

93201Q



932012



NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

Scholarship 2012 Statistics and Modelling

9.30 am Saturday 10 November 2012

Time allowed: Three hours

Total marks: 40

QUESTION BOOKLET

There are FIVE questions in this booklet. Answer ALL questions.

Pull out Formulae and Tables Booklet S–STATF from the centre of this booklet.

Write your answers in Answer Booklet 93201A.

Show ALL working. Start your answer to each question on a new page. Carefully number each question.

Grids are provided on pages 25 and 26 of the Answer Booklet for answering Question Five (b).

Check that this booklet has pages 2–11 in the correct order and that none of these pages is blank.

YOU MAY KEEP THIS BOOKLET AT THE END OF THE EXAMINATION.

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You have three hours to complete this examination.

The questions in this paper are all based on a gym. Questions 1, 2, 3(a) and 3(b) are about a movement therapy programme the gym provides for people who have mobility problems due to injuries.

QUESTION ONE

Two of the activities in the movement therapy programme are a stair climb and a walk.

- (a) Two hundred and fifty people take part in the movement therapy programme. A random sample of 32 participants was selected, and the time each took to complete the stair climb was measured. The mean was 14.3 seconds with a standard deviation of 3.1 seconds.
- (i) Based on this sample, construct and interpret a 95% confidence interval for the mean time to complete the stair climb for all 250 participants in the programme.
- (ii) Suppose that for all participants, the proportion that completes the stair climb in less than 14 seconds needs to be estimated to within 0.05, with 95% confidence.
Determine the sample size that should be used.
- (iii) Outline the main concern you have about the answer in (a) (ii).
Describe briefly how you would try to complete the task stated in (a) (ii).
- (b) A new movement therapy programme is being trialled at the gym with a different group of people who have mobility problems due to injuries. Researchers want to compare the mean time to walk 20 metres for all participants in the new programme, with that of all participants in the current programme.

Two random samples were selected; one from participants in the current programme and one from participants in the new programme. For each selected participant, the time taken to walk 20 metres was measured.

Summary statistics are shown in Table 1 below.

Table 1

Programme	Sample Size	Mean	Standard Deviation
Current	35	31.4 s	9.5 s
New	40	28.1 s	7.6 s

Can it be claimed that the mean time to walk 20 metres for all participants in the new programme is different from the mean time for all participants in the current programme?

Justify your answer.

QUESTION TWO

A balancing activity is part of the current movement therapy programme. A random sample of 31 participants was selected. The time, in seconds (s), that each participant could balance on their left leg was measured twice; once balancing with both eyes open, and once balancing with both eyes closed. The data is shown in Table 2 below.

Table 2

Time, eyes open (s)	Time, eyes closed (s)	Time, eyes open (s)	Time, eyes closed (s)
10	15	32	27
11	10	31	26
13	30	31	25
12	18	42	29
13	22	40	30
9	2	45	31
16	15	40	32
18	7	39	24
22	21	48	34
18	10	52	35
25	20	56	37
28	30	60	36
30	28	36	20
32	20	24	29
36	25	60	10
38	32		

Two scatter plots of the data are shown in Figures 1 and 2 opposite. The difference between the scatter plots is that the variables have been reversed on the axes.

