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On the propagation of ideal, linear Alfvén waves in radially stratified stellar atmospheres and winds

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Abstract. The propagation of Alfvén waves through isothermal, radially stratified, spherically symmetrical models of stellar atmospheres and winds is discussed. The transmission coefficient for the waves is calculated as a function of frequency, magnetic field base intensity, surface gravity and atmospheric temperature. When a wind is present, the wave energy flux is no longer conserved, but the conservation of the wave-action flux (Heinemann & Olbert 1980) allows the definition of an analogous transmission coefficient, giving the relative amount of waves reaching the super-Alfvénic regions of the wind. It is shown that for high-frequency waves the transmission coefficient for static and wind models is identical, while for low-frequency waves the presence of a wind enhances the transmission considerably. The latter are however totally reflected asymptotically, far from the stellar surface, a behaviour which is reminiscent of the observed evolution of the "Alfvénicity" of turbulence in the solar wind. Recent isotropic models for Alfvénic turbulence which display the same qualitative behaviour are compared to the low-frequency limit of the linear equations. Finally, it is argued that models for the mass loss of cool giants and supergiants which properly treat the reflection of Alfvén waves might overcome the difficulties inherent in standard (WKB) wave-driven models.

Key words: stars: atmospheres – stars: magnetic fields – Sun: atmosphere – Sun: magnetic fields – plasmas

1. Introduction

The problem of the propagation and transmission of magnetohydrodynamic waves through solar and stellar atmospheres and winds has attracted considerable interest for some time, given its relevance to the questions of chromospheric and coronal heating and wind acceleration (see e.g. Leer et al. 1982; Hollweg 1991; Narain & Ulmschneider 1991; Velli et al. 1991 for reviews of different aspects of the subject). In order for waves to efficiently accelerate a wind they must be able to propagate without much reflection or attenuation to the sonic point, because any addition in momentum below that point essentially goes to increase the mass flux but not the asympotic wind speed (Leer et al. 1982). On the other hand, waves that suffer either significant reflection and/or nonlinear evolution in the lower atmospheric layers may contribute to the nonradiative heating there through turbulent decay (and shock dissipation).

In the case of the solar wind, it is well known that given the 10^6 K solar corona, thermal driving is sufficient to explain only the slowest solar wind streams, with speeds of order $300\,\mathrm{km\,s^{-1}}$, while an additional source of momentum is necessary to boost the high speed streams to their asymptotic velocities of around $700\,\mathrm{km\,s^{-1}}$. Among the MHD wave-modes generated (presumably) in abundance through convection and magnetic footpoint shuffling at the photospheric levels, Alfvén waves have been the prime candidate both for the acceleration of high speed wind streams and for wave-based theories of coronal heating. The main reason is that Alfvén waves propagating away from the Sun have been shown to be a dominant component of the observed MHD turbulence in the solar wind (see Mangeney et al. 1991; Marsch 1991 for recent reviews). Also, estimates based on the simplest properties of the other wavemodes (slow and fast magnetoacoustic waves) indicate that the latter have much difficulty in even reaching the solar corona: (slow) sound waves with periods greater than 5 min are evanescent, while waves with shorter periods appear to steepen into shocks within the chromosphere (Ulmschneider et al. 1978). Fast magnetoacoustic modes which are not strictly propagating along the magnetic field become evanescent in the chromosphere and corona due to the rapid increase of Alfvén speed with height (Hollweg 1978). Because of their highly anisotropic dispersion relation which allows them to propagate along the magnetic field independently of the wave-vector, Alfvén waves do not suffer (at lowest order) from either of the above effects.

It should be kept in mind however that dispersion might have an important effect on sound waves close to the Lamb cut-off, and that Alfvén waves couple nonlinearly into compressional waves, and therefore also develop into shocks (and rotational discontinuities) (Cohen & Kulsrud 1974; Hollweg et al. 1982). For the latter process what really counts is the ratio of the typical nonlinear to propagation time: for wave amplitudes in the range 20–30 $\mathrm{km\,s^{-1}}$, and coronal Alfvén speeds in the range 500–2000 $\mathrm{km\,s^{-1}}$ this ratio lies between 15 and 100, so that the waves should travel 6–200 solar radii (for frequencies between 1 min and 1 h) without significant nonlinear distortion [over such distances however damping of both Alfvén and fast-mode waves through thermal conduction is probably important (Habbal & Leer 1982)].

The actual propagation of Alfvén waves is fairly complex, even in the linear case: although the waves propagate along the magnetic field, they still feel the very steep gradient in the Alfvén speed, which, as first shown by Ferraro & Plumpton (1958), can lead to significant reflection if the wavelength is of the same order or greater than the region over which the Alfvén speed varies significantly. An evaluation of the maximum critical frequency below which Alfvén waves become evanescent as a function of temperature and magnetic flux-tube expansion for isothermal atmospheres has been recently presented by Moore et al. (1991): the critical frequency has a very sensitive exponential dependence on the reciprocal of the temperature. This has led to the interesting suggestion that the Alfvén waves might contribute to the heating of the solar corona in a self-regulating way: waves with frequency below the critical one would heat coronal holes through a turbulent cascade (involving outward and reflected waves), until the temperature necessary for them to escape into the solar wind is reached. Also on the basis of the dependence of the critical frequency on temperature (as well as stellar radius and mass) Rosner et al. (1991) have conjectured that the X-ray and mass loss dividing lines for red giants might be explained by the combined effect of Alfvén wave reflection and a change in dominant magnetic topology (from closed to open fields) on the surface of the star: for stars with low atmospheric temperature, Alfvén wave reflection in the lower layers would become substantial, leading to large wave-pressures below the sonic critical point. The open magnetic field topology would hence allow the formation of massive winds.

The criterion for evanescence discussed above, based on the comparison of wavelength and scale-height, is clearly local: on the other hand, in the case of a spherically symmetric static isothermal atmosphere, the Alfvén speed, after an initial exponential increase, decreases as R^{-2}; as a consequence, waves of all frequencies have a wavelength which becomes smaller than the Alfvénic scale-height beyond a certain distance from the star, i.e. all waves are asymptotically propagating. A global solution is therefore necessary for accurate predictions of wave behaviour.

It is the aim of this paper to reconsider the question of the propagation of Alfvén waves in detail, both for static atmospheric models and models allowing for the presence of winds. We shall phrase the discussion in terms of transmission and reflection coefficients, which give the fraction of the energy flux incident from below an atmospheric layer that is able to propagate through to the top of the layer. The plan of the paper is as follows: in the next section we first review a number of well known properties of Alfvén wave propagation in vertically stratified media. We then discuss the transmission in a spherically symmetric isothermal atmosphere with radial magnetic field, as a function of frequency, temperature and magnetic field. This is worthwhile because estimates based on the maximal critical frequency mentioned above (and discussed in more detail below), though fairly accurate in determining the frequency above which WKB theory is valid, miss out completely the significant transmission in an intermediate, low frequency band.

In Sect. 3 we consider propagation in the presence of a stellar wind. As the basic equations are those used by Heinemann & Olbert (1980); (hereafter HO) and more recently by Barkhudarov (1991), we shall omit the details on the method of solution, focusing again on the transmission coefficient, defined as the ratio of the wave action flux (per unit frequency) reaching the superalfvénic portions of the wind to the flux incident on the base of the atmosphere. At low frequencies, the transmission is significantly enhanced over the static case, while at higher frequencies the transmission coefficients in the two cases become identical. We also compare the predicted evolution in the superalfvénic wind regions with the evolution described by recent isotropic turbulence models given by Zhou & Matthaeus (1989, 1990), and illustrate some of the difficulties involved with developing the latter. In Sect. 4 we discuss the relevance of the results obtained to the questions of solar and stellar atmospheric heating and wind acceleration.

2. Alfvén wave propagation in vertically stratified static media

The basic equations for transverse magnetic field ($\boldsymbol{b}$) and incompressible velocity ($\boldsymbol{v}$) fluctuations, in a plasma of density ρ, may be conveniently expressed in terms of Elsässer variables

$$z^{\pm} = \boldsymbol{v} \mp \operatorname{sign}(\boldsymbol{B}_0)\boldsymbol{b}/\sqrt{(4\pi\rho)},$$

which in a homogeneous medium describe Alfvén waves propagating in opposite directions along the average magnetic field $\boldsymbol{B}_0$:

$$\frac{\partial z^{\pm}}{\partial t} \pm \boldsymbol{V}_{\mathrm{a}} \cdot \nabla z^{\pm} \mp z^{\mp} \cdot \nabla \boldsymbol{V}_{\mathrm{a}} \pm \frac{1}{2}(z^{\mp} - z^{\pm})\nabla \cdot \boldsymbol{V}_{\mathrm{a}} = 0, \tag{1}$$

where $\boldsymbol{V}_{\mathrm{a}}$ is the average Alfvén velocity. The first two terms in (1) describe wave propagation; the third term describes the reflection of waves by the gradient of the Alfvén speed along the fluctuations (which vanishes for a vertical field in a planar atmosphere, but is different from zero in the more realistic case of a spherically or supraspherically diverging flux tube); the fourth term describes the WKB losses and the isotropic part of the reflection. In Eq. (1) gravity and terms involving the gradients of the average density along the fluctuation polarisation are absent: this is because the average magnetic field and gravity are assumed to be collinear. Equation (1) then describes the parallel propagation of fluctuations in the plane perpendicular to $\boldsymbol{B}_0$, or in the case of spherical or cylindrical symmetry, the propagation of toroidal fluctuations in the equatorial plane. An immediate consequence of Eq. (1) is the conservation of net upward energy flux, which may be written as

$$S^+ - S^- = S_\infty, \qquad S^\pm = F V_a |z^\pm|^2/8, \tag{2}$$

where S_∞ is constant and $F = \rho R^\sigma$ (R is the radial distance from the base of the stellar atmosphere and σ is the infinitesimal flux tube expansion factor: $\sigma = 0$, 2 in a plane and spherical atmosphere respectively). The constancy of the energy flux allows one to define the transmission coefficient across an atmospheric layer bounded by regions of constant Alfvén speed. The correct boundary condition to impose is that only an outward propagating wave should exist above the layer in question, and the transmission T is defined as

$$T = S_\infty / S_0^+, \tag{3}$$

where S_∞ is the energy flux carried by the outwardly propagating wave, while S_0^+ is the outward propagating energy flux at the atmospheric base.

For waves of frequency ω and wavevector $k = \omega/V_a$, Eq. (1) becomes, after elimination of the systematic amplitude variation of $z^\pm$ through the renormalization $z^\pm = \rho^{1/4} z^\pm$,

$$z^{\pm\prime} \mp ikz^\pm - \frac{1}{2}\frac{k'}{k} z^\mp = 0 \tag{4}$$

(distance is normalized to the stellar radius R_0, i.e. $r = R/R_0$ throughout; a prime denotes differentiation with respect to r). The corresponding equations for the velocity and magnetic field fluctuations (also in velocity units) are

$$v'' + k^2 v = 0, \qquad b'' + (k^2 + k''/k - 2k'^2/k^2) b = 0. \tag{5}$$

The criterion for local evanescence of these fields may be found by applying the Liouville Green transformation to Eq. (5) (see e.g. Nayfeh 1973), which consists in replacing the independent variable r with $\xi = \int dx/V_a$ and the dependent variables v, b with $v/\sqrt{V_a}$, $b/\sqrt{V_a}$. From the resulting Schrodinger equations one may directly read off the critical frequencies (Moore et al. 1991)

$$\omega_c^{2v} = V_a'^2/4 + V_a V_a''/2, \qquad \omega_c^{2b} = V_a'^2/4 - V_a V_a''/2, \tag{6}$$

for the velocity and magnetic field respectively. The critical frequencies are a function of position, but Eq. (6) implies that waves will certainly be evanescent when $\omega < \min(\omega^v, \omega^b)$, and Moore et al. (1991) therefore conclude that for a given Alfvén speed profile, transmission of waves with frequencies below the maximum value (within the atmosphere) of the smaller of ω_c^v, ω_c^b is negligible. This conclusion must be taken with some caution, for at least two reasons: first, the independent variable is no longer x, but the travel time at the Alfvén speed, while the potential is essentially a logarithmic derivative of the Alfvén speed, which means that exponential behaviour in the latter variable does not translate into spatial evanescence as a function of heliocentric distance (as may be seen from Eq. (6) at zero frequency the two independent solutions for the velocity are $v = $ const. and $v \sim r$). Second, the effective potential tends to zero high in the corona, which implies significant low frequency tunneling, as we shall see further below.

The general solution to Eq. (4) may be written formally as (Kholkunov 1970)

$$z^\pm = H(r, r_0) z_0^\pm,$$
$$H(r, r_0) = T_{\exp}\left(\int_{r_0}^r dr \left[ik\hat{\sigma}_3 + \frac{1}{2}\frac{k'}{k}\hat{\sigma}_1\right]\right), \tag{7}$$

where $T_{\exp}$ denotes the time-ordered exponential operator, while $\hat{\sigma}_{1,3}$ denote the Pauli matrices. The time-ordered propagator is quasi-unitary, i.e. the matrix elements of H satisfy

$$H_{11} = H_{22}^0 \quad H_{12} = H_{21}^0 \quad |H_{11}|^2 - |H_{12}|^2 = 1, \tag{8}$$

which expresses again the conservation of net upward energy flux. See also Lontano & Luzin (1991) for a useful collection of formulae concerning the properties of the time-ordered propagator. When the Alfvén speed gradient is small, the lowest order solutions to Eq. (4) are given by oppositely propagating waves. The WKB or multiple scale analyses however do not preserve the energy flux constraint Eq. (2). On the other hand, Magnus (1956) showed that the expansion

$$H(r, r_0) = \exp\left(\int_{r_0}^r dr' \hat{p}(r') + \frac{1}{2}\int_{r_0}^r dr' \int_{r_0}^{r'} dr'' [\hat{p}(r'), \hat{p}(r'')] + \cdots\right), \tag{9}$$

where $\hat{p} = ik\hat{\sigma}_3 + \frac{1}{2}k'/k\hat{\sigma}_1$ and square brackets denote the commutator, preserves the quasi-unitary character of H to all orders. This expansion may be used to obtain approximate solutions when the Alfvén speed profile may be divided into sections where propagation (first term of $\hat{p}$) or reflection (second term of $\hat{p}$) dominate, as will be shown below.

Let us consider now specific atmospheric models and the consequent profiles for the Alfvén speed. Ferraro & Plumpton (1958) were the first to consider the case of a uniform vertical magnetic field in an isothermal plane atmosphere extending to infinity. In this case, the Alfvén speed increases exponentially with height, $V_a = V_{a0}\exp(r/2h)$ where $h = R\Theta/\mu g R_0$ is the (adimensional) scale height, (g is the surface gravity, R the gas constant, the atomic weight is $\mu \approx 1/2$ for a fully ionized plasma and Θ is the temperature) and the travel time for a wave to infinity is finite (twice the time necessary for a wave to travel one scale height at the base Alfvén velocity). While the Alfvén velocity scale height is constant, the wavelength increases to infinity, and hence the scale height is smaller than the wavelength from a certain point upwards no matter what the frequency: the wave propagation term in Eq. (4) may then be neglected, leading to the asymptotic solutions

$$z^+ = z^- \sim k^{1/2}, \qquad z^+ = -z^- \sim k^{-1/2}. \tag{10}$$

Since k tends to zero with height, requirement of finite energy implies that $z^+ = z^-$ there. Hence, the atmosphere forms a closed cavity and stationary solutions must have a vanishing net upward energy flux, $S_\infty = 0$. The exact solution may be expressed in terms of Bessel functions,

$$u = u_0 J_0[2hk_0\exp(-r/2h)], \quad b = \mathrm{i}u_0 J_1[2hk_0\exp(-r/2h)], \tag{11}$$

where u_0, k_0 are the velocity fluctuation and wave-vector at the atmospheric base respectively. If one attempts to impose the amplitude of the velocity (or magnetic field fluctuation) at the base, then clearly problems arise if the frequency is such that the value of the argument of J_0 (J_1) at the base corresponds to a root of $J_0 = 0$ ($J_1 = 0$): there is no stationary solution, and as An et al. (1990) have shown, in the time-dependent case forcing with the base velocity causes the energy in the cavity to grow with time as t^2, as typical of a resonantly excited oscillator. If however one imposes any other linear combination of u, b at the base, such as for example the amplitude of the outwardly propagating mode z^+, then one always obtains stationary solutions, since J_0, J_1 never vanish simultaneously (Zugzda & Locans 1982). Nonetheless the model as just described has features which make it unphysical and unrealistic: mainly the infinite extension of the isothermal atmosphere and the uniformity of the magnetic field.

Better models, where the atmosphere is divided into one or more isothermal layers, with differing scale heights, were developed by Hollweg (1972, 1978) and Leroy (1981). As stressed beforehand in order to define a transmission coefficient it is necessary to assume the Alfvén speed to be uniform (or at least slowly varying so that the lowest order WKB expansion is valid) above the layer considered. The models of Hollweg and Leroy were developed for wave propagation in the solar chromosphere, transition region, and corona, and the atmospheres were taken to terminate at the height where the Alfvén speed reaches a value around 2000 km s^{-1}.

The transmission coefficient was found to present a series of maxima at certain frequencies (referred to as "resonances" in the literature, though they do not coincide precisely with the real resonant frequency discussed above in the context of the infinite isothermal atmosphere) which are related to the typical scales defined in the problem: thickness of the isothermal layers considered, and Alfvén speed scale heights of the two layers. Outside the resonances the transmission coefficient, for typical solar chromospheric, transition region and coronal parameters was found to be very low. The origin and interpretation of the resonant peaks has since been the subject of some debate: Hollweg (1978) attributed them to discontinuities in scale height separating the various atmospheric layers, on the contrary Leroy (1981) and Leer et al. (1982) attributed them to the process of continuous reflection. The variation of the transmission with frequency is clearly an integrated effect, which depends on the whole profile of the Alfvén speed with height. However scale height discontinuities, where the derivatives in velocity and magnetic field fluctuations are also discontinuous, contribute significantly to the appearance of resonances in the transmission, because they define additional characteristic length scales, namely the distance between successive discontinuities. Consider for example the linearly varying Alfvén speed profile $V_a = V_{a0}(1 + r/a)$. The commutators in Eq. (9) all vanish in this case, yielding immediately inside the layer in question:

$$z^+(r) = \frac{\mathrm{e}^{\mathrm{i}\phi} - (\tilde{\omega} - \Omega)^2 \mathrm{e}^{-\mathrm{i}\phi}}{1 - (\tilde{\omega} - \Omega)^2} z_0^+ + \frac{\mathrm{i}(\tilde{\omega} - \Omega)(\mathrm{e}^{\mathrm{i}\phi} - \mathrm{e}^{-\mathrm{i}\phi})}{1 - (\tilde{\omega} - \Omega)^2} z_0^-,$$

$$z^-(r) = \frac{\mathrm{e}^{-\mathrm{i}\phi} - (\tilde{\omega} - \Omega)^2 \mathrm{e}^{\mathrm{i}\phi}}{1 - (\tilde{\omega} - \Omega)^2} z_0^- + \frac{\mathrm{i}(\tilde{\omega} - \Omega)(\mathrm{e}^{\mathrm{i}\phi} - \mathrm{e}^{-\mathrm{i}\phi})}{1 - (\tilde{\omega} - \Omega)^2} z_0^+, \tag{12}$$

where $\tilde{\omega} = 2\omega a/V_{a0}$ is the adimensional frequency, $\Omega = \sqrt{\tilde{\omega}^2 - 1}$ and $\phi = (\Omega \log(1 + r/a))/2$. For a layer of thickness $\bar{r}$, the transmission coefficient is given by

$$T = \frac{S_\infty}{S_0} = \frac{\tilde{\omega}^2 - 1}{\tilde{\omega}^2 - \cos^2(\phi)}. \tag{13}$$

For this profile we have discontinuities in scale height at the boundaries of the layer, defining its total thickness: the transmission coefficient equals one at the "resonant" frequencies where $\bar{\phi} = [\Omega \log(1 + \bar{r}/a)]/2 = (2n+1)\pi/2$. If we replace the Alfvén speed profile defined above with a profile matched continuously to the values of V_a at the extrema, for example $V_a = V_{a0}\,[1 + \bar{r}/a(1 - \cos(\pi r/\bar{r})]/2$, the maxima in the transmission disappear [Fig. 1 shows the transmission for the two profiles described]. More generally, roughness in the profile of the Alfvén speed and/or periodicity tend to promote resonant-type behaviour in the transmission coefficient.

