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Phy207 Final Exam (Form1) Professor Zuo Spring 2015

*"On my honor, I have neither received nor given aid on this examination"*

Signature:____________________

Name:________________________ ID Number:_______________________

Enter your name and Form 1 (FM1) in the scantron sheet.

Do all 28 multiple choice questions and 2 out of 3 essay questions. Multiple choice questions are worth 2 1/2 pts each and the essay 15 pts each. Attempt only one of the last two essay problems. There is a formula sheet attached at the end.

This is a closed book exam, you must work independently! No collaboration is allowed.

Prohibited items: any electronic devices including cell phones and calculators, pens, backpacks, notes, books.

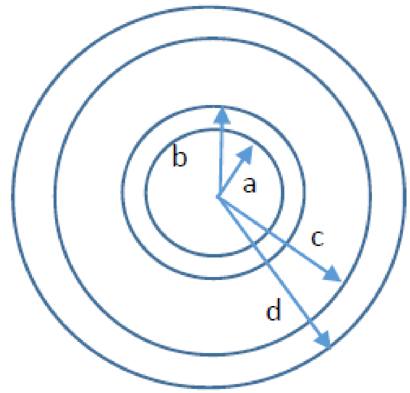
Anyone found cheating during the exam will automatically receive an "**F**" grade for the course and sent to the honor's court.

Put an X next to your discussion section:
[ ] Dr. Mezincescu 5O, 9:30 – 10:20 am
[ ] Dr. Dr. Zuo 5P 11:00 – 11:50 a.m.
[ ] Dr. Zuo 5R, 2:00 – 2:50 p.m.

For the next seven questions, consider two concentric ***spherical*** conducting shells with inner radius *a* and outer radius *b* for the inner shell and inner radius c and outer radius d for the outer shell are charged with *q* and *2q*, respectively.

1. Find the surface charge density at $r$=$d$

A) $\frac{q}{4\pi d^2}$ B) $\frac{2q}{4\pi d^2}$ C) $\frac{3q}{4\pi d^2}$ D) $\frac{-q}{4\pi d^2}$ E) 0




2. Find the electric field for $r$>$d$

A) $\frac{kq}{r^2}$ B) $\frac{2kq}{r^2}$ C) $\frac{3kq}{r^2}$ D)$-\frac{kq}{r^2}$ E) 0

3. Find the electric field for $b$<$r$<$c$

A) $\frac{kq}{r^2}$ B) $\frac{2kq}{r^2}$ C) $\frac{3kq}{r^2}$ D) $-\frac{kq}{r^2}$ E) 0

4. Find the electric potential at $r$=$c$, assuming $V(r=\infty)$=0

A) $\frac{kq}{d}$ B) $\frac{kq}{c}$ C) $\frac{3kq}{d}$ D) $\frac{3kq}{c}$ E) 0

5. Find the electric potential difference between the two shells.

A) $kq(\frac{1}{b}-\frac{1}{c})$ B) $kq(\frac{1}{a}-\frac{1}{c})$ C) $kq(\frac{1}{b}-\frac{1}{d})$ D) $kq(\frac{1}{b^2}-\frac{1}{c^2})$ E) 0

6. Find the electric field energy contained within a thin shell of radius *r* (*b*<*r*<*c*) and thickness dr.

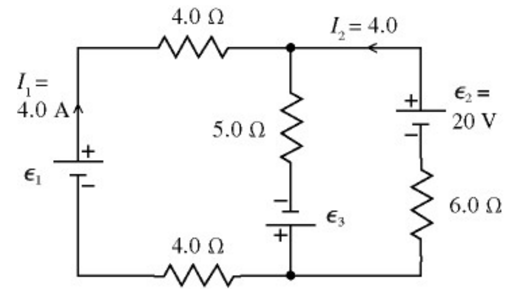
A) $\frac{1}{2}\varepsilon_o(\frac{kq}{r^2})^2 2\pi r dr$ B) $\frac{1}{2}\varepsilon_o(\frac{kq}{r^2})^2 4\pi r^2 dr$ C) $\frac{1}{2}\varepsilon_o(\frac{3kq}{r^2})^2 2\pi r dr$ D) $\frac{1}{2}\varepsilon_o(\frac{3kq}{r^2})^2 4\pi r^2 dr$
E) 0

