Symbolic Logic: Theory and Applications

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Chapter 1

Basic Concepts

1.1 What is Logic?

I don't know that there is any accepted definition of logic. Here are four that I've heard:

1. The study of reasoning
2. The analysis and evaluation of arguments
3. The study of formal languages
4. The study of patterns of truth

Although I don't think that any of the four are successful definitions, it is certainly the case that they each represent some important part of the study of logic. We will see all four of these topics, in various forms, in later chapters. This chapter introduces some important concepts that are fundamental to the study of logic. The first is truth preservation.

The study of logic provides a set of tools for reasoning. As you continue in the study of logic, you will find that there are different systems of logic, each system being a different tool that can be used. There are two obvious, but important, things to note — first, the particular tool that should be used is relative to the context or task at hand. Hammers and saws are both tools used in carpentry. Which one should you use? Is your goal to drive a nail or cut a board? We'll see that the system of logic that should be used depends on the context in which one is reasoning. A good rule of thumb is that one should use the simplest tool that is adequate for the task. For example, a doctor shouldn't use an electrocardiogram or MRI when a stethoscope will do. Systems of logic also differ in degrees of complexity, and we should use the simplest system that will be effective in the circumstances.

Second, a tool that leaves you in a worse situation than you were in before you used it is certainly *not* good. The purpose of a toaster is to make a nicely toasted piece of bread. A toaster that produces a smoldering, inedible, piece of charcoal is not a good toaster. You were better off just eating the non-toasted bread. A central purpose of a logic system is to preserve truth. When the tools of logic are applied to truths, the results should still be true. A system of logic that turns truth into falsehoods is a failure as a tool of reasoning. Another way to state this is that the rules of our logic systems should be truth-preserving.

1.2 Arguments

One fundamental concept in logic is that of the argument. For a good example of what we are not talking about, consider a bit from a famous sketch by *Monty Python's Flying Circus* (Cleese and Chapman 1980):

> Man: (Knock)
> Mr. Vibrating: Come in.
> Man: Ah, Is this the right room for an argument?
> Mr. Vibrating: I told you once.
> Man: No you haven't.
> Mr. Vibrating: Yes I have.
> Man: When?
> Mr. Vibrating: Just now.
> Man: No you didn't.
> Mr. Vibrating: Yes I did.
> Man: You didn't!
> Mr. Vibrating: I did!
> Man: You didn't!
> Mr. Vibrating: I'm telling you I did!
> Man: You did not!!
> Mr. Vibrating: Oh, I'm sorry, just one moment. Is this a five minute argument or the full half hour?

People often use ”argument” to refer to a dispute or quarrel between people. For our purposes, an argument is defined as

Argument A set of statements, one of which is taken to be supported by the remaining sentences.

Conclusion The statement in the argument that is being supported.

Premises The statements in the argument that provide the support for the conclusion.

There are three important things to remember here:

1. Arguments contain statements.
2. They have a conclusion.
3. They have at least one premise

This might get more complicated later, but for now, think of a statement as a sentence that has a truth-value. Also, for now, we will assume that there are only two truth values: true and false, which we will abbreviate with **T** and **F**. Think of a statement as an attempt to describe the world; when it gets it right, the statement is true, otherwise, it is false. Since George Washington was in fact the first President of the United States, the statement ‘George Washington was the first President of the United States’ has the truth value of **T**. Likewise, the statement ‘John Adams was the first President of the United States’ has the truth value of **F**.

Some sentences don’t have truth values, that is to say that they are neither true or false. Examples are questions (‘What did you do last summer?’) and commands (‘Please take a seat.’). Such sentences can be neither premises nor conclusions of arguments. It is important to note that, even though a statement has a truth value, we may not necessarily know what that truth value is. The rules of logic will still apply, though.

An inference is the process of reasoning from the premises to the conclusion. That is, inference is the psychological process by which one draws a conclusion from some given premises. The argument just is the premises and conclusion.

A distinction needs to be made between deductive arguments and inductive arguments. It’s very hard to make this distinction precise, but it’s not hard to have an informal understanding of the difference. Think of the difference in terms of what the argument is intended to establish. If the argument is only intended to establish the probable truth of the conclusion, then it is inductive. If the argument is intended to guarantee the truth of the conclusion, then it is deductive.

Every argument has exactly one conclusion. Very complex arguments may have sub-conclusions, which are themselves inferred from premises. These sub-conclusions then serve as premises for the main conclusion of the argument. Let's keep things simple for now. Consider this argument:

> Calculus II will be no harder than Calculus I. Susan did well in Calculus I. So, Susan should do well in Calculus II.

Here the conclusion is that Susan should do well in Calculus II. The other two sentences are premises. These premises are the reasons offered for believing that Susan should do well in the course..

