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NON-WKB EVOLUTION OF SOLAR WIND FLUCTUATIONS: A TURBULENCE MODELING APPROACH

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Abstract. Recent observations indicate systematic changes in the interplanetary inertial range velocity-magnetic field correlation with increasing heliocentric distance. Here, we outline a multiple length scale model for the evolution of the small scale fluctuations due to local nonlinear couplings as well as couplings to large scale gradients, an approach similar to that used in turbulence modeling of inhomogeneous shear flows. A simple special case solution is given, indicating that couplings to large scale flow gradients can cause radial evolution of the type seen in observations. The strongest couplings of this type are linear and do not appear in the usual WKB orderings. The relationship of our approach to WKB theory is not fully understood at present.

Introduction

The dynamical state of solar wind magnetohydrodynamic (MHD) scale fluctuations has remained a topic of considerable debate, with the most controversial issue remaining the degree to which they might be described principally as passive wave-like remnants of coronal processes, or alternatively, as an active, MHD turbulent medium. On the other hand, observations also indicate the presence of correlations between the velocity and magnetic fields that are highly suggestive of MHD waves, often of the "outward-traveling" type [Coleman 1968; Belcher and Davis 1971].

Active solar wind turbulence was invoked some time ago [Coleman, 1968] as a means of explaining the heating of the interplanetary magnetofluid. A number of observational studies [Matthaeus and Goldstein, 1982a,b] also indicated that the state of solar wind fluctuations is quite consistent, in a number of ways, with expections of MHD turbulence theory. However, until recently [Roberts et al., 1987a,b], what was lacking was clear observational indication that the interplanetary plasma undergoes specific dynamical changes that can be attributed to MHD turbulence. The latter studies showed that, on average, the frequency of occurrence of Alfvénic periods is highest in the inner heliosphere and decreases substantially with increasing heliocentric distance. By two AU or so, the preponderance of "outward traveling" fluctuations over "inward" ones is marginal. This was suggested to be a consequence of stream shear driven turbulence.

In another important recent development, Tu and coworkers [Tu et al., 1984; Tu, 1988] showed how the heating effects of a turbulent cascade can be incorporated into the WKB-like transport equations for the fluctuations that admit correlations of the outward traveling type. This theoretical approach met with some success in terms of predicting the heliocentric distance dependence of the magnetic energy spectrum shape, as well as heating properties of the type suggested by Coleman [1968]. However, there also appear to be some limitations in the Tu approach. The fluctuations are assumed to be almost all of the type with an outward traveling sense of velocity - magnetic field correlation; the admixture of fluctuations with the opposite type of correlation is assumed small. It is clear that the framework of the transport equations needs to be generalized, especially with regard to including mixed correlations, both to achieve additional measures of internal consistency as well as to describe known radial dependencies of the properties of the turbulence [Roberts et al., 1987a,b]. Recently [Zhou and Matthaeus, 1988a,b] we have begun to explore such extensions of the Tu models, which we develop in the spirit of turbulence modeling calculations familiar in hydrodynamic studies [Zhou et al., 1988] and in mean field electrodynamics studies of MHD systems [Krause and Radler, 1980; Moffatt, 1978]. Here, we outline our solar wind turbulence transport theory and present simple solutions that indicate its capability to explain radial variations of interplanetary Alfvénic fluctuations [Roberts et al., 1987a,b]. Surprisingly, the simplest form of the theory appears not to reduce to WKB prediction [Hollweg, 1973].

Outline of the Model

The basis of the model is a two-length scale expansion of the equations of compressible MHD, which may be written in terms of the fluid plasma velocity $\underline{V}$, the magnetic field $\underline{B}$ and the plasma mass density ρ, as an equation of motion

$$\rho(\frac{\partial \underline{V}}{\partial t} + \underline{V}\cdot\nabla\underline{V}) = -\nabla p+\underline{J}x\underline{B}/c+\underline{D}, \qquad (1)$$

an induction equation,

$$\frac{\partial \underline{B}}{\partial t} = \nabla \times (\underline{V}x\underline{B})+\underline{D}', \qquad (2)$$

and a continuity equation,

$$\frac{\partial \rho}{\partial t} = -\nabla\cdot(\underline{V}\rho). \qquad (3)$$

In the above, t is the time, c is the speed of light and p is the pressure. The electric current density is $\underline{J} = (c/4\pi)\nabla x\underline{B}$, and the magnetic field satisfies $\nabla\cdot\underline{B} = \underline{0}$. Viscous and Ohmic dissipation terms D and D', respectively, in (2) and (3), are assumed to be significant only at wavenumbers considerably

