capital of New York.

They essentially assert exactly the same thing, that both of those conjuncts are true. So, changing order of the conjuncts or disjuncts does not change the meaning of the sentence, and if meaning doesn't change, then true value doesn't change.

That's not true of conditionals. Note the difference between these two sentences:

- If you drew a diamond, then you drew a red card.
- If you drew a red card, then you drew a diamond.

The first sentence *must* be true. if you drew a diamond, then that guarantees that it's a red card. The second sentence, though, could be false. Your drawing a red card doesn't guarantee that you drew a diamond, you could have drawn a heart instead. So, we need to be able to specify which sentence goes before the arrow and which sentence goes after. The sentence before the arrow is called the antecedent, and the sentence after the arrow is called the consequent.

Look at those three examples again:

1. "If Sarah makes an A on the final, then she will get an A for the course."
2. "Your car will last many years, provided you perform the required maintenance."
3. "You can light that match only if it is not wet."

The antecedent for the first sentence is "Sarah makes an A on the final." The consequent is "She will get an A for the course." Note that the `if` and the `then` are not parts of the antecedent and consequent.

In the second sentence, the antecdent is "You perform the required maintenance." The consequent is "Your car will last many years." This tells us that the antecedent won't always come first in the English sentence.

The third sentence is tricky. The antecedent is "You can light that match." Why? The explanation involves something called necessary and sufficient conditions.

4.4.1 Necessary and Sufficient Conditions

A sufficient condition is something that is enough to guarantee the truth of something else. For example, getting a 95 on an exam is sufficient for making an A, assuming that exam is worth 100 points. A necessary condition is something that must be true in order for something else to be true. Making a 95 on an exam is not necessary for making an A—a 94 would have still been an A. Taking the exam is necessary for making an A, though. You can't make an A if you don't take the exam, or, in other words, you can make an a only if you enroll in the course.

Here are some important rules to keep in mind:

- 'If' introduces antecedents, but `Only if` introduces consequents.
- If A is a sufficient condition for B, then $A \to B$.
- If A is a necessary condition for B, then $B \to A$.

4.5 Biconditional

We won't spend much time on biconditionals. There are times when something is both a necessary and a sufficient condition for something else. For example, making at least a 90 and getting an A (assuming a standard scale, no curve, and no rounding up). If you make at least a 90, then you will get an A. If you got an A, then you made at least a 90. We can use a double arrow to translate a biconditional, like this:

- $N \to A$

For biconditionals, as for conjunctions and disjunctions, order doesn't matter.

Here are some English phrases that signify biconditionals:

- it and only if
- when and only when
- just in case
- is a necessary and sufficient condition for

4.6 Translations

Propositional logic is language. Like other languages, it has a syntax and a semantics. The syntax of a language includes the basic symbols of the language plus rules for putting together proper statements in the language. To use propositional logic, we need to know how to translate English sentences into the language of propositional logic. We start with our sentence letters, which represent simple English sentences. Let's use three borrowed from an elementary school reader:

T: Tom hit the ball.
J: Jane caught the ball.
S: Spot chased the ball.

We then build complex sentences using the sentence letters and our five logical operators, like this:

English Sentence	PL Translation
Tom did not hit the ball	$\neg T$
Either Tom hit the ball or Jane caught the ball	$T \vee J$
Spot chased the ball, but Jane caught it.	$S \,\&\, J$
If Jane caught the ball, then Spot did not chase it.	$L \rightarrow \neg S$
Spot chased the ball if and only if Tom hit the ball.	$S \leftrightarrow T$

We can make even more complex sentences, but we will soon run into a problem. Consider this example:

$$T \,\&\, J \rightarrow S$$

We don't know this means. It could be either one of the following:

1. Tom hit the ball, and if Jane caught the ball, then Spot chased it.
2. If Tom hit the ball and Jane caught it, then Spot chased it.

The first sentence is a conjunction, T is the first conjunct and $M \rightarrow S$ is the second conjunct. The second sentence, though, is a conditional, $T \,\&\, M$ is the antecdent, and S is the consequent. Our two interpretations are not equivalent, so we need a way to clear up the ambiguity. We can do this with parentheses. Our first sentence becomes:

$$T \,\&\, (J \rightarrow S)$$

The second sentence is:

$$(T \,\&\, J) \to S$$

If we need higher level parentheses, we can use brackets and braces. For instance, this is a perfectly good formula in propositional logic:

$$[(P \,\&\, Q) \vee R] \to \{[(\neg P \leftrightarrow Q) \,\&\, S] \vee \neg P\}$$

[2]

Every sentence in propositional logic is one of six types:

1. Simple
2. Negation
3. Conjunction
4. Disjunction
5. Conditional
6. Biconditional

What type of sentence it is will be determined by its main logical operator. Sentences can have several logical operators, but they will always have one, and only one, main operator. Here are some general rules for finding the main operator in a symbolized formula of propositional logic:

1. If a sentence has only one logical operator, then that is the main operator.
2. If a sentence has more than one logical operator, then the main operator is the one outside the parentheses.
3. If a sentence has two logical operators outside the parentheses, then the main operator is not the negation.

Here are some examples:

Formula	Main Operator	Sentence Type
P	None	Simple
$\neg P \,\&\, Q$	&	Conjunction
$\neg(P \,\&\, Q)$	$\neg$	Negation
$P \vee (Q \to R)$	$\vee$	Disjunction

[2] It may be a good formula in propositional logic, but that doesn't mean it would be a good English sentence.

Formula	Main Operator	Sentence Type
$[(P \mathbin{\&} \neg Q) \leftrightarrow R] \rightarrow P$	$\rightarrow$	Conditional

Chapter 5

Truth Tables

Translations in propositional logic are only a means to an end. Our goal is to use the translated formulas to determine the validity of arguments. To do this, we will use a tool called a truth table. Basically, a truth table is a list of all the different combinations of truth values that a sentence, or set of sentences, can have.

5.1 Single Sentences

Before we can analyze arguments with truth tables, we need to know how to construct truth tables for single sentences. Let's begin with a truth table for the negation. First, write the formula to be analyzed at the top.

$$\neg P$$

To the left of the formula, list the simple sentence letters in alphabetical order. In this case, we only have one sentence letter.

$$P \quad \neg P$$

Now draw a horizontal line underneath all of that, and a vertical line separating the formula from the sentence letters, like this:

P	$\neg P$

The next step is to list all of the possible combinations of truth-values of the simple sentence letters. In this case, we only have one letter, and can be either true or false.

P	$\neg P$
T	
F	

Finally, fill in the truth values of the formula for each line, given the truth values of the simple sentences on that line. Since the negation just changes the truth value of the simple sentence, our truth table will look like this:

P	$\neg P$
T	F
F	T

Now, let's construct a truth table for a conjunction. Again, we'll write the formula at the top:

$$P \quad \& \quad Q$$

We'll then write the simple sentence letter to the left, and draw the lines.

P	Q	P & Q

Next, we need to write all the different possible combinations of truth values of those simple sentence letters. First, they could both be true.

P	Q	P & Q
T	T	

Then, P could be true and Q false.

P	Q	P & Q
T	T	
T	F	

For the next line, P could be false and Q true.

P	Q	P & Q
T	T	
T	F	
F	T	

Last, they could both be false.

P	Q	P	$\&$	Q
T	T			
T	F			
F	T			
F	F			

Now, we just fill in the rest. The conjunction is true when both conjuncts are true, and false otherwise, So, the completed truth table looks like this.

P	Q	P	$\&$	Q
T	T		T	
T	F		F	
F	T		F	
F	F		F	

Here is the truth table for the disjunction. Remember that disjunctions are true when at least one disjunct is true, and false otherwise. So, the disjunction is only false on the bottom line.

P	Q	P	$\vee$	Q
T	T		T	
T	F		T	
F	T		T	
F	F		F	

This is what the truth table for the conditional looks like. Conditionals are false whenever the antecedent is true and the conclusion is false, but they are true any other time.

P	Q	P	$\rightarrow$	Q
T	T		T	
T	F		F	
F	T		T	
F	F		T	

Finally, here is the truth table for the biconditional. Biconditionals are true whenever both sides have the same truth value. That will be the first line, where they are both true, and the last line, where they're both false.

P	Q	P	$\leftrightarrow$	Q
T	T		T	
T	F		F	
F	T		F	
F	F		T	

Let's do one that is slightly longer. Here's a truth table for $P \,\&\, (Q \vee R)$:

We'll go ahead and write the formula and sentence letters, and draw the lines.

P	Q	R	P	$\&$	$(Q$	$\vee$	$R)$

It gets more difficult to fill in the combinations of truth values for the sentence letters as the tables get larger. Done one entire row at a time, it's easy to miss a combination. The best way is to do it an entire column at a time. Start with the rightmost column, and alternate T's and F's.

P	Q	R	P	$\&$	$(Q$	$\vee$	$R)$
		T					
		F					
		T					
		F					
		T					
		F					
		T					
		F					

Then, move to the next column to the left. Here, alternate pairs of T's and pairs of F's.

P	Q	R	P	$\&$	$(Q$	$\vee$	$R)$
	T	T					
	T	F					
	F	T					
	F	F					
	T	T					
	T	F					
	F	T					
	F	F					

Maybe you can see the pattern now. We'll then move to the next column and put four T's and four F's.

P	Q	R	P	$\&$	$(Q$	$\vee$	$R)$
T	T	T					
T	T	F					
T	F	T					
T	F	F					
F	T	T					
F	T	F					
F	F	T					
F	F	F					

Notice that we have eight rows. If there were four different simple sentences, we'd have sixteen, thirty-two for five, and so on. The general formula is this: if there are n simple sentences, then there will be 2^n rows.

Next, we fill in the rest of the truth table. With longer tables, it can be easier to first copy the columns of the sentence letters, like this:

P	Q	R	P	$\&$	$(Q$	$\vee$	$R)$
T	T	T	T		T		T
T	T	F	T		T		F
T	F	T	T		F		T
T	F	F	T		F		F
F	T	T	F		T		T
F	T	F	F		T		F
F	F	T	F		F		T
F	F	F	F		F		F

Then, we start working inside the parentheses. Since it is a disjunction, it will be true whenever at least one of Q and R are true, and false when they are both false.

P	Q	R	P	$\&$	$(Q$	$\vee$	$R)$
T	T	T	T		T	T	T
T	T	F	T		T	T	F
T	F	T	T		F	T	T
T	F	F	T		F	F	F
F	T	T	F		T	T	T
F	T	F	F		T	T	F
F	F	T	F		F	T	T
F	F	F	F		F	F	F

Now, we can ignore the columns under Q and R. We're focused on P and the column under the disjunction symbol. To make it clear, I'll remove the others.

P	Q	R	P	$\&$	$(Q$	$\vee$	$R)$
T	T	T	T			T	
T	T	F	T			T	
T	F	T	T			T	
T	F	F	T			F	
F	T	T	F			T	
F	T	F	F			T	
F	F	T	F			T	
F	F	F	F			F	

Now, we complete the column for the conjunction. It's true whenever P and $Q \vee R$ are both true.

P	Q	R	P	$\&$	$(Q$	$\vee$	$R)$
T	T	T	T	T		T	
T	T	F	T	T		T	
T	F	T	T	T		T	
T	F	F	T	F		F	
F	T	T	F	F		T	
F	T	F	F	F		T	
F	F	T	F	F		T	
F	F	F	F	F		F	

Ultimately, the column I'm really interested is the one underneath the main connective. I'll make it bold to be clear. Our complete truth table with all the columns looks like this:

P	Q	R	P	$\&$	$(Q$	$\vee$	$R)$
T	T	T	T	**T**	T	T	T
T	T	F	T	**T**	T	T	F
T	F	T	T	**T**	F	T	T
T	F	F	T	**F**	F	F	F
F	T	T	F	**F**	T	T	T
F	T	F	F	**F**	T	T	F
F	F	T	F	**F**	F	T	T
F	F	F	F	**F**	F	F	F

Notice that the column of the main connective has a mixture of T's and F's. This is called a contingency. A contingent statement is true on some rows and false on others. Some sentences are true on all rows. They are called tautologies. Here is a simple example:

P	P	$\vee$	$\neg P$
T	T	**T**	F
F	F	**T**	T

If a sentence has an F on every row of the table, it is a contradiction.

P	P	$\&$	$\neg P$
T	T	**F**	F
F	F	**F**	T

Tautologies can't possibly be false, contradictions can't possibly be true, and contingencies can be true or false.

5.2 Logical Equivalence

It is sometimes useful to put a pair of sentences on the same truth table. If the columns under their main connectives are identical, then the sentences are *logically equivalent*. That means that they always have the same truth value.

Here is an example. The sentences are separated by a forward slash.

P	Q	$\neg$	$(P$	$\&$	$Q)$	/	$\neg P$	$\vee$	$\neg Q$
T	T	**F**	T	T	T		F	**F**	F
T	F	**T**	T	F	F		F	**T**	T
F	T	**T**	F	F	T		T	**T**	F
F	F	**T**	F	F	F		T	**T**	T

It should be no surprise that those sentences are equivalent. The first one essentially claims that it's not the case that P and Q are both true, and the second one claims that at least one of them is false. They are just two ways of saying the same thing.

5.3 Truth Tables and Validity

To evaluate an argument using a truth table, put the premises on a row separated by a single slash, followed by the conclusion, separated by two slashes.[1]

Here's a simple argument, called Modus Ponens:

[1] The only function of the slashes is to help visualize where one sentence ends and another begins.

1. P → Q
2. P
3. Q

We'll begin the truth table like this:

P	Q	$(P$	$\rightarrow$	$Q)$	$/$	P	$//$	Q

Then fill in our possible truth values for the simple sentences on the left.

P	Q	$(P$	$\rightarrow$	$Q)$	$/$	P	$//$	Q
T	T							
T	F							
F	T							
F	F							

We can easily fill in the columns for the second premise and the conclusion, since they involve just copying over the P and Q columns.

P	Q	$(P$	$\rightarrow$	$Q)$	$/$	P	$//$	Q
T	T					T		T
T	F					T		F
F	T					F		T
F	F					F		F

Finally, we'll fill in the column for the first premise. Remember that a conditional is false only when the antecdent is true and the consequent is false. So, the first premise is false on the second line, and true on the other lines.

P	Q	$(P$	$\to$	$Q)$	$/$	P	$//$	Q
T	T		T			T		T
T	F		F			T		F
F	T		T			F		T
F	F		T			F		F

Now, what tells us that the argument is valid? Remember that an argument is valid if it is impossible for the premises to be true and the conclusion to be false. So, we check to see if there is a row on the truth table that has all true premises and a false conclusion. If there is, then we know the argument is invalid. In this argument, the only row where all the premises are true is the line 1. On that line, however, the conclusion is also true. So, this argument is valid.

There's often no need to fill out the whole truth table to determine validity. Let's look at shortcut, using the same argument. I can see immediately that there is really only one row that I need to work. See if you can figure out which one it is.

P	Q	$(P$	$\to$	$Q)$	$/$	P	$//$	Q
T	T							
T	F							
F	T							
F	F							

I only need to focus on rows where I know the premises might be true or the conclusion might be false. So, I can safely ignore rows 3 and 4, because the second premise is false on those rows. When I look at rows 1 and 2, I see that the conclusion is true on line 1. So, the only row that has a chance of showing this argument to be invalid is row 2. So, I'll work that one.

P	Q	$(P$	$\to$	$Q)$	$/$	P	$//$	Q
T	T							
T	F		F			T		F
F	T							
F	F							

After working it, I see that one of the premises turned out to be false. So, I know there's no row that has all true premises and a false conclusion.

Now, let's see what happens when we switch the second premise with the conclusion. On which rows do you think we should focus?

P	Q	$(P$	$\rightarrow$	$Q)$	/	Q	//	P
T	T							
T	F							
F	T							
F	F							

Notice that the conclusion is false only on rows 3 and 4. On row 4, though, the second premise is false. So, the only row that could make this invalid is row 3. Let's work it and see what results.

P	Q	$(P$	$\rightarrow$	$Q)$	/	Q	//	P
T	T							
T	F							
F	T		T			T		F
F	F							

Since a conditional with a false antecedent is true, the first premise if true on line 3. The second premise is also true, but the conclusion is false. So, this argument is invalid. In fact, this is such a common invalid argument that it has a name: "Assuming the Consequent."

Here is another example:

1. P → Q
2. ¬ Q
3. ¬ P

P	Q	$(P$	$\rightarrow$	$Q)$	/	$\neg Q$	//	$\neg P$
T	T							
T	F							
F	T							
F	F							

We'll just go ahead and fill in the whole thing:

P	Q	$(P \rightarrow Q)$	/	$\neg Q$	//	$\neg P$
T	T	T		F		F
T	F	F		T		F
F	T	T		F		T
F	F	T		T		T

There is no line with all true premises and a false conclusion, so the argument is valid. This argument type is called by the Latin name, *Modus Tollens*. Let's again switch the second premise and the conclusion, and see what happens.

P	Q	$(P \rightarrow Q)$	/	$\neg P$	//	$\neg Q$
T	T	T		F		F
T	F	F		F		T
F	T	T		T		F
F	F	T		T		T

The third line has all true premises and a false conclusion, so this argument is invalid. This is called "Denying the Antecedent."

Let's try a truth table for a more complex argument.

1. A v B
2. A → (B v C)
3. ¬(C & A)
4. B

The table begins like this:

A	B	C	A	$\vee$	B	/	A	$\rightarrow$	$(B$	$\vee$	$C)$	/	$\neg$	$(C$	$\&$	$A)$	//	B
T	T	T																
T	T	F																
T	F	T																
T	F	F																
F	T	T																
F	T	F																
F	F	T																
F	F	F																

I'll talk you through the first line, then just fill out the rest. On the first line, since A and B are both true, the first premise is true. The second premise is a conditional with a true antecdent and a true consequent (B and C are both true, making $B \vee C$ true). So, the second premise is also true. The third premise is false, since it is a negation of a true conjunction. Finally, the conclusion is true.

A	B	C	A	$\vee$	B	/	A	$\rightarrow$	$(B$	$\vee$	$C)$	/	$\neg$	$(C$	&	$A)$	//	B
T	T	T	T	T	T		T	T	T	T	T		F	T	T	T		T
T	T	F																
T	F	T																
T	F	F																
F	T	T																
F	T	F																
F	F	T																
F	F	F																

The completed truth table looks like this:

A	B	C	A	$\vee$	B	/	A	$\rightarrow$	$(B$	$\vee$	$C)$	/	$\neg$	$(C$	&	$A)$	//	B
T	T	T	T	T	T		T	T	T	T	T		F	T	T	T		T
T	T	F	T	T	T		T	T	T	T	F		T	F	F	T		T
T	F	T	T	T	F		T	T	F	T	T		F	T	T	T		F
T	F	F	T	T	F		T	F	F	F	F		T	F	F	T		F
F	T	T	F	T	T		F	T	T	T	T		T	T	F	F		T
F	T	F	F	T	T		F	T	T	T	F		T	F	F	F		T
F	F	T	F	F	F		F	T	F	T	T		T	T	F	F		F
F	F	F	F	F	F		F	T	F	F	F		T	F	F	F		F

There is no line with all true premises and a false conclusion, so the argument is valid.

5.4 Short Truth Tables

Truth tables can be used to determine the validity of any argument in propositional logic. If you carefully follow the rules, *without making any* *careless mistakes*, you're guaranteed to get the right answer. The only drawback is that they get very large, very quickly. A truth table for an argument with six simple sentences in it has 64 rows—not something that most of us would look forward to doing.

It would be nice if there was a way that we could go straight to the row that showed an argument to be invalid, if there was one. Fortunately, there is, although it can be tricky at times.

Let's try this argument:

1. A & B
2. ¬ [A → (C v D)]
3. C v D

The first step is to set up the truth table as we've done in the past, but we'll only need one row.

A	B	C	A	$\&$	B	$/$	$\neg$	$[A$	$\rightarrow$	$(C$	$\vee$	$D)]$	$//$	C	$\vee$	D

The next step is to put a 'T' underneath the main operator of all the premises and a 'T' under the main operator of the conclusion.

