Assessment Schedule – 2012

Scholarship Statistics and Modelling (93201)

Evidence Statement

General Principles:
1. Ignore incorrect answers if alongside correct answers. The exception is contradictory statements.
2. Ignore minor copying errors.

QUESTION ONE

Tasks Q1 (a) (i)

Evidence:

Point estimate for the mean time taken to complete the stair climb = 14.3 seconds
95% confidence interval for the mean time taken to complete the stair climb is given by 14.3s +/– 1.07 s

With 95% confidence, it is estimated that, for all participants in the programme, the mean time taken to climb the stairs is between 13.23 s and 15.37 s.
OR
The probability that the interval 13.23s to 15.37s contains the mean time for all participants in the programme to climb the stairs is 0.95.

Note:
1. No penalty if confidence interval is to nearest one decimal place.
2. Use of the sample mean symbol scores a P.

Judgement:

S: Confidence interval is calculated and interpreted correctly in context.
P: Either the confidence interval is calculated correctly or interpreted correctly in context.

Tasks Q1 (a) (ii)

Evidence:

Take the prior estimate of the population proportion **p** as 0.5 due to the fact that we have no idea about its value.
Now $\boldsymbol{n} = (1.96 / 0.05)^2 \times 0.5 \times 0.5 = 384.16$ so 385
So sample size that should be used = 385

Note:
1. n = 382 acceptable due to variations on 1.96 or using mean and standard deviation with Normal in (a) p is deduced as 0.49755 which is an acceptable alternative.

Task Q1 (a) (iii)

Evidence:

A main concern with the answer in Q1 (a) (ii) is that we have only 250 participants in the population and we are required to select 385 participants in order to achieve a margin of error of 0.05.

To complete the task stated in Q1 (a) (ii) possibilities include the following:
1. Take an initial sample of 50 participants and estimate the population proportion. Then the margin of error could be compared to 0.05 or the sample size could be recalculated to give a lower value compared to 385.
2. Take n = 250 and deduce the population proportion which will have no margin of error.
3. Scale n = 385 down to a required sample size of (385 x 250) / (385 + 250) = 132 by using a finite population correction factor.
4. The sample size could be recalculated in (a) (ii) by using a larger margin of error or a lower level of confidence (one of these is acceptable).

Note:
1. To achieve a required sample size of less than 250, the prior estimate of the proportion would need to be either below 0.19 or greater than 0.81.
2. Other reasonable points made about completing the task are acceptable.
3. Accept n = 384.
4. Resampling to get calculated n was accepted.

Judgement:

O: The calculated sample size is correct and concern about its value is stated along with a description of how to complete the task.
P: The calculated sample size is correct **or** correct concern plus description for incorrect sample size > 250.

Tasks Q1 (b)

Evidence:

Construct a 95% confidence interval for the difference in the population mean time taken to walk 20 metres. This is given by:

$\mu_{\text{current}} - \mu_{\text{new}} = (31.4 - 28.1) \pm 1.96\,(9.5^2 / 35 + 7.6^2 / 40)^{0.5}$

So $-0.63\text{ s} < \mu_{\text{current}} - \mu_{\text{new}} < 7.23\text{s}$ or $\mu_{\text{current}} - \mu_{\text{new}} = 3.30\text{ s} \pm 3.93\text{ s}$

Note:

1. Rounding errors accepted.
2. Can have a confidence interval for $\mu_{\text{new}} - \mu_{\text{current}} = -3.30\text{ s} \pm 3.93\text{ s}$
3. Can have any level of confidence 90% and above. A 90% confidence interval -3.30s $\pm$ 3.299 or a 99% confidence interval 3.30s $\pm$ 5.17 with appropriate conclusion relating to the interval.

As zero is contained within this confidence interval we cannot conclude that the mean time taken to walk 20 metres by the participants in the new programme is different from the mean time taken to walk 20 metres by the participants in the current programme.

Note:

1. Confidence interval estimates calculated to one decimal place are acceptable.
2. Can use 90% confidence interval with appropriate conclusion.
3. Accept two separate 95% or 99% confidence interval calculations with an examination of overlap.
4. Accept margin of error is larger than the difference (3.93 > 3.30).

Judgement:

S: Confidence interval correctly calculated along with a correct conclusion backed up by evidence.
P: Confidence interval correctly calculated or a correct conclusion based on an incorrect confidence interval (carried error).

