On the spatio-temporal behavior of magnetohydrodynamic turbulence in magnetized plasma

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Contents

Contents

Introduction

- The simplest case correspond to incompressible MHD with uniform background magnetic field $\mathbf{B_0}$, for which the linear dispersion relation $\omega = \mathbf{k} \cdot \mathbf{v_A}$ describes Alfvén waves, with $\mathbf{v_A} = \mathbf{B_0}/\sqrt{4\pi/rho}$.
- $\mathbf{v}(\mathbf{k}) \cdot \mathbf{k} = \mathbf{b}(\mathbf{k}) \cdot \mathbf{k} = 0$, where $\mathbf{b}(\mathbf{k})$ are the magnetic field fluctuations.
- When non-linear terms are taken into account, the system delevops turbulence, and the waves coexist with eddies.
- For MHD turbulence, in addition to the global nonlinear time τ_{nl}, there are also time scales associated with scale-dependent nonlinear effects, nonlocal sweeping and wave propagation.

Introduction

- In the early 70's, investigations of hydrodynamic turbulence conclude that sweeping dominates the temporal decorrelation in the inertial range.
- Recently, Servidio *et al* (2011) studied the MHD case. They concluded that, for isotropic turbulence, the temporal decorrelation in MHD is governed by nonlocal interactions, but they could not distiguish between the effect of sweeping and Alfvén distortion.

Contents

Objectives

- Study decorrelation times through various scales in inertial range.
- Understand the temporal decorrelation of the fluctuations.
- This way, differentiate between non-linear, sweeping and Alfvén effects.
- Clark di Leoni *et al* (2014) proposed to consider the fluctuations at more than one length scale in a rotating fluid, to discern between the different phenomena thar are associated with temporal decorrelation.

Contents

The MHD equations

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{1}{\rho} \nabla p + \mathbf{j} \times \mathbf{B} + \frac{1}{R} \nabla^2 \mathbf{v}$$

$$\frac{\partial \mathbf{b}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{1}{R_m} \nabla^2 \mathbf{b}$$

The MHD equations

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$$\frac{\partial \mathbf{b}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{1}{R_m} \nabla^2 \mathbf{b}$$

- $\mathbf{B} = \mathbf{b} + \mathbf{B_0}$, with a fluctuating part $\mathbf{b}$ and a mean DC field $\mathbf{B_0} = B_0 \hat{\mathbf{x}}$
- The units are based on a characteristic speed v_0, which for MHD is chosen to be the typical Alfvén speed of the magnetic field fluctuations, $v_0 = \sqrt{\langle b^2 \rangle / (4\pi\rho)}$.
- R and R_m are the the kinetic and magnetic Reynolds numbers.
- The unit time is $t_0 = L/v_0$, which for MHD becomes the Alfvén crossing time based on magnetic field fluctuations.

Characteristic times

Non-linear time

- Local eddy turnover.

$$\tau_{nl} \sim [kv(k)]^{-1} \Rightarrow \tau_{nl} \approx \left[v_{rms} L^{-1/3} \left(\sqrt{k_\perp^2 + k_\parallel^2} \right)^{2/3} \right]^{-1}$$

Sweeping time

- Advection of small scale structures by the large scale flow.

$$\tau_{sw} \approx \left(v_{rms} \sqrt{k_\perp^2 + k_\parallel^2} \right)^{-1}$$

Alfvén time

$$\tau_A \approx (B_0 k_\parallel)^{-1}$$

Decorrelation function

- The statistic of, for example, the magnetic field mey be characterized by the spatio-temporal two-point autocorrelation function $R(\mathbf{r}, \tau) = \langle \mathbf{b}(\mathbf{x}, t) \cdot \mathbf{b}(\mathbf{x} + \mathbf{r}, t + \tau) \rangle / \left\langle \mathbf{b}^2 \right\rangle$
- Fourier transformig in r leads to a time-lagged spectral density $S(\mathbf{k}, \tau) = S(\mathbf{k}) \Gamma(\mathbf{k}, \tau)$
- $\Gamma(\mathbf{k}, \tau)$ represents the dynamical decorrelation effects describing the time decorrelation of each spatial mode $\mathbf{k}$.
- $\Gamma(\mathbf{k}, \tau)$ is thus the temporal correlation function of the Fourier mode $\mathbf{k}$.
- In the presence of a guide field, $\Gamma = \Gamma(\mathbf{k}_\perp, \mathbf{k}_\parallel, \tau)$ can help us to understand the dynamics of different regions in Fourier space.
- This gives us information on how the memory in one direction affects the other and how to distinguish between random sweeping and Alfvén propagation.

