

CSC 473, Fall 2020

# Automata, Grammars, and Languages

Eric Anson



# CSC 473: Automata, Grammars, Languages

**Class website** <https://piazza.com/arizona/fall2020/csc473/home>

D2L – videos, quizzes, calendar

GradeScope – turning in assignments and exams

- **Professor: Eric Anson**

- [eanson@email.arizona.edu](mailto:eanson@email.arizona.edu)

- Office Hours: Mon 2-3, Wed 3-4 – via zoom

- **Teaching Assistants:**

- Jesse Liu

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- Jack Zhang

- Office Hours: TBA

- **Class email:**

- [cs473f20@cs.arizona.edu](mailto:cs473f20@cs.arizona.edu)

# CSC 473: Learning Outcomes

From the **ACM-IEEE 2013 Curriculum Guidelines**

- Discuss the concept of finite state machines.
- Design a deterministic finite state machine to accept a specified language.
- Generate a regular expression to represent a specified language.
- Design a context-free grammar to represent a specified language.
- Define the classes P and NP.
- Explain the significance of NP-completeness.
- Explain why the halting problem has no algorithmic solution.

- We will be online until phase 3 begins (date???)
- Then those who wish will see lectures live in Chemistry 134
- Lectures will be recorded for viewing at another time.
- Lectures will also be on zoom.
- **Because they are being recorded, please leave your cameras off.**
  - I miss seeing students faces, but for the recording it is better to have just one video source up.
- After Thanksgiving all classes will be online again for the remainder of the semester.

- When (if) we are in class together everyone must wear a face mask.
- It is actual school policy that everyone wear a mask everywhere on campus. (except in your own room/office or outside where a distance of 6ft can be maintained)
- Also, please use social distancing. No students should sit next to any other student (the room is huge)
- This is a crazy time and I will be SOOO glad when it is over, but until then we all have to do our part to slow the spread of the disease.

# About CSC 473: Grading

- **Two Midterms (each 22%)** **44%**
  - Dates: ~September 30 and November 4
- **Assignments** **20%**
  - These will be written assignments that consist mostly of questions from the text.
- **Final** **30%**
  - On all the material covered in the course
- **Quizzes** **6%**

- Required:
- *Sipser, Michael; Introduction to the Theory of Computation, 3<sup>rd</sup> Edition; (Cengage Learning)*

# Assignments

- There will be written assignments that will be 20% of your grade. These assignments will be mostly (if not entirely) from the book.
- Homework is for you to learn the material and most of your grade comes from tests so . . .
- Unless you are told otherwise, working on homework with your friends is NOT cheating for this class. BUT NEVER copy your work from any source.
- So to restate copying is cheating but discussing the problems (unless told otherwise) is not.
- You must create a pdf of your assignment and upload it to GradeScope.



- This is a strange semester. I hope some of you will want to attend class when/if it is given in person.
- It would also be beneficial to attend the zoom session live so you can ask questions.
- You will be responsible for material covered in class even if it does not appear in the book.
- 6% of the final grade will come from quizzes. Most of the time quizzes will refer directly from the lectures, so attending or listening to the lectures will be required.

# Professor: Dr. Eric Anson

## CSC 473: Automata, Grammars, and Languages

- BS Math/Comp Sci - Pepperdine University - 1985
- MS Mathematics - University of Arizona - 1993
- PhD Comp Sci - University of Arizona – 2000
- Joined faculty of University of Arizona in fall 2015
- Have taught 101, 210, 245, 345, 352, 473
- Also taught math courses in 1988-1993 as a TA
- Took the graduate version of this class while in the math department in 1991

# Why Theory For Me?

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2. There is nothing “fuzzy” and there are no calculation errors.
3. Your lab is your brain.
4. Theory never becomes obsolete.

# What will we be studying in this class?

1. **Finite Automata and Regular Expressions**
2. **Pushdown Automata and Context Free Grammars**
3. **Turing Machines and Complexity**

# Finite Automata and Regular Languages

Applications to search, compilers, software verification, protocols, circuits, ...

- String matching and searching
- Lexical analysis in compilers
- Software Verification
- Communication Protocols
- Software for designing and checking the behavior of digital circuits.
- Software for scanning large bodies of text
- Vending machines
- Video games



# Why Study Grammars and PDAs?

**Applications to programming language specifications, compilers, manuals**

- **Computer Languages**
- **Compiler Front Ends**
- **Code Generation**
- **Natural Language Processing**
- **Computational Biology**

# Computational Complexity (Turing Machines)

**Theory provides the tools for dealing with inherently hard problems**

- **There are limitations on what software can do**
  - **Intractable** problems: there are programs, but no known fast programs
  - **Undecidable** problems: there can be no program to solve them
- **The class of seemingly unrelated intractable problems**
  - Precise term: the class of NP-complete problems
  - A fast solution to one implies fast solutions to all of them
  - Examples of NP-complete problems:  
Traveling Salesman, Multiple Sequence Alignment
- **What can we do?**
  - Settle for a less than perfect or approximate answer

# Preparation

- **Read Chapter 0 and 1.1**
- **This class will involve many proofs, especially inductive proofs. You may want to review those.**

- **A set is a group of objects.**

- unordered
- No notion of multiple occurrences of an element

- **Subsets**

- Given two sets A and B we say A is a subset of B ( $A \subset B$ ) iff  $\forall x, x \in A \Rightarrow x \in B$
- A is a proper subset of B iff  $A \subset B$  AND  $\exists x \in B$  s.t.  $x \notin A$
- Make sure you also remember the definitions for union, intersection, etc.

