RM 294 - Optimization I

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Project 1 - Linear Programming

Group 15

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Introduction

The goal of this project was to use linear programming to analyze how a company should allocate its $10M marketing budget among several marketing mediums to maximize its return on investment (ROI). Two outside consulting firms were consulted to estimate the ROI of each marketing medium under consideration. The results given by each firm are shown below.

Consulting Firm #1

Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
ROI	3.1%	4.9%	2.4%	3.9%	1.6%	2.4%	4.6%	2.6%	3.3%	4.4%

Consulting Firm #2

Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
ROI	4.9%	2.3%	2.4%	3.9%	4.4%	4.6%	2.6%	1.9%	3.7%	2.6%

Problem Solutions

As we began formulating our linear program, we found there to be 10 decision variables, $(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10})$, one for each marketing medium. Then, we wrote our objective function which is to maximize the aggregate ROIs from each marketing medium. After that, we identified four constraints as shown below.

1. The sum of the values allocated to the marketing mediums can not exceed $10M.
$$x_1 + x_2 + x_3 + x_4 \quad + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} \leq 10$$

2. The combined values allocated to print, and TV can not be more than that of the amount allocated to Facebook and Email combined.
$$x_1 + x_2 \leq x_5 + x_{10}$$

3. The total amount used in social media (Facebook, LinkedIn, Instagram, Snapchat, and Twitter) has to be at least twice that of SEO and AdWords combined.
$$x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} \geq 2(x_3 + x_4)$$

4. The amount allocated to any individual platform can not be more than $3M.
$$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10} \leq 3$$

We need to maximize the ROI for the model. Given the ROI matrix for the problem:

The Objective Function with consulting Firm #1 ROI Estimates is as follows:

$$0.031\ x1 + 0.049\ x2 + 0.024\ x3 + 0.039\ x4 + 0.016\ x5 + 0.024\ x6 + 0.046\ x7 + 0.026\ x8 + 0.033\ x9 + 0.044\ x10$$

We began by loading the files with ROI estimates:

```
obj = pd.read_csv('ROI_data.csv') #Read file into a dataframe
```

```
obj = obj.set_index(obj.columns[0]) #Set the first column as index for easier access
```

Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
ROI	0.031	0.049	0.024	0.039	0.016	0.024	0.046	0.026	0.033	0.044
Second Firms ROI Estimate	0.049	0.023	0.024	0.039	0.044	0.046	0.026	0.019	0.037	0.026

We constructed our matrix with decision variables, constraints, direction, and right-hand-side values so that we could import all our data into the Gurobi software. Our formulation for setting up the linear program is shown below.

```
A = np.zeros((3,10))
A[0,:]=[1,1,0,0,-1,0,0,0,0,-1] #First constraint Print+TV < Facebook + Email
A[1,:]=[0,0,-2,-2,1,1,1,1,1,0] # Social Media > 2(SEO + Adwords)
A[2,:]=[1,1,1,1,1,1,1,1,1,1] # Total Budget constraint
```

```
b = np.array([0,0,10]) # Right Hand Side Limits of constraints
sense = np.array(['<','>','<'])
```

We need to maximize the ROI for the model. Given the ROI matrix for the problem:

The Objective Function is as follows:

$$0.031\ x_1 + 0.049\ x_2 + 0.024\ x_3 + 0.039\ x_4 + 0.016\ x_5 + 0.024\ x_6 + 0.046\ x_7 + 0.026\ x_8 + 0.033\ x_9 + 0.044\ x_{10}$$

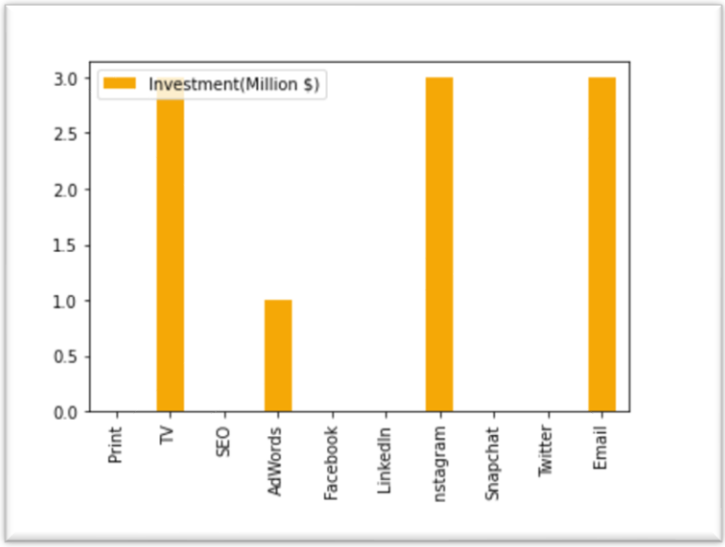
```
pModel = gp.Model() # initialize an empty model

pModX = pModel.addMVar(10,ub=np.array([3,3,3,3,3,3,3,3,3,3])) #Decision Variables add the upper bounds as 3million USD for each
pModCon = pModel.addMConstrs(A, pModX, sense, b) # add the constraints to the model
pModel.setMObjective(None,obj.iloc[0],0,sense=gp.GRB.MAXIMIZE) # Maximize the ROI for the model

pModel.Params.OutputFlag = 0 # tell gurobi to shut up!!
pModel.Params.TimeLimit = 3600
```

