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Sweeping decorrelation in isotropic turbulence

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Tennekes [J. Fluid Mech. **67**, 561 (1975)] estimated the time decorrelation of inertial-range excitation in isotropic turbulence by assuming effective statistical independence of the one-time distributions of inertial-range and energy-range excitation. This picture has been challenged by Yakhot, Orszag, and She [Phys. Fluids A **1**, 184 (1989)], who studied forced turbulence by renormalization-group (RNG) methods. The analysis given in the present paper leads to the conclusion that (a) precise coherence between energy-range and inertial-range excitation is needed to inhibit sweeping effects; (b) in the case of randomly forced turbulence, this coherence is impossible and Tennekes' picture is unavoidable; and (c) the RNG analysis does not demonstrate inhibition of sweeping; instead, it discards sweeping effects at the outset. To augment the present study, an advected passive scalar is examined by computer simulation. Sweeping effects on small scales survive even in the case of long-time advection by a frozen velocity field. The observed probability distributions resemble those for the alignment of vorticity and velocity observed in flow simulations.

I. INTRODUCTION

The local similarity arguments of Kolmogorov[1] lead not only to the celebrated wavenumber spectrum in the inertial range of Navier–Stokes (NS) turbulence but also to a prediction for the frequency spectrum of the Lagrangian velocity field.[2] On the other hand, elementary arguments based on near-Gaussian statistics suggest that local similarity alone does not correctly give the frequency spectrum of the Eulerian velocity field in the inertial range: sweeping effects by the energy-range excitation are expected to play a dominant effect,[3] and this brings the total rms velocity v_0 into the dimensional analysis along with the mean energy dissipation per unit mass ϵ.

Yakhot, Orszag, and She analyze a forced inertial-range by renormalization-group methods and conclude that the Tennekes analysis is incorrectly applied.[4] In the limit where the wavenumber spectrum has the Kolmogorov form, they find that the Eulerian frequency spectrum strictly obeys local similarity and has the form classically associated with the Lagrangian frequency spectrum. Nelkin and Tabor[5] have examined some implications of this conclusion for scaling laws of fourth-order moments in the inertial range. Their work helps motivate the present paper, which asks what physical phenomena could upset Tennekes' analysis and whether these phenomena are in fact addressed by the renormalization-group analysis.

In order to give some added insight into the sweeping question, we also examine numerically the tendency of threadlike blobs of an advected passive scalar to align with streamlines of a prescribed Gaussian velocity field. The observed probability distributions of the angle between local velocity field and the strained scalar threads have some resemblance to distributions of angle between velocity and vorticity that have been observed in a number of flow simulations.[6,7] The underlying mechanisms that build and destroy alignment may be similar in the two cases.

II. SOME KOLMOGOROV SCALING RELATIONS

Kolmogorov's 1941 theory (K41) of the turbulent inertial range[1] leads to a well-known prediction for the wavenumber spectrum of the velocity field in the inertial range:

$$E(k) = C\epsilon^{2/3}k^{-5/3}, \tag{1}$$

where k is wavenumber, $\int_0^\infty E(k)dk$ is the mean kinetic energy per unit mass, C is a dimensionless constant, and ϵ is the mean rate of kinetic energy dissipation by viscosity, per unit mass. Equation (1) can be obtained in a number of ways. It follows immediately from dimensional analysis under Kolmogorov's assumption that ϵ is the only global parameter that is felt in the inertial range.

The dimensional analysis that yields (1) does not extend unaltered to prediction of the frequency spectrum of the velocity field in the inertial range. Instead, the rms velocity of the energy range appears in the final formula together with ϵ. This comes about as follows. The K41 theory appeals to effective statistical independence of the one-time probability distributions of energy-range and inertial-range excitation: at any instant, the two ranges know about each other only through ϵ. Tennekes[3] has pointed out that this implies a statistical form of Taylor's hypothesis so that inertial-range components of the velocity field suffer advective sweeping by the energy-range excitation. Consequently, the many-time distribution of the inertial-range excitation involves the magnitude of the sweeping as well as ϵ.

