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A model for the three-dimensional magnetic field correlation spectra of low-frequency solar wind fluctuations during Alfvénic periods

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Abstract. Using statistical homogeneity, magnetic field fluctuations in the solar wind during Alfvénic periods are analyzed in terms of the two independent polarizations allowed for each wave vector **k**. It is shown that the energy spectra of the two polarizations can be related both to the correlation tensor and to the variance matrix, which is generally used to characterize the anisotropy of the turbulence. Assuming simple anisotropic power law models, the parameters defining the spectra of the two polarizations are determined by fitting the eigenvalues of the variance matrix on the corresponding eigenvalues evaluated by *Bavassano et al.* (1982) for Helios 2 data. Then the corresponding form of the correlation tensor is obtained. In particular, we found that the spectrum of polarization [1] fluctuations (corresponding, in a weak turbulence theory approach, to the Alfvén mode) is steeper than the polarization [2] spectrum (corresponding to the magnetosonic modes). While the former spectrum is dominated by wave vectors parallel to $\mathbf{B}_0$, the latter is strongly flattened on the plane containing the radial and the mean magnetic field $\mathbf{B}_0$ directions. A discussion of these results in connection with other observational and theoretical issues is outlined.

Introduction

The study of the nature and of the properties of solar wind low-frequency fluctuations in the frequency range 10^{-7} Hz $< f <$ 1 Hz not only is important in itself, since it represents a way to understand the behavior of the magnetohydrodynamic (MHD) turbulence in a parameter region which is not accessible in terrestrial laboratories, but it also furnishes information that is useful in many different astrophysical problems. In the last years a considerable amount of work has been done concerning both the analysis of the solar wind data and the theoretical understanding of the physical mechanisms which determine the features observed in the solar wind turbulence. It is now well established that, at least in the inner heliosphere and for the range of frequencies 10^{-4} Hz $< f < 10^{-1}$ Hz, interplanetary fluctuations in the trailing edges of solar wind high-speed streams (Alfvénic periods) display the following main properties [*Coleman*, 1968; *Belcher and Davis*, 1971; *Barnes*, 1979]:

1. A power law range is present in the energy spectrum; i.e., the energy spectrum is proportional to $f^{-\alpha}$, where α is often close to the values typical of a fully developed turbulence.

2. The fluctuations are essentially noncompressive: During long periods the density fluctuations $\delta\rho$ are extremely reduced, and the magnetic field intensity remains almost constant. In particular, $\delta\rho/\rho < 0.1$ and $\delta|\mathbf{B}|/|\mathbf{B}| \simeq 0.06$, where $\mathbf{B}$ is the magnetic field, whereas the normalized fluctuations $\delta|\mathbf{B}|/|\mathbf{B}|$ of the $\mathbf{B}$ components are of the order of unity. The fluctuations $\delta\rho/\rho$ and $\delta|\mathbf{B}|/|\mathbf{B}|$ are smaller for large-scale structures and slightly increase with increasing frequency.

3. A strong correlation between velocity $\delta\mathbf{v}$ and magnetic field $\delta\mathbf{B}$ fluctuations is present in the form $\delta\mathbf{v} \simeq \pm\delta\mathbf{B}/(4\pi\rho)^{1/2}$. The sign is that corresponding to fluctuations which propagate away from the Sun.

These fluctuations are also strongly anisotropic over all the power law range. The anisotropy has been revealed through a minimum variance analysis [*Bavassano et al.*, 1978, 1982; *Chang and Nishida*, 1973]. This method gives information about the energy distribution among the different components of the magnetic fluctuations. It is based on the determination of the eigenvalues and the eigenvectors of the variance matrix S_{ij} of the magnetic fluctuations,

$$S_{ij} = \langle \delta B_i \delta B_j \rangle \tag{1}$$

where the angle brackets indicate a time average. This analysis has been carried out by *Bavassano et al.* [1982] for the solar wind magnetic fluctuations during Alfvénic periods; these authors found that one of the eigenvalues of S_{ij} is typically much smaller than the other two, say, $\lambda_3 \ll \lambda_2 < \lambda_1$ (for example, typical values for the ratios of the eigenvalues of S_{ij} are $\lambda_1 : \lambda_2 : \lambda_3 = 10 : 3 : 1$). The minimum variance direction turns out to be almost parallel to the average magnetic field $\mathbf{B}_0$, so the magnetic field fluctuations $\delta\mathbf{B}$ lie in a plane approximately

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perpendicular to $\mathbf{B}_0$. This result is also confirmed by the direct analysis of *Tu et al.* [1989] for the Alfvénic periods of magnetic fluctuations in the interplanetary space. The maximum variance direction is perpendicular to the plane containing $\mathbf{B}_0$ and $\mathbf{v}_0$, $\mathbf{v}_0$ being the solar wind velocity [*Bavassano et al.*, 1982].

A great amount of theoretical work has been done to explain the simultaneous occurrence of a power law spectrum, which is the characteristic result of a nonlinear turbulent energy cascade, with a strong $\delta \mathbf{v} - \delta \mathbf{B}$ correlation in the interplanetary magnetic fluctuations [*Dobrowolny et al.*, 1980a; *Meneguzzi et al.*, 1981; *Grappin et al.*, 1982, 1983; *Matthaeus et al.*, 1983; *Grappin*, 1986; *Pouquet et al.*, 1986; *Ting et al.*, 1986]. But very few attempts to explain the associated anisotropy have been performed [*Shebalin et al.*, 1983; *Matthaeus et al.*, 1986; *Carbone and Veltri*, 1990].

The purpose of this paper is to make a further step toward a full understanding of the solar wind MHD fluctuations by deriving a description of the observed anisotropy in terms of a model for the three-dimensional (3-D) energy spectra of solar wind magnetic fluctuations. Since these spectra have a direct physical meaning, this could help to single out the mechanisms responsible for the anisotropy and could increase the possibility to build up a self-consistent model. Moreover, the model of the 3-D autocorrelation spectrum of the magnetic field so obtained could be used, for example, in the context of cosmic ray diffusion. This problem has been previously considered by *Matthaeus et al.* [1990]. These authors calculated the two-point magnetic field correlation tensor by a direct analysis of solar wind data sets, selected on the basis of statistical stationarity [*Matthaeus et al.*, 1986]. They find that two populations are present in the solar wind fluctuations with wave vectors parallel and perpendicular to $\mathbf{B}_0$, respectively. Due to the paucity of the stationary data sets for which the mean magnetic field is highly inclined to the ecliptic, they assume a wave vector distribution which is axisymmetric around $\mathbf{B}_0$. Moreover, they do not consider the polarization of the fluctuations.

Our approach to obtain some information on the two-point magnetic field correlation tensor is different. The statistical homogeneity hypothesis allows us to relate the integral of the Fourier transform of the two-point correlation tensor to the variance matrix. Moreover, for each wave vector $\mathbf{k}$, the Fourier transform of the magnetic field $\mathbf{B}(\mathbf{k})$ can be decomposed along two directions orthogonal both to each other and to $\mathbf{k}$. In the following we will refer to these directions as the independent polarizations of the magnetic field fluctuations. We have then assumed a suitable phenomenological model for the 3-D energy spectra of the two polarizations. This model depends on a limited number of parameters, which we have determined by a best fit technique. In the present paper, the fit has been performed on the results by *Bavassano et al.* [1982], who calculated the eigenvalues of the variance matrix evaluated over five time intervals (τ_i = 168 s, 8 min, 22.5 min, 1 hour, and 3 hours) for observations at three heliocentric distances (R = 0.29 AU, 0.65 AU, and 0.87 AU) of Helios 2 spacecraft. It is clear that such a method does not univocally determine the full 3-D two-point correlation tensor, but it selects among those corresponding to the postulated anisotropic power law spectra the one which fits best the minimum variance data. A similar approach was used by *Dobrowolny et al.* [1980b] but using much more restrictive hypotheses: statistically normal correlations, an unrealistic gaussian spectrum, and a magnetosonic polarization much smaller than the Alfvénic one.

The plan of the paper is the following: in section 2 we express the correlation tensor in terms of the energy spectra of two independent polarizations of the magnetic field fluctuations. In section 3 we relate these energy spectra to the variance matrix. Assuming a model for the spectra of the two polarizations, in section 4 we determine the parameters which characterize the model by fitting the eigenvalues of the variance matrix on the corresponding values determined by *Bavassano et al.* [1982]. In section 5 we discuss the results obtained.

A Model for the Correlation Spectrum

The Magnetic Field Correlation Tensor

A statistical description of a turbulent magnetic field can be given in terms of the quantity

$$\langle B_i(\mathbf{r}_1, t_1) B_j(\mathbf{r}_2, t_2) \rangle$$

($i, j = 1, 2, 3$) where $B_i(\mathbf{r}, t)$ is the ith component of the magnetic field at the location $\mathbf{r}$ and at the time t, and angle brackets indicate an ensemble average. Under the hypothesis of statistical homogeneity of the turbulence in time and space, the above expression depends on the space and time variables only through the differences $\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$ and $\tau = t_2 - t_1$, so that the two-point correlation tensor can be defined:

$$C_{ij}(\mathbf{r}, \tau) = \langle B_i(\mathbf{r}_1, t_1) B_j(\mathbf{r}_1 + \mathbf{r}, t_1 + \tau) \rangle \qquad (2)$$

($i, j = 1, 2, 3$). The knowledge of such a quantity furnishes a detailed description of the statistical properties of the turbulence, and in particular it gives information about the anisotropy of the turbulence. A direct evaluation of $C_{ij}(\mathbf{r}, \tau)$ for the solar wind MHD turbulence would require simultaneous measures of the magnetic field at different space locations, along at least three independent directions. Up to now, only measures taken along one-dimensional lines (the spacecraft trajectories) are available. In the frame of reference comoving with the solar wind, the spacecraft velocity is much larger than the typical wave propagation velocities; then in the evaluation of S_{ij} only information at $\tau \simeq 0$ is available, since the magnetic fluctuations appears to the spacecraft as being "frozen" in the plasma. *Matthaeus et al.* [1990] performed a direct calculation of

$$C_{ij}(\mathbf{r}, 0) = \langle B_i(\mathbf{r}_1, t_1) B_j(\mathbf{r}_1 + \mathbf{r}, t_1) \rangle \qquad (3)$$

based on the ISEE 3 data set; since a small number of stationary data sets with $\mathbf{B}_0$ highly inclined with respect to the ecliptic plane were available, *Matthaeus et al.* [1990] assumed cylindrical symmetry with respect to $\mathbf{B}_0$. We will follow a different approach: We introduce the space Fourier transform of the magnetic field

$$\mathbf{B}(\mathbf{k},t) = \int d^3\mathbf{r}\ \mathbf{B}(\mathbf{r},t)\ \exp(-i\mathbf{k}\cdot\mathbf{r})$$

and we assume that the magnetic field fluctuations are statistically homogeneous and stationary. The statistical stationarity of the data sets we will use in our analysis has been checked according the criterion of weakly time stationary process [*Matthaeus and Goldstein*, 1982]. We have found that for all three data sets used, the duration of which is 17 to 30 times longer than the corresponding correlation time (the correlation time of the fluctuations is of the order of 10^4 s), the above criterion is satisfied. Due to Taylor hypothesis [*Taylor*, 1938], statistical stationarity also ensures statistical homogeneity [*Matthaeus and Goldstein*, 1982]. Stationarity implies

$$\langle B_i(\mathbf{k}_1,t)B_j(\mathbf{k}_2,t)\rangle = (2\pi)^3\ T_{ij}(\mathbf{k}_1)\ \delta^3(\mathbf{k}_1+\mathbf{k}_2) \quad (4)$$

so that the correlation tensor $C_{ij}(\mathbf{r},0)$ can be expressed as follows

$$C_{ij}(\mathbf{r},0) = \frac{1}{(2\pi)^3}\int d^3\mathbf{k}\ T_{ij}(\mathbf{k})\ \exp(-i\mathbf{k}\cdot\mathbf{r})$$