7. Find the total electric field energy between the two shells.

A) $\frac{1}{2}\varepsilon_o(\frac{kq}{r^2})^2\frac{4\pi}{3}(c^3-b^3)$ B) $2\pi\varepsilon_o(kq)^2(\frac{1}{b}-\frac{1}{c})$ C) $2\pi\varepsilon_o(kq)^2(\frac{1}{a}-\frac{1}{d})$

D) $2\pi\varepsilon_o(kq)^2(\frac{1}{a}-\frac{1}{c})$ E) 0

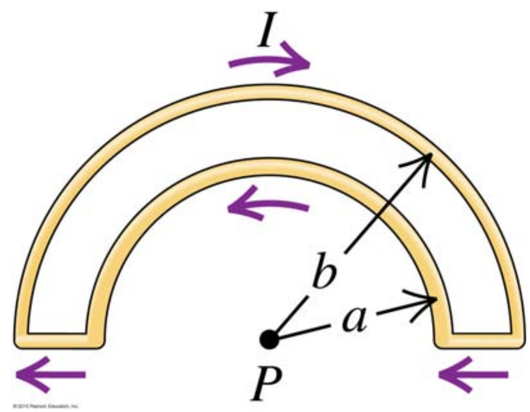
For the next two questions, consider the circuit shown on the right.
$I_1 = I_2 = 4.0$ A




8. What is the current going through the 5Ω resistor?
A) 2 A B) 4 A C) 6 A D) 8 A E) unknown

9. What is $\epsilon_3$ ?
A) 24V B) 44 V C) 64 V D) 84V E) unknown

For the next two questions, consider a current I going through the path shown.
10. Find the magnetic field at point P.

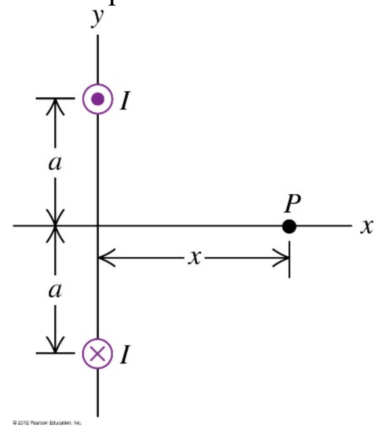



A) $\frac{\mu_o I}{2\pi}(\frac{1}{a}-\frac{1}{b})$
B ) $\frac{\mu_o I}{2\pi}(\frac{1}{a}+\frac{1}{b})$
C) $\frac{\mu_o I}{4}(\frac{1}{a}-\frac{1}{b})$
D) $\frac{\mu_o I}{4}(\frac{1}{a}+\frac{1}{b})$
E) 0

11. Find the magnetic dipole moment for the current configuration.
A) $\pi(b^2-a^2)I$ into the page
B) $\pi(b^2-a^2)I$ out of the page
C) $\frac{\pi}{2}(b^2-a^2)I$ into the page
D) $\frac{\pi}{2}(b^2-a^2)I$ out of the page
E) 0

12. Two long wires carrying current I as shown. What is the magnetic field at point P?

A) $\frac{\mu_o I}{\pi a}$ B) $\frac{\mu_o I}{\pi\sqrt{a^2+x^2}}$ C) $\frac{\mu_o I a}{\pi(a^2+x^2)}$ D) $\frac{\mu_o I x}{\pi(a^2+x^2)}$ E) 0

