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TOP SCHOLAR



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MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Scholarship 2015 Physics

9.30 a.m. Monday 16 November 2015

Time allowed: Three hours

Total marks: 40

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should answer ALL the questions in this booklet.

For all 'describe' or 'explain' questions, the answers should be written or drawn clearly with all logic fully explained.

For all numerical answers, full working must be shown and the answer must be rounded to the correct number of significant figures and given with the correct SI unit.

Formulae you may find useful are given on page 2.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–19 in the correct order and that none of these pages is blank.

You are advised to spend approximately 35 minutes on each question.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Question	Mark
ONE	
TWO	
THREE	
FOUR	
FIVE	
TOTAL	/40

ASSESSOR'S USE ONLY

QUESTION ONE: PARTICLES AND WAVES

- (a) (i) Describe the photoelectric effect.

In your answer you should include a derivation of the relationship between the incident photon's frequency and the electron's kinetic energy, and how these relate to the work function of the metal.

The photoelectric effect is when an electromagnetic wave (infrared, visible, UV, etc) hits metal, causing a photoelectron to be released. The energy of the photon is proportional to its frequency (by Planck's constant \hbar). The work function is the minimum amount of energy required for the release of the photoelectron. Hence, the E_k of the photoelectron is the energy of the photon ($\hbar f$) minus the work function (ϕ). The photoelectric effect was unable to be fully explained using classical physics.

- (ii)

Comment on this statement.

Classical physics treated light as a wave, which predicted that photoelectrons would have greater E_k when brightness increased, and that frequency would have no effect. In fact, increasing brightness increases the number of photoelectrons, and increasing frequency increases E_k . These observations were explained through modern physics, and the idea of photons and quanta.

- (b) Describe the similarities and the differences between the orbit of the Moon around the Earth and the orbit of an electron around a proton in a hydrogen atom.

In both cases, there are attractive forces between the two objects. In the case of the Moon and the Earth, there is a gravitational force, and in the case of the atom, there is an electrostatic force due to the opposite charges.



However, the circular orbit of the Moon around the Earth has a (roughly) constant radius, where F_g is acting as a centripetal force. In the case of the atom, the position of the electron is not defined, and it can move closer to and further from the nucleus (through energy levels) by taking in energy from, and releasing, photons.

- (c) Sound from a small loudspeaker L reaches a point P by two paths, which differ in length by 1.2 m. When the frequency of the sound is gradually increased, the resultant intensity at P goes through a series of maxima and minima. A maximum occurs when the frequency is 1000 Hz, and the next maximum occurs at 1200 Hz.

- (i) Explain what causes the maxima and minima to occur.

As the frequency increases, the wavelength decreases due to a constant velocity. As the wavelength decreases, the path difference of 1.2 m means that at certain wavelengths there is maximum constructive interference (maxima), and at others there is maximum destructive interference (minima)



- (ii) Calculate the speed of sound in the medium between L and P.

$$v = f \lambda \quad \text{For maxima } 1.2 = \text{integer number of}$$

$$v = 1000 \lambda_1, \quad v = 1200 \lambda_2 \quad \text{wavelengths (in phase,}$$

$$1000 \lambda_1 = 1200 \lambda_2, \quad \text{see diagram below}$$

$$\frac{1000}{1200} = \frac{\lambda_1}{\lambda_2} = \frac{5}{6}$$

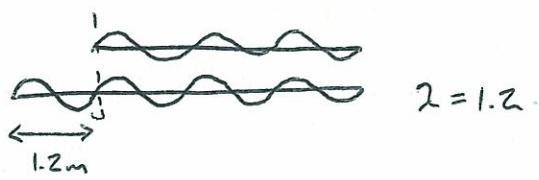
$$1.2 = \frac{\lambda_1}{\lambda_2} \quad \text{where } \lambda_1 \text{ and } \lambda_2 \text{ are two successive } \lambda \text{ values}$$

$$1.2 = \frac{0.24}{0.2} \quad \text{in the sequence below}$$

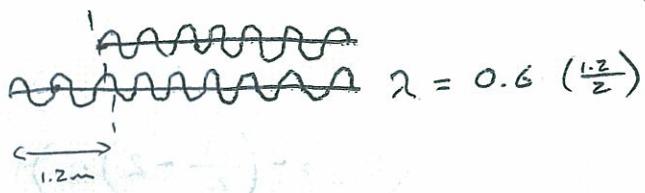
$$\lambda_1 = 0.24 \quad \lambda_2 = 0.2$$

