1. CFL:

   1. $\neg\mathtt{S}(x_1) \lor \mathtt{I}(x_1)$
   2. Negated goal: $\neg\forall x \neg\mathtt{I}(x) \supset \neg\mathtt{S}(x))$ becomes:
   2. a. $\neg\mathtt{I(a)}$
   2. b. $\mathtt{S(a)}$

[The alternative gives : 1. a. $\neg\mathtt{S}(x_1, y_1) \lor \neg\mathtt{M}(y_1) \lor \mathtt{I(f(}x_1\mathtt{))}$. 1. b. $\neg\mathtt{S}(x_2, y_2) \lor \neg\mathtt{M}(y_2) \lor \mathtt{E}(x_2,\ \mathtt{f}(x_2))$. 2. Negated goal: $\exists u\mathtt{I}(u) \supset \forall v\forall w((\mathtt{S}(v,w) \supset \neg\mathtt{M}(w))$ becomes: 2. a. $\neg\mathtt{I}(u)$. 2. b. $\mathtt{S(a,\ b)}$. 2. c. $\mathtt{M(b)}$.]

2. CFL:

   1. $\neg\mathtt{MAN}(x_1) \lor \mathtt{MORTAL}(x_1)$ MAN(Conf)
   2. Negated goal: $\neg\mathtt{MORTAL(Conf)}$ which doesn't change in CFL.

3. CFL:

   1. $\neg\mathtt{P}(x) \lor \neg\mathtt{M}(x)$ P(J)
   2. Negated goal: $\neg\mathtt{M(J)}$ which doesn't change in CFL.

4. CFL:

   1. Same as in FOPC.
   2. Negated goal: $\neg\exists x\mathtt{HUSB}(x,\ \mathtt{M})$ which becomes $\neg\mathtt{HUSB}(y,\ \mathtt{M})$ which itself becomes $\neg\mathtt{HUSB}(y,\ \mathtt{M}) \lor \mathtt{ANS}(y)$

5. CFL:

   1. $\neg\mathtt{F}(x_1, y_1) \lor \neg\mathtt{F}(z_1, x_1) \lor \mathtt{GF}(z_1, y_1)$
   2. $\mathtt{F(f(}w\mathtt{)},\ w)$
   3. Negated goal: $\neg\exists x\exists y\mathtt{GF}(y, x)$ which becomes $\neg\mathtt{GF}(y_2, x_2) \lor \mathtt{ANS}(y_2)$

6. CFL:

   1. a. $\neg M_gO \lor \neg H_2 \lor M_g$,
   1. b. $\neg M_gO \lor \neg H_2 \lor H_2O$
   2. $\neg C \lor \neg O_2 \lor CO_2$
   3. $\neg CO_2 \lor \neg H_2O \lor H_2CO_3$
   4. M_gO
   5. H_2
   6. O_2
   7. C
   8. Negated goal: $\neg H_2CO_3$ becomes $\neg H_2CO_3$ in CFL

7. CFL:

1. a. $\texttt{P(a)}$
1. b. $\neg\texttt{D}(y) \vee \texttt{L(a,}\ y\texttt{)}$
2. a. $\neg\texttt{P}(x_1) \vee \neg\texttt{Q}(y_1) \vee \neg\texttt{L}(x_1, y_1)$
2. b. $\neg\texttt{P}(x_2) \vee \neg\texttt{Q}(y_2) \vee \neg\texttt{L}(x_2, y_2)$
3. Negated goal: $\neg\forall x(\texttt{D}(x) \supset \neg\texttt{Q}(x))$ becomes
3. a. $\texttt{D(b)}$
3. b. $\texttt{Q(b)}$

8. CFL:

1. a. $\neg\texttt{E}(x_1) \vee \texttt{V}(x_1) \vee \texttt{S}(x_1,\ \texttt{f}(x_1))$
1. b. $\neg\texttt{E}(x_2) \vee \texttt{V}(x_2) \vee \texttt{C(f}(x_2))$
2. a. $\texttt{P(a)}$
2. b. $\texttt{E(a)}$
2. c. $\neg\texttt{S(a,}\ y) \vee \texttt{P}(y)$
3. $\neg\texttt{P}(x_3) \vee \neg\texttt{V}(x_3)$
4. Negated goal: $\neg\exists x(\texttt{P}(x) \wedge \texttt{C}(x))$ becomes $\neg\texttt{P}(x_4) \vee \neg\texttt{C}(x_4)$

