Exercise 2.2

Are the random variables X and Y in the joint ensemble of figure 2.2 independent?

Solution:

No, $P(x,y) \neq P(x)P(y)$ since each row or column are not proportional to each other.

Exercise 2.4

An urn contains K balls, of which B are black and $W = K - B$ are white. Fred draws a ball at random from the urn and replaces it, N times.

(a) What is the probability distribution of the number of times a black ball is drawn, n_B?

(b) What is the expectation of n_B? What is the variance of n_B? What is the standard deviation of n_B? Give numerical answers for the cases $N = 5$ and $N = 400$, when $B = 2$ and $K = 10$.

Solution:

(a) $P(n_B) = \binom{N}{n_B}(\frac{B}{K})^{n_B}(\frac{K-B}{K})^{N-n_B}$

(b) $\frac{B}{K} = \frac{1}{5}$, $\frac{K-B}{K} = \frac{4}{5}$ so the distribution is $B(n, \frac{1}{5})$.

Hence, $\mathrm{E}[n_b] = \frac{1}{5}n$, $\mathrm{Var}[n_B] = \frac{1}{5}n(1 - \frac{1}{5})$, $\mathrm{Std}[n_B] = \sqrt{\mathrm{Var}[n_B]}$

For n = 5: $\mathrm{E}[n_b] = 1$, $\mathrm{Var}[n_B] = \frac{4}{5}$, $\mathrm{Std}[n_B] = \sqrt{\frac{4}{5}}$

For n = 400: $\mathrm{E}[n_b] = 80$, $\mathrm{Var}[n_B] = 64$, $\mathrm{Std}[n_B] = 8$

Exercise

Solution: