

The analytical solution to the Mean Variance Efficient (MVE) Portfolio: The case with two risky assets and one risk-free asset

This note provides the analytical solution to finding the mean variance efficient portfolio when the investment opportunity consists of two risky assets and a risk-free asset.

Recall that the mean variance efficient portfolio is the optimal risky portfolio on the mean variance frontier that maximizes the Sharpe ratio. Put differently, it is the risky portfolio that gives us the steepest Capital Allocation Line when we combine the risk-free asset with this optimal risky portfolio.

We are therefore solving for the following mathematical problem:

$$\max_{w_1, w_2} \frac{[E(r_p) - r_f]}{\sigma_p}$$

where

$$E(r_p) = w_1 E(r_1) + w_2 E(r_2)$$

$$\sigma_p = \sqrt{w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \rho_{12} \sigma_1 \sigma_2}$$

and of course

$$w_1 + w_2 = 1$$

The solution to this problem is given by:

$$w_{MVE} = w_1 = \frac{E[r_1 - r_f] \sigma_2^2 - E[r_2 - r_f] \sigma_{12}}{E[r_1 - r_f] \sigma_2^2 + E[r_2 - r_f] \sigma_1^2 - (E[r_1 - r_f] + E[r_2 - r_f]) \sigma_{12}}$$

Now let's plug in numbers using just US and Japan from the data below. Let the US equity market be risky asset #1 and the Japanese equity market risky asset #2.

	RETURN	DEVIATION
US	0.1355	0.1535
UK	0.1589	0.2430
France	0.1519	0.2324
Germany	0.1435	0.2038
Japan	0.1497	0.2298

Risk-free rate	0.0500
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	US	UK	France	Germany	Japan
US	1.0000	0.5003	0.4398	0.3681	0.2663
UK	0.5003	1.0000	0.5420	0.4265	0.3581
France	0.4398	0.5420	1.0000	0.6032	0.3923
Germany	0.3681	0.4265	0.6032	1.0000	0.3663
Japan	0.2663	0.3581	0.3923	0.3663	1.0000

You can solve for the weight of the US equity market in the mean-variance efficient portfolio as

$$w_{US,MVE} = 0.698$$

The weight of the Japanese equity market is then given by:

$$w_{JP,MVE} = 1 - w_{US,MVE} = 0.302$$