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NZQA

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

**QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!**

Scholarship 2018
Statistics

2.00 p.m. Monday 12 November 2018

FORMULAE AND TABLES BOOKLET

Refer to this booklet to answer the questions for Scholarship Statistics 93201Q.

Check that this booklet has pages 2–4 in the correct order and that none of these pages is blank.

YOU MAY KEEP THIS BOOKLET AT THE END OF THE EXAMINATION.

STATISTICS – USEFUL FORMULAE AND TABLES

Permutations and Combinations

$$^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = \,^nC_r = \frac{n!}{(n-r)!\,r!}$$

Expectation Algebra

$$E[aX + b] = aE[X] + b$$

$$\text{Var}[aX + b] = a^2\text{Var}[X]$$

$$E[aX + bY] = aE[X] + bE[Y]$$

$$\text{Var}[aX + bY] = a^2\text{Var}[X] + b^2\text{Var}[Y]$$
$$\text{if } X,\, Y \text{ are independent}$$

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Mean and Variance of a Discrete Random Variable

$$\mu = E(X) \qquad \sigma^2 = \text{Var}(X)$$
$$= \Sigma x.P(X = x) \quad \sigma = \text{SD}(X)$$
$$= \sqrt{\Sigma(x - \mu)^2.P(X = x)}$$
$$= \sqrt{E(X^2) - [E(X)]^2}$$
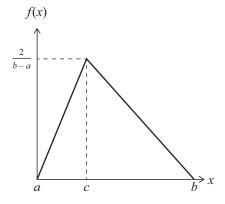
Continuous Uniform Distribution

The probability density function, $f(x)$, for a continuous uniform distribution is defined as:

$$f(x) = \begin{cases} \dfrac{1}{b-a} & \text{for } a \le x \le b \\ 0 & \text{elsewhere} \end{cases}$$

Standard Normal Distribution table and Triangular Distribution.
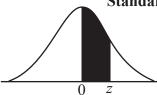
Triangular Distribution

The probability density function, $f(x)$, for a triangular distribution is defined as:

$$f(x) = \begin{cases} 0 & x < a \\ \dfrac{2(x-a)}{(b-a)(c-a)} & a \le x \le c \\ \dfrac{2(b-x)}{(b-a)(b-c)} & c \le x \le b \\ 0 & x > b \end{cases}$$

$$\text{Area of a triangle} = \frac{1}{2}\text{base} \times \text{height}$$

Standard Normal Distribution

$$\left(z = \frac{x - \mu}{\sigma} \right)$$

Each entry gives the probability that the standardised normal random variable Z lies between 0 and z.

Differences

z	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359	4	8	12	16	20	24	28	32	36
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0754	4	8	12	16	20	24	28	32	36
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141	4	8	12	15	19	22	27	31	35
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517	4	8	11	15	19	22	26	30	34
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879	4	7	11	14	18	22	25	29	32
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224	3	7	10	14	17	21	24	27	31
0.6	.2258	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549	3	6	10	13	16	19	23	26	29
0.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852	3	6	9	12	15	18	21	24	27
0.8	.2881	.2910	.2939	.2967	.2996	.3023	.3051	.3078	.3106	.3133	3	6	8	11	14	17	19	22	25
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389	3	5	8	10	13	15	18	20	23
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621	2	5	7	9	12	14	16	18	21
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830	2	4	6	8	10	12	14	16	19
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015	2	4	5	7	9	11	13	15	16
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177	2	3	5	6	8	10	11	13	14
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319	1	3	4	6	7	8	10	11	13
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441	1	2	4	5	6	7	8	10	11
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545	1	2	3	4	5	6	7	8	9
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633	1	2	3	3	4	5	6	7	8
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706	1	1	2	3	4	4	5	6	6
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767	1	1	2	2	3	4	4	5	5
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817	0	1	1	2	2	3	3	4	4
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857	0	1	1	2	2	2	3	3	4
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890	0	1	1	1	2	2	2	3	3
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916	0	0	1	1	1	2	2	2	2
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936	0	0	1	1	1	1	1	2	2
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952	0	0	0	1	1	1	1	1	1
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964	0	0	0	0	1	1	1	1	1
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974	0	0	0	0	0	1	1	1	1
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981	0	0	0	0	0	0	0	0	1
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986	0	0	0	0	0	0	0	0	1
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990	0	0	0	0	0	0	0	0	0
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993	0	0	0	0	0	0	0	0	0
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995	0	0	0	0	0	0	0	0	0
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997	0	0	0	0	0	0	0	0	0
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998	.4998	0	0	0	0	0	0	0	0	0
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	0	0	0	0	0	0	0	0	0
3.6	.4998	.4998	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	0	0	0	0	0	0	0	0	0
3.7	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.4999	0	0	0	0	0	0	0	0	0
3.8	.4999	.4999	.4999	.4999	.4999	.4999	.4999	.5000	.5000	.5000	0	0	0	0	0	0	0	0	0
3.9	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	0	0	0	0	0	0	0	0	0

Binomial Distribution

Each entry gives the probability that a binomial random variable X, with the parameters n and π, has the value x.

$$P(X = x) = \binom{n}{x}\pi^x(1-\pi)^{n-x}$$

$$\mu = n\pi, \qquad \sigma = \sqrt{n\pi(1-\pi)}$$

n	x	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5
4	0	0.8145	0.6561	0.5220	0.4823	0.4096	0.3164	0.2401	0.1975	0.1785	0.1296	0.0915	0.0625
	1	0.1715	0.2916	0.3685	0.3858	0.4096	0.4219	0.4116	0.3951	0.3845	0.3456	0.2995	0.2500
	2	0.0135	0.0486	0.0975	0.1157	0.1536	0.2109	0.2646	0.2963	0.3105	0.3456	0.3675	0.3750
	3	0.0005	0.0036	0.0115	0.0154	0.0256	0.0469	0.0756	0.0988	0.1115	0.1536	0.2005	0.2500
	4		0.0001	0.0005	0.0008	0.0016	0.0039	0.0081	0.0123	0.0150	0.0256	0.0410	0.0625
5	0	0.7738	0.5905	0.4437	0.4019	0.3277	0.2373	0.1681	0.1317	0.1160	0.0778	0.0503	0.0313
	1	0.2036	0.3281	0.3915	0.4019	0.4096	0.3955	0.3602	0.3292	0.3124	0.2592	0.2059	0.1563
	2	0.0214	0.0729	0.1382	0.1608	0.2048	0.2637	0.3087	0.3292	0.3364	0.3456	0.3369	0.3125
	3	0.0011	0.0081	0.0244	0.0322	0.0512	0.0879	0.1323	0.1646	0.1811	0.2304	0.2757	0.3125
	4		0.0005	0.0022	0.0032	0.0064	0.0146	0.0284	0.0412	0.0488	0.0768	0.1128	0.1563
	5			0.0001	0.0001	0.0003	0.0010	0.0024	0.0041	0.0053	0.0102	0.0185	0.0313
6	0	0.7351	0.5314	0.3771	0.3349	0.2621	0.1780	0.1176	0.0878	0.0754	0.0467	0.0277	0.0156
	1	0.2321	0.3543	0.3993	0.4019	0.3932	0.3560	0.3025	0.2634	0.2437	0.1866	0.1359	0.0938
	2	0.0305	0.0984	0.1762	0.2009	0.2458	0.2966	0.3241	0.3292	0.3280	0.3110	0.2780	0.2344
	3	0.0021	0.0146	0.0415	0.0536	0.0819	0.1318	0.1852	0.2195	0.2355	0.2765	0.3032	0.3125
	4	0.0001	0.0012	0.0055	0.0080	0.0154	0.0330	0.0595	0.0823	0.0951	0.1382	0.1861	0.2344
	5		0.0001	0.0004	0.0006	0.0015	0.0044	0.0102	0.0165	0.0205	0.0369	0.0609	0.0938
	6					0.0001	0.0002	0.0007	0.0014	0.0018	0.0041	0.0083	0.0156
7	0	0.6983	0.4783	0.3206	0.2791	0.2097	0.1335	0.0824	0.0585	0.0490	0.0280	0.0152	0.0078
	1	0.2573	0.3720	0.3960	0.3907	0.3670	0.3115	0.2471	0.2048	0.1848	0.1306	0.0872	0.0547
	2	0.0406	0.1240	0.2097	0.2344	0.2753	0.3115	0.3177	0.3073	0.2985	0.2613	0.2140	0.1641
	3	0.0036	0.0230	0.0617	0.0781	0.1147	0.1730	0.2269	0.2561	0.2679	0.2903	0.2918	0.2734
	4	0.0002	0.0026	0.0109	0.0156	0.0287	0.0577	0.0972	0.1280	0.1442	0.1935	0.2388	0.2734
	5		0.0002	0.0012	0.0019	0.0043	0.0115	0.0250	0.0384	0.0466	0.0774	0.1172	0.1641
	6			0.0001	0.0001	0.0004	0.0013	0.0036	0.0064	0.0084	0.0172	0.0320	0.0547