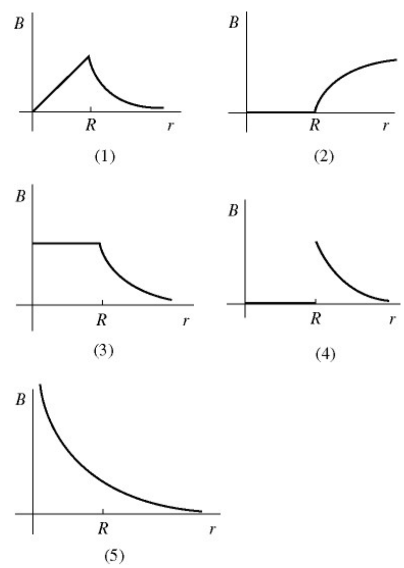



13. If one is to carry out the integral $\int_{-\infty}^{\infty} B(x)dx$, what will it equal to ? (There is no need to integrate it, but if you insist, you may find this useful $\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1} x$ )

A) 0 B) $\mu_o I$ C) $-\mu_o I$ D) $2\mu_o I$ E) None of above

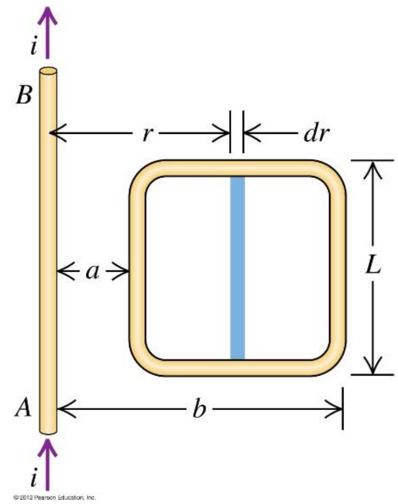
14. A very long, solid, conducting cylinder of radius $R$ carries a current along its length uniformly distributed throughout the cylinder. Which one of the graphs shown in the figure most accurately describes the magnitude $B$ of the magnetic field produced by this current as a function of the distance $r$ from the central axis?

A) 1
B) 2
C) 3
D) 4
E) 5



For the next four questions, consider a current in the long straight wire AB is **decreasing** steadily at a rate of $di/dt = -\alpha$.

15. At an instant when the current is i, what is the magnetic flux going through the narrow, shaded strip?



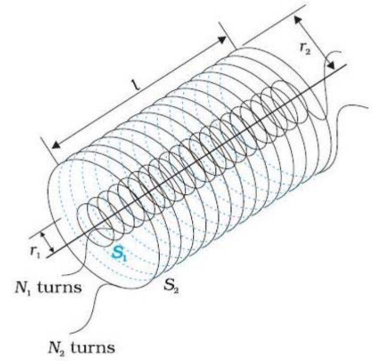

A) $\frac{\mu_o i}{2\pi r} Ldr$ B) $\frac{\mu_o i}{2r} Ldr$ C) $\frac{\mu_o i}{4\pi r^2} Ldr$
D) $\frac{\mu_o i}{2\pi r} L(b-a)$ E) 0

16. What is the total flux going through the loop?
A) $\frac{\mu_o i}{2} Lln(\frac{b}{a})$ B) $\frac{\mu_o i}{2\pi} Lln(\frac{b}{a})$ C) $\frac{\mu_o i}{2\pi r} L(b-a)$
D) $\frac{\mu_o i}{2\pi a} L(b-a)$ E) 0

17. What is the magnitude and direction of the induced current $I_{ind}$ in the loop if the loop has a resistance $R$?
A) $\frac{\mu_o \alpha}{2\pi R} Lln(\frac{b}{a})$, c.w
B) $\frac{\mu_o \alpha}{2\pi R} Lln(\frac{b}{a})$, c.c.w.
C) $\frac{\mu_o \alpha}{2R} Lln(\frac{b}{a})$, c.w.
D) $\frac{\mu_o i}{2\pi R} Lln(\frac{b}{a})$, c.w.
E) 0

18. What is the force acting on the loop in terms of $I_{ind}$?
A) $\frac{\mu_o i}{2\pi} LI_{ind}(\frac{1}{a} - \frac{1}{b})$, to the left
B) $\frac{\mu_o i}{2\pi} LI_{ind}(\frac{1}{a} - \frac{1}{b})$, to the right
C) $\frac{\mu_o i}{2\pi r} LI_{ind}$, to the left
D) $\frac{\mu_o i}{2\pi} LI_{ind} ln(\frac{b}{a})$, to the right
E) 0

For the next four questions, consider a small long solenoid $S_1$ placed inside a large long solenoid $S_2$ as shown. $S_1$ has $N_1$ turns and a length *l* and $S_2$ has the same length *l* but with $N_2$ turns with radius $r_1$ and $r_2$, respectively. If $S_1$ carries a time dependent current $i_1$ through it.