Now, to make the argument easier to evaluate, we will put it into what is called "standard form." To put an argument in standard form, write each premise on a separate, numbered line. Draw a line underneath the last premise, the write the conclusion underneath the line.

1. Calculus II will be no harder than Calculus I.
2. <u>Susan did well in Calculus I.</u>

∴ Susan will do well in Calculus II.

Now that we have the argument in standard form, we can talk about premise 1, premise 2, and all clearly be referring to the same statement.

Unfortunately, when people present arguments, they rarely put them in standard form. So, we have to decide which statement is intended to be the conclusion, and which are the premises. Don't make the mistake of assuming that the conclusion comes at the end. The conclusion is often at the beginning of the passage, but could even be in the middle. A better way to identify premises and conclusions is to look for indicator words. Indicator words are words that signal that statement following the indicator is a premise or conclusion. The example above used a common indicator word for a conclusion, 'so.' The other common conclusion indicator, as you can probably guess, is 'therefore.' This table lists the indicator words you might encounter.

Each argument will likely use only one indicator word or phrase. When the conlusion is at the end, it will generally be preceded by a conclusion indicator. Everything else, then, is a premise. When the conclusion comes at the beginning,

Conclusion	Premise
Therefore	Since
So	Because
Thus	For
Hence	Is implied by
Consequently	For the reason that
Implies that	
It follows that	

the next sentence will usually be introduced by a premise indicator. All of the following sentences will also be premises.

For example, here's our previous argument rewritten to use a premise indicator:

> Susan should do well in Calculus II, because Calculus II will be no harder than Calculus I, and Susan did well in Calculus I.

Sometimes, an argument will contain no indicator words at all. In that case, the best thing to do is to determine which of the premises would logically follow from the others. If there is one, then it is the conclusion. Here is an example:

> Spot is a mammal. All dogs are mammals, and Spot is a dog.

The first sentence logically follows from the others, so it is the conclusion. When using this method, we are forced to assume that the person giving the argument is rational and logical, which might not be true.

One thing that complicates our task of identifying arguments is that there are many passages that, although they look like arguments, are not arguments. The most common types are:

1. Explanations
2. Mere asssertions
3. Conditional statements
4. Loosely connected statements

Explanations can be tricky, because they often use one of our indicator words. Consider this passage:

> Abraham Lincoln died because he was shot.

If this were an argument, then the conclusion would be that Abraham Lincoln died, since the other statement is introduced by a premise indicator. If this is an argument, though, it's a strange one. Do you really think that someone would be trying to prove that Abraham Lincoln died? Surely everyone knows that he is dead. On the other hand, there might be people who don't know how he died. This passage does not attempt to prove that something is true, but instead attempts to explain why it is true. To determine if a passage is an explanation or an argument, first find the statement that looks like the conclusion. Next, ask yourself if everyone likely already believes that statement to be true. If the answer to that question is yes, then the passage is an explanation.

Mere assertions are obviously not arguments. If a professor tells you simply that you will not get an A in her course this semester, she has not given you an argument. This is because she hasn't given you any reasons to believe that the statement is true. If there are no premises, then there is no argument.

Conditional statements are sentences that have the form "If…, then…" A conditional statement asserts that *if* something is true, then something else would be true also. For example, imagine you are told, "If you have the winning lottery ticket, then you will win ten million dollars." What is being claimed to be true, that you have the winning lottery ticket, or that you will win ten million dollars? Neither. The only thing claimed is the entire conditional. Conditionals can be premises, and they can be conclusions. They can be parts of arguments, but that cannot, on their own, be arguments themselves.

Finally, consider this passage:

> I woke up this morning, then took a shower and got dressed. After breakfast, I worked on chapter 1 of the logic text. I then took a break and drank some more coffee….

This might be a good description of my day, but it's not an argument. There's nothing in the passage that plays the role of a premise or a conclusion. The passage doesn't attempt to prove anything. Remember that arguments need a conclusion, there must be something that is the statement to be proved. Lacking that, it simply isn't an argument, no matter how much it looks like one.

1.3 Deductive Validity

Deductive arguments are intended to be fully truth-preserving. A deductively valid argument is one that is in fact completely truth-preserving. That is, a deductively valid argument will never have all true premises and a false conclusion. It is important to understand how strong this claim is. It is not merely the case a valid argument happens to have true premises and a true conclusion. The relationship between the premises and the conclusion is so strong that it is not possible for the premises to be true and the conclusion false.

Deductive validity An argument is deductively valid if and only if it is not possible for the premises to be true and the conclusion to be false.

Deductive invalidity An argument is deductively invalid if and only if it is not deductively valid.

Here is an example of a deductively valid argument:

1. All dogs are mammals.
2. Lucy is a dog.

∴ Lucy is a mammal.

Since the first premise is true, being a dog is enough to guarantee that the animal is a mammal. So, the only way that the conclusion could be false is if Lucy is not a dog. It is impossible that the premises be true and the conclusion false.

Compare the previous argument to this one:

1. All dogs are mammals.
2. Lucy is a mammal.

∴ Lucy is a dog.

Since Lucy is the name of my dog, both the premises and the conclusion are in fact true. Note, though, that the premises are not enough to guarantee the truth of the conclusion. Lucy could have been the name of a cat. If so, the premises would have been true and the conclusion false. This argument is therefore invalid.

If we know that an argument is invalid, then we know that there is a special logical relationship between the premises and the conclusion such that, if the premises are true, then the conclusion must also be true. It is important to understand that validity alone does not mean that the premises and conclusion are true. If I know only that an argument is valid, I know that the *if* the premises are true, *then* the conclusion must also be true. Valid arguments cannot have a combination of true premises and false conclusion, but they can have any other combination. It can be reasonable to doubt that a conclusion is true, even if the argument is valid. What is not reasonable is to grant the argument is valid and has true premises and still doubt that the conclusion is true. This means that validity alone is not enough to guarantee that a conclusion is true. What guarantees that a conclusion is true is deductive validity along with true premises. This is called deductive soundness.

Deductive soundness An argument is deductively sound if and only if it is deductively valid and has all true premises.

1.4 Inductive Arguments

Later chapters will cover inductive arguments. For now it is enough to say that inductive arguments are not intended to be deductive valid. That is, inductive arguments are those whose premises do not guarantee the truth of the conclusion. For inductive arguments, it is always possible that the premises be true and the conclusion false. Here is an example of an inductive argument:

1. A random sample of 100 students at the university unanimously reported preferring traditional classes to online instruction.

∴ The majority of all students at the university prefer traditional classes to online instruction.

The truth of the premise does not guarantee the truth of the conclusion. It is certainly possible that the sample managed to include the only students that don't prefer online courses. Still, though, it seems that, if the premise is in fact true, then the conclusion should be highly likely to be true. So, this is a good inductive argument, one that we call inductively strong.

Inductive strength An argument is inductively strong to the extent that the conclusion is probably true given the truth of the premises.

Another difference between inductive and deductive arguments is that inductive strength is a matter of degree. The argument above is inductively strong, but doubling the sample size would make it even stronger.

1.5 Logical Consistency and Logical Truth

Consistency is a property of sets of statements:

Logical consistency A set is logically consistent if and only if it is possible for all of the members of the set to be true at the same time.

Logical inconsistency A set is logically inconsistent if and only if it is not logically consistent.

It is not necessary for all of the statements to be true in order for the set to be consistent. Here is an example:

Oklahoma is south of Texas. There are 125 members of the U.S. Senate.

Neither of the statements in this set are true. The set is consistent, though, because there is nothing about either sentence that prevents the other from possibly being true. Here is an example of an inconsistent set:

No student will make an A in Logic this semester. At least one student will make an A in Logic this semester.

The truth of one of those statements is incompatible with the truth of the other. So, the set containing both is inconsistent. Logical consistency is a very important concept, because, once defined, other logical concepts can be defined in terms of consistency. We'll do this in a later chapter.

For the most part, logic alone is not enough to determine if a statement is true. It is not logic that makes it true that Topeka is the capital of Kansas. What makes that true is something about the political structure and history of Kansas. There are two important exceptions to this, though. There are some statements that are true simply because of their logical structure. An important example is something like this: Either Susan will pass logic this semester or Susan will not pass logic this semester. No matter how well Susan does in the class, it must be true that she either passes or not. Sentences like these are called logical truths, or tautologies. Logical truths *must* be true. On the other hand, there are sentences that *cannot* be true. For example, Susan will both pass logic this semester and not pass logic this semester. Sentences like these are called logical falsehoods, or contradictions. Most statements are neither logical truths nor logical falsehoods. Such statements are logically indeterminate.

Logical truth A statement is logically true if and only if it is not possible for the statement to be false.

Logical falsity A statement is logically false if and only if it is not possible for the statement to be true.

Logical indeterminacy A statement is logically indeterminate if and only if it is neither logicall true nor logically false.

Finally, there are sentences that are related in such a way so that, if one is true, the second must also be true, and vice versa. These sentences are called logically equivalent. Here is a simple example:

Susan will pass.
Susan will not fail.

Since failing is just not passing, then any situation in which Susan passes is a situation in which she does not fail. So, the two statements are true in exactly the same situations, and false in exactly the same situations. They always have the same truth values.

Logical equivalence Two sentences are logically equivalent if and only if it is impossible for one to be true and the other to be false.

