1ST
- First Order Predicate logic (FOPL)
- Clausal form Logic (CFL)
- Resolution Refutation
- Pages 11-14 from 1ST notes. ch.4.

1. Everyone who saves money earns interest. Show if there is no interest, then nobody saves money.

2. Every man is mortal. Confucius is a man. Is Confucius mortal?

3. If one is in Paris, then one is not in Moscow. John is in Paris. Is John in Moscow?

4. John is Mary's husband. Who is Mary's husband?

5. For all x, y and z, if x is the father of y and z is the father of x, then z is the grandfather of y. Everyone has a father. For all x, who is the grandfather of x?

6. Chemical Synthesis. Suppose we can perform the following chemical reactions:

$$M_gO + H_2 \longrightarrow M_g + H_2O$$
$$C + O_2 \longrightarrow CO_2$$
$$CO_2 + H_2O \longrightarrow H_2CO_3$$

Suppose we have some quantities of M_gO, H_2, O_2 and C. Show that we can make H_2CO_3.

7. Some patients like all doctors. No patients like any quack. Show that no doctor is a quack.

8. The customs officials searched everyone who entered this country who was not a VIP. Some of the drug pushers entered this country and they were only searched by drug pushers. No drug pusher was a VIP. Show that some of the officials were drug pushers.

9. Either it rains or it is hot. If it rains, then it is hot. If it does not rain, then it is not hot. Is it true that if it is hot, then it must not be raining.

10. All jokes are meant to amuse. No act of Parliament is a joke. Can you answer: is it true that every act of Parliament is not meant to amuse? If you can answer it, give the answer; if not, explain.

11. If an engine misfires and the spark at the plugs is intermittent, then the ignition leads are loose or the battery leads are loose. My engine misfires. The spark at my plugs is intermittent. My ignition leads are not loose. Show that my battery leads are loose.

12. Anyone married to a queen is male, and is a monarch or a prince. A male monarch is a king or a prince. Show that anyone married to a queen is a king or a prince.

13. The police computer recorded that Mr. Smallfry had not paid his parking fine. When he did pay it, the computer recorded the fact, but due to poor program design, did not wipe the statement that he had not. Show how the computer concluded that the Prime Minister was a spy.

14. Whoever can read is literate. Dolphins are not literate. Some dolphins are intelligent. Show that some who are intelligent cannot read.

15. Heads I win; tails you lose. Show that I win.

16. If a course is easy, some students are happy. If a course has an exam, no students are happy. Show that if a course has an exam, it is not easy.

17. Victor has been murdered, and Arthur, Bertram and Carleton are suspects. Arthur says he did not do it. He says that Bertram was the victim's friend but that Carleton hated the victim. Bertram says he was out of town the day of the murder, and besides he didn't even know the guy. Carleton says he is innocent and he saw Arthur and Bertram with the victim just before the murder. Assuming that everyone — except possibly for the murderer — is telling the truth, who committed the crime?

(Hints: You will need to add some axioms to capture world knowledge not given in the question. In particular, you will need to add the fact that only one person is guilty, i.e. `(I(A) ∨ I(B)) ∧ (I(A) ∨ I(C)) ∧ (I(B) ∨ I(C))`, where $I(x)$ means x is innocent. You also want to make each person's statements conditional on their innocence.)

Notes: There are often alternative (equivalent and hence equally correct) translations into FOPC to those given below. So the answers below are not "gospel". Furthermore, since I might have got the answers wrong, this is another reason not to treat the following as gospel. Come to see me if you're having problems.

1. FOPC Representation:

   1. $\forall x(\mathtt{S}(x) \supset \mathtt{I}(x))$
   2. To show $\forall x \neg \mathtt{I}(x) \supset \neg \mathtt{S}(x))$.

[An alternative is: 1. $\forall x(\exists y(\mathtt{S}(x,y) \land \mathtt{M}(y)) \supset \exists z(\mathtt{I}(z) \land \mathtt{E}(x,z)))$. 2. To show $\neg u \mathtt{I}(u) \supset \forall v \forall w((\mathtt{S}(v,w) \supset \neg \mathtt{M}(w))$].

2. FOPC Representation:

   1. $\forall x\ (\mathtt{MAN}(x) \supset \mathtt{MORTAL}(x))$ MAN(Conf)
   2. To show $\mathtt{MORTAL(Conf)}$.

3. FOPC Representation:

   1. $\forall x(\mathtt{P}(x) \supset \neg \mathtt{M}(x))$ P(J)
   2. To show $\mathtt{M(J)}$.

4. FOPC Representation:

   1. $\mathtt{HUSB(J,\ M)}$
   2. To show $\exists x \mathtt{HUSB}(x,\ \mathtt{M})$.

5. FOPC Representation:

   1. $\forall x \forall y \forall z((\mathtt{F}(x,y) \land \mathtt{F}(z,x)) \supset \mathtt{GF}(z,y))$
   2. $\forall x \exists y \mathtt{F}(y,x)$
   3. To show $\exists x \exists y \mathtt{GF}(y,x)$.

6. FOPC Representation:

   1. $(M_gO \land H_2) \supset (M_g \land H_2O)$
   2. $(C \land O_2) \supset CO_2$
   3. $(CO_2 \land H_2O) \supset H_2CO_3$
   4. M_gO
   5. H_2
   6. O_2
   7. C
   8. To show H_2CO_3.