- **Cartesian Product**

- The cartesian product of two sets A and B ( $A \times B$ ) is  $\{(x,y) : x \in A \text{ and } y \in B\}$

- **Cardinality**

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  - What if  $A$  is infinite?

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- **Question:**

- If  $A$  and  $B$  are finite, what is the cardinality of  $A \times B$ ? ( $|A \times B|$ )

- **Definition**

- A function or mapping from set A to set B ( $f: A \rightarrow B$ ) is an assignment of a single element of from B for each element of A.
  - How do we formalize this?



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- **1-1 (one to one)**

- A function  $f: A \rightarrow B$  is 1-1 or an injection iff
$$f(a) = f(b) \Rightarrow a = b$$
  - In other words f doesn't map two different elements of A to the same element of B

- **Onto**

- A function  $f: A \rightarrow B$  is onto or a surjection iff
$$\forall b \in B, \exists a \in A \text{ s.t. } f(a) = b$$
  - In other words every element in B is mapped to by some element of A

- **1-1 Correspondence**

- A function f is a 1-1 correspondence or bijection if it is 1-1 and onto

- **Comparing cardinality**

- For sets  $A$  and  $B$ , we say  $|A| = |B|$  iff  $\exists$  bijection  $f: A \rightarrow B$ 
  - This works for finite AND infinite sets
- We say  $|A| \leq |B|$  iff  $\exists$  1-1 function  $f: A \rightarrow B$

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  - Note all finite sets are countable
  - Many infinite sets are also countable (we often refer to them as countably infinite)

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- **Thm: Given sets  $A, B$  then  $|A| = |B|$  iff  $|A| \leq |B|$  and  $|B| \leq |A|$**

- In other words if there is an injection from  $A$  to  $B$  and an injection from  $B$  to  $A$ , then there is a bijection from  $A$  to  $B$ .
- We will skip the proof of this

- **Alphabet**

- An alphabet is a finite, nonempty set of symbols.
- We will denote the set by  $\Sigma$ 
  - For example  $\Sigma = \{0, 1\}$
  - or  $\Sigma = \{0, 1, 2, \dots, 9\}$

- **String**

- A string is a finite sequence (possibly empty) over some alphabet
- The empty string is denoted by  $\epsilon$
- The length of a string  $w$ , denoted by  $|w|$  is the number of symbols in  $w$  (counting repeats)
- The concatenation of two strings  $s$  and  $t$  is the string formed by appending  $t$  to  $s$

- **String (cont)**

- if  $w$  is a string and  $n$  is a nonnegative integer then  $w^n$  is the string  $w$  concatenated  $n$  times  
or defined recursively:
  - $w^0 = \epsilon$
  - $w^{i+1} = w^i w$
- if  $w$  is a string then define the reverse of  $w$  denoted  $w^R$  as
  - if  $|w| = 0$ ,  $w = \epsilon$  and  $w^R = w$
  - if  $|w| \geq 1$  then  $\exists a \in \Sigma$  s.t.  $w = xa$ ,  $w^R = (xa)^R = ax^R$
- Theorem: If  $w$  and  $x$  are strings then  $(wx)^R = x^R w^R$

- Def: Let  $\Sigma^i$  be all the strings over  $\Sigma$  of length  $i$
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- Def:  $\Sigma^* = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \dots$
- So  $\Sigma^*$  is all the strings over  $\Sigma$   
(\* is called the Kleene Star)
- How big is  $\Sigma^*$  ?

- Def: A language over an alphabet  $\Sigma$  is a subset of  $\Sigma^*$

- **Def: A language over an alphabet  $\Sigma$  is a subset of  $\Sigma^*$**
- **How many languages are there?**

- Languages are sets, so we can perform any set operation on them.
- We will can define concatenation and Kleene Star on languages.
  - Given languages  $L_1, L_2$  define
$$L_1L_2 = \{ w_1w_2 : w_1 \in L_1 \text{ and } w_2 \in L_2 \}$$
  - Now define  $L^* = L^0 \cup L^1 \cup L^2 \cup L^3 \cup \dots$

# Problem

- Is the following true or false (prove your answer)
- $\forall L_1, L_2 (L_1 = L_2 \text{ iff } L_1^* = L_2^*)$