Chapter 2

Categorical Logic

Now we turn to some structured logic systems. The first, categorical logic, is one of the oldest. It dates back at least to Aristotle (384–322 BCE). Categorical logic is a fairly simple logic of categories or classes. A class is a group of things that we designate with a common noun: students, teachers, dogs, politicians, etc. Each sentence will use two different classes. One is the subject class, and the other is the predicate class. In this logic, we can say something about all members of a class, called a universal sentence, or we can say something about some members of a class, called a particular sentence. We can also make a positive claim, called an affirmation, or we can make a negative claim, called a negation.

With these two distinctions, universal/particular and affirnation/negation, we can make four kinds of sentences. S and P stand for the subject class and the predicate class, respectively.

A : All S are P (universal affirmation)

E : No S are P (universal negation)

I : Some S are P (particular affirmation)

O : Some S are not P (particular negation)[1]

Here are some examples of categorical statements, some true and some false.

[1] The letters A, E, I, and O, are thought to come from the first two vowels of the Latin words *affirmo* and *nego*, meaning "I affirm" and "I deny."

1. All dogs are mammals.
2. All mammals are dogs.
3. No reptiles are dogs.
4. No politicians are honest people.
5. Some politicians are honest people.
6. Some cats are amphibians.
7. Some dogs are not beagles.
8. Some beagles are not dogs.

Look at the sentences carefully. You should be able to tell that the odd-numbered ones are true and the even-numbered ones are false.

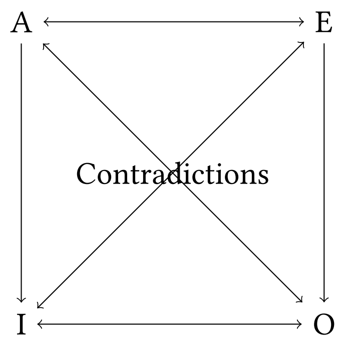
2.1 The Square of Opposition

We can visualize interesting logical relationships between these four types of sentences with something called "The Square of Opposition."

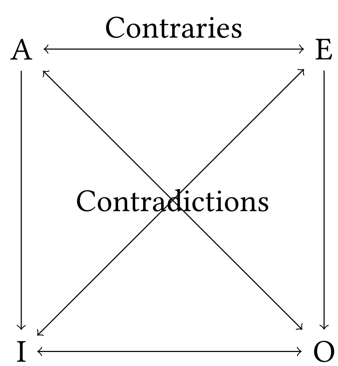
The first step is to place the sentence types in the corners of an imaginary square. A is at the upper left; E, the upper right; I, the lower left, and O, the lower right. Next, draw arrows on the diagonals, pointing to the sentences in the corners. Then, draw an arrow between the two at the top, and another one between the two at the bottom. Finally, draw an arrow on each side, going from top to bottom. When finished, you should have something like this:

$$
\begin{tikzcd}
\mathrm{A} \arrow[r, leftrightarrow] \arrow[d] \arrow[dr, leftrightarrow] & \mathrm{E} \arrow[d] \arrow[dl, leftrightarrow] \\
\mathrm{I} \arrow[r, leftrightarrow] & \mathrm{O}
\end{tikzcd}
$$

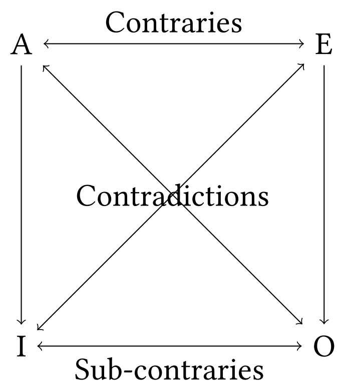
The next step is to note the relationship between the diagonals. The diagonals are contradictories, meaning they always have opposite truth values. They can't both be true, and they also can't both be false. If the A sentence is true, the O sentence must be false—if it is true that all dogs are mammals, it cannot be true that some dogs are not mammals. If the O sentence is true, then the A sentence must be false. It is the same for the E and the I.



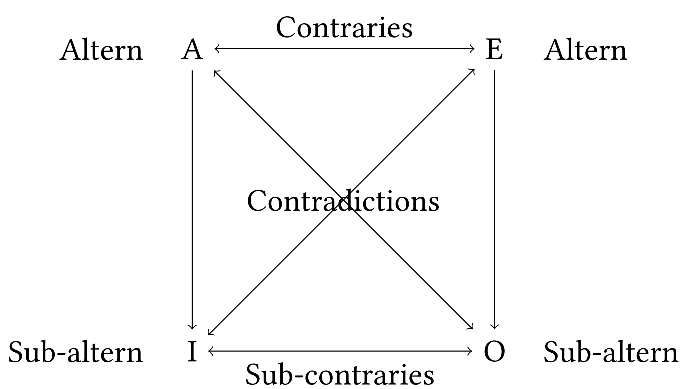
Next, note the relationship between the A sentences and the E sentences, called contraries. Like the contradictories, they cannot both be true. Unlike the contradictories, they can both be false. If it's true that all critical thinking students are good students, then it must be false that no critical thinking students are good students. If it's false that all critical thinking students are good students, then it can be false that critical thinking students are good students. In fact, they are both false, because some critical thinking students are good and others are not.