Consulting Firm #1

The ROI achieved with Consulting Firm #1 Estimates (max) given the constraints, is 0.456 million ~ 456000 USD

The Optimal Investments for Consulting Firm #1 Estimates are given by:

	Investment(Million $)
Print	0.0
TV	3.0
SEO	0.0
AdWords	1.0
Facebook	0.0
LinkedIn	0.0
Instagram	3.0
Snapchat	0.0
Twitter	0.0
Email	3.0



We observe that investments are made in only four mediums with TV, Instagram and Email getting 3 million $ investments and AdWords getting a 1 million $ investment.

Consulting Firm #2

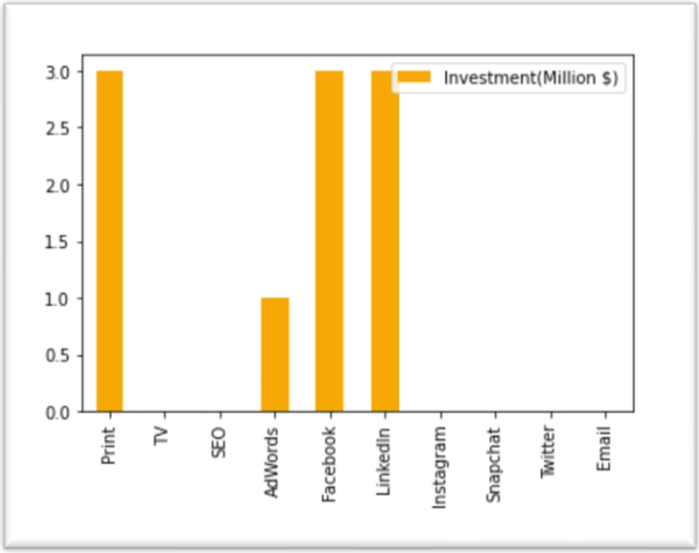
The Objective Function with consulting Firm #2 ROI Estimates is as follows:

$$0.049\ x1 + 0.023\ x2 + 0.024\ x3 + 0.039\ x4 + 0.044\ x5$$
$$+ 0.046\ x6 + 0.026\ x7 + 0.019\ x8 + 0.037\ x9 + 0.026\ x10$$

We follow a similar process as in the first case. We observe that the Objective function or the ROI observed in both cases is the same, 456000 USD.

However, the investments are different compared to the case 1. We observe investments in Print, Facebook, LinkedIn ($ 3 Million) and AdWords ($ 1 Million).

	Investment(Million $)
Print	3.0
TV	0.0
SEO	0.0
AdWords	1.0
Facebook	3.0
LinkedIn	3.0
Instagram	0.0
Snapchat	0.0
Twitter	0.0
Email	0.0



Intuitively it makes sense as investments go to the channels with higher ROI. We can observe the side ROI and Investments side by side to see the same trend:

	ROI	Investment(Million $)
Print	0.031	0.0
TV	0.049	3.0
SEO	0.024	0.0
AdWords	0.039	1.0
Facebook	0.016	0.0
LinkedIn	0.024	0.0
Instagram	0.046	3.0
Snapchat	0.026	0.0
Twitter	0.033	0.0
Email	0.044	3.0

Consulting Firm 1

	ROI	Investment(Million $)
Print	0.049	3.0
TV	0.023	0.0
SEO	0.024	0.0
AdWords	0.039	1.0
Facebook	0.044	3.0
LinkedIn	0.046	3.0
Instagram	0.026	0.0
Snapchat	0.019	0.0
Twitter	0.037	0.0
Email	0.026	0.0

Consulting Firm 2

The Allocations are different as the ROI estimates are different for channels. For ex, the Print ROI was increased from 3.1 percent to 4.9 percent and hence we see 3 million allocated to Print in the second case unlike the first case.