19. What is the magnetic field inside $S_1$?

A) $\mu_o \frac{N_1 i_1}{2r_1}$    B) $\mu_o \frac{N_1 i_1}{4\pi r_1^2}$    C) $\mu_o \frac{N_1 i_1}{l}$    D) $\mu_o \frac{N_2 i_1}{l}$    E) 0

20. What is the total flux going through $S_1$ in terms of magnetic field $B_1$ inside the $S_1$?

A) $\pi r_1^2 B_1$    B) $N_1 \pi r_1^2 B_1$    C) $N_2 \pi r_2^2 B_1$    D) 0    E) None of above

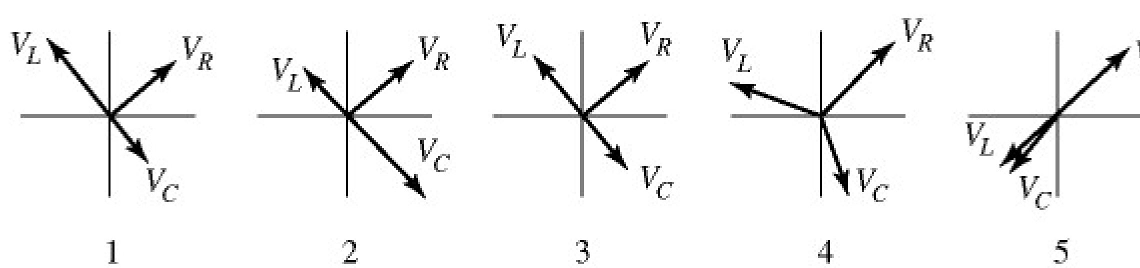
21. Now calculate the total flux going through the large solenoid $\emptyset_2$ due to $i_1$. What is the mutual inductance $M = \frac{\emptyset_2}{i_1}$ ?

A) $\frac{N_1 \pi r_1^2 B_1}{i_1}$    B) $\frac{N_2 \pi r_2^2 B_1}{i_1}$    C) $\frac{N_2 \pi r_1^2 B_1}{i_1}$    D) $\frac{N_1 \pi r_2^2 B_1}{i_1}$    E) None of above

22. If the voltages across $S_1$ and $S_2$ are $V_1$ and $V_2$, respectively. What is the relationship between the two voltages (neglect the resistance of the wires)?

A) $\frac{V_1}{V_2} = \frac{N_1}{N_2}$    B) $\frac{V_1}{V_2} = \frac{N_2}{N_1}$    C) $\frac{V_1}{V_2} = \frac{r_1}{r_2}$    D) $\frac{V_1}{V_2} = 0$    E) None of above

23. Which one of the phasor diagrams shown below best represents a series *LRC* circuit driven at resonance?




A) 1    B) 2    C) 3    D) 4    E) 5

24. Consider a *LRC* circuit that has a reactance (due to its capacitance) of 2 kΩ, a reactance (due to its inductance) of 5 kΩ, and a resistance of 4 kΩ. What is the phase shift between the total voltage and the current?
A) $-\tan^{-1}\frac{3}{4}$ (current leads voltage)
B) $+\tan^{-1}\frac{3}{4}$ (voltage leads current)
C) $\frac{\pi}{2}$
D) 0
E) None of above