The knowledge of the correlation spectrum $T_{ij}(\mathbf{k})$ is equivalent to the direct knowledge of $C_{ij}(\mathbf{r},0)$. Since $T_{ij}(\mathbf{k})$ is an hermitian tensor, only six of its components are independent. In the following, using the fact that the magnetic field is divergenceless, we will show that the number of independent components can be reduced to three. We consider two frames of reference: the first one (frame I) moves with the spacecraft velocity; the second frame of reference (frame II) moves with the local average solar wind velocity. We indicate by $\mathbf{B}$ the magnetic field which is measured in frame I, by $\mathbf{b}$ the magnetic field in frame II, and by $\mathbf{v}_0$ the average solar wind velocity measured with respect to frame I. Since $v_0 \ll c$ (c being the velocity of the light), the magnetic field in these two frames of reference is related by

$$\mathbf{B}(\mathbf{r},t) = \mathbf{b}(\mathbf{r}-\mathbf{v}_0 t,t) \quad (5)$$

or, in terms of Fourier components,

$$\mathbf{B}(\mathbf{k},t) = \mathbf{b}(\mathbf{k},t)\exp(-i\mathbf{k}\cdot\mathbf{v}_0 t) \quad (6)$$

The magnetic field $\mathbf{b}$ must satisfy the equation $\nabla\cdot\mathbf{b} = 0$. This condition implies

$$\mathbf{k}\cdot\mathbf{b}(\mathbf{k},t) = 0 \qquad \forall\mathbf{k} \quad (7)$$

Then, a given Fourier amplitude $\mathbf{b}(\mathbf{k},t)$ lies in a plane perpendicular to the corresponding wave vector $\mathbf{k}$ and has only two independent components at any time. For any wave vector $\mathbf{k}$ we define the following unity vectors perpendicular to $\mathbf{k}$:

$$\mathbf{e}^{[1]}(\mathbf{k}) = i\frac{\mathbf{k}\times\mathbf{B}_0}{|\mathbf{k}\times\mathbf{B}_0|} \quad (8a)$$

$$\mathbf{e}^{[2]}(\mathbf{k}) = i\frac{\mathbf{k}}{|\mathbf{k}|}\times\mathbf{e}^{[1]}(\mathbf{k}) \quad (8b)$$

where $\mathbf{B}_0$ represents the average magnetic field. These definitions cannot be used for $\mathbf{k}$ parallel to $\mathbf{B}_0$. In such a case we define $\mathbf{e}^{[1]}(\mathbf{k})$ and $\mathbf{e}^{[2]}(\mathbf{k})$ as two arbitrary unity vectors which are perpendicular to each other and to $\mathbf{B}_0$. According to equation (7), we decompose the magnetic field Fourier amplitude $\mathbf{b}(\mathbf{k},t)$ as a superposition of these two polarizations:

$$\mathbf{b}(\mathbf{k},t) = \sum_{s=1}^{2} b^{[s]}(\mathbf{k},t)\mathbf{e}^{[s]}(\mathbf{k}) \quad (9)$$

Statistical homogeneity implies

$$\left\langle b^{[r]}(\mathbf{k}_1,t)b^{[s]}(\mathbf{k}_2,t)\right\rangle = (2\pi)^3 I^{rs}(\mathbf{k}_1)\delta^3(\mathbf{k}_1+\mathbf{k}_2) \quad (10)$$

Using (6), (9), and (10) to calculate the left-hand side of (4) and comparing with the right-hand side of (4), we obtain the correlation spectrum in terms of the matrix $I^{rs}(\mathbf{k})$, which is defined by equation (10):

$$T_{ij}(\mathbf{k}) = \sum_{r,s=1}^{2} I^{rs}(\mathbf{k})e_i^{[r]}(\mathbf{k})e_j^{[s]^*}(\mathbf{k}) \quad (11)$$

The last relation shows that $T_{ij}(\mathbf{k})$ and then $C_{ij}(\mathbf{r},0)$ can be expressed in terms of the independent components of the matrix $I^{rs}(\mathbf{k})$, which are only three. In the following we will discuss the fact that in the framework of both weak and strong turbulence, $I^{rs}(\mathbf{k})$ is diagonal, and we will give an explicit model for the two spectra $I^{11}(\mathbf{k})$ and $I^{22}(\mathbf{k})$.

Let us introduce the time Fourier transform $\mathbf{b}(\mathbf{k},\omega)$,

$$b^{[s]}(\mathbf{k},t) = \int\frac{d\omega}{2\pi}b^{[s]}(\mathbf{k},\omega)\exp(-i\omega t)$$

Taking into account that magnetic fluctuations are supposed to be statistically homogeneous and stationary, we can write the following expression for the correlation between independent Fourier components:

$$\left\langle b^{[r]}(\mathbf{k}_1,\omega_1)b^{[s]}(\mathbf{k}_2,\omega_2)\right\rangle = (2\pi)^4 I^{rs}(\mathbf{k}_1,\omega_1)\delta^3(\mathbf{k}_1+\mathbf{k}_2)\delta(\omega_1+\omega_2) \quad (12)$$

It can be easily seen that $I^{rs}(\mathbf{k})$ and $I^{rs}(\mathbf{k},\omega)$ are related through

$$I^{rs}(\mathbf{k}) = \int\frac{d\omega}{2\pi}I^{rs}(\mathbf{k},\omega) \quad (13)$$

Weak Turbulence Approach

In the framework of the MHD weak turbulence theory [*Tsytovich*, 1970; *Akhiezer et al.*, 1975a], $\mathbf{b}(\mathbf{k},\omega)$ can be written as a superposition of modes

$$\mathbf{b}(\mathbf{k},\omega) = 2\pi\sum b^{\sigma}(\mathbf{k})\mathbf{w}^{\sigma}(\mathbf{k})\delta\left[\omega-\omega^{\sigma}(\mathbf{k})\right] \quad (14)$$

where σ = A, F, S indicates the Alfvén, fast, and slow magnetosonic modes, respectively and $\omega^{\sigma}(\mathbf{k})$ and $\mathbf{w}^{\sigma}(\mathbf{k})$ are the corresponding dispersion relations and polarization vectors. It is worth noting that the polarization vectors can be expressed in terms of $\mathbf{e}^{[1]}(\mathbf{k})$ and $\mathbf{e}^{[1]}(\mathbf{k})$ as follows [*Akhiezer et al.*, 1975b]:

$$\mathbf{w}^{\mathrm{A}}(\mathbf{k}) = \mathbf{e}^{[1]}(\mathbf{k}) \qquad \mathbf{w}^{\mathrm{F}}(\mathbf{k}) = \mathbf{w}^{\mathrm{S}}(\mathbf{k}) = \mathbf{e}^{[2]}(\mathbf{k}) \quad (15)$$

Using (14), we can write the correlation between independent Fourier components of the magnetic field as

$$\begin{aligned}\left\langle b^{[r]}(\mathbf{k}_1,\omega_1)b^{[s]}(\mathbf{k}_2,\omega_2)\right\rangle = \\ (2\pi)^2 \sum_{\alpha,\sigma=1}^{3} \langle b^{\alpha}(\mathbf{k}_1)b^{\sigma}(\mathbf{k}_2)\rangle \\ \times \left[\mathbf{w}^{\sigma}(\mathbf{k}_2)\cdot\mathbf{e}^{[s]}(\mathbf{k}_2)\right]\left[\mathbf{w}^{\alpha}(\mathbf{k}_1)\cdot\mathbf{e}^{[r]}(\mathbf{k}_1)\right] \\ \times\delta\left[\omega_1-\omega^{\alpha}(\mathbf{k}_1)\right]\delta\left[\omega_2-\omega^{\sigma}(\mathbf{k}_2)\right] \quad (16)\end{aligned}$$

Since

$$\langle b^{\alpha}(\mathbf{k}_1)b^{\sigma}(\mathbf{k}_2)\rangle = (2\pi)^3 F^{\alpha\sigma}(\mathbf{k}_1)\delta^3(\mathbf{k}_1+\mathbf{k}_2)$$

and $\omega^{\sigma}(-\mathbf{k}_1) = -\omega^{\sigma}(\mathbf{k}_1)$, we can see by comparing (12) with (16) that the only nonvanishing terms in (16) must be those with $\omega_1 = -\omega_2 = \omega^{\alpha}(\mathbf{k}_1) = \omega^{\sigma}(\mathbf{k}_1)$. This means that $F^{\alpha\sigma}(\mathbf{k}) = F^{\alpha}(\mathbf{k})\delta_{\alpha\sigma}$, i.e., $F^{\alpha\sigma}$ is diagonal. In this case, due to (15), $I^{rs}(\mathbf{k}_1,\omega_1)$ assumes the following form:

$$\begin{aligned}I^{rs}(\mathbf{k},\omega) = \delta_{rs}\sum_{\alpha=1}^{3} F^{\alpha}(\mathbf{k})\left|\mathbf{w}^{\alpha}(\mathbf{k})\cdot\mathbf{e}^{[r]}(\mathbf{k})\right|^2 \\ \times 2\pi\delta\left[\omega-\omega^{\alpha}(\mathbf{k})\right] = \delta_{rs}I^{[r]}(\mathbf{k},\omega) \quad (17)\end{aligned}$$

Using this last expression, we can write $I^{rs}(\mathbf{k})$ as

$$\begin{aligned}I^{rs}(\mathbf{k}) = \delta_{rs}\sum_{\alpha=1}^{3} F^{\alpha}(\mathbf{k})\left|\mathbf{w}^{\alpha}(\mathbf{k})\cdot\mathbf{e}^{[r]}(\mathbf{k})\right|^2 \\ = \delta_{rs}I^{[r]}(\mathbf{k}) \quad (18)\end{aligned}$$

showing that only the diagonal elements of $I^{rs}(\mathbf{k})$ are different from zero,

$$\begin{aligned}I^{[1]}(\mathbf{k}) &= I^{11}(\mathbf{k}) = F^{\mathrm{A}}(\mathbf{k}) \\ I^{[2]}(\mathbf{k}) &= I^{22}(\mathbf{k}) = F^{\mathrm{F}}(\mathbf{k}) + F^{\mathrm{S}}(\mathbf{k}) \quad (19)\end{aligned}$$

Strong Turbulence Approach

Equation (14) is valid only if the turbulence is weak, i.e., the fluctuation amplitudes are much smaller than the background magnetic field for any wave vector $\mathbf{k}$. This hypothesis is not strictly verified for the solar wind fluctuations, so it is necessary to outline how $I^{rs}(\mathbf{k},\omega)$ is modified in this case. One of the most fruitful approaches to the theory of homogeneous MHD strong turbulence has been founded by *Kraichnan* [1958] on two main statistical hypotheses: (1) weak dependence and (2) direct interaction. The hypothesis of weak dependence states that if the dimension of the cyclic box where Fourier analysis is performed is increased without limit (this corresponds to the passage from a discrete mode description to a continuous wavenumber description), "the statistical dependencies among any finite number of modes should decrease without limit" [*Kraichnan*, 1958]. In other words, the cross moments, which necessarily vanish in a rigorously independent joint distribution (skew moments), are supposed to be infinitesimally small. This means that the correlation matrix among independent Fourier variables can safely assumed to be diagonal:

$$I^{rs}(\mathbf{k},\omega) = \delta_{rs}I^{[r]}(\mathbf{k},\omega) \qquad I^{rs}(\mathbf{k}) = \delta_{rs}I^{[r]}(\mathbf{k}) \quad (20)$$

The weak dependence hypothesis does not conflict with the well-established fact that in a turbulent flow, measured distributions are strongly nonnormal [*Kraichnan*, 1958]. In fact, it is easily seen that the skew moments involved in the calculation of the correlations in the physical space "outnumber the non-skew by a factor of the total number of modes excited. Thus, the weak dependence hypothesis does not implies the neglect of cross-dependencies in the evaluation of extended sums over Fourier modes" [*Kraichnan*, 1958].