The amplitude of the transmission at the resonances, remains, of course, smaller than or equal to one. In this sense the energy flux remains below its WKB value (corresponding to $S^- = 0$, i.e. $T = 1$). If however we choose to define the WKB energy flux, following Hollweg (1972), as

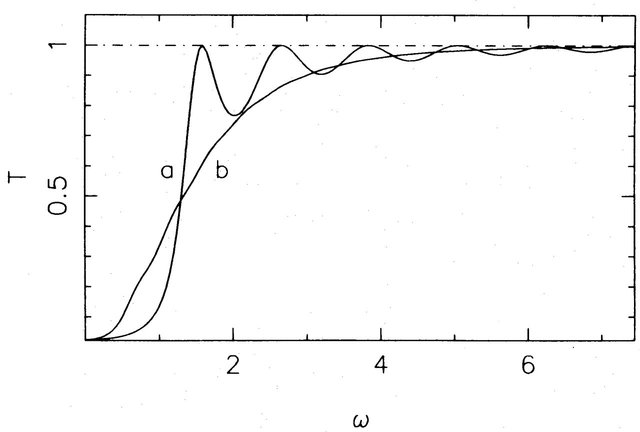

Fig. 1. Transmission coefficient as a function of frequency (in arbitrary units) for the linear Alfvén speed profile (curve a) and the cosine profile (curve b)

that based on the observed rms velocity amplitude at the solar photospheric levels, then the actual flux can be both greater than or smaller than the WKB value. This is because we have no information on the magnetic field fluctuation at the same position. Denote this observationally based WKB estimate of the energy flux (which considers $v=b$ at the photosphere) as S_{WKB}, which may be written $S_{\mathrm{WKB}}=F_0 V_{a0}|u|_0^2/2$, while the actual energy flux is given by Eq. (2)

$$S_\infty = F/4V_a(E^+ - E^-) = F_0/4V_{a0}E_0^+ T, \tag{14}$$

and $E^\pm = |z^\pm|^2/2$. In terms of the residual energy $E^{\mathrm{R}} = |u|^2 - |b|^2$, the ratio reads

$$\frac{S_\infty}{S_{\mathrm{WKB}}} = \frac{T}{2-T+E^{\mathrm{R}}/E^+}. \tag{15}$$

The Schwartz inequality applied to the $z^\pm$ fields implies that

$$-2(1-T)^{1/2} \leq E^{\mathrm{R}}/E^+ \leq 2(1-T)^{1/2},$$

(E^{R}/E^+ is equal to twice the cosine of the phase shift between outward and reflected wave). The only definite statement one can make, independently of the Alfvén speed profile, is that as $T\to 0$ the upward energy flux tends to vanish (S_{WKB}, on the other hand, may or may not vanish, depending on whether the velocity at the base has a node). The use of the transmission coefficient as a useful indicator of the energy flux propagating upwards through the solar atmosphere has been recently criticized by Hollweg (1991) as misleading. The reason is an apparent contradiction found for the transmission by a slab of isothermal atmosphere: while the Poynting flux remains finite in the static limit, the transmission coefficient appears to tend to zero as $T \sim \omega$. The discrepancy is due to a different estimate of the low-frequency transmission, which may be calculated from the static solutions of Eq. (4), for any finite layer and independently of the detailed profile within the layer (provided it is continuous), to be

$$T = \frac{4V_{\mathrm{al}}V_{\mathrm{ar}}}{(V_{\mathrm{al}}+V_{\mathrm{ar}})^2}, \tag{16}$$

(where $V_{\mathrm{al,r}}$ indicate the Alfvén speed on either side). This result is the same as the transmission across a δ-function potential, because a perturbation of infinite wavelength sees the variation in propagation speed as a discontinuity. Now if the thickness of the layer across which the transmission is calculated is held fixed as the frequency is decreased to zero, the transmission tends to a constant for vanishing frequency, in a way consistent with the Poynting flux. If, however, the thickness of the isothermal slab is varied to take into account the different depths at which a low frequency wave becomes WKB (a calculation in a similar spirit is carried out below for a spherically symmetric isothermal atmosphere) then the Alfvén speed at the lower boundary must decrease as $V_{\mathrm{al}} \sim \omega$. In this case however, both the Poynting flux and the transmission tend to zero, and the discrepancy discussed above exists no longer.

We conclude by stressing that within the framework of linear theory the basic quantity characterizing the energy flux is the transmission coefficient: for any given Alfvén speed profile one may compare the WKB–non WKB energy fluxes on the basis of the full solutions to Eq. (4). Clearly, if it were possible to obtain simultaneous observations of velocity and magnetic field at different heights in the solar atmosphere, then one would be able to ascertain the presence of nonlinear interactions and or dissipation from the observed differences with the predictions from the transmission problem, and hence obtain a quantitative estimation of the wave damping in the solar atmosphere.

The model atmospheres described above, though giving a more appropriate description of a stellar atmosphere than the plane, infinite, isothermal slab, still lack a number of features typical of the structure of stars with transition regions and coronae: first, the intermittent structure of the flux-tube confined photospheric magnetic field, and its rapid expansion through the chromosphere and transition region; second, the decrease of the magnetic field far from the stellar surface due to the spherical expansion of the atmosphere. Recently, Similon & Zargham (1991) have solved Eq. (4) for transverse waves along the central fieldline of a strongly diverging fluxtube (appropriately modelling the upper chromosphere and transition region of the Sun), in a 2-D (slab) geometry: Alfvén wave reflection is substantially decreased, essentially because the expansion of the flux tube reduces the rate of increase of the Alfvén speed [see also Hollweg 1981, for a piecewise exponential model of an expanding flux tube]. Although for the outer field lines the increase in the transmission should be even greater, the coupling to other wave modes due to the field line curvature will then become important (mode-conversion, see e.g. Hollweg & Lilliequist 1978; Scheurwater 1990). Here we do not consider the wave propagation in this extremely complex and dynamic region; the problem requires a detailed model for the magnetic field and density of a chromospheric flux tube extending into the corona, which is at present unavailable. We take our base as the base of the corona, where an assumption of spherical symmetry (e.g. for a coronal hole) is probably not totally absurd. We remark in passing that the base of the corona may be a region where significant power is channelled into Alfvén waves by reconnection (see Parker 1991 and references therein) and turbulent magnetic footpoint motions.