Numerical simulations

- Standard pseudospectral code to solve numerically the incompressible 3D-MHD equations with a guide field.
- Second-order Runge-Kutta time integration scheme.
- Aliasing removed by the two-thirds rule truncation method.
- Periodic boundaries.
- Size of the box is $2\pi L$, where $L = 1$ is the initial correlation length of all fluctiations.
- $N^3 = 512^3$ grid points.
- $B_0 = 0.25$, 1 and 8, in units of the initial r.m.s. magnetic fluctuations value.
- Driving force in $0.9 \leq k \leq 1.8$, with random and time-coherent component ($\tau_f \sim 1$).

Contents

Energy spectra and dominant time scales

Reduced perpendicular energy spectrum

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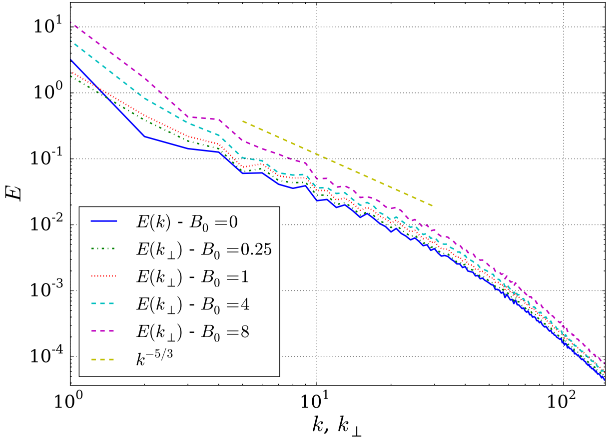
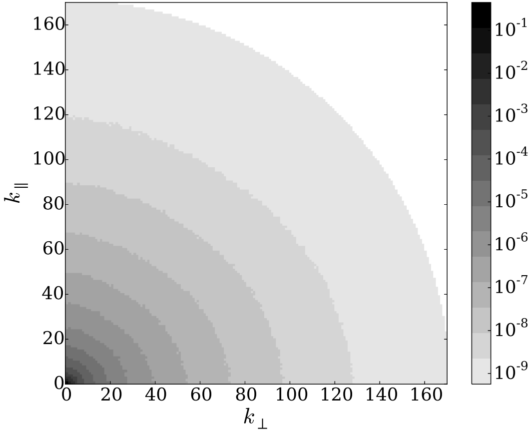
Reduced perpendicular energy spectrum



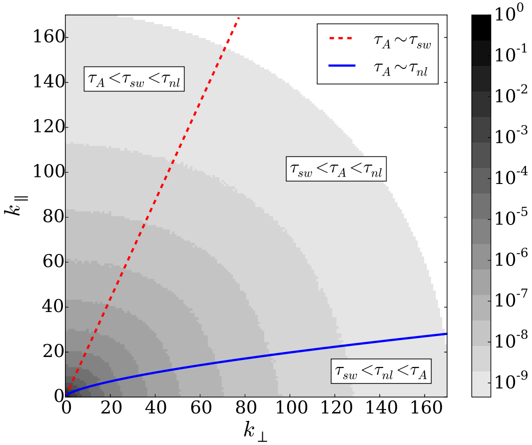
Isocontours of the axisymmetric energy spectrum

Isocontours of the axisymmetric energy spectrum :

