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# Scale Interactions in Magnetohydrodynamic Turbulence

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## Keywords

magnetohydrodynamics, modeling, simulation, isotropy, universality

## Abstract

This article reviews recent studies of scale interactions in magnetohydrodynamic turbulence. The present-day increase of computing power, which allows for the exploration of different configurations of turbulence in conducting flows, and the development of shell-to-shell transfer functions, has led to detailed studies of interactions between the velocity and the magnetic field and between scales. In particular, processes such as induction and dynamo action, the damping of velocity fluctuations by the Lorentz force, and the development of anisotropies can be characterized at different scales. In this context we consider three different configurations often studied in the literature: mechanically forced turbulence, freely decaying turbulence, and turbulence in the presence of a uniform magnetic field. Each configuration is of interest for different geophysical and astrophysical applications. Local and nonlocal transfers are discussed for each case. Whereas the transfer between scales of solely kinetic or solely magnetic energy is local, transfers between kinetic and magnetic fields are observed to be local or nonlocal depending on the configuration. Scale interactions in the cascade of magnetic helicity are also reviewed. Based on the results, the validity of several usual assumptions in hydrodynamic turbulence, such as isotropy of the small scales or universality, is discussed.

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# 1. INTRODUCTION

Turbulence is a multiscale phenomenon ubiquitous in geophysical and astrophysical flows. In many of these flows, the coupling of a conducting fluid with electromagnetic fields requires consideration of the magnetohydrodynamic (MHD) equations (see, e.g., Moffatt 1978). The equations describe the dynamics of nonrelativistic conducting fluids as, e.g., in the Earth's core or in industrial applications, and under some approximations they can also describe the large-scale behavior of magnetospheric, space, and astrophysical plasmas. In these latter cases, care must be taken to consider only the scales in which a one-fluid approximation holds, as scales small enough may require consideration of kinetic plasma effects such as ambipolar diffusion in weakly ionized plasmas as the interstellar medium, or the Hall current for highly ionized media such as small scales in the solar wind. However, in those cases the MHD equations still give a good description of large scales, and the approximation gives a useful approach to obtain lowest-order physical insight into the fate of the flows.

In the simplest case, that of an incompressible flow with constant mass density, the equations give the evolution of the bulk fluid velocity **u** and of the magnetic field **b**:

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \mathbf{b} \cdot \nabla \mathbf{b} + \nu \nabla^2 \mathbf{u}, \tag{1}$$

$$\partial_t \mathbf{b} + \mathbf{u} \cdot \nabla \mathbf{b} = \mathbf{b} \cdot \nabla \mathbf{u} + \eta \nabla^2 \mathbf{b}, \tag{2}$$

where the magnetic field is written in Alfvénic units, the density is set to unity, and $p$ is the (fluid plus magnetic) pressure. The kinematic viscosity $\nu$ and magnetic diffusivity $\eta$ control the viscous and Ohmic dissipation, respectively. These equations are constrained by the incompressibility condition and by the solenoidal character of the magnetic field,

$$\nabla \cdot \mathbf{u} = 0, \quad \nabla \cdot \mathbf{b} = 0. \tag{3}$$

Two different Reynolds numbers can be defined in MHD flows: the mechanical Reynolds number

$$Re = \frac{UL}{\nu}, \tag{4}$$

which is the ratio of convective to viscous forces (where $U$ is the root-mean-square velocity and $L$ is a characteristic length scale of the flow), and the magnetic Reynolds number

$$Rm = \frac{UL}{\eta}, \tag{5}$$

which can be interpreted as the ratio of induction to Ohmic dissipation. In many flows these Reynolds numbers are very large, and the flows are in a turbulent regime.

Whereas in the hydrodynamic case the phenomenological theory of Kolmogorov (K41) predicts to a good approximation (although without intermittency corrections) the power law of the energy spectrum, no clearly established phenomenological counterpart exists in MHD. This has many implications as the energy dissipation rate (required to predict, e.g., heating rates in solar and space physics) depends on the slope of the energy spectrum. Also, subgrid models, required to do numerical modeling in astrophysics and geophysics given the large scale separation involved in such flows, are less developed in MHD as a result of the lack of detailed knowledge of its energy spectrum.

In the Kolmogorov description of hydrodynamic turbulence, the interactions of similar-size eddies play the basic role of cascading the energy to smaller scales on a scale-dependent timescale $\tau_\ell \sim \ell/u_\ell$, where $\ell$ is the examined length scale and $u_\ell$ is the characteristic velocity at that scale. This timescale, which is proportional to the eddy turnover time at scale $\ell$, is the only

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timescale available on dimensional grounds in the inertial range, provided enough scale separation exists between forcing and dissipation. In this context, interactions between scales are local (in spectral space) as dominant interactions are between eddies of similar sizes. One then expects the statistical properties of sufficiently small scales to be independent of the way turbulence is generated and therefore to have universal character. Recent experiments showed deviations from this behavior even for simple hydrodynamic flows [e.g., slower than expected recovery of isotropy, or the presence of long-time correlations in the small scales (see Carlier et al. 2001; Poulain et al. 2006; Shen & Warhaft 2000; Wiltse & Glezer 1993, 1998)]. Numerical simulations also gave evidence of the presence of nonlocal interactions with the large-scale flow playing a role in the cascade of energy (Alexakis et al. 2005a, Domaradzki 1988, Domaradzki & Rogallo 1990, Zhou 1993). In numerical simulations with Reynolds numbers as high as $R_\lambda \approx 800$, it was observed that 20% of the energy flux in the small scales resulted from interactions with the large-scale flow (Mininni et al. 2006). However, more recent simulations with Reynolds numbers up to $R_\lambda \approx 1{,}300$ using spatial resolutions of $2{,}048^3$ grid points showed that as the Reynolds number is increased, the percentage of the nonlocal flux decreases as a power law of the Reynolds number, suggesting that the flux in hydrodynamic turbulence may be predominantly local for very large Reynolds numbers (Mininni et al. 2008). Recent theoretical results put this on firmer grounds (Aluie & Eyink 2009, Eyink & Aluie 2009), showing that the energy flux in hydrodynamic turbulence is local in the limit of infinite Reynolds number and obtaining bounds on the scaling of the nonlocal contribution to the flux with Reynolds number, which are in agreement with the numerical results.

The case for MHD turbulence is less clear and has given rise to more controversy. Several attempts have been made to extend the phenomenological arguments of Kolmogorov to conducting flows (see, e.g., Boldyrev 2006, Goldreich & Sridhar 1995, Iroshnikov 1963, Kraichnan 1965, Matthaeus & Zhou 1989). However, the MHD equivalent of the 4/5 law in hydrodynamic turbulence [the Politano-Pouquet relations (see Politano & Pouquet 1998a,b)] couples the velocity and the magnetic field in a way that can be compatible with several power-law behaviors; in three dimensions these relations read

$$\langle \delta z_{\parallel}^{\mp}(\mathbf{l}) |\delta \mathbf{z}^{\pm}(\mathbf{l})|^2 \rangle = -\frac{4}{3}\epsilon^{\pm} l, \tag{6}$$

where $\epsilon^{\pm}$ are the dissipation rates of the Elsässer variables $\mathbf{z}^{\pm} = \mathbf{u} \pm \mathbf{b}$, and the subindex $\parallel$ denotes the increment of the field along the displacement vector $\mathbf{l}$.