7. FOPC representation:

   1. $\exists x(\mathtt{P}(x) \land \forall y(\mathtt{D}(y) \supset \mathtt{L}(x,y)))$
   2. $\forall x(\mathtt{P}(x) \supset \forall y(\mathtt{Q}(y) \supset \neg \mathtt{L}(x,y)))$
   3. To show $\forall x(\mathtt{D}(x) \supset \neg \mathtt{Q}(x))$.

8. FOPC representation:

   1. $\forall x((\mathtt{E}(x) \land \neg \mathtt{V}(x)) \supset \exists y(\mathtt{S}(x,y) \land \mathtt{C}(y)))$
   2. $\exists x(\mathtt{P}(x) \land \mathtt{E}(x) \land \forall y(\mathtt{S}(x,y) \supset \mathtt{P}(y)))$
   3. $\forall x(\mathtt{P}(x) \supset \neg \mathtt{V}(x))$
   4. To show $\exists x(\mathtt{P}(x) \land \mathtt{C}(x))$.

9. FOPC representation:

   1. If you interpret "or" as inclusive: RAIN $\lor$ HOT; if you interpret it as exclusive: (RAIN $\lor$ HOT) $\land \neg$(RAIN $\land$ HOT)
   2. RAIN $\supset$ HOT
   3. $\neg$RAIN $\supset$ $\neg$HOT
   4. To show HOT $\supset$ $\neg$RAIN.

10. FOPC representation:

    1. $\forall x\ (\mathtt{J}(x) \supset \mathtt{MTA}(x))$
    2. $\forall x\ (\mathtt{AOP}(x) \supset \neg \mathtt{J}(x))$
    3. To show $\forall x\ (\mathtt{AOP}(x) \supset \neg \mathtt{MTA}(x))$.

11. FOPC representation:

    1. (MISFIRE $\land$ INT) $\supset$ (IGLOOSE $\lor$ BATLOOSE)
    2. MISFIRE
    3. INT
    4. $\neg$IGLOOSE
    5. To show BATLOOSE.

12. FOPC representation:

    1. $\forall x \forall y((\mathtt{MAR}(x,y) \land \mathtt{Q}(y)) \supset (\mathtt{MALE}(x) \land (\mathtt{MON}(x) \lor \mathtt{PR}(x)))$
    2. $\forall x((\mathtt{MON}(x) \land \mathtt{MALE}(x)) \supset (\mathtt{K}(x) \lor \mathtt{PR}(x))$
    3. To show: $\forall x \forall y((\mathtt{MAR}(x,y) \land \mathtt{Q}(y)) \supset (\mathtt{K}(x) \lor \mathtt{PR}(x)))$.

13. FOPC representation:

1. Originally in computer $\neg$PAID(Smf)
2. Added: PAID(Smf)
3. To show SPY(PM).

14. FOPC representation:

1. $\forall x$(R(x) $\supset$ L(x))
2. $\forall x$(D(x) $\supset \neg$L(x))
3. $\exists x$(D(x) $\wedge$ I(x))
4. To show: $\exists x$(I(x) $\wedge\neg$R(x)).

15. FOPC representation:

1. HEADS $\supset$ WIN(me)
2. TAILS $\supset$ LOSE(you)
3. $\neg$HEADS $\supset$ TAILS
4. LOSE(you) $\supset$ WIN(me)
5. To show WIN(me).

[This example will work out more concisely later if the FOPC is as follows: HEADS $\supset$ I-WIN; $\neg$HEADS $\supset$ I-WIN; To show: I-WIN.]

16. FOPC representation:

1. $\forall x$((C(x) $\wedge$ E(x)) $\supset \exists y$(S(y) $\wedge$ H(y)))
2. $\forall x$((C(x) $\wedge$ HASEXAM(x)) $\supset$ ($\forall y$ (S(y) $\supset \neg$H(y))
3. To show: $\forall x$((C(x) $\wedge$ HASEXAM(x)) $\supset \neg$E(x)).

17. Victor has been murdered, and Arthur, Bertram and Carleton are suspects. [Since only one is guilty, we get: 1. (I(A) $\vee$ I(B)) $\wedge$ (I(A) $\vee$ I(C)) $\wedge$ (I(B) $\vee$ I(C))]. [The things A says will be dependent on I(A).] Arthur says he did not do it. [i.e. (IA) $\supset$ I(A): we can ignore this.] He says that Bertram was the victim's friend [i.e. 2. I(A) $\supset$ FRIEND(B, V)] but that Carleton hated the victim [i.e. 3. I(A) $\supset \neg$ FRIEND(C, V)]. Bertram says he was out of town the day of the murder, [i.e. 4. I(B) $\supset \neg$IN-TOWN(B)] and besides he didn't even know the guy [i.e. 5. I(B) $\supset \neg$ KNOW(B, V)]. Carleton says he is innocent [i.e. I(C) $\supset$ I(C), which we can ignore] and he saw Arthur and Bertram with the victim just before the murder [i.e. 6. I(C) $\supset$ SAW-WITH(A, V) and 7. I(C) $\supset$ SAW-WITH(B, V)]. Assuming that everyone — except possibly for the murderer — is telling the truth, who committed the crime? [i.e. the goal is $\exists x\neg$I(x)].

Other world knowledge is that anyone who was with Victor must have been in town: 9. $\forall x$(SAW-WITH(x, V) $\supset$ IN-TOWN(x)). And that friends know each other: 10. $\forall x \forall y$FRIEND(x,y) $\supset$ KNOW(x,y).