If the velocity vector of the total velocity field changes little over the distance $1/k$, a velocity component of magnitude v parallel to $\mathbf{k}$ induces oscillation at frequency vk because of advection of the excitation at wave vector $\mathbf{k}$. An average, over the velocity distribution, of wavenumber-fre-

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quency spectra of the form $\delta(\omega - vk)E(k)$ then yields the inertial-range scaling relation

$$E(k,\omega) = k^{-1}P(\omega/v_0 k)E(k), \quad (2)$$

where $\int_0^\infty E(k,\omega)d\omega = E(k)$, $v_0^{-1}P(v/v_0)$ is the probability distribution of the absolute value of any vector component of the total velocity field, and v_0 is the rms value of that component. If P falls off fast enough at infinity, (1) and (2) yield an inertial range of values of ω in which

$$F(\omega) = C_\omega v_0^{-1}\epsilon^{2/3}(\omega/v_0)^{-5/3}, \quad (3)$$

where $F(\omega) = \int_0^\infty E(k,\omega)dk$ and

$$C_\omega = C\int_0^\infty P(x)x^{2/3}\,dx. \quad (4)$$

III. MECHANISMS FOR INHIBITION OF SWEEPING

The authors have been able to think of only two basic ways in which the sweeping proposed by Tennekes can be inhibited: Either the small-scale excitation is confined to the immediate neighborhood of nodes in the energy-range velocity, or the energy-range velocity is almost perpendicular to the local velocity gradient of small-scale excitation almost everywhere. In the latter case, the small-scale excitation could be stretched out along streamlines of the energy-range velocity field. In either event, an exquisitely precise coherence between energy-range and inertial-range excitation is needed to so strongly inhibit the sweeping, at inertial-range wavenumbers large compared to the typical energy-range wavenumber k_0, that the v_0 dependence in (2) and (3) is removed.

In order to express these matters mathematically, let the total velocity field be decomposed into band-filtered fields each of which has a width of, say, one decade in wavenumber. Denote the velocity in the nth band by $\mathbf{u}_n$, the rms value of a vector component of $\mathbf{u}_n$ by v_n, and the central wavenumber of the band by k_n, where the energy range comprises the band $n = 0$. Equation (1) implies $v_n/v_0 \approx n^{-1/3}$. The term in the Navier–Stokes equation that expresses advection of band n by the energy range is $\mathbf{u}_0\cdot\nabla\mathbf{u}_n$. If $\mathbf{u}_0(\mathbf{x},t)$ and $\mathbf{u}_n(\mathbf{x},t)$ are statistically independent at each time t, the variance of this term is

$$\langle|\mathbf{u}_0\cdot\nabla\mathbf{u}_n|^2\rangle = O(k_n^2 v_0^2 v_n^2). \quad (5)$$

On the other hand, if the energy-range sweeping is sufficiently inhibited to remove the v_0 dependence from (2) and (3), restoring ϵ as the sole relevant parameter, then (5) must be replaced by

$$\langle|\mathbf{u}_0\cdot\nabla\mathbf{u}_n|^2\rangle = O(k_n^2 v_n^4). \quad (6)$$

If (6) is achieved by segregation of band-n excitation into the nodes of the energy-range field, then

$$\langle|\mathbf{u}_0|^2|\mathbf{u}_n|^2\rangle \ll v_0^2 v_n^2 \quad (n \gg 1). \quad (7)$$

If, instead, Eq. (6) is a consequence of the alignment of gradients of $\mathbf{u}_n$ normal to streamlines of $\mathbf{u}_0$, then $|\mathbf{u}_0|^2$ and $|\mathbf{u}_n|^2$ can be statistically independent. The field $\mathbf{u}_0$ in (5)–(7) properly should be replaced by $\mathbf{u}_{<n}$, defined as the total velocity field in all bands m such that $m/n < \alpha$, for some $\alpha < 1$.

The cross term between $\mathbf{u}_0$ and $\mathbf{u}_n$ in the pressure term of the NS equation has been ignored in discussing (5)–(7). In a Gaussian, or near-Gaussian distribution, the variance of this cross term is smaller in magnitude than (5) by a factor $O(k_0^2/k_n^2)$. The existence of the pressure term therefore cannot alter the conclusion that a strong departure from Gaussian statistics and a precise coherence between band 0 and band n is needed to suppress sweeping effects on band n when $n \gg 1$.