9. CFL:

1. If inclusive: RAIN $\vee$ HOT; if exclusive: 1.a. RAIN $\vee$ HOT; 1.b. $\neg$RAIN $\vee\neg$HOT
2. $\neg$RAIN $\vee$ HOT
3. RAIN $\vee\neg$HOT
4. Negated goal: $\neg$(HOT $\supset$ $\neg$RAIN) becomes:
4. a. HOT
4. b. RAIN

10. CFL:

1. $\neg\texttt{J}(x) \vee \texttt{MTA}(x)$
2. $\neg\texttt{AOP}(y) \vee \neg\texttt{J}(y)$
3. Negated goal: $\neg\forall x\ (\texttt{AOP}(x) \supset \neg\texttt{MTA}(x))$ becomes:
3. a. $\texttt{AOP(a)}$
3. b. $\texttt{MTA(a)}$

11. CFL:

1. $\neg$MISFIRE $\vee$ $\neg$INT $\vee$ IGLOOSE $\vee$ BATLOOSE
2-4. as in the original FOPC.
5. Negated goal : $\neg$BATLOOSE doesn't change either.

12. CFL:

1. a. $\neg$MAR(x_1, y_1) $\vee\neg$Q(y_1) $\vee$ MALE(x_1)
1. b. $\neg$MAR(x_2, y_2) $\vee\neg$Q(y_2) $\vee$ MON(x_2) $\vee$ PR(x_2)
2. $\neg$MON(x_3) $\vee\neg$MALE(x_3) $\vee$ K(x_3) $\vee$ PR(x_3)
3. Negated goal: $\neg\forall x\forall y$ ((MAR(x,y) $\wedge$ Q(y)) $\supset$ (K(x) $\vee$ PR(x)))
becomes:
3. a. MAR(a, b)
3. b. Q(b)
3. c. $\neg$K(a)
3. d. $\neg$PR(a)

13. CFL:

1-2. as in the orginal FOPC.
3. Negated goal: $\neg$SPY(PM) doesn't change either.

14. CFL:

1. $\neg$R(x_1) $\vee$ L(x_1)
2. $\neg$D(x_2) $\vee\neg$L(x_2)
3. a. D(flipper)
3. b. I(flipper)
4. Negated goal: $\neg\exists x$(I(x) $\wedge\neg$R(x)) becomes: $\neg$I(x_3) $\vee$ R(x_3)

15. CFL:

1. $\neg$HEADS $\vee$ WIN(me)
2. $\neg$TAILS $\vee$ LOSE(you)
3. HEADS $\vee$ TAILS
4. $\neg$LOSE(you) $\vee$ WIN(me)
5. Negated goal: $\neg$WIN(me) doesn't change in CFL.

16. CFL:

1. a. $\neg$C(x_1) $\vee\neg$E(x_1) $\vee$ S(f(x_1))
1. b. $\neg$C(x_2) $\vee\neg$E(x_2) $\vee$ H(f(x_2))
2. $\neg$C(x_3) $\vee\neg$HASEXAM(x_3) $\vee\neg$S(y) $\vee\neg$ H(y)
3. Negated goal: $\neg\forall x$((C(x) $\wedge$ HASEXAM(x)) $\supset\neg$E(x)) becomes
3. a. C(a)
3. b. HASEXAM(a)
3. c. E(a)

17.

In CFL:

1. a. I(A) $\vee$ I(B)
1. b. I(A) $\vee$ I(C)
1. c. I(B) $\vee$ I(C)
2. $\neg$I(A) $\vee$ FRIEND(B, V)
3. $\neg$I(A) $\vee\neg$FRIEND(C, V)
4. $\neg$I(B) $\vee\neg$IN-TOWN(B)
5. $\neg$I(B) $\vee\neg$KNOW(B, V)
6. $\neg$I(C) $\vee$ SAW-WITH(A, V)
7. $\neg$I(C) $\vee$ SAW-WITH(B, V)
8. Negated goal: $\neg\exists x \neg$I(x) becomes I(x_3) which itself becomes I(x_3) $\vee$ ANS(x_3)
9. $\neg$SAW-WITH(x_1, V) $\vee$ IN-TOWN(x_1)
10. $\neg$FRIEND(x_2, y_2) $\vee$ KNOW(x_2, y_2)

As I have said in lectures, proofs should be shown as trees in exam answers!

