

Exercise 1.2

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Show that the error probability is reduced by the use of R_3 by computing the error probability of this code for a binary symmetric channel with noise level f .

Answer:

Without R_3 , we have error probability f .

With R_3 , we have error probability

$${}_3C_2(1-f)f^2 + f^3 = 3f^2 - 3f^3 + f^3 = 3f^2 - 2f^3$$

Now,

$$\begin{aligned} f - (3f^2 - 2f^3) &= f - 3f^2 + 2f^3 \\ &= f(1 - 2f)(1 - f) > 0 \quad (\because f > \frac{1}{2}) \end{aligned}$$

so $f > 3f^2 - 2f^3$ hence error probability reduced.