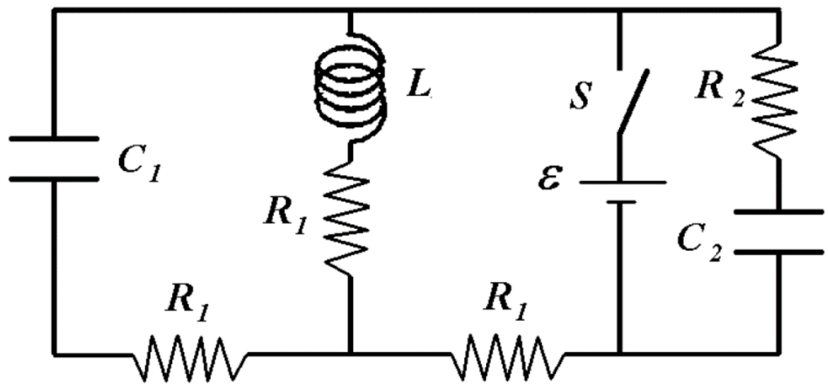
25. When a current of 2.0 A flows in the 100-turn primary of an ideal transformer, this causes 14 A to flow in the secondary. How many turns are in the secondary?
A) 700
B) 356
C) 114
D) 14
E) 4

26. If an electromagnetic wave has components $E_y = E_0 \sin(kx - \omega t)$ and $B_z = B_0 \sin(kx - \omega t)$, in what direction is it traveling?
A) $-x$
B) $+x$
C) $+y$
D) $-y$
E) $+z$

27. A laser with an average power of *P* has a beam radius of *R*. What is the peak value of the magnetic field in that beam?
A) $\sqrt{\frac{2c\mu_o P}{\pi R^2}}$ B) $\sqrt{\frac{2\mu_o P}{\pi R^2}}$ C) $\sqrt{\frac{2\mu_o P}{c\pi R^2}}$ D) $\sqrt{\frac{2c\varepsilon_o P}{\pi R^2}}$ E) $\sqrt{\frac{2\varepsilon_o P}{\pi R^2}}$

28. A very small source of light that radiates uniformly in all directions produces an electric field with an amplitude of $E_m$ at a distance *R* from the source. What is the amplitude of the magnetic field at a point *2R* from the source?
A) $\frac{E_m}{2c}$ B) $\frac{E_m}{4c}$ C) $\frac{E_m}{\sqrt{2}c}$ D) $\frac{E_m c}{2}$ E) $\frac{E_m}{c}$

29. In the circuit shown, switch *S* is closed at time *t=0*. Calculate:
A) The current going though $C_1$ immediately after S is closed.
B) The voltage cross the inductor *L* immediately after S is closed.
C) The energy stored in the inductor a long time after the switch is closed.
D) The energy stored in capacitor $C_1$ a long time after the switch is closed.
E) The power provided by the battery a long time after the switch is closed.




A) The current going through $C_1$ immediately after S is closed is given by

$I_{C1}=\varepsilon/2R_1$

B) Voltage across the inductor immediately after S is closed is given by

$V(t=0)=\frac{\varepsilon}{2R_1} * R_1 = \frac{\varepsilon}{2}$

C) Longtime after S is closed, the current going through the inductor is $\varepsilon/2R_1$, the energy stored by an inductor is $U_L = \frac{1}{2}LI^2 = \frac{1}{2}L(\frac{\varepsilon}{2R_1})^2 = \frac{L}{8}(\frac{\varepsilon}{R_1})^2$

D) Energy stored in a capacitor is $U_C = \frac{1}{2}C_1V^2 = \frac{1}{8}C_1\varepsilon^2$

E) Power provided by the battery is $P = I\varepsilon = \frac{\varepsilon^2}{2R_1}$

**<u>Do only one of next two problems</u>**

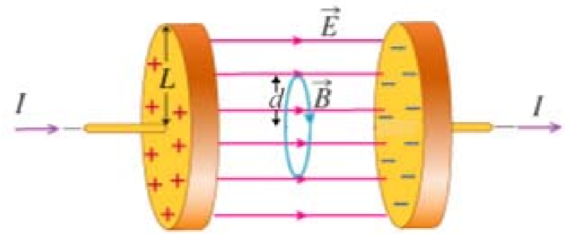
30. Consider a capacitor consisting of two very close parallel circular plates is initially uncharged. A current $I(t) = I_o e^{-t/\tau}$ is switched on at time t=0 and begins charging the capacitor. $L$ is the radius of the plate.

A) What is the amount charge $q$ on the capacitor at time t?