$$v = 1000 \lambda_1 = 1000 \cdot 0.24$$

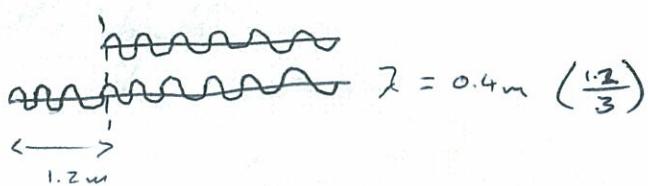
$$= 1200 \lambda_2 = 1200 \cdot 0.2 = 240 \text{ ms}^{-1}$$



$$\lambda_s = 1.2, 0.6, 0.4, 0.3, 0.24, 0.2$$



$$\lambda = 0.4 \text{ m } \left(\frac{1.2}{3} \right)$$



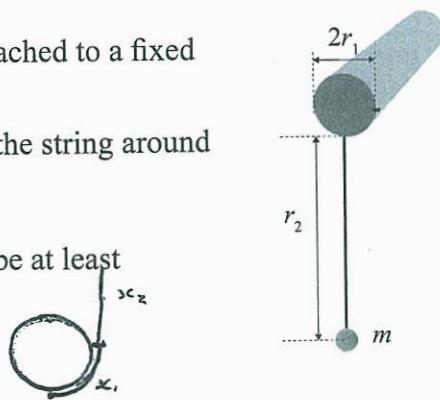
QUESTION TWO: THE VERTICAL CIRCLE

A small ball of mass m , hangs from a light, inextensible string attached to a fixed horizontal post of radius r_1 , as shown.

The ball is hit horizontally with a large bat so that the ball wraps the string around the post.

- (a) Show that the ball's speed at the top of its first swing must be at least

$$v_{\text{top}} = \sqrt{g \left(r_2 - \frac{\pi r_1}{2} \right)}$$
 so that the string remains taut.



At top length = r_2 $x_1 = \frac{1}{4} \cdot 2\pi r_1 = \frac{\pi r_1}{2}$

$$x_2 = r_2 - \frac{\pi r_1}{2}$$

$a_c = \frac{v^2}{r}$ For minimum v , $a_c \approx g$

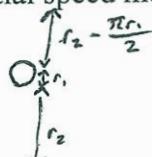
$$g = \frac{v^2}{r} \quad \text{where} \quad r = r_2 - \frac{\pi r_1}{2}$$

$$v^2 = g \left(r_2 - \frac{\pi r_1}{2} \right) \quad v = \sqrt{g \left(r_2 - \frac{\pi r_1}{2} \right)}$$

✓

- (b) For the speed of the ball in (a), show that the initial speed must be at least

$$v_{\text{initial}} = \sqrt{g \left(5r_2 - \left(\frac{3\pi}{2} - 2 \right) r_1 \right)}.$$



$$h = 2r_2 - \frac{\pi r_1}{2} + r_1$$

$$E_{\text{final}} = \frac{1}{2} mv^2 + hgm \quad \text{where} \quad v = \sqrt{g \left(r_2 - \frac{\pi r_1}{2} \right)} \quad \text{and} \quad E_{\text{final}} = E_{\text{initial}}$$

$$m \left(\frac{g \left(r_2 - \frac{\pi r_1}{2} \right)}{2} + g \left(2r_2 - \frac{\pi r_1}{2} + r_1 \right) \right) = \frac{1}{2} mv^2 + hgm$$

where mv cancels and $h = 0$:-

$$\frac{g \left(r_2 - \frac{\pi r_1}{2} \right)}{2} + g \left(2r_2 - \frac{\pi r_1}{2} + r_1 \right) = \frac{1}{2} v^2$$

$$\frac{gr_2}{2} - \frac{g\pi r_1}{4} + 2r_2g - \frac{\pi r_1 g}{2} + gr_1 = \frac{1}{2} v^2$$

$$g \left(\frac{r_2}{2} - \frac{\pi r_1}{4} + hr_2 - \frac{\pi r_1}{2} + r_1 \right) = \frac{1}{2} v^2 \quad g \left(\frac{r_2}{2} - \frac{\pi r_1}{4} + 2r_2 - \frac{\pi r_1}{2} + r_1 \right) = \frac{1}{2} v^2$$