One could worry about the fact that, since during Alfvénic periods magnetic field fluctuations are characterized by the relation $B^2 \simeq const$, this would introduce some correlation among the Fourier amplitudes $b^{[1]}(\mathbf{k},t)$ and $b^{[2]}(\mathbf{k},t)$, which are supposed to be statistically independent. Actually, when this relation is written in terms of the Fourier amplitudes, it corresponds to a nonlinear equation, which involves an infinite number of Fourier modes through the convolution products. Then no direct relation between $b^{[1]}(\mathbf{k},t)$ and $b^{[2]}(\mathbf{k},t)$ can be obtained from it. Trying to determine the second-order correlation between $b^{[1]}(\mathbf{k},t)$ and $b^{[2]}(\mathbf{k},t)$ gives rise to an infinite hierarchy of higher-order moments.

A Parametric Expression for the Correlation Spectra

Equations (11), (19), and (20) show that the correlation spectrum $T_{ij}(\mathbf{k})$ depends only on two scalar quantities, $I^{[1]}(\mathbf{k})$ and $I^{[2]}(\mathbf{k})$, which are proportional to the spectral magnetic energy densities polarized along $\mathbf{e}^{[1]}(\mathbf{k})$ and $\mathbf{e}^{[2]}(\mathbf{k})$, respectively.

$$T_{ij}(\mathbf{k}) = \sum_{s=1}^{2} I^{[s]}(\mathbf{k})e_i^{[s]}(\mathbf{k})e_j^{[s]^*}(\mathbf{k}) \quad (21)$$

Expression (21) corresponds to a correlation spectrum which is real and symmetric. In fact, from equation (11) it follows that the spectra $I^{[s]}(\mathbf{k})$ are real quantities, while from the definitions in (8) it is seen that the tensors $\{e_i^{[s]}(\mathbf{k})e_j^{[s]^*}(\mathbf{k})\}$ are real and symmetric. Then the assumption that the matrix $I^{rs}(\mathbf{k})$ is diagonal allows us to drop the skew-symmetric part of $T_{ij}(\mathbf{k})$. Let us note also that the polarization spectra $I^{[s]}(\mathbf{k})$ must satisfy the symmetry condition $I^{[s]}(\mathbf{k}) = I^{[s]}(-\mathbf{k})$. In fact, using the reality condition for C_{ij} we can easily see that $T_{ij}(\mathbf{k}) = T_{ij}^*(-\mathbf{k})$. Looking now at equation (11) and taking into account the definitions in (8), one sees also $I^{rs}(\mathbf{k}) = I^{rs^*}(-\mathbf{k})$. This implies, in particular, that the diagonal terms $I^{[1]}(\mathbf{k})$ and $I^{[2]}(\mathbf{k})$, which are real, must be symmetric with respect to the change $\mathbf{k} \to -\mathbf{k}$. It is worth noting that the propagation direction of a given Fourier component is determined by both $\mathbf{k}$ and ω. In other words, the information about the propagation direction is contained in $I^{rs}(\mathbf{k},\omega)$. So the above mentioned simmetry does not imply any assumption at all about the energy ratio between forward and backward propagating waves. Relation (21) shows that two different sources can be responsible for the anisotropy of the magnetic fluctuations: (1) a different energy content in the polarizations [1] and [2] and (2) an anisotropic energy distribution in the wave vector space for each polarization. In order to take into account both effects, we will set up a parametric model for the spectra $I^{[1]}(\mathbf{k})$ and $I^{[2]}(\mathbf{k})$. In particular, we are interested in giving a model of the power law range of the spectrum, where measures of anisotropy in the solar wind fluctuations have essentially been made.

Some indications can be found looking at the results of theoretical studies on the anisotropic MHD turbulence. One approach to this problem is based on numerical simulations in which the full two-dimensional MHD equations are solved [*Shebalin et al.*, 1983; *Grappin*, 1986; *Weisshaar*, 1988]. In such an approach, relatively low values of the Reynolds number (of the order of 10^3) can in practice be used. As a consequence, the power law range is quite short, and a detailed description of the corresponding spectrum is not easy to obtain. Situations with higher values of the Reynolds number (up to 10^8) have been considered by *Carbone and Veltri* [1990]; these authors studied the anisotropic MHD turbulence, using a set of statistical equations which have been derived from the MHD equations in a three-dimensional configuration. The relevant results can be summarized as follows: (1) in both spectra of polarization [1] and [2], power law ranges form with given spectral indices μ_1 and μ_2; (2) the energy distribution in the wave vector space is anisotropic for both spectra; and (3) the two spectra in general can have different energy contents. Features 2 and 3 both give origin to an anisotropy of the turbulence.

On this basis, we use a phenomenological expression for the spectra $I^{[1]}(\mathbf{k})$ and $I^{[2]}(\mathbf{k})$ which accounts for features 1, 2, and 3; in particular, we assume the following anisotropic power law for the spectra of our model:

$$I^{[s]}(\mathbf{k}) = C^{[s]}\left[\left(\ell_x^{[s]}k_x\right)^2+\left(\ell_y^{[s]}k_y\right)^2+\left(\ell_z^{[s]}k_z\right)^2\right]^{-1-\frac{\mu^{[s]}}{2}} \quad (22)$$

where k_x, k_y, and k_z are the cartesian components of $\mathbf{k}$, while $C^{[s]}$, $\ell_i^{[s]}$, and $\mu^{[s]}$ ($s=1,2$; $i=x,y,z$) are free parameters of the model. In particular, $C^{[s]}$ furnish a measure of the energy contents on the two polarizations; $\ell_i^{[s]}$ represent the spectral extensions along the direction of a given system of coordinates, and $\mu^{[s]}$ are the two spectral indices. Since expression (22) has no rotational symmetry, it is necessary to specify the orientation of the coordinate axes, which should depend on the relevant directions in the physical space. These directions are given by the average magnetic field $\mathbf{B}_0$ and the solar wind velocity $\mathbf{v}_0$, which coincides with the radial direction. Then we assume the x axis perpendicular to the $\mathbf{v}_0-\mathbf{B}_0$ plane. In accordance with the results by *Shebalin et al.* [1983], *Grappin* [1986], *Weisshaar* [1988], and *Carbone and Veltri* [1990] we orientate the z axis parallel to $\mathbf{B}_0$. As a consequence, the y axis lies on the $\mathbf{v}_0-\mathbf{B}_0$ plane. It is worth noting that the directions of the coordinate axes are the same as those of the eigenvectors of the variance matrix calculated by *Bavassano et al.* [1982], the x and z axes corresponding to the directions of maximum and minimum variance, respectively.

Determination of the Variance Matrix

Equation (21) relates the correlation spectrum $T_{ij}(\mathbf{k})$ to the spectra $I^{[s]}(\mathbf{k})$ of the polarizations [1] and [2]. In this section we will show that the quantities $I^{[s]}(\mathbf{k})$ are also related to the magnetic field variance matrix S_{ij}. In particular, the knowledge of the variance matrix for different time bases allows us to get information on both $I^{[1]}(\mathbf{k})$ and $I^{[2]}(\mathbf{k})$ in different ranges of the $\mathbf{k}$ space, and to determine the values of the parameters in the model spectra (22).

The determination of the variance matrix S_{ij} is based on magnetic field measures performed by a spacecraft in the solar wind. The measurements are performed on a discrete set of times $\{t_n = n\Delta t,\ n=0,1,2,...,N-1\}$, where Δt is the duration of the measured time interval and $N=T/\Delta t$ is the total number of measures in the data set. The corresponding values of the magnetic field at the time t_n is an average over the time interval $[t_n, t_{n+1}]$,

$$\mathbf{B}(\mathbf{r}_S,t_n)=\frac{1}{\Delta t}\int_{t_n}^{t_{n+1}}\mathbf{B}(\mathbf{r}_S,t)\ dt \quad (23)$$

where $\mathbf{r}_S$ indicates the spacecraft location in frame I. This quantity can be expressed in terms of the Fourier transform

$$\mathbf{B}(\mathbf{r}_S,t_n)=\int\frac{d^3\mathbf{k}d\omega}{(2\pi)^4}\mathbf{B}(\mathbf{k},\omega)f_{\Delta t}(\omega) \times\exp[i(\mathbf{k}\cdot\mathbf{r}_S-\omega t_n)] \quad (24)$$

where

$$f_{\Delta t}(\omega)=\frac{1-\exp(-i\omega\Delta t)}{i\omega\Delta t} \quad (25)$$

The function (25) can be approximated by $f_{\Delta t}(\omega)\simeq 1$ for $|\omega|\ll\omega_{max}$, where $\omega_{max}=2\pi/\Delta t$, while it vanishes in the limit $|\omega|\to\infty$. Then the factor $f_{\Delta t}(\omega)$ in expression (24) filters out the contribution of the frequencies $|\omega|\gg\omega_{max}$.

In order to obtain information in various frequency ranges along the spectrum, the magnetic fluctuations over various time bases are calculated by using the following technique [see *Bavassano et al*, 1982]. A certain time interval $[t_m, t_m+\tau]$ is chosen, where $\tau = M\Delta t$

(M being an integer) is a time basis shorter than T ($\Delta t < \tau < T$ or $M < N$). The following running average over the interval $[t_m, t_m + \tau]$ is calculated:

$$< \mathbf{B} >_\tau (\mathbf{r}_S, t_m) \equiv \frac{1}{M} \sum_{j=m}^{m+M-1} \mathbf{B}(\mathbf{r}_S, t_j) = \int \frac{d^3\mathbf{k} d\omega}{(2\pi)^4} \mathbf{B}(\mathbf{k}, \omega) f_{\Delta t}(\omega) \exp(i\mathbf{k}\cdot\mathbf{r}_S) \times \frac{1}{M} \sum_{j=m}^{m+M-1} \exp(-i\omega t_j)$$

The sum in the above expression gives

$$\frac{1}{M} \sum_{j=m}^{m+M-1} \exp(-i\omega t_j) = \frac{f_\tau(\omega)}{f_{\Delta t}(\omega)} \exp(-i\omega t_m) \quad (26)$$

thus the running average takes the form

$$< \mathbf{B} >_\tau (\mathbf{r}_S, t_m) = \int \frac{d^3\mathbf{k} d\omega}{(2\pi)^4} \mathbf{B}(\mathbf{k}, \omega) f_\tau(\omega) \times \exp[i(\mathbf{k}\cdot\mathbf{r}_S - \omega t_m)] \quad (27)$$

The magnetic fluctuation $\delta^{(\tau)}\mathbf{B}$ relative to the time interval $[t_m, t_m + \tau]$ is calculated as

$$\delta^{(\tau)}\mathbf{B}(\mathbf{r}_S, t_n; t_m) = \mathbf{B}(\mathbf{r}_S, t_n) - < \mathbf{B} >_\tau (\mathbf{r}_S, t_m) \quad (28)$$

($n = m, ..., m + M - 1$). In practice, frequencies lower than τ^{-1} do not contribute to $\delta^{(\tau)}\mathbf{B}$. Inserting the expressions (24) and (27) into (28), we find

$$\delta^{(\tau)}\mathbf{B}(\mathbf{r}_S, t_n; t_m) = \int \frac{d^3\mathbf{k} d\omega}{(2\pi)^4} \mathbf{B}(\mathbf{k}, \omega) \exp(i\mathbf{k}\cdot\mathbf{r}_S) \times [f_{\Delta t}(\omega) \exp(-i\omega t_n) - f_\tau(\omega) \exp(-i\omega t_m)] \quad (29)$$

This expression is used to evaluate the variance matrix relative to the time interval $[t_m, t_m+\tau]$, which is defined as

$$S_{ij}^{(\tau)}(\mathbf{r}_S, t_m) = \left\langle \delta^{(\tau)} B_i \delta^{(\tau)} B_j \right\rangle_\tau \equiv \frac{1}{M} \sum_{n=m}^{n+M-1} \delta^{(\tau)} B_i(\mathbf{r}_S, t_n; t_m) \delta^{(\tau)} B_j(\mathbf{r}_S, t_n; t_m) \quad (30)$$