$B_0 = 0$

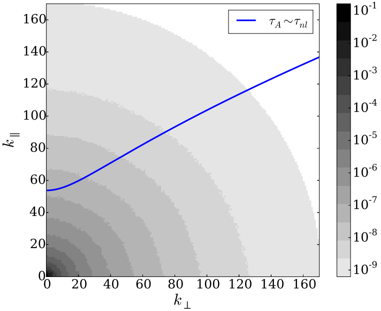


Isocontours of the axisymmetric energy spectrum : $B_0 = 1$



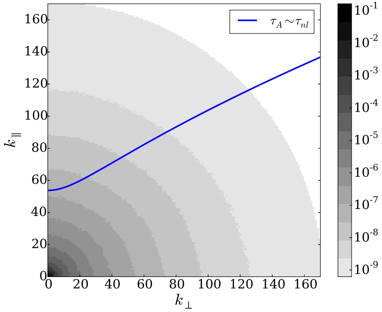
Isocontours of the axisymmetric energy spectrum

$$B_0 = 0.25$$

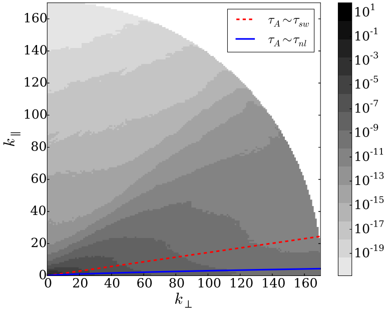


Isocontours of the axisymmetric energy spectrum

$B_0 = 0.25$



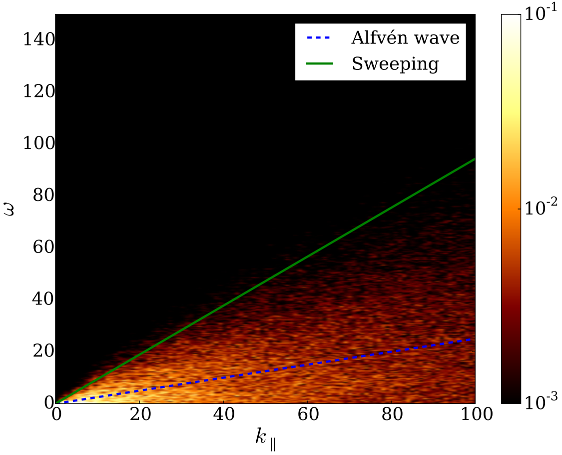
$B_0 = 8$



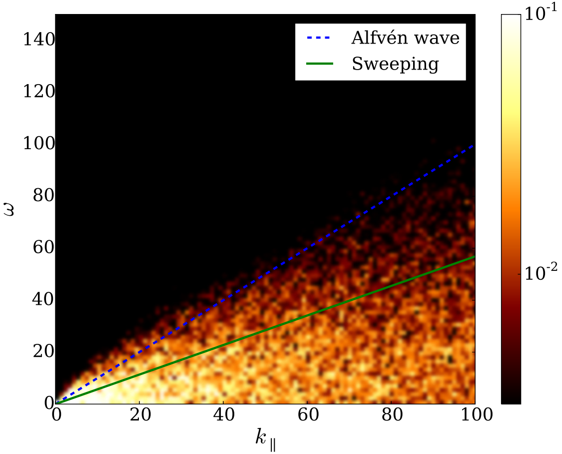
Normalized energy spectrum $E(\mathbf{k}, \omega)/E(\mathbf{k})$

Spatio-temporal spectra

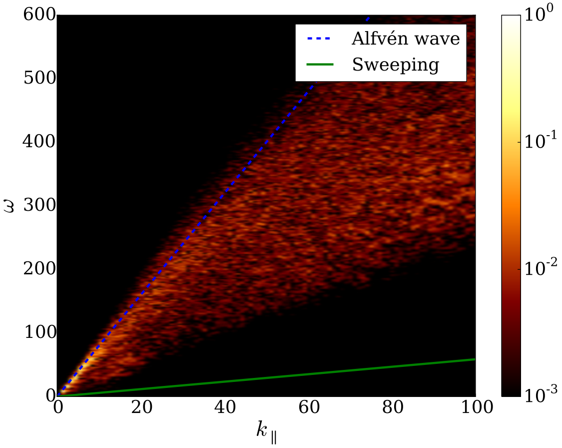
Normalized energy spectrum $E(\mathbf{k},\omega)/E(\mathbf{k})$: $B_0 = 0.25$