Suppose that in decaying isotropic turbulence the dynamics induces either the segregation of intensity described by (7) or a precise normal alignment of the gradients of small scales so that, in either event, dependence of $F(\omega)$ on v_0 is removed and ϵ is the only remaining parameter. Then dimensional analysis yields

$$F(\omega) = C'\epsilon\omega^{-2}, \quad (8)$$

where C' is another dimensionless constant. Equation (8) is the power law classically associated with an inertial range of ω in the frequency spectrum of the Lagrangian velocity.[2]

Whatever the case for freely decaying isotropic turbulence, (8) is an impossible result for steady turbulence supported by white-noise random forcing of the energy range, or by power-law forcing throughout a modified inertial range. The reason is simply that the exquisitely precise coherence between energy-range and inertial-range excitation needed to produce (6) would be continuously upset by the random forcing that, at each instant, is statistically independent of the velocity field. The time required to set up the needed coherence is at least of order $1/(v_0 k_0)$, the reciprocal of the strain rate associated with $\mathbf{u}_0$ and felt by all higher bands. This is also the time in which the field $\mathbf{u}_0$ is substantially replaced by totally uncorrelated new excitation. If there is power-law forcing throughout a modified inertial range, this forcing directly randomizes the inertial-range bands $\mathbf{u}_n$ and further destroys any coherence between energy range and inertial range. In general, power-law forcing can be expected to inhibit departure from Gaussian statistics, to an extent that is best determined by comparing simulations.

The more plausible of the two mechanisms for inhibition of sweeping discussed above is strong alignment of small-scale excitation along the streamlines. This would be closely linked to a preference for parallel or antiparallel orientation of velocity and vorticity. A tendency toward such orientation, has, in fact, been observed in a number of flow simulations.[6,7] It is a weak tendency, however, in contrast to the almost-perfect alignment that would be required to inhibit sweeping of small scales at high Reynolds numbers.

IV. COMPARISON WITH EXPERIMENTS

It must be emphasized that the discussion in Secs. II and III is concerned with asymptotic behavior of inertial-range bands in the limit $k_n/k_0 \to \infty$. At finite Reynolds numbers, a variety of effects can alter frequency spectra. First of all, $v_n \propto n^{-1/3}$. This slow dependence on n implies that at finite k_n/k_0 substantial contributions to the velocity in bands $m < n$ come from wavenumbers that do not simply advect the velocity field of band n without distortion. Also, the energetic coupling within mode triads with wavenumber ratios that are finite, but substantially different from unity, affects the frequency spectrum of modes with high k.

This is exhibited, for example, by solutions of the direct-interaction approximation (DIA) for decaying isotropic turbulence at finite Reynolds number: there is substantial depression of the characteristic frequencies of high-k modes below the values expected from kinematic arguments, and this depression closely reproduces full numerical simulation results.[8,9] But within DIA, sweeping effects at infinite Reynolds number are easily shown analytically to satisfy Tennekes' assumptions. Clearly, then, the depression of characteristic frequencies below values appropriate to Gaussian statistics, which appears in the simulations, cannot be taken as a predictor of asymptotic behavior. The dangers of making asymptotic extrapolations are emphasized by Yeung and Pope,[10] who also find finite-Reynolds-number deviations from Tennekes' hypothesis in simulations of isotropic turbulence.

Panda *et al.*[11] have published a simulation of isotropic turbulence driven in a statistically steady state by forcing that has a wavenumber spectrum $\propto k^{-1}$ and is a white noise in time. The simulation covered a wavenumber range of $k = 1$ to $k = 64$ (results for a smaller simulation with the range 1–32 also were reported) and the k^{-1} forcing was imposed for all $k < 30$. A nearly steady state was reached with rms velocity component $v_0 = 1.29$ and Taylor microscale $R_\lambda = 64.2$. The results reported include a wavenumber spectrum $E(k)$ that osculates a line with $-\frac{5}{3}$ slope on a log–log plot and an overall frequency spectrum $F(\omega)$ that, after smoothing, is to a remarkable extent proportional to ω^{-2}. The authors cite these results as a confirmation of a renormalization-group prediction[4] that sweeping of small scales by large scales is asymptotically inhibited.