Inserting expression (29) into (30) and using relations (26), we find

$$S_{ij}^{(\tau)}(\mathbf{r}_S, t_m) = \int \frac{d^3\mathbf{k} d\omega}{(2\pi)^4} \int \frac{d^3\mathbf{k}' d\omega'}{(2\pi)^4} B_i(\mathbf{k}, \omega) B_j(\mathbf{k}', \omega') F_\tau(\omega, \omega') \times \exp\{i[(\mathbf{k}+\mathbf{k}')\cdot\mathbf{r}_S - (\omega+\omega')t_m]\} \quad (31)$$

where

$$F_\tau(\omega, \omega') = \frac{f_{\Delta t}(\omega) f_{\Delta t}(\omega') f_\tau(\omega+\omega')}{f_{\Delta t}(\omega+\omega')} - f_\tau(\omega) f_\tau(\omega') \quad (32)$$

Finally, we take the average of expression (31) over all the values of the times $t_m = 0, \Delta t, ..., T$. Since, as discussed in section 2, T is sufficiently long to ensure statistical stationarity, we can consider such average in the limit $T \to \infty$. Then we define

$$S_{ij}^{(\tau)}(\mathbf{r}_S) = \lim_{N\to\infty} \frac{1}{N} \sum_{m=0}^{N-1} S_{ij}^{(\tau)}(\mathbf{r}_S, t_m) \quad (33)$$

This expression can be evaluated taking into account that

$$\lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} \exp[-i(\omega+\omega')t_n] = \sum_{p=-\infty}^{\infty} \delta_{\omega+\omega', p\omega_{max}}$$

thus obtaining the following expression for the averaged variance matrix:

$$S_{ij}^{(\tau)}(\mathbf{r}_S) = \sum_{p=-\infty}^{\infty} \int \frac{d^3\mathbf{k} d^3\mathbf{k}' d\omega d\omega'}{(2\pi)^8} B_i(\mathbf{k}, \omega) B_j(\mathbf{k}', \omega') \times F_\tau(\omega, \omega') \exp\{i[(\mathbf{k}+\mathbf{k}')\cdot\mathbf{r}_S]\} \delta_{\omega+\omega',\, p\omega_{max}} \quad (34)$$

The terms with $p \neq 0$ represent "aliasing" contributions due to interference among frequencies higher than ω_{max} in determining the quadratic quantity $S_{ij}^{(\tau)}$. Let us note that for $\omega = p\omega_{max}$

$$F_\tau(\omega, p\omega_{max} - \omega) = 0$$

whereas for $\omega \neq p\omega_{max}$

$$F_\tau(\omega, p\,\omega_{max} - \omega) = \frac{\omega}{\omega - p\,\omega_{max}} G_\tau(\omega)$$

where

$$G_\tau(\omega) = |f_{\Delta t}(\omega)|^2 - |f_\tau(\omega)|^2 \quad (35)$$

As seen in Figure 1, the function $G_\tau(\omega)$ represents a filter which is sensibly nonvanishing only in the frequency range

$$\frac{2\pi}{\tau} \leq |\omega| \leq \omega_{max}$$

In the same range the ratio $\omega/(\omega - p\,\omega_{max})$ decreases for increasing $|p|$. Then the dominant terms in expression (34) are those corresponding to $p = 0$, and we retain only these terms:

$$S_{ij}^{(\tau)}(\mathbf{r}_S) \simeq \int \frac{d^3\mathbf{k} d\omega}{(2\pi)^4} \int \frac{d^3\mathbf{k}' d\omega'}{(2\pi)^4} B_i(\mathbf{k}, \omega) B_j(\mathbf{k}', \omega') \times G_\tau(\omega) \exp\{i[(\mathbf{k}+\mathbf{k}')\cdot\mathbf{r}_S]\} \delta_{\omega, -\omega'}$$

To introduce in this expression the model of turbulence derived in the previous section, we perform now an en-

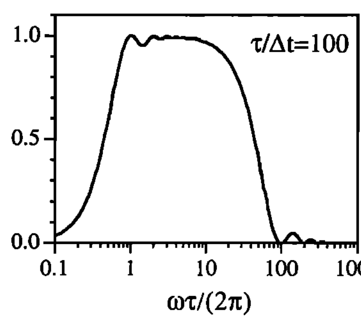

Figure 1. The filter function $G_\tau(\omega)$ is represented for $\tau/\Delta t = 100$.

semble average of $S_{ij}^{(\tau)}(\mathbf{r}_S)$, and we use (6), (9), and (12) to calculate

$$\begin{aligned}&\langle B_i(\mathbf{k},\omega)B_j(\mathbf{k}',\omega')\rangle = \\ &\quad\sum_{r,s} e_i^{[r]}(\mathbf{k})e_j^{[s]}(\mathbf{k}') \\ &\quad\times \left\langle b^{[r]}(\mathbf{k},\omega-\mathbf{k}\cdot\mathbf{v}_0)b^{[s]}(\mathbf{k}',\omega'-\mathbf{k}'\cdot\mathbf{v}_0)\right\rangle = \\ &\quad(2\pi)^4\delta^3(\mathbf{k}+\mathbf{k}')\delta(\omega+\omega') \\ &\quad\times \sum_{r,s=1}^{2} I^{rs}(\mathbf{k},\omega-\mathbf{k}\cdot\mathbf{v}_0)e_i^{[r]}(\mathbf{k})e_j^{[s]}(\mathbf{k}')\end{aligned}$$

The ensemble average of $S_{ij}^{(\tau)}(\mathbf{r}_S)$ is then given by

$$\begin{aligned}S_{ij}^{(\tau)} =< S_{ij}^{(\tau)}(\mathbf{r}_S) >= \sum_{r,s=1}^{2}\int \frac{d^3\mathbf{k}d\omega}{(2\pi)^4}G_\tau(\omega) \\ \times I^{rs}(\mathbf{k},\omega-\mathbf{k}\cdot\mathbf{v}_0)e_i^{[r]}(\mathbf{k})e_j^{[s]^*}(\mathbf{k})\end{aligned}$$

Taking into account that $I^{rs}(\mathbf{k},\omega)$ is always diagonal (see equations (17) and (20)) and performing the variable change $\omega \longrightarrow \omega + \mathbf{k}\cdot\mathbf{v}_0$, we obtain

$$\begin{aligned}S_{ij}^{(\tau)} = \sum_{r,s=1}^{2}\int \frac{d^3\mathbf{k}d\omega}{(2\pi)^4} I^{[s]}(\mathbf{k},\omega)e_i^{[r]}(\mathbf{k})e_j^{[s]^*}(\mathbf{k}) \\ \times G_\tau(\omega+\mathbf{k}\cdot\mathbf{v}_0) \qquad (36)\end{aligned}$$

In the framework of weak turbulence, introducing (17) in (36), the integration over the frequencies can be performed, thus obtaining

$$\begin{aligned}S_{ij}^{(\tau)} = \sum_{s=1}^{2}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\sum_{\alpha=1}^{3}F^\alpha(\mathbf{k})\left|\mathbf{w}^\alpha(\mathbf{k})\cdot\mathbf{e}^{[s]}(\mathbf{k})\right|^2 \\ \times G_\tau\left(\omega^\alpha(\mathbf{k})+\mathbf{k}\cdot\mathbf{v}_0\right)e_i^{[s]}(\mathbf{k})e_j^{[s]^*}(\mathbf{k}) \quad (37)\end{aligned}$$

According to the Taylor hypothesis [*Taylor*, 1938], since the solar wind velocity in the spacecraft frame of reference $\mathbf{v}_0$ is much higher than the phase velocity of the MHD waves, we neglect the term $\omega^\alpha(\mathbf{k})$ in the filter function G_τ as compared with the Doppler term. Taking into account equation (18), we finally obtain

$$S_{ij}^{(\tau)} = \sum_{s=1}^{2}\int \frac{d^3\mathbf{k}}{(2\pi)^3} I^{[s]}(\mathbf{k})e_i^{[s]}(\mathbf{k})e_j^{[s]^*}(\mathbf{k})G_\tau(\mathbf{k}\cdot\mathbf{v}_0) \quad (38)$$

Considering now the strong turbulence case, it is reasonable to assume that perturbations propagate with typical velocities which are much lower than the solar wind speed $\mathbf{v}_0$. In other words, for a given wave vector $\mathbf{k}$ the function $I^{[s]}(\mathbf{k},\omega)$ is substantially different from zero only in the frequency range $|\omega| \ll |\mathbf{k}\cdot\mathbf{v}_0|$. Then we neglect ω in the argument of the filter function G_τ in equation (36), and perform the integration over the frequencies. Taking into account the definition of $I^{[r]}(\mathbf{k})$ given in equation (20), we obtain, also in the strong turbulence framework, an expression for the variance matrix $S_{ij}^{(\tau)}$ which is formally identical to that given by equation (38). In summary, both in the strong and in the weak turbulence approach we can use expression (38), which relates the variance matrix $S_{ij}^{(\tau)}$ calculated in the spacecraft frame of reference to the energy spectra of the magnetic field polarizations in the solar wind frame of reference.

It is useful to explicitly calculate the eigenvalues of the variance matrix (38), which will be fitted on the corresponding measured values. Let us introduce the polarization tensors $A_{ij}^{[s]}(\mathbf{k}) = e_i^{[s]}(\mathbf{k})e_j^{[s]^*}(\mathbf{k})$, $s = 1,2$, which appear in expression (38). In the system of coordinates defined in the previous section these matrices have the following forms:

$$A_{ij}^{[1]}(\mathbf{k}) = \frac{1}{k_\perp^2}\begin{pmatrix} k_y^2 & -k_xk_y & 0 \\ -k_xk_y & k_x^2 & 0 \\ 0 & 0 & 0\end{pmatrix} \qquad (39)$$

$$A_{ij}^{[2]}(\mathbf{k}) = \begin{pmatrix} \frac{k_x^2k_z^2}{k^2k_\perp^2} & \frac{k_xk_yk_z^2}{k^2k_\perp^2} & \frac{-k_xk_z}{k^2} \\ \frac{k_xk_yk_z^2}{k^2k_\perp^2} & \frac{k_y^2k_z^2}{k^2k_\perp^2} & \frac{-k_yk_z}{k^2} \\ \frac{-k_xk_z}{k^2} & \frac{-k_yk_z}{k^2} & \frac{-k_\perp^2}{k^2}\end{pmatrix} \qquad (40)$$

where $k^2 = k_x^2 + k_y^2 + k_z^2$ and $k_\perp^2 = k_x^2 + k_y^2$. In the considered coordinate system the solar wind velocity $\mathbf{v}_0$ has no x component:

$$\mathbf{v}_0 = (0, v_{0y}, v_{0z})$$

This implies that the filter function $G_\tau(\mathbf{k}\cdot\mathbf{v}_0)$ does not depend on k_x. Taking into account that both $I^{[1]}(\mathbf{k})$ and $I^{[2]}(\mathbf{k})$ are even functions of k_x and that the matrix elements $A_{12}^{[1]}$, $A_{21}^{[1]}$, $A_{12}^{[2]}$, $A_{21}^{[2]}$, $A_{13}^{[2]}$, and $A_{31}^{[2]}$ are odd functions of k_x, it follows that $S_{12}^{(\tau)} = S_{21}^{(\tau)} = 0$ and $S_{13}^{(\tau)} = S_{31}^{(\tau)} = 0$. Then the eigenvalues of the variance matrix $S_{ij}^{(\tau)}$ are given by

$$\lambda_1^{(\tau)} = S_{11}^{(\tau)} \qquad (41)$$

$$\begin{aligned}\lambda_{2,3}^{(\tau)} = \frac{1}{2}\left[S_{22}^{(\tau)} + S_{33}^{(\tau)}\right] \pm \frac{1}{2} \\ \times\sqrt{\left(S_{22}^{(\tau)} + S_{33}^{(\tau)}\right)^2 - 4\left(S_{22}^{(\tau)}S_{33}^{(\tau)} - S_{23}^{(\tau)}\right)} \quad (42)\end{aligned}$$

where

$$\begin{aligned}S_{11}^{(\tau)} = \int d^3\mathbf{k}\,G_\tau(\mathbf{k}\cdot\mathbf{v}_0) \\ \times\left[\left(\frac{k_y}{k_\perp}\right)^2 I^{[1]}(\mathbf{k}) + \left(\frac{k_xk_z}{kk_\perp}\right)^2 I^{[2]}(\mathbf{k})\right] \quad (43)\end{aligned}$$

$$\begin{aligned}S_{22}^{(\tau)} = \int d^3\mathbf{k}\,G_\tau(\mathbf{k}\cdot\mathbf{v}_0) \\ \times\left[\left(\frac{k_x}{k_\perp}\right)^2 I^{[1]}(\mathbf{k}) + \left(\frac{k_yk_z}{kk_\perp}\right)^2 I^{[2]}(\mathbf{k})\right] \quad (44)\end{aligned}$$

$$S_{33}^{(\tau)} = \int d^3\mathbf{k} \frac{(k_x^2 + k_z^2)}{k^2} I^{[2]}(\mathbf{k}) G_\tau(\mathbf{k}\cdot\mathbf{v}_0) \quad (45)$$

$$S_{23}^{(\tau)} = -\int d^3\mathbf{k} \frac{k_y k_z}{k^2} I^{[2]}(\mathbf{k}) G_\tau(\mathbf{k}\cdot\mathbf{v}_0) \quad (46)$$

