Mathematics - Sample Document

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Chapter 1

Matrices and Linear Algebra

1.1 Sample text

The transpose of a matrix reverses the element indices i, j:

$$\begin{pmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,m} \\ a_{2,1} & a_{2,2} & & \\ \vdots & & \ddots & \\ a_{n,1} & & & a_{n,m} \end{pmatrix}^T = \begin{pmatrix} a_{1,1} & a_{2,1} & \cdots & a_{n,1} \\ a_{1,2} & a_{2,2} & & \\ \vdots & & \ddots & \\ a_{1,m} & & & a_{m,n} \end{pmatrix} \tag{1.1}$$

To tranpose a matrix product, transpose each matrix and reverse the order:

$$(\mathbf{AB})^T = \mathbf{B}^T \mathbf{A}^T \tag{1.2}$$

1.1.1 Example 1

Show that for a generic matrix $\mathbf{A}$ containing scalar values, $\mathbf{A}^T\mathbf{A}$ results in a symmetric matrix:

$$\begin{pmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,m} \\ a_{2,1} & a_{2,2} & & \\ \vdots & & \ddots & \\ a_{n,1} & & & a_{n,m} \end{pmatrix} \begin{pmatrix} a_{1,1} & a_{2,1} & \cdots & a_{n,1} \\ a_{1,2} & a_{2,2} & & \\ \vdots & & \ddots & \\ a_{1,m} & & & a_{m,n} \end{pmatrix} = \begin{pmatrix} \sum_k^{k=m} (a_{1,1})^2 & \sum_k^{k=m} a_{1,k}a_{2,k} & \cdots & \sum_k^{k=m} a_{1,k}a_{n,k} \\ \sum_k^{k=m} a_{2,k}a_{1,k} & \sum_k^{k=m} (a_{2,2})^2 & & \\ \vdots & & \ddots & \\ \sum_k^{k=m} a_{n,k}a_{1,k} & & & \sum_k^{k=m} (a_{1,k})^2 \end{pmatrix}$$

Since $a_{1,2}a_{2,1} = a_{2,1}a_{1,2}$, etc., the off-diagonals are mirror images of each other, so $\mathbf{A}^T\mathbf{A} = \left(\mathbf{A}^T\mathbf{A}\right)^T$, as expected.