B) What is the electric field between the plates in terms of $q$ at time $t$? Assuming the plates are very large and the separation is very small)

C) What is the displacement current going through a loop of radius d, as shown?

D) Using the generalized Ampere's law, find the magnitude of the magnetic field at a distance $d$ from the center.




A) $q(t) = \int_0^t I(t)dt = \tau I_o(1 - e^{-t/\tau})$

B) The electric field inside a parallel plate capacitor is given by

$$E = \frac{\sigma}{\varepsilon_0} = \frac{q}{\varepsilon_0 \pi L^2}$$

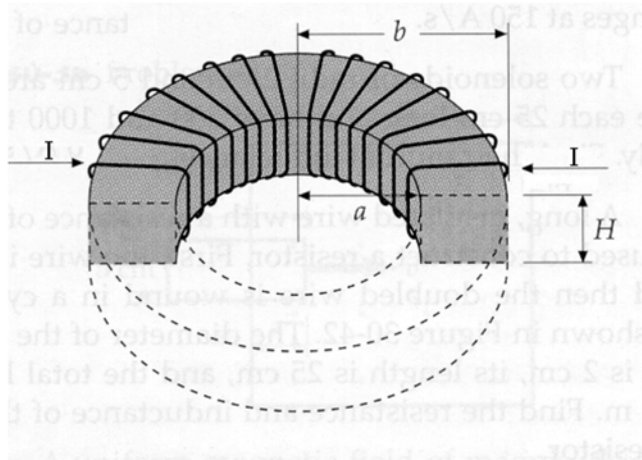
C) The displacement current is defined as

$$I_d = \varepsilon_0 \frac{d\emptyset}{dt} = \varepsilon_0 \pi d^2 \frac{dE}{dt} = \varepsilon_0 \pi d^2 \frac{d(\frac{q}{\varepsilon_0 \pi L^2})}{dt} = (\frac{d}{L})^2 \frac{dq}{dt} = \left(\frac{d}{L}\right)^2 I(t)$$

D) The generalized Ampere's law consider displacement current as equivalent to the conduction current, in the case here, there is only displacement current, so

$$B(d) = \frac{\mu_o I_d}{2\pi d} = \frac{\mu_o d}{2\pi L^2} I(t) = \frac{\mu_o d I_o}{2\pi L^2} e^{-t/\tau}$$

31. Consider a toroid of rectangular cross section as shown. The inner wall of the toroid is a distance a from the center and the outer wall a distance b from the center. The height of the toroid is H. The toroid is tightly wound with N turns of wire carrying a current I.




A) Find the magnitude of the magnetic field at distance r (a<r<b).

B) Find the total magnetic flux and the self-inductance of the toroid.

C) Find the magnetic field energy contained within a very small section of toroid of radius r and thickness dr and height H.

D) Use the result from C) to find the total magnetic field energy.

E) Find the self-inductance by using the result from D) and compare it with what you found in B).

A) Using Ampere's law, the magnetic field inside the toroid is given by

$$B(r) = \frac{\mu_o NI}{2\pi r}$$

B) The total flux is N times of the flux of a single loop, it can be calculated by integration over the loop

$$\emptyset = N\int_a^b BHdr = NH\frac{\mu_o NI}{2\pi}ln\frac{b}{a} = \frac{\mu_o N^2 H}{2\pi}ln\frac{b}{a}\ I$$

$$L = \frac{\emptyset}{I} = \frac{\mu_o N^2 H}{2\pi}ln\frac{b}{a}$$

C) The magnetic field energy contained in a volume element is given by

$$dU = \frac{B^2}{2\mu_o}2\pi r drH = \frac{\mu_o N^2 I^2 H}{4\pi r}dr$$

D) The total energy can be integrated to give $U_B = \int_a^b dU = \frac{\mu_o N^2 I^2 H}{4\pi}ln\frac{b}{a}$

E) The magnetic energy can also be expressed in terms of $U = \frac{1}{2}LI^2$

So $L = \frac{2U}{I^2} = \frac{\mu_o N^2 H}{2\pi}ln\frac{b}{a}$

Consistent with the calculation from B), as expected.