A	B	C	A	$\&$	B	$/$	$\neg$	$[A$	$\rightarrow$	$(C$	$\vee$	$D)]$	$//$	C	$\vee$	D
				T			T								F	

When we do this, we're assuming that there is a line on which all of the premises of the argument are true and the conclusion is false. Now, we'll see if that assumption leads to a contradiction. If it does, then there can't be a line like that, and the argument is valid. If it does not lead to a contradiction, then there will be a line like that, and the argument will be invalid.

Now, we'll start filling in what must be true if our assumptions are correct. The first premise is a true conjunction, so both conjuncts must be true.

A	B	C	A	$\&$	B	$/$	$\neg$	$[A$	$\rightarrow$	$(C$	$\vee$	$D)]$	$//$	C	$\vee$	D
			T	T	T		T								F	

The conclusion is a false disjunction, so both disjuncts must be false.

A	B	C	A	$\&$	B	$/$	$\neg$	$[A$	$\rightarrow$	$(C$	$\vee$	$D)]$	$//$	C	$\vee$	D
			T	T	T		T							F	F	F

Now, we know what A, B, C, and D have to be. Let's transfer those values to the second premise.

A	B	C	A	$\&$	B	$/$	$\neg$	$[A$	$\rightarrow$	$(C$	$\vee$	$D)]$	$//$	C	$\vee$	D
			T	T	T		T	T		F		F		F	F	F

Now, we need to finish completing the second premise. We'll start with the disjunction in the antecedent of the conditional.

Then, the conditional itself:

Now, we check to see if there were any problems. We're looking for something like a letter that has different truth values, or a false disjunction with a true disjunct. Here, each letter has the same truth value wherever it occurs. We have a conjunction with two true conjunctions, two false disjunctions, both with two false disjuncts, a false conditional with a true antecedent and a false consequent, and a true negation with a false negated sentence. Everything looks fine, which means it *is* possible for the argument to have true premises and a false conclusion, and is definitely invalid. The problem row would be the one where A is true, B is true, C is false, and D is false. That would be row 4 of the entire truth table.

Let's try another. Here is a classic argument called a constructive dilemma:

1. A B
2. C D
3. A v C
4. B v D

Start by constructing the basic table heading.

A	B	C	D	A	$\rightarrow$	B	$/$	C	$\rightarrow$	D	$/$	A	$\vee$	C	$//$	B	$\vee$	D

Then, fill in T and F under the main connective of the premises and conclusion, respectively.

A	B	C	D	A	$\to$	B	/	C	$\to$	D	/	A	$\vee$	C	//	B	$\vee$	D
					T				T				T				F	

Then, we'll fill in what we can figure out, given those assumptions. We don't know anything about the premises yet. The first two are true conditionals, and there are three different ways a conditional can come out true. Likewise for the true disjunction in the third premise. So, let's start with the conclusion. Since it is a false disjunction, then both B and D must be false.

A	B	C	D	A	$\to$	B	/	C	$\to$	D	/	A	$\vee$	C	//	B	$\vee$	D
					T				T				T			F	F	F

Then, we can fill in those values wherever B and D occur.

A	B	C	D	A	$\to$	B	/	C	$\to$	D	/	A	$\vee$	C	//	B	$\vee$	D
					T	F			T	F			T			F	F	F

Now, we can do the first two premises. Notice that we have true conditionals with false consequents. That means that both of the antecedents must be false.

A	B	C	D	A	$\to$	B	/	C	$\to$	D	/	A	$\vee$	C	//	B	$\vee$	D
				F	T	F		F	T	F			T			F	F	F

Now, we can fill that in where A and C occur in the third premise.

A	B	C	D	A	$\to$	B	/	C	$\to$	D	/	A	$\vee$	C	//	B	$\vee$	D
				F	T	F		F	T	F		F	T	F		F	F	F

Now, we have a problem. We have a true disjunction in the third premise with two false disjuncts. That's a contradiction. That means we cannot make this argument have true premises and a false conclusion. We've proved that the argument is valid.

5.5 Argument Forms

Before we leave propositional logic, here are some important argument forms that might be useful.

5.5.1 Valid

Modus Ponens

1. $P \to Q$
2. P
3. Q

Modus Tollens

1. $P \to Q$
2. $\neg Q$
3. $\neg P$

Disjunctive Syllogism

1. A v B
2. $\neg$ B
3. A

Hypothetical Syllogism

1. A B
2. B C
3. A C

Constructive Dilemma

1. A B
2. C D
3. A v C
4. B v D

Destructive Dilemma

1. A B
2. C D
3. $\neg$B v $\neg$ D

4. $\neg$A v $\neg$ C

5.5.2 Invalid

Affirming the Consequent

1. P $\rightarrow$ Q
2. Q
3. P

Denying the Antecedent

1. P $\rightarrow$ Q
2. $\neg$ P
3. $\neg$ Q

Chapter 6

Sense Perception

6.1 Evaluating Sources of Information

Determining validity is only half of the task of evaluating an argument. As we've seen, a valid argument can still be very bad. Here's a valid argument:

1. If I tossed heads with that coin, then the world will end tomorrow.
2. I tossed heads with that coin.
3. The world will end tomorrow.

No one should take this argument to be evidence for an impending apocalypse. A valid argument is only as good as its premises, and the implausibility of the first premise makes the argument, although valid, unconvincing. In other words, our reasoning is only as good as the information with which we use to reason.

So, good critical thinking requires the ability to determine whether the claims used as premises should be believed. Given the amount of information that bombards us today, we are rarely in a position to directly determine if a claim is true. For example, imagine that a news source reports that a terrorist attack occured in London. I'm not there, so I can't directly verify the claim. So, should I believe it?

We have three options when presented with a claim that is made. We can either accept it as true, reject it as false, or suspend judgment. Now, consider these claims:

1. Either it will rain today or not.
2. At least 90% of critical thinking students will pass this semester.
3. The first person to enroll in critical thinking next semester will make an A.
4. The United States will adopt a universal health care system within five years.
5. Everyone in critical thinking this semester will both pass the course and not.

I accept both of the first two claims, but not to the same degree. The first claim I accept to a maximal degree, since it cannot possibly be false. I believe the second claim is true, but I have to admit that this early in the semester, I could be wrong.

The third claim could be true, but just as easily could be false. So, I have to admit I don't know whether to accept it or not. I simply suspend judgment.

The fourth claim, I believe, is likely to be false, but, then again, I have to admit that events could surprise me. So, I reject the claim, but not to the same degree as the fourth claim. We should accept claims that are likely to be true and reject claims that are likely to be false. So, how do we determine if a claim is likely to be true or false? Here are some conditions:

1. The plausibility of the claim.
2. How strongly the claim coheres with other claims we accept as true.
3. The trustworthiness of the source of the information.

We fail to think critically when we automatically accept claims from unreliable sources and automatically reject claims from reliable sources. So, it is important to know how to evaluate sources of information. Some important sources of information are our senses, memory, other people, and now, the Internet. We will begin by examining human sense perception.

6.2 Don't Take Your Senses For Granted

Computational vision researchers have made some remarkable advances over the last fifty years. Software on your mobile phone can read text and recognize faces fairly well, yet the most powerful computers that we currently have can't keep up with the average two year old toddler. Seeing, smelling, touching, and tasting seem effortless and automatic to us, yet the processes behind those tasks are unbelievably complex. Learning about those processes will help us to understand when they work well and when they sometimes deceive us. We will focus on vision, but many of the issues related to vision apply to the other senses as well.

When we take our senses for granted, we risk making two mistakes. First, we don't appreciate how amazing they are. In what is called a rapid serial visual presentation test (RSVP), test subjects are shown a series of pictures at a rate of about 10 per second. (If you were taking the test, you would have seen about fifty pictures in the time that it took you to read this sentence.) After several repetitions of the series, the test subject is asked to write down what she saw. Even though the images are presented at an unbelievably fast rate, most people can remember almost all of the pictures in the series. Whenever you start to feel disappointed in your abilities, think about this — scientists have not found any information processing systems that can do these operations as fast as your brain. The world's fastest supercomputer just can't keep up with you when doing these complex tasks. Your brain can do some unbelievably complex tasks, and do them amazingly well. That's the good news.

Now, for the bad news... the second mistake we make when we take our senses for granted is to be overconfident in their accuracy. The reason that our brains can do those difficult tasks at the speed that they do is because they receive sensory input, then basically make educated guesses about the world around us. We can demonstrate this with a simple experiment called the blind spot test. Take a blank piece of paper, draw a dot on the left side, and an X on the right. Make them about 4-5 inches apart. Now, hold the paper in your hand and extend your arm in front of you. Cover your left eye, and focus on the dot with your right eye. You should see the X in your peripheral vision. Now, slowly move the paper toward you. At some point, the X will vanish, then reappear as you continue to bring the paper closer to you.

What has happened? Basically, each eye has a blind spot where the optic nerve attaches to the retina. When you can't see the dot, it's because it is in the blind spot. Why do we not just have a hold in the visual field? Why do we see a solid sheet of paper? It's because the brain makes a guess: it decides that what is in the blind spot is probably like what is in the area around it. Your brain made a guess — a better way to state this it that it made an inference from the data surrounding the blind spot. These inferences are fast, automatic, and outside our conscious awareness. Very often, they are correct, but sometimes, occasionally tragically so, they get it wrong.

6.3 Is What You See What You Get?

The blind spot test tells us that we should be at least a bit skeptical that the way we see things is the way they are. We all can think of times when we "saw" something that wasn't there or failed to see something that was there. The latter is often called selective perception. Some of that selectivity is "hard-wired" into us — we only see light and hear sounds within certain frequency ranges. Dogs, for example, can hear higher pitched sounds than we do.

Other instances of perceptual selectivity are not simply functions of our physiology, but rather of our beliefs, emotions, desires, etc. Think of the filtering that occurs when you are in a noisy, crowded room. At first, the noise level is very high, and you can't understand anything anyone is saying. Then, after you begin to have a conversation with a particular person, the noise level seems to decrease and you can easily hear the voice of the person with whom you are engaged in conversation. That is, until you hear someone across the room say your name, then the person in front of you might as well just be moving their lips. We hear what we think is important and ignore the rest.

6.3.1 Constancies and Ambiguities

When you walk closer to your desk, does the desk appear to change size? As you watch someone open a door, does the door appear to change shape? For most of us, the answer would be no. What we see tends to stay the same size and shape. These are called perceptual constancies. The retinal images, hwoever, change. As we approach the desk, the retinal image increases in size, although what we see does not change. As the door swings open, the retinal image goes from rectangular to more trapezoidal. Again, what we see does not change shape.

If we think of the retinal image as input, and the image in our minds as output. Then, perceptual constancies teach us that we can have the same output for different inputs. That is, sameness of input is not necessary for sameness of output.

Ambiguous figures teach us something else, that we can have the same input with different outputs, or that sameness of input is not sufficient for sameness of output. A classic case, one that you are probably familiar with, is called the Necker Cube:

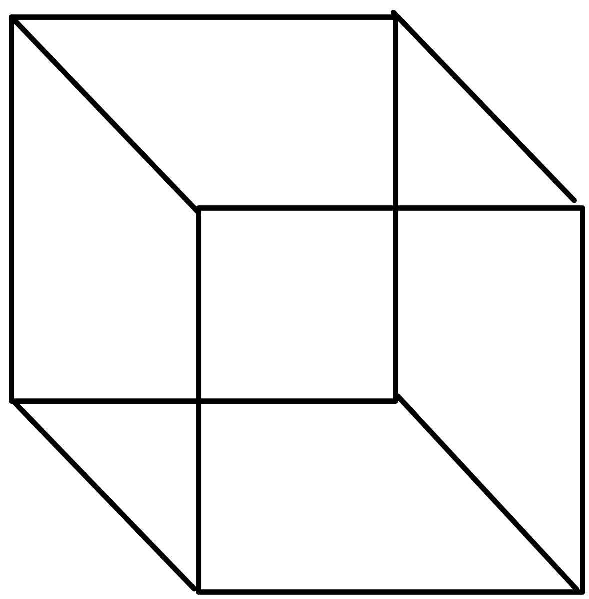

Figure 6.1: Necker cube

The interesting thing about the Necker cube is that we can, at will, change which side of the cube faces front. That is, we can change the way the image looks to us in our minds, although we know that the printed image on the page is not changing. Here, we have same inputs with different outputs. So, sameness of input is not sufficient for sameness of output. That is, the same sensory input does not guarantee that we will see the same thing.

6.3.2 Perceptual Set

It's particularly interesting when what we see is determined by context, expectations, beliefs, desires etc. These things form what is called the perceptual set.

Look at this next figure:

.images/abc123.pdf

Figure 6.2: B or 13?

Here we have an ambiguous figure in the center, which could either be the letter “B” or a “13”. You can see how a viewer would report seeing a “B” in one context, and a “13” in another. There’s a sense in which what was seen would not change, but how the experience would be interpreted certainly would. There are plenty of more familiar cases of times that our perceptual set affects what we see:

- Have you ever proof-read an essay that you wrote and found no mistakes, just to have a friend look it over and instantly see a host of spelling and typographical errors?
- Have you ever yelled at a friend that you saw in the distance just to find out that it was a stranger who looked nothing like your friend?

We’ll discuss other examples in class. The point is that there are factors that influence the way we see the world. We should always be aware of how much these our influencing us, and careful not to naively trust our senses beyond what they deserve.

Chapter 7

Memory

In the last chapter, we discussed the potential problems with sense perception as a source of information. In this chapter, we'll do the same for memory. Although the details differ, the main problem with both perception and memory is the same. That is, they are active, not passive. Memory is not like a machine that simply records what happened. Our memory adds new information, fills in the gaps, revises what was remembered, and invents new information.

7.1 Stages of Memory

There are three stages of memory. The first is the encoding stage; this is the point at which the person had the initial sense perception that is later remembered. So, all of the problems with sense perception are also problems with memory. The second stage is the storage stage, and the final stage is the retrieval stage. Errors can happen at all three stages.

7.2 Retrieval

There are two important types of retrieval. The first is recall. This is what happens in most instances when you are trying to remember something. For example, remembering someone's name, answering an essay question on an exam, and describing what someone looks like are all examples of recall.

Recognition is when you see the thing you remember and are able to recognize it as the thing remembered. For example, seeing a friend that you haven't seen in years and recognizing her as your friend would be an instance of recognition. Answering a multiple choice question is also recognition. The answer is right there in front of you, all you need to do is recognize it as the answer.

Retrieval is context dependent, state dependent, and mood congruent. Context dependence means that a person will tend to remember information when they are in the same context as when they initially learned that information. For example, students can recall information on a test more easily when they are taking the test in the same room in which they initially learned the information.

State dependence means that information is more likely to be remembered when the subject is in the same psychological state as when the information was initially learned. These states included synthetic states of consciousness induced with certain drugs (as mild as caffeine), or moods.

Mood-congruence means that when in a certain mood, subjects will tend to remember things that match that mood. For example, if a person is feeling depressed, they will tend to remember depressing things, while if happy, they will tend to remember happy things.

7.3 Framing Effects

Another thing that can affect what we remember is the wording of the questions used to prompt the retrieval. This is called the framing effect. In one study, some subjects were asked "How frequently do you have headaches?" The average response was 2.2 headaches per week. Other subjects were asked, "If you have them, how often?" Their responses averaged 0.7 headaches per week. The first question assumes that the subjects have headaches, and their answers reflected that assumption.

7.4 Source Misattribution

Source misattribution occurs when a person can remember some information, but can't remember from whom they got the information. A famous case was Ronald Reagan, who repeatedly told a story about a pilot who heroically went down with his plane in World War II. The scene, though, was actually from a film called *A Wing and a Prayer*.

There are two particular dangers caused by source misattribution. The first is plagiarism. A person might think that the idea is their own, but, in reality, he heard it from another person. The second danger is in believing information to have a higher degree of credibility that it really does. Someone might believe that she learned the information from a credible source, but actually she learned it from someone who she wouldn't really trust.

7.5 Confidence and Accuracy

So, how do we know when a memory is accurate. Unfortunately, there's no easy answer. There are memories that seem to us to be "fuzzy." We're not very confident that those memories are true. On the other hand, there are memories that seem so vivid, that we can say exactly where we were when we initially experienced the event that we remember. Psychologists call such events "flashbulb events," and we tend to have an extreme level of confidence in those memories. It turns out, though, that confidence is not a reliable indicator of accuracy.

After one such event, a psychologist had the students in a large class write down what they were doing when the event occurred. Later, he contacted them, and had them again write down what they were doing when the event occurred. Only in 1/3 of the cases did their memories of the event match what they wrote immediately after the event. Another 1/3 were wrong in some important details, while the remaining 1/3 were wildly wrong. It is interesting that, when the psychologist showed the initial report to the people who were significantly wrong, they made up a story to justify why the initial thing they wrote was wrong.

Chapter 8

Other People as Sources of Information

Other people are an important, and inevitable, source of information. We trust our doctors to tell us what is wrong with us when we aren't feeling well. I have to trust my auto mechanic to tell me what is wrong when my car isn't running right. We trust absolute strangers to give us directions when traveling in unfamiliar places. When is it rational to rely on someone else for information? Who should we ask? Who should we *not* ask?

8.1 Expertise

We should ask the person who is likely to know the answer, that is, the expert. We will define expertise very generally — an expert is someone who knows more about a field than most people do. This does not mean that an expert is always right. Experts are certainly not foolproof. Expertise also comes in degrees. There could be a person that knows more than the average person and another person that knows much more than the average person. The first is an expert, but the second has a higher degree of expertise. The final thing to keep in mind is that expertise is always in a field. If the claim is outside the expert's area of expertise, then there is not reason to think that she knows any more than the average person on the subject.

8.2 Evaluating Claims to Expertise

An argument from expertise has the following structure:

1. Some person, P, is an expert on a subject, S.
2. C is a claim in S.
3. P says that C is true.
4. C is true.

Are such arguments valid? What would it mean if they were? Since valid arguments are such that, if the premises are true, the conclusion must be true. So, in this case, if these arguments from expertise were valid, then anything an expert said that was in her field would have to be true. That would mean, however, that experts were foolproof, which they certainly are not. So, arguments from expertise are invalid. That doesn't mean they are bad, however. Arguments from expertise can be strong inductive arguments. That is, if the premises are true, then the conclusion is likely to be true.

These arguments can fail in several different ways, at least one for each premise:

1. The person who made the claim might not be an expert.
2. The person might be an expert, but the claim is outside the expert's area of expertise.
3. The person never made the claim. That is, they have been misquoted, or the quote is taken out of context.
4. The person might be an expert, the claim might be in their area of expertise, they actually made the claim, but, for some reason, they are biased.

How does one determine if a person is an expert? What do you think that experts have that non-experts lack? Some things that come to mind are education, professional licensing, and experience.

8.3 Faking Expertise

People tend to listen to experts, so if someone wants to influence others, it is in their best interests to appear to be an expert. Things that give a person an aura of expertise are called halo effects. These can include props, clothing, jargon, and certificates that look like degrees or licenses.

8.4 The Internet

More and more of our information is coming from online sources. These sources should be evaluated in the same ways that we evaluate other sources of information. Online information is often anonymous, however, which means that the source cannot be evaluated for expertise in the usual ways. Some things to consider are:

- Has the site been linked to from a known, reputable site?
- Was it recommended by someone who is a known expert in the area?
- Was it cited in some reputable, scholarly source?

Chapter 9

Informal Fallacies

A fallacy is a mistake in reasoning. A formal fallacy is a fallacy that can be identified merely by examining the argument's form or using a tool like a truth table. An informal fallacy cannot be detected from the argument's form. There are no foolproof tools for detecting informal fallacies. Unlike validity, these fallacies can occur in degrees. Sometimes, it is clear that a fallacy has been committed, at other times, there can be legitimate questions about whether a fallacy has been committed.

9.1 Emotions and Critical Thinking

There are two important uses of language that play an important role in critical thinking. The first is to convey information, or what is sometimes called cognitive content. The premises and the conclusion of an argument all have cognitive content. It is on the basis of the information they convey that we can evaluate them as true or false. Another use of language, however, is to express emotion, which often tends to evoke similar feelings in the audience. The emotions that are expressed by the language is its emotive content.