Moreover, even in the simplest incompressible case, at least two timescales can be identified in the inertial range of MHD turbulence. Besides the eddy turnover time, incompressible MHD flows are also characterized by the period of Alfvén waves $\tau \sim (B_0 L)^{-1}$, where $B_0$ is the amplitude of the large-scale magnetic field in Alfvénic units. In a first attempt to derive a phenomenological theory, Iroshnikov (1963) and Kraichnan (1965) assumed that the large-scale magnetic field acts as a uniform field for the small-scale fluctuations, which then behave as Alfvén waves. In that case, small scales can interact not only through the eddies but also through Alfvén packages, which reduce the energy flux to small scales by increasing its transfer time. This introduces in practice a nonlocal interaction as the waves propagate along the large-scale field (see Gomez et al. 1999 for a discussion). From dimensional analysis, Iroshnikov and Kraichnan then derived an isotropic energy spectrum proportional to $k^{-3/2}$. Later, extensions were considered to take into account the anisotropy induced at small scales by the large-scale magnetic field (Boldyrev 2006; Galtier et al. 2000, 2005; Goldreich & Sridhar 1995). Some of these extensions, after accounting for the anisotropy, rely on some form of a balance between the two fields that leaves only the turnover time as the relevant timescale and can therefore be considered local or nonlocal, depending on the authors.

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At the core of the early disquisitions is the fact that in MHD the roles of a large-scale flow and of a large-scale magnetic field are different. Whereas a (uniform) large-scale flow can be removed by a Galilean transformation, a large-scale magnetic field cannot. As a remarkable coincidence, the lack of Galilean invariance is at the basis of the $\sim k^{-3/2}$ spectrum for hydrodynamic turbulence within the framework of the direct interaction approximation by Kraichnan (1959), a flaw later corrected by the development of the test field model and the Lagrangian history direct interaction approximation. However, in MHD magnetic fields are not Galilean invariant, and for this reason the associated Alfvén waves have to be taken into account in phenomenological theories and are also considered when studying nonlocal effects in the eddy-damped quasi-normal Markovian closure (Pouquet et al. 1976) or in weak turbulence theory (Nazarenko et al. 2001). However, although phenomenological descriptions assume that a large-scale field has the effect of reducing the energy cascade rate, the transfer of energy (and the cascade) in many cases still takes place between eddies of similar size, presumably allowing for recovery of universal statistical properties at small scales.

In recent years, this universal behavior has been questioned by different authors. Because energy can be injected in MHD by a mechanical forcing or by an electromotive forcing, MHD turbulence is characterized by a larger number of regimes than hydrodynamic turbulence even in its simplest configurations. Magnetic fields in planets and stars are believed to be generated by dynamo action, in which turbulent motions sustain magnetic fields against Ohmic dissipation (Brandenburg & Subramanian 2005, Krause & Raedler 1980, Moffatt 1978, Pouquet et al. 1976). This regime is often studied in numerical simulations [and recently in experiments (see Monchaux et al. 2007)] by mechanically stirring the flow. Depending on the amount of mechanical helicity in the flow (the alignment between the velocity and the vorticity), or in the presence of large-scale shear, the magnetic field generated may have large- or small-scale correlation (compared with the integral scale of the flow), giving a steady state that may be dominated by mechanical or magnetic energy. Alternatively, plasmas in the solar corona and in the solar wind are dominated by magnetic energy and are often studied numerically by stirring the flow with electromotive forces or using simulations of freely decaying turbulence. Finally, the amount of cross-correlation between the velocity and magnetic fields depends on the flow (e.g., on the heliocentric distance in the solar wind) and can also be varied in the simulations.

The questioning of universality was accompanied by recent detailed studies of scale interactions in MHD turbulence. Many of the studies considered the so-called shell-to-shell transfer functions and partial energy fluxes, either in numerical simulations, observations, and closures or from the theoretical point of view. In the following sections we review the results in this area, considering the several regimes studied by different authors, and also some examples of possible sources of nonlocality in MHD. Finally, we discuss the results in the context of universality and of phenomenological theories for MHD. To briefly summarize the results, several authors have shown that the locality of energy transfer is in question in MHD flows. In particular, it was shown from simulations that the transfer of energy in MHD has two components: a local one that shares similar properties with hydrodynamic turbulence and a component coupling the velocity and magnetic fields for which energy from the large scales can be, under some circumstances, injected directly into the small scales without the intervention of intermediate scales.

## 2. INDIRECT EVIDENCE OF NONLOCALITY

Some theoretical, phenomenological, and (more recently) numerical results indicate that scale interactions in MHD can be, under some conditions, of a different nature than in hydrodynamic turbulence. In this section we review early theoretical indications of nonlocality in MHD turbulence, as well as numerical results that support the theoretical arguments without directly measuring scale interactions.

Early studies of dynamo action, and of magnetic field evolution under flows with simple strain, show that a large-scale flow can excite, through field line stretching, magnetic fields at widely separated scales. One of the first works along these lines is Batchelor (1950) in which he considered the similarity between the induction equation and the vorticity equation ($\boldsymbol{\omega} = \nabla \times \mathbf{u}$):

$$\frac{\partial \boldsymbol{\omega}}{\partial t} + \mathbf{u} \cdot \nabla \boldsymbol{\omega} = \boldsymbol{\omega} \cdot \nabla \mathbf{u} + \nu \nabla^2 \omega. \tag{7}$$

Whereas the second term on the left-hand side advects the vorticity, the first term on the right-hand side (in three dimensions) produces vorticity by vortex stretching. For $P_M = \eta/\nu > 1$ (the magnetic Prandtl number), Batchelor then concluded that the magnetic field would grow as magnetic field–line stretching overcomes Ohmic dissipation. Later works considering stretching by uniform straining motion (Moffatt & Saffman 1964, Zel'Dovich et al. 1984) showed that a large-scale magnetic field can directly create small-scale magnetic fields. The work of Kazanstev (1968) considered a similar process under a random velocity field and described a nonlocal coupling that sustains the so-called small-scale dynamo, in which magnetic fields are amplified at scales smaller than the integral scale of the flow. Several numerical simulations support these results and show that smooth motions at the viscous scale give exponential growth of magnetic fields that can peak at the magnetic diffusion scale (Schekochihin et al. 2002a,b, 2004).

The opposite limit, when the magnetic Prandtl number is much smaller than unity (a case of interest for industrial flows), is sometimes studied using the quasi-static approximation (see Knaepen & Moreau 2008 for a review). In this case, an external uniform magnetic field is applied, and the magnetic Reynolds number is chosen small enough that magnetic field fluctuations are rapidly damped. In that limit the Lorentz force in the momentum equation reduces to linear Joule damping

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \sigma B_0^2 \nabla^{-2} \frac{\partial^2 \mathbf{u}}{\partial z^2} + \nu \nabla^2 \mathbf{u}, \tag{8}$$

where $\sigma$ is the conductivity of the medium, and the uniform magnetic field $B_0$ is in the $z$ direction. The Joule damping, although anisotropic in spectral space, is roughly independent of the wave number and, unlike viscous damping, is not concentrated at small scales but rather acts at all scales. As a result, the large-scale magnetic field in this approximation exerts work over all scales in the velocity field (damping turbulent fluctuations) in a nonlocal way. We see below that the shell-to-shell transfers indicate in some cases similar behavior of the Lorentz force even in cases far from this approximation.

Another important example concerns Alfvén waves, which are also nonlinear solutions of the ideal MHD equations. Alfvénic states with $\mathbf{u} = \pm \mathbf{b}$ make the nonlinear terms in Equations 1 and 2 zero, leaving only interactions with the large-scale fields to transport energy across scales. Finally, it is worth mentioning here some recent attempts to build shell models of MHD turbulence (see, e.g., Lessinnes et al. 2009, Plunian & Stepanov 2007, Stepanov & Plunian 2008). In these models, it was found that many features of steady-state MHD turbulence can be reproduced using local coupling between shells, but nonlocal transfers have to be considered to reproduce the small-scale dynamo and turbulence at $P_M \gg 1$ (Stepanov & Plunian 2008).

## 3. THE SHELL-TO-SHELL TRANSFER

In recent years, the increase in computing power has allowed numerical exploration of MHD turbulence in different regimes. The development of shell-to-shell transfers (see Alexakis et al.



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2005b, Dar et al. 2001, Debliquy et al. 2005) allowed for explicit computation of detailed scale interactions in MHD turbulence using the output stemming from the simulations and without the need to compute the more expensive triadic interactions. In this section we briefly introduce the isotropic shell-to-shell energy transfer functions and describe how fluxes can be obtained from them.

A shell filter decomposition of the two fields is introduced as

$$\mathbf{u}(\mathbf{x}) = \sum_K \mathbf{u}_K(\mathbf{x}), \tag{9}$$

$$\mathbf{b}(\mathbf{x}) = \sum_K \mathbf{b}_K(\mathbf{x}), \tag{10}$$

where

$$\mathbf{u}_K(\mathbf{x}) = \sum_{K_1 < |\mathbf{k}| \le K_2} \tilde{\mathbf{u}}(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}} \tag{11}$$

and

$$\mathbf{b}_K(\mathbf{x}) = \sum_{K_1 < |\mathbf{k}| \le K_2} \tilde{\mathbf{b}}(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}}. \tag{12}$$

Here $\tilde{\mathbf{u}}(\mathbf{k})$ and $\tilde{\mathbf{b}}(\mathbf{k})$ are the Fourier transforms of the velocity and magnetic fields with wave number $\mathbf{k}$, respectively. The shell-filtered fields $\mathbf{u}_K$ and $\mathbf{b}_K$ are therefore defined as the field components whose Fourier transforms contain only wave numbers in a given shell $K$. These shells can be defined with linear binning using $K_1 = K$ and $K_2 = K + 1$, or alternatively with logarithmic binning using $K_1 = \gamma^n K_0$ and $K_2 = \gamma^{n+1} K_0$ for some positive $\gamma > 1$ and integer $n$ ($\gamma = 2$ is often used). The latter definition has the advantage of being conceptually closer to the idea of scale of eddies in turbulence, which in general implies the logarithmic division of wave numbers. The former has the advantage of having a direct association with Alfvén waves, which are of the form $\mathbf{u} = \pm\mathbf{b} \sim e^{i(\mathbf{k}\cdot\mathbf{x}\pm\omega t)}$ in periodic boxes or in infinite domains, and which are more akin to the linear treatment of spectral space. Note that the transfer among logarithmic shells can be reconstructed by summing over the linearly spaced shells.

Another variant when defining the shell filter decomposition has to do with the choice of using sharp filters (as in the equations above) or smooth filters (Eyink 1994, 2005). This issue has raised some controversy in the hydrodynamic case, with claims that nonlocalities observed in simulations may result from the commonly used sharp filters. Recent numerical comparisons (Domaradzki & Carati 2007a,b) have shown that results are only weakly dependent on the shape of the filter used, except in the case in which a very broad smooth filter is considered. Moreover, recent theoretical results were able to show locality of hydrodynamic turbulence in Fourier space in the limit of infinite Reynolds number for both smooth and sharp filters (Aluie & Eyink 2009, Eyink & Aluie 2009).

Based on the shell filter decomposition, the evolution of the kinetic energy in a shell $K$, $E_u(K) = \int \mathbf{u}_K^2/2\, dx^3$, can be derived from Equation 1 as

$$\frac{\partial E_u(K)}{\partial t} = \sum_Q [T_{uu}(Q, K) + T_{bu}(Q, K)] - \nu D_u(K), \tag{13}$$

and for the magnetic energy, $E_b(K) = \int \mathbf{b}_K^2/2\, dx^3$, from Equation 2 as

$$\frac{\partial E_b(K)}{\partial t} = \sum_Q [T_{bb}(Q, K) + T_{ub}(Q, K)] - \eta D_b(K), \tag{14}$$



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where the functions $D_u(K)$ and $D_b(K)$ express the kinetic and magnetic energy dissipation in shell $K$, respectively. The transfer functions $T_{uu}(Q, K)$, $T_{ub}(Q, K)$, $T_{bb}(Q, K)$, and $T_{bu}(Q, K)$ that express the energy transfer between different fields and shells are given by

$$T_{uu}(Q, K) = -\int \mathbf{u}_K(\mathbf{u} \cdot \nabla)\mathbf{u}_Q \, dx^3, \tag{15}$$

$$T_{bu}(Q, K) = \int \mathbf{u}_K(\mathbf{b} \cdot \nabla)\mathbf{b}_Q \, dx^3, \tag{16}$$

$$T_{bb}(Q, K) = -\int \mathbf{b}_K(\mathbf{u} \cdot \nabla)\mathbf{b}_Q \, dx^3, \tag{17}$$

$$T_{ub}(Q, K) = \int \mathbf{b}_K(\mathbf{b} \cdot \nabla)\mathbf{u}_Q \, dx^3. \tag{18}$$

The function $T_{uu}(Q, K)$ measures the transfer rate of kinetic energy in shell $Q$ to kinetic energy in shell $K$ due to the advection term in the momentum equation given in Equation 1. This is the nonlinear transfer that is also present in hydrodynamic turbulence. Similarly, $T_{bb}(Q, K)$ expresses the rate of magnetic energy transferred from shell $Q$ to magnetic energy in shell $K$ due to the magnetic advection term. The Lorentz force is responsible for the transfer of energy from the magnetic field in shell $Q$ to the velocity field in shell $K$, as measured by $T_{bu}(Q, K)$. Finally, the term responsible for the stretching of magnetic field lines, the first term on the right-hand side of Equation 2, results in the transfer of kinetic energy from shell $Q$ to magnetic energy in shell $K$ and is expressed by $T_{ub}(Q, K)$. This is the term that describes magnetic induction and dynamo action.

All these transfer functions satisfy

$$T_{vw}(Q, K) = -T_{wv}(K, Q), \tag{19}$$

where the subindices $v$ and $w$ stand for $u$ or $b$. This expression indicates that the rate at which shell $Q$ gives energy to shell $K$ is equal to the rate shell $K$ receives energy from shell $Q$, and is a necessary condition to define shell-to-shell transfers that satisfy a detailed energy balance between shells. Then, the contribution of these transfers to the total energy flux can be computed as

$$\Pi_{vw}(k) = -\sum_{K=0}^{k} \sum_{Q} T_{vw}(K, Q). \tag{20}$$

Besides the total energy, the MHD equations have two more ideal invariants: the cross-helicity $C = \int \mathbf{u} \cdot \mathbf{b} \, dx^3$ and the magnetic helicity $H = \int \mathbf{a} \cdot \mathbf{b} \, dx^3$, where $\mathbf{a}$ is the vector potential such as $\mathbf{b} = \nabla \times \mathbf{a}$. These quantities also satisfy detailed balance equations equivalent to Equations 13 and 14. Shell-to-shell transfer functions for the magnetic helicity have been defined in Alexakis et al. (2006). Its transfer from shell $Q$ to shell $K$ is measured by

$$T_H(K, Q) = \int \mathbf{b}_K \cdot (\mathbf{u}_K \times \mathbf{b}_Q) \, dx^3. \tag{21}$$

The energy transfer functions were also generalized in recent works to consider the flux of energy in terms of the Elsässer variables (Alexakis et al. 2005b, 2007a; Carati et al. 2006), anisotropic transfers (Alexakis et al. 2007a, Teaca et al. 2009), forward and backward transfers in an attempt to quantify the backscatter required for subgrid models (Carati et al. 2006, Debliquy et al. 2005), extensions to consider compressibility effects (Graham et al. 2010) and kinetic plasma effects as in two-fluid MHD approximations (Mininni et al. 2007).