To check whether the third constraint, i.e., the individual investment less than 3 million USD, we tested both cases with no upper bounds to check how the investments vary.

1) First Case ROI estimates with no upper bound

The ROI has increased to ~ 465000 USD (1.9 percent increase from 456000 USD).

The investments have changed. We observe investments only in TV and Email which have the highest ROI percentages, 4.9% and 4.4% respectively.

	Investment(Million $)
Print	0.0
TV	5.0
SEO	0.0
AdWords	0.0
Facebook	0.0
LinkedIn	0.0
Instagram	0.0
Snapchat	0.0
Twitter	0.0
Email	5.0

2) Second Case ROI Estimates with No upper Bound constraint (3 million for individual investment).

	Investment(Million $)
Print	5.0
TV	0.0
SEO	0.0
AdWords	0.0
Facebook	5.0
LinkedIn	0.0
Instagram	0.0
Snapchat	0.0
Twitter	0.0
Email	0.0

We see that though the objective/ROI has increased. The problem solves such that unfair amounts of funds get allocated to either TV and Email (first ROI estimates) or Print and Facebook (based on second ROI estimates).
This could lead to lack of presence/marketing on other channels so keeping the upper bound on the individual investments would lead to a fair distribution among the mediums. Hence, we believe that the third constraint is very useful in optimal and fair allocation of funds among all marketing mediums.

Sensitivity Analysis

Sensitivity Analysis is the systematic study of how sensitive the solution is to the changes in the data. How much will the optimal solution change if we change the ROI per medium?

Note: We take the Consulting Firm #1's estimates to compute further calculations

We use Gurobi Model's parameters to obtain the lower threshold and upper threshold for each channel's ROI for the optimal solution to remain the same. These thresholds are the answers to how much the ROI (Objective function) can change for the optimal solution (investment in each channel) to be the same.

```
pModX.SAObjLow #Lower threshold Sensitivity Analysis
```

```
array([ -inf, 0.039,  -inf, 0.033,  -inf,  -inf, 0.039,  -inf,  -inf,
       0.029])
```

```
pModX.SAObjUp #Upper threshold of individual ROI estimates
```

```
array([0.049, 0.062, 0.039, 0.046, 0.029, 0.039,   inf, 0.039, 0.039,
         inf])
```

The advertising medium's ROI can be decreased by the below amount for each channel.

Note: Inf refers to Infinity i.e. the irrespective of the decrease in such channels, the optimal solution remains the same.

We observe that optimal solution remains the same when:

- ROI of TV can be decreased from 4.9% to 3.9%
- ROI of Email can be decreased from 4.4% to 2.9%
- ROI of AdWords can be decreased from 3.9% to 3.3%
- ROI of Instagram can be decreased from 4.6% to 3.9%

Print	inf
TV	0.010
SEO	inf
AdWords	0.006
Facebook	inf
LinkedIn	inf
Instagram	0.007
Snapchat	inf
Twitter	inf
Email	0.015

The advertising medium's ROI can be increased by the below amount for each channel
Note: Inf refers to Infinity i.e., the irrespective of the increase in such channels, the optimal solution remains the same.

We observe that optimal solution remains the same when:

- ROI of Print can be increased from 3.1% to 4.9%
- ROI of TV can be increased from 4.9% to 6.2%
- ROI of SEO can be increased from 2.4% to 3.6%
- ROI of AdWords can be increased from 3.9% to 4.6%
- ROI of Facebook can be increased from 1.6% to 2.9%
- ROI of LinkedIn can be increased from 2.4% to 3.9%
- ROI of Snapchat can be increased from 2.6% to 3.9%
- ROI of Twitter can be increased from 3.3% to 3.9%

Print	0.018
TV	0.013
SEO	0.015
AdWords	0.007
Facebook	0.013
LinkedIn	0.015
Instagram	inf
Snapchat	0.013
Twitter	0.006
Email	inf

Reinvestment of Return

Given the ROI estimates for an entire year, we model to find allocations for each month of the year. Our budget would not be a static 10 million USD but would increase according to the return observed in each month.

For example, if the marketing obtains a 4% return in January, the budget of February will be $10M + $10M × 4% × 50% = $10.2M.

We need to iterate the modelling over each month as our budget constraint changes every month. We first read the estimates for the entire year

Month	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
January	0.040	0.036	0.024	0.039	0.030	0.035	0.036	0.0225	0.035	0.035
February	0.040	0.039	0.027	0.038	0.043	0.032	0.027	0.0180	0.037	0.035
March	0.035	0.029	0.031	0.038	0.024	0.041	0.037	0.0260	0.042	0.025
April	0.038	0.031	0.024	0.044	0.024	0.038	0.037	0.0250	0.036	0.029
May	0.035	0.032	0.019	0.034	0.027	0.027	0.039	0.0220	0.045	0.039
June	0.040	0.032	0.027	0.034	0.034	0.030	0.045	0.0210	0.038	0.041
July	0.039	0.036	0.020	0.044	0.039	0.037	0.043	0.0180	0.040	0.038
August	0.042	0.033	0.028	0.042	0.020	0.037	0.036	0.0150	0.044	0.043
September	0.041	0.028	0.025	0.042	0.029	0.037	0.028	0.0250	0.040	0.034
October	0.030	0.030	0.031	0.046	0.031	0.033	0.032	0.0230	0.025	0.032
November	0.048	0.033	0.027	0.041	0.029	0.036	0.042	0.0300	0.031	0.041
December	0.048	0.040	0.019	0.037	0.042	0.036	0.026	0.0290	0.036	0.037

The Decision Variables now would be the form of

$$x_{i,j}$$

where, i stands for the month
j stands for the Marketing medium

Hence, we will have 12 months * 10 marketing mediums = 120 decision variables.

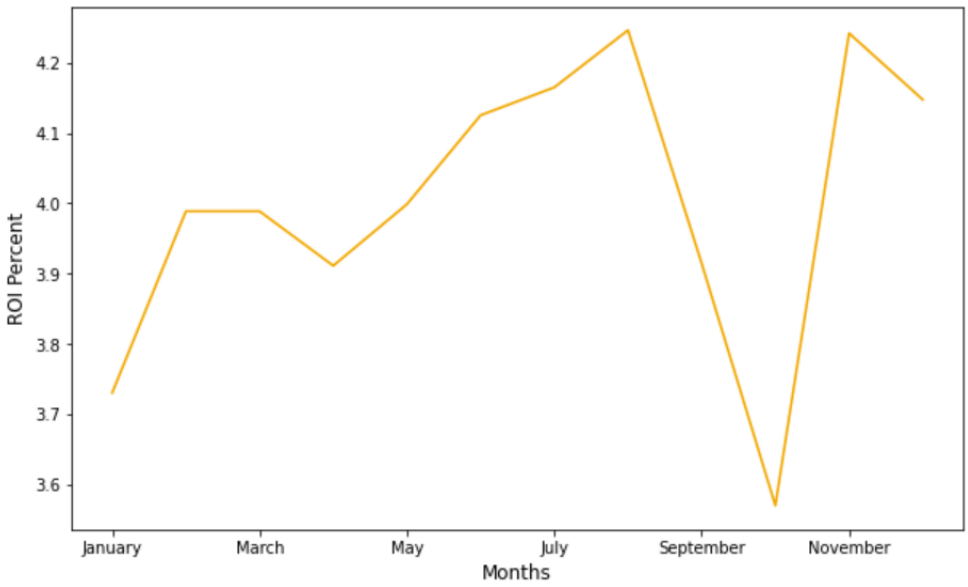
We would iterate one model over the 12 months with the same constraints except for the budget constraint (as the RHS for budget would change every month)

The optimal allocation for the entire year as follows (in Million $)

Month	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
January	3.000000	0.0	0.0	1.333333	0.000000	0.000000	2.666667	0.0	0.000000	3.000000
February	3.000000	0.0	0.0	2.395500	3.000000	0.000000	0.000000	0.0	1.791000	0.000000
March	0.000000	0.0	0.0	3.000000	0.000000	3.000000	1.393437	0.0	3.000000	0.000000
April	0.000000	0.0	0.0	3.000000	0.000000	3.000000	3.000000	0.0	1.608870	0.000000
May	1.828962	0.0	0.0	0.000000	0.000000	0.000000	3.000000	0.0	3.000000	3.000000
June	3.000000	0.0	0.0	0.000000	0.000000	0.000000	3.000000	0.0	2.063416	3.000000
July	1.157944	0.0	0.0	3.000000	1.157944	0.000000	3.000000	0.0	3.000000	0.000000
August	3.000000	0.0	0.0	1.860853	0.000000	0.721706	0.000000	0.0	3.000000	3.000000
September	1.433704	0.0	0.0	3.000000	0.000000	3.000000	0.000000	0.0	3.000000	1.433704
October	0.000000	0.0	0.0	3.000000	0.143045	3.000000	3.000000	0.0	0.000000	3.000000
November	3.000000	0.0	0.0	2.135402	0.000000	1.270804	3.000000	0.0	0.000000	3.000000
December	3.000000	3.0	0.0	0.732675	3.000000	0.000000	0.000000	0.0	0.000000	3.000000