Good critical thinkers are persuaded by relevant cognitive content, and are not unduly persuaded by irrelevant emotive content.

9.2 Slanters

Slanters are words or phrases that are used to manipulate by using emotive language. Slanters can have both positive and negative connotations. Slanters are types of non-argumentative persuasion. Sometimes, people will use them innocently, maybe because they have passionate feelings about the subject being discussed. Other time, though, they are used because the speaker knows that he does not have a good argument for his position. They are ways for people to affect the beliefs of others without offering reasons for their positions.

9.2.1 Euphemisms and Dysphemisms

Different words and phrases passages can have the same cognitive content but differ in their emotive content. A euphemism is a positive synonym for some neutral term. A dysphemism is a negative synonym. For example, think of the words that we use to talk about the death of a pet:

- Neutral: euthanize
- Euphemism: put to sleep
- Dysphemism: ?

Euphemisms are common in the military and in advertising. Here are some examples:

- Enhanced interrogation methods
- Collateral damage
- Depopulated area
- Pre-owned, Pre-loved
- Genuine imitation leather

In other cases, a person may use a euphemism when they have been caught doing something wrong, but want to minimize the wrongness of the action. I heard a politician once, who was caught telling a falsehood, admit to having "committed terminological inexactitude."

Dysphemisms convey a negative attitude towards something. Examples of dysphemisms include:

- Snail mail
- Cancer stick

- Egghead
- Worm food
- Pig
- Dead tree edition

9.2.2 Innuendoes

Innuendoes imply something by what is not said. A common scene in crime films has a gangster go into a store and say something like "Nice store you got. It would be a shame if anything happened to it." Notice that he didn't actually say he would damage the store, although he certainly implied it.

Another use of an innuendo is to condemn with faint praise. Imagine receiving a reference letter for a student applying to graduate school. The letter only says that the student was never late to class. The implication is that her punctuality is the best that can be said of her.

Finally, there is the apophasis, which is mentioning something by saying that it won't be mentioned. For example, "I'm not going to talk about your failure to turn assignments in on time."

9.2.3 Weaselers

A weaseler is a way of qualifying a claim in order to avoid criticism. Weaselers include words and phrases like "perhaps," "there's a good chance that," and "it's possible that." A weaseler that is common in advertising is "up to" — "By using our diet plan, you can lose up to ten pounds in a month." Notice that "up to ten pounds" means "no more than ten pounds." So, if a customer loses no weight at all, then there is no grounds for a lawsuit.

9.2.4 Downplayers

A downplayer is a way of making something seem less important than it is. The most common downplayers are "mere, merely, and so-called." A person might say, "That's merely your opinion" to avoid having to respond with facts. A politician might talk about his opponent's "so-called" plan to cut spending, implying that it isn't much of a plan at all. Improper use of quotation marks can also serve as a downplayer.

9.2.5 Proof surrogate

A proof surrogate offers no real support, but just claims that support exists. Examples are using "studies show" without saying what those studies are and where they can be found. Another proof surrogate is just to say that "It's obvious that. . . ." Doing so implies that proof is simply not needed.

9.2.6 Hyperbole

Hyperbole is an inappropriate or extreme exaggeration. "Taking critical thinking is the most exciting thing you'll do in your whole life!" Since it is an extreme exaggeration, no one will be fooled into believing it, so what's the danger? The danger of hyperbole is that once the exaggeration is made, the listener is then prepared to accept a weaker version of the statement. The weaker version, compared to the extreme exaggeration, sounds more believable.

9.3 Fallacies of Ambiguity and Vagueness

An ambiguous word or phrase is one that has more than one meaning. "Bank" is an ambiguous term, it can refer to a financial institution, a riverbank, a kind of basketball or pool shot, etc. A vague term is one that does not have a precise meaning. That is, there will be cases where a vague term clearly applies, cases in which it clearly does not apply, and cases in the middle where it's just not clear whether it applies. Terms like "rich" and "heap" are vague terms.

9.3.1 Equivocation

The fallacy of equivocation is committed by using the same term in two different senses in the same argument. Here is my favorite example:

1. God is love.
2. Love is blind
3. Ray Charles is blind.
4. Ray Charles is God.

There are several things wrong with this argument, one of them is equivocating on "blind." To say that love is blind, is to say that people overlooks the faults of those they love. To say that Ray Charles is blind is to say that he cannot physically see anything, not that he just overlooks things.

9.3.2 Amphiboly

Amphibolies rely on syntactic ambiguities. Those are ambiguities that result from the arrangement of the words. Church bulletin bloopers are good places to find amphibolies: "The Rev. Adams spoke briefly, much to the delight of his audience."

9.3.3 Accent

The fallacy of accent is an equivocation resulting from accenting different words in a sentence. Think about the different meanings that are implied from accenting different words in this sentence: "I didn't take the exam yesterday."

9.3.4 Division and Composition

The last two fallacies of ambiguity are division and composition. The fallacy of division improperly attributes a property of the whole to its parts. The fallacy of division improperly attributes a property of the parts to the whole. Here is an example of division: "That wall weighs more than 500 pounds, so each brick in it weighs more than 500 pounds." A similar example of composition is "Each brick in that wall weighs less than a pound, so the entire wall weighs less than a pound."

Some properties, however, can be attributed from the parts to the whole or the whole to the parts. For example, "Each link in that chain is solid gold, so the whole chain is solid gold."

9.3.5 Line-Drawing Fallacy

The line-drawing fallacy is a fallacy of vagueness, having the form "Since there is no precise line that can be drawnd between A and not-A, there is no real difference between A and not-A." Example: "Since no one can say where the line should be drawn between legitimate uses of force and excessive uses of force, then no one can honestly claim that any use of force is excessive."

9.4 Fallacies of Relevance

A good critical thinker will offer arguments that have premises that are logically relevant to their conclusions. A fallacy of relevance is committed when the premises of the argument are not logically relevant to the truth of the conclusion. They may be, however, *psychologically* relevant, so that we can be deceived in thinking that the argument is valid, when in fact it is not.

9.4.1 Ad Hominem

The Ad Hominem fallacy is committed by attacking the person giving the argument, rather than responding to the argument itself. There are four common types:

9.4.1.1 Personal Attack

This is also known as an ad hominem abusive. This is committed when one verbally attacks the person giving the argument instead of responding to the argument itself. For example:

Jack: “There are so many unexplored planets out there. Surely, there must be life somewhere out there”

Jill: “You can’t be right, you’re just a loser who watches too much television.”

9.4.1.2 Circumstantial Ad Hominem

A person commits the circumstantial ad hominem by referring to circumstances that discredit the arguer. This is often a charge of bias or vested interest. A person has a vested interest when they stand to gain, financially or otherwise, by something. For example, a doctor has a vested interest in a pharmaceutical study when she owns stock in the company that produces the drug. Bias or vested interest is a good reason to examine an argument carefully, but not a good reason to simply dismiss it.

Jill: John has made an excellent case for increasing the budget of the church’s youth program.

Jack: Of course he would say that — he’s the youth minister! You can just forget everything he said.

9.4.1.3 Tu Quoque

This is also known as a pseudorefutation. It accuses the arguer of hypocrisy. Example: “Don’t tell me I shouldn’t start smoking. I know how many packs a day you inhale!”

A person’s behavior may very well be inconsistent with their argument, but that doesn’t mean that the argument is bad.

9.4.1.4 Poisoning the Well

Poisoning the well is an ad hominem committed before the arguer has spoken. The goal is to provide harmful information about the speaker to preemptively discredit anything that the speaker might say. Any of the previous examples can be turned into examples of poisoning the well. For example: “John is about make his case for increasing the church’s youth budget. Don’t pay any attention to him — he’s the youth minister, what else would he say?”

9.4.2 Appeal to Force

The appeal to force, also called scare tactics, is a threat, either explicit or implicit. For example, imagine a student saying to a professor, “I deserve an A because my father is a major donor to this university and a very good friend of the dean.” The appeal to force tries to instill fear in the listener, and to be fallacious, the fear must be irrelevant to the truth of the claim. These are common in both advertising and politics. An example of an appeal to force in advertising would be an ad for a Medicare supplement policy with an elderly woman weeping in front of a pile of unpaid bills. The advertisement works by making the viewer afraid of ending up like the person in the commercial.

9.4.3 Appeal to Pity

The appeal to pity is like the appeal to force, except that the goal is to evoke pity, not fear. For example: “I deserve an A because my mother is very ill, and I had to spend most of my time caring for her this semester.”

9.4.4 Popular Appeal

This is also called appeal to the people. Here, the goal is to use the desire to be loved, admired, accepted, etc. to get others to accept the conclusion. Two important types are the bandwagon fallacy and the appeal to vanity.

9.4.4.1 Bandwagon

The bandwagon fallacy tells the listener that since everyone does, or believes, something, then they should too.

Example: “Everyone supports Smith for president. You need to get with the program and support him too!”

9.4.4.2 Appeal to Vanity

This is a claim that you will be admired if you do this. Unlike bandwagon, which claims that everyone does this, the appeal to vanity is usually about something that not everyone can do or have.

Example: “Wear a Rolex — that way everyone will know that you’re not just somebody.”

9.4.5 Appeal to Ignorance

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Concluding that since something has never been proved true (false), it must b
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9.4.5.1 Burden of Proof

On most issues, one side will have the burden of proof. That means that if that side fails to make its case, then the other side wins by default. There are two standard rules for determining burden of proof:

1. Especially for existence claims, the side making the positive case has the burden of proof.
2. The side making the more implausible claim has the burden of proof.

Sometimes, these conditions can conflict. Here is an example:

Jill: Surely, there are species of insects that we have not yet discovered.

Jack: I don't think that's true.

Who has the burden of proof? Jill is making a positive existence claim, but it is one that very plausible. That makes Jack's claim very implausible. In this case, I'd say that Jack has the burden of proof.

9.4.5.2 The Law

One area where these rules do not apply is the American legal system. There, the prosecution *always* has the burden of proof. That is, if the prosecution fails to make its case against the defendant, then the defense wins.

Burden of proof should not be confused with standard of proof. Burden of proof is concerned with who needs to make their case. Standard of proof is concerned with how strong a case needs to be made. There are four different levels of standard of proof in the law:

1. Beyond a reasonable doubt
2. Clear and convincing evidence
3. Preponderance of evidence
4. Probable cause

Criminal cases use the highest standard of proof, which is "beyond a reasonable doubt." This is a high degree of probability. It does not mean that no doubt at all is possible, but that any doubt, given the evidence, would be unreasonable. The next two level are used in civil cases. Most civil cases are tried at the "preponderance of evidence" level. This means that, given the evidence, it is more likely that the defendant is liable than not. Clear and convincing evidence is a standard of proof between preponderance of evidence and beyond a reasonable doubt. It is used in civil cases that involve the potential loss of important rights or interests, such as the termination of potential rights. The lowest standard of proof is probable cause. This is used to determine if a search or arrest is warranted, and also used by grand juries to issue indictments.

9.4.6 Straw Man

The straw man fallacy Distorts a position so that it can be easily attacked. It does not address the actual view held by the opponent, but responds to a weaker version. It is often committed by making the conclusion of an argument more extreme than it actually is, since extreme views are often easy to attack.

Example: “Senator Snodgrass has argued that there be a mandatory waiting period before any handgun purchase. Obviously, the senator wants to make all firearm ownership illegal.”

Here some subtle ways of committing the fallacy:

- Taking words out of context.
- Treating extreme views as representative.
- Criticizing early versions of a position.
- Criticizing deliberately simplified versions of a position.

9.4.7 Red Herring

The goal of the red herring fallacy is to lead the opponent off the track, by subtly changing the issue being discussed. The arguer changes the subject to a different but related one. To determine if something is a red herring, ask yourself if the issue at the beginning of the argument is the same as the issue at the end. Here is an example:

> The American Cancer Society has argued that smoking is bad for your health. Many people in the Southeastern United States are dependent upon the tobacco industry for their jobs. Making smoking illegal would have a devastating economic effect on many states. Therefore, the ACS is simply wrong.

Notice the original isse is whether smoking has bad health consequences. By the end of the paragraph, the issue has been changed to the economic impact of making smoking illegal.

9.4.8 Horse Laugh

This occurs when someone simply ridicules the position held, and offers no real response to the argument at all. For example: “Mr. Jones has argued that watching television is emotionally unhealthy. If you believe that, then I’ve got a great deal on some swampland for you.”

9.5 Fallacies of Unwarranted Assumptions

9.5.1 Begging the Question

An argument begs the question if it is impossible to believe at least one of the premises unless one already believes the conclusion. Note that if this is the case, then the premises cannot serve as reasons to believe the conclusion, since believing the premises requires already believing the conclusion. There are three common types of arguments that beg the question.

The first is a circular argument. That occurs when one explicitly uses the conclusion as support for one of the premises. Here is an example:

1. The Bible says that God exists.
2. The Bible is the inspired word of God.
3. God exists.

The Bible can't be the inspired word of God unless God exists, so the argument begs the question.

Another type of argument that begs the question is one that simply rephrases the conclusion and uses it as a premise. Example: "If such actions were not illegal, then they would not be prohibited by the law." In this case, the conclusion is synonymous with one of the premises.

The last type is one that generalizes the conclusion and uses the generalized rule as a premise. Example: "Spanking children is wrong because corporal punishment is wrong."

9.5.2 Appeal to Authority

The fallacy of appeal to authority is committed by using an pseudo-authority to support a claim. Note that it is not committed by merely appealing to an authority, but by appealing to an unqualified authority. Always ask, "Should this person know more about this subject than the average person?"

9.5.3 Loaded Question

A loaded question suggests something with the question. "Whem will you stop cheating on exams?" is a loaded question, the question implies that the person is cheating. Notice that there is no way to directly answer the question without admitting to cheating on exams.

9.5.4 False Dilemma

This is sometimes called the either-or fallacy. This happens when a person asserts a disjunction, a sentence of the form "either A or B, when there is at least one more option that is true. Disjunctions are true whenever at least one of the disjuncts, the sentences joined by the 'or', are true. A False dilemma asserts that one of the two sentences must be true when there is really a third alternative. Here are some examples:

"Either buy our personal financial guide or never have control of your finances."

Child to parent: "Either let me go to the party or I'll just die."

False dilemmas are often expressed in pithy slogans on bumper stickers: "It's my way or the highway" or "America, love it or leave it."

Consider this example: "My opponent voted against the public schools spending bill. He must think educating our children is not important." The claim is that either one votes for the bill or one believes that education is not important. This is a false dilemma since there may be many other reasons to vote against a particular bill.

It's important to remember that a disjunction can be expressed as a conditional: "Either let me go to that party or I'll die" is equivalent to "If you don't let me go to that party, then I'll die." In general, P or Q is equivalent to if not-P then Q.

9.5.5 Slippery Slope

Slippery slopes rest a conclusion on a chain reaction that is not likely to occur. They generally have this form:

1. *AßB*
2. *BßC*
3. *CßD*
4. D is bad.
5. A is bad.

In order for this to be a fallacy, at least one of the conditional statements in the premises must not be likely to be true. Here's an example: "If I fail this test, then I will fail the course. If I fail the course, then I'll be expelled from school. If I'm expelled from school, then I'll never be able to have a good job. If I can't get a good job, then I can't support a family…"

To test for a slippery slope, just ask, are there any weak links in this chain of conditionals? Is it really the case that one failed exam will result in an F for a course grade?

Chapter 10

Probability

Students tend to find these two chapters on probability the most difficult material in the course. It looks hard, but it's really not as complicated as it looks. It is important to understand how probabilities work in order to reason well. We rarely have conclusive evidence for or against any claim. Imagine that you're on a jury trial, you have been tasked with determining the probability of the defendant's guilt given the evidence. To do this well requires that a person have a basic understanding of how probability works.

Given the example about serving on a jury, it's more than a little disturbing that our intuitions about probabilities are extremely flawed. Here's a classic example called the Monty Hall Problem: In the game show *Let's Make a Deal*, the host, Monty Hall, would select a person to play for the big prize. The contestant would have a choice of three doors. After choosing a door, the host, who knows which door the prize is behind, would open one of the other doors and show the contestant that that door did not reveal the prize. The contestant would then be offered the choice to stick with his original choice or to switch to the third door.

So, you are the contestant. You choose door number 1. Let's say that Monty opens door 2 and shows you that it has nothing behind it. What should you do? Stick with 1 or switch to 3? You should do what will increase the probability of your winning. Which has the higher probability? Most people will answer that, since there are only two doors, neither has a higher probability than the other. So, the common answer goes, the odds of your winning are simply 50/50.

The correct answer, though, is that you should switch. If you switch, the probability of winning doubles. Is this intuitive? Absolutely not.

10.1 Calculating Probabilities

First, a few preliminaries. Probabilities are numbers between 0 and 1. Unfortunately, it will be necessary to be able to add, multiply, and divide fractions. If you can't remember how, look at the review in section 13.6 of the text.

We won't worry about the morality of gambling, but it's easiest to learn basic probability in the context of cards, dice, and coin tosses. Basic probability questions are often about cards and dice. So, a few facts to keep in mind:

1. Each die has six sides.
2. A standard deck of cards has 52 cards, divided into 4 suits (clubs, diamonds, hearts, and spades). Each suit has an Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, and King. We'll never have Jokers in our imaginary decks.
3. $\Pr(S) = n$ means that the probability that the sentence S is true is equal to n.
4. Probabilities range from zero to one, inclusive. The answer to a probability problem will never be less than zero nor greater than one.

10.2 Calculating Simple Events

Examples of simple events include tossing heads with one toss of a single coin, getting a six with a roll of a single die, and drawing a heart on one draw from a standard deck of cards. The rule for calculating the probability of a simple event is this:

$$\Pr(A) = \frac{\text{favorable outcomes}}{\text{total possible outcomes}}$$

That's easy enough. We just have to determine how many possible ways this scenario could work out, and how many of those ways get us the outcome that we're looking for.

10.2.1 Examples

What is the probability of tossing heads with a single coin? If we toss a coin once, there are only two possible outcomes (to keep things simple, we rule out the very, very slim possibility that it lands and stays on edge). Of those two outcomes. only one is heads. So, the probability of tossing heads is equal to 1/2. In probability notation, $\Pr(H) = 1/2$

What is the probability of rolling two on one roll with a single die? There are six possible outcomes, only one is a two. $\Pr(2) = 1/6$

What is the probability of drawing the Ace of Spades on one draw from a deck of cards? There is one favorable outcome out of 52 total possible: $\Pr(A\spadesuit) = 1/52$

What is the probability of drawing an Ace on a single draw? Now there are four favorable outcomes in the 52 total possible: $\Pr(A) = 4/52 = 1/13$

What is the probability of drawing a Heart on a single draw? Since there are thirteen Hearts, there are 13 favorable outcomes, but still only 52 cards. $\Pr(A) = 13/52 = 1/4$

10.3 Calculating Complex Events

Complex probabilities are probabilities of negations, conjunctions, or disjunctions. A negation is a "not" sentence. The sentence "I will not go to the movies tonight" is the negation of the sentence "I will go to the movies tonight." A conjunction is an "and" sentence. An example is the sentence "I will go to dinner and I will go to the movies." A disjunction is an "or" sentence, as in "I will go to dinner or I will go to the movies." Unless specified otherwise, disjunctions are always inclusive disjunctions. So, "I will go to dinner or I will go to the movies" means that I will do one, or the other, or both.

There are also some symbols that you need to know. "Not" is symbolized by "¬" and "and" is symbolized by "&"

1. The probability of $\neg P$ is the probability that P is not true.
2. The probability of $P \& Q$ is the probability that both P and Q are true.

3. The probablity of P or Q the probability that either P or Q or both are true.

10.3.1 Necessities and Impossibilities

$$\text{If S cannot be true, then} \Pr(S) = 0$$

$$\text{If S must be true, then} \Pr(S) = 1$$

What is the probability of rolling a 7 with one die? Since it is impossible to roll more than a six with one die, $\Pr(7) = 0$.