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## 4. DIRECT STUDIES OF MULTISCALE INTERACTIONS

The shell-to-shell energy transfers have been studied extensively (Alexakis et al. 2005b, Carati et al. 2006, Dar et al. 2001, Debliquy et al. 2005, Mininni et al. 2005a, Verma 2004) for a variety of mechanically forced and decaying MHD flows in two and three dimensions. Depending on the configuration, different degrees of nonlocality were reported. In the following subsections we present a short summary of the results discriminating by the forcing configuration. Overall, we can say that in all cases examined in the literature the transfers $T_{uu}$ and $T_{bb}$ have a local behavior: Energy is transferred forward between nearby shells, in a fashion similar to what is observed in hydrodynamic turbulence (Alexakis et al. 2005a, Domaradzki & Rogallo 1990, Mininni et al. 2006, Ohkitani & Kida 1992, Yeung et al. 1995, Zhou 1993). Alternatively, the $T_{bu}$ and $T_{ub}$ transfers that express the energy exchange between the velocity and the magnetic field have a rather different behavior.

### 4.1. Forced Isotropic and Homogeneous Turbulence

As mentioned above, forced simulations of MHD turbulence can be attained by forcing both fields, or by forcing only the velocity field (in which case magnetic fields are sustained by dynamo action; a distinction must be made then between the kinematic regime, where the magnetic field has no backreaction on the flow, and the turbulent steady state, where the magnetic field modifies the flow through the Lorentz force). The mechanically forced case is of more interest as it is closer to astrophysical and geophysical configurations, and as it is consistent with the constraint of magnetic flux conservation. The first studies of shell-to-shell transfer from simulations in such a configuration were presented in Alexakis et al. (2005b). In the simulations with a resolution of $256^3$ grid points, the velocity field was forced with time-independent mechanical forcing until a hydrodynamic turbulent steady state was reached. Two different forcing functions were studied: one nonhelical and one helical. Magnetic Prandtl numbers of unity and smaller were considered. Once a hydrodynamic steady state was reached for each forcing function, a small magnetic field was introduced, and after the transient kinematic dynamo amplification, the system reached a steady-state MHD turbulent regime. In such a state the transfer functions described in the previous section were computed. Typical results are illustrated in **Figure 1**.

The $T_{uu}$ and $T_{bb}$ transfers were observed to behave in a similar fashion, giving direct and local transfer of energy. In **Figure 1**, this is indicated by the negative and positive peaks, which show that energy is removed by these transfer functions from smaller wave numbers and given to slightly larger wave numbers. However, for $T_{ub}$ a distinct behavior appeared. The large-scale flow injected energy (through stretching) directly into the magnetic field at all scales. This manifests itself as a peak at the mechanical forcing scale for all receiving shells, and as an extended positive plateau (note that positive $T_{ub}$ indicates energy given by the velocity field at shell $Q$ to magnetic field at shell $K$). In other words, at a given shell $K$, the magnetic field receives energy from the velocity field in all shells $Q < K$ and gives energy to the velocity field in shells $Q > K$. This result, reminiscent of the theoretical arguments by Batchelor (1950) and Zel'Dovich et al. (1984), was interpreted as the sustainment of the magnetic field against Ohmic dissipation by dynamo action: To maintain the magnetic field when only the velocity field is stirred, a nonzero flux from the velocity field to the magnetic field is required at all times. It is worth pointing out here that in the steady state this nonlocal transfer is small compared with the local transfers (approximately 10%–20% at the resolutions studied). When considering Elsässer variables, the transfer functions were observed to become more local.

The case of random forcing with magnetic Prandtl number of unity was studied in Carati et al. (2006) using $512^3$ simulations. The analysis, which used logarithmic binning, confirmed the previous results, showing local transfer in $T_{uu}$ and $T_{bb}$, and nonlocal coupling between the velocity

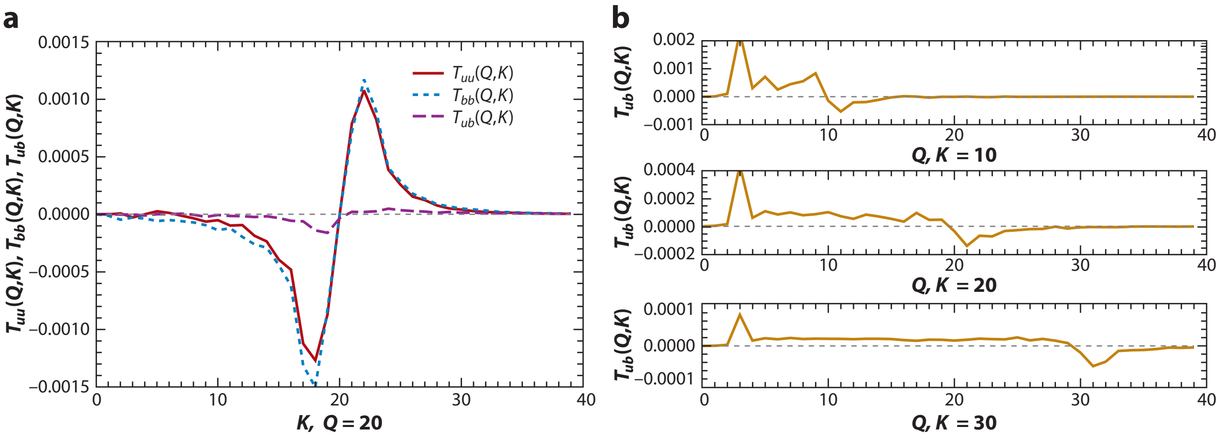


**Figure 1**

(*a*) Transfer functions in mechanically forced magnetohydrodynamic turbulence, for $Q = 20$. The $T_{uu}$ and $T_{bb}$ functions are local, with a negative peak for $K < Q$ and a positive peak for $K > Q$, which indicate that energy is removed by these transfer functions from smaller wave numbers and given to slightly larger wave numbers. The transfer between magnetic to kinetic energy is of smaller amplitude and also seems local. (*b*) The $T_{ub}$ transfer, for different values of $K$. This function is nonlocal, with a strong peak at the forcing scale and with a constant positive plateau that extends up to $K \approx Q$. Figure adapted from Alexakis et al. (2005b).

and the magnetic field. This indicates that the phenomenon may be independent of the type of forcing and associated with the stretching process that sustains the magnetic field. The work also discussed the possibility of splitting the transfer functions to discriminate between forward and backward contributions, which were used to discuss implications of the shell-to-shell transfers for large-eddy simulation models. Similar results were obtained for forced two-dimensional MHD turbulence (Dar et al. 2001).

A different approach was considered by Yousef et al. (2007), who studied the steady state of small-scale dynamo action for $P_M \leq 1$. Instead of using transfer functions to measure the different components of the energy flux, they considered the Politano-Pouquet relations given in Equation 6 in terms of the velocity and the magnetic field

$$\langle \delta u_\parallel (|\delta \mathbf{u}|^2 + |\delta \mathbf{b}|^2)\rangle \mp \langle \delta b_\parallel (|\delta \mathbf{u}|^2 + |\delta \mathbf{b}|^2)\rangle \pm 2\langle \delta \mathbf{u} \cdot \delta \mathbf{b}(\delta u_\parallel \mp \delta b_\parallel)\rangle = -\frac{4}{3}\epsilon^{\pm} l, \tag{22}$$

together with Chandrasekhar's (1951) law

$$\langle \delta u_\parallel^3 \rangle - 6\langle b_\parallel^2 \delta u_\parallel \rangle = -\frac{4}{5}\epsilon l, \tag{23}$$

where $\epsilon$ is the total energy flux. The authors discriminated between the different terms to see how they balanced to give rise to the direct flux. Each term in these expressions can indeed be associated with a counterpart in real space of the $\Pi_{uu}$, $\Pi_{bb}$, and $\Pi_{ub} + \Pi_{bu}$ fluxes in Fourier space.