Figure 1

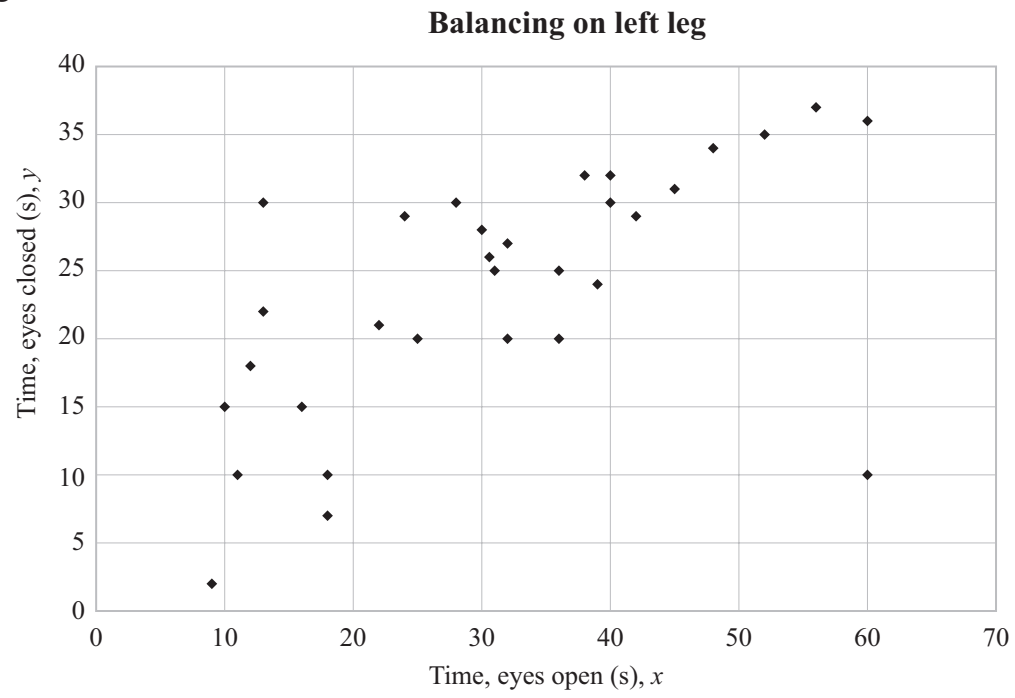
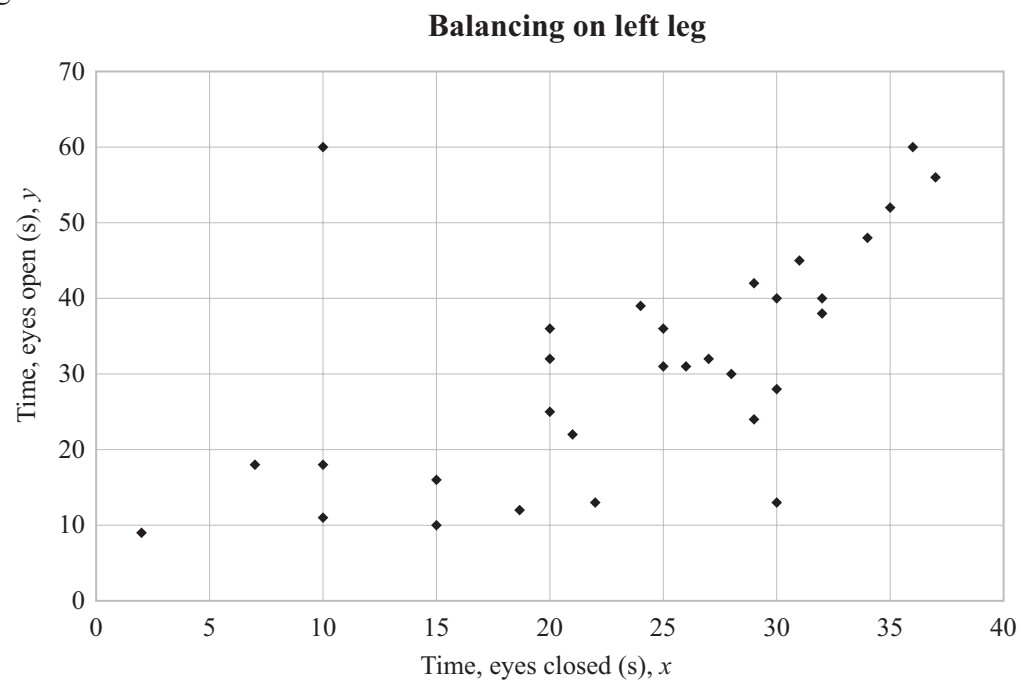


Figure 2



- (a) Use the scatter plots to make four distinct comments about this data.
- (b) Using regression analysis, a model is fitted to the data in Figure 1 and in Figure 2.
The equations of the fitted models are:
Figure 1: $y = 0.3727x + 11.923$
Figure 2: $y = 0.04534x^2 - 0.8879x + 23.280$
- (i) By selecting an appropriate model in each case:
- For a person who balanced on their left leg for 65 seconds with both eyes open, predict the time they would balance on their left leg with both eyes closed.
 - For a person who balanced on their left leg for 23 seconds with both eyes closed, predict the time they would balance on their left leg with both eyes open.
- (ii) Comment on the validity of each of your predictions in (i).
- (c) Other variables may have an effect on the time people can balance on their left leg.
- (i) Suggest two other variables.
- (ii) State how each of your variables could be measured.
- (iii) State, with justification, the probable relationship each of your variables would have with the balancing time.

QUESTION THREE

- (a) One task used to assess a participant's movement is to have them sit down in a chair, reach to pick up a cup and have a drink, put the cup down again and then stand up. The time, at the start of the programme, to complete this task can be modelled by a normal distribution with a mean of 16.0 seconds and a standard deviation of 0.75 seconds.

At the end of the programme, a random sample of six participants was selected and the time for each participant to complete the same task was measured. The sample mean was 15.2 seconds.

State a conclusion you can make from this information.

Justify your answer.

- (b) The participants in the movement therapy class engage in one of three activities: stair climbing, balancing or walking. The participants engage in these activities in the ratio 1:2:1 respectively and, in particular, the male participants engage in these activities in the ratio 4:2:3 respectively.

If the probability that a randomly chosen participant in the walking group is female equals $\frac{1}{6}$, find the probability that a randomly chosen female participant is in the walking group.

- (c) The gym has five treadmills and four rowing machines. For each treadmill, the probability that it is in use is 0.5. For each rowing machine, the probability that it is in use is 0.4.

If only two of these nine pieces of equipment are in use at a particular time, determine the probability that they are both rowing machines.

Assume that the use of each piece of equipment is independent of the others.

QUESTION FOUR

The graphs in Figures 3, 4 and 5 below show the number of people who attended the gym over three different time periods for the 65 days from Monday 16 July 2012 to Tuesday 18 September 2012.

The time periods are:

- early morning (5 am to 9 am)
- mid-day (10 am to 2 pm)
- evening (4 pm to 8 pm).

Each graph also shows a series of 7-point moving means with a trend line or curve fitted to the series of moving means.

For each graph, $x = 1$ for Monday 16 July, $x = 2$ for Tuesday 17 July, etc.

The equation of the fitted trend is shown on each graph.