Determination of the Parameters of the Turbulence Spectra

In this section we determine the parameters of the energy spectra of the magnetic field polarizations $I^{[1]}(\mathbf{k})$ and $I^{[2]}(\mathbf{k})$ given in the form (22) by fitting the variance matrix eigenvalues $\lambda_i^{(\tau)}$ into the corresponding eigenvalues measured in the solar wind turbulence. Expression (22) is suitable to describe the energy spectra only in the power law range. However, the filter function $G_\tau(\mathbf{k}\cdot\mathbf{v}_0)$, which appears in the variance matrix elements (43)-(46), selects the wave vector range $2\pi/\tau \leq \mathbf{k}\cdot\mathbf{v}_0 \leq \omega_{max}$, which is in practice comprised in the power law range of the solar wind fluctuation spectrum. In their analysis of the anisotropy of the solar wind turbulence, *Bavassano et al.* [1982] considered three data sets taken by Helios 2 spacecraft at three different distances from the Sun. For each of these data sets, they calculated the eigenvalues of the variance matrix, using five different values for the time basis τ (τ_1 = 168 s, τ_2 = 8 min, τ_3 = 22.5 min, τ_4 = 1 hour, and τ_5 = 3 hours) and obtaining a corresponding set of 15 numerical values, which we indicate by $\{\lambda_i^{(\tau_j)}$, $i = 1,2,3$ and $j = 1,..,5\}$. In particular, *Bavassano et al.* [1982] gave the sum and the ratios of the eigenvalues for each time basis, i.e., the following quantities:

$$\sigma_j = \lambda_1^{(\tau_j)} + \lambda_2^{(\tau_j)} + \lambda_3^{(\tau_j)} \quad (47a)$$

$$\rho_j = \frac{\lambda_2^{(\tau_j)}}{\lambda_1^{(\tau_j)}} \quad (47b)$$

$$\beta_j = \frac{\lambda_3^{(\tau_j)}}{\lambda_1^{(\tau_j)}} \quad (47c)$$

with $j = 1, ..., 5$. The numerical values of the quantities (47) are given in Table 1, along with the corresponding incertitudes $\Delta\sigma_j$, $\Delta\rho_j$, and $\Delta\beta_j$. The expressions, predicted by our model for the sum and the ratios of the eigenvalues $\lambda_i^{(\tau_j)}$ and corresponding to σ_j, ρ_j, and β_j, can be calculated by using equations (41) and (42) and will be indicated by s_j, r_j, and b_j, respectively.

The quantities s_j, r_j, and b_j depend on 10 parameters $C^{[s]}$, $\ell_i^{[s]}$, and $\mu^{[s]}$ ($s = 1,2$; $i = x,y,z$), which define the spectra of the two polarization. These parameters will be determined by fitting the expressions (41) and (42) on the corresponding values (47) by a χ^2 method. The number of free parameters to be determined by the fitting procedure can be reduced from 10 to seven. Actually, $C^{[s]}$ and $\ell_i^{[s]}$ in expression (22) for a given value of s are not independent of each other, so we can safely set $\ell_z^{[s]} = 1$ ($s = 1,2$), without any loss of generality. Moreover, in order to reduce the correlations between the parameters to be determined, we have defined χ^2 as follows:

$$\chi^2 = \sum_{j=1}^{5} \left[\frac{(r_j - \rho_j)^2}{(\Delta\rho_j)^2} + \frac{(b_j - \beta_j)^2}{(\Delta\beta_j)^2} \right] + \sum_{j=1}^{4} \frac{(s_j/s_5 - \sigma_j/\sigma_5)^2}{[\Delta(\sigma_j/\sigma_5)]^2} \quad (48)$$

This χ^2 function depends on the parameters $C^{[1]}$ and $C^{[2]}$ only through their ratio $C^{[1]}/C^{[2]}$, so that the minimization of χ^2 allows us to determine the seven parameters $C^{[1]}/C^{[2]}$, $\ell_x^{[1]}$, $\ell_y^{[1]}$, $\ell_x^{[2]}$, $\ell_y^{[2]}$, $\mu^{[1]}$, and $\mu^{[2]}$; once the minimization is performed, from the values of these parameters the value of $C^{[1]}$ is simply obtained through the relation

$$C^{[1]} = \sigma_5/s_5$$

The search of the minimum for expression (48) by a numerical procedure requires several evaluations of the integrals (43)-(46). The functions in square brackets in the integrals (43)-(46) display some symmetries with respect to the coordinate planes. However, these sym-

Table 1. Sum and Ratios of Variance Matrix Eigenvalues Calculated From Helios 2 Data for Different Time Bases τ_j and Distances R

	R, AU	τ_1 = 168 s	τ_2 = 8 min	τ_3 = 22.5 min	τ_4 = 1 hour	τ_5 = 3 hours
σ_j	0.29	0.166 ± 0.102	0.239 ± 0.111	0.307 ± 0.110	0.351 ± 0.099	0.382 ± 0.080
	0.65	0.090 ± 0.071	0.152 ± 0.091	0.239 ± 0.121	0.307 ± 0.091	0.392 ± 0.096
	0.87	0.071 ± 0.062	0.128 ± 0.090	0.214 ± 0.119	0.298 ± 0.111	0.407 ± 0.103
ρ_j	0.29	0.288 ± 0.150	0.384 ± 0.158	0.456 ± 0.160	0.533 ± 0.151	0.626 ± 0.135
	0.65	0.234 ± 0.139	0.299 ± 0.152	0.389 ± 0.163	0.489 ± 0.168	0.601 ± 0.101
	0.87	0.230 ± 0.149	0.271 ± 0.163	0.316 ± 0.155	0.327 ± 0.113	0.458 ± 0.166
β_j	0.29	0.038 ± 0.022	0.077 ± 0.037	0.126 ± 0.047	0.168 ± 0.044	0.204 ± 0.040
	0.65	0.023 ± 0.014	0.045 ± 0.025	0.081 ± 0.037	0.136 ± 0.047	0.199 ± 0.055
	0.87	0.022 ± 0.014	0.036 ± 0.023	0.063 ± 0.029	0.110 ± 0.044	0.176 ± 0.061

From *Bavassano et al.* [1982].

metries are broken by the presence of the filter function $G_\tau(\mathbf{k}\cdot\mathbf{v}_0)$ and they cannot be used to simplify the calculation of the integrals. The relevant integration domain D_τ in the $\mathbf{k}$ space is roughly given by the inequalities $2\pi/\tau \le \mathbf{k}\cdot\mathbf{v}_0 \le \omega_{max}$, i.e.,

$$D_\tau = \{\mathbf{k} : k_\tau \le \mathbf{k}\cdot\mathbf{u} \le k_{max}\} \tag{49}$$

where $k_\tau = 2\pi/(v_0\tau)$, $k_{max} = \omega_{max}/v_0 = 2\pi/(v_0\Delta t)$, and $\mathbf{u} = \mathbf{v}/v_0$.

The domain D_τ is represented in Figure 2 in the $k_y k_z$ plane; its orientation with respect to the k_y and k_z axes depends on the angle between the solar wind velocity $\mathbf{v}_0$ and the mean magnetic field $\mathbf{B}_0$ (which has been chosen parallel to the k_z axis). In consequence of this complicated geometry, the numerical calculation of the integrals (43)-(46) during the minimization procedure requires very long computational times. In order to simplify this problem we replaced the integrals (43)-(46) by approximated forms which are simpler to evaluate. In particular, we replaced the filter function $G_\tau(\mathbf{k}\cdot\mathbf{v}_0)$ by the following function:

$$\begin{aligned} G'_\tau(\mathbf{k}) &= 1 \qquad k_\tau \le |\mathbf{k}| \le k_{max} \\ G'_\tau(\mathbf{k}) &= 0 \qquad |\mathbf{k}| \le k_\tau \ , \quad |\mathbf{k}| \ge k_{max} \end{aligned} \tag{50}$$

This is equivalent to the following simplifications:

1. The relevant integration domain D_τ is changed to D'_τ, defined by

$$D'_\tau = \{\mathbf{k} : k_\tau \le |\mathbf{k}| \le k_{max}\} \tag{51}$$

2. The smooth filter function is changed into a step function which is equal to 1 on the domain D'_τ and vanishes out of this domain.

The latter point clearly does not introduce important modifications in the integrals (43)-(46); the former point, i.e., the change of the relevant domain, deserves a more detailed discussion, which is given in the appendix. Anyway, this approximation will be checked at the end of the whole procedure.

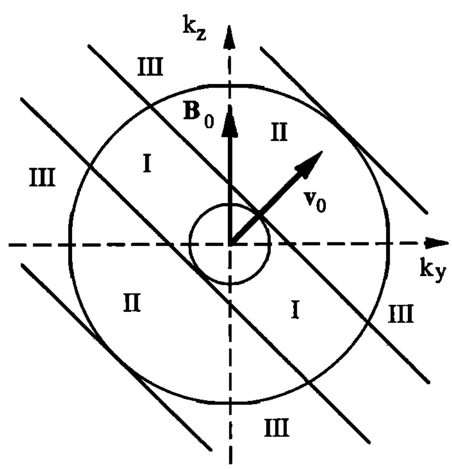

Figure 2. The integration domains D_τ and D'_τ are drawn in the $k_y k_z$ plane. The domain D_τ is the union of region II with region III, while D'_τ is the union of region I with region II. The radii of the small and large circles are k_τ and k_{max}, respectively. The directions of the vectors $\mathbf{v}_0$ and $\mathbf{B}_0$ are also drawn.

