Exercise 1.2

Show that the error probability is reduced by the use of R_3 by computing the error probability of this code for a binary symmetric channel with noise level f.

Solution:

Without R_3, we have error probability f.
With R_3, we have error probability

$$\binom{3}{2}(1-f)f^2 + f^3 = 3f^2 - 3f^3 + f^3 = 3f^2 - 2f^3$$

Now,

$$\begin{aligned} f - (3f^2 - 2f^3) &= f - 3f^2 + 2f^3 \\ &= f(1-2f)(1-f) > 0 \quad (\because f < \frac{1}{2}) \end{aligned}$$

so $f > 3f^2 - 2f^3$ hence error probability reduced.

Exercise 1.3

(a) Show that the probability of error of R_N, the repetition code with N repetitions, is

$$p_{\mathrm{b}} = \sum_{n=(N+1)/2}^{N} \binom{N}{n} f^n (1-f)^{N-n}, \tag{1.24}$$

for odd N.

(b) Assuming $f = 0.1$, which of the terms in this sum is the biggest? How much bigger is it than the second-biggest term?

(c) Use Stirling's approximation (p.2) to approximate the $\binom{N}{n}$ in the largest term, and find, approximately, the probability of error of the repetition code with N repetitions.

(d) Assuming $f = 0.1$, find how many repetitions are required to get the probability of error down to 10^{-15}. [Answer: about 60.]

Solution: