MEAN-VARIANCE OPTIMIZATION: OPTIMAL RISKY PORTFOLIO WITH TWO RISKY ASSETS

WHAT WILL YOU LEARN?

▶What is the optimal portfolio of risky assets?
  ▶What is the best risky asset portfolio?
  ▶Mean-variance efficient (MVE) portfolio

▶What is the optimal asset allocation?
  ▶How should we allocate our wealth between this optimal risky portfolio and the risk-free asset?
  ▶Capital allocation line (CAL)
  ▶Sharpe ratio

RECAP THE ANSWER TO THE SECOND QUESTION

▸ The CAL describes the menu of possible risk/return trade-offs between a risk-free and risky asset.

▸ Its slope equals the Sharpe ratio.

▸ The optimal allocation with a risky and risk-free asset for any investor is the tangency point of that investor's indifference curves with the CAL.

NOW TURN TO THE FIRST QUESTION

▸ What is the optimal portfolio of risky assets?

  ▸ What is the best risky asset portfolio?
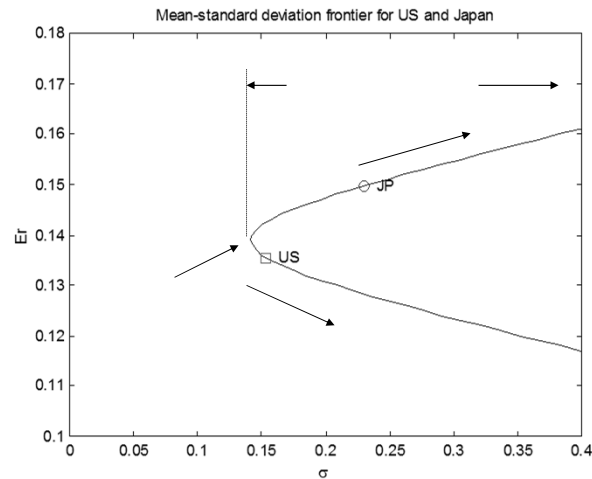
  ▸ Mean-variance efficient (MVE) portfolio

WHAT IF WE HAVE TWO RISKY ASSETS?

▸ With two risky assets, different CALs are available.

▸ What is the composition of the best risky portfolio?

PORTFOLIO OF TWO RISKY ASSETS

Mean-standard deviation frontier for US and Japan
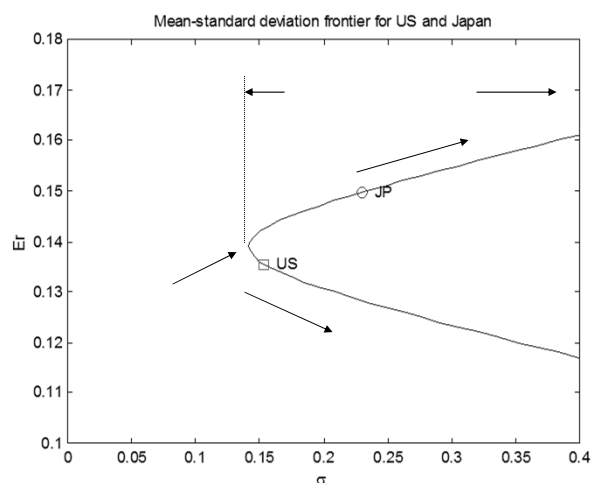
LET'S ADD THE RISK-FREE ASSET

Mean-standard deviation frontier for US and Japan

TWO RISKY ASSETS AND ONE RISK-FREE ASSET

▸The "tangency portfolio" provides the "steepest" CAL.

▸It maximizes the Sharpe ratio for all possible risky portfolios.

▸This portfolio is known as the mean-variance efficient portfolio or MVE.

MEAN-VARIANCE EFFICIENT PORTFOLIO

▸ Mean-variance efficient (MVE) portfolio determines the optimal risky portfolio.

▸ Everyone will hold the same risky portfolio – the MVE – and mix it with the risk-free asset according to his or her preferences, namely risk aversion.