Tasks Q2 (a)

Evidence:

- The correlation between "eyes open" and "eyes closed" is both positive and moderate.
- There is an outlier at (60, 10) in Figure One and the same person as an outlier at (10, 60) in Figure Two.
- The fit of the points in Figure One appear to be curved and also a quadratic (non-linear curve) fit appears to be more appropriate for the "fit of the points" in Figure Two.
- The time spent balancing with "eyes open" ranges from 10 s to 60 s while the time spent balancing with "eyes closed" ranges from 3 s to 38 s. (Approximate limits acceptable).
- In Figure One, as the time spent balancing with "eyes open" increases, the amount of variation (scatter) in the time spent balancing with "eyes closed" is reduced. In Figure Two there appears to be a greater amount of variation in the times for "eyes open" when the times for "eyes closed" ranges between 20 s and 34 s.
- Can refer to the percentage that are above or below the "eyes open" = "eyes closed" line in either figure.

Note:

1. Can score half points within each dot point.

Judgement

S: Two points in total.
P: One point.

Tasks Q2 (b)

Evidence:

(i) Figure 1: $y = 0.3727 \times 65 + 11.923 =$ **36 s** as a prediction of the time spent balancing on the left leg with both eyes closed.

Figure 2: $y = 0.04534 \times 23^2 - 0.8879 \times 23 + 23.28 =$ **27 s** as a prediction of the time spent balancing on the left leg with both eyes open.

(ii) Both predictions are suspect. In Figure One, $x = 65$ s is outside the data range and/or the majority of the points are above the fitted line due to the outlier. In Figure Two, $x = 23$ s is in a section of the scatter where there is a high variation in the "eyes open" times. This is an effect of the outlier.

Note:

1. Vague validity comments aren't acceptable.

Judgement

S: At least one prediction is correct along with one corresponding validity comment corresponding to the prediction.
P: One or both predictions correct or one good validity comment.

Tasks Q2(c)

Evidence:

Note: (i) and (ii) score one point, (iii) scores two points, one for direction and the other for strength.

(i) Variables	(ii) Method of Measurement	(iii) Relationship (direction & strength)
Level of Fitness	Beep test / fitness test	Strong /Positive The greater the time spent balancing implies a higher level of fitness.
Weight	Use scales	Weak/Negative More weight would cause some lowering of the time spent balancing.

Note:

1. Answers can involve a description of other variables given in (i) e.g. age, length of time on programme, arm span, height, time since injury
2. Variables have to be continuous or on a continuous scale.
3. Any sensible variable is acceptable with a strong justification. For example centre of mass to floor, foot size, and balance on right leg, arm span or length of time since sustaining injury.
4. If (i) isn't accepted nor is (ii) and (iii).

Judgement

O: Five or Six answers correct.
S: Three or Four answers correct.
P: One or Two answers correct.

Tasks Q3 (a)

Evidence:

Prob (z < (15.2 – 16) / 0.75 / √6) = prob (z < –2.613) = 0.5 – 0.4955 = 0.0045.
As this probability is very small, it is unlikely that the mean is 16 seconds and is probably lower.

Note:

1. Another conclusion could be that the amount of variation (standard deviation) in the times taken to complete the task has decreased.
2. A 95% confidence interval 15.2 ± 0.6 is calculated and 16.0 are compared to it. As 16.0 > 15.8, the same conclusion is reached.
3. A 99% confidence interval 15.2 ± 0.79 can be used instead to reach conclusion.
4. Conclude that six is a very small sample and results could be biased.
5. Use limits around μ = 16 that would be expected. As 15.2 is outside conclude no change.
6. A formal hypothesis test is acceptable.
7. Confidence interval for difference in means not acceptable.

Judgement

S: An appropriate conclusion is provided with correct evidence.
P: Evidence is correct or a conclusion is correct based on answer.

Tasks Q3 (b)

Evidence:

Let x = number of males in total and T = total number in the group.
From the given ratios the following table is constructed:

Gender	**Climbing**	**Balancing**	**Walking**	**TOTAL**
Male	4x / 9	2x / 9	3x / 9	x
Female			T / 4 – 3x / 9	T – x
TOTAL	T / 4	T / 2	T / 4	T

Given prob (Female 1 Walking) = 1 / 6 we require prob (Walking 1 Female).

So 1 / 6 = ((T / 4 – 3x / 9) / T) / ((T / 4/T))
Hence T / 24 = T / 4 – x / 3 so x = 5T / 8

Now prob (Walking 1 Female) = ((T / 4 – x / 3) / T) / ((T – x) / T) = (T / 4 – 5T / 24)/ (T – 5T / 8) = (T / 24) / (3T / 8) = 1 / 9.