The dominant balance was identified between $(4/5)\epsilon l$ and $6\langle b_\parallel^2 \delta u_\parallel \rangle$, and they concluded that, at their available resolution, the local direct cascade of energy was "short-circuited" by the transfer of kinetic energy into magnetic energy. They also associated this nonlocal coupling with the folded structure of the small-scale magnetic field. Using the shell-to-shell transfer approach, Alexakis et al. (2007a) further showed that the nonlocal effects disappear if phases are randomized for the two fields, which also make the current sheet and folded structures disappear.

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The nonlocal effects play a more important role in the kinematic dynamo regime (Mininni et al. 2005a), as in that case the turbulence is not in a steady regime and $T_{ub}$ accounts for all mechanisms that amplify the magnetic field. In that case, the $T_{ub}$ transfer has been shown to be useful in identifying and quantifying scale-by-scale sources of dynamo action (Alexakis et al. 2007b, Mininni et al. 2005a).

## 4.2. Freely Decaying Turbulence

The nonlocal effects observed in forced turbulence are either absent or negligible in the freely decaying case. Debliquy et al. (2005) considered $512^3$ simulations of freely decaying MHD turbulence. The $T_{uu}$ and $T_{bb}$ transfers are similar to the forced case (see **Figure 2**) and indicate local direct transfer. However, the $T_{ub}$ and $T_{bu}$ transfer functions were also observed to be local, with most of the transfer between the velocity and the magnetic field taking place between the same shell. The remaining transfer (for non-neighboring shells) was observed to decay more slowly than in the $T_{uu}$ and $T_{bb}$ functions; except for this detail, no other indications of nonlocality were reported.

Similar results were obtained from analysis of solar wind turbulence (Strumik & Macek 2008a,b). Solar wind turbulence is often considered the MHD equivalent of hydrodynamic freely decaying wind-tunnel turbulence (see Bruno & Carbone 2005 for a review). From 1996 Ulysses magnetometers time series and using a Markov process approach, Strumik & Macek (2008b) concluded that the transfer of magnetic to magnetic energy was local. Then, using velocity and magnetic field time series from ACE spacecraft from 1999 to 2006 and performing the same analysis on the remaining transfers, they concluded that all transfers were local.

The differences between the forced and freely decaying cases can be understood by noting that, in the mechanically forced runs, the velocity field has to continuously supply energy to the magnetic field to sustain it against Ohmic dissipation. This is not necessarily the case for freely decaying runs in which both fields are dissipated in time.

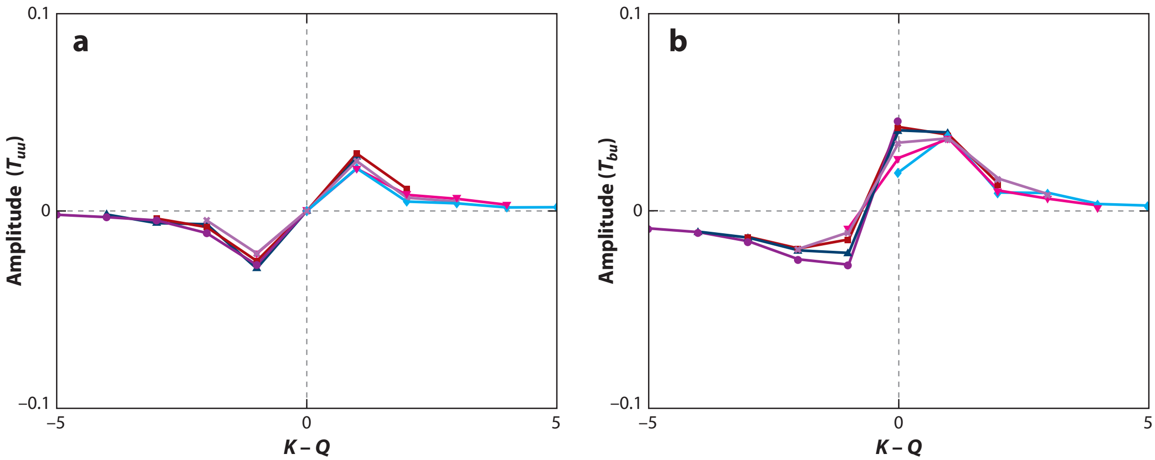


**Figure 2**

(*a*) $T_{uu}$ transfer function in freely decaying magnetohydrodynamic turbulence, for different shells. The $T_{bb}$ transfer function is similar but has twice the amplitude. (*b*) $T_{bu}$ transfer function in the same simulation. Note that the peak for $K - Q = 0$, indicating that most interchange of energy between the velocity and the magnetic field takes place between similar scales. The shells are logarithmically binned. Figure adapted from Debliquy et al. (2005).



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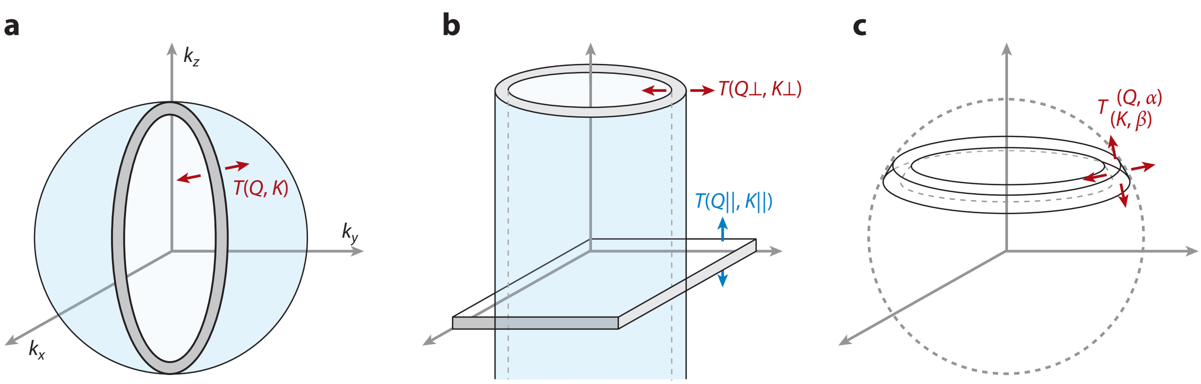


**Figure 3**

Isotropic (spherical) shells (*a*) and anisotropic foldings of shells in Fourier space. The uniform magnetic field is assumed to be in the $z$ direction. Cylindrical and planar shells are shown in panel *b*, and ring shells are shown in panel *c*. The transfer of energy across planes is denoted by $T(Q_\parallel, K_\parallel)$, and transfer across cylinders is denoted by $T(Q_\perp, K_\perp)$. For ring-to-ring transfers, the notation $T^{\{Q,\alpha\}}_{\{K,\beta\}}$ denotes that transfer can be measured between $K$ and $Q$ spherical shells, as well as between two azimuthal angles $\alpha$ and $\beta$.

## 4.3. Anisotropic Turbulence

Recently, the shell-to-shell transfers were extended to consider anisotropies when an external uniform magnetic field is imposed. This case is of interest as in many astrophysical problems a strong large-scale magnetic field is present, creating small-scale anisotropy. Unlike hydrodynamic turbulence, MHD turbulence does not recover isotropy at small scales, and theoretical and numerical results indicate that anisotropy becomes stronger at smaller scales.