▸ CAL combining the risk-free asset and the MVE becomes the set of efficient portfolios.

SUMMARY

▸ Step 1: Create the mean variance (minimum standard-deviation) frontier. (For two risky assets, this is all the possible combinations of the two assets.)

▸ Step 2: Find the portfolio that maximizes Sharpe ratio. This is the mean variance efficient (MVE) portfolio.

▸ Step 3: Construct the efficient frontier: the line starting from the risk-free asset, intersecting the frontier at the MVE.

▸ Step 4: Mix the MVE portfolio (fund) with the risk-free asset according to your preferences.

MEAN-VARIANCE OPTIMIZATION: OPTIMAL RISKY PORTFOLIO WITH MULTIPLE RISKY SECURITIES

OPTIMAL PORTFOLIO CHOICE PROBLEM

▸Step 1: Create the mean variance (minimum standard-deviation) frontier. (For two risky assets, this is all the possible combinations of the two assets.)

▸Step 2: Find the portfolio that maximizes Sharpe ratio. This is the minimum variance efficient (MVE) portfolio.

▸Step 3: Construct the efficient frontier: the line starting from the risk-free asset, intersecting the frontier at the MVE.

▸Step 4: Mix the MVE portfolio (fund) with the risk-free asset according to your preferences.

WHAT WILL YOU LEARN?

▸ What is the optimal portfolio of risky assets you have multiple securities?

  ▸ How do you construct the mean variance frontier when you have multiple securities?

MULTIPLE RISKY SECURITIES

MEAN-VARIANCE FRONTIER

$E(r_p)$

μ*=12%

μ*=10%

σ_p
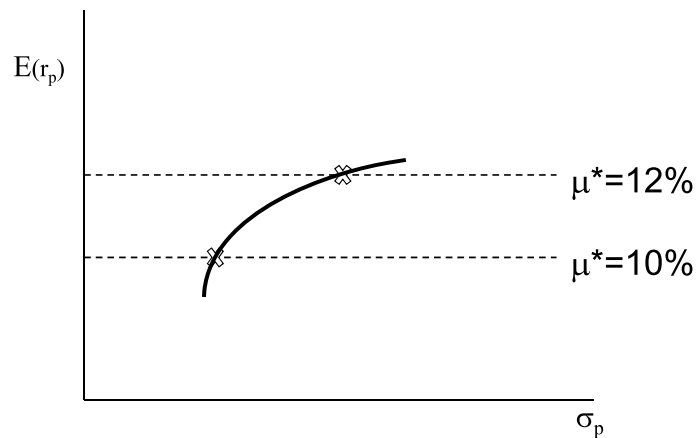
MEAN VARIANCE FRONTIER WITH MULTIPLE RISKY SECURITIES

▶ Well, how many target levels do we have to try out to obtain the mean-variance frontier?

HOW DO WE CONSTRUCT THE MEAN VARIANCE FRONTIER ?

▸ Find two portfolios on the frontier…

▸ If we can find two portfolios on the frontier that we can take all possible combinations of those to trace the mean variance frontier.

▸ What two portfolios could you use???

HOW DO WE CONSTRUCT THE MEAN VARIANCE FRONTIER?

HOW DO WE CONSTRUCT THE MEAN VARIANCE FRONTIER

▸ Take all combinations of these two portfolios
   ▸ Let w be the weight in the MVE and (1-w) in the minimum variance portfolio for various weights and draw the frontier.

▸ Use the formulas to compute the means and variances to trace the risk-return combinations of the two portfolios

▸ To actually find two frontier portfolios, Excel or some other optimizer can be used.

SUMMARY

▸ Mean variance frontier with multiple risky securities

▸ Two portfolios on the frontier generate the whole frontier

▸ Given a risk-free asset, the set of efficient portfolios has one risky portfolio that delivers the steepest CAL – the MVE.

▸ The optimal asset allocation involves mixing the risk free asset with the MVE according to your preferences