Note:

1. Can use numbers instead of letters in the table, i.e. start with T = 96 or any other number in bottom right corner.
2. Ratio 4:2:3 in question should have been 2:4:3 which doesn't affect the answer however if candidate gets a contradiction (more male climbers than the total number of climbers) with the above table then award an O.
3. Can construct table below based on a grand total of 96:

Gender	**Walking**	**TOTAL**
Male	20	60
Female	4	36
TOTAL	24	96

So prob (Walking 1 Female) = 4/36 = 1/9.

Judgement

O: Correct answer with appropriate method.
P: Some indication of correct method.

Tasks Q3(c)

Evidence:

Let R = event that a rowing machine is in use and T = event that a treadmill is in use.

Prob (two of the nine pieces of equipment are in use) = Prob (R and R) + Prob (T and R) + Prob (T and T)

Now prob (two rowing machines are use) = Binomial (n = 4, x = 2, π = 0.4) = 0.3456
and prob (no treadmills are in use) = Binomial (n = 5, x = 0, π = 0.5) = 0.0313
and prob (one rowing machine is in use) = Binomial (n = 4, x = 1, π = 0.4) = 0.3456
and prob (one treadmill is in use) = Binomial (n = 5, x = 1, π = 0.5) = 0.1563
and prob (two treadmills are in use) = Binomial (n = 5, x = 2, π = 0.5) = 0.3125
and prob (no rowing machines are in use) = Binomial (n = 4, x = 0, π = 0.4) = 0.1296

So prob (two of the nine pieces are in use) = 0.3456 × 0.0313 + 0.3456 × 0.1563 + 0.3125 × 0.1296 = 0.1053

So prob (R and R given that two of the nine pieces are in use at a particular time) = 0.3456 × 0.0313 / 0.1053 = 0.1027 (4/39).

Note:

1. Answer can be expressed to three decimal places as 0.103
2. Normal approximation to the Binomial not acceptable as n is small.

Judgement

S: Correct answer
P: Some evidence of correct method like choice of Binomial with some parameters correct.

Task Q4 (a)

Evidence:

Trend

- There is an average of about 30 people at the gym in each daily 4-hour time period.
- The early morning trend shows an overall slight average increase of 0.2 people (one person every five days) attending per day with a slight decline during July, increasing during August and relatively constant over the last three weeks.
- The mid-day trend shows a decline in attendance of approximately five in July through to mid-August followed by an increase from 25 to 29.
- The evening attendance shows a constant trend with an average attendance of about 34 per day.

Seasonal

- The seasonal component for early morning and evenings shows a lower attendance (10 to 15 per day) in the weekends.
- The seasonal component shows a high attendance in early morning (30 to 40 per day) and evening (40 to 50 per day) during the week.
- The seasonal component for mid-day shows a high attendance on weekends (about 35 per day) and fewer (about 20–25 per day) on mid-week days.
- The attendance has the greatest fluctuation (10 to 50) in the evening compared to the early morning and mid-day.

Income

- The gym income is constant through 2009 and 2010 at about \$13 500 per month with a sharp drop in January 2011.
- The gym income shows a steady increase through January 2011 to September 2012 of about \$7 000 over the 20 months (\$350 per month increase on average).

Note:

1. Vague comments not accepted.
2. Maximum of two points score from each graph.
3. Need **<u>numbers and context</u>** in each point.
4. Points need to be distinct to score.
5. A maximum of three points per feature is allowed to be counted e.g. trend, seasonal,

Judgement

S + P: Five points in total with at least one comment about both attendance and income.
S: Three or four points in total.
P: One or two or three points in total.

Task Q4 (b)

Evidence:

Saturday 29 September 2012 corresponds to $x = 76$.
Early Morning Forecast = $0.2089 \times 76 + 23.196 - 10.27 = 29$ people
This forecast is only two weeks ahead of known data. It might be optimistic given that the trend seems to have levelled off.

Mid-day Forecast = $0.0032 \times 76^2 - 0.2097 \times 76 + 29.709 + 6.76 = 39$ people
This forecast of 39 people might be optimistic as the parabolic curve fit to the centred moving mean is unlikely to continue into the future as the end of the moving mean graph shows a levelling off.

Note:

1. Vague answers not accepted.
2. Method of forecast calculation needs to be shown.
3. MEI if $x = 75$ or 77.
4. Accept 28 for Saturday's forecast.

Judgement

S: At least one correct forecast with an appropriate corresponding validity comment about it.
P: Correct forecasts only or one appropriate validity comment resulting from incorrectly calculated forecasts.

Task Q4(c)

Evidence:

Let p_1 represent the number of members in June 2009 and p_2 represent the number of members in June 2012.
Let s_1 represent the subscription in June 2009 and s_2 represent the subscription in June 2012.
In June 2009: $p_1 \times s_1 = 12\,740$ and in June 2012: $p_2 \times s_2 = 17\,850$.
Now $s_2 = 1158 \times s_1 / 1075 = 1.077s_1$
Hence $p_2 / p_1 = (17\,850 / 1.077s_1) / (12740 / s_1) = 1.30$

The percentage change in membership from June 2009 to June 2012 is 30%.