We find that this conclusion melts away under examination. The wavenumber at the osculation, which reasonably can be taken as an approximation to the center of the putative inertial range, is $k_i \approx 10$. The nominal overall sweeping frequency at this wavenumber is $v_0 k_i \approx 12.9$. The rms velocity component associated with all kinetic energy at wavenumbers $< k_i$ is $v_{<i} = 1.06$ and the associated sweeping frequency by "large scales" acting at k_i is $v_{<i} k_i \approx 10.6$. The rms velocity component associated with all kinetic energy at wavenumbers between 5 and 20 is $v_i \approx 0.77$. This yields an estimate $v_i k_i \approx 7.7$ for the characteristic frequency expected at wavenumber k_i from "local" interactions. None of these three frequencies differs greatly from the others.

Now, the plot of $\omega^2 F(\omega)$ [Ref. 11, Fig. 9(a)], shows a rising portion from $\omega = 1$ to $\omega \approx 13.5$ and a flat portion from $\omega \approx 13.5$ to $\omega = 50$, with both portions exhibiting substantial oscillations. In the rising portion, $F(\omega) \propto \omega^{-5/3}$ fits better than $F(\omega) \propto \omega^{-2}$. The part of the curve with $F(\omega) \propto \omega^{-2}$ thus lies wholly at frequencies *higher* even than the frequencies that are associated with the center of the "inertial range" by the crudest sweeping estimates. The frequencies in this part of the curve are further yet above frequencies obtained either by correcting the sweeping effect downward by factors like those that appear in the cited DIA calculations[8,9] or by realistically estimating the local dynamical frequencies appealed to by the authors. We conclude that, whatever the cause of the nearly ω^{-2} behavior (dissipation-range contributions, forcing effects, etc.), the measured overall function $F(\omega)$ does not represent an inertial-range spectrum.

In order to carry the analysis of this simulation further, it is necessary to examine the measured frequency spectra for a localized band of wavenumbers $\approx k_i$ and compare the actual absolute values of the characteristic frequencies thus found with those expected under various approximations to sweeping and local dynamics. Reference 11 does not present the data needed for this. We would also like to see statistics for the angle between the velocity field and vorticity. Strong inhibition of the sweeping of high wavenumbers would require that this angle be effectively confined to small values. There may be value in doing a DIA calculation with forcing, using the parameters of the simulation, and comparing the frequency spectra in various bands with those of the simulation.

V. SWEEPING OF A PASSIVELY ADVECTED SCALAR

The arguments made in Sec. III concerning sweeping effects on small scales can be restated, and with greater simplicity, for a scalar field passively advected by a prescribed velocity field. Consider a Gaussianly distributed isotropic velocity field with rms velocity component v_0 in any direction and with spectral support confined to a thin spherical shell at wavenumber k_0. At first, let the field be frozen in time. It is known that such fields exhibit Lagrangian turbulence in the sense that a given fluid particle executes a random walk on scales large compared to $1/k_0$.[12] Let a passively advected and nondiffusive scalar field consist initially of long thin threads of thickness $1/k_s$ and the length $l < 1/k_0$ that accurately follow the streamlines of the velocity field. The characteristic wavenumber of the scalar field is then k_s. The threads will move along themselves like snakes, and consequently the characteristic decorrelation time of the scalar field will be l/v_0 rather than the much shorter sweeping time $1/v_0 k_s$.

Now let the velocity field change in time with a characteristic frequency $\omega_0 \approx v_0 k_0$. As the velocity field changes, the threads will no longer lie along streamlines and the decorrelation time will shrink to the sweeping time $1/v_0 k_s$.

A passive scalar advected by a frozen, random velocity field is very different from NS turbulence. Nevertheless it is a relevant model because the frozen-field advection seems to us the situation most hospitable to the development of alignment with streamlines. We note later that the scalar results carry over directly to the straining of weak, thin vortex tubes.