What is the probability of rolling at least a 1 with one die? No matter what you roll, you will get at least a 1, so $\Pr(\text{at least } 1) = 1$.

10.3.2 Negations

Now, let's pause and think for a moment. Remember that the probability of an event that must occur is equal to one. For any event, it must be the case that some outcome occurs. For example, if you toss a coin, you have to get either heads or tails. So, if you add up the probabilities of all the possible outcomes for an event, they have to add up to 1. Now, let's imagine an event that has three possible outcomes, A, B, and C. By our reasoning, $\Pr(A) + \Pr(B) + \Pr(C) = 1$. That is, we can think of the probability of an event as represented by a big pie. Each possible outcome is a piece of the pie. The size of the whole pie is 1, so when we add up the areas of each the pieces, they have to total 1. Now, let's I want to know the probability of $\neg A$. The probability of A is just the size of A in the pie. The probability of $\neg A$ then is the size of the remainder of the pie, once we take out A. Since the size of all of the pieces add up to 1, the probability of A not occuring is equal to $1 - \Pr(A)$. Thus, we get the negation rule:

$$\Pr(\neg S) = 1 - \Pr(S)$$

For example, what is the probability of not rolling a 6 on one roll of a die? It must be equal to 1 minus the probability of rolling a six. Since there a six sides, the probability of rolling a six equals $1/6$. So, $\Pr(\neg 6) = 1 - 1/6 = 5/6$

What is the probability of not drawing the King of Hearts? $1 - Pr(K\heartsuit) = 11/52 = 51/52$

10.3.3 Compatibility

Two statements are compatible if they can both be true, and two events are compatible if they can both occur. There are certain events such that one's occurring automatically rules out the other's occurring. For example, if I get heads on one coin toss, that means that I didn't get tails. There's no way for both of those to occur on the same toss. They are incompatible events. Other events are compatible. Let's say I draw one card from a deck. Can I get both a King and a Heart? Yes, if I get the King of Hearts. So, getting a King and getting a Heart on the same draw are compatible events.

So, compatible or incompatible?

1. Tossing heads on one coin toss and tossing tails on the same toss. Incompatible
2. Tossing heads on one toss and tossing tails on the next. Compatible.
3. Drawing the ace of spades on both of two draws, if
4. The first card is put back into the deck (with replacement). Compatible
5. The first card is not put back into the deck (without replacement). Incompatible

10.3.4 Incompatible Disjunctions

If A and B are incompatible, then $\Pr(A \text{ or } B) = \Pr(A) + \Pr(B)$

What is the probability of getting either heads or tails on one coin toss? The two events are incompatible, you can't get both, so $\Pr(H \text{ or } T) = \Pr(H) + \Pr(T) = 1/2 + 1/2 = 1$. Of course it equals one, since you must get one or the other.

What is the probability of getting either a king or a queen on one draw from a deck? They are incompatible, you can't get both, so $\Pr(K \text{ or } Q) = \Pr(K) + \Pr(Q) = 1/13 + 1/13 = 2/13$.

What is the probability of getting the Ace of Spades or a heart on one draw? Incompatible, so $\Pr(A\spadesuit \text{ or } \heartsuit) = \Pr(A\spadesuit) + \Pr(\heartsuit) = 1/52 + 13/52 = 14/52 = 7/26$.

10.3.5 Compatible Disjunctions

What about compatible events? First, let's see why that formula will not work. What is the probability of getting heads at least once on two coin tosses? If we use the formula for incompatible events, we have $\Pr(H1 \text{ or } H2) = \Pr(H1) + \Pr(H2) = 1/2 + 1/2 = 1$. This cannot be right! Why not? If it has a probability of one, it must occur, but we know it's possible to get tails on both tosses. In fact, we know the answer should be 3/4. (I'll let you figure out why. Think in terms of favorable and total possible.)

So what went wrong with the formula? Essentially, we counted the same thing twice. If we toss a coin twice, there are four possible outcomes: HH, HT, TH, and TT. Out of those, there are two ways to get H on toss 1 and two ways to get H on toss 2. Adding those, it looks like we have four favorable outcomes. The problem is that we have counted one of those favorable outcomes (HH) twice, so we need to #tract one of them.

That gives us the rule for compatible disjunctions:

$$\text{If A and B are compatible, then } \Pr(A \text{ or } B) = \Pr(A) + \Pr(B) \Pr(A\&B)$$

The problem now is that $\Pr(A\&B)$ is the probability of a conjunction. So, before we can calculate compatible disjunctions, we need to learn to calculate the probabilities of conjunctions.

10.3.6 Independent Conjunctions

To calculate disjunctions, we have to determine whether they are compatible or incompatible. For conjunctions, we're concerned about dependence and independence. Two statements are independent if the truth value of one has no bearing on the truth value of the other. Two events are independent if the occurrence of one has no bearing on the truth value of the other.

For example, tossing heads on one toss and tossing tails on the same toss are dependent events. If the first happens, the probability of the second becomes zero. Tossing heads on one toss and then tossing tails on the *next* toss are independent events. he second has a probability of 1/2 whether the first occurs or not. Card draws are independent if you put the cards back as you draw them (called with replacement). Card draws are dependent if the cards are not placed back in the deck (without replacement).

$$\text{If A and B are independent, then } \Pr(A\&B) = \Pr(A) \times \Pr(B)$$

Here are some examples. What is the probability of getting heads on two consecutive tosses? That means getting heads on the first toss and heads on the second toss. The two are independent events, so $\Pr(H1\&H2) = \Pr(H1) \times \Pr(H2) = 1/2 \times 1/2 = 1/4$.

What's the probability of getting a king on two consecutive draws with replacement? Again, these are independent events. $\Pr(K1\&K2) = \Pr(K1) \times \Pr(K2) = 1/13 \times 1/13 = 1/169$.

Chapter 11

Conditional Probabilities

Probabilities change as circumstances change. The probability of drawing an Ace from a full deck of cards is 4/52. If you draw an Ace out and don't replace it, the probability of drawing an ace changes. Now, there are only 51 total possible outcomes, only three of which are favorable. So, the conditional probability of drawing an ace given that one ace has been removed is 3/51

Now, a bit more symbolization: $\Pr(A \text{ given } B)$ is written as $\Pr(A|B)$

11.1 Calculating Conditional Probabilities

The probability of A given B is the probability of the conjunction of A and B, divided by the probability of B, provided $\Pr(B)$ is not 0. This is called the Definition of Conditional Probability. Symbolized, this looks like:

$$\Pr(A|B) = \frac{\Pr(A\&B)}{\Pr(B)}$$

This formula is rarely used. It's usually only necessary to think in terms of favorable outcomes over total possible. Conditional probabilities change one or more of these outcomes. For example, you draw a king from a standard deck, keep it, and draw a second card. What is the probability of getting a king on your second draw given that you got a king on the first draw? That is, what is $\Pr(K2|K1)$? Now there are 51 total cards, three of which are kings. So, it is 3/51.

You roll a die and it goes off table. Your friends tells you that it's an even number, but you can't see. Given this extra information, what's the probability that you got a 2? Now, you can eleimate all of the odd numbers. So, your only possibilities are 2, 4, and 6. So, the probability of rolling a two given that you rolled an even number is 1/3.

Now, we can add one more conjunction rule:

$$\text{If A and B are dependent, } \Pr(A\&B) = \Pr(A) \times \Pr(B|A)$$

Draw two cards from a full deck and don't replace the first card before drawing the second. What is the probability of drawing two Kings? $\Pr(K1\&K2) = \Pr(K1) \times \Pr(K2|K1) = 1/13 \times 3/51 = 1/221$

A and B are independent events just in case $\Pr(A) = \Pr(A|B)$. This is just what you would expect. In this case, the extra information that B is true did not change the probability of A. This can only be the case if B makes no difference, in other words, they are independent.

11.2 The Rules

Here are all the rules up to this point:

11.2.1 Simple Events

$$\Pr(A) = \frac{\text{favorable outcomes}}{\text{total possible outcomes}}$$

11.2.2 Negations

$$\Pr(\neg A) = 1 - \Pr(A)$$

11.2.3 Disjunctions

$$\Pr(A \text{ or } B) = \Pr(A) + \Pr(B) \quad \text{(Incompatible Events)}$$

$$\Pr(A \text{ or } B) = \Pr(A) + \Pr(B) - \Pr(A \,\&\, B) \quad \text{(Compatible Events)}$$

11.2.4 Conjunctions

$$\Pr(A \,\&\, B) = \Pr(A) \times \Pr(B) \quad \text{(Independent Events)}$$

$$\Pr(A \,\&B) = \Pr(A) \times \Pr(B|A) \quad \text{(Dependent Events)}$$

11.2.5 Definition of Conditional Probability

$$\Pr(A|B) = \frac{\Pr(A\&B)}{\Pr(B)}$$

11.3 Odds

Odds and probabilities are different, but closely related. Probabilities are numbers between 0 and 1. Odds are not numbers, but ratios between two numbers. Odds are defined as the ratio of favorable outcomes to unfavorable outcomes. The top number will always be the same in probabiliteis and odds. The bottom number will be smaller for the odds than for the corresponding probability.

$$\text{If } Pr(X) = m/n, \text{ the odds in favor of X} = m \text{ to } (n - m)$$

The odds against something occurring are the same as the odds for it occurring, except reversed. So, if the odds for A are 2/3, the odds against A are 3/2.

Converting odds to probabilities are just as easy. The top number stays the same, and you add the two numbers together to get the bottom. If odds for S are m to n, probability of S is m/(m + n)

If the odds of OBU's beating SNU are 1:5, then the probability that OBU wins is 1/6. The proability that SNU wins is 5/6.

Why are odds useful? One reason is that they can be used to easily determine what a fair bet would be. – To make a fair bet a two will come up when you roll a die, you bet 1 dollar you will get a two and your opponent bets

5 dollars you won't. If you both bet these amounts, over the long run you both tend to break even. Since the probability of rolling a two is 1/6, you should expect to win one out of every six times. On that time that you win, you'll win five dollars. Of course, you'll lose the other five times, and will lose a dollar each time. So, in six turns, you should expect to win five dollars and lose five dollars. Notice how this follows naturally from the odds. The odds of rolling a two are 1/5. If the person betting that the event will occur bets the top number and the other person bets the bottom, it's a fair bet. Gamblers call such a bet an even-up proposition.

Casinos couldn't turn a profit with even-up bets. So, you will never get a fair bet in a casino. The house essentially takes a percentage of the winnings. They do this by paying the winners less than the actual odds would require. A good example is roulette. A roulette wheel has 38 compartments: 1-36, 0 and 00. You bet the ball will land on number 10. The probability that the ball lands on number 14 is 1/38. So, the odds of the ball landing on 14 are 1/37. If you win, the house should be willing to pay you 37 dollars on a one dollar bet. At these odds, the true odds, you break even. The house, though doesn't pay according to the true odds, they pay according to house odds. The house odds pretend that the 0 and 00 squares aren't there. So, as far as the house is concerned, the odds of your winning are 1/35. The pay 35 dollars when, in fairness, the winner deserves 37 dollars. Another way of stating this is that they are taking two dollars of your 37 dollar winnings, a percentage of 2/37 or 5.4%. Now, since they are taking two dollars of your winnings every time, you will lose in the long run.

Chapter 12

Bayes' Theorem

12.1 Introduction

Now that you have an idea of how simple, complex, and conditional probabilities work, it is time to introduce a new formula called Bayes' Theorem. This formula, although a bit more complicated than the others, can be incredibly useful. Sometimes, we know the probability of A given B, but need to know the probability of B given A. Bayes' Theorem provides a way of converting one to the other.

For example, imagine that you have recently donated a pint of blood to your local blood bank. You receive in the mail a letter informing you that your blood has tested positive for HIV antibodies. The letter informs you that you could have AIDS. How worried should you be?

You need to know the probability that you have the disease given a positive test for the disease, or $\Pr(D|+)$. Now, if you contact the company that produces the test, they will be glad to give you some information about the test. Each test has certain true positive, false positive, true negative, and false negative rates. These rates have been determined by extensive testing.

The true positive rate is the percentage of times that the test will correctly identify the samples that have the disease. The false positive rate is the percentage of times that the test will say that the disease is present when it really is not. The true negative rate is the percentage of times that the test says negative when the #ject does not have disease. The false negative rate is the rate of positive tests when the #ject did not have the disease.

In terms of probabilities, it looks like this:

True Positive: $\Pr(+|D)$ **False Positive**: $\Pr(+|\neg D)$ **True Negative**: $\Pr(-|\neg D)$ **False Negative** $\Pr(-|D)$

If you know the true positive rate and the true negative rate, you can figure out the other two. The false negative rate is equal to one minus the true positive rate, and so on.

The blood bank is concerned about contamination of the blood supply. Therefore, they want a test that has a false negative rate of zero. They aren't as concerned about the false positive rate, though. They don't want a high false positive rate, because then they would end up throwing out blood that was just fine. If they have to needlessly dispose of a few pints out of the many that they receive, though, it's no great loss. The other thing they need is an inexpensive test. They have to test every single donation, so the test they use must be one they can afford. It turns out that tests with good false negative rates are fairly inexpensive, but tests with low false positive rates are not. The blood bank takes the cheaper test, because it adequately keeps the blood supply safe, even though a few donations test positive that really were not contaminated.

So, the blood bank gives you the false positive rate: $\Pr(+|\neg D)$, but you need to know $\Pr(D|+)$. What do you do? You use Bayes' Theorem.

12.2 The Theorem

The short form of Bayes' Theorem is this:

$$\Pr(A|B) = \frac{\Pr(A) \times \Pr(B|A)}{\Pr(B)}$$

Here is an expanded version of the same formula:

$$\Pr(A|B) = \frac{\Pr(A) \times \Pr(B|A)}{\Pr(A) \times \Pr(B|A) + \Pr(\neg A) \times \Pr(B|\neg A)}$$

Believe it or not, it's often easier to use the expanded version than the shorter version. In fact, you will very rarely use the shorter version. It's generally easier to just automatically begin working the problem with the longer version.

12.3 Examples

Let's look at some examples, beginning with a very easy one. This is a problem that you wouldn't need the formula to solve, but it helps us understand how the formula works.

1. What is the probability that a card is a heart given that it is red?

We know that the answer to this problem is 1/2. Half of the red cards are hearts. Just to get used to the formula, we'll solve it using Bayes' Theorem:

$$\Pr(H|R) = \frac{\Pr(H) \times \Pr(R|H)}{\Pr(R)}$$

Since, 1/4 of all cards in a deck are hearts, $\Pr(H) = 1/4$. All hearts are red, so $\Pr(R|H) = 1$, and half of the cards are red, so $\Pr(R) = 1/2$.

So,

$$\begin{aligned} \Pr(H|R) &= \frac{\Pr(H) \times \Pr(R|H)}{\Pr(R)} \\ &= \frac{\frac{1}{4} \times 1}{\frac{1}{2}} \\ &= \frac{\frac{1}{4}}{\frac{1}{2}} \\ &= \frac{1}{2} \end{aligned}$$

The longer formula will give us the same answer:

$$\Pr(H|R) = \frac{\Pr(H) \times \Pr(R|H)}{\Pr(H) \times \Pr(R|H) + \Pr(\neg H) \times \Pr(R|\neg H)}$$

The only additions are the probability that a card is not a heart, which is 3/4; and the probability that a card is red, given that it is not a heart, which is 1/3 (If we don't include the hearts, there are three suits, one of which is red).

So,

$$
\begin{aligned}
\Pr(H|R) &= \frac{\Pr(H) \times \Pr(R|H)}{\Pr(H) \times \Pr(R|H) + \Pr(\neg H) \times \Pr(R|\neg H)} \\
&= \frac{\frac{1}{4} \times 1}{\left(\frac{1}{4} \times 1\right) + \left(\frac{3}{4} \times \frac{1}{3}\right)} \\
&= \frac{\frac{1}{4}}{\left(\frac{1}{4} + \frac{1}{4}\right)} \\
&= \frac{1}{2}
\end{aligned}
$$

Now, for a more difficult one. Here's a question you might remember from the pre-test:

> Exactly two cab companies operate in Belleville. The Blue Company has blue cabs, and the green Company has Green Cabs. Exactly 85% of the cabs are blue and the other 15% are green. A cab was involved in a hit-and-run accident at night. A witness, Wilbur, identified the cab as a Green cab. Careful tests were done to ascertain peoples' ability to distinguish between blue and green cabs at night. The tests showed that people were able to identify the color correctly 80% of the time, but they were wrong 20% of the time. What is the probability that the cab involved in the accident was indeed a green cab, as Wilbur says?

The probability that a cab is blue is 0.85, and the probability that it is green is 0.15. The probability that Wilbur will say it is green if it is in fact green is 0.80. The probability that Wilbur will not say it is in green if it is in fact green is 0.20. Symbolized, if G = "The cab was green" and W = "Wilbur says the cab was green", this is $\Pr(G) = .15$, $\Pr(\neg G) = .85$, $\Pr(W|G) = .80$, and $\Pr(W|\neg G) = .20$.

Using the expanded formula:

$$
\begin{aligned}
\Pr(G|W) &= \frac{\Pr(G) \times \Pr(W|G)}{\Pr(G) \times \Pr(W|G) + \Pr(\neg G) \times \Pr(W|\neg G)} \\
&= \frac{.15 \times .80}{(.15 \times .80) + (.85 \times .20)} \\
&= \frac{.12}{.12 + .17} \\
&= \frac{.12}{.29} \\
&= .41(\text{rounded off})
\end{aligned}
$$

12.4 The Blood Donation

Now, we can solve the problem of the blood donor's positive test. The probability that he has the disease give a positive test is a function of the probability that a person in the population has the disease, which is called the base-rate of the disease, the probability that a person tests positive if they have disease, which is the true positive rate for the test, and the probability that a person will test positive.

One current estimate that I read is that one million people in the USA now have AIDS. The most recent census reports the population as 304,059,724. This means that the base-rate, or the percentage of the population that has AIDS is 0.32%. False-positive rates for the most common, low-cost, AIDS test vary. They range from from 50% to 90%. A more expensive test, the Western Blot test appears to have a false positive rate of 4.8% of Western blood donors.

Let's plug some data into the expanded formula:

$$\Pr(D|+) = \frac{\Pr(D) \times \Pr(+|D)}{\Pr(D) \times \Pr(+|D) + \Pr(\neg D) \times \Pr(+|\neg D)}$$

Let's go with the base-rate of .32%. So, $\Pr(D) = .0032$. We'll also assume the more expensive test, so $\Pr(+|\neg D) = .048$. Let's also assume that the test is very good at catching all cases of the disease, so $\Pr(+|D) = 1$.

$$\begin{aligned}
\Pr(D|+) &= \frac{\Pr(D) \times \Pr(+|D)}{\Pr(D) \times \Pr(+|D) + \Pr(\neg D) \times \Pr(+|\neg D)} \\
&= \frac{.0032 \times 1}{(.0032 \times 1) + (.9968 \times .048)} \\
&= \frac{.0032}{.0032 + .0478} \\
&= \frac{.0032}{.051} \\
&= .063
\end{aligned}$$

So, assuming these numbers, there is only a 6.3% chance that you have AIDS given that you got a positive test for the disease, and this is assuming that the Western Blot was used and not the ELISA test, which has a much worse false-positive rate! There are more expensive and reliable ways to test for the disease, so if a person gets a positive result on one of these screening tests, they should not panic, but get the more expensive test. There have been tragic reports of people committing suicide because they got a positive result on one of the initial screening tests.

12.5 Hints

As with other probability problems, once the right numbers are plugged into the right formula, then the answers are generally easy to find. The most common problem is finding the right values in what looks like a complex paragraph.

Here's an example conditional probability problem requiring Bayes' Theorem:

> 1% of OBU students are philosophy majors. 90% of OBU philosophy majors are accepted into their preferred graduate program. 30% of OBU non-philosophy majors are accepted into their preferred graduate program. Jane is an OBU student that was accepted into her preferred graduate program. What is the probability that she is a philosophy major?

Let, P = "An OBU student is a philosophy major" and A = "An OBU student was accepted into her preferred graduate program."