The optimal investment along with the ROI and Budget for each month is given as below:

Month	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email	ROI (Million $)	Budget(Million $)
January	3.000000	0.0	0.0	1.333333	0.000000	0.000000	2.666667	0.0	0.000000	3.000000	0.373000	10.000000
February	3.000000	0.0	0.0	2.395500	3.000000	0.000000	0.000000	0.0	1.791000	0.000000	0.406296	10.186500
March	0.000000	0.0	0.0	3.000000	0.000000	3.000000	1.393437	0.0	3.000000	0.000000	0.414557	10.393437
April	0.000000	0.0	0.0	3.000000	0.000000	3.000000	3.000000	0.0	1.608870	0.000000	0.414919	10.608870
May	1.828962	0.0	0.0	0.000000	0.000000	0.000000	3.000000	0.0	3.000000	3.000000	0.433014	10.828962
June	3.000000	0.0	0.0	0.000000	0.000000	0.000000	3.000000	0.0	2.063416	3.000000	0.456410	11.063416
July	1.157944	0.0	0.0	3.000000	1.157944	0.000000	3.000000	0.0	3.000000	0.000000	0.471320	11.315889
August	3.000000	0.0	0.0	1.860853	0.000000	0.721706	0.000000	0.0	3.000000	3.000000	0.491859	11.582559
September	1.433704	0.0	0.0	3.000000	0.000000	3.000000	0.000000	0.0	3.000000	1.433704	0.464528	11.867408
October	0.000000	0.0	0.0	3.000000	0.143045	3.000000	3.000000	0.0	0.000000	3.000000	0.433434	12.143045
November	3.000000	0.0	0.0	2.135402	0.000000	1.270804	3.000000	0.0	0.000000	3.000000	0.526300	12.406206
December	3.000000	3.0	0.0	0.732675	3.000000	0.000000	0.000000	0.0	0.000000	3.000000	0.528109	12.732675



Stable Budget

A stable budget is defined as a monthly allocation such that for each platform the monthly change in spend is no more than $1M. We check whether our optimal budget allocation across mediums is stable by taking the difference for each medium and each month.

Month	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
January	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN	NaN
February	0.000000	0.0	0.0	1.062167	3.000000	0.000000	-2.666667	0.0	1.791000	-3.000000
March	-3.000000	0.0	0.0	0.604500	-3.000000	3.000000	1.393437	0.0	1.209000	0.000000
April	0.000000	0.0	0.0	0.000000	0.000000	0.000000	1.606563	0.0	-1.391130	0.000000
May	1.828962	0.0	0.0	-3.000000	0.000000	-3.000000	0.000000	0.0	1.391130	3.000000
June	1.171038	0.0	0.0	0.000000	0.000000	0.000000	0.000000	0.0	-0.936584	0.000000
July	-1.842056	0.0	0.0	3.000000	1.157944	0.000000	0.000000	0.0	0.936584	-3.000000
August	1.842056	0.0	0.0	-1.139147	-1.157944	0.721706	-3.000000	0.0	0.000000	3.000000
September	-1.566296	0.0	0.0	1.139147	0.000000	2.278294	0.000000	0.0	0.000000	-1.566296
October	-1.433704	0.0	0.0	0.000000	0.143045	0.000000	3.000000	0.0	-3.000000	1.566296
November	3.000000	0.0	0.0	-0.864598	-0.143045	-1.729196	0.000000	0.0	0.000000	0.000000
December	0.000000	3.0	0.0	-1.402727	3.000000	-1.270804	-3.000000	0.0	0.000000	0.000000

We observe that it isn't a stable budget as the monthly allocation changes by way more than 1 million USD for each platform.

To make it stable, we will take the array of optimal solution for each month. In this case X[i,:] stores each month's optimal allocation (Model.X values). While running the modelling for February we specify the lower bounds and upper bounds as the optimal solution for January ± 1. This would give us a stable allocation model such that monthly change isn't more than $1M

So, for February Lower Bound of investment per medium, lb = X[0,:] - 1

for February Upper Bound of investment per medium ub = X[0,:] + 1

Note:
*1. X[0, :] would contain the optimal budget allocation per medium for Januar*y
2. *lb refers to Lower Bounds which are added while initializing the number of decision variables in model.*
3. *ub refers to Upper Bounds of the decision variables which were set to a constant 3 in the above questions*