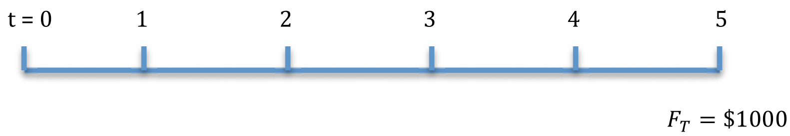
Review of Bond Valuation and Yield-to-maturity

Zero-coupon bond valuation

Suppose you were offered an opportunity to purchase a zero-coupon bond with a face value of $1000 with five years to maturity. Suppose you require a 10% rate of return to invest in this bond. How much will you pay for the bond?

Let's first describe the promised cash flows. The only promised cash flow a zero-coupon bond makes is the face value at maturity, F_T = $1000.



Let B_0 denote the value of the bond today. It is equal to the present value of the promised cash flows discounted at the required rate of return $r = 10\%$.

$$B_0 = \frac{\$1000}{(1.10)^5} = \$620.92$$

In general,

$$B_0 = \frac{F_T}{(1+r)^T}$$

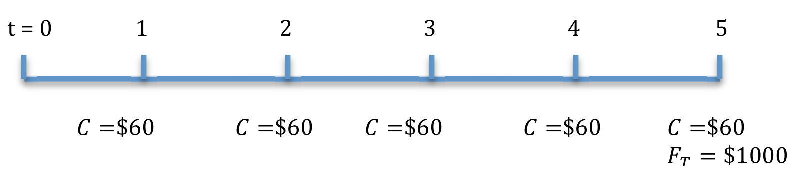
Annual coupon bond valuation

Suppose you were offered an opportunity to purchase a 6% annual coupon bond with a face value of $1000 with five years to maturity. Suppose you require a 10% rate of return to invest in this bond. How much will you pay for the bond?

Let's first describe the promised cash flows.

First, let C denote the dollar amount of each coupon payment. The coupon amount is determined by the coupon rate: $C = Face\ Value \times Coupon\ rate = \$1000 \times 6\% = \$60$.

In addition to the coupon payments, the bond will pay the face value at maturity. Let F_T denote the face value payment at maturity. $F_T = \$1000$.



Let B_0 denote the value of the bond today. It is equal to the present value of the promised cash flows discounted at the required rate of return $r = 10\%$.

$$B_0 = \frac{\$60}{(1.10)} + \frac{\$60}{(1.10)^2} + \frac{\$60}{(1.10)^3} + \frac{\$60}{(1.10)^4} + \frac{\$1060}{(1.10)^5} = \$848.37$$

In general,

$$B_0 = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \cdots + \frac{C + F_T}{(1+r)^T}$$

where T denotes the years to maturity and r is the annual discount rate.

Semi-annual coupon bond valuation

What if the coupons were paid semi-annually? Note that each coupon payment will now be $C = Face\ Value \times \frac{Coupon\ rate}{2} = \$1000 \times \frac{6\%}{2} = \$30$. There will now be ten coupon payments.

The value of the bond today would then be given by

$$B_0 = \frac{\$30}{(1.05)} + \frac{\$30}{(1.05)^2} + \frac{\$30}{(1.05)^3} + \frac{\$30}{(1.05)^4} + \cdots + \frac{\$30}{(1.05)^9} + \frac{\$1030}{(1.05)^{10}} = \$845.56$$

In general,

$$B_0 = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \cdots + \frac{C + F_T}{(1+r)^T}$$

where T denotes the number of periods to maturity and r is the per period discount rate.

Finding the yield-to-maturity (YTM)

Suppose there is a bond with a face value of \$1000 and an annual coupon rate of 10% and four years to maturity. The current price of the bond is \$1200. What is the yield-to-maturity of this bond if you purchase this bond at the current price and hold it until maturity?

The yield-to-maturity is the internal rate of return that you earn on the bond. Put differently, it is the discount rate that sets the current price of the bond to the present value of the promised cash flows.

$$B_0 = 1200 = \frac{\$100}{(1+r)} + \frac{\$100}{(1+r)^2} + \frac{\$100}{(1+r)^3} + \frac{\$1100}{(1+r)^4}$$

where $C = \$1000 \times 10\% = \$100$ and $F_T = \$1000$. Solving for r gives the yield to maturity: $r = 4.43\%$. Typically, you will need a calculator or Excel to solve for r.