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higher than the wavenumbers of the "inertial range" fluctuations which we focus on. It is assumed that there exists an ensemble averaging operator $<\ldots>$ that separates dependencies on the large scale (slowly varying) spatial coordinate from the small scale (rapidly varying, or inertial range) coordinate. This kind of operation is familiar in the mean field theory of the MHD dynamo [Krause and Radler, 1980; Moffatt, 1978]. Therefore we decompose the velocity and magnetic fields into fluctuations and spatially slowly varying means, according to $\underline{B} = \underline{B}_o + \underline{b}$ and $\underline{V} = \underline{U} + \underline{v}$, where we write $\underline{U} = <\underline{V}>$ and $\underline{B}_o = <\underline{B}>$. The small scale fluctuations are assumed to be incompressible, so the density ρ is slowly varying in position and admits no fluctuations, and $\nabla \cdot \underline{v} = 0$. Observational studies based on ergodic theory [Matthaeus and Goldstein, 1982b] indicate that time averaging in the spacecraft frame is an adequate means of performing the above decomposition, and that fluctuations properly computed in this way are approximately time stationary and locally spatially homogeneous.

In terms of the above decomposition into fast and slow-varying fields, evolution equations are computed separately for large and small scale fields. Large scale evolution consists of large scale-large scale couplings and large scale-small scale couplings. Similarly, the small scale equations include small scale-small scale nonlinearities and small scale-large scale couplings. This basic structure is familiar in "turbulence modeling" formalisms [e.g., Zhou et al., 1988; Moffatt, 1978; Krause and Radler, 1980]. Most often two-scale decomposition is used to find closures for the small scale evolution, so that the evolution of the large scale fields may be computed. Our interest, for the present, is in computing approximations to the behavior of the small scale fluctuations. Thus, we will "freeze" the large scale dynamics, and consider the large scale fields to be given functions of position in the heliosphere; in the simplest case $\underline{U}$ is a spherically symmetric, constant solar wind flow, and $\underline{B}_o$ is the Parker magnetic field.

We are interested only in certain properties of the fluctuations, and not their detailed evolution. For example, the incompressible energy (per unit mass) $E = < v^2 + b^2/(4\pi\rho) >/2$ and the cross helicity, $H_c = < \underline{v} \cdot \underline{b}/\sqrt{4\pi\rho} >/2$ and their inertial range wavenumber (k) spectra, $E(k)$ and $H_c(k)$ are of importance. In addition, the quantity $\sigma_c(k) = 2H_c(k)/E(k)$ is a sensitive diagnostic for the presence of Alfvénic fluctuations near wavenumber k. In our two-scale calculations the dependent variables and spectra are functions of time and the (slowly varying) large scale heliocentric coordinate. Spectra are Fourier transforms with respect to the local rapidly varying spatial coordinate associated with the fluctuations.

To account for a finite heating rate in a consistent way, and to capture other effects of turbulence and its interaction with large scale interplanetary fields, we allow for arbitrary admixtures of inward and outward-type fluctuations. This is facilitated by introduction of the small scale Elsasser [1950] fields $\underline{z}^{\pm} = \underline{v} \pm \underline{b}/\sqrt{4\pi\rho}$, which are treated on equal footing. The Elsasser variables are a convenient representation of incompressible MHD turbulence [e.g., Kraichnan, 1965] with a large scale magnetic field and/or nonzero cross helicity, and have recently [Marsch and Mangeney, 1987] been shown to be useful for compressible flows. Even though the fluctuations are incompressible, $\nabla \cdot \underline{z}^{\pm}$ is nonzero but slowly varying. A useful heuristic interpretation of the z-fields is that they represent "inward" and "outward" propagating wavepackets [Kraichnan, 1965]. In fact, this representation is completely general and includes nonpropagating structures.

With these assumptions, we compute the time evolution equations for the fluctuating fields by subtracting the ensemble average of the MHD equations from the exact forms (Eqs. 1-3). We ignore the dissipative terms in (1) and (2). Next, we calculate evolution equations for covariance tensors such as

$$H^{\pm}_{ij} = < z_i^{\pm} z_j^{\pm\prime} >,$$

where the primed and unprimed variables are evaluated at distinct positions, separated in heliocentric radial distance. The trace of these equations gives

$$\frac{\partial H^{\pm}_{ii}}{\partial t} + \underline{Z}^{\mp} \cdot \nabla H^{\pm}_{ii} + \left(\frac{\nabla \cdot \underline{U}}{2} \pm \nabla \cdot \underline{V}_A \right) H^{\pm}_{ii} + S^{\pm}_{ii} = NL^{\pm}, \quad (4)$$

where repeated indices are summed and we have defined $\underline{V}_A = \underline{B}_o/\sqrt{4\pi\rho}$ and $\underline{Z}^{\pm} = \underline{U} \pm V_A$.

