S

S–CALCF

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Scholarship 2015
Calculus

2.00 p.m. Tuesday 17 November 2015

FORMULAE AND TABLES BOOKLET

Refer to this booklet to answer the questions for Scholarship Calculus 93202Q.

Check that this booklet has pages 2–4 in the correct order and that none of these pages is blank.

YOU MAY KEEP THIS BOOKLET AT THE END OF THE EXAMINATION.

CALCULUS – USEFUL FORMULAE

ALGEBRA

Quadratics

If $ax^2 + bx + c = 0$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Logarithms

$y = \log_b x \Leftrightarrow x = b^y$

$\log_b(xy) = \log_b x + \log_b y$

$\log_b\left(\dfrac{x}{y}\right) = \log_b x - \log_b y$

$\log_b\left(x^n\right) = n\log_b x$

$\log_b x = \dfrac{\log_a x}{\log_a b}$

Complex numbers

$z = x + \mathrm{i}y$

$\quad = r\,\mathrm{cis}\,\theta$

$\quad = r(\cos\theta + \mathrm{i}\sin\theta)$

$\bar{z} = x - \mathrm{i}y$

$\quad = r\,\mathrm{cis}\,(-\theta)$

$\quad = r(\cos\theta - \mathrm{i}\sin\theta)$

$r = |z| = \sqrt{z\bar{z}} = \sqrt{(x^2 + y^2)}$

$\theta = \arg z$

where $\cos\theta = \dfrac{x}{r}$

and $\sin\theta = \dfrac{y}{r}$

De Moivre's Theorem

If n is any integer, then

$(r\,\mathrm{cis}\,\theta)^n = r^n\,\mathrm{cis}\,(n\theta)$

Binomial Theorem

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + \binom{n}{n}b^n$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{(n-r)!r!}$$

Some values of $\binom{n}{r}$ are given in the table below.

n \ r	0	1	2	3	4	5	6	7	8	9	10
0	1										
1	1	1									
2	1	2	1								
3	1	3	3	1							
4	1	4	6	4	1						
5	1	5	10	10	5	1					
6	1	6	15	20	15	6	1				
7	1	7	21	35	35	21	7	1			
8	1	8	28	56	70	56	28	8	1		
9	1	9	36	84	126	126	84	36	9	1	
10	1	10	45	120	210	252	210	120	45	10	1
11	1	11	55	165	330	462	462	330	165	55	11
12	1	12	66	220	495	792	924	792	495	220	66

COORDINATE GEOMETRY

Straight Line

Equation $y - y_1 = m(x - x_1)$

Circle

$(x-a)^2 + (y-b)^2 = r^2$

has a centre (a, b) and radius r

Parabola

$y^2 = 4ax$ or $(at^2, 2at)$

Focus $(a, 0)$ Directrix $x = -a$

Ellipse

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ or $(a\cos\theta, b\sin\theta)$

Foci $(c, 0)$ $(-c, 0)$ where $b^2 = a^2 - c^2$

Eccentricity: $e = \dfrac{c}{a}$

Hyperbola

$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ or $(a\sec\theta, b\tan\theta)$

asymptotes $y = \pm\dfrac{b}{a}x$

Foci $(c, 0)$ $(-c, 0)$ where $b^2 = c^2 - a^2$

Eccentricity: $e = \dfrac{c}{a}$

CALCULUS

Differentiation

$y = f(x)$	$\frac{\mathrm{d}y}{\mathrm{d}x} = f'(x)$
$\ln x$	$\frac{1}{x}$
e^{ax}	$a\mathrm{e}^{ax}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\cot x$	$-\operatorname{cosec}^2 x$

Integration

$f(x)$	$\int f(x)\mathrm{d}x$
x^n	$\frac{x^{n+1}}{n+1} + c$ $(n \neq -1)$
$\frac{1}{x}$	$\ln\lvert x\rvert + c$
$\frac{f'(x)}{f(x)}$	$\ln\lvert f(x)\rvert + c$

First principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Parametric Function

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \cdot \frac{\mathrm{d}t}{\mathrm{d}x}$$

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) \cdot \frac{\mathrm{d}t}{\mathrm{d}x}$$

Product Rule

$(f.g)' = f.g' + g.f'$ or if $y = uv$ then $\frac{\mathrm{d}y}{\mathrm{d}x} = u\frac{\mathrm{d}v}{\mathrm{d}x} + v\frac{\mathrm{d}u}{\mathrm{d}x}$

Quotient Rule

$\left(\frac{f}{g}\right)' = \frac{g.f' - f.g'}{g^2}$ or if $y = \frac{u}{v}$ then $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Composite Function or Chain Rule

$(f(g))' = f'(g).g'$

or if $y = f(u)$ and $u = g(x)$ then $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \cdot \frac{\mathrm{d}u}{\mathrm{d}x}$

NUMERICAL METHODS

Trapezium Rule

$$\int_a^b f(x)\ \mathrm{d}x \approx \frac{1}{2}h\left[y_0 + y_n + 2(y_1 + y_2 + \ldots + y_{n-1})\right]$$

where $h = \frac{b-a}{n}$ and $y_r = f(x_r)$

Simpson's Rule

$$\int_a^b f(x)\ \mathrm{d}x \approx \frac{1}{3}h\left[y_0 + y_n + 4(y_1 + y_3 + \ldots + y_{n-1}) + 2(y_2 + y_4 + \ldots + y_{n-2})\right]$$

where $h = \frac{b-a}{n}$, $y_r = f(x_r)$ and n is even.

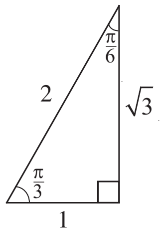
TRIGONOMETRY

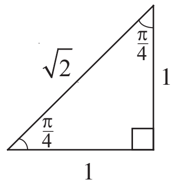
$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$





Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine Rule

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Identities

$$\cos^2\theta + \sin^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\cot^2\theta + 1 = \operatorname{cosec}^2\theta$$

General Solutions

If $\sin\theta = \sin\alpha$ then $\theta = n\pi + (-1)^n\alpha$

If $\cos\theta = \cos\alpha$ then $\theta = 2n\pi \pm \alpha$

If $\tan\theta = \tan\alpha$ then $\theta = n\pi + \alpha$

where n is any integer

Compound Angles

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

Double Angles

$$\sin 2A = 2\sin A\cos A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

$$\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A\\ &= 2\cos^2 A - 1\\ &= 1 - 2\sin^2 A\end{aligned}$$

Products

$$2\sin A\cos B = \sin(A+B) + \sin(A-B)$$

$$2\cos A\sin B = \sin(A+B) - \sin(A-B)$$

$$2\cos A\cos B = \cos(A+B) + \cos(A-B)$$

$$2\sin A\sin B = \cos(A-B) - \cos(A+B)$$

Sums

$$\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$$

$$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$$

$$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$$

$$\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$$

MEASUREMENT

Triangle

Area $= \frac{1}{2}ab\sin C$

Trapezium

Area $= \frac{1}{2}(a+b)h$

Sector

Area $= \frac{1}{2}r^2\theta$

Arc length $= r\theta$

Cylinder

Volume $= \pi r^2 h$

Curved surface area $= 2\pi rh$

Cone

Volume $= \frac{1}{3}\pi r^2 h$

Curved surface area $= \pi rl$ where $l =$ slant height

Sphere

Volume $= \frac{4}{3}\pi r^3$

Surface area $= 4\pi r^2$

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