	7						0.0001	0.0002	0.0005	0.0006	0.0016	0.0037	0.0078
8	0	0.6634	0.4305	0.2725	0.2326	0.1678	0.1001	0.0576	0.0390	0.0319	0.0168	0.0084	0.0039
	1	0.2793	0.3826	0.3847	0.3721	0.3355	0.2670	0.1977	0.1561	0.1373	0.0896	0.0548	0.0313
	2	0.0515	0.1488	0.2376	0.2605	0.2936	0.3115	0.2965	0.2731	0.2587	0.2090	0.1569	0.1094
	3	0.0054	0.0331	0.0839	0.1042	0.1468	0.2076	0.2541	0.2731	0.2786	0.2787	0.2568	0.2188
	4	0.0004	0.0046	0.0185	0.0260	0.0459	0.0865	0.1361	0.1707	0.1875	0.2322	0.2627	0.2734
	5		0.0004	0.0026	0.0042	0.0092	0.0231	0.0467	0.0683	0.0808	0.1239	0.1719	0.2188
	6			0.0002	0.0004	0.0011	0.0038	0.0100	0.0171	0.0217	0.0413	0.0703	0.1094
	7					0.0001	0.0004	0.0012	0.0024	0.0033	0.0079	0.0164	0.0313
	8							0.0001	0.0002	0.0002	0.0007	0.0017	0.0039
9	0	0.6302	0.3874	0.2316	0.1938	0.1342	0.0751	0.0404	0.0260	0.0207	0.0101	0.0046	0.0020
	1	0.2985	0.3874	0.3679	0.3489	0.3020	0.2253	0.1556	0.1171	0.1004	0.0605	0.0339	0.0176
	2	0.0629	0.1722	0.2597	0.2791	0.3020	0.3003	0.2668	0.2341	0.2162	0.1612	0.1110	0.0703
	3	0.0077	0.0446	0.1069	0.1302	0.1762	0.2336	0.2668	0.2731	0.2716	0.2508	0.2119	0.1641
	4	0.0006	0.0074	0.0283	0.0391	0.0661	0.1168	0.1715	0.2048	0.2194	0.2508	0.2600	0.2461
	5		0.0008	0.0050	0.0078	0.0165	0.0389	0.0735	0.1024	0.1181	0.1672	0.2128	0.2461
	6		0.0001	0.0006	0.0010	0.0028	0.0087	0.0210	0.0341	0.0424	0.0743	0.1160	0.1641
	7			0.0001	0.0001	0.0003	0.0012	0.0039	0.0073	0.0098	0.0212	0.0407	0.0703
	8					0.0001	0.0004	0.0009	0.0013	0.0035	0.0083	0.0176
	9							0.0001	0.0001	0.0003	0.0008	0.0020
10	0	0.5987	0.3487	0.1969	0.1615	0.1074	0.0563	0.0282	0.0173	0.0135	0.0060	0.0025	0.0010
	1	0.3151	0.3874	0.3474	0.3230	0.2684	0.1877	0.1211	0.0867	0.0725	0.0403	0.0207	0.0098
	2	0.0746	0.1937	0.2759	0.2907	0.3020	0.2816	0.2335	0.1951	0.1757	0.1209	0.0763	0.0439
	3	0.0105	0.0574	0.1298	0.1550	0.2013	0.2503	0.2668	0.2601	0.2522	0.2150	0.1665	0.1172
	4	0.0010	0.0112	0.0401	0.0543	0.0881	0.1460	0.2001	0.2276	0.2377	0.2508	0.2384	0.2051
	5	0.0001	0.0015	0.0085	0.0130	0.0264	0.0584	0.1029	0.1366	0.1536	0.2007	0.2340	0.2461
	6		0.0001	0.0012	0.0022	0.0055	0.0162	0.0368	0.0569	0.0689	0.1115	0.1596	0.2051
	7			0.0001	0.0002	0.0008	0.0031	0.0090	0.0163	0.0212	0.0425	0.0746	0.1172
	8					0.0001	0.0004	0.0014	0.0030	0.0043	0.0106	0.0229	0.0439
	9							0.0001	0.0003	0.0005	0.0016	0.0042	0.0098
	10	(all other entries < 0.0001)									0.0001	0.0003	0.0010

Poisson Distribution

Each entry gives the probability that a Poisson random variable X, with parameter λ, has the value x.

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$\mu = \lambda, \qquad \sigma = \sqrt{\lambda}$$

x \ λ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493	0.4066	0.3679
1	0.0905	0.1637	0.2222	0.2681	0.3033	0.3293	0.3476	0.3595	0.3659	0.3679
2	0.0045	0.0164	0.0333	0.0536	0.0758	0.0988	0.1217	0.1438	0.1647	0.1839
3	0.0002	0.0011	0.0033	0.0072	0.0126	0.0198	0.0284	0.0383	0.0494	0.0613
4		0.0001	0.0003	0.0007	0.0016	0.0030	0.0050	0.0077	0.0111	0.0153
5				0.0001	0.0002	0.0004	0.0007	0.0012	0.0020	0.0031
6							0.0001	0.0002	0.0003	0.0005
7										0.0001

x \ λ	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
0	0.3329	0.3012	0.2725	0.2466	0.2231	0.2019	0.1827	0.1653	0.1496	0.1353
1	0.3662	0.3614	0.3543	0.3452	0.3347	0.3230	0.3106	0.2975	0.2842	0.2707
2	0.2014	0.2169	0.2303	0.2417	0.2510	0.2584	0.2640	0.2678	0.2700	0.2707
3	0.0738	0.0867	0.0998	0.1128	0.1255	0.1378	0.1496	0.1607	0.1710	0.1804
4	0.0203	0.0260	0.0324	0.0395	0.0471	0.0551	0.0636	0.0723	0.0812	0.0902
5	0.0045	0.0062	0.0084	0.0111	0.0141	0.0176	0.0216	0.0260	0.0309	0.0361
6	0.0008	0.0012	0.0018	0.0026	0.0035	0.0047	0.0061	0.0078	0.0098	0.0120
7	0.0001	0.0002	0.0003	0.0005	0.0008	0.0011	0.0015	0.0020	0.0027	0.0034
8			0.0001	0.0001	0.0001	0.0002	0.0003	0.0005	0.0006	0.0009
9							0.0001	0.0001	0.0001	0.0002