These equations contain transport and expansion terms of the type obtained in WKB theory [e.g., Hollweg, 1973, 1974] and in Tu's model [Tu et al., 1984; Tu, 1988]. Numerical factors superficially differ from WKB theory due to carrying along the $1/\sqrt{4\pi\rho}$ factor with the magnetic field. Also present are the nonlinear terms designated as $NL^{\pm}$. However, completely new kinds of effects are included in the present model through the appearance of $S^{\pm}_{ii}$ which has the structure

$$S^{\pm}_{ii} = F^{\pm}_{ik} \left[\frac{\partial U_i}{\partial x_k} \pm \frac{1}{\sqrt{4\pi\rho}} \frac{\partial B_{oi}}{\partial x_k} \quad \delta_{ik} \frac{1}{2} \nabla \cdot \left(\frac{1}{2} \underline{U} \pm \underline{V}_A \right) \right] \quad (5)$$

where we have defined $F^{\pm}_{ik} = < z_i^{\pm\prime} z_k^{\mp} + z_i^{\pm} z_k^{\mp\prime} >$. Several features of $S^{\pm}_{ii}$ warrant mention. First, it involves the cross covariance tensors $F^{\pm}_{ik}$, which are in general completely independent of $H^{\pm}_{ii}$. Therefore, the model must be closed either by including additional dynamical equations for $F^{\pm}_{ik}$, or by adopting approximations that relates $F^{\pm}_{ik}$ to $H^{\pm}_{ii}$. The "cross-tensors" $F^{\pm}_{ik}$ have the physical interpretation as the correlation of inward- and outward-type fluctuations. Such correlations may be generated either through nonlinear small scale effects, or by couplings to large scale gradients. In the most general case $F^{\pm}_{ik}$ can introduce effects such as nonequipartition of kinetic and magnetic energies, as well as an electric field related to dynamo activity and differences in the helicities of the magnetic and velocity fields. Consequently, to include a broad class of phenomena in this kind of model, we need to resort to either complex closures of a two-equation model (for H^{+}_{ii} and H^{-}_{ii}), or extend the model to include time advancement of additional fundamental tensors, such as $F^{\pm}_{ik}$.

In spite of the above complexities, it is possible to deduce some simple but relevant physics from a two equation model. Specifically, we consider the Fourier tranforms of the trace of the equations for $H^{\pm}_{ii}$, arriving at

$$\frac{\partial P^{\pm}}{\partial t} + \underline{z}^{\mp}\cdot\nabla P^{\pm} + \left(\frac{\nabla\cdot\underline{U}}{2} \pm \nabla\cdot\underline{V}_A \right) P^{\pm} + C^{\pm} = \frac{\partial G^{\pm}}{\partial k} \qquad (6)$$

where $C^{\pm}$ is the trace of the Fourier transform of $S^{\pm}_{ii}$. The term on the right hand side of (6) is the Fourier transform of the nonlinear term in (4), representing the flux of ± "energy" due to small scale turbulence. $G^{\pm}$ may be modeled, for isotropic inertial range turbulence, according to either Kolmogoroff [Batchelor, 1970] or Kraichnan [1965] phenomenology [e.g., Tu et al, 1984; Tu, 1988]. Since $P^{\pm}(k)$ is the spectrum of $z^{\pm 2}$, these equations contain information about the radial and time dependence of the fluctuation energy and cross helicity spectra, provided that the appropriate boundary and initial data are supplied at some surface near the sun (say, at $r = r_o$). Each of these equations is similar in structure to the single equation in the Tu model, but also includes a possibly wide variety of new effects through the new term $C^{\pm}$. Notably, couplings of the turbulence to the large scale fields and their spatial derivatives can easily be produced by this new term, whenever the z^{+} and z^{-} become correlated with one another, as can be seen from the following highly simplified example.

Example: Generation of "Inward" Waves

Consider the approximation $C^{\pm} = \beta(P^{+}+P^{-})(\nabla\cdot\underline{U})$. This amounts to a closure of the model by approximating the diagonal components of the cross covariance $F^{\pm}_{ik}$ in terms of a quantity proportional to the energy spectrum, while ignoring other elements of $F^{\pm}_{ik}$. We have calculated an approximate value of β based on the structure of an isotropic $k^{-3/2}$ energy spectrum, using standard techniques of homogeneous turbulence theory [Batchelor, 1970]. We find that

$$\beta = \frac{3}{28}\,\frac{r_A(k)-1}{r_A(k)+1}\;.$$

We assume the condition that $\beta<0$, which can be analytically motivated, as will be discussed in a subsequent paper. However, $\beta<0$ is also equivalent to the property that the Alfvén ratio $r_A(k) = E_V(k)/E_B(k)$ is less than unity in the inertial range, which is consistent with observations [Roberts et al., 1987b]. Notice that, in general, there is an implicit restriction on the value of β, since whenever $|\sigma_c|\to 1$, r_A also $\to 1$ and $\beta\to 0$.