To study anisotropic transfers, one can implement different foldings of the shells in Fourier space. **Figure 3** shows the possible options. Alexakis et al. (2007a) introduced anisotropic shell-to-shell transfer functions by folding Fourier shells in cylinders (associated with wave numbers $k_\perp$ perpendicular to the mean magnetic field) and in planes (associated with parallel wave numbers $k_\parallel$). Shell-to-shell transfers were only considered for the Elsässer variables, but the fluxes were reconstructed from these functions to measure the relative contribution of nonlocality to the total flux. Freely decaying simulations with spatial resolution of $256^3$ grid points were analyzed, and the amplitude of the imposed magnetic field was varied from 0 to 15 (in units of the initial small-scale fluctuations). The transfer functions of the two Elsässer energies were found local in both the parallel and perpendicular directions, irrespective of the amplitude of the external field. However, interactions between the counterpropagating Alfvén waves were reported to become nonlocal. For strong magnetic fields, most of the energy flux in the perpendicular direction was found to result from interactions with modes with $k_\parallel = 0$ (see **Figure 4**). In the parallel direction, however, $k_\parallel = 0$ modes cannot transfer energy, and most of the interactions were observed to take place with modes near $k_\parallel \approx 0$. The results are in qualitative agreement with predictions from weak turbulence theory (Galtier et al. 2000) and with recent nonlocal phenomenological models (Alexakis 2007).

A different approach to studying anisotropic transfers was presented by Teaca et al. (2009), who decomposed the spectral space into rings, studying then transfers along radial and angular directions in spectral space (which they termed "ring-to-ring" transfers). They considered forced simulations of MHD turbulence with an imposed magnetic field with a spatial resolution of $512^3$ grid points and varied the imposed magnetic field from 0 to $\sqrt{10}$ (in units of the small-scale magnetic field fluctuations). They also observed the dominance of energy transfer in the direction perpendicular to the uniform magnetic field and suppression of the transfer in the

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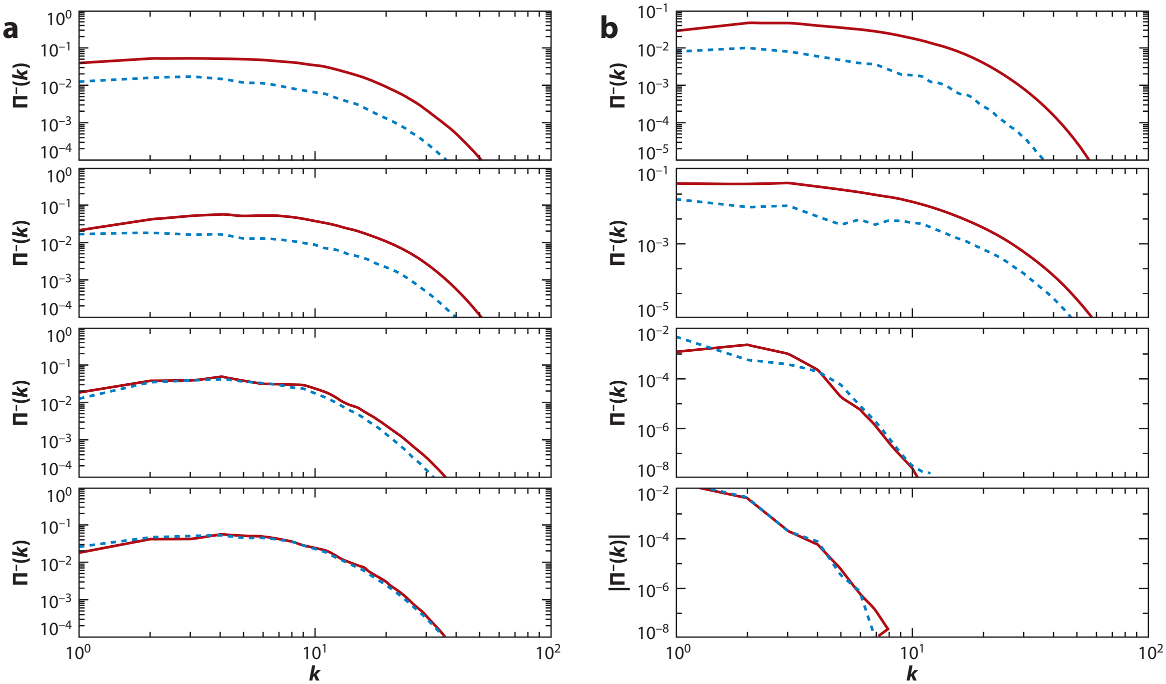


**Figure 4**

(*a*) Total energy flux (*solid red lines*) across cylinders and partial flux associated with interactions with modes with $k_{\parallel} = 0$ (*dashed blue lines*), with four different values of the external magnetic field $B_0$ from 0 to 15 (*from top to bottom*). (*b*) Same as in panel *a*, but with the total flux and partial flux associated with interactions with modes with $k_{\parallel} = 1$ across planes. Figure adapted from Alexakis et al. (2007a).

parallel direction. Their approach is useful to understand how energy is angularly distributed in spectral space to create anisotropy. Nonlocal effects with the forcing shell were observed in the shell-to-shell transfers, but in the angular ring-to-ring transfers they were too weak to be noticed.

## 4.4. Magnetic Helicity and the Inverse Cascade

Nonlocal transfers were also reported in investigations of the cascade of magnetic helicity. Magnetic helicity is an ideal invariant in MHD that is known to cascade inversely (to the large scales) in a turbulent flow (Alexakis et al. 2006, Brandenburg 2001, Brandenburg & Subramanian 2005, Gómez & Mininni 2004, Meneguzzi et al. 1981, Pouquet et al. 1976). The generation of large-scale magnetic fields in galaxies and other astrophysical bodies is sometimes attributed to this inverse cascade. Alexakis et al. (2006) considered magnetically and mechanically forced simulations. In both cases, both local and nonlocal transfers were observed. At early times, magnetic helicity was observed to cascade inversely and locally from the closest neighbor shells, and nonlocally from the forced shells. When the correlation length became the size of the box, the direct input from the forced scales became dominant, and a local direct transfer of helicity from large to small scales also developed. This latter effect was speculated to be dependent on boundary conditions and therefore nonuniversal.

In the mechanically forced case, the inverse cascade of helicity was associated with the large-scale dynamo $\alpha$-effect (Brandenburg 2001, Brandenburg & Subramanian 2005, Krause & Raedler 1980, Pouquet et al. 1976, Steenbeck et al. 1966). In that case, the mechanical forcing creates

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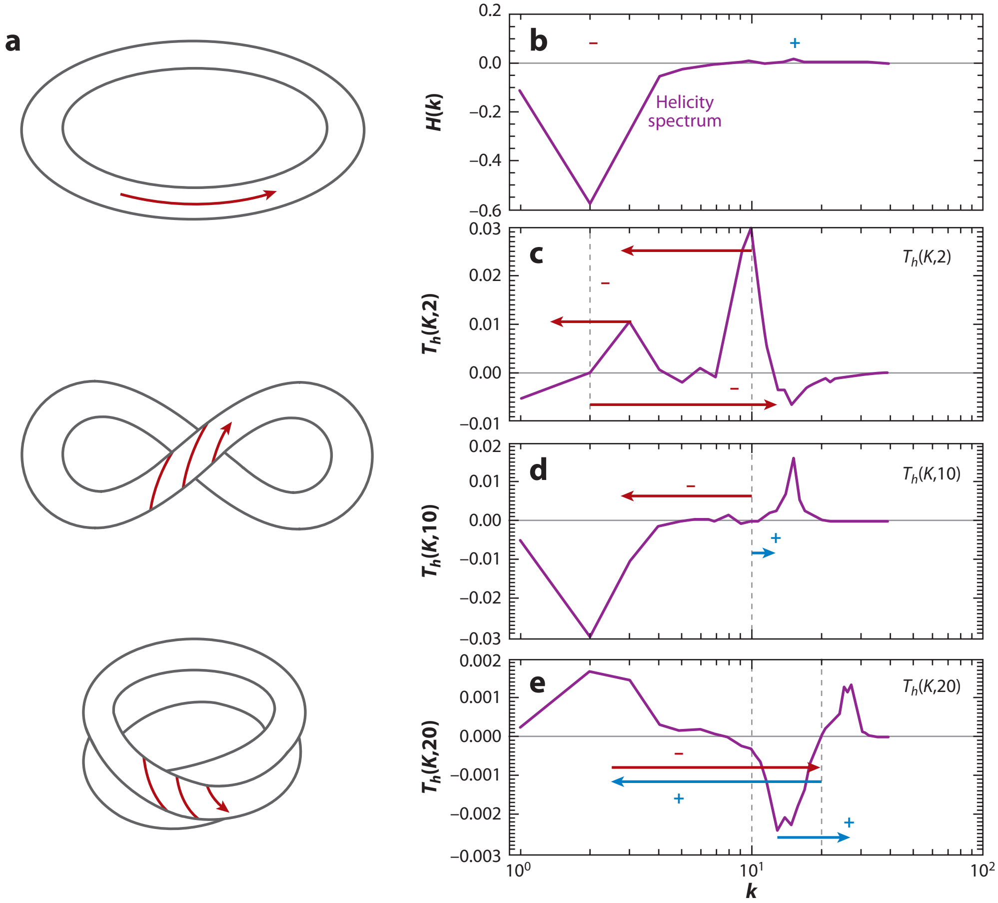