Normalized energy spectrum $E(\mathbf{k},\omega)/E(\mathbf{k})$: $B_0 = 1$

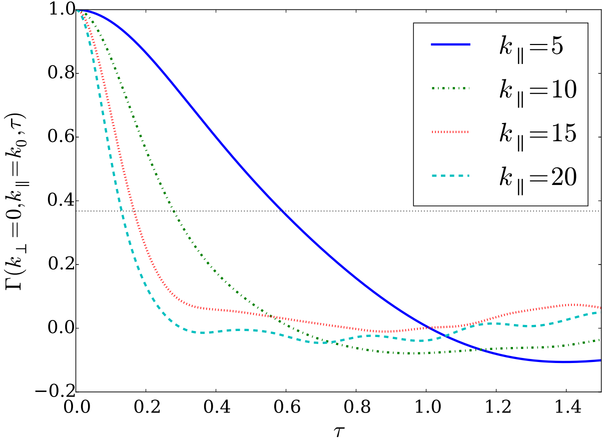


Normalized energy spectrum $E(\mathbf{k}, \omega)/E(\mathbf{k})$: $B_0 = 8$

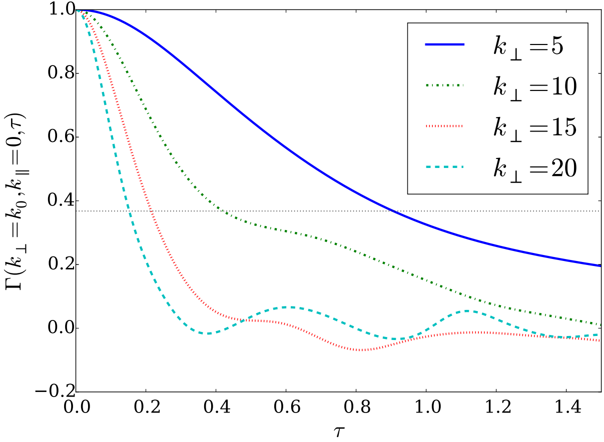


Correlation functions

Correlation function $\Gamma(k_\perp = 0, k_\parallel = k_0, \tau)$

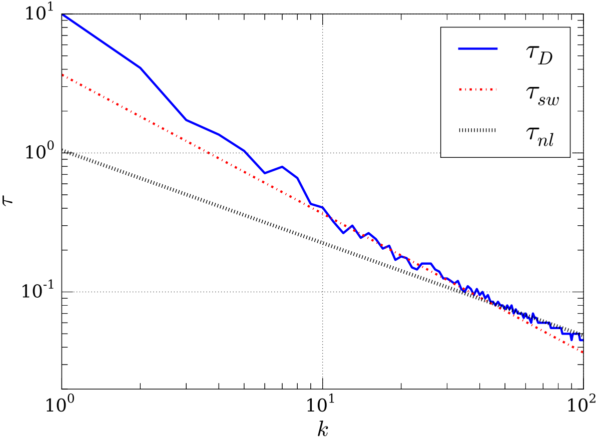


Correlation function $\Gamma(k_\perp = k_0, k_\parallel = 0, \tau)$



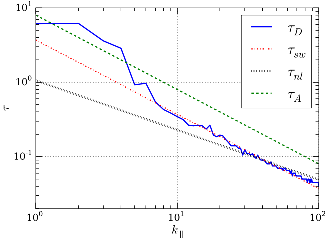
Decorrelation times

Decorrelation times for the isotropic case $B_0 = 0$

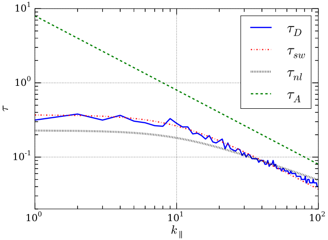


Decorrelation times: $B_0 = 0.25$ and $k_\perp = k_0$

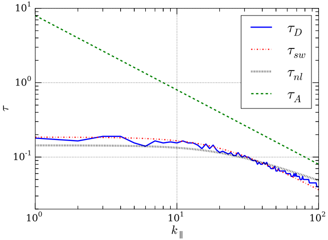
$k_\perp = 0$



$k_\perp = 10$

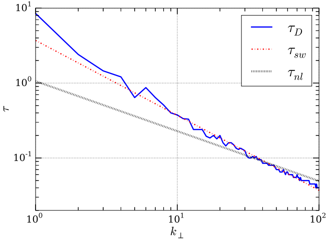


$k_\perp = 20$

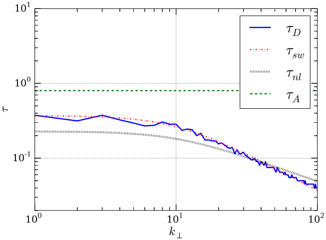


Decorrelation times: $B_0 = 0.25$ and $k_\parallel = k_0$

$k_\parallel = 0$



$k_\parallel = 10$



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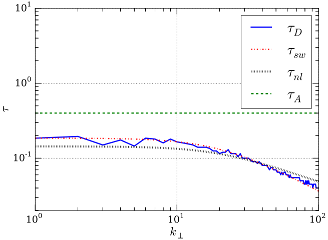
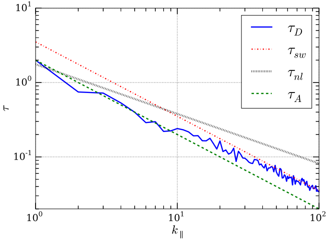
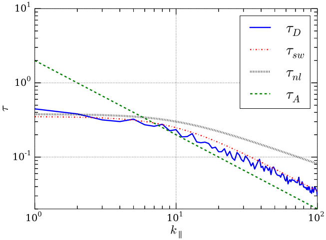