The form (50) for the filter function makes it easier to calculate the integrals (43)-(46) by using spherical variables ($k_x = k\sin\theta\cos\phi$, $k_y = k\sin\theta\sin\phi$, $k_z = k\cos\theta$). The integration with respect to k is readily performed, and we found the following expressions for the eigenvalues $\lambda^{(\tau)}_{1,2,3}$:

$$\lambda_1^{(\tau)} \simeq \int_{-1}^{1} d(\cos\theta) \int_0^{2\pi} d\phi \left[g^{[1]}\sin^2\phi F^{[1]}(\theta,\phi) + g^{[2]}\cos^2\theta\cos^2\phi F^{[2]}(\theta,\phi)\right] \tag{52}$$

$$\lambda_2^{(\tau)} \simeq \int_{-1}^{1} d(\cos\theta) \int_0^{2\pi} d\phi \left[g^{[1]}\cos^2\phi F^{[1]}(\theta,\phi) + g^{[2]}\cos^2\theta\sin^2\phi F^{[2]}(\theta,\phi)\right] \tag{53}$$

$$\lambda_3^{(\tau)} \simeq \int_{-1}^{1} d(\cos\theta) \int_0^{2\pi} d\phi g^{[2]}\sin^2\theta F^{[2]}(\theta,\phi) \tag{54}$$

where

$$g^{[s]} = \frac{C^{[s]}\left[k_\tau^{(1-\mu^{[s]})} - k_{max}^{(1-\mu^{[s]})}\right]}{\mu^{[s]} - 1}$$

$$F^{[s]}(\theta,\phi) = \left\{\left(\ell_z^{[s]}\cos\theta\right)^2 + \sin^2\theta\left[\left(\ell_x^{[s]}\cos\phi\right)^2 + \left(\ell_y^{[s]}\sin\phi\right)^2\right]\right\}^{-(1+\mu^{[s]}/2)}$$

($s = 1, 2$). The approximation that we have introduced in the expressions for $\lambda_i^{(\tau)}$ allows us to perform the χ^2 minimization in shorter computing times, since double integrals (52)-(54), instead of triple integrals (43)-(46), are now to be calculated. Moreover, when all wave vectors are normalized to $k_{min} = 2\pi/(v_0\tau_5)$, the results obtained turn out to be independent of the solar wind velocity magnitude and direction. It is worth noting that assuming $v_0 \simeq 600$ km/s, k_{min} is given by $k_{min} \simeq 10^{-11}$ cm^{-1} (this corresponds to choosing a unit length equal to $k_{min}^{-1} \simeq 10^{11}$ cm). The minimization of χ^2 allows us to calculate the values of the parameters which determine the spectra (22) of both the first and the second polarization. This procedure has been performed for the three data set of Helios 2 [*Bavassano et al.*, 1982], which have been taken at different distances from the Sun. The results are given in Tables 2 and 3 along with the corresponding distance R from the Sun and the value of the normalized total energy $E^{[s]}$,

$$E^{[s]} = \frac{1}{B_0^2}\int d^3\mathbf{k} I^{[s]}(\mathbf{k})$$

of both polarizations.

A check of the goodness of approximation (50) has been performed: The eigenvalues $\lambda_i^{(\tau)}$ have been calculated by using the original filter function $G_\tau(\mathbf{k}\cdot\mathbf{v}_0)$

(i.e., expressions (41)-(46)) in which the spectral energy densities $I^{[s]}$ are determined by using the values of the parameters given in Tables 2 and 3. In Figure 3 we show the eigenvalues calculated by using both filter functions G_τ and G'_τ, and the eigenvalues derived by *Bavassano et al.* [1982] from the Helios 2 data set. In the former case the angle between $\mathbf{v}_0$ and $\mathbf{B}_0$ has been calculated by evaluating, for each data set, the orientation of the average magnetic field with respect to the radial direction. It can be seen that both theoretical results fit remarkably well the measures, showing that approximation (50) leads to fully acceptable results.

Results and Conclusions

Using the following hypotheses, (1) statistical homogeneity and stationarity of the medium (2) the fact that the magnetic field $\mathbf{B}$ is divergenceless and (3) the Taylor hypothesis [*Taylor*, 1938] about the frequency measurements in the solar wind, we have shown that both the correlation spectra of the magnetic field fluctuations $T_{ij}(\mathbf{k})$ and its variance matrix S_{ij} can be expressed as functions of only two scalar quantities: the magnetic energy density spectra $I^{[1]}(\mathbf{k})$ and $I^{[2]}(\mathbf{k})$ of the two allowed polarizations. A parametric expression for these energy spectra, which is able to describe anisotropic distributions of the wave vectors, has been assumed. The procedure described in the previous section has allowed us to determine the values of the parameters which fit the eigenvalues of the variance matrix calculated by *Bavassano et al.* [1982]. In so doing, we have found a solution $I^{[s]}(\mathbf{k})$ of equations (38). Clearly, this is not the unique solution, but it represents the best choice, with respect to the used data set, among the family of spectra defined by the model (22). In the following we will describe the physical meaning of the obtained results, and we will compare, as far as possible, these results with other observational issues, in order to check the goodness of the model.

The eigenvalues of the variance matrix determined by *Bavassano et al.* [1982] have been calculated for different time bases τ_j ranging from $\tau_1 = 168$ s to $\tau_5 = 3$ hours. This data set contains detailed information on the energy distribution at different wavelengths in the spectrum. As a consequence, the fitting procedure allows an accurate determination of the spectral indices $\mu^{[1]}$ and $\mu^{[2]}$, the incertitude on the resulting values being of the order of some percent. From Table 2 it is seen that the spectral index $\mu^{[1]}$ of the polarization [1] ranges between 1.1 and 1.31, corresponding to rather flat spectra. In fact, these spectra are less steep than both the Kraichnan ($\mu = 3/2$) and the Kolmogorov ($\mu = 5/3$) spectra. The spectral index $\mu^{[2]}$ ranges between 1.46 and 1.81 (Table 3); i.e., it is larger by a factor of 1.3 - 1.4 than the corresponding value of $\mu^{[1]}$; then the polarization [2] spectra are always steeper than the corresponding polarization [1] spectra. The spectral indices of both polarizations increase with increasing distance from the Sun, showing a tendency of both spectra to become steeper at larger distances.

Table 2. Parameters of the Polarization [1] Magnetic Fluctuation Energy Spectrum

R, AU	$C^{[1]}$	$\mu^{[1]}$	$\ell_x^{[1]}$	$\ell_y^{[1]}$	$\ell_z^{[1]}$	$E^{[1]}$
0.29	7.2	1.10	100	30	1.0	0.25
0.65	7.8	1.23	70	20	1.0	0.26
0.87	5.3	1.31	50	10	1.0	0.30

Table 3. Parameters of the Polarization [2] Magnetic Fluctuation Energy Spectrum

R, AU	$C^{[2]}$	$\mu^{[2]}$	$\ell_x^{[2]}$	$\ell_y^{[2]}$	$\ell_z^{[2]}$	$E^{[2]}$
0.29	1.1	1.46	110	1.3	1.0	0.13
0.65	1.7	1.73	100	1.2	1.0	0.12
0.87	0.9	1.81	90	0.7	1.0	0.10

The energy contents $E^{[1]}$ and $E^{[2]}$ of the two polarizations listed in Tables 2 and 3 have also been determined with a good accuracy (to within some percent). The values of $E^{[1]}$ and $E^{[2]}$ are both normalized to B_0^2. The square root of $E^{[1]}$ and $E^{[2]}$ represents a measure of the relative fluctuation level for the two polarization, at the longest considered time scale ($\tau_5 = 3$ hours). Comparing $(E^{[1]} + E^{[2]})$ with the normalized trace σ_5 of the variance matrix for the longest time basis reported in Table 1, we find a good agreement at all the considered distances R, which confirms the goodness of the fitting procedure. Note that B_0 varies with R, so the normalization of $E^{[1]}$ and $E^{[2]}$ changes at different distances from the Sun. However, the ratio $E^{[1]}/E^{[2]}$ is independent of this normalization. It can be seen that polarization [1] is always more energetic than polarization [2], the ratio $E^{[1]}/E^{[2]}$ being comprised between 2 and 3. This ratio tends to increase with the distance from the Sun.

The other parameters of the energy spectra given in Tables 2 and 3 are $\ell_x^{[s]}$ and $\ell_y^{[s]}$ (we set $\ell_z^{[s]} = 1$, as previously said), which determine the shape of the energy distributions of the two polarizations in the $\mathbf{k}$ space. The fitting procedure did not allow us to determine such parameters in a manner as accurate as that of $\mu^{[s]}$ and $E^{[s]}$; thus the values of $\ell_x^{[s]}$ and $\ell_y^{[s]}$ given in Tables 2 and 3 should be considered as order of magnitude estimates. However, within the uncertainties we can say the following:

1. Parameter $\ell_x^{[1]} > \ell_y^{[1]} \gg \ell_z^{[1]}$. These inequalities indicate that in the energy distribution of polarization [1] fluctuations, wave vectors quasi-parallel to $\mathbf{B}_0$ (z direction) largely dominate. The corresponding contour surfaces in the $\mathbf{k}$ space are sort of "cigars" aligned along the $\mathbf{B}_0$ direction; in particular, the first inequality shows that these surfaces are rather flat in the $\mathbf{B}_0 - \mathbf{v}_0$ plane.

2. Concerning polarization [2], we found that $\ell_x^{[2]} \gg \ell_y^{[2]}, \ell_z^{[2]}$, indicating that the spectrum $I^{[2]}(\mathbf{k})$ is strongly flat on the $\mathbf{B}_0 - \mathbf{v}_0$ plane. Within this plane the energy distribution does not present any relevant anisotropy, since the characteristic lengths $\ell_y^{[2]}$ and $\ell_z^{[2]}$ are of the same order.

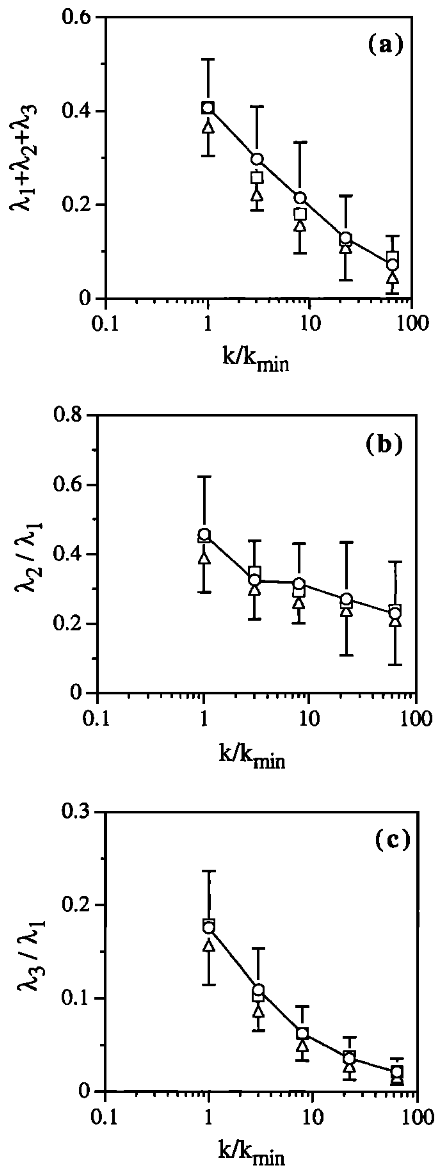

Figure 3. The sum (a) $\lambda_1+\lambda_2+\lambda_3$, and the ratios (b) λ_2/λ_1 and (c) λ_3/λ_1 of the eigenvalues of the variance matrix are plotted for $R = 0.87$. Circles indicate the values determined in the analysis of solar wind data by *Bavassano et al.* [1982], along with the respective error bars. The squares and the triangles indicate the same quantities calculated with formulae (52)-(54) and(41)-(45), respectively. The values of the spectral parameters given in Tables 2 and 3 have been used.

Let us see how polarizations [1] and [2] contribute to the fluctuating energy along the directions of the eigenvectors of the variance matrix. From equations (8) it is seen that the fluctuating component δB_z (parallel to $\mathbf{B}_0$) is only due to polarization [2], fluctuations belonging to polarization [1] being strictly perpendicular to $\mathbf{B}_0$. Thus since the minimum variance direction is close to $\mathbf{B}_0$, the corresponding eigenvalue represents a lower limit for the energy content of polarization [2]: $E^{[2]} \geq \lambda_3^{(\tau_5)}$. From Tables 1 and 2 it can be seen that $E^{[2]} \simeq 2\lambda_3^{(\tau_5)}$. As discussed above, almost all the energy of polarization [2] is localized on wave vectors parallel to the $\mathbf{B}_0-\mathbf{v}_0$ plane; this means that these fluctuations do not contribute to the energy on the fluctuating component δB_x (perpendicular to the $\mathbf{B}_0-\mathbf{v}_0$ plane). Then the eigenvalue $\lambda_1^{(\tau_5)}$ must represent a lower limit for $E^{[1]}$; actually, $E^{[1]} \simeq (6/5)\lambda_1^{(\tau_5)}$. Since $\lambda_3^{(\tau_5)} \simeq (1/5)\lambda_1^{(\tau_5)}$, we can conclude that polarizations [1] and [2] contribute almost equally to the fluctuating energy in the y direction.

