Lab 2 Report: Mine Crafting
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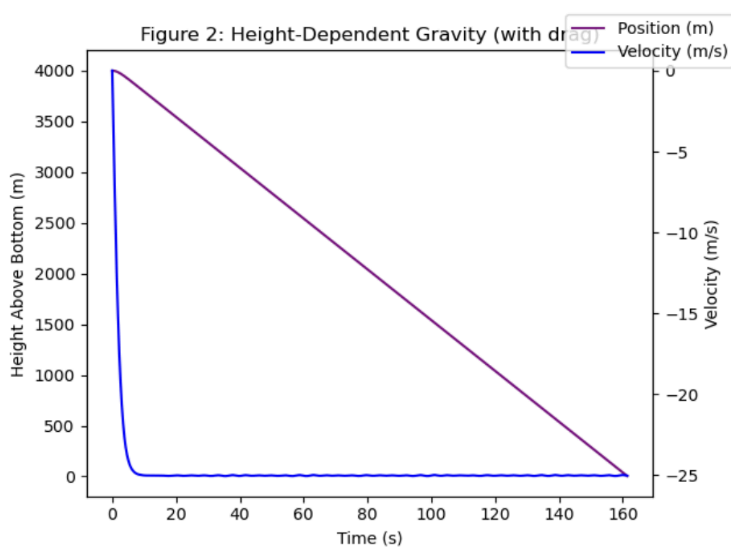
Introduction:

This study evaluates the depth of a vertical shaft that is estimated to be roughly 4 km at the equator through a series of calculations regarding dropping a 1 kg test mass down the shaft and recording the fall time. The first physical model was ideal free fall, considering solely gravity, and the models became increasingly more accurate by including other physical considerations like variable gravity, drag, Coriolis effect, and non-uniform Earth density. These considerations led to other hypothetical scenarios, such as a tunnel through Earth's core and a lunar shaft. The results that were obtained through this research were all done with Python's differential equation integrator.

Fall Time:

In the ideal scenario (no drag or variable gravity), a simple kinematic equation was used to calculate the fall time of the 1 kg test mass: $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2*4000}{9.81}}$ which yielded a result of 28.6 seconds. Using numerical integration, I refined this by using a height-dependent gravitational acceleration given by $g(y) = g_0(\frac{r}{r_{earth}})$ which resulted in a fall time of 28.6 seconds. Lastly, I considered drag in addition to height-dependent gravity to calculate the true fall time of the 1 kg mass. With a terminal velocity of 50 m/s, it took 161.5 seconds to reach the bottom of the shaft as shown in Figure 2. When gravity is modeled as a function of distance from the center of the Earth, it decreases linearly as the object goes deeper. The object accelerates more slowly over time, which results in a longer fall time. Drag increases as the square of the velocity, which also greatly increases the fall time of the object. The combination of height-dependent gravity and drag increases the fall time by approximately 130 seconds.



Feasibility of Depth Measurement:

In order to determine the feasibility of using a mine shaft to measure Earth's rotation with Coriolis deflection, I modeled the motion of the falling mass considering both vertical and horizontal acceleration due to gravity and Coriolis acceleration, respectively. The Coriolis force was modeled by the equation $F_c = -\ 2m(\Omega x v)$ where Ω is Earth's rotation rate along the z axis and m is the mass of the object (1 kg in this case). In the case with no drag, the maximum transverse displacement was 5.5 meters and hit the wall after descending 3997.6 meters, just 2.4 meters above the ground as shown in Figure 3. Drag does have an effect on the motion of the falling mass in regards to the Coriolis force. Because the falling time is longer as a result of force, there is more transverse displacement. With drag, the transverse displacement would be 941.8 meters and would strike the wall after descending only 1161.9 meters. This feasible approach measures whether the shaft is wide enough to accommodate for the horizontal drift of the object.