At the bottom, we have sub-contraries. They can both be false, but cannot both be true.



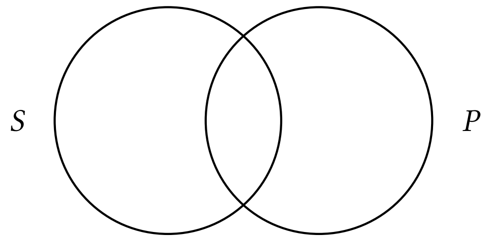
Finally, we have the relationship between the top level sentences and the bottom level sentences on the same side. This is called alternation. The universal is called the superaltern and the particular is called the subaltern. If the superaltern is true, then the subaltern must also be true. If the superaltern is false, then the subaltern can be either true or false. If the subaltern is false, then the superaltern must be false. If the subaltern is true, then the superaltern can be either true or false. It is easy to remember this way: truth goes down, falsity goes up.



2.2 Diagramming Sentences

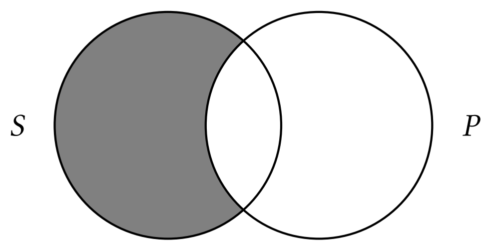
We diagram sentences and arguments in categorical logic using Venn diagrams. You've probably used these in a math class at some time. Before we can use these to evaluate arguments in categorical logic, we first have to learn how to diagram individual sentences.

The first step is to draw two interlocking circles. Label the left circle with an "S" and the right circle with "P"—standing for the subject term and predicate term, respectively.



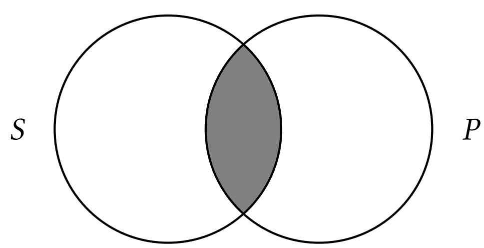
A-Sentences

Remember that the A-sentence has the form All S are P. That means that everything that is in the S circle must also be in the P circle. To diagram this, we shade the region of the S circle that is not contained in the P circle. If a region is shaded, that means that nothing is in that region.



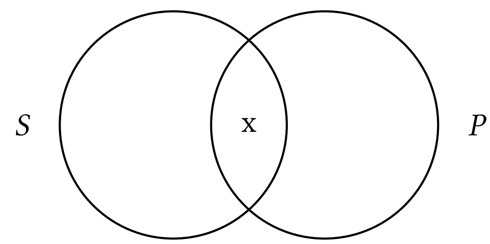
E-Sentences

To shade the universal negation, we shade the region that is shared by both S and P:



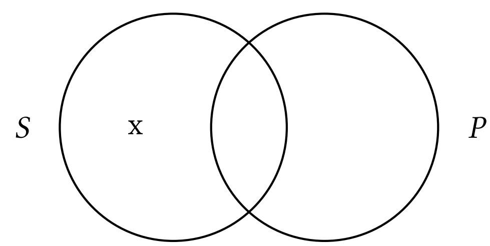
I-Sentences

To diagram a particular affirmation, we place an x in the region shared by S and P:



O-Sentences

Finally, to diagram an O-sentence, we place an x in S, but not in P:



Evaluating Categorical Syllogisms

A syllogism is an argument that has two premises and a conclusion. A categorical syllogism is a syllogism that contains only categorical sentences. Here is an example:

1. All Dogs are mammals.
2. All mammals are animals.

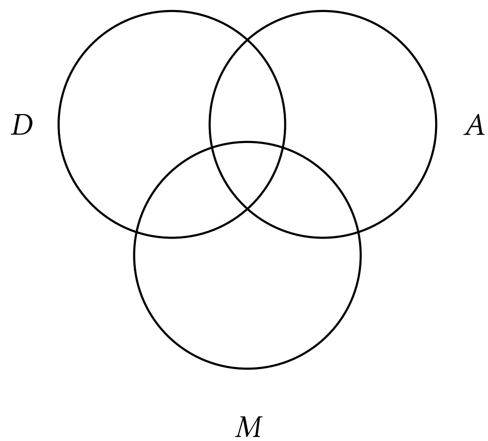
∴ All dogs are animals

Both premises and the conclusion are A-sentences. Notice that we have three terms in the argument: dogs, mammals, and animals. Every categorical syllogism, in proper form, has three terms. Each term occurs in two sentences. Two of those terms will be found in the conclusion, and one term is only in the premises. The predicate term of the conclusion is called the major term. The subject of the conclusion is called the minor term. The term that is not in the conclusion is called the middle term.