Decorrelation times: $B_0 = 1$ and $k_\perp = k_0$

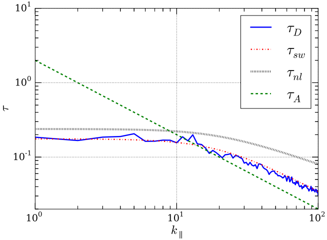
$k_\perp = 0$



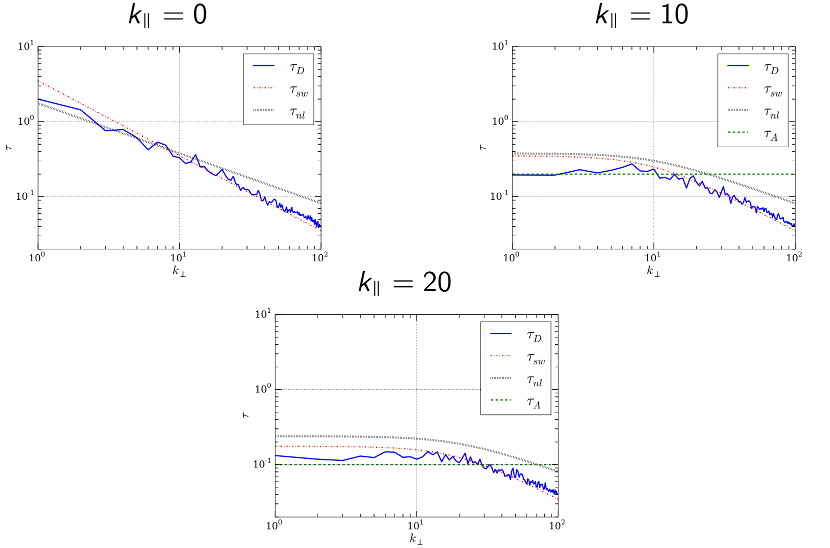
$k_\perp = 10$



$k_\perp = 20$

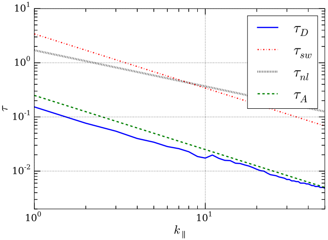


Decorrelation times: $B_0 = 1$ and $k_\parallel = k_0$

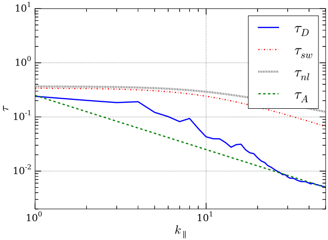


Decorrelation times: $B_0 = 8$ and $k_\perp = k_0$

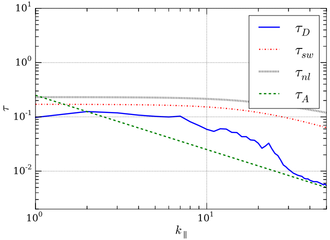
$k_\perp = 0$



$k_\perp = 10$

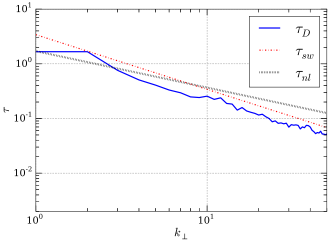


$k_\perp = 20$

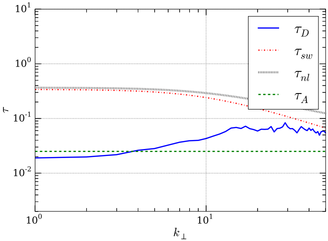


Decorrelation times: $B_0 = 8$ and $k_\parallel = k_0$

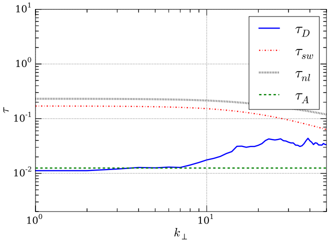
$k_\parallel = 0$



$k_\parallel = 10$



$k_\parallel = 20$



Contents

Conclusions

Conclusions

- We have studied the time correlations that enter into magnetohydrodynamics in the incompressible approximation.
- The results support the conclusion that non-local effects (in spectral space) play an important role in MHD turbulence and that decorrelations are mainly dominated by the sweeping and Alfvénic interactions.
- The analysis presented here can further distinguish between sweeping and Alfvénic effects, and the results support the conclusion that the sweeping interaction dominates the decorrelation for moderate values of B_0, while for large values of the mean field B_0 and at large scales (low perpendicular wavenumbers) the decorrelations are more controlled by the Alfvénic interactions.

Conclusions