The first step is to determine the conditional probability that the problem is asking us to solve. The first part is generally easy, just look for the question. In this case, "What is the probability that she is a philosophy major?" To find the given, look for the one thing that is known for certain; it won't be a probability or percentage. We know that she was accepted into her preferred graduate program. So, we want to know the probability that Jane is a philosophy major given that she was accepted into her preferred graduate program, or $\Pr(P|A)$. Once that is determined, then simply write out the formula:

$$\Pr(P|A) = \frac{\Pr(P) \times \Pr(A|P)}{\Pr(P) \times \Pr(A|P) + \Pr(\neg P) \times \Pr(A|\neg P)}$$

Now, we have to find the numbers to plug into the formula. Many, if not most, of the problems are stated in terms of percentages. The probability of A given B is a function of the percentage of B's that are A's. That is, if A's comprise half of the B's, then $\Pr(A|B) = 0.5$

So,

1. $\Pr(A|P) = 0.9$
2. $\Pr(A|\neg P) = 0.3$
3. $\Pr(P) = .01$

$$\begin{aligned}
\Pr(P|A) &= \frac{\Pr(P) \times \Pr(A|P)}{\Pr(P) \times \Pr(A|P) + \Pr(\neg P) \times \Pr(A|\neg P)} \\
&= \frac{.01 \times .9}{(.1 \times .9) + (.99 \times .03)} \\
&= \frac{.009}{.009 + .0297} \\
&= \frac{.009}{.0387} \\
&= .258
\end{aligned}$$

Chapter 13

Applying Probabilities

13.1 What Do They Mean?

In the last chapter, we defined simple probabilities as the number of favorable outcomes divided by the number of total possible outcomes. This is called the *Classical Theory* of probability. This works only when all cases are equally probable, as in drawing cards, throwing dice, tossing coins, and so on. There are other cases, though, for which the classical theory seems to fail. For example, what's the probability that the Democratic candidate will beat the Republican candidate in the U.S. Presidential election in 2020, or what's the probability that I will live to at least age 90? For these, we have two other ways of assessing simple probabilities.

The first is the frequency method. On the frequency method, a simple probability is the number of favorable observed outcomes divided by total observed outcomes. To determine the probability that I will leave at least to age 90, take a large group of people that are a lot like me, and determine what percentage of them lived at least to 90.

The second alternate method is to use degrees of confidence.[1] On this account, the simple probability is determined by the level of confidence you have that the event will occur. No matter which method is used, however, the rules

[1]This is sometimes called "degrees of belief." I prefer degrees of confidence, since it seems to me that one either believes something or not, but can have different levels of confidence in those beliefs.

governing complex probabilities are all the same.

13.2 Expected Value

It should be obvious that, when it comes to making decisions, it's important to know the probabilities involved. Is there any other information required? Think about it this way, would you ever make a bet that you were almost certain to lose? When I ask that question, most people say no. What if some offered you this wager: they pick a number between 1 and 1,000,000. If you guess it correctly, you win; otherwise, you lose. The probability of winning is $\frac{1}{1{,}000{,}000}$, so you are almost certain to lose. However, if you win, you win 10,000,000 dollars, and if you lose, you lose 1 cent. Now, everyone I ask is willing to take this bet.

So, it helps to know the probabilities when making good decisions, but it also helps to know the costs and benefits. This obviously applies to gambling situations. Imagine a dice game, in which you roll a single die. If you get a six, you win 6 dollars. If you get anything else, you pay 1 dollar. If you remember from the last chapter, we know that we can determine if the bet is fair by converting the probability of winning into odds. Here, the probability of winning is $\frac{1}{6}$, which is equal to odds of $\frac{1}{5}$. So, a fair one dollar bet should pay 5 dollars. Since it pays 6, a winning player will be paid more than she deserves. There is a simple formula that lets us determine expected value, what one should expect to gain or lose from a decision. It is

$$EV = \Pr(Success) \times Payoff + \Pr(Failure) \times Cost$$

In this case, the probability of success is $\frac{1}{6}$, the payoff is 6 dollars, the probability of failure is $\frac{5}{6}$, and the cost is 1 dollar. So,

$$EV = \frac{1}{6} \times 6 + \frac{5}{6} \times -1 = 1 - \frac{5}{6} = \frac{1}{6}$$

That is, when playing this game, you should expect to come out ahead $\frac{1}{6}$ of a dollar, or nearly 17 cents, each time you plan. Obviously, no casino will offer a game like this.

There are other, non-gambling, applications for expected value. Some standardized exams discourage guessing by assessing a penalty for every wrong answer. So, imagine you are taking such an exam, and you come to question 32, a multiple choice question with four options. You have no idea which is correct, and would have to guess. If you guess correctly, you get one point, if you guess incorrectly, you lose $\$5\frac{}{12 of a point. Should you guess? It depends on the expected value.}$

$$EV = \frac{1/4}{\times}1 + \frac{3}{4} \times -\frac{5}{12} = -\frac{3}{48}$$

Since, the expected value is negative, guessing is not in your best interests. Now, imagine that on the next question, you still don't know which option is correct, but you know that it can't be the first one. The payoffs are the same, but the probabilities have now changed.

$$EV = \frac{1/3}{\times}1 + \frac{2}{3} \times -\frac{5}{12} = -\frac{3}{48} = \frac{1}{18}$$

Now, the expected value is positive, and guessing pays.

13.3 Four Common Mistakes

13.3.1 The Gambler's Fallacy

The gambler's fallacy is committed when a person treats independent things as if they were dependent. When flipping a coin, each toss is independent — the coin can't remember what happened on the previous tosses. If a person tossed twenty heads in row, though, it's natural to think that the next toss is more likely to be tails, since it's very unlikely to get 21 tails in a row. The probability of getting tails on the 21st toss is the same as getting tails on the first toss, 0.5.

13.3.2 The Conjunction Fallacy

The conjunction fallacy is judging a conjunction to be more probable than its conjuncts. Here is a common experiment used to demonstrate the conjunction fallacy:

> Susan is 27 years old, and very intelligent. She majored in philosophy at the University of California, Berkeley. There, she was active in many different political, conservation, and justice issues. Which of the following is more likely?
>
> A: Susan is a bank teller. B: Susan is a bank teller and a feminist activist.

The answer is A, although people are inclined to answer B. Here is one way to think about these questions, every time you add another condition that must be met, the probability of meeting all of those conditions goes down (assuming the probability is not 1). Imagine that the probability of Susan being a bank teller is $\frac{1}{100}$ and the probability of her being a feminist activist is \$9$\frac{}{10.Theprobabilityofherbeingboth,assumingtheyareindependent,is\$\frac{9}{1,000},whichislowerthan\frac{1}{100}}$.

13.3.3 Failure and Success

People seem to have a strong tendency to overestimate the probability of success and underestimate the probability of failure. Consider a complex system that has many different subsystems. In order for the entire system to work, each subsystem has to work. This can be represented as a conjunction: $S\&S\&S\&\ldots$ Failure requires only the failure of one of those subsystem, now represented as a disjunction: $S \vee S \vee S \vee \ldots$ With each additional conjunct, the probability of the conjunction goes down, but with each additional disjunct, the probability of the disjunction goes up. This also applies to anything that occurs repeatedly; for example, the probability of having an accident sometime during your commute over the years.

13.3.4 Regression to the Mean

In 1953, the University of Oklahoma lost its opening game to Notre Dame, tied the next game wit Pittsburgh, then began a remarkable winning streak that continued for the next four seasons. In November of 1957, the team was featured on the cover of *Sports Illustrated*, with the title "Why Oklahoma is Umbeatable." Just a few days after the magazine cover was printed, and before the actual date of that particular issue, Oklahoma lost to Notre Dame. This began something that has come to be known as the *Sports Illustrated* curse. Apparently, after athletes get their pictures on the cover of the magazine, they are doomed to a bad season or poor performance.

There is some truth to this, but it's probably not a curse. Athletes are featured on the cover when they are having better than average performances, and it's no surprise that a better than average series is followed by a below average series. That is, for any series that has variation, some parts of the series will be above the average and others below the average. The above average parts will be followed by below average parts to bring the whole back to the average. This is called regression to the mean.

This makes it difficult to predict future performance when we just have one encounter with a person. Imagine that you are interviewing two candidates for a position in your company. One candidate has an exceptional performance and the other has a lackluster performance. Who do you hire? Could it be the case that the first is having an above average day and the second a below average day? It would be natural to hire the first, but it would be also natural to expect some regression to the mean.

Regression to the mean also makes it difficult to assess the effectiveness of policies. If a city were to have a period with higher than average rates of crime, it would be natural for citizens to demand that the mayor take action. Unsurprisingly, after initiating curfews, hiring more police officers, and instituting neighborhood watch programs, the crime rate goes down. Did it go down because of the policies, or was it simply regression to the mean? It's difficult to say.

Chapter 14

Inductive Arguments

The goal of an inductive argument is not to guarantee the truth of the conclusion, but to show that the conclusion is probably true. Three important kinds of inductive arguments are

- Inductive generalizations,
- Arguments from analogy, and
- Inferences to the best explanation.

14.1 Inductive Generalizations

Sometimes, we want to know something about some group, but we don't have access to the entire group. This may be because the group is too large, we can't reach some members of the group, etc. So, we instead study a subset of that group. Then, we infer that the entire group is probably like the subset. The group we are interested in is called the population, and the observed subset of the population is called the sample.

Imagine that I wanted to know the level of current student satisfaction with access to administration at the university. I would probably survey students to get this information. The population would be students currently enrolled at the university, and the sample would be students who were surveyed. The sample is guaranteed to be a subset of the population, since, even if I give every student a chance to take the survey, we know that not all students will participate. Some students will return the survey, giving me an answer for the sample. I then conclude that the answer for the population is about what it is for the sample.

There are some terms that are important to know when dealing with data values. The mean is the mathematical average. To find the mean, add up all the values of the data points and divide by the number of data points. For example, the mean of 1, 2, 3, 5, 9 is 4. The median is the value that is in the center, such that half of the numbers are less than it and have are greater. In this case, the median is 3. The mode is the value that occurs most often. The mode of 1, 2, 4, 2, 7, 2 is 2.

Another thing that is important to keep in mind is how spread out the values are. The average annual temperature in Oklahoma City is about the same as the average annual temperature in San Diego, leading one to conclude that the two cities have about the same comfort level. The difference is that the average monthly highs and lows range from 45 to 76 in San Diego and 29 to 94 in Oklahoma City. Three ways to talk about data dispersal are

- Range: the distance between the greatest and the smallest value,
- Percentile rank: the percentage of values that fall below some value, and
- Standard deviation: how closely things are grouped the mean.

14.1.1 Random Samples

In an inductive generalization, the premises will be claims about the sample, and the conclusion will be a claim about the population. Although such arguments are not valid, they can be inductively strong if the sample is good. Good samples are first, not too small, and second, not biased. The ideal sample is representative, which means that it matches the population in every respect. Of course, reasoning from a representative sample to a population would always be perfect, since they would be, except for size, mirror images of each other. Unfortunately, there is no way to guarantee that a sample is representative, nor is there any way, presumably, to know that a sample is representative. To know that our sample was representative, we would already have to know everything about the population. If that were the case, what's the use taking a sample?

Since we can't do anything to guarantee a representative sample, our best way to ensure our sample is not biased is for it to be random. A random sample is one such that every member of the population had an equal chance of being included in the sample. Randomness is very difficult to achieve in practice. For example, if I send out an email invitation to participate in the university survey, it looks like every student has an equal chance of being included in the sample. Actually, though, there are several groups that are guaranteed to not be included: students who have forgotten their email password, students who don't check email, students who don't really care, etc. Even if I have a truly random sample, it is still possible for it to be a biased sample. This is called random sampling error. Random samples, though, are less likely to be biased than non-random samples.

14.1.2 Margins of Error

The other feature of a good sample is that it needs to be big enough. How big is big enough? It often depends on what we want to know and the result that we get from the sample. This is because of something called the margin of error. Let's assume that I have a random sample from a population. I get a value from the population, and I can be pretty confident that the value in the population is within the margin of error from the value in the sample. How confident? It depends on how big the margin of error is.

Does this sound confusing? It's really not. Imagine that a friend is coming to visit you at your home on Monday. You, wanting to be prepared, asked her when she would arrive. Here are some possible responses that she might give:

1. "Exactly 9:00"
2. "About 9:00"
3. "Sometime Monday morning"
4. "Sometime on Monday"

Now, which of these can you be most confident is true? It's easy to see that the first is the one in which we should be the least confident, and the fourth is one in which we should be the most confident. The first is very precise, and then the answers become increasingly more vague, and thus more likely to be true. Margins of error function the same way. The greater the margin of error, the more vague the claim. The more vague the claim, the greater the likelihood of being true.

There is a trade-off, though. Your friend could tell you that she will be there sometime this year. That's very likely to be true, but not very helpful, because it's so imprecise. The trade-off is between precision and likelihood. The more precise the claim, the less likely it is to be true. What we need to find is the best balance between the two.

For inductive generalizations, precision is a function of the margin of error. Likelihood is expressed by something called the confidence level. The confidence level of a study is a measure of how confident we can be that the right answer in the population is within the margin of error of the value in the sample. Here is a chart with confidence levels and their respective margins of error, expressed in standard deviations (SD).

Margin of Error	Confidence Level
1 SD	67%
2 SD	95%
3 SD	99%

So, if my margin of error is ± 1 standard deviation, then I can be 67% confident that the value in the population is within that margin of error. If I increase the margin of error by another standard deviation, my confidence level leaps a whole 32% from 67% to 95%. Increasing it by another margin of error only gives me an additional 4% confidence level. So, the best balance between likelihood and precision seems to be at the 95% confidence level, and most, if not almost all, studies are done at the 95% confidence level.

The margin of error is a function of the sample size. As the sample size gets larger, the margin of error gets smaller. Statisticians use complicated formulas to calculate standard deviations and margins of error. If the population is very large, though, we can estimate them fairly simply: $1SD = \frac{1}{2 \times \sqrt{N}}$, where N is the sample size. So, at the 95% confidence level, the margin of error is $\$1\frac{}{\sqrt{N}.Thisgivesusthefollowingmarginsoferrorforafew,easytocalculate,samplesizes:}$

Sample Size	Margin of Error
100	± 10
400	± 5
10,000	± 1

Remember when I said that how large a sample needs to be depended on what we wanted to know and the result we got from the sample? Now, that should make more sense. Let's say you were conducting a survey to determine which of two candidates were going to win an upcoming election. You somehow managed to get a random sample of 100, 70% of whom were going to vote for candidate A. So, you conclude that between 60% and 80% of the population were going to vote for candidate A. Since your range does not overlap the 50% mark, you rightfully conclude that candidate A will win. Now, had 55% of your sample intended to vote for candidate A, you could only infer that between 45% and 65% of the population intended to vote for that candidate. To conclude something definite, you will need to shrink the margin of error, which means that you'll need to increase your sample size.

14.1.3 Bad Samples

Since a good sample is unbiased and large enough, there are two ways for samples to be bad. Generalizing from sample that is too small is called committing the fallacy of hasty generalization. Here are some examples of hasty generalizations:

1. I've been to two restaurants in this city and they were both bad. There's nowhere good to eat here.
2. Who says smoking is bad for you? My grandfather smoked a pack a day and live to be 100!

Cases like the second example are often called fallacies of anecdotal evidence. This happens when evidence is rejected because of a few first-hand examples. (I know someone who had a friend who...)

We're often not very aware of the need for large enough samples. For example, consider this question:

A city has two hospitals, one large and one small. On average, 6 babies are born a day in the small hospital, while 45 are born a day in the large hospital. Which hospital is likely to have more days per year when over 70% of the babies born are boys?

1. The large hospital
2. The small hospital
3. Neither, they would be about the same.

The answer is "the small hospital." Think of this as a sampling problem. Overall, in the world, the number of boys born and girls born is roughly the same.[1] A larger sample is more likely to be close to the actual value than a smaller sample, so the small hospital is more likely to have more days when the births are skewed one way or another.

We'll call drawing a conclusion from a biased sample the fallacy of biased generalization.[2] Imagine a study in which 1,000 different households were randomly chosen to be called and asked about the importance of regular church attendance. The result was that only 15% of the families surveyed said that regular church attendance was important. On the surface, it seems that a study like this would be good — it's certainly large enough and the families were chose randomly. Let's imagine that the phone calls were made between 11:00 and 12:00 on Sunday morning? Would that make a difference?

The classic example is the 1936 U.S. presidential election, in which Alfred Landon, the Republican governor of Kansas, ran against the incumbent, Franklin D. Roosevelt. *The Literary Digest* conducted one of the largest and most expensive pools ever done. They used every telephone directory in the country, lists of magazine subscribers, and membership rosters of clubs and associations to create a mailing list of 10 million names. Everyone on the list was sent a mock ballot that they were asked to complete and return to the magazine. The editors of the magazine expressed great confidence that they would get accurate results, saying, in their August 22 issue,

> Once again, [we are] asking more than ten million voters – one out of four, representing every county in the United States – to settle November's election in October.
>
> Next week, the first answers from these ten million will begin the incoming tide of marked ballots, to be triple-checked, verified, five-times cross-classified and totaled. When the last figure has been totted and checked, if past experience is a criterion, the country will know to within a fraction of 1 percent the actual popular vote of forty million [voters].

[1] There are slightly more boys born than girls. Worldwide, the ratio of boys to girls is 107:100. This is partially explained by sex-selective abortion in countries where sons are more desired than daughters. If we eliminate those cases, the ration is still 105:100.

[2] There is no general agreement on this. Sometime "hasty generalization" is used for both. I think it's useful to have two terms to distinguish the two different errors.

2.4 million people returned the survey and the magazine predicted that Landon would get 57% of the vote to Roosevelt's 43%.

The election was a landslide victory for Roosevelt. He got 62% of the vote Landon's 38%. What went wrong?

The problem wasn't the size of the sample, although only 24% of the surveys were returned, 2.4 million is certainly large enough for an accurate result. There were two problems. The first was that 1936 was the end of the Great Depression. Telephones, magazine subscriptions, and club memberships, were all luxuries. So, the list that the magazine generated was biased to upper and middle-class voters.

The second problem was that the survey was self-selected. In a self-selected survey, it is the respondents who decide if they will be included in the sample. Only those who care enough to respond are included. Local news stations often do self-selected surveys. They will ask a question during the broadcast, then have two numbers to dial, one for "Yes" and another for "No." There's never a number for "Don't really care," because those people wouldn't bother calling in anyway. The 1936 survey failed to include people who didn't care enough to respond to the survey, but they very well might have cared enough to vote.

14.1.4 Bad Polls

Good surveys are notoriously difficult to construct. There are a number of ways that surveys can be self selected — think of what you do when you see someone standing in the mall holding a clipboard. Caller ID now makes telephone surveys self-selected. If your caller ID read, "ABC Survey Company", would you answer the phone?

Today, telephone surveys are almost guaranteed to be biased. Most telephone surveys are conducted by calling traditional "landline" phones, not mobile phones. More and more, though, people are rejecting such phones in favor of only having mobile phones. So, by having a telephone survey, pollsters are limiting their responses to mostly older generations.

Another example of a bad poll is the push-poll. Here, the goal is not to pull information from the sample, but to push information to the people in the sample. A few years ago, I received a call from the National Rifle Association today. A recorded message from the NRA Executive Vice-President concerning the U.N. Small Arms Treaty was followed by the following single question survey:

> Do you think it's OK for the U.N. to be on our soil attacking our gun rights?

I was instructed to press "1" if I did not think was OK for the U.N. to be on our soil attacking our gun rights. That was followed by a repeat instruction to press "1" if I did not think it was OK. I was then instructed to press "2" if I did think was OK for the U.N. to attack our gun rights. (Note that I was only given that instruction once.)

This survey was a classic example of a push-poll. It was designed simply to push a message out to the population. This is evident from the question. What useful information do we expect to gain from asking people if they think it's OK for the U.N to attack our gun rights. Do we really not know how people will answer that question? It's no different from my polling my students to find out if they would like to get out of class early. As far as information gathering goes, it's a complete waste of time and money. For propaganda pushing, on the other hand, it's very effective.