Let us compare these results with some other observations. The first check for our model is a comparison with the results by *Matthaeus et al.* [1990], who calculated the trace of the correlation tensor of the magnetic fluctuations by a direct analysis of solar wind data. These authors found that two distinct populations are present in the magnetic fluctuations: the first one has wave vectors nearly parallel to the average magnetic field ("slablike" Alfvénic fluctuations), while the second one has wave vectors nearly perpendicular to $\mathbf{B}_0$ (quasi-two-dimensional turbulence). In their analysis, *Matthaeus et al.* [1990] assumed that the correlation tensor is axisymmetric around the direction of $\mathbf{B}_0$. The assumption of axisymmetry is not compatible with our data set, since the eigenvalues λ_1 and λ_2 of the variance matrix, relative to directions perpendicular to $\mathbf{B}_0$, are not equal [*Bavassano et al.*, 1982]. Nonetheless, since in most of the data set used by *Matthaeus et al.* [1990] $\mathbf{B}_0$ is not highly inclined to the ecliptic plane (so that the $\mathbf{B}_0 - \mathbf{v}_0$ plane is close to that containing the spacecraft trajectory), a comparison between the results by *Matthaeus et al.* [1990] and those of our model restricted to the $\mathbf{B}_0 - \mathbf{v}_0$ (yz) plane will be significant.

In our calculation we decomposed the magnetic fluctuations as a superposition of two independent polarizations [1] and [2], compatible with the divergenceless condition. For this comparison we consider the trace of the correlation tensor Fourier transform, which, on account of equation (21), is given by

$$T_{ii}(\mathbf{k}) = I^{[1]}(\mathbf{k}) + I^{[2]}(\mathbf{k}) \qquad (55)$$

The contour levels of $T_{ii}(0, k_y, k_z)$ are plotted in Figure 4. The result is similar to that obtained by *Matthaeus et al.* [1990]. Actually, from Figure 4 two populations of fluctuations, with wave vectors nearly parallel and nearly perpendicular to $\mathbf{B}_0$, respectively, can be identified. The first population is formed by all the polarization [1] fluctuations and by the fluctuations with $\mathbf{k}$ quasi-parallel to $\mathbf{B}_0$ belonging to polarization [2]. The latter fluctuations are physically indistinguishable from the former, in that when $\mathbf{k}$ is nearly parallel to $\mathbf{B}_0$, both polarization vectors are quasi-perpendicular to $\mathbf{B}_0$. On the contrary, the second population is almost entirely formed by fluctuations belonging to polarization [2].

While it is clear that fluctuations with $\mathbf{k}$ nearly parallel to $\mathbf{B}_0$ are mainly polarized in the plane perpendicular to $\mathbf{B}_0$ (this is an obvious consequence of the condition $\nabla \cdot \mathbf{B} = 0$), our results allow us to assign a definite polarization also to the second population ($\mathbf{k}$ nearly perpendicular to $\mathbf{B}_0$): in particular, since they

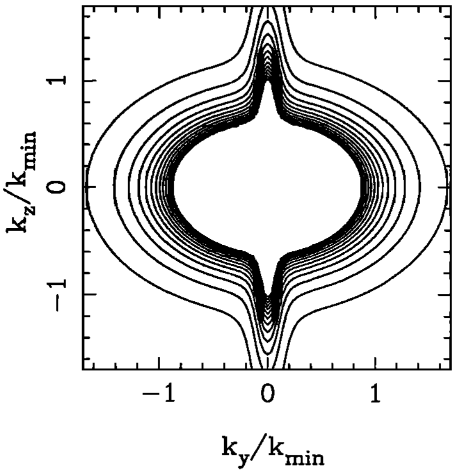

Figure 4. Contour levels of the Fourier transform of the correlation tensor trace $T_{ii}(\mathbf{k})$ in the plane $k_x = 0$. The plot has been drawn by using the spectral parameters given in Tables 2 and 3 for $R = 0.87$.

essentially belong to polarization [2], according to equation (8b), they are polarized nearly parallel to $\mathbf{B}_0$.

It is worth noting that in the introduction of their paper, *Matthaeus et al.* [1990] give an interpretation of their results, which is in contrast with our finding. Namely, they suggest that a nearly two-dimensional (2-D) incompressible turbulence characterized by wave vectors and magnetic field fluctuations both perpendicular to $\mathbf{B}_0$ is present in the solar wind. This interpretation, however, does not arise from the analysis they performed on the solar wind magnetic field fluctuations, but has been suggested only on the basis of 2-D simulations of the decay of anisotropic incompressible turbulence [*Shebalin et al.*, 1983] and of analytical studies of quasi 2-D turbulence in presence of strong dc magnetic field [*Montgomery*, 1982]. Let us note, however that in the former approach, which is strictly 2-D, when $\mathbf{k}$ is perpendicular to $\mathbf{B}_0$, magnetic field fluctuations are necessarily parallel to $\mathbf{B}_0$. In the latter one, along with incompressibility, it is assumed that the energy in the fluctuations is much less than in the dc magnetic field; both hypotheses do not apply to the solar wind case. On the contrary, our results are directly related to the observational data.

The presence in the solar wind of magnetic field fluctuations with both $\mathbf{k}$ and $\delta\mathbf{B}$ perpendicular to $\mathbf{B}_0$ has been also suggested by *Tu and Marsch* [1991]. Analyzing solar wind data, these authors found static structures satisfying

$$(\mathbf{B}\cdot\nabla)\mathbf{B} = 0 \qquad\qquad \delta|\mathbf{B}| = 0$$

Concerning these results we can note the following:

1. These structures have been observed during an interval of a non Alfvénic period (almost zero correlation between velocity and magnetic field fluctuations) where they seem to represent the main component of magnetic field fluctuations. There is no reason to expect that the same structures are always present also during Alfvénic periods like those we considered.

2. The above mentioned structures give rise to fluctuations which have both $\mathbf{k} \perp \mathbf{B}_0$ and $\delta\mathbf{B} \perp \mathbf{B}_0$ (i.e., fluctuations belonging to polarization [1]) only in the linear case, i.e., when $|\delta\mathbf{B}| \ll |\mathbf{B}_0|$. In the nonlinear case (which is the case of solar wind) the terms which should be neglected to derive the polarization of these static structures are of the same order of or greater than the retained terms. Then nothing can be said about the polarization ([1] or [2]) of these structures.

Matthaeus et al. [1990] also found that the first (slab-like) population becomes dominant with respect to the second one with decreasing wavelength. The same tendency is present in our results: Since the spectral index $\mu^{[2]}$ is larger than $\mu^{[1]}$, with increasing $\mathbf{k}$, wave vectors parallel to $\mathbf{B}_0$ tend to dominate with respect to those perpendicular.

A further characterization of the second population of magnetic fluctuations can be obtained considering other observations. In the weak turbulence framework this population would result from the superposition of slow and fast magnetosonic waves at quasi-perpendicular wave vectors. Actually, slow magnetosonic waves with $\mathbf{k}$ perpendicular to $\mathbf{B}_0$ are characterized by [e.g., *Akhiezer et al.*, 1975b] (1) velocity and magnetic field fluctuations parallel to $\mathbf{B}_0$; (2) vanishing phase velocity; i.e., they are stationary in the plasma reference frame; and (3) equilibrium between magnetic and thermal pressure fluctuations. On the contrary, fast waves display a positive magnetic-thermal pressure correlation.

One important feature of the "compressive" fluctuations observed in the solar wind is the presence of a distinct anticorrelation between proton density and magnetic field intensity [*Burlaga*, 1968; *Burlaga and Ogilvie*, 1970; *Vellante and Lazarus*, 1987; *Roberts*, 1990; *Burlaga et al.*, 1990], which has been interpreted, as being due to the presence of quasi-static pressure-balanced structures. Thus in our analysis the magnetic field fluctuations with wave vectors quasi-perpendicular to $\mathbf{B}_0$ and belonging to polarization [2] (second population) are probably due to nearly pressure balanced structure with a smaller amount of fast magnetosonic waves.

The usefulness of these last considerations is somewhat limited by the fact that the amplitude of magnetic field fluctuations in the solar wind is of the same order as B_0. Nonetheless, quasi-static pressure-balanced structures, stationary in the plasma reference frame (convected structures), represent also a nonlinear solution of MHD equations; in the limit of small characteristic lengths they are also known as "tangential discontinuities." These discontinuities [e.g., *Akhiezer et al.*, 1975b] are one-dimensional stationary structures in which both the magnetic field and the mass flow are perpendicular to the modulation direction. Magnetic pressure variations are balanced by opposite thermal pressure variations, while no magnetic tension is generated by the perturbation. The departures $\delta\mathbf{B}$ from the mean magnetic field $\mathbf{B}_0$ associated with such structures are parallel to $\mathbf{B}_0$, while their wave vector $\mathbf{k}$ is perpendicular to $\mathbf{B}_0$, just as we find in our polarization analysis for the second population fluctuations.

It would be interesting now to compare our model, which directly concerns only magnetic field fluctuations, with the observations of the energy level and the spectra of compressive quantities, i.e., density and magnetic field intensity. Correlations between the energy spectrum of the magnetic fluctuations and the spectra of these compressive quantities do exist only in the weak turbulence framework, i.e., in the small-amplitude limit. This is not the case for solar wind magnetic fluctuations; for instance, the rms magnetic fluctuations amplitude $(E^{[1]} + E^{[2]})^{1/2} = (\lambda_1 + \lambda_2 + \lambda_3)^{1/2} \simeq 0.6$. In this situation, considering the turbulence as a superposition of linear modes (Alfvén and magnetosonic) is only a rough approximation and none of the correlations predicted by the linear theory can expected to be satisfied a priori. In particular, within the weak turbulence theory the polarization [2] magnetic field fluctuations can be identified as magnetosonic (fast or slow) fluctuations. On the contrary, the nonlinear Alfvén wave solution in a compressible medium is characterized by $\delta \mathbf{v} = \pm \delta \mathbf{B}/(4\pi\rho)^{1/2}$ and B^2 and ρ both uniform, regardless of the polarization. This means that in the presence of such type of solution we can have any value of $E^{[2]}$ and, at the same time, $\delta\rho = \delta|\mathbf{B}| = 0$. Then in the solar wind case the relationship between the density and magnetic field intensity fluctuations and the polarization [2] fluctuations deduced from the linear theory of magnetosonic waves can at most be considered as a first-order approximation. As a consequence, the following comparison with the observations of compressive fluctuations cannot be used to directly check the results of our model but can only verify to what extent the correlations predicted by the linear theory actually survive in the solar wind MHD turbulence.