**Figure 5**

(*a*) The stretch, twist, and fold dynamo mechanism. Each time a closed magnetic flux tube is twisted, magnetic helicity of opposite sign is created at large and small scales. The folding creates regions where helical magnetic fields can reconnect. (*b*) The helicity spectrum in a simulation with (positive) helical mechanical forcing at $k = 10$. Magnetic helicity is negative at larger scales and positive at smaller scales. (*c–e*) The transfer of helicity for $Q = 2$, 10, and 20. The red arrows indicate the transfer of negative helicity, and the blue arrows represent the transfer of positive helicity. At large scales (*c*), negative magnetic helicity inversely cascades locally between neighboring shells and nonlocally from the forced shell to the small-scale shells. At the forced shell (*d*), the forcing injects opposite signs of helicity at large and small scales. At small scales (*e*), positive magnetic helicity has a local direct transfer of helicity, while the small scales also remove negative magnetic helicity from the large scales. Note that the direct transfer of negative helicity is equivalent to the inverse transfer of positive helicity.

equal amounts of magnetic helicity of opposite signs at large and small scales. The process can be understood using the conceptual stretch, twist, and fold dynamo mechanism (Childress & Gilbert 1995, Vainshtein & Zeldovich 1972). Each time a closed magnetic flux tube is twisted by the helical velocity field, magnetic helicity is created at large scales, while small-scale magnetic field lines are twisted in the opposite direction, thus creating an equal amount of magnetic helicity of opposite sign in the small scales. As the stretch, twist, and fold process is repeated, the large-scale helicity is transferred inversely both locally and nonlocally (with constant flux), while the small-scale helicity is pushed toward smaller scales (see **Figure 5**). This latter process removes magnetic helicity from the large scales and allows the magnetic field to disentangle through reconnection events, destroying magnetic helicity in that way (Alexakis et al. 2006, 2007b). It is presently unclear whether these processes should be associated with a cascade (i.e., if the process takes place with constant flux), although results in Alexakis et al. (2007b) and Mininni & Pouquet (2009) suggest this may not be the case.

## 5. NONLOCAL INTERACTIONS AND UNIVERSALITY OF MAGNETOHYDRODYNAMIC TURBULENCE

The above considerations led several authors to consider whether some of the usual assumptions in hydrodynamic turbulence hold in the MHD case. From the shell-to-shell transfer, the scenario pictured in **Figure 6** seems to arise for the energy: Interactions between the same fields are mostly local, and interactions between the velocity and the magnetic field can have different degrees of nonlocality depending on whether the turbulence is forced or freely decaying, depending on how the velocity and the magnetic fields are maintained against dissipation in the forced case, and depending on the presence of an external magnetic field. It is unclear presently whether the varying degree of nonlocality with the configuration will converge to a universal solution for very large Reynolds numbers.

Theoretical arguments considering interactions in MHD turbulence also obtained conflicting results. Using the eddy-damped quasi-normal Markovian closure, Pouquet et al. (1976) reported nonlocal interactions, which were associated with Alfvén waves. Verma (2003, 2004) and Verma et al. (2005) used field-theoretic calculations to compute the shell-to-shell transfers and concluded that they were local, except for the transfer between the velocity and the magnetic field, which was found to be somewhat nonlocal. The helicity transfer was also found to be nonlocal. Recently, Aluie & Eyink (2010) gave strict bounds for fluxes in MHD turbulence under the assumptions

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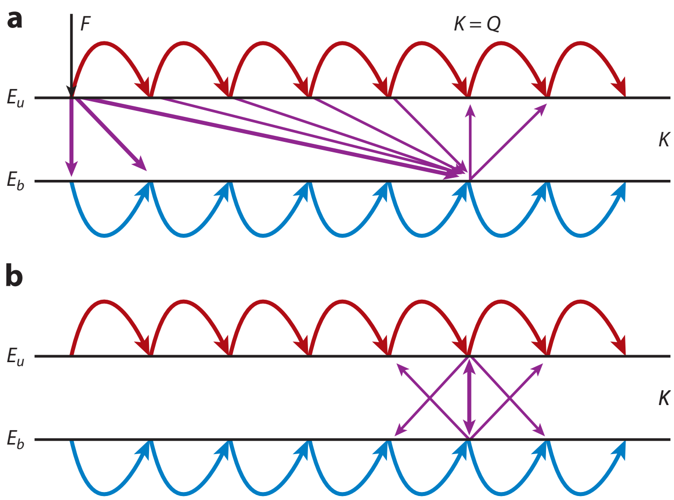


**Figure 6**

Sketch of the several shell-to-shell energy transfers identified in simulations of isotropic and homogeneous magnetohydrodynamic turbulence. The $T_{uu}$ transfers are shown in red, $T_{bb}$ transfers in blue, and $T_{ub}$ and $T_{bu}$ in purple. The thickness of the arrows roughly indicates the strength of the transfers. (*a*) Mechanically forced simulations. At shell $K$, the magnetic field receives energy from the velocity field at all larger scales and gives energy to the velocity field at slightly smaller scales. (*b*) Freely decaying turbulence. The $T_{ub}$ and $T_{bu}$ transfers only interchange energy between similar scales. In both cases, the $T_{uu}$ and $T_{bb}$ transfers are local and give the largest contribution to the flux.

that both the velocity and the magnetic energy follow power laws in the inertial range between $k^{-1}$ and $k^{-3}$. The velocity-to-velocity and magnetic-to-magnetic fluxes were found to be local in the limit of infinite Reynolds number, and the fluxes coupling velocity and magnetic fields were found to be local, although counterexamples to their proof (such as the ones mentioned in Section 2) were acknowledged. However, these results shed light on why some simulations were found to be more local than others, as mechanisms such as the small-scale dynamo can be expected to be less relevant in freely decaying turbulence in approximate equipartition between the two fields.

At presently attainable spatial resolutions, other indications of possible nonuniversal behavior have been reported in numerical simulations. Dmitruk et al. (2003) presented simulations of forced reduced MHD in which the energy spectrum changed its power law depending on the timescale of the external forcing. Spectra compatible with Kolmogorov, Iroshnikov-Kraichnan, and weak turbulence theory, or even steeper laws, were observed. The reduced MHD equations correspond to an approximation of the MHD equations when a strong external magnetic field is imposed. Similar results were reported by Mason et al. (2008), who considered forced MHD with an imposed magnetic field. Other numerical simulations of forced MHD turbulence (see, e.g., Beresnyak & Lazarian 2009, Haugen et al. 2003, Müller & Grappin 2005, Müller et al. 2003) also reported conflicting results. In freely decaying isotropic turbulence, some simulations were observed to develop Iroshnikov-Kraichnan scaling, whereas others developed Kolmogorov-like scaling (Mininni & Pouquet 2007, 2009; Müller & Grappin 2005). Recently, large-resolution simulations of freely decaying MHD flows showed that, depending on the amplitude of the dynamically consistent large-scale magnetic field, different power laws can be realized (Lee et al. 2009). Finally, recent studies of spectral laws in solar wind data (Podesta et al. 2007) indicate that many of these power laws can also be identified in space plasmas.

Although the main aim of this review is to consider studies of scale interactions in MHD, in this context it is worth mentioning some of the existing phenomenological theories for MHD turbulence. Although Iroshnikov and Kraichnan considered small-scale fluctuations as isotropic, it is clear now that MHD turbulence does not recover isotropy at small scales (Goldreich & Sridhar 1995, Milano et al. 2001, Shebalin et al. 1983) and may become even more anisotropic as the scales are decreased. To take this into account, Goldreich & Sridhar (1995) advocate for a different MHD spectrum, whereby the anisotropy of the flow induces a Kolmogorov-like spectrum in the perpendicular direction. A balance between linear and nonlinear timescales (the Alfvén and turnover times) is assumed, which leads to a critical balance of the form $k_{\parallel} B_0 \sim k_{\perp} b_l$. Another anisotropic model based on dynamic alignment of the velocity and magnetic fields (Boldyrev 2006) gives Iroshnikov-Kraichnan-like scaling in the perpendicular direction. In this case, the angle between the two fields decreases (and therefore the fields become more aligned) with the scale as $\sim l^{1/4}$. Consideration of this alignment in the Politano-Pouquet relations leads to the aforementioned scaling for the energy spectrum. Early extensions to flows with sizable cross-correlations can be found in Galtier et al. (2000) and Grappin et al. (1983). Other models have considered transitions from Kolmogorov to Iroshnikov-Kraichnan scaling by taking different combinations of the nonlinear and Alfvén timescales (Matthaeus & Zhou 1989) or by taking into account nonlocality (Alexakis 2007).

Therefore, although the assumptions of locality and of isotropization of the small scales common in hydrodynamic turbulence allow for a simpler phenomenological treatment of MHD, the development of local anisotropies, the variety of timescales in the problem (see Zhou et al. 2004 for a review), and the different simulations showing scaling consistent with different phenomenological theories led some authors to question some of these assumptions. Schekochihin et al. (2008) considered nonlocality, anisotropy, and nonuniversality as defining properties of MHD turbulence. The authors argued that the small-scale dynamo, a fundamental process in MHD

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turbulence, shows clear signatures of nonlocality (Haugen et al. 2003, 2004; Mininni et al. 2005a; Schekochihin et al. 2002a,b, 2004). They also argued that anisotropy is intrinsic to MHD and that nonuniversality manifests itself just from the needed distinction between MHD turbulence in the presence and in the absence of a strong mean field. Similar concerns about universal behavior in MHD were discussed in Lee et al. (2009) for the case of freely decaying turbulence. Beresnyak & Lazarian (2009) considered the lack of a bottleneck in MHD (an accumulation of energy at the beginning of the viscous range observed in hydrodynamic turbulence) as evidence of nonlocality (see also Graham et al. 2009). In some sense, some of these discussions can be tracked back to early considerations of freely decaying MHD turbulence and the processes of selective decay (Kinney et al. 1995, Matthaeus & Montgomery 1980, Mininni et al. 2005b, Ting et al. 1986) and dynamic alignment (Ghosh et al. 1988, Grappin et al. 1983, Mininni et al. 2005c, Pouquet et al. 1986). MHD, having three ideal invariants, is known to decay for very long times into different attractors, depending on the initial ratio of these invariants (Stribling & Matthaeus 1991, Ting et al. 1986). Although these solutions involve final stages of the decay, recent numerical simulations showed that these relaxed states can be realized locally in the flow in very short timescales (Mason et al. 2006, Matthaeus et al. 2008, Perez & Boldyrev 2009, Servidio et al. 2008), giving rise to different regimes.

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## 6. CONCLUDING REMARKS

Since the success of Kolmogorov's phenomenological theory in hydrodynamic turbulence, several attempts have been made to apply similar considerations to MHD turbulence. The presence of waves, several timescales, and several ideal invariants limited these approaches, giving rise to many possible models. Solar wind observations and numerical simulations later showed that assumptions such as isotropy of the small scales, or equipartition between the fields, may not hold in the MHD case. More recently, the increase in computing power allowed for some exploration of the parameter space, giving rise to conflicting results for scaling laws in the energy spectrum.

The recent introduction of shell-to-shell transfers allowed for detailed studies of scale interactions in MHD turbulence and opened the door for the discussion of another hypothesis: that of the locality of interactions between scales. The results, at intermediate spatial resolutions and Reynolds numbers, show different degrees of nonlocality depending on the configuration studied: for example, forced or freely decaying turbulence, and in the presence or in absence of an external magnetic field. Nonlocal transfers, when observed, involve the coupling between the velocity and the magnetic field, or the transfer of magnetic helicity. In the former case, the nonlocal transfers were not larger than 10%–20% of the total, although they played fundamental roles, e.g., sustaining the magnetic field by dynamo action against Ohmic dissipation.

Despite some conflicting results in the simulations and theory, there is growing consensus that MHD turbulence is less local than hydrodynamic turbulence, although to what extent is a matter of debate. It is unclear at the moment whether these effects will go away for larger Reynolds numbers, or if they stay, what impact they will have in the flow dynamics, and under what conditions. However, the different degrees of nonlocality observed at present resolutions, and the existence of nonlocal processes in MHD (as, e.g., the small-scale dynamo), call for a discussion about the validity of the hypothesis of locality of interactions, and whether there is only one kind of MHD turbulence or many. This raises the question of the definition of MHD turbulence in phenomenological or theoretical approaches. If only configurations such as the ones in solar wind (with an imposed magnetic field) are to be considered, then a universal scaling (or several classes of universality) may be possibly identified. However, if processes such as the small-scale dynamo, the large-scale dynamo, and inverse cascades are to be considered as manifestations of MHD turbulence, nonlocal interactions and nonuniversal behavior may persist even for very large Reynolds numbers. In this context, many of the works reviewed here may have to be revisited in the following years, as experiments and increased computing power will allow us to explore new regions of the parameter space of MHD turbulence.

### SUMMARY POINTS

1. Simply applying properties of hydrodynamic turbulence to the MHD case may not be possible. In particular, assumptions of scale locality of MHD turbulence must be tested in experiments and simulations.
2. Shell-to-shell transfer functions allow for detailed studies of coupling between fields and scales in numerical simulations. The shell-to-shell transfers can also be associated with physical processes such as Alfvén wave interactions, Joule damping, and dynamo action.
3. The degree of nonlocality observed at the presently attainable spatial resolutions depends on the configuration.
4. Mechanically forced turbulence shows local transfer of magnetic and kinetic energy, but the coupling between the velocity and magnetic field that sustains the latter against Ohmic dissipation is nonlocal.
5. In freely decaying MHD turbulence, nonlocal effects seem to be negligible.
6. Studies of the energy transfer in the presence of an imposed magnetic field show that most of the transfer takes place in the direction perpendicular to the external field, with strong nonlocal interactions with modes with $k_\parallel = 0$.
7. The transfer of energy for the Elsässer variables is more local than the transfer in terms of the velocity and magnetic fields.
8. The shell-to-shell transfer of magnetic helicity is more complex, with superimposed direct and inverse transfers. The inverse transfer has a local component and a nonlocal one that moves energy from the forced scale directly to the largest scales in the system.

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## DISCLOSURE STATEMENT

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