In spherical geometry the Alfvén speed profile for an isothermal static atmosphere may be written

$$V_a = \frac{V_{a0}}{r^2} \exp\left[\left(\frac{\alpha}{2}\left(1-\frac{1}{r}\right)\right)\right], \quad \alpha = \frac{GM_0}{R_0 C_s^2}, \quad 1 < r < \infty, \tag{17}$$

where C_s is the isothermal sound speed and the parameter α, for the Sun, typically lies in the range $4 \le \alpha \le 15$ for coronal temperatures between $3.0\ 10^6$ and $8.0\ 10^5$ K. For this family of profiles, the Alfvén speed first increases exponentially, has a maximum in $r = \alpha/4$ and then decreases asymptotically as $V_a \sim r^{-2}$. An et al. (1990) investigated the propagation of Alfvén waves in this configuration numerically as an initial value problem, and showed that, as expected, local reflection becomes strong where the Alfvén speed gradients are greatest. They did not determine the solutions of the stationary problem however because they stopped their computations before the waves reached the outer edge of their (finite) computational domain. To calculate the transmission coefficient, we integrate Eq. (4) backwards numerically, starting at a heliocentric distance large enough so that an outwardly propagating second order WKB solution is adequate ($\omega \gg \omega_c^v, \omega_c^b$, the general form in terms of $z^{\pm}$ may be found in Velli et al. 1989 and Hollweg 1991). In Fig. 2 we plot the transmission coefficient as a function of frequency as the temperature of the atmosphere is decreased (α is varied from 4.0 to 14.0 in integer steps) but the ratio β of kinetic to magnetic pressure at the coronal base is held constant at a value of 8%. The frequency plotted is normalized to $\omega_* = cC_s(\alpha=4)/R_0$, where for the Sun $C_s(\alpha=4) \simeq 215\ \mathrm{km\,s^{-1}}$ and the constant c($\simeq 5.572$) has been chosen so that $\omega_* = 2\pi\ \mathrm{h}^{-1}$, i.e. $\omega/\omega_* = 1$ corresponds to a wave period of one hour. A strong dependence of transmission on temperature for waves of periods greater than one hour is apparent, while waves with periods less than about 15 min are completely transmitted by the solar corona. The maximum of the critical frequencies Eq. (6), defining evanescence for the magnetic and velocity fluctuations (denoted by the stars on the transmission curves), gives a rough estimate of the frequency above which transmission is 100%. For a sufficiently low temperature, the transmission coefficient displays a low-frequency maximum, which is the result of a significant tunneling effect (to be discussed in more detail below).

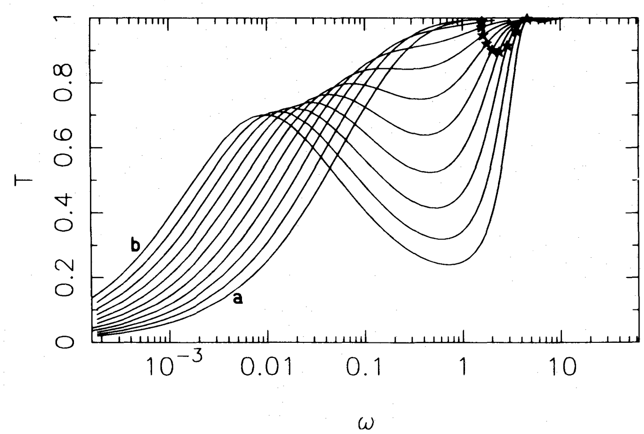

Fig. 2. Transmission coefficient as a function of frequency for the isothermal wind models of various temperature described in the text. α varies in integer steps from $\alpha=4$ (curve a) to $\alpha=14$ (curve b). Transmission is calculated for a constant value of $\beta=8\%$ at the base of the atmosphere. Frequencies are normalized to $\omega_* = cC_s(\alpha=4)/R_0 = 2\pi\ \mathrm{h}^{-1}$, ($c=5.572$), so that $\omega=1$ corresponds to a period of 1 h for the Sun. Stars denote the maximal critical frequency (see text)

The dependence of the transmission on the magnetic field base-intensity may be recovered by noticing that the frequency appears in the equations only in the adimensional combination $\Omega = \omega R_0/V_{a0}$. This implies that the curves in Fig. 2 should be shifted to the right (left) as the magnetic field base intensity is increased (decreased). Thus, if the magnetic pressure were to decrease tenfold at the base of the solar corona, we would have total transmission of waves with periods below about one hour. The most crucial parameter is of course α, which appears in the exponential. When moving from the Sun to stars of greater mass and radius, but lower temperature, such as red giants and supergiants, α tends to grow, for a typical K5 supergiant ($M = 16M_\odot$, $R = 400R_\odot$) α increases from 10 to 30 as Θ varies from 46 000 to 15 000 K. The transmission curves as α is varied from 14 to 24 are shown in Fig. 3. Note that the wiggles in the transmission at high frequencies are real. The physical frequency corresponding to $\omega=1$ in Fig. 3 of course depends on the mass and radius of the star considered. Since $\omega=1$ corresponds to a period of 1 h for the Sun, $\omega=1$ will correspond to a period of $(M_\odot/M_0)^{1/2}(R_0/R_\odot)^{3/2}$ h for other stars: e.g. for the K5 supergiant a period of about 83 d. The most evident effect of increasing α is the drop in the transmission, in agreement with the heuristical arguments based on the critical frequencies [Eq. (6)] (Rosner et al. 1991; Moore et al. 1991).

The low frequency scaling of the transmission may be understood by remarking that the quasi-static solution of Eq. (4) [Eq. (10)] will be approximately valid out to a distance where $k \simeq k'/2k$, i.e., $\omega \simeq V_a'/2$. For the given profile [Eq. (17)], we find that this occurs where (Ω is the adimensional frequency defined above) $r \simeq r_\omega = \Omega^{-1/3}\exp(\alpha/6)$. Beyond this point, we may neglect (to

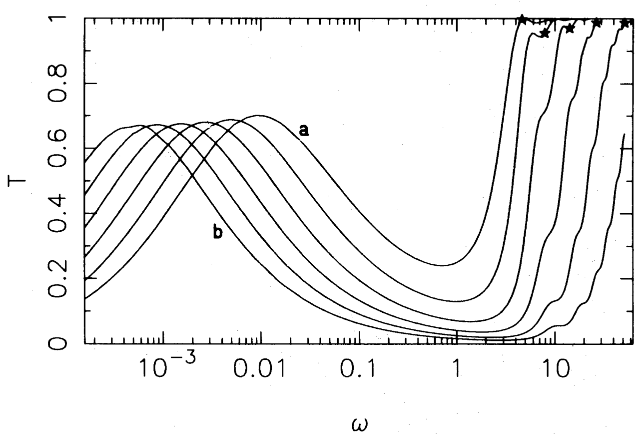

Fig. 3. Same as Fig. 2 for stars with greater mass and cooler atmospheres than the Sun. α is an even integer varying from $\alpha = 14$ (curve a) to $\alpha = 24$ (curve b)