This is also a good example of a slanted question. When I looked up the purpose of the U.N. Small Arms Treaty, it's stated purpose was to keep firearms out of the hands of terrorists. If the question had been, "Do you think it's OK that the U.N. negotiate a treaty designed to prevent guns from falling into the hands of terrorists?", I would expect a very different result.

One reason it is very difficult to construct good surveys is because of order effects. The order that questions appear in affects how people will respond to them. A study conducted a survey that included these two questions:

1. Should U.S. allow reporters from a fundamentalist country like Iraq come here and send back reports of the news as they see it to their country?
2. Should an Islamic Fundamentalist country like Iraq let US news reporters come in and send back reports of the news as they see it to the US?

When question 1 was asked first, 55% of respondents said yes. When question 1 was asked second, however, 75% of the respondents answered yes. What seems to happen here is a basic commitment to fairness. Once I have already said that other countries should let in our reporters, then there's no fair reason for me not to allow their reporters into my country.

To summarize, here is a list of bad polls:

- Self-selected
- Ignore order effects
- slanted questions
- push polls
- loaded questions

14.2 Arguments from Analogy

Another common type of inductive argument is the argument from analogy. Let's say that you are shopping for a car, so that you can have transportation to school, work, and so on. Since it's important that you get to the places on time, you need to buy a reliable car. You find a good deal on a 2013 Honda Civic, but how do you know that it will be reliable? One way to judge reliability is to look at reliability reports from owners of other 2013 Honda Civics. The more cases in which they reported that their cars were reliable, the more you can conclude that yours will be also.

With inductive generalizations, we were reasoning from a sample to a population. Arguments from analogy reason from a sample to another individual member of the population, called the target. The members of the sample have a number of properties in common; they are all Honda Civics made in 2013. They also have another property in common that we will call the property in question, in this case, reliability. Our target has all of the other properties, so it probably also have the property in question. The more similar our target is to the sample in some respects, the more similiar it is likely to be in other respects. Here is the basic structure:

1. members of s have properties $P \ldots P$ and P_Q.
2. The target has $P \ldots P$.
3. The target probably also has P_Q

These arguments are weak when

1. The similarities stated aren't relevant to the property in question. (In our example, the color of the car would not be relevant to its reliability.)

2. There are relevant dissimilarities. (If all the members of the sample had excellent maintenance records, but our target had very poor maintenance, then we wouldn't expect the target to be reliable just because the members of the sample were.)
3. There are instances of the sample that do not have the property in question. (The more 2013 Honda Civics we find that are unreliable, the weaker the argument becomes.)

So, the arguments are stronger when there are

1. More relevant similarities,
2. Fewer relevant dissimilarities, and
3. Fewer known instances of things that have the shared properties but lack the property in question.

14.3 Inferences to the Best Explanation

Our final type of inductive argument to discuss in this chapter is the inference to the best explanation, also called abductive reasoning. Very simply, this is used when we have a situation that needs explanation. You consider the possible explanations, and it's rational for you to believe the best one.

How do we decide which explanation is best, though? Here are some critiera:

1. It must explain the data, that is, tell us why the data is true.
2. It must be a good explanation, which means be
    1. Plausible
    2. Simple
3. Of the good explanations, be the best.

Chapter 15

Reasoning about Causation

Much of our reasoning is about causes and effects. We argue about proposed solutions to social problems, but how can we evaluate a solution if we haven't identified the cause? Reasoning about causation is difficult but important, if we're wrong about causes, then we won't be able to bring about our desired effects. So, it's important to be able to accurately detect causes and to avoid the common mistakes in causal reasoning.

15.1 Correlations

An important 18th century philosopher, David Hume, pointed out that we don't directly observe the necessary causal link between two events. What we observe is that the two events are correlated, then we infer that they are causally related. The stronger the correlation, the greater the evidence that there is some causal link.[1]

Correlation is a measure of the degree to which two variables are related. A variable is something which can have different values, like age, GPA, or nationality. Some variables, called dichotomous variables, only have two values, pass/fail, enrolled, not enrolled, etc. If the variables are positively correlated, then higher values of one tend to go with higher values of the other, and lower values of one tend to go with lower values of the other. Examples

[1] We'll see later that, even with very strong correlation, it can be very difficult to say exactly what the causal relation is.

of positively correlated variables include height and weight, and number of years in school and income. Variables that are negatively correlated tend to go in opposite directions, like absences and GPA. None of these examples are *perfect* correlations. Perfect positive correlations always go in the same direction, and perfect negative correlations always go in opposite directions. An example of a perfect positive correlation is height in inches and height in centimeters. Almost all perfect correlations are as trivial as this example. If there is no correlation between two variables, then they are independent, like state of residence and GPA. Correlations are measured from -1 to 1. -1 is a perfect negative correlation, 1 is a perfect positive correlation, and 0 is no correlation.

We make predictions on the basis of correlations. If regular attendance is positively correlated with doing well in a class. Then, if you have good attendance, we can predict that you will do well. The higher the correlation, the stronger the prediction. We can represent correlation in probability terms. Assume that A = regular attendance and P = passing the course. If the two are positively correlated, then $\Pr(P|A) > \Pr(P|\neg A)$. It will also be true that $\Pr(P|A) > \Pr(P)$. Both of these probability statements will be true if either is.

Correlation is symmetrical, meaning it goes both ways. If A is positively correlated with B, then B is positively correlated with A. In probability terms, if $\Pr(A|B) > \Pr(A)$, then $\Pr(B|A) > \Pr(B)$. Causation, however, is not symmetrical. If A is the cause of B, then B is not the cause of A. The important point here is that correlation is evidence for causation, but correlation is not causation.

15.2 Causal Fallacies

15.2.1 Post Hoc, Ergo Propter Hoc

The Latin phrase, "Post hoc, ergo propter hoc" means "After this, therefore caused by this."[2] This happens when one believes that two events have a causal relationship simply because they have a temporal relationship, or that A is caused by B just because A was followed by B. A trivial example would be believing that the crowing of roosters cause the sun to rise just because

[2] This is often called simply the "Post hoc" fallacy.

the crowing precedes the rising. I have heard about people who wear the same shirt to every game that their favorite team plays. They wore it once and their team one, therefore their wearing the shirt some caused the team to win. Many superstitions and stereotypes are rooted in post hoc fallacies.

15.2.2 Ignoring a Common Cause

Sometimes two events are correlated, not because one causes the other, but because they are both effects of the same thing. A person might have a bad headache and begin to feel nauseated. It is natural to think that the headache caused the nausea, but it might well be the case that they are both symptoms of the flu.

An interesting example is that the bigger a child's shoe size is, the better the child's handwriting will be. I don't think we would conclude that big feet causes better penmanship. It's more likely that having bigger feet is caused by increased physical development of a child, which makes the motor control possible that good penmanship requires.

15.2.3 Confusing Cause and Effect

This happens when you think the cause is really the effect, and the effect is really the cause. You might think that this fallacy should be difficult to commit, since causes generally precede their effects. No one should observe that the number of firefighters on the scene is positively correlated with the size and intensity of the fires, then conclude that more firemen cause bigger fires.

Sometimes, though, it's hard to separate causes from effects. A sports announcer might say that the team is moving the ball well because they got their confidence back. Could it be that they are more confident now that they began to play well? A family therapist might face a counseling situation with a dysfunctional family that has a child with severe behavioral problems. Is the dysfunction a cause of the behavioral problems, or do the behavioral problems cause the dysfunction. These cases are particularly difficult, because they generate what is called a "feedback loop." As the behavioral problems increase, so does the dysfunction, which causes even greater behavioral problems, which causes the dysfunction to become greater, and so on.

15.3 Mill's Methods

Mill's methods are ways that are used to determine genuine causes. They aren't foolproof — they certainly won't work in every situation, but they can be very useful in helping to identify the most likely cause. There are four different methods, each to be used in different situations. Unfortunately, the easiest examples are cases of food poisoning.

15.3.1 The Method of Agreement

Imagine that you and a friend go out to dinner on Friday night. When you wake up Saturday morning, you aren't feeling well. Naturally, you call your friend and find out that she is also ill. So, what did you eat last night that's causing the illness? Since you're both ill, it must be something you both ate. Ideally, there will be only one thing that you both ate. Let's expand the example to a family of four:

Person	Oysters	Beef	Salad	Cake	Ill
Mom	Yes	Yes	Yes	Yes	Yes
Dad	Yes	No	No	Yes	Yes
Sister	Yes	Yes	No	No	Yes
Brother	Yes	No	Yes	No	Yes

Everyone is ill, what's the cause? The oysters are the only thing that they all ate, so that's the most likely culprit.

This is called the Method of Agreement. It's used to detect the cause when there are multiple cases of the effect.

15.3.2 The Method of Difference

The Method of Difference is used when the effect occurs in some cases but not in others. The cause must be a difference between the two. If there's only one difference, then we should conclude that the single difference is the cause. Look at this example:

Person	Oysters	Beef	Salad	Cake	Ill
Mom	Yes	Yes	Yes	Yes	Yes
Dad	Yes	Yes	Yes	Yes	Yes
Sister	Yes	Yes	Yes	Yes	Yes
Brother	Yes	Yes	No	Yes	None

The brother did not get ill, but everyone else did. So, the cause must be something the rest ate, but he did not. The only thing is the salad, so that must be it.

15.3.3 The Joint Method

Person	Oysters	Beef	Salad	Cake	Ill
Mom	Yes	Yes	Yes	Yes	Yes
Dad	Yes	Yes	No	Yes	Yes
Sister	Yes	Yes	Yes	No	Yes
Brother	Yes	No	No	Yes	No

The Joint Method is simply applying both the Method of Agreement and the Method of Difference in a case. Note here that the brother is not ill, but everyone else is. Applying the Method of Difference, we decide that it is either the beef or the salad. Now, we apply the method of agreement, and see that, of those two things, the beef is the only one that everyone who got ill ate. So, the cause is the beef.

15.3.4 The Method of Concomitant Variation

Person	Oysters	Beef	Salad	Cake	Ill
Mom	Yes	Yes	Yes	Yes	Yes
Dad	Yes	Yes	Yes	Yes	Yes
Sister	Yes	Yes	Yes	Yes	Yes
Brother	Yes	Yes	Yes	Yes	Yes

Here, everyone is ill, and everyone had an identical meal. So, neither the method of difference nor the method of agreement will help identify the cause. They are not, however, ill to the same extent. The brother and the sister feel a bit queasy, the mom is fairly ill, but the dad is critically ill. Everyone had identical portion sizes of everything except the oysters. The brother and the sister ate one oyster, the mom had four, and the dad ate a dozen. The degree to which they ate the oysters matches the degree to which they are ill.

This is called the method of concomitant variation. If a quantitative change in the effect is correlated with quantitative changes in some given factor, then that factor should be cause.

15.4 Causal Studies

15.4.1 Types

Mill's methods are handy tools to determine causes in fairly simple situations. For cases like finding the cause of a disease, or determining the causal efficacy of a drug, we'll need some more complex, and more careful, tools.

Two basic types of studies are cross-sectional studies and longitudinal studies. A cross-sectional study collects data at one point in time, for instance, the end-of-semester course evaluation is a cross-sectional study. A longitudinal study collects data over time.

There are two types of longitudinal studies. A prospective longitudinal study tracks data forward in time. Subjects are chosen, then tracked over time to see if the effect occurs. A retrospective study, on the other hand, looks backward in time to gather data. Subjects are chosen by effect, then the study looks back to see if the cause was present.

Both prospective and retrospective are useful for different purposes. Let's imagine a study to determine if smoking causes heart disease. A prospective study will pick subjects, some who smoke and others who do not, and trace them over time to determine if they develop heart disease. There are two problems, here. First, this study will take a long time. It won't be the case that someone will start smoking one week and develop heart disease the next. It will likely take decades to get the data. The second problem is related, the study will be very expensive, since it requires medical testing of many subjects over many years.

Retrospective studies, in comparison, are short and inexpensive. An example of a retrospective study would be finding some people who have heart disease and others who do not, then looking back to see if the rates of past smoking in the two groups. We would expect there to be more smokers in the heart disease group than in the non-disease group.

So, why should one do a prospective study, if retrospective studies are cheaper and faster. Retrospective studies also have two significant problems. First, they often depend on subjects accurately reporting their past histories. We tend to not be entirely honest about our bad habits. Second, retrospective studies are testing for the wrong probability. In the smoking and heart disease case, what we want to know is $\Pr(D|S)$, the probability of getting heart disease given that the subject is a smoker. What they tell us is the $\Pr(S|D)$, the probability of being a smoker, given that the subject has heart disease.

So, retrospective studies can be unreliable, and they are testing for the wrong thing. Why do them? Why not just do the prospective study that will give us the information we need? One answer is that prospective studies, since they are so long and expensive, will require funding before they can begin. Before the researchers can get the funding, however, they will have to give some reason to believe that the study will be fruitful. A retrospective study, although far from conclusive, can show that there is some reason to believe that a prospective study would be successful.

15.4.2 Controlled Studies

Sometimes, just believing that one is receiving treatment can lead to improvement. This is called the placebo effect. To compensate for the placebo effect, scientists use controlled experiments.

A controlled study has a control group — a group that does not receive the purported cause. To eliminate the placebo effect, the control group has to receive something, most likely a causally inert substance called a placebo. The group that gets the purported cause is called the experimental group. So, every person in the study receives something, and they do not know which group they are in. This is called a blind study. Ideally, the researchers who immediately interact with subjects will not know which group the subjects are in. This is called a double-blind study.

In an interventional (or experimental) study, the researchers assign the groups. In a randomized interventional study, the groups are randomly assigned by the researchers. In an observational study, the subjects assign themselves to groups. There are observational studies with control groups, but there are also observational studies without control groups. In general, from most reliable to least, studies are ranked in this order:

1. Interventional study
2. Observational study with controls
3. Observational study with no controls

A researcher may choose to do an observational study for very good reasons. Some people hypothesized that having an abortion increased the risk of breast cancer. For ethical reasons, a researcher would not want to divide women randomly into two groups, those who would be given induced abortions and those who would not.

It is important to understand that what the control group receives depends on the nature of the hypothesis being tested. If the hypothesis is that the substance being tested has no causal effect, then the control group will receive a placebo. If the hypothesis is that the new drug has no more causal effect than the current one, then the control group will receive the current drug.

Chapter 16

Heuristics and Cognitive Biases

16.1 Heuristics

Ideally, when we are pressed to make a decision about what to do or believe, we would be able to gather and assess the evidence required to make a good decision. Sometimes, though, we're just not in a position to do that. We are finite creatures with limited attention spans, limited computational abilities, and even limited interest. Besides that, we very often do not have the time or energy necessary to gather evidence. So, when we are pressed for quick decision, we just have to work with what is available. What we use in those cases are reasoning shortcuts, called inferential heuristics. They are quick, and often get us close enough to the truth. Given our cognitive limitations, quick and close is sometimes better than slow and precise.

We use these inferential heuristics often, and we're usually not away of them. They allow us to draw very quick inferences without having to gather evidence or compute probabilities, and they work nearly automatically. it's important, though, to understand them, because there are certain situations in which they tend to fail us in systematic ways. Here are some examples:

1. Do more people in the United States die from murder or suicide?
2. Are there more six letter words in English that end in 'ing' or that have 'n' as their fifth letter?

The answer to the first question is that there are nearly twice as many suicides as there are murders. There have to be more six letter words that have ‘n’ as their fifth letter. If you got these wrong, it was likely from using the availability heuristic.

16.1.1 The Availability Heuristic

We use the availability heuristic when we draw conclusions about frequencies or proportions based on what we can easily recall in our memories or imaginations. When we do this, we’re conducting our own sampling, but that sampling is all in our heads. This heuristic is often accurate. For instance, if asked “Are there more Fords or Jeeps on the road?”, you will naturally think of what you have seen in the past. having seen more fords than jeeps, you’ll conclude correctly that there are in fact more Fords than Jeeps. You are using the data that is readily available to you, because it is present in memory.

When the data is available because it really occurs more frequently, then the heuristic works find. The problem, though, is that there are many other reasons for availability. Something might be more available because it’s reported more frequently in the media, not because it actually occurs more frequently. This is the case with suicides and murders. All murders are reported, but few suicides are reported. This leads us to mistakenly believe that there are more murders than suicides.

This also makes us underestimate some things and overestimate others, which, in turn, causes us to make bad decisions concerning risks. Every terrorist attack is reported, but few deaths by heart attack or stroke are reported. Shark attacks are reported nationwide, but deaths caused by other animals are not. Cases of winning the lottery are always reported, but no one mentions the number of people who lost. We are led to believe, then, that winning the lottery is not the near statistical impossibility that it is.

The easier it is to recall things, the more likely we tend to think they are. How many six letter English words ending in ‘ing’ can you recall? (Flying, skiing, typing, boring,. . .) Are there more famous people from Oklahoma or from Kansas? What can you easily recall? A person from Oklahoma is likely to answer that differently than a person from Kansas.

16.1.2 The Representativeness Heuristic

"Representativeness" is an awkward word, but, unfortunately, we're stuck with it. We use the representativeness heuristic when we conclude that something is more closely something resembles the typical example of some type, the more likely it is to be of that type. For example, Joe is very physically fit, over six-feet tall, 250 pounds, and played football in college. Which of the following is more likely?

1. Joe is an NFL linebacker.
2. Joe works in the insurance industry.

The answer is that Joe is more likely to be an insurance agent. If you answered that Joe is more likely to be an NFL linebacker, then you were using the representativeness heuristic. Some uses of the representativeness heuristic are perfectly reasonable, maybe even wise The snake in front of me has vertical slits for pupils, rough scales, a triangular shaped head, and distinct fangs. It looks like the typical poisonous snake, so I'll treat it like it's dangerous.

The instances when the representativeness heuristic goes wrong are often because it leads us to ignore base rates. The base rate for a characteristic is the frequency, proportion, or percentage of things in the population that have that characteristic. In probability terms, this is called the prior probability of the characteristic. 79% of the world's population have brown eyes, so the base rate of brown-eyed people is 0.79. A quick Internet search reveals that there are currently 357 linebackers in the NFL. I don't know how many people work in the insurance industry, but it's many times more than 357. So, given the base rates, and the fact that the description is compatible with working in insurance, it is more likely that Joe is in insurance than the NFL.

Ignoring base rates when estimating probabilities and making predictions is called the base rate fallacy.

16.1.3 The Affect Heuristic

A third heuristic is the affect heuristic. We won't spend much time on it, since it's something that we all have probably used in the past. If you've ever decided to do something foolish because you were angry, then you've likely used the affect heuristic. The affect heuristic is basing a decision on emotion, not on a reasonable assessment of the risks and payoffs. Examples of the affect heuristic might be a gambler playing against the odds because he feels lucky, or posting something you regret on social media because you are angry.

Of course, it's not always wrong to base decisions on our emotions. It may even be better at times to base those decisions on emotions rather than a calculation of the costs and payoffs. What is a better reason to buy flowers for my wife, a cost-benefit analysis, or because I love her? There are other times, though, when our emotions keep us from making good decisions, that's when we need to be careful.

16.2 Cognitive Biases

Cognitive biases are systematic ways that we are prone to reason badly and make irrational decisions. That we have these biases has been confirmed by much research. So, it's important to understand what they are and when they are likely to be used to avoid falling into the traps that they set for us. Psychologists have identified many different cognitive biases, I will limit our discussion to some that I think are particularly important.

16.2.1 Confirmation Bias

Confirmation bias is the tendency to recognize only the evidence that supports what we already believe, or interpret any evidence in a way that confirms our preconceptions.

16.2.2 Belief Bias

Belief bias is tendency to judge arguments based on whether we believe their conclusions or not.

16.2.3 Anchoring and Adjustment

This is the tendency to begin with a given piece of information and adjust insufficiently from there. (So, this is also called insufficient adjustment.)

16.2.4 Contrast Effect

The contrast effect is the tendency to determine the value of something based, not on its inherent qualities, but by comparing it to the things around it.