x \ λ	2.2	2.4	2.6	2.8	3.0	3.2	3.4	3.6	3.8	4.0
0	0.1108	0.0907	0.0743	0.0608	0.0498	0.0408	0.0334	0.0273	0.0224	0.0183
1	0.2438	0.2177	0.1931	0.1703	0.1494	0.1304	0.1135	0.0984	0.0850	0.0733
2	0.2681	0.2613	0.2510	0.2384	0.2240	0.2087	0.1929	0.1771	0.1615	0.1465
3	0.1966	0.2090	0.2176	0.2225	0.2240	0.2226	0.2186	0.2125	0.2046	0.1954
4	0.1082	0.1254	0.1414	0.1557	0.1680	0.1781	0.1858	0.1912	0.1944	0.1954
5	0.0476	0.0602	0.0735	0.0872	0.1008	0.1140	0.1264	0.1377	0.1477	0.1563
6	0.0174	0.0241	0.0319	0.0407	0.0504	0.0608	0.0716	0.0826	0.0936	0.1042
7	0.0055	0.0083	0.0118	0.0163	0.0216	0.0278	0.0348	0.0425	0.0508	0.0595
8	0.0015	0.0025	0.0038	0.0057	0.0081	0.0111	0.0148	0.0191	0.0241	0.0298
9	0.0004	0.0007	0.0011	0.0018	0.0027	0.0040	0.0056	0.0076	0.0102	0.0132
10	0.0001	0.0002	0.0003	0.0005	0.0008	0.0013	0.0019	0.0028	0.0039	0.0053
11			0.0001	0.0001	0.0002	0.0004	0.0006	0.0009	0.0013	0.0019
12					0.0001	0.0001	0.0002	0.0003	0.0004	0.0006
13								0.0001	0.0001	0.0002
14										0.0001

x \ λ	4.2	4.4	4.6	4.8	5.0	5.2	5.4	5.6	5.8	6.0
0	0.0150	0.0123	0.0101	0.0082	0.0067	0.0055	0.0045	0.0037	0.0030	0.0025
1	0.0630	0.0540	0.0462	0.0395	0.0337	0.0287	0.0244	0.0207	0.0176	0.0149
2	0.1323	0.1188	0.1063	0.0948	0.0842	0.0746	0.0659	0.0580	0.0509	0.0446
3	0.1852	0.1743	0.1631	0.1517	0.1404	0.1293	0.1185	0.1082	0.0985	0.0892
4	0.1944	0.1917	0.1875	0.1820	0.1755	0.1681	0.1600	0.1515	0.1428	0.1339
5	0.1633	0.1687	0.1725	0.1747	0.1755	0.1748	0.1728	0.1697	0.1656	0.1606
6	0.1143	0.1237	0.1323	0.1398	0.1462	0.1515	0.1555	0.1584	0.1601	0.1606
7	0.0686	0.0778	0.0869	0.0959	0.1044	0.1125	0.1200	0.1267	0.1326	0.1377
8	0.0360	0.0428	0.0500	0.0575	0.0653	0.0731	0.0810	0.0887	0.0962	0.1033
9	0.0168	0.0209	0.0255	0.0307	0.0363	0.0423	0.0486	0.0552	0.0620	0.0688
10	0.0071	0.0092	0.0118	0.0147	0.0181	0.0220	0.0262	0.0309	0.0359	0.0413
11	0.0027	0.0037	0.0049	0.0064	0.0082	0.0104	0.0129	0.0157	0.0190	0.0225
12	0.0009	0.0013	0.0019	0.0026	0.0034	0.0045	0.0058	0.0073	0.0092	0.0113
13	0.0003	0.0005	0.0007	0.0009	0.0013	0.0018	0.0024	0.0032	0.0041	0.0052
14	0.0001	0.0001	0.0002	0.0003	0.0005	0.0007	0.0009	0.0013	0.0017	0.0022
15			0.0001	0.0001	0.0002	0.0002	0.0003	0.0005	0.0007	0.0009
16						0.0001	0.0001	0.0002	0.0002	0.0003
17								0.0001	0.0001	0.0001

(all other entries < 0.0001)