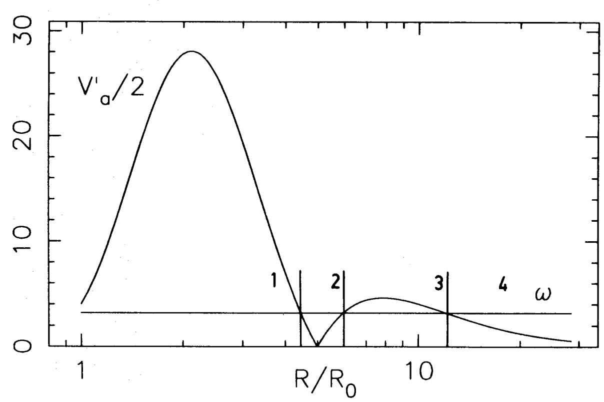

Fig. 4. The profile of the Alfvén speed derivative for the spherically symmetric isothermal atmospheres of Figs. 1 and 2, showing regions of evanescence, (1, 3) and propagation (2, 4), for a given frequency (denoted by the straight line)

lowest order) the reflected wave: imposing that z^- vanish in r_ω then yields $T \sim \Omega^{2/3}$, as may be seen from Eq. (16). A better understanding of the low frequency behaviour of the curves presented in Figs. 2 and 3 may be obtained by keeping terms up to second order in the Magnus (1956) expansion and dividing the atmosphere into regions dominated respectively by propagation and reflection. As mentioned before the Alfvén speed profile presents a maximum at $r = \alpha/4$: here the propagation terms of Eq. (4) dominate, so that 4 regions, as illustrated in Fig. 4, must be considered. We write the solution as $[V_{\rm am} = V_{\rm a}(\alpha/4), V_{\rm a\omega} = V_{\rm a}(r_\omega)]$

$$z^\pm(r) = H(r, r_\omega)H(r_\omega, r_2)H(r_2, r_1)H(r_1, r_0)z_0^\pm,$$

$$r_{1,2} = \alpha/4 \mp \delta r, \qquad \delta r \simeq \Omega(\alpha/4)^2/V_{\rm am}. \tag{18}$$

A tedious, but straightforward calculation then yields

$$T = \left[f \cos^2\left(2\frac{\Omega\delta r}{V_{\rm am}}\right)\cosh^2\left(\frac{1}{2}\ln\frac{V_{\rm a0}}{V_{\rm a\omega}}\right) + g \sin^2\left(2\frac{\Omega\delta r}{V_{\rm am}}\right)\cosh^2\left(\frac{1}{2}\ln\frac{V_{\rm am}^2}{V_{\rm a\omega}V_{\rm a0}}\right)\right]^{-1}, \tag{19}$$

where the functions f and g contain the contributions from the second order terms in the expansion, and depend only weakly on the frequency. As the frequency is increased from zero, the point r_ω moves inwards, and the value of $V_{\rm a\omega}$ also increases. When $\alpha \geq 8$, a point is reached where $V_{\rm a\omega} \approx V_{\rm a0}$, and the first hyperbolic cosine in the denominator for T has a minimum (equal to one) there. This explains the low frequency maximum in Figs. 2 and 3. The result follows also from the static transmission formula Eq. (16), which shows that T has a maximum ($=1$) when the Alfvén speed has the same value on the right and left hand side of the layer in question. In Eq. (19) the maximum is less than one because of the function f. In the limit of large $\alpha \to \infty$, all the integrals in the Magnus expansion may be calculated analytically, the sine term in Eq. (19) may be neglected, and $f \to 1.5525$, giving $T \approx 0.644$, which is close to the low frequency maximum value seen in Fig. 3. For large values of α the transmission for frequencies beyond the maximum decays in a symmetrical way $T \sim \Omega^{-2/3}$, as follows also from Eq. (16).

3. Propagation in stellar winds

For the majority of stars the problem of the transmission of Alfvén waves cannot be solved correctly unless we take into account the presence of winds which at a distance of a few stellar radii replace the static gravitational stratification. The propagation equation for incompressible waves in this case becomes

$$\partial\frac{z^\pm}{\partial t} + (\boldsymbol{U} \pm \boldsymbol{V}_{\rm a}) \cdot \nabla z^\pm + z^\mp \cdot \nabla(\boldsymbol{U} \mp \boldsymbol{V}_{\rm a}) + \frac{1}{2}(z^- - z^+)\nabla \cdot \left(\boldsymbol{V}_{\rm a} \mp \frac{1}{2}\boldsymbol{U}\right) = 0, \tag{20}$$

where $\boldsymbol{U}$ is the average wind velocity.

We will consider here only the simplest, isothermal, spherically symmetric wind models (see e.g. Parker 1979); the mach number $M = U/C_{\rm s}$ and the Alfvén speed satisfy the well known equations

$$\left(M - \frac{1}{M}\right)M' = \frac{2}{r} - \frac{\alpha}{r^2},$$

$$V_{\rm a} = V_{\rm a0}/r(U/U_0)^{1/2}. \tag{21}$$

As shown by HO, the asymptotic boundary condition of propagating Alfvén waves is replaced when the wind is included by the condition of regularity at the Alfvénic critical point where $U = V_{\rm a} = V_{\rm ac}$. The flux conservation Eq. (2) still holds if we reinterpret S as a wave action per unit frequency:

$$S^+ - S^- = S_\infty, \qquad S^\pm = F\frac{(U \pm V_{\rm a})^2}{V_{\rm a}}\frac{|z^\pm|^2}{8}, \tag{22}$$

and F is unchanged. The transmission coefficient is again

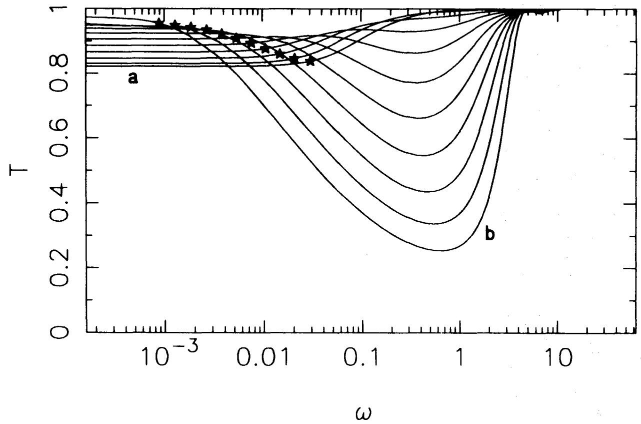

Fig. 5. Transmission coefficient as a function of frequency for the isothermal wind models Eq. (19). Normalization of frequency is the same as in Figs. 2 and 3. α varies from $\alpha=4$ (curve a) to $\alpha=14$ (curve b), in integer steps, while stars denote the cutoff frequency for total asymptotic reflection

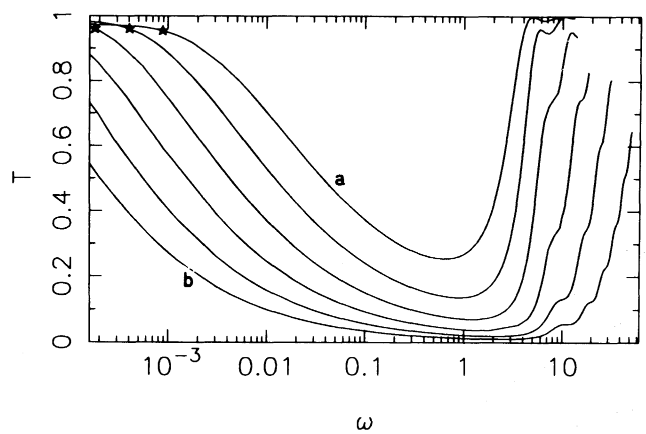

Fig. 6. Same as Fig. 5 for stars with greater mass and cooler atmospheres than the Sun. α is an even integer varying from $\alpha=14$ (curve a) to $\alpha=24$ (curve b)