16.2.5 Endowment Effect

This is our tendency to think that something is more valuable simply because it is ours.

16.2.6 Loss Aversion

This explains why a loss seem greater than a gain of the same size

16.2.7 Status Quo Bias

This is a preference for things remaining the way they are now.

16.2.8 Framing Effect

This is the tendency for people to make different choices when given the same options expressed in different language.

16.2.9 Self-Fulfilling Prophecy

A self-fulfilling prophecy a the tendency of one's expectations about the future to influence the future in such a way that makes those expectations come true.

This is related to the Pygmalion effect, the tendency for people to live up, or down, to our expectations.

16.2.10 Wishful Thinking

This is believing something simply because you want it to be true.

16.2.11 Primacy and Recency Effects

The primacy effect is the tendency to remember the first members of a series. The recency effect is a tendency to remember the last members of a series.

16.2.12 Just-world Hypothesis

This is a belief that the world is basically just, that is, people get what they deserve.

16.2.13 Sunk Costs

A sunk cost is a cost that has already been paid and cannot be recouped. For example, imagine that you have paid 20 dollars for a ticket to a movie. After an hour, you find that you are really not enjoying the movie, and you're deciding whether to stay for the second hour. Many people would stay because, otherwise, they would be wasting 20 dollars. The twenty dollars, though, is gone either way, and shouldn't play a role in the deliberation.

16.2.14 Magic Numbers

A magic number is a relatively arbitrary target. The problem is that people will set targets, and, if they finish just short of the target, consider themselves failures. A larger problem is that once the target is reached, the tendency is to quit, rather than try to exceed the target.

16.2.15 Lake Woebegone Syndrome

This is the tendency that we all have to believe that we are above average.

16.2.16 Dunning-Kruger Effect

This is a bias in which people who have low levels of ability at some task believe that they have high levels of ability.

16.2.17 Validity Effect

The validity effect is the fact that mere repetition of a claim increases the tendency of people to believe it.

16.2.18 Mere Exposure Effect

This is our tendency to like things more the more times that we are exposed to them.

Chapter 17

The Effects of the Situation

Why do people who know better do foolish things? Why do good people do things that seem callous, or even immoral? One answer is that they don't really know better, or that they really aren't as moral as they once seemed. Another answer is that maybe they wouldn't have done those things had they not been in their respective situations. We'll see experiments that show that situations can somehow lead people to do things they would never otherwise do.

17.1 The Fundamental Attribution Error

There are two kinds of explanations of human behavior. First, we can explain a person's behavior by appealing to internal causes, like desires, beliefs, and character states. On the other hand, there are external causes, features of the situation the person is in at the time. It should be no surprise to anyone that both of these causes are at work whenever we act in a particular situation. When we underestimate the role that the external causes play, we commit what psychologists call the fundamental attribution error.

17.2 Experiments

Here are a few of the experiments that show the various ways that situations affect behavior

17.2.1 Game Show

In a 1977 study, subjects were assigned to be either questioners or contestants. The questioners were tasked to write challenging general knowledge questions and then test the contestants with them. Afterwards, both the questioner and the contestant were asked to rate the general knowledge of themselves and the other person. Questioners were encouraged to ask difficult questions from their own background knowledge, which gave them a significant advantage. As one would expect given the situation, the questioners were able to ask question that the contestants could not answer. Interestingly, both the questioner and the contestant rated the questioner as more intelligent than the contestant. (?)

17.2.2 Good Samaritan Study

John Darley and Daniel Batson recruited seminary students for a study that was purportedly on religious education. The subjects first completed some personality surveys, they were then told to go to another building to continue the study. Some subjects were told they would be giving a talk on job opportunities for seminary graduates, and others were told they would be giving a talk on the parable of the Good Samaritan. Another variable in the study was the "hurry condition." The subjects were divided into three further groups that were told:

High-hurry: "Oh, you're late. They were expecting you a few minutes ago."

Intermediate-hurry: "The assistant is ready for you, so please go right over."

Low-hurry: "It'll be a few minutes before they're ready for you, but you might as well head on over. If you have to wait over there, it shouldn't be long."

They were given directions and a map to the other building. On the way there, they encountered a man who was slumped over in a doorway, not moving, with his head down and eyes closed. As they passed by, the person would cough twice and groaned. The subjects were rated on whether, and to what degree, they helped the person.

40% of the subjects offered some help to victim. 63% in the low-hurry group helped, 45% of the medium-hurry group, and 10% of the high-hurry group. A person who is not in a hurry may very well stop to help. A person who is in a hurry is very likely to just keep going. Even those who were in a hurry and were going to talk about the Good Samaritan were very likely to not help. The researchers reported that some students on their way to talk about the Good Samaritan literally had to step over the victim to hurry on their way.

17.2.3 Helping Studies

In 1964, Kitty Genovese was stabbed to death in front of an apartment building in New York City. The *New York Times* reported that 38 people witnessed the attack, yet no one called the police or came to her aid. (**?**) Even though the original story has been shown to be false in some important details, it prompted some important psychological research into what is called "bystander apathy," the theory that people are less likely to offer help when there are other people present.[1]

John Darley and Bibb Latané hypothesized that the presence of other onlookers would incline people to not intervene for several reasons:

1. The responsibility to intervene was diffused among the observers.
2. Any blame for not intervening was also diffused among the observers.
3. When an observer cannot see the reactions of other observers, it was possible that someone had already acted.

Test subjects were placed in individual rooms, where they were told that they would participate in a discussion of problems in college life. They were told that they were in a group of a certain size, and would communicate electronically with microphones and headphones to the other participants, who were also in individual rooms. Each person would have two minutes to speak, and while they were speaking, all other microphones would be off. (Actually, all of the other "participants" were recordings.) The first person to speak talked about his difficulty adjusting, specifically mentioning that he was prone to siezures. As he talked, he became louder and more incoherent, saying,

> I-er-um-I think I-I need-er-if-if could-er-er-somebody er-er-er-er-er-er-er give me a liltle-er-give me a little help here because-er-I-er-I'm-er-er- h-h-having a-a-a real problem-er-right now and I-er-if somebody could help me out it would-it would-er-er s-s-sure be-

[1] Two people claimed to have called the police, and one woman rushed downstairs to give aid. For details see **?** and **?**.

> sure be good . . . because- er-there-er-er-a cause I-er-I-uh-I've got a-a one of the-er-sei—er-er-things coming on and-and-and I could really-er-use some help so if somebody would-er-give me a little h-help-uh-er-er-er-er-er c-could somebody-er-er-help-er-uh-uh-uh (choking sounds). . . . I'm gonna die-er-er-I'm . . . gonna die-er-help-er-er-seizure-er-[chokes, then quiet].

The experimenter began timing the subject's response time from the beginning of the victim's speech. Some subjects thought that they were in a two-person group, others in a three-person group, and the rest in a six-person group. Here are the results of people reporting by the end of the victim's message:

Group size	% responding	Time to respond in seconds
2	85	52
3	62	93
6	31	166

Notice that the larger the group, not only were the subjects less likely to report at all, but they were slower to report when they did. Every subject in the two-person groups eventually reported, some after the victim finished his message.

There are reasons that would explain this behavior. First, since we can't see or hear wha the other people are doing (or, more importantly, _not_doing), we tend to assume that someone else has already intervened. The second reason is something called diffusion of responsibility. If there is one person besides the victim present, then that person feels 100% of the responsibility for acting. If there are 10 people present, then the responsibility is diffused throughout the group, an each person feels only 10% of the responsibility. Feeling 10% percent of of the responsibility might very well not be enough to motivate the person to act.

17.2.4 Stanford Prison Study

Philip Zimbardo's famous Stanford Prison Experiment divided volunteer subjects randomly into two groups, prisoners and guards. Prisoners were picked up at their homes, charged, cuffed, and put into police cars, then taken to a station where they were informed of their rights, booked, finger printed, blindfolded, and placed in a holding cell. The "prison" was constructed in the basement of the psychology building at Stanford University. Guards were given no training, and were free to do whatever they thought was necessary to maintain order. The guards began to increasingly harass and humiliate the prisoners, forcing them to do things like clean out toilet bowls with their bare hands. Some guards were strict, but fair, others lenient, but some were hostile and creative in devising ways to humiliate the prisoners.

The experiment was planned for two weeks, but ended after six days for two reasons. First, the guards began escalating their abuse in the middle of the night when they thought no experimenters were watching, Second, a person brought in from outside to interview the guards and prisoners was outraged at what she saw. Given her response, it became clear that they needed to end the experiment[2].

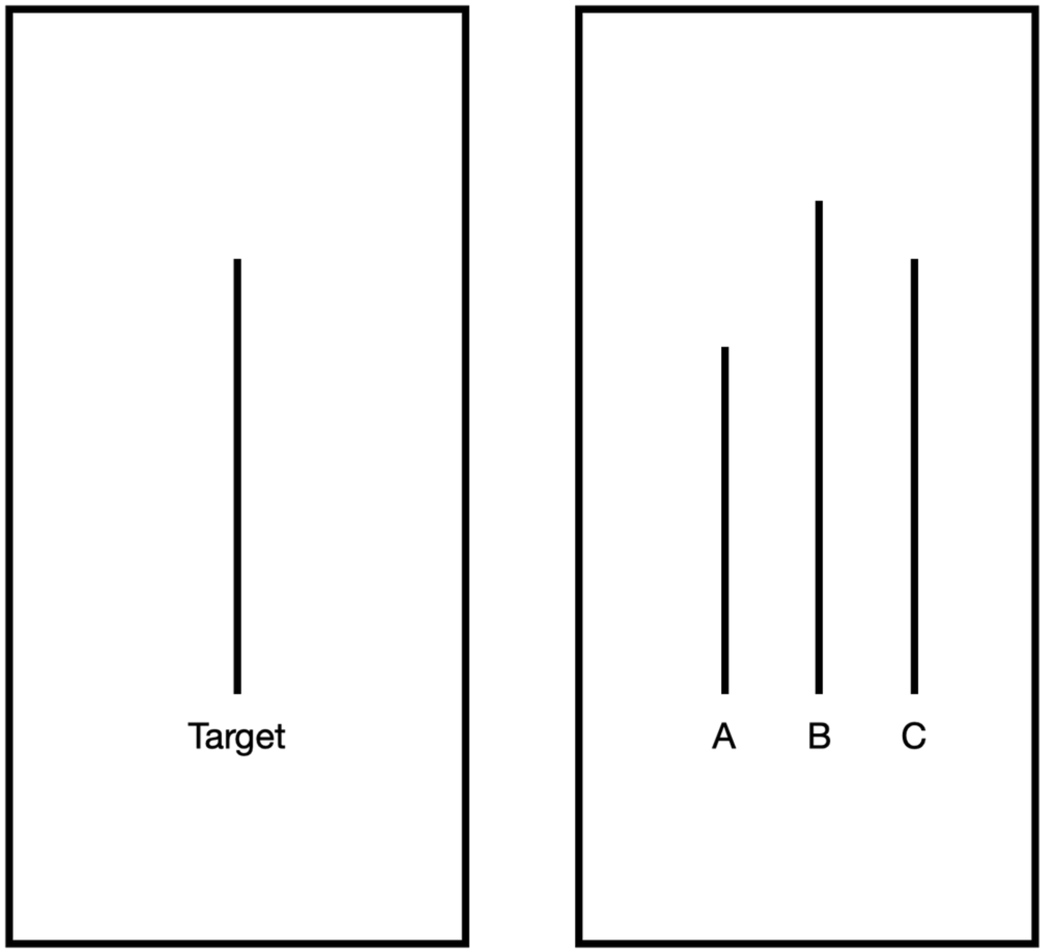
17.2.5 Asch Conformity Study

The autokinetic effect is common perceptual illusion — if you are in a dark room in which you can see only a small, stationary point of light, the light will appear to move. In the 1930's, Muzafer Sheri, a pioneer of social psychology, found that people in a group who experience this effect develop a group norm; that is, after a time, they begin to see the point of light moving the same direction and the same distance. Interestingly, as people in the group are replaced, the new members will adopt the same group norm. This is true, even after several generations of new members. (**?**)

Sherif's results were provocative, but they did have a problem, as Solomon Asch later pointed. Since the point of light is actually stationary, there isn't a correct answer to how far it is moving. If there is not correct answer, then how is it possible to be sure that the subjects are conforming? So, Asch devised an experiment in which it would be clear if subjects were conforming.

Students from Swarthmore College were chosen to participate in the study, and informed that it was a vision test. Each subject was placed in a room with seven other people. who were really confederates (people working with the experimenter.) The people were shown figures like this:

[2] For more details see **?**



Each person was to state aloud which line (A, B, or C) was closest in size to the target line — the answer was intended to be obvious. The test subject always sat at the end of the row and would answer last. The confederates would give a clearly incorrect answer, and about 75% of the test subjects conformed to that wrong opinion at least once. The times that the confederates gave the right answer, less than 1% of the subjects gave the wrong answer. Later, when asked why they gave the wrong answer, most subjects said that they didn't believe the answer was correct, but went along with the group to avoid being ridiculed. (**?**)

It is important to note that Asch discovered that if there was only one confederate then the test subject would not conform. Disagreeing with one person seems to be no problem, but disagreeing with an otherwise unanimous group is different. More important, even in the larger group, if even one of the confederates refused to conform to the others, then the test subject would also not conform.

17.2.6 Milgram Experiment

The final experiment we'll discuss is a famous one by Stanley Milgram, published in 1963. (**?**) The subjects were 40 men, ages 20–50. Two people were brought in, one was the actual test subject, and the other was an accomplice. They were told that the experiment was to study the effects of punishment on learning. A drawing was held to determine who would be the teacher and who would be the learner. Since the drawing was rigged, the test subject was always the teacher. The accomplice was strapped into a chair with an electrode attached to his wrist.

The subject read a series of word pairs to the learner. He then read the first word of a pair, along with four other words. The learner was to signal which of the four had been paired with the first word by pressing one of four switches. The teacher was at an instrument panel consisting of 30 switches in a line. Each switch was labeled with a voltage ranging from 15 to 450 volts. Groups of four switches were labeled with these designations:

- Slight Shock
- Moderate Shock
- Strong Shock
- Very Strong Shock
- Intense Shock
- Extreme Intensity Shock
- Danger: Severe Shock.

The last two groups were simply labeled "XXX"

With each successive wrong answer by the learner, the teacher was to increase the level of the shock, by announcing the new voltage level, and throwing the switch. The learners would give three wrong answer to one correct answer, and upon reaching 300 volts, pound on the wall. The subject would usually then turn to the experimenter for direction, who would tell the subject to wait 5–10 seconds for an answer, then keep increasing the shock level until the learner responded. If the subject wanted to quit, the experimenter would say these lines, beginning with the first and going on as necessary:

1. Please continue. or Please go on.
2. The experiment requires that you con- tinue.
3. It is absolutely essential that you con- tinue.
4. You have no other choice, you must go on.

It was predicted that only a small minority of the subjects would continue to 450 volts, and that almost all would go no further than 240 volts, which was labeled "Very Strong Shock." The actual results were that no one quit before 300 volts, and 26 continued all of the way to 450 volts.

17.3 Actor-Observer Difference

We have seen many ways in which experiments have shown that features of the situation prompt our behavior. Those external causes are more powerful than we usually think. People do tend to give external explanations in two different cases: first, for their own *bad* actions, and second, for others' *good* actions. On the other hand, they tend to give internal explanations for their own *good* actions and for others' *bad* ones. This is called the "Actor-observer difference."

It is important to note that the Fundamental Attribution Error does *not* mean that people are not responsible for their actions. Even though the situation inclines us to act wrongly, that doesn't mean that we have to do so. We must ask ourselves if *everyone* in this situation do the same thing? If the answer is no, then the external reason, although true, does not excuse.

Chapter 18

Group Reasoning

We often find ourselves, whether we like it or not, having to work in groups, whether it is a school group project, committee work, a social club, or a community service organization. It's important, therefore, to understand the situations in which groups tend to reason badly, so that we can guard against them.

18.1 Supposed Advantages

Since groups are made up by individuals, all of the cognitive biases that we discussed in chapter 16 affect group reasoning. There are particular biases, though, that apply only in the contexts of reasoning in groups or about groups. So, if groups are susceptible to special group biases, and tend to reason badly in certain situations, why are we so often asked to use them?

We tend to think that groups are more likely to come to more reasonable and unbiased decisions than people working alone, because groups *seem* to have the following advantages:

1. Groups should have access to more information than single individuals do, because everyone brings something to the table, so to speak.
2. Since every individual has their own point of view, group decisions will represent more perspectives than decisions made by single individuals.
3. We have all had times when we have overlooked something important, or just not noticed a flaw in one of our plans. The more people that

are considering a proposed solution, the more likely it should be that some notices a problem.

4. We are all either risk-averse or risk-seekers to various degrees. Do we really want the people on the extremes making the decisions? If those decisions are made by a group, the risk-seekers and the risk-averse members should balance each other out to form a more reasonable decision-making body.
5. There are times when we want our decisions to reflect the will of the majority of the members of the organization. Given what we know about sampling, it seems right that a group decision would more accurately reflect the general will than the decision of a single individual.

These five points should be true of groups, but they are not true of *all* groups. We will examine particular weaknesses that groups can have, and think about ways to structure groups so that we can avoid the weaknesses while maintaining the advantages.

18.2 Social Loafing

There are people who are driven to succeed, and then, there are the rest of us. If you are one of those people who are driven to succeed, my guess is that you're also a person who doesn't like group work. You're probably afraid that you'll end up doing all of the work, and pulling the weight of those who fail to do their fair share.

Social loafing is the tendency for members of a group to do less work, and to do it less well, than they would if they were working alone. One reason for this is the diffusion of responsibility that we discussed in the last chapter. If all members of the group are equally responsible for the work, then the feeling of responsibility is diffused throughout the group, so that the individuals in the group each feel less responsible than they would have had they been working alone.

There are ways to reduce social loafing in group projects. They include:

1. Grading each person on how much they contributed to the project.
2. Assigning each member a particular task, so that 100% of the responsibility for completing that task falls on some particular person.
3. Including peer reviews as part of the evaluation process.

18.3 Group Polarization

Group polarization, also called choice shift, is the tendency for people to make decisions about risks differently when in a group than when alone. When they become more risk-seeking, this is called the risky shift. When they become more risk averse, it is called the conservative, or cautious, shift.

That this occurs should surprise no one. We have all heard stories about people who have done things in mobs that they never would have considered if acting alone. For both types of shifts, being in a group magnifies the qualities of the individuals.

There have been a number of proposed explanations for the risky shift:

1. Diffusion of responsibility might again be the cause. Feeling completely responsible for the decision should prompt one to take few chances, that is, to become more risk averse. As perceived responsibility is shared with others, the risk is also perceived as shared. (**?**)
2. Risk-seekers naturally are perceived as being more confident, and thus may persuade others to also accept the greater risk. (**?**)
3. It may be that risk-seekers tend to have greater status within the group, leading people to follow their lead and supporting riskier choices. (**?**)
4. As groups discuss a proposed action, group members become more comfortable with the proposal, making it seem less risky. (**?**)

Group polarization has also been shown to affect our attitudes. Students that had low levels of racial prejudice became even less prejudiced after discussing issues of race with each other. The reverse happened with more highly prejudiced students. The became even more prejudiced after discussing the same issues with each other. (**?**)

18.4 Group Accuracy

The accuracy of a group depends, of course, on the type of problem and the characteristics of the particular members of the group. It's not surprising that on problems with clear right answers, a math problem, for example, groups tend to do better than the average member of the group, but worse than the best member.

18.5 Groupthink

Groupthink is the tendency for groups to make poor decisions and judgments. This is common in groups that are highly cohesive, have dynamic leaders, and are isolated from any external input or information. Several of the studies discussed in the last chapter are relevant here, such as the Asch conformity study, the Milgram experiment, and the Stanford prison experiment. (Remember that the Stanford study was only stopped after it was viewed by an external evaluator.) Members of such groups have a strong tendency to want to please the group leader, to agree on any decision that is made, and to feel both a strong sense of invulnerability and a strong sense of being right.