1. Proof:

   1 + 2a = 3: $\neg$`S(a)`
   3 + 2b = $\Box$

2. Proof:

   1 + 2 = 4: `MORTAL(Conf)`
   4 + 3 = $\Box$

3. Proof:

   1 + 2 = 4: $\neg$`M(J)`
   4 + 3 = $\Box$

4. Proof:

   1 + 2 = `ANS(J)`

5. Proof:

   1 + 3 = 4: $\mathtt{ANS}(y_3) \lor \neg\mathtt{F}(x_4, x_5) \lor \neg\mathtt{F}(y_3, x_4)$
   4 + 2 = 5: $\mathtt{ANS}(\mathtt{f}(x_6)) \lor \neg\mathtt{F}(x_6, x_7)$
   5 + 2 = 6: $\mathtt{ANS}(\mathtt{f}(\mathtt{f}(x_8)))$

6. Proof:

   8 + 3 = 9: $\neg CO_2 \lor \neg H_2O$
   9 + 2 = 10: $\neg C \lor \neg O_2 \lor \neg H_2O$
   10 + 6 = 11: $\neg C \lor \neg H_2O$
   11 + 7 = 12: $\neg H_2O$
   12 + 1b = 13: $\neg M_gO \lor \neg H_2$
   13 + 4 = 14: $\neg H_2$
   14 + 5 = $\Box$

7. Proof:

   3a + 1b $\{b/y\}$ = 4: `L(a, b)`
   4 + 2b $\{a/x_1, b/y_1\}$ = 5: $\neg$`P(a)` $\lor\neg$`Q(b)`
   5 + 3b = 6: $\neg$`P(a)`
   6 + 1a = $\Box$

8. Proof:

1a + 2c $\{a/x_1, f(x_1)/y\}$ = 5: $\neg$E(a) $\vee$ V(a) $\vee$ P(f(x_1))
5 + 2b = 6: V(a) $\vee$ P(f(x_1))
6 + 4 $\{f(x_1/x_4\}$ = 7: V(a) $\vee\neg$C(f(x_1))
7 + 1b = $\{f(x_2)/f(x_1)\}$ = 8: V(a) $\vee\neg$E(x_2) $\vee$ V(x_2)
8 + 2b = 9: V(a) $\vee$ V(x_2)
9 + 3 $\{a/x_3\}$ = 10: $\neg$P(a) $\vee$ V(x_2)
10 + 3 $\{x_2/x_3\}$ = 11: $\neg$P(a) $\vee\neg$P(a)
11 + 2a = 12: $\neg$P(a)
12 + 2a = $\square$

9. Proof:

The answer is "no" (i.e. not provable) if "or" is inclusive: the resolvents we generate are no different from the original clauses, so we get no nearer to $\square$. But if "or" is exclusive:
1b + 4a = 5: $\neg$RAIN
5 + 4b = $\square$

10. Proof:

3a + 2 $\{a/y\}$ = 4: $\neg$J(a)

$\square$ not found, so not proved. Reason: things other than jokes might be meant to amuse.

11. Proof:

5 + 1 = 6: $\neg$MISFIRE $\vee\neg$INT $\vee$ IGLOOSE
6 + 2 = 7: $\neg$INT $\vee$ IGLOOSE
7 + 3 = 8: IGLOOSE
8 + 4 = $\square$

12. Proof:

3a + 1b $\{a/x_2, b/y_2\}$ = 4: $\neg$Q(b) $\vee$ MON(a) $\vee$ PR(a)
4 + 3b = 5: MON(a) $\vee$ PR(a)
5 + 3d = 6: MON(a)
6 + 2 $\{a/x_3\}$ = 7: $\neg$MALE(a) $\vee$ K(a) $\vee$ PR(a)
7 + 3d = 8: $\neg$MALE(a) $\vee$ K(a)
8 + 3c = 9: $\neg$MALE(a)
9 + 1a $\{a/x_1\}$ = 10: $\neg$MAR(a, y_1) $\vee\neg$Q(y_1)
10 + 3a $\{b/y_1\}$ = 11: $\neg$Q(b)
11 + 3b = $\square$