defined as $T=S_\infty/S_0^+$, where now however the wave action flux is determined by the amplitude of the outwardly propagating wave at the critical point as $S_\infty=S_c^+ =F|z^+|_c^2/2$. The transmission coefficient as a function of frequency is shown in Figs. 5 and 6 (the wind is taken to be isothermal all the way out to the Alfvén critical point). Again, the coronal base β is fixed at a value of 8%, so the models are still defined completely, for a star of given mass and radius, by the single parameter α [varying from 4 to 14 in Fig. 5, from 14 to 24 in Fig. 6]. The shape of the transmission curves is completely different from that of the static case at low frequencies, where the transmission is enhanced, while at higher frequencies the curves become similar in shape. That the presence of the wind increases the transmission at low frequencies may be understood by examining the static limit of Eq. (20), which, rewritten in terms of $y_r^\pm=(U\pm V_a)z^\pm$, becomes

$$y_r^{\pm\,\prime}-\frac{1}{2}\frac{V_a'}{V_a}y_r^\mp-\frac{1}{2}\left(\frac{U'}{U}+\frac{V_a'}{V_a}\right)y_r^\pm=0, \tag{23}$$

with solutions, (as in the static case), given by $y^+=\pm y^-$. Imposing that y^- vanish at the critical point then gives (HO, Leer et al. 1982; Hollweg & Lee 1989)

$$z^\pm(r)\sim\frac{1}{U\pm V_a}\left(\frac{U}{U_c}\right)^{1/2}\left(\frac{V_a}{V_{ac}}\pm 1\right), \tag{24}$$

and the subscript denotes quantities calculated at the Alfvénic critical point. For the low frequency limit of the transmission we obtain

$$T=\frac{4U_0V_{a0}}{(U_0+V_{a0})^2}\frac{z_c^{+2}}{z_0^{+2}}=\frac{4V_{a0}V_{ac}}{(V_{a0}+V_{ac})^2}. \tag{25}$$

Thus we see that it is possible to have perfect transmission at low frequencies, if the Alfvén speed at the coronal base and the critical point are "tuned" close to the same value. We remark that the above result is independent of the position of the coronal base, provided the geometry allows for the propagation of a pure Alfvén-type wave. If we follow the fluctuations down below the transition region, the problem becomes that of correctly estimating the Alfvén speed within a chromospheric or photospheric flux-tube. Note that there is a complete formal analogy in the expression for the transmission at low frequencies in the static and expanding models. The difference in the behaviour lies in the fact that, while for the static case the position at which the outward propagating wave boundary condition is imposed varies with frequency (leading to a decrease of T with decreasing ω), in the expanding case the boundary position is fixed at the Alfvénic point, and T therefore tends to a constant as $\omega\to 0$.

At intermediate frequencies, the transmission still depends very sensitively on the wind temperature. In the solar case this is reminiscent of the strong observed correlation, (at least at solar minimum), between the level of the so called Alfvénic turbulence and the stream temperature at periods around one hour (Grappin et al. 1991); the wind models described above are rather crude, not taking into account important effects such as flux tube expansion. However it would be interesting to develop more realistic models to test whether the observed correlation may be attributed, at least partially, to the transmission properties of the ambient medium.

There is a major difference between the propagation properties in the static and expanding case, and in the significance of Figs. 2, 3 and 5, 6: in the static case, waves of all frequencies are asymptotically outwardly propagating. This may conveniently be expressed by introducing the normalized cross-helicity σ

$$\sigma=\frac{z^{+2}-z^{-2}}{z^{+2}+z^{-2}}\to 1, \qquad r\to\infty. \tag{26}$$

In the spherically expanding case, Figs. 5 and 6 show the proportion of outward waves that reach the superalfvénic wind, but the asymptotic behaviour of σ depends on the frequency. In isothermal winds the wind speed grows logarithmically with distance from the stellar base. On the other hand, in realistic wind models the speed levels off at a value $U=U_\infty$, so the Alfvén profile goes asymptotically as $V_a=V_{a\infty}/r$. Equation (20) may now be rewritten as

$$(U_\infty \pm V_{a\infty}/r)z^{\pm'} - i\omega R_0 z^\pm + \frac{1}{2r}(z^+ + z^-)(U_\infty \mp V_{a\infty}/r) = 0 \tag{27}$$

An eikonal expansion which treats the boundary condition at the critical point correctly (see Barkhudarov 1991) then shows that at large distances

$$z^+ \simeq -[(1-4\omega^2 V_{a\infty}^2 R_0^2/U_\infty^4)^{1/2} - 2i\omega V_{a\infty} R_0/U_\infty]^{-1} z^- + O(1/r)z^- . \tag{28}$$

For all frequencies greater than $\omega_0 = U_\infty^2/2V_{a\infty}R_0$ the normalized cross helicity σ increases with distance beyond the Alfvén critical point to a frequency-dependent limiting value which tends to one at high frequencies as $1-(\omega_0/\omega)^2$. At lower frequencies however σ decreases with distance and tends asymptotically to 0, i.e., we have total reflection at infinity. In the solar case the critical frequency is of the order of a few days, and in the case of the giants and supergiants described above, this rises to a period of the order of a few weeks. Hence it might appear that results within this frequency domain are of only academic interest (it depends on whether one can take stellar winds to be stationary over the periods considered), were it not for the fact that observations of Alfvénic turbulence in the solar wind also show a decrease of σ with distance from the Sun (Roberts et al. 1987). This is difficult to explain within the framework of homogeneous MHD turbulence (see Mangeney et al. 1991). The observed decrease in σ however occurs at all frequencies, in contradiction with predictions from linear theory. The observed decrease of the (specific) energy with distance, $E = E^+ + E^- \sim r^{-1-\varepsilon}$, where ε is a small positive quantity depending on frequency and radius, is also in contradiction with the predictions of the low-frequency linear theory [Eq. (27) showing that $E \sim$ const]. Zhou & Matthaeus (1989, 1990) have suggested a way in which the combined effects of MHD turbulent evolution and the solar wind expansion might lead to the observed behaviour. Their model is based on an isotropized version of the system Eq. (20), written in terms of the energies $E^\pm$ in the $\pm$ modes and the residual energy E^R:

$$(U \pm V_a)E^{\pm'} \mp E^\pm \nabla\cdot(V_a \mp U/2) \pm E^R \xi/2\nabla\cdot(V_a \pm U/2) = 0,$$

$$U E^{R'} + E^R \nabla\cdot U + \xi(E^+ + E^-)\nabla\cdot U/2 - \xi(E^+ - E^-)\nabla\cdot V_a = 0, \tag{29}$$