The assumed initial condition with scalar threads precisely aligned with the streamlines is a very special one. If scalar blobs are placed at random in the frozen velocity field, will they, in fact, be drawn out into structures aligned with the streamlines? A simple argument says that they cannot. For simplicity, consider a scalar blob that consists initially of a rod randomly placed and oriented and with length small compared to $1/k_0$. With the passage of time, the rod will eventually be strained into a structure of length long compared to $1/k_0$. A given pair of particles in the rod eventually will execute random walks relative to each other. It is then impossible that the pair lie on the same streamline. Equivalently, one may argue that a strained substructure produced in passage through one eddy of the field finds itself randomly rotated relative to velocity in a distant eddy to which it eventually migrates.

These remarks have been corroborated by a simple numerical experiment. The described frozen velocity field was constructed by a previously described algorithm.[12] A set of 200 fluid particles was chosen that initially were equally spaced along a randomly placed and oriented line segment of initial length $0.01/k_0$. The particles were followed for a total time $20/(v_o k_0)$, by which time the mean length of the (then highly distorted) thread of fluid particles, averaged over 2000 independent realizations, had increased exponentially to $\approx 25/k_0$ while the rms distance between neighboring particles had increased to $\approx 0.1/k_0$. At each time step, the angle θ between the local velocity and the line element connecting each neighboring pair of particles was measured. The experiment was repeated for a Gaussian velocity field with the wavenumber spectrum again confined to a thin shell at k_0 but with the time covariance

$$\langle \mathbf{u}(\mathbf{x},t)\cdot\mathbf{u}(\mathbf{x},t')\rangle = 3v_0^2 \exp(-\tfrac{1}{2}\omega_0^2|t-t'|^2) \qquad (9)$$

and $\omega_0 = v_0 k_0$.

Some of the results are shown in Figs. 1–3. The effective mean-square sweeping velocity locally normal to the thread is $v_0^2\langle \sin^2\theta\rangle$, which has the value $\frac{2}{3}v_0^2$ if θ is random. The probability distribution of $P(x)$ of $x = |\cos\theta|$ is flat $[P(x) = 1]$ if θ is randomly distributed. Figure 1 shows clearly that there is an initial tendency toward alignment of the thread with streamlines, as straining proceeds, but this alignment saturates when elements of the thread are carried to distant eddies. The equilibrium value of the effective mean-square sweeping velocity is about one-half the value for random orientation in the frozen-field case and about 0.85 of the random-orientation value in the time-varying case.

The curves in Fig. 2 have cusps at $x = 1$ and are nearly flat at smaller values of x. They may be interpreted as follows: At any given time, some regions of the thread of the advected scalar are subject to strong stretching and consequent near-alignment with the local velocity field. Later, these regions are rotated relative to the then-local velocity and the alignment is lost. The cusps correspond to the regions undergoing strong stretching and the flat parts correspond to regions that have never been aligned or have lost their alignment. If the probability distribution for θ itself were plotted, there would be no cusp at $\theta = 0$, but there would be a nonzero value. In contrast, the distribution of θ for purely random orientation ($t = 0$) is $\propto \sin\theta$ and vanishes at $\theta = 0$.

Figure 3 shows the typical form assumed by the initially

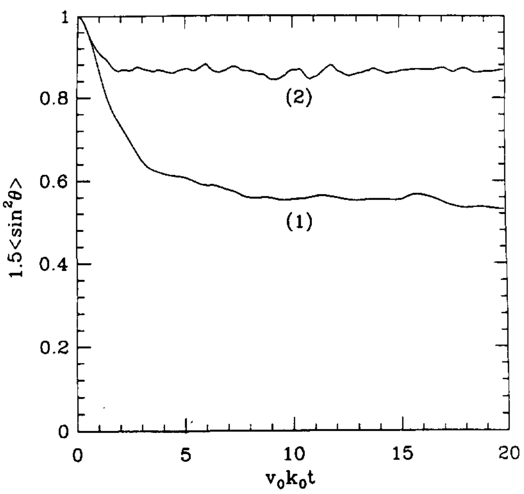

FIG. 1. Alignment of scalar thread with streamlines for Gaussian velocity field confined to a single shell in k space. The mean square of the sine of the angle between the thread and the local velocity field is plotted against time for a frozen field (curve 1) and for a field with decorrelation in time according to (9) with $\omega_0 = v_0 k_0$ (curve 2).

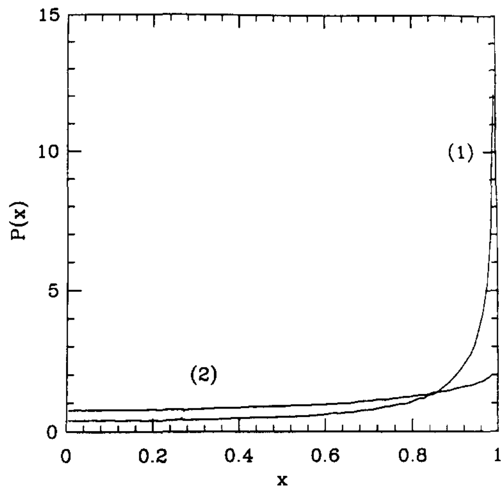

FIG. 2. Probability distribution $P(x)$, where $x = |\cos\theta|$, for the frozen (curve 1) and time-varying (curve 2) cases shown in Fig. 1. The plotted curves represent means over the period $10 \leqslant v_0 k_0 t \leqslant 15$.

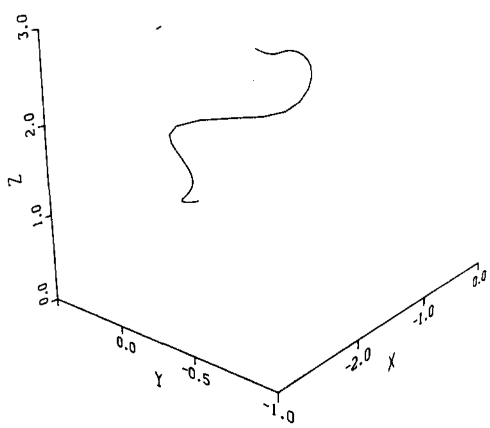

FIG. 3. Perspective view of a typical stretched scalar thread at $t = 15/(v_0 k_0)$ for the frozen velocity field. By this time the thread has been stretched so that it extends over several correlation lengths of the velocity field. Axes are marked in units of $1/k_0$.

straight thread of scalar after it has been stretched for several Lagrangian correlation times of the frozen velocity field.

It is clear from the numerical results that (a) there is a tendency toward alignment with streamlines but the alignment saturates with an effective mean-square sweeping velocity equal to an $O(1)$ fraction of the value for purely random orientation; and (b) the degree of alignment is reduced markedly by time variation of the velocity field. A further reduction of alignment would be expected if the velocity spectrum were to have broader support. This is because the vorticity in small-scale eddies would then tend to rotate portions of the scalar thread relative to the large-scale velocity field. This effect should be markedly stronger for a NS field than for a Gaussian field. In the former case, the small eddies are carried by the large-scale velocity and have a longer time to act than in the Gaussian case, where, because of total statistical independence, the large scales advect the scalar field rapidly through the small eddies.

The described simulations of strained scalar threads also can be taken to represent the straining of very thin vortex tubes with circulation weak enough that the strain within the tubes is dominated by that of the prescribed large-scale velocity field. (For consistency, the initial threads should be closed loops, but this does not affect the results.) The total local stretching at any time, as exhibited by the separation of neighboring marked fluid particles, is a measure of the increase of vorticity within the stretched tube.

It is of interest that Fig. 2 resembles plots of the distribution of angle between velocity and vorticity in simulations of NS turbulence[6,7] and that the degree of alignment shown in Fig. 1 for the time-varying case is like that found for velocity–vorticity alignment. Very possibly both the alignment mechanism and the mechanisms that destroy alignment are essentially similar in the two cases.