In many cases, though, groups are tasked to work on problems for which there are no clear right answers. For example, a university committee might be tasked with develop a strategy for increasing enrollment, or revise the core curriculum. Groups working on problems like these can work well, but often do not. The reason is that the group decision process naturally tends to go like this:

1. Once the problem or task has been identified, a group member proposes a solution.
2. Objections are then raised to the proposed solution.
3. Another solution is proposed, and new objections are raised. This may be repeated several times.
4. Finally, a solution is proposed against which no strong objections are raised.
5. Since no one can think of a serious objection, the proposal is accepted.

There are two problems with this process. The first is that the process is biased in favor of ideas that are discussed early. The second problem is that the solution that is accepted is the first minimally acceptable solution. There may very well be better solutions that never had a chance of being considered.

18.6 Successful Groups

Since there seem to be so many problems with groups, it's natural to wonder if there is any reason to use them, especially in education. One reason is, since graduates, once they join the work force, will have to sometimes participate in groups, students should learn the skills of successful group interaction.

Another reason is that, although group work does not seem to enhance abilities to learn material in a basic sense, discussing and explaining the material does enhance the understanding of the material.

By understanding when groups reason badly, we can now identify some characteristics of successful groups. Successful groups

1. Protect the brain-storm process. That is, they do not evaluate the ideas until *all* of the ideas have been proposed. If this isn't feasible, then another option is to initially split into sub-groups for the initial group reasoning process.
2. Set specific goals. A group cannot identify the best solution if the problem has not been made clear.
3. Get feedback, especially from outside the group.
4. Encourage dissent. If no one disagrees, then appoint someone to raise objections to the group's decision (known as the devil's advocate).
5. Reward success, to motivate finding the best solution.

18.7 Voting

The final step in any group decision-making process is to determine the will of the group, usually done by voting. If there are only two options, then voting presents no problems — if there is a winner, then it will be absolutely clear which option it is. If there are more than two options, however, things get complicated.

Sometimes the group's preferences are simply inconsistent. Let's imagine a small group with three people, who are considering three different options, A, B, and C. The three people rank their preferences like this:

1. A>B>C
2. B>C>A
3. C>A>B

Both persons 1 and 3 prefer A to B, so A should win over B. Persons 1 and 2 both prefer B to C, so B should win over C. A consistent preference structure should be transitive, so if A beats B, and B beats C, then A should also beat C. In this case, however, persons 2 and 3 both prefer C to A, so the ranked order looks like this: A>B>C>A, which means that A should be preferred over A!

Let's look at a fairly simple group of nine people considering three options. Imagine that the people order their preferences like this:

2	3	4
A	B	C
B	A	A
C	C	B

That is, two people prefer A to B to C, and 3 people, B to A to C, and 4 people, C to A to B. Now, let's look at a few reasonable voting methods and see which option wins on each.

18.7.1 Plurality Wins

This is the method that we use for large-scale elections in the United States. There is a single round of voting and the option with the most first place vote wins. In this case, C wins with four votes, compared to three votes vor B and 2 votes for A. The problem here is that the majority of the people place C last. C is the winner, but the majority want anything else.

18.7.2 Weighted Voting

One way to avoid this problem is by using weighted voting. Here, voters, list all of their preferences. First-place vote gets the most points, second-place votes get a little less, and so on. The most common weighted voting scheme is called *Borda's scheme*, where for n options, first-place votes get $n-1$ points, second-pace votes get $n-2$ points, and so on, all the way to 0 points for last-place.

In this case, A wins with 11 points, B and C are tied for second place with 8 points each.

18.7.3 Preferential Voting

Here, again, voters rank all of the options. If there is an option that receives a majority of first place votes, then that option wins. If not, then the option with the fewest first-place votes is eliminated. We then look at the rankings that have the eliminated option first, and give those votes to whatever option those people ranked second. That sounds complicated but it should be easy to understand with the example. Look at the scenario again:

2	3	4
A	B	C
B	A	A
C	C	B

A is eliminated, having only 2 first-place votes. Those two votes to to B, so B then wins 5 to 4 over C. The intuition here is that, had A not been an option, those two people would have voted for B instead.

18.7.4 Approval and Negative Voting

We already have three apparently reasonable voting methods that each produces a different winner from the same set of social preferences. There are many more methods, however. For example, approval voting, where you get to give one vote to as many options as you like, like voting for all the candidates that you wouldn't mind winning.

There is also negative voting. In this case, you can give a +1 to the option you prefer, or you can give a -1 to the option that you absolutely wouldn't want to win. That provides a way for people to express their preferences who really don't care which options wins, but really care that one of them not win.

18.7.5 Runoff Voting

This is common in some smaller scale elections, and avoids the problem of possibly electing the candidate that the majority would place last. It still can be problematic in certain situations.

Imagine three candidates for office, with 17 voters having these preferences:

6	5	4	2
A	C	B	B
B	A	C	A
C	B	A	C

A and B each have 6 of the 17 votes, but a majority requires 9. So, there's a runoff between those two candidates, and, if nothing were to happen, then C's voters would switch to A and A would win.

Now, imagine instead that there is a debate just before the election, and A has a particularly impressive performance. The two voters in the rightmost column decide to switch their votes from B to A.

6	5	4	2
A	C	B	A
B	A	C	B
C	B	A	C

Now, A has 8 votes, B only 4, and C has 5. So, the runoff now is between A and C. The four voters in the third column that supported B give their votes to C, and C wins with 9 votes compared to the 8 votes for A. Curiously, A would have won had A not gained the extra support before the election!

18.7.6 Best Method?

All of the methods we've discussed have their particular strengths and weaknesses. Economist Kenneth Arrow proved in 1950 that there is no method that meets certain plausible minimal conditions. These conditions are:

1. The method will produce an ordering of options for any logically possible ordering of individual preferences.
2. If voters initially prefer A to B, changing their mind about the ranking of anything else shouldn't affect A and B.
3. If every individual prefers A to B, then the method should rank A over B.
4. The method should not be sensitive only to one person's rankings.

Arrow proved that no method meets all of these conditions. (**?**) So, since there is no perfect method, which method should be used? It really depends on the circumstances. Some things that will need to be considered include the cost and complexity of the voting system and the type and extent of information that needs to be gathered. The more complex the system, the less likely voters will feel they can trust the system. A system that is too expensive is useless no matter how accurate it is.

Chapter 19

Social Dilemmas

Sometimes, what is rational for you to do in a particular situation depends on what another person does. In those cases, in order to decide what you should do, you have to make assumptions about what the other person will do. Many board games, like checkers or chess, are played like this. A player deliberates about how to play by thinking like this, "If I move here, then she will move there, and I won't be able to save my queen, but if I move there, then she will have to move here, and I'll be able to put her in check. . . ."

When we reason like this, we generally assume two things:

1. The other player wants to win, and
2. The other player knows how to play the game well.

These situations happen, not just in when playing board games, but in the rest of life as well. For example, imagine that you are the first person who is asked to help put on some event. You certainly don't mind helping, but you don't want to do it all yourself. Should you agree? It depends on whether you think other people can be depended on to help also.

There is a branch of study that focuses on these situations, called *game theory*. Game theory is the study of the ways in which the choices of interacting persons result in outcomes that are described in terms of the preferences of those persons. (What this means will become clearer later.) The situations that game theorists study range from fairly simple to very complex. We'll only discuss four simple examples, including the most famous, the prisoner's dilemma. As we do when playing a board game, we will make two assumptions:

1. Each person wants the outcome that is best for that person.
2. Each person will act intentionally to achieve that outcome.

Most board games are what are called *zero-sum* games. In a zero-sum game, if there is a winner, there must be a loser. Tic-tac-toe is the classic example. If we give +1 point to the winner, -1 to the loser, and 0 to each for a tie, then the sum of each players score for a game will always be 0. Our personal interactions in life, however, are not always zero-sum. In a non-zero-sum game, both players can win. Sometimes, what is best for me can always be best for you.

In the cases that we will look at, all simple social dilemmas involving two persons, each player will have two choices, to cooperate or to defect. We can think of cooperating as keeping an agreement, or doing what is best for the other person. Defecting is breaking an agreement, or doing what is best for you at the expense of the other person. Each person decides to cooperate with, or defect against, the other, without knowing what the other person will do. There are, then, four possible outcomes:

1. Mutual cooperation (CC)
2. Mutual defection (DD)
3. Mixed (CD)
4. Mixed (DC)

The type of social dilemma is determined by how the persons involved rank these four preferences. Before we look at some examples, I should define two important concepts.

Pareto Optimality: An outcome is Pareto optimal when any other outcome that would be better for one party is worse for the other party. In other words, there is no other outcome where everyone would be better off.

Stability: An outcome is stable when no party would have been better off by acting in a different way, given what the other person did. This means that I wouldn't have any regrets acting the way I did, considering how you acted.

19.1 Deadlock

In a deadlock, the parties have this preference structure:

1. DC
2. DD
3. CC
4. CD

Each person's first preference is that they defect and the other person cooperate, followed by mutual defection, then mutual cooperation, and finally that they cooperate and the other person defect. Notice that, strictly speaking, this is not a dilemma. Each player prefers defection to cooperation, so there is no question about what will happen.

Negotiations often take the form of deadlocks. Cooperation means that the party will have to give something up in the negotiation. Of course, the first preference is that they not have to give anything up, while the other party does. Given the endowment effect, what a party gives up seems greater in value than what they gain in return. So, neither party is inclined to see cooperation as a benefit. Thus, the final outcome is mutual defection. Negotiations break down, and the result is maintaining the status quo.

19.2 Chicken

This dilemma gets its name from a foolish game in which two people drive at high rates of speed directly toward each other. The goal is to force the other person to swerve, showing your bravery (or foolishness), and their cowardice. In this case, cooperation is swerving, and defection is maintaining the direct course. Here are the preferences:

1. DC
2. CC
3. CD
4. DD

The first preference is to seem brave, while the other person is a coward. The second preference is to both be cowards, and never mention that this happened. The third preference is to be the coward and still alive. The last preference is that you are both remembered as brave, and kind words are etched on your tombstone.

So, what should you do, swerve or drive straight? (Obviously, the answer is that you shouldn't play foolish games like this, but we'll ignore that for now.) It depends on what you think your opponent will do. If you have good reason to think your opponent is suicidal, then you definitely should swerve. Otherwise, it's hard to say.

Chicken dilemmas don't just involve foolish games that can end in death. Many volunteer situations are examples of chicken dilemmas. Imagine that you are a parent who is asked to help provide refreshments for an elementary school party. You're very busy now, so you would prefer that someone else do it. If that can't happen, then your next preference is that you provide some, but not have to provide all of the refreshments. You do want the kids to be able to have the party, though, so your third preference is that you do everything. This is better than no one doing anything, because then there would be no party.

19.3 Stag Hunt

A stag hunt is any situation with these preferences:

1. CC
2. DC
3. DD
4. CD

The name comes from a story used as an example. Imagine two people, Fred and Ethel, hunting with primitive weapons. Their goal is to kill a stag, because that will provide them with food for the winter. To kill the stag, though, Fred has to hide at one location, while Ethel tries to direct the stag there from another location. While separated, they cannot see each other. So, after waiting for some time, Fred begins to wonder if Ethel is still looking for the stag. Then, he sees a rabbit nearby. Now he has a dilemma: should he wait in his hiding place, or go chase the rabbit? If he waits, they might be able to get a stag, that is, if Ethel really is out there chasing one. On the other hand, if Ethel has abandoned him, he's better off with the rabbit than with nothing. So what should he do? His preferences are:

1. Get the stag, which requires mutual cooperation.
2. Get the rabbit, and Ethel gets nothing. (DC)
3. They both get a rabbit. (DD)
4. Ethel gets a rabbit, and he gets nothing (CD)

In a stag hunt, mutual cooperation is ideal. The extent to which one is inclined to cooperate, though, is determined by the amount of trust one has in the other person.

19.4 The Prisoner's Dilemma

A prisoner's dilemma is a social dilemma with this preference structure:

1. DC
2. CC
3. DD
4. CD

This is the most famous, and important, social dilemma. Now, imagine that Fred and Ethel have been arrested on suspicion of robbing a bank. The district attorney, unfortunately, does not have enough evidence for a conviction, unless one of them confesses. So, she offers both of them the same deal. If one confesses and the other does not, the one who confesses goes free, and the other gets 10 years. If neither confesses, they will be convicted on lesser charges, which will get them 3 years apiece. If both confess, then they both get 7 years. So, should they each confess or stay quiet? Here's a table with the possible outcomes:

	T	Q
T	7,7	0,10
Q	10,0	3,3

Assuming that each person wants only what is best for that person, then what should they do? An understandable first inclination is to say that they should both stay quiet. That's the only way they can both get 3 years, which is really the best practical outcome. On the other hand, if Fred is going to stay quiet, then shouldn't Ethel confess? Then her sentence would drop from

3 to 0 years. Also, if Fred decides to talk, then Ethel certainly should confess. Her sentence would drop from 10 to 7 years. Basically, each person is three years better off by confessing, regardless what the other does. But if it's rational for Ethel to confess, then it's rational for Fred to also. So, it seems that they're both doomed to 3 years apiece.

They both would have been better off had they just cooperated with each other by staying quiet. What makes the prisoner's dilemma particularly interesting is that, by acting rationally in terms of utility maximization, both parties are worse off than they would have otherwise been. Acting for their own individual good was, in fact, self-defeating.

Prisoner's dilemma situations happen whenever a person can be better off by defecting, providing that everyone else cooperates, but, if everyone defects, then everyone is made worse off compared to mutual cooperation. So, if everyone is cooperating, then each person has a strong motivation to defect, which results in everyone defecting, which is worse for everyone.

19.4.1 Real-Life Prisoner's Dilemmas

Prisoner's dilemmas can happen more often than you might think. A classic example is called "the tragedy of the commons." Centuries ago, villages had a central pasture that everyone used for grazing called the "commons." Obviously, the more cattle a family had, the more prosperous it was. One family adding an extra cow or two wouldn't affect the commons, but if every family did it, then the pasture would be over-grazed, and everyone suffers.

That same situation can happen in 21st century cities. Imagine a group of competing builders in a city. So long as the builders do not build too many houses, then they can sell the houses they build for a good price. It's in each builder's interests to build a few more houses to make more money. If every builder does this, though, there are too many houses for the market, prices drop, and every builder suffers.

Advertising can put businesses in a prisoner's dilemma situation. Take a product that everyone buys, something like toilet paper. Advertising is unlikely to attract new customers who have never used the product. (Can you imagine someone seeing a toilet paper commercial and thinking "I never thought of that—that stuff really looks handy!"?) The ads that a company buys are intended just to lure customers away from its competitors. Selling more advertising is good for a company, but only if its competitors don't increase their advertising. But, what is good for one, should be good for everybody, so everyone increases their ad sales, expenses rise, and profits drop.

The same thing happens with salary caps in sports. An agreement to cap salaries is good for the owners, but if everyone else keeps the agreement, it's good for one owner to exceed the cap to attract the best players. If one breaks the agreement, though, the others will have to also, so the owners find themselves in the same situation as they were in before, except for spending a lot more money.

The use of performance-enhancing drugs, or doping, is another case in sports. If only one athlete is doping, then that athlete has a competitive advantage over others. So, to compete, the others also have to use the performance-enhancing drugs. In the end, no one has an advantage, and everyone suffers from the detrimental effects of using the drugs.

19.4.2 Iterated Prisoner's Dilemmas

There are circumstances in which it wouldn't be a good idea to take the deal from the district attorney and send a partner in crime to prison for 10 years, even if it meant no prison time for you. What's going to happen in 10 years when the other person is released? What you should do depends on whether the other person will have a chance to get even.

Let's switch from thinking about the prisoner's dilemma in terms of prison sentences to a game with points. The goal here is not to get more points than your opponent, but to get the most points for yourself. (This is like life, if you are underpaid in your job, it shouldn't satisfy you that you are less underpaid than any other employee—you are all still underpaid.) For each turn, the players get to select to either cooperate or defect. The points given are determined by this payoff table:

	C	D
C	2,2	0,3
D	3,0	1,1

The first number in a pair is the points given to the row player, and the second number is the points given to the column player. So, if a person cooperates and the other defects, the defector is given 3 points and the other Son gets nothing. Mutual cooperation is awarded 2 points apiece, and mutual defection 1 point apiece. So, how should you play? If you're only going to play one round, the answer is clear: you should defect. Let's assume that your opponent will cooperate. In that case, you are one point better off by defecting. If your opponent defects, you're also one point better off by defecting. So, you're one point better off by defecting, no matter what your opponent does. The rational strategy is for both players to defect.

What if you're going to play more than one round? Should you cooperate or defect? If you adopted a strategy of defecting every turn, you and your opponent will end up getting one point per turn, which is only half what you would get if you both cooperated. On the other hand, if you cooperated every turn, you would be in trouble as soon as your opponent realized what your strategy was. Somehow, you need a way of encouraging your opponent to cooperate. The best strategy for a game with an unknown number of turns, called an iterated prisoner's dilemma with an unknown number of iterations, is one called *tit-for-tat*. You cooperate on the first turn, then, on every successive turn, do what your opponent did on the previous turn. So, you reward your opponent for cooperating, and punish your opponent for defecting. If both players are rational, then this strategy results in mutual cooperation in the long run. The opponent realizes that it's impossible to get ahead by defecting, because the other person just gets even. What if your opponent is irrational, that is, someone who doesn't try to maximize their points? This can happen in a number of ways—your opponent may just want to "stick it to you" on every turn, or maybe he flips a coin to decide what to do. In these cases, players just has to protect themselves by defecting every turn.

If you're playing a iterated game with a *known* number of iterations, then things get surprising. Let's say two people are playing a game with only five rounds. To determine what they should do let's work backwards from the end of the game. The fifth round is essentially just like playing a single round game. Each player is one point better off by defecting no matter what the other person does, and there is no way for the other person to get even. So, both players will defect on the fifth round:

	1	2	3	4	5
Player A					D
Player B					D

If my opponent is going to defect in round 5 no matter what I do in round 4, then there is no reason for me to cooperate in round 4. So, we'll both defect in round 4:

	1	2	3	4	5
Player A				D	D
Player B				D	D

But what is true of round 4 is also true of rounds 1–3, so we should defect on all turns:

	1	2	3	4	5
Player A	D	D	D	D	D
Player B	D	D	D	D	D

19.5 Public Goods

Prisoner's dilemmas and chicken dilemmas are both dangers in situations that involve public goods. A public good is one that meets all of these three conditions:

1. The contribution of many members of a group is necessary for providing the good.
2. If the good is provided, then it will be available for the use of everyone in the group, regardless of whether they contributed. (There's no practical way to prevent non-contributors from using the good.)
3. Contribution is a cost to those members that do so.

Public television is a public good. Most of the revenue for public television comes from voluntary viewer support. Viewers still have access to public television even if they did not contribute anything during the annual fundraising drives. National defense is also a public good. People contribute by paying taxes or serving in the armed forces. People who don't pay taxes for whatever reason still enjoy the benefits of national defense. Other examples are highways and bridges, public health by immunizations, environmental protection, and so on.

These goods require the contribution of many people, but they don't require the contribution of everyone. So long as enough people contribute to the fund drive, I'll still be able to watch public television, even if I don't contribute. So, I have a strong motivation to be a free rider. If I can get the good for free, then why should I pay? Here's an argument that I shouldn't pay:

1. Production of the good requires contributions by some large number of people.
2. It's very unlikely that exactly the number of people required will contribute.
3. So, either more than enough people will contribute, or not enough people will.
4. If more than enough people contribute, I can receive the good anyway.
5. If not enough people do, then the good will not produced.
6. If I can receive the good without contributing, I shouldn't contribute.
7. If the good will not be produced, then I shouldn't contribute
8. I shouldn't contribute.

Most of us don't reason this way, because we're not what economists call "rational utility maximizers." We're happy to contribute our fair share, *if* we can be assured that others will also. The latest research on prisoner's dilemma situations shows that people defect because they fear being taken advantage of by others. So, the more free-riders there are, the less motivated others are to contribute.