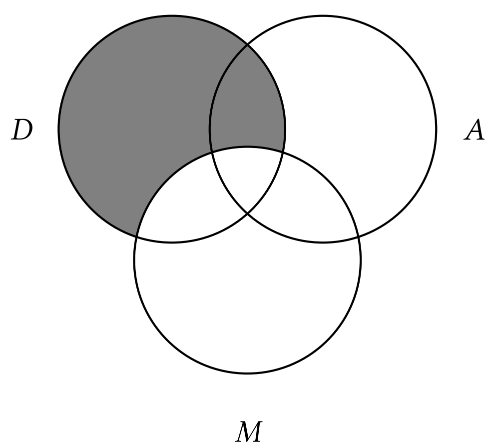
There are two ways to determine if a categorical syllogism is valid. One way uses Venn diagrams, and the other involves applying some simple rules.

Diagram Method

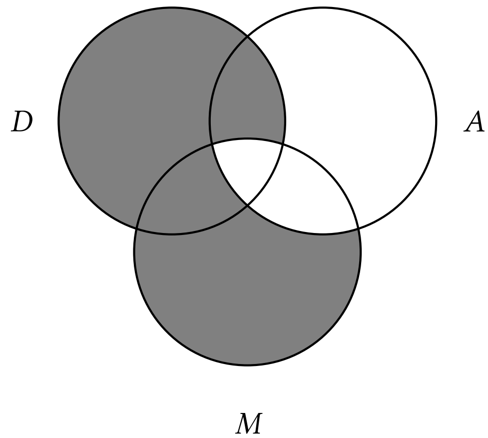
Since we have three terms in the argument, we'll need three intersecting circles. We'll start by drawing two circles for the conclusion, just as we did before. Then, in the middle and below, we'll draw another circle for the middle term. For labels, use letters that correspond to the classes in the argument. Here, we'll use D for dogs, M for mammals, and A for animals.



Next, we finish diagramming the premises by shading or placing an x. Since our first premise is "All dogs are mammals", we need to shade everything in the D circle that is not in the M circle.



Next, we diagram the second premise by shading everything that is in the M circle but not in the A circle.

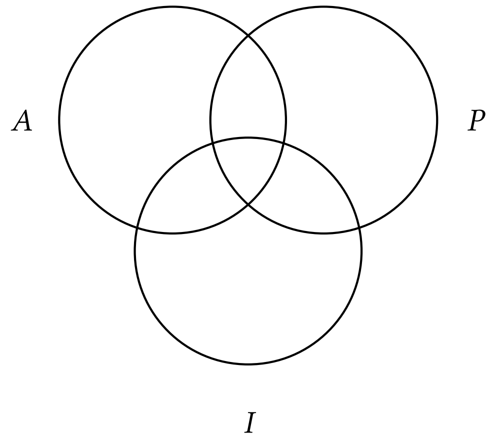


If there is any circle that has only one region left unshaded, you can place an 'X' in that region. This is because categorical logic assumes that there are no empty categories, meaning that every category has at least one thing in it. This is really only important for arguments that have an I or an O-sentence for a conclusion. In this case, we won't worry about it. Now that the premises are diagrammed, check to see if the conclusion has also been diagrammed, which in this case means that everything in the D circle that is not also in the A circle is shaded out. If so, then the argument is valid. This shows that making the premises true was enough to make the conclusion true also.

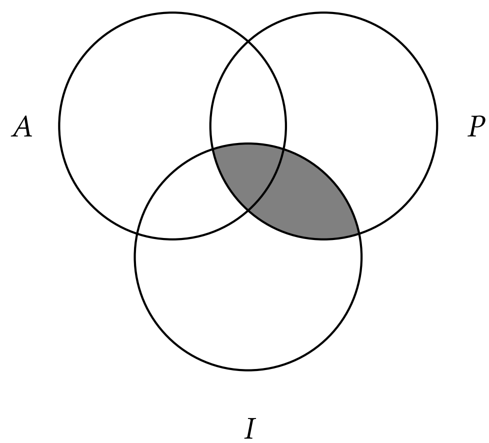
Let's try to diagram this argument:

1. No introverts are politicians
2. <u>All artists are introverts</u>
3. No artists are politicians

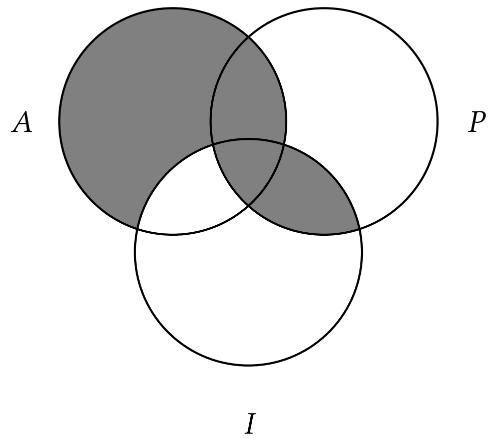
First, we draw and label the circles:



Then we diagram the premises, always doing the universals before any particulars. In this case, we have two universal premises, so we will just begin with the first premise:



Now, we'll diagram the second premise:

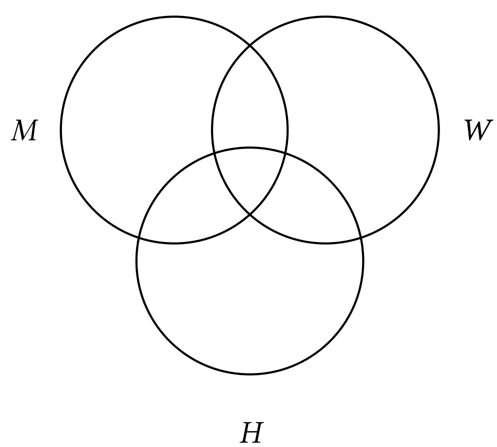


Diagramming the conclusion would require the intersection of A and P to be shaded. Notice, though, that the region between A and P has already been shaded by just diagramming the premises. That means that making the premises true was enough to guarantee that the conclusion would also be true, and the argument is valid.

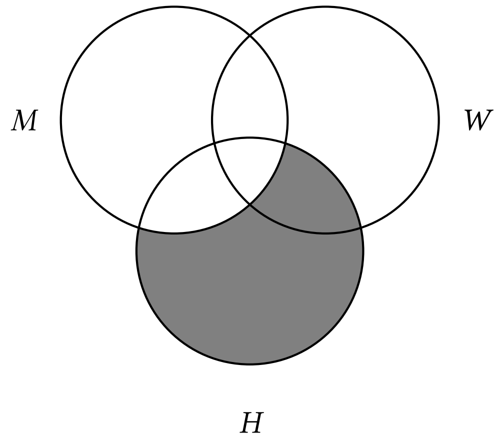
Let's try one more argument.

1. Some horses are things that weigh over 2,000 pounds.
2. <u>All horses are mammals.</u>
3. Some mammals are things that weigh over 2,000 pounds.

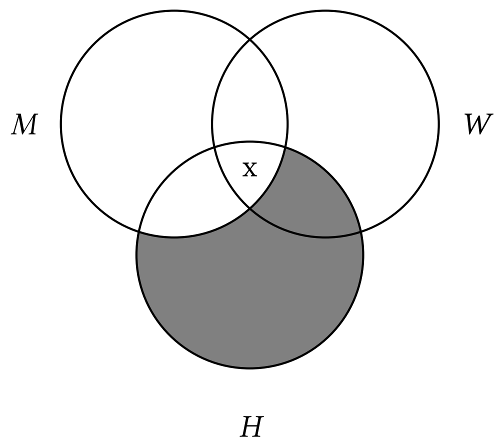
Again, we begin by drawing and labeling the circles.



The we diagram any universal premises, which, in this case, is the second premise.



Then, we diagram any particular premises.



Finally, we check to see if diagramming the premises was enough to make the conclusion also diagrammed. In this case, it was, so the argument is valid.

Hints for Diagramming Categorical Syllogisms

1. Diagram universals before particulars (shade before making an x.)
2. If it is not clear where the x goes, then put it on the line.

2.3 Rules for Categorical Syllogisms

There is another way to determine validity for categorical syllogisms. Every valid syllogism must meet three conditions:

1. There must be the same number of negations in the conclusion as in the premises.
2. The middle term must be distributed at least once.
3. Any term distributed in the conclusion must be distributed in the premises.

Before these rules can be applied, we'll have to explain what distribution is. Every categorical statement says something about a category or class. A statement distributes a term just in case what it says about that class is true of every subset of the class. For example, it is true that all dogs are mammals. It's also true that all members of any subset of the set of dogs are mammals—all dogs in Oklahoma are mammals, and all dogs in Greece are mammals, and so on. All dogs are not necessarily members of every subset of the class of mammals, however. The class of cats is a subset of the class of mammals, and no dog is a cat. So, the subject of an A-sentence is distributed, but the predicate is not. To remember when something is distributed, keep this in mind:

1. Universals distribute subjects, and
2. Negations distribute predicates.

So, A-sentences distribute the subject, E-sentences distribute both terms, I-sentences don't distribute anything, and O sentences distribute the predicate.