Figure 3

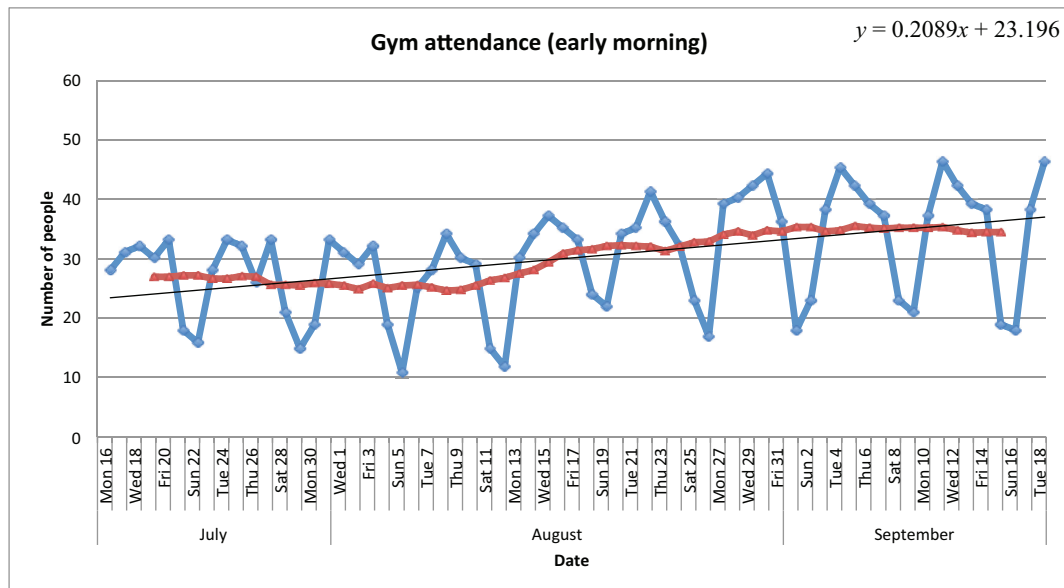


Figure 4

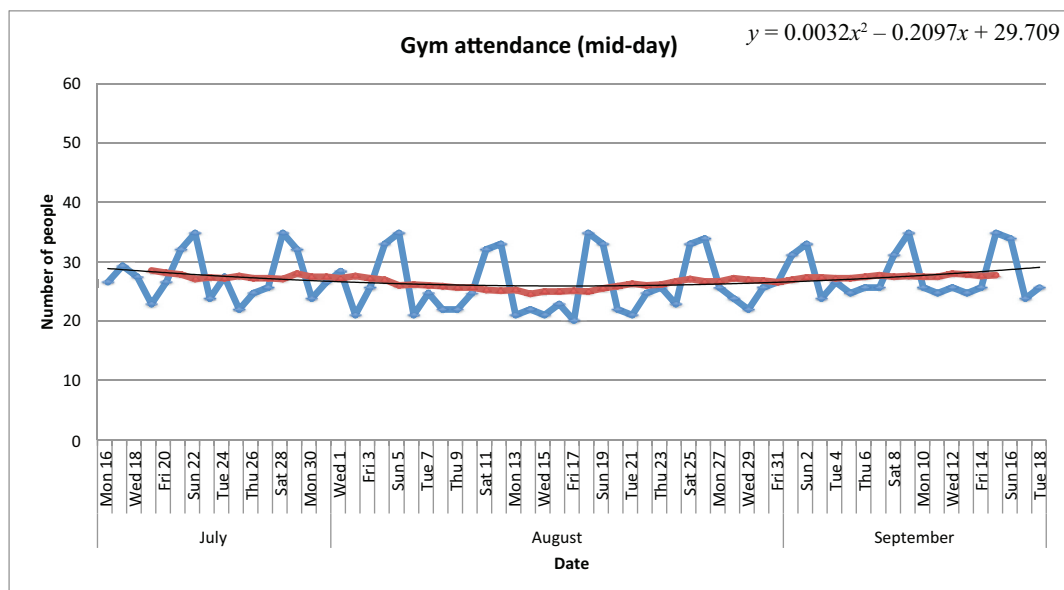
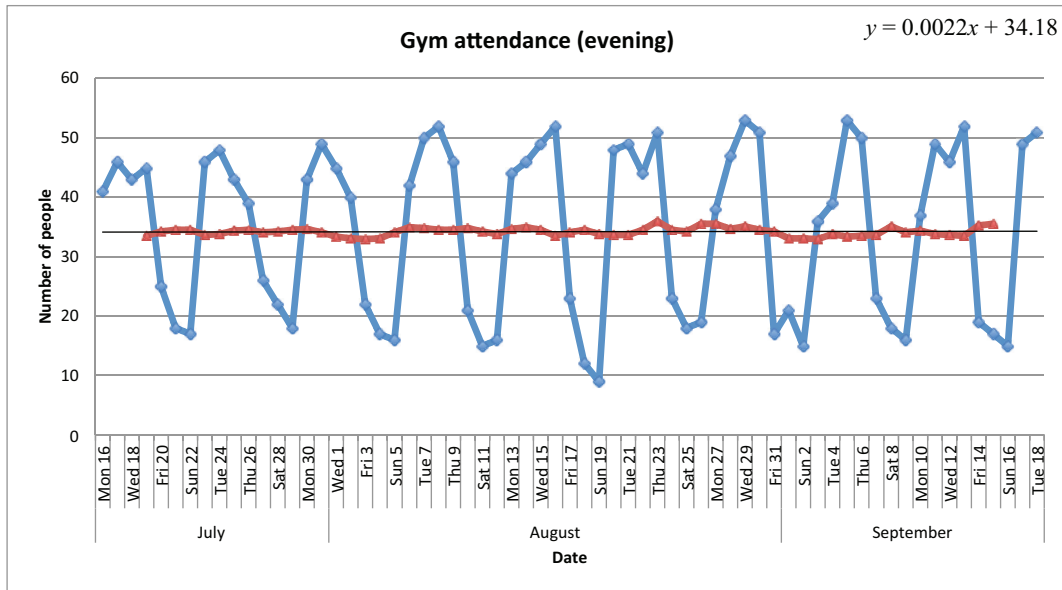