By linearizing the expression for B^2, the following relation between magnetic field intensity and polarization [2] fluctuations is obtained:

$$\frac{\delta|\mathbf{B}|}{B_0} \simeq \frac{\mathbf{B}_0 \cdot \delta \mathbf{B}^{[2]}}{B_0^2} \tag{56}$$

This furnishes a level of magnetic field intensity fluctuation of the order of $\delta|\mathbf{B}|/B_0 \simeq (E^{[2]}/2)^{1/2} \simeq 0.22$. The corresponding values calculated by *Bavassano et al.* [1982] for the same data sets range from 0.05 to 0.07. However, to obtain expression (56), we have neglected nonlinear terms like $\{\delta \mathbf{B}^{[1]}\}^2$, $\{\delta \mathbf{B}^{[2]}\}^2$, $2\delta\mathbf{B}^{[1]} \cdot \delta\mathbf{B}^{[2]}$, which can be estimated to be of the same order as the linear term we have retained. This indicates that the nonlinear terms tend to counterbalance the linear term, to keep the magnetic intensity fluctuations to the very low observed level. A similar conclusion can be drawn also for the density fluctuations. For the magnetosonic modes the linear relation between density and magnetic field fluctuations is given by [e.g., *Akhiezer et al.*, 1975b]

$$\frac{\delta B^{[2]}(\mathbf{k})}{B_0} = -\frac{\delta\rho(\mathbf{k})}{\rho_0} \sin\theta \times \frac{\left\{1 + \beta \pm \left[(1+\beta)^2 - 4\beta\cos^2\theta\right]^{1/2}\right\}}{1 + \beta - 2\cos^2\theta \pm \left[(1+\beta)^2 - 4\beta\cos^2\theta\right]^{1/2}} \tag{57}$$

where $\beta = c_S^2/c_A^2$, c_S and c_A are the sound and the Alfvén velocity, respectively, θ is the angle between $\mathbf{k}$ and $\mathbf{B}_0$, and the upper (lower) sign refers to the fast (slow) magnetosonic mode. Expression (57) shows that in propagation quasi-parallel to $\mathbf{B}_0$ we can find either Alfvén waves ($\delta\rho \simeq 0$) or acoustic waves ($\delta\mathbf{B}^{[2]} \simeq 0$); i.e., density fluctuations are decoupled from magnetic field fluctuations. Thus in order to establish a link between $\delta\rho$ and $\delta\mathbf{B}^{[2]}$ we must evaluate expression (57) for $\theta \neq 0$. Recalling that nearly pressure balanced structures (slow mode) prevail over fast mode fluctuations, the following estimation can be obtained:

$$\frac{\Delta\rho}{\rho} \simeq \frac{1}{\beta}\frac{\Delta B^{[2]}}{B_0} \simeq \frac{\sqrt{E^{[2]}}}{\beta} \tag{58}$$

On the one hand this expression depends critically on β, a parameter which is not easily deduced from solar wind measurements; on the other hand, it overestimates the density fluctuations, since a certain fraction of the polarization [2] fluctuations are in quasi-parallel propagation. However, the typical measured level $\Delta\rho/\rho \simeq 0.1$ [*Matthaeus et al.*, 1991] is obtained from equation (58) for $\beta \simeq 3$. Lower values of β, which are likely to be found in the solar wind, would yield an estimation of $\Delta\rho/\rho$ which is higher than the observed value. Then also in the case of the density fluctuations, nonlinear terms neglected in the above approach should play a role in keeping these fluctuations to a level lower that that predicted by the linear theory. A somewhat more direct comparison is possible between spectral indices, in that they do not depend on other unknown parameters, such as β, θ, or the ratio between the energy levels of fast and slow waves. *March and Tu* [1990] have calculated several spectra of proton density and magnetic field magnitude in the inner solar wind, based on the Helios data. These authors found that in high-speed streams the spectra of $\delta\rho/\rho$ and $\delta|\mathbf{B}|/|\mathbf{B}|$ (ρ being the proton density) in the frequency range 2×10^{-5} Hz $< f < 3 \times 10^{-4}$ Hz are quite steep, the spectral indices being typically close to the Kolmogorov value ($\mu = 5/3$) for both quantities. For higher frequencies (5×10^{-4} Hz $< f < 2 \times 10^{-3}$ Hz) the density spectrum becomes much flatter, the spectral index being typically close to 0.5 - 0.7. In the higher-frequency range the $\delta|\mathbf{B}|/|\mathbf{B}|$ spectra are also flatter than in the lower-frequency range, but the change in slope from one range to the other is less evident than in the density spectra; this holds especially at larger distances from the Sun.

The spectra calculated by *March and Tu* [1990] at 0.29 AU are based on a subset of the data set used by *Bavassano et al.* [1982] to calculate the variance matrix at the same distance from the Sun. Then a comparison can be performed between the spectra shown in Figures 1e and 3e of *March and Tu* [1990] and our results relative to $R = 0.29$ AU. It can be seen that in the frequency range which we considered ($f \geq 10^{-4}$ Hz) the slope of the polarization [2] energy spectrum ($\mu^{[2]} = 1.46$) fits quite well on the $\delta|\mathbf{B}|/|\mathbf{B}|$ power spectrum (Figure 3e of *March and Tu* [1990]). At larger distances from the Sun we cannot make other direct comparisons. However, we found that $\mu^{[2]}$ increases with increasing R, and also

March and Tu [1990] report the same trend in the slope of $\delta|\mathbf{B}|/|\mathbf{B}|$ power spectra.

Concerning the density spectra, the slope of the polarization [2] spectrum which we found at $R = 0.29$ AU ($\mu^{[2]} = 1.46$) fits well on the density spectrum shown in Figure 1e of *March and Tu* [1990] only in the lower-frequency range (10^{-4} Hz $\leq f \leq 2 \times 10^{-3}$ Hz). The higher-frequency part (5×10^{-4} Hz $\leq f \leq 2 \times 10^{-3}$ Hz) of the density spectrum is much flatter, the slope being neatly lower than the value found for polarization [2] spectral index. The discrepancy found in the higher-frequency range between the polarization [2] spectrum and the density spectrum can be interpreted assuming that a population of soundlike fluctuations, propagating parallel to $\mathbf{B}_0$, is present in this frequency range. This idea is supported by the fact that in this frequency range the magnetic field magnitude spectrum is steeper than the density spectrum [*March and Tu*, 1990, Figures 1e and 3e]. In fact, since sound waves do not involve magnetic field fluctuations, they can affect neither the energy spectra of the magnetic fluctuations calculated by our model, nor the magnetic field magnitude fluctuation spectrum reported by *March and Tu* [1990], while they would give a contribution to the density fluctuation spectrum. On the contrary, if only pressure-balanced structures and/or magnetosonic waves were present in this frequency range, it is expected that the slopes of the density and the magnetic field intensity spectra should be almost equal, which is in contrast with the results reported by *March and Tu* [1990].

In conclusion, the analysis we have performed allows us to calculate the parameters of a model for the full 3-D energy spectra for the two components of the MHD turbulence in the solar wind. This contributes to furnish a physical description of the solar wind turbulence which could help to understand the behavior of compressible MHD turbulence in the solar wind. Such a quantitative theoretical analysis is beyond the purpose of the present paper; however, some general indications can be derived about the effect of different physical mechanisms in the development of the solar wind turbulence.

One of the main features found by our analysis is the strong flattening of both the polarization [1] and [2] spectra on the $\mathbf{B}_0 - \mathbf{v}_0$ plane. This flattening can be interpreted as a signature of the solar wind expansion. Indeed, let us consider a space domain sufficiently small with respect to its mean distance from the Sun, which is convected in the radial direction with velocity $\mathbf{v}_0$. In consequence of the expansion, the scale length of a given structure inside this domain is progressively enhanced perpendicularly to the radial direction; correspondingly, the wave vectors tends to become aligned with the direction of $\mathbf{v}_0$ [*Barnes*, 1969; *Völk and Alpers*, 1975; *Grappin et al.*, 1992].

The results of our analysis show that while this tendency is effective in flattening the wave vector distribution onto the $\mathbf{B}_0 - \mathbf{v}_0$ plane, it is not sufficient to realize the wave vector alignment along $\mathbf{v}_0$. Indeed, in the plane $\mathbf{B}_0 - \mathbf{v}_0$ the situation is much more complicated: the wave vector distribution of polarization [1] is peaked in the $\mathbf{B}_0$ direction, while that of polarization [2] is almost isotropic. The anisotropy should be related to the presence of an average magnetic field. This presence introduces many different physical mechanisms, which can compete with the expansion and which could in principle be different for the two polarizations. Few theoretical results in such a regard exist and they are limited to incompressible MHD turbulence. In this framework it has been shown by direct numerical simulations [*Shebalin et al.*, 1983; *Weisshaar*, 1988] and by integration of a set of averaged statistical equations [*Carbone and Veltri*, 1990] that the presence of an average magnetic field slows down nonlinear interactions between wave vectors parallel to $\mathbf{B}_0$ with respect to those between wave vectors perpendicular to $\mathbf{B}_0$ (Alfvén effect [*Pouquet et al.*, 1976]). This is due to the fact that in an incompressible medium, two Fourier modes propagating parallel to $\mathbf{B}_0$ in opposite directions go out of phase faster than Fourier modes propagating almost perpendicular to $\mathbf{B}_0$. When the decay of an initially isotropic turbulence is considered [*Shebalin et al.*, 1983; *Carbone and Veltri*, 1990], this effect produces an alignment of the wave vectors along $\mathbf{B}_0$ in the large-scale range and a dominance of wave vectors perpendicular to $\mathbf{B}_0$ in the small-scale range [*Carbone and Veltri*, 1990].

Even if these simulations show that the presence of an average magnetic field represents an important source of anisotropy, they cannot be directly applied to the solar wind, which is a compressible medium and where the situation is extremely complex. Moreover, it is not clear why the anisotropy observed on the data should be that found in the large-scale range by *Carbone and Veltri* [1990].

Since the magnetic fluctuations we have analyzed are characterized by $\delta|\mathbf{B}| \simeq 0$, it is tempting to suppose that Landau damping could play a role in keeping almost constant the magnetic field intensity [*Veltri et al.*, 1992]. The analytical theory of Landau damping [*Barnes*, 1979] predicts that the Alfvénic and the magnetosonic waves are characterized by considerable differences in the damping rate, the former being essentially unaffected and the latter being damped at a rate which is roughly proportional to the wave frequency. In applying this theory to the case of solar wind low-frequency turbulence, one must keep in mind that it has been obtained in the small-amplitude limit. So one can refer to this application only as a rough approximation. Nevertheless, supposing that the energy level of magnetosonic waves at a given wave vector results from an equilibrium between production by local nonlinear interactions of Alfvén waves and dissipation through the Landau damping and taking into account that the Landau damping of magnetosonic waves grows linearly with the wave frequency, one could perhaps understand why the spectrum of the polarization [2] fluctuations is systematically steeper than that of the polarization [1] fluctuations [*Veltri et al.*, 1992]. This conjecture, however, requires a more quantitative analysis, for example, based on nonlinear kinetic simulations, which is beyond the aim of this paper.

Appendix

In this appendix we want to show that the integrations (43)-(46) on the domain D_τ, defined in section 4, give rise to approximately the same results as those on the domain D'_τ. With reference to Figure 2 the borders of the domains D_τ and D'_τ divide the **k** space into several subdomains; these regions are represented in Figure 2 in the $k_y k_z$ plane. The domain D_τ is the union of region II with region III, while D'_τ is the union of region I with region II. Then the differences due to the modification of the filter function result from the contributions of the regions I and III.

Let us indicate with I_I, I_{II}, I_{III} one of the integrals (43)-(46) in which the integration domain is limited to region I, II, and III, respectively. If $k_\tau \ll k_{max}$, simple geometrical considerations show that $(I_I/I_{II}) \simeq (k_\tau/k_{max})$; in the cases which we consider the ratio k_τ/k_{max} ranges from 3.6×10^{-2} to 9.2×10^{-5}; thus we always have $I_I \ll I_{II}$.

In order to estimate the ratio (I_{III}/I_{II}), let us neglect the angular dependence in the integrals (43)-(46) so that the functions in brackets can be approximated by the power law $k^{-\gamma}$, where $\gamma = 2 + \mu^{[s]}$. Typical values for the spectral indices $\mu^{[s]}$ found by *Carbone and Veltri* [1990] are in the range $1 < \mu^{[s]} < 2$; assuming $\mu^{[s]} \simeq 1.5$, we have

$$I_{II} \simeq \int_{k_\tau}^{k_{max}} k^{-\gamma} k^2 dk \simeq \frac{1}{k_\tau^{\gamma-3}}$$

$$I_{III} \simeq \int_{k_{max}}^{\infty} k k^{-\gamma} k_{max} dk \simeq \frac{1}{k_{max}^{\gamma-3}}$$

with $\gamma \simeq 3.5$. Then we find

$$\frac{I_{III}}{I_{II}} \simeq \left(\frac{k_\tau}{k_{max}}\right)^{\gamma-3} \simeq \left(\frac{k_\tau}{k_{max}}\right)^{0.5} \ll 1$$

In conclusion, the main contribution to the integrals (43) - (46) results from region II of the **k** space, which belongs both to the domain D_τ and to the domain D'_τ. Then, modifying the filter function from $G_\tau(\mathbf{k} \cdot \mathbf{v}_0)$ to $G'_\tau(\mathbf{k})$ does not introduce relevant changes in the integrals (43)-(46). Using the latter filter allows us to obtain the simplified expressions (52)-(54), which were finally assumed in the procedure of minimization of χ^2.

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