- Our results further indicate that the system selects, in effect, the shortest decorrelation time available. As a result, even for large values of the guide field B_0, for sufficiently small scales in which the sweeping time becomes faster than the Alfvénic time, after a broad range of scales dominated by Alfvén waves the system transitions to a sweeping dominated behaviour.
- In MHD, both sweeping and Alfvén wave propagation contribute to the total time variation at a point (Eulerian frequency spectrum), and are therefore influential in limiting prediction.
- The observed behavior of MHD time decorrelation have applications in a number of subjects, including charged particle scattering theory (Schlickeiser 1993, Nelkin 1990), interplanetary magnetic field and magnetospheric dynamic (Miller 1997), and interpretation of spacecraft data from historical and future missions (Matthaeus 2016).

Apendix

Apendix: Axisymmetric energy spectrum

- Axisymmetric energy spectrum $e(k_\perp, k_\parallel, t)$

$$e(k_\perp, k_\parallel, t) = \sum_{\substack{k_\perp \leq |\mathbf{k}\times\hat{\mathbf{x}}| < k_\perp + 1 \\ k_\parallel \leq k_x < k_\parallel + 1}} |\hat{\mathbf{u}}(\mathbf{k}, t)|^2 + |\hat{\mathbf{b}}(\mathbf{k}, t)|^2 =$$

$$= \int \left(|\hat{\mathbf{u}}(\mathbf{k}, t)|^2 + |\hat{\mathbf{b}}(\mathbf{k}, t)|^2 \right) |\mathbf{k}| \sin\theta_k \; d\phi_k$$

- Reduced perpendicular energy spectrum $E(k_\perp)$

$$E\left(k_\perp\right) = \frac{1}{T} \int \int e(|\mathbf{k}_\perp|, k_\parallel, t)\, dk_\parallel \; dt,$$