The rules are easy to apply. First, put the argument in standard form:

1. All A are B.
2. All B are C.

∴ All A are C.

Then, circle all of the distributed terms.

1. All A are B.
2. All B are C.

∴ All A are C.

Now, just check to see if there are any violations of the rules:

1. Are there the same number of negations in the conclusion as in the premises? Yes, since there are no negations at all.
2. Is the middle term distributed at least once? Yes, the middle term is B and it is distributed in the second premise.

3. Is any term that distributed in the conclusion also distributed in the premises? Yes, A is distributed in the conclusion, but it is also distributed in the first premise.

So, since the argument breaks none of the rules, it is valid.

2.4 Relations of Equivalence

Properly formed categorical syllogisms have only three terms. Unfortunately, some arguments that you will encounter won't always be in proper form. One common way this happens is for a person to use a term like "Americans" in one premise, but use "non-Americans" in another. This can result in a syllogism with four or more terms, making it impossible to evaluate using either of our two methods. What we then need to do is to convert the sentence using one of the terms into a logically equivalent sentence that uses the other term.

There are three operations that can be applied to categorical sentences: conversion, obversion, and contraposition. It is important to know both how to apply them and in what cases does an operation result in an equivalent sentence. We're particularly interested in the conditions that those different operations are *truth-preserving*. An operation is truth preserving when, applied to a true sentence, it always results in a true sentence.

Conversion

Conversion is the simplest of the three. The converse of a sentence simply exchanges the subject and predicate terms of the original sentence. Conversion applied to A-sentences is *not* truth-preserving. "All dogs are mammals" is true, but "All mammals are dogs" is not. Conversion is truth-preserving for E-sentences and I-sentences. If it is true that no dogs are reptiles, it must be true that no reptiles are dots. Likewise, if it is true that some dogs are brown things, it must be true that some brown things are dogs.

Another way to think about this is to consider what the diagrams would like before the change and after the change. Before the change, the diagram looks like figure below, with the intersection of the S and P circles shaded.

After the change, the diagram looks like figure , with the intersection of the S and P circles shaded. Essentially, there's been no change. Imagine what it would like to view the first diagram from behind, or upside-down. In either case, what you would see is the same as the first diagram.

Obversion

Take another look at the square of opposition in figure 4.1. Note that the A and the E are straight across from each other, as are the I and the O. The first step in forming the obverse is to first change the sentence into the type that is straight across the square of opposition. That is, if you started with an A-sentence, then make it into and E. The O becomes and I, and so on.

Once you've changed the sentence type, the next step is to change predicate into its complement. The complement of a class *C* is the class of everything that is not in *C*. The easiest way to form a complement is to prefix the class with 'non'. For example, the complement of the class of students is the class of non-students.

So, the obverse of all dogs are mammals is no dogs are non-mammals. The obverse of no OBU students are martians is all OBU students are non-martians. Obversion is truth-preserving in all cases.

Contraposition

The last of our three relations is contraposition. To form the contrapositive of a sentence, first form the converse, then exchange both terms for their complements.

The contrapositive of all dogs are mammals is all non-mammals are non-dogs. Contraposition is truth-preserving for A-sentences and O-sentences only.

Original	Converse	Obverse	Contrapositive
All S are P	All P are S	**No S are non-P**	**All non-P are non-S**
No S are P	**No P are S**	**All S are non-P**	No non-P are non-S
Some S are P	**Some P are S**	**Some S are not non-P**	Some non-P are non-S
Some S are not P	Some P are not S	**Some S are non-P**	**Some non-P are not non-S**

Here's a table to help keep this straight (operations that are truth-preserving are in bold type):

Example

Look at the following argument:

1. All Catholics are non-Protestants.
2. <u>All Lutherans are Protestants.</u>
3. No Catholics are Lutherans.

Note that this argument has four terms:

1. Catholics
2. Non-Protestants
3. Lutherans
4. Protestants

To evaluate the argument, we will first have to either change "non-Protestants" to "Protestants" in the first premise, or "Protestants" to "non-Protestants" in the second premise and conclusion. To minimize errors, we should probably try the option requiring the fewest changes. The only two truth-preserving operations on A-sentences are obversion and contraposition. The contrapositive of "All Catholics are non-Protestants" is "All non-non-Protestants are non-Catholics." The double-non will cancel out, which will fix our original problem, but it will leave us with a new term, "non-Catholic." So, let's try the obverse. The obverse of "All Catholics are non-Protestants" is "No Catholics are Protestants." So, using that for our first premise, the argument becomes:

1. No Catholics are Protestants.
2. <u>All Lutherans are Protestants.</u>

3. No Catholics are Lutherans.

Now, we can check for validity — I'll leave that for you.

Chapter 3

Sentential Logic: Syntax

Chapter 4

Predicate Logic: Syntax

Bibliography

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