where ξ is a parameter which depends on whether we consider the E's as the laterally integrated (i.e. the energies as measured in situ by the satellites) part of the energy tensors, or the modal energy of isotropic spectra (in the former case $\xi = 3/14$, while in the latter $\xi = 1/3$). When the Alfvén speed is neglected everywhere, Eqs. (29) and (27), with $\omega = 0$ are structurally identical, [Eqs. (27) corresponding to $\xi = 1/2$], and thus the model can reproduce the decrease in σ (but not the observed decrease in specific energies) far from the Alfvénic critical point. Closer, where the Alfvén speed may not be neglected, Eqs. (29) and (27) are fundamentally different: by writing (27) in terms of the second order moments, we notice that the Alfvénic singularity appears both in the equation for E^- and E^R. This ensures that the Schwartz inequality $E^{R^2} \le 4E^+E^-$ is satisfied. The system (29) on the other hand has lost the singularity in the E^R equation (E^R is convected by the wind), and the Schwartz inequality is no longer enforced by the equations, as a simple calculation demonstrates. The isotropy condition used to obtain (29), as discussed in greater detail in Velli et al. (1992b), is at the origin of this discrepancy: radially aligned perturbations are not Alfvén waves but sound waves; in the incompressible limit [implied by Eq. (20)] they are simply convected by the wind, and cannot be "forced" into behaving as Alfvén waves, without losing physical consistency. We must conclude that a satisfactory quantitative model of the evolution of Alfvén waves in an expanding wind, dropping the constraints of incompressibility and isotropy, which appear to be too severe, has yet to be developed.

4. Conclusions

In the previous sections we have discussed the propagation of Alfvén waves in radially stratified atmospheres and winds. Whether such computations may be used as an accurate guide to understand the problems of atmospheric heating and wind acceleration is a topic for some discussion. Let us compare the variation with distance, for example, of the wave pressure force in the WKB (reflectionless) and non-WKB limit. A number of diagrams concerning the WKB–non-WKB comparison for different stellar wind models may be found in HO and Barkhudarov (1991). Their results for the solar wind show that the wave acceleration is accentuated (weakly) with respect to WKB values in the subsonic (coronal) regions for intermediate frequencies corresponding to the transmission minimum. This moderate enhancement has negligible effects however at greater distances. When going to models of massive stellar winds, the main problem with WKB acceleration theories is that too much acceleration occurs beyond the sonic point, yielding not only large mass fluxes, but also too large asymptotic speeds. The same problem appears if a phenomenological damping length is included, to dissipate the waves immediately beyond the sonic point (where the work goes exclusively into wind acceleration). In Figs. 7 and 8 we plot the ratio of the finite wave

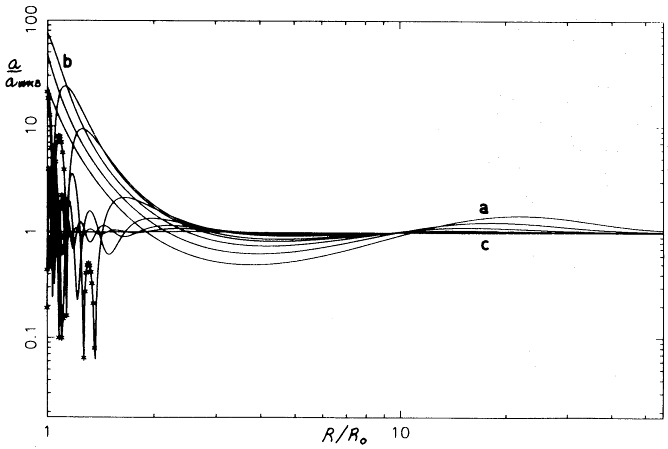

Fig. 7. Ratio of wave pressure to the WKB (reflectionless) value, for the static isothermal atmosphere. $\alpha = 20$ and the period varies from 1.1 h (using the solar mass and radius as normalization) (curve a) to 33 s (curve c). The ratio becomes distance independent at high frequencies. The period for which the acceleration is strongest close to the stellar surface is (curve b) 17 min. Note that as the frequency is increased, the acceleration begins to oscillate with distance, and can become negative over an intermediate frequency domain (regions marked with an asterisk)

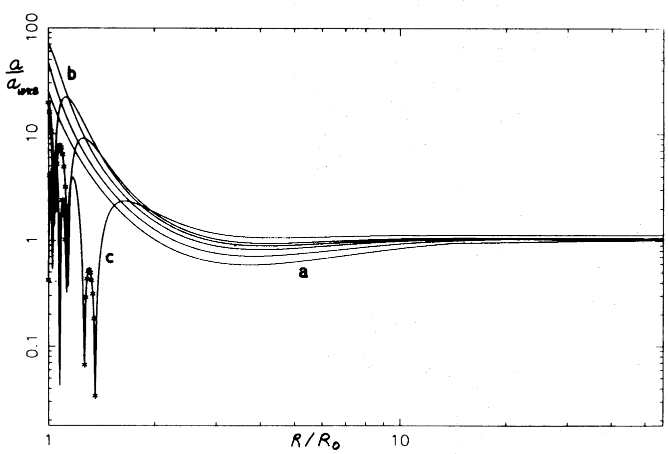

Fig. 8. Ratio of wave pressure to the WKB (reflectionless) value for the case of an isothermal wind. $\alpha = 20$ and the period varies from 1.1 h (curve a) to 165 s (curve c). The results are identical to those of the static case within $10 R_0$. Again, the waveperiod for which the acceleration is strongest close to the stellar surface is (curve b) 17 min

acceleration to WKB acceleration for $\alpha = 20$ respectively in the cases of a static atmosphere and an isothermal wind (the sonic critical point is located at $r = 10$). Since in the static case the asymptotic radial dependence is the same in the WKB non-WKB cases, the ratio is normalized to one at infinity, in Fig. 7; for the expanding case the ratio is normalized at the Alfvénic radius, in Fig. 8. Note that the ratio becomes very large, close to the stellar surface, for waves in an intermediate frequency band (well above the low-frequency cutoff described in the last section), around the transmission minimum (which for massive cool stars becomes extremely pronounced). These waves might be well suited to obtain the formation of a massive, low speed wind, as suggested by Rosner et al. (1991). To be able to give a definite answer one would like to know the shape of the initial spectrum of Alfvén waves generated at the stellar surface. Observationally, this point is still very uncertain even for the Sun, given the impossibility of differentiating between the different wave-modes. The spectra observed in the solar wind display a fully developed turbulence with a change of slope at periods around 1 h (roughly corresponding to the transmission minima in Fig. 5), which might be attributed to the atmospheric filtering, but again observational progress on the photospheric base spectrum is necessary before definite answers can be given.

Theoretically, to go further in assessing the capability of Alfvén wave to accelerate massive winds requires the solution of the self-consistent non-WKB problem. The coupled wind and wave equations, even neglecting wave-damping, now contain two critical points, which in the small amplitude limit reduce to the Alfvénic and sonic radii. The observations we have from radio-scintillation data (Scott et al. 1983), show however that at least in the solar case, the observed fluctuations are of the same order of magnitude as the average wind speed precisely in the domain of interest, so that significant modifications are to be expected (in WKB acceleration theories, where the Alfvénic point plays no role, the presence of waves shifts the effective sonic point closer to the stellar surface). This is a problem we are currently investigating. Another, related aspect, is of course the development of more realistic models of MHD turbulence relevant to outer stellar atmospheric layers and winds. As previously discussed, work has only recently begun, and with only partial success, for the case of the solar wind.

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