VI. RENORMALIZATION-GROUP TREATMENT OF SWEEPING

In a recent paper, Yakhot, Orzsag, and She[4] make a renormalization-group (RNG) analysis of $F(\omega)$ for the forced NS equation and obtain a result that reduces to (8) in the limit where the wavenumber spectrum is (1). More precisely, they find that white-noise forcing with the wavenumber spectrum

$$\mathscr{F}(k) \propto k^{-1+\mu/2} \tag{10}$$

yields

$$E(k) \propto k^{-5/3+\mu/3} \tag{11}$$

and

$$F(\omega) \propto \omega^{-2+3\mu/(4+\mu)}, \tag{12}$$

where μ is a positive parameter. The Kolmogorov limit is $\mu \to 0$.

These results squarely conflict with the conclusion in Sec. III that inhibition of sweeping effects on $F(\omega)$ in the classical or modified inertial range is impossible for the forced NS equation. It will be argued here that the RNG analysis in fact does not address the question of whether sweeping effects are inhibited in the inertial range. Instead, the RNG analysis discards sweeping effects at the outset. Some of the matters involved have been discussed in an earlier paper.[13]

The RNG analysis makes use of two concurrent expansions: expansion about a Gaussian statistical state by perturbative treatment of nonlinearities and expansion in powers of the deviation of the forcing-spectrum power-law exponent from the reference value $\mu = 8$ ("epsilon" expansion). The reference state is one in which $\mathscr{F}(k) \propto k^3$ and $E(k) \propto k$. There is a profound qualitative difference between Gaussian or near-Gaussian states with the spectrum $E(k) \propto k$ and such states with a power-law spectrum near the Kolmogorov form $E(k) \propto k^{-5/3}$. In the former case, the leading-order contribution to the nonlinear term that is seen by a band $\mathbf{u}_n$ well into the power-law range comes from interaction with modes at the high-k end of the power-law range. The latter modes contain most of the total energy. This interaction gives rise, under perturbation analysis, to eddy damping felt by $\mathbf{u}_n$. In the case of Kolmogorov or near-Kolmogorov spectrum shape, most of the energy is at the low-k end of the spectrum and the nonlinear term is dominated by the sweeping interaction with energy-range modes if the statistics are nearly Gaussian. The eddy damping is present but now comes principally from interactions local in wavenumber.

Yakhot, Orszag, and She note that sweeping effects remain small for power laws in the vicinity of the reference state. On this basis they ignore sweeping. It is hard to justify such a procedure in the face of the fact that sweeping is the dominant dynamical effect in near-Gaussian states with spectra of actual interest: those with power laws for $E(k)$ near the Kolmogorov value $-\frac{5}{3}$. What would be needed to demonstrate inhibition of sweeping in the physical case would be instead to *keep* the terms that describe sweeping and show that, at or near the Kolmogorov exponent value, strong coherence effects were generated that make the sweeping terms in the NS equation small. At the least, this would require reliable calculation of the cumulant contributions to the fourth-order moments that appear in (5)–(7).

The reference state $E(k) \propto k$ seems a singularly inappropriate starting point for such a program since it has a relevant qualitative physics so different from that of the state of physical interest. On the other hand, this reference state does serve for the calculation of eddy damping. Eddy damping appears in the lowest order of self-consistent perturbation theory about Gaussian statistics both for the reference spectrum and for the spectrum (1). Only lowest-order perturbation theory is used in the RNG treatment so that, in any event, only near-Gaussian states can be described. Such a perturbation treatment cannot produce strong coherence effects between large and small scales, whether or not such effects exist in the actual physical system.

Energy-range sweeping effects can be consistently removed from the NS equation by altering the interaction coefficients between Fourier modes.[14] This can be done while eddy damping of the energy range by the inertial-range bands is retained. The RNG analysis gives precisely the same predictions for this butchered NS equation as it does for the original equation. This is because the only interaction between distant wavenumbers that is kept in the RNG analysis is the eddy-damping effect. The frequency spectra obtained in the RNG analysis can be logically interpreted only as spectra in some kind of effective Lagrangian coordinate frame (see Ref. 13, Sec. IV).

RNG analysis like that used for the velocity field can also be applied to the advection of a passive scalar by a Gaussian velocity field with a Kolmogorov spectrum. Again, the sweeping effects get discarded.

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