Note:

1. Must use CPI in calculation.
2. If only answer is given score N.
3. Use CPI adjustments with 17850/1158 = 15.4 then 12740/1075 =11.9 leading to % = (15.4-11.9)/11.9 x 100 = 30%.

Judgement

O: Correct answer with appropriate method.
S: 130% MEI.
P: Some indication of correct method.

Tasks Q5 (a)

Evidence:
Let z = number of rowing machines
Floor Area: $3x + 2y + 4z = 132$
Now $z \geq 3$ with minimum required usage so $4z \geq 12$ so $132 - 3x - 2y \geq 12$
Rearranging gives $3x + 2y \leq 120$

Judgement
P: Correct reasoning leading to given answer.

Tasks Q5 (b)

Evidence:
Constraints are:
From Q 5(a): $3x + 2y \leq 120$
Minimum required Usages: $x \geq 8$ and $y \geq 5$
Concurrent Usages: $y \leq 1.5x$ and $y \geq x$

Cost Function C is given by: C = k ($2x + 3y + 5z$) where k = constant of proportionality
So expressing C in terms of x and y we get C = k ($2x + 3y + 5(132 - 3x - 2y) / 4$) = k ($165 - 1.75x + 0.5y$)

The feasible region in the (x, y) plane has corner points at (8, 8), (8, 12), (24, 24) and (20, 30)
This gives rise to the C values: 155k, 157k, 135k and 145k respectively.
Cost is minimised when $x = 24$ and $y = 24$.
Hence $4z = 132 - 3(24) - 2(24) = 12$ so $z = 3$.
Optimal point (x, y, z) = (24, 24, 3) which represents 24 treadmills, 24 steppers and 3 rowing machines.

Note:
1. Can miss k off cost function C and get S.
2. Can leave C in terms of x,y and z and for every corner point (x,y) in the feasible region calculate z then C. Choose minimum C to get optimal point and score S. Evidence must be shown.

Judgement
S: Correct optimal point with all constraints and cost function correct in terms of x and y only.
P: All constraints are correct (either written or graphically) or optimal point correct.

Tasks Q5 (c)

Evidence:
Cost Function C is given by: C = k ($2x + 3y + cz$) where k = constant of proportionality and c is to be determined.
So expressing C in terms of x and y we get C = k ($2x + 3y + c(132 - 3x - 2y) / 4$) = k ($33c - x(2 - 3c / 4) + y(3 - c / 2)$)
Multiple solutions occur when the cost function is parallel to one of the sides of the feasible region and the cost is minimised.
The cost function can be rearranged to give $y = (2 - 3c / 4) / (3 - c / 2)\, x + (C / k - 33c) / (3 - c / 2)$

Side 1: $y = x$ so gradient = 1 hence $(3c / 4 - 2) / (3 - c / 2) = 1$ so c = 4. Then cost C = 132k.
Side 2: $3x + 2y = 120$ so gradient = $-3 / 2$ hence $(3c / 4 - 2) / (3 - c / 2)$ so $c = -3 / 2$ and this gives rise to no solution for c.
Side 3: $y = 3x / 2$ so gradient = $3 / 2$ hence $(3c / 4 - 2) / (3 - c / 2) = 3 / 2$ so $c = 13 / 3$. Then cost C = 143k.
Side 4: $x = 8$ so gradient has no solution. Hence $3 - c / 2 = 0$ so c = 6. Then cost C = 218k.

The solution is c = 4

Note:
1. If c = 4 obtained with only one or two sides of feasible region being considered then score S.
2. If c = 4 only then score P.

Judgement
O: Correct value of c with full evidence (all costs must be evaluated for each value of c).
S: Correct value of c with partial evidence.
P: Correct reworked cost function C or some evidence of correct method.

Scoring for each Question

Each question part within a question is scored as:

N = No meaningful work, insufficient or incorrect answer.
P = partially correct to a predetermined level.
S = totally correct to a scholarship level.
O = totally correct to an outstanding level.

The codes are put together for each question and then converted to a mark out of eight according to the following table:

Mark	Codes
8	O + 2S, O + S + P, O + S + 2P, O + 2S + P
7	O, O + P, O + 2P, O + S, 3S
6	2S, 2S + P, 2S + 2P
5	S + P, S + 2P, S + 3P
4	S
3	3P
2	2P
1	P
0	N

The marks for each question are totalled to give an overall mark. Best possible overall mark is 40.