For a first case consider the leading order effects when $V_A \ll U$. We drop terms of order V_A/U, and assume that $\underline{U}$ represents a spherically symmetric, constant wind. We also drop the nonlinear terms $\partial G^{\pm}/\partial k$ for illustration in the present linear, leading order calculation. With these assumptions, the steady state solution to the two equation model (Eq. 6) is

$$P^{+} + P^{-} = (P_0^{+} + P_0^{-}) \left[\frac{r_0}{r} \right]^{1+4\beta}$$

and

$$P^{+} - P^{-} = (P_0^{+} - P_0^{-}) \left[\frac{r_0}{r} \right]$$

where $P_0^{\pm}$ denotes the specified time independent value of the spectra at the inner boundary at r_o

To examine the behavior of the relative admixture of inward and outward-type fluctuations, we look at the inertial range fractional cross helicity $\sigma_c(k) = 2H_c(k)/E(k) = (P^{+}-P^{-})/(P^{+}+P^{-})$. From the above solution we see that

$$\sigma_c(k) = \sigma_{co} \left[\frac{r_o}{r} \right]^{-4\beta} \to 0 \text{ as } r\to\infty,$$

where σ_{co} is the value of σ_c at $r = r_o$. Thus, whenever $\beta < 0$, any initial overpopulation of "outward traveling waves", will decay as a powerlaw in heliocentric distance, as the "energies" in z^{+} and z^{-} approach equality. Using $r_A(k) \approx 0.5$ (Matthaeus and Goldstein, 1982a; Roberts et al. 1987a,b), this gives, for the leading order heliocentric distance dependence of the spectrum of fluctuating energy per volume, $W(k) = \rho(P^{+} + P^{-}) \propto r^{-20/7}$, and for the normalized cross helicity spectrum, $\sigma_c \propto r^{-1/7}$. This appears to be roughly consistent with the observed decay of the preponderance of outward traveling waves [Roberts et al., 1987a,b], and indicates that the effect can be produced by couplings to the large scale fields. In the present highly simplified case, this coupling is solely due to the expansion of $\underline{U}$, and may be considered to be a linear effect. The mechanism for the effect is simple: Couplings to the expansion delay the decay of the energy spectrum relative to the lowest order WKB ($W(k) \propto r^{-3}$, $(P^{+} + P^{-}) \propto r^{-1}$) expectation, presumably due to reflection from the large scale density gradient. Thus, relative to WKB, energy is locally built-up. However, the rate of radial decrease of cross helicity is unchanged by the new effect, and one can think of the new coupling term as injecting no net H_c. Thus, equal amounts of the two z fields are injected, and eventually the ratio of the two kinds of waves approaches equality. The addition of dissipation terms that are symmetric [Tu et al., 1984] in $P^{\pm}$ may slow this effect, but cannot eliminate it if the driving is strong enough.

The production of mixed cross helicities in the above three dimensional expansion-driven example appears to be related to the one dimensional mechanisms described by Heinemann and Olbert [1980] and Hollweg and Lilliequist [1978]. We are primarily interested in extensions of the present example to the case of sheared large scale solar wind flows. In such cases, the couplings to the large scale fields would be through the large scale vorticity, which might be written as $|\nabla\times\underline{U}|\approx\Delta U/Z$. It is reasonable to estimate $\Delta U\approx 200$km/sec as the velocity jump near the lateral boundary of a high speed stream, while Z, the thickness of the vorticity layer, is no more than the half width of the stream, and should be much less than the local heliocentric coordinate r. This type of shear term in the transport equations is expected to be of significance compared to the above treated expansion term, which is proportional to $\nabla \cdot \underline{U} = 2U/r$. Consequently, it seems almost certain that inward- and outward-type fluctuations will be "mixed" when there are strong shear-type gradients in the large scale flow. We have not yet attempted to solve the realistic sheared $\underline{U}$ case, owing to the difficulties introduced by the lack of isotropy.

In conclusion, we have presented the simplest form of a coupled transport equation model for the radial evolution of solar wind fluctuations, fashioned after two-length scale expansions of the type used in turbulence modeling theory. We find that when the model includes at least two transport equations, it can qualitatively account for observations of the radial dependence of the interplanetary cross helicity. Moreover, we have argued that several different types of couplings, including those involving expansion and shear can contribute to the mixing of cross helicity states in solar wind fluctuations. The most notable consequence of the present simple example is that inward-type fluctuations are generated from an initially dominant outward-type population at a reasonably rapid rate, solely due to linear couplings that appear in the lowest order versions of the present model. The couplings that produce this effect are absent in WKB-like expansions. Since the relationship of our procedure to WKB theory is both poorly understood and controversial, further investigation along these lines is needed. We envision that further extensions and applications of this model may provide a tool that complements observational and direct simulation studies in our future attempts to understand more fully the phenomenon of interplanetary MHD turbulence.

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