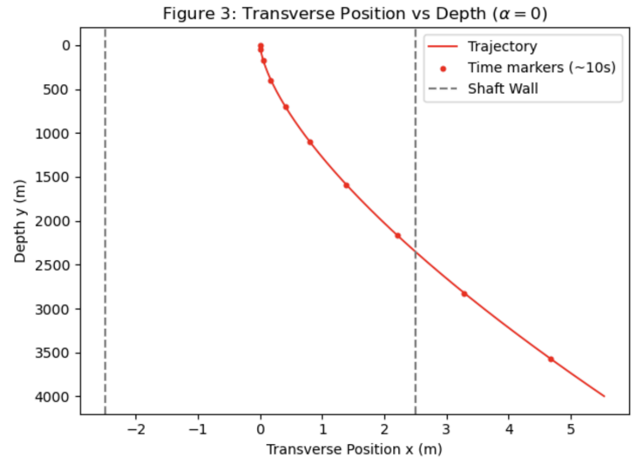


Figure 3: Transverse Position vs Depth ($\alpha = 0$)

Calculation of Crossing Times:

To understand the impact of internal density on fall times through a planetary body, I modeled the falling of an object through the Earth and Moon using the radial density profile $p(r) = \rho_n(1 - \frac{r^2}{R^2})^{\ n}$ where n=0 is a uniform density and n=9 is a core-concentrated density. With varying density profiles, I calculated the fall time and the maximum speed at the center of the planetary body. Figure 5 shows that a higher concentration of mass near the center leads to a weaker gravitational field near the surface. This affects the total crossing time. On Earth, when n=0, the time to center is 4492.6 seconds and the speed at the center is 2230 m/s. For n=1, n=2, and n=9, the time to center values are 3887 seconds, 3669.3 seconds, and 3345.6 seconds respectively. The speeds at the center are 2950 m/s, 3436.7 m/s, and 5182.3 m/s respectively. As n

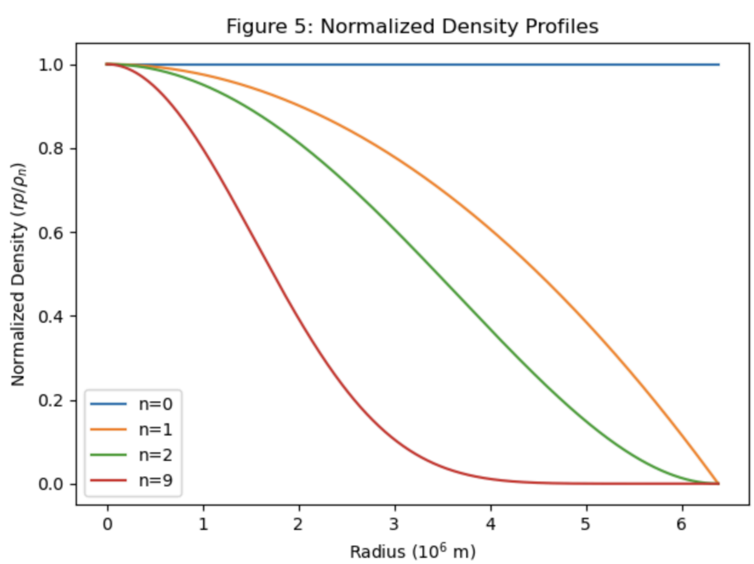


Figure 5: Normalized Density Profiles

increases, the mass of the planet is much more concentrated near the core. Earth's density is 5494.9 kg/m^3 and the Moon's density is 3341.8 kg/m^3, and the ratio between the two is .608. The fall time depends on the density of the planetary body because it scales like 1 over the square root of the density. This is not a linear relationship.

Discussion:

Throughout this report I investigated the dynamics of free-fall under varying physical conditions on different planetary bodies. My findings revealed that falling time through Earth depends greatly on density distribution and height-dependent gravity. I found that air drag suppresses lateral motion that is caused by the Coriolis effect.