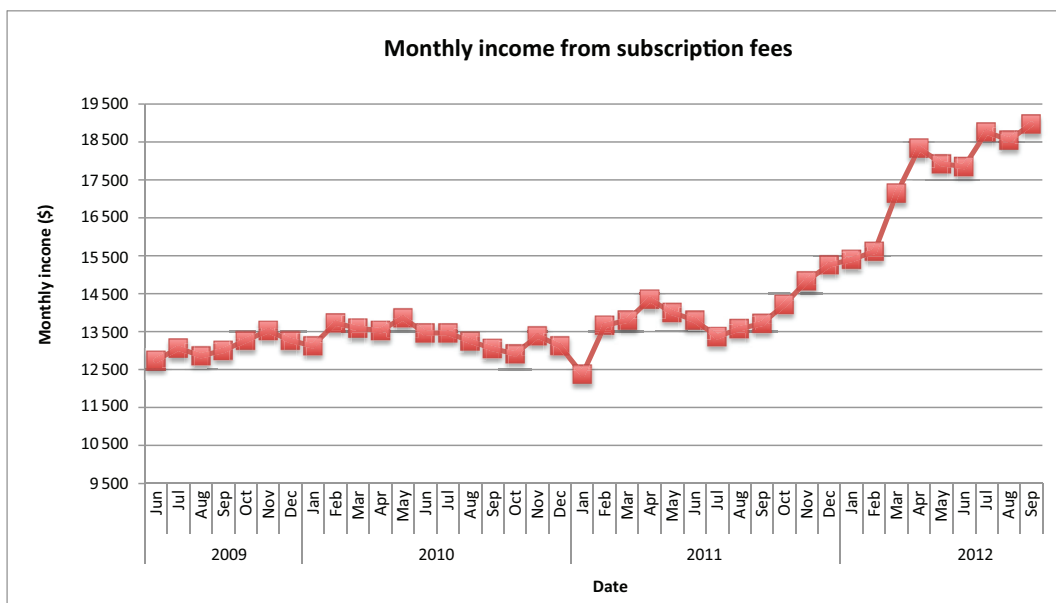
Figure 5



The graph in Figure 6 below shows the monthly income for the gym from the members' subscription fees from June 2009 to September 2012.

At the beginning of 2011, the gym began an advertising campaign to attract new members, and later that year started to purchase some new exercise equipment.

Figure 6



- (a) Write a one-page report to the gym management about daily gym attendance and monthly income from subscription fees.

- (b) The estimated seasonal effects for the early morning and mid-day periods are shown in Table 3 below.

Table 3

Day	Estimated Seasonal Effects	
	Early Morning	Mid-day
Mon	1.14	−3.61
Tues	6.55	−2.22
Wed	7.41	−3.15
Thu	4.35	−3.24
Fri	3.63	−2.19
Sat	−10.27	6.76
Sun	−12.82	7.68

Forecast the number of people who would attend the gym in the early morning period and in the mid-day period for Saturday 29 September 2012.

Show how each forecast was made and comment on the validity of each forecast.

- (c) In June of each year the gym increased its subscription fee in line with inflation. Table 4 below lists the monthly income from members' subscription fees, along with values of the Consumer Price Index (CPI) for June of each year. The CPI is a measure of inflation.

Table 4

Time	Monthly Income (\$)	CPI
June 2009	12 740	1075
June 2010	13 464	1097
June 2011	13 804	1137
June 2012	17 850	1158

By considering index numbers, or by using other calculations, estimate the percentage change in gym membership from June 2009 to June 2012.

QUESTION FIVE

The gym is planning on purchasing some new treadmills, steppers and rowing machines.

In order to meet usage requirements, there are to be a minimum of eight treadmills, five steppers and three rowing machines. The floor area needed for each piece of equipment is 3 m^2 per treadmill, 2 m^2 per stepper and 4 m^2 per rowing machine. After allowing for room between each piece of equipment, 132 m^2 is available. All of this area must be used. Data collected on concurrent usages in peak periods determined that there should be no more than three steppers to every two treadmills and at least one stepper to every treadmill.

The cost of each piece of equipment is in the ratio 2:3:5 for treadmills, steppers and rowing machines respectively.

Let x be the number of treadmills and y be the number of steppers.

- (a) Show that $3x + 2y \leq 120$.
- (b) Find the number of treadmills, steppers and rowing machines that should be purchased and installed in order to minimise the cost.

Grids are provided on pages 25 and 26 in your answer booklet to help you answer this question.

- (c) Suppose that the cost of each piece of equipment had been in the ratio 2:3: c for treadmills, steppers and rowing machines respectively.
- Given that all the other constraints still apply and that the cost is to be minimised, find the value of c that gives multiple solutions.