$$g \left(r_2 - \frac{\pi r_1}{2} + 4r_2 - \pi r_1 + 2r_1 \right) = v^2 \quad g \left(r_2 - \frac{\pi r_1}{2} + 4r_2 - \pi r_1 + 2r_1 \right) = v^2$$

$$v^2 = g \left(5r_2 - \frac{\pi r_1}{2} - \frac{2\pi r_1}{2} + 2r_1 \right)$$

$$= g \left(5r_2 - \left(\frac{3\pi}{2} - 2 \right) r_1 \right)$$

✓

✓

- (c) Assuming an elastic collision, show that the speed of the bat is approximately half that of the ball's initial speed.

State any other assumptions made, and the reasons for them.

In an elastic collision, kinetic energy and momentum are both conserved. This requires the additional assumption of no energy loss to friction in the string or air resistance. Initially the ball has $0 E_k$ and $0 p$. * All the energy and momentum are initially in the bat. Assuming that the bat is ~~decently~~ more massive than the ball, the same amount of kinetic energy will have the ball move faster than the bat. Assuming the bat comes to a stop, this means that the bat would have a mass 4 times greater than the ball (if the speed is $\frac{1}{2}$), which is believable.

- (d) As the ball completes its first orbit around the post, explain why the ball appears to be travelling at a speed greater than its initial value.

As the ball completes its first orbit, the length of the string that is not wrapped around the post is reduced, indicating that the effective radius of rotation has been reduced (to $r_2 - \frac{3\pi r_1}{2}$). Assuming no major energy loss to friction/air resistance, energy is conserved. Because the ball is higher, it will have more E_{pgrav} , so actually slightly less E_k . However the effect is not significant, and the ball appears to be going faster because of the increased angular velocity (more rotations per unit of time), which in this context is what the person observes as being its speed.

20/20

✓

10/10

QUESTION THREE: CRICKET – THROW IN FROM THE BOUNDARY

Acceleration due to gravity = 9.81 m s^{-2}

- (a) Show that the range, R , of a projectile thrown from ground level at angle, ϕ , to the horizontal with starting velocity, v , is $\frac{v^2 \sin 2\phi}{g}$.



(Note that $2 \sin \phi \cos \phi = \sin 2\phi$.)

$$\begin{aligned} R &= vt_{\text{flight}} \text{ where } [v_x = v \cos \phi] \text{ and } t_{\text{flight}} = \text{time in flight} \\ v_i &= v \sin \phi \quad v_f = 0 \quad a = -9.81 \quad t = ? \quad = 2t \quad \text{where } t \text{ is time to max.} \\ 0 &= v \sin \phi - 9.81 t \quad v \sin \phi = gt \quad t = \frac{v \sin \phi}{g} \quad \left[t_{\text{Flight}} = \frac{2v \sin \phi}{g} \right] \\ R &= v \cos \phi \cdot \frac{2v \sin \phi}{g} \\ &= \frac{v^2 \cdot (2 \sin \phi \cos \phi)}{g} \quad \left[= \frac{v^2 \sin(2\phi)}{g} \right] \end{aligned}$$

- (b) A cricket ball is thrown from ground level with a velocity 28.0 m s^{-1} , and hits a target on the ground 80.0 m away.

Show that the time of flight of the ball is 4.04 s.

The effects of air resistance can be ignored.

$$\begin{aligned} R &= \frac{v^2 \sin(2\phi)}{g} \quad R = \frac{28^2 \sin(2\phi)}{9.81} \\ 80 &= 28 \cos \phi \cdot t \quad 80 = 28 \cos 45 \cdot t \quad \therefore \phi \approx 45^\circ \\ R &= v \cos \phi \cdot t \quad R = 28 \cos 45 \cdot t \\ v \sin \phi &= \frac{gt}{2} \quad 80 = 28 \sin \phi \cos \phi \quad 0.5005 = \sin \phi \cos \phi \\ \frac{2 \sin \phi}{g} &= t \quad 80 = 28 \cdot \frac{0.5005}{\cos \phi} \quad t = \frac{80}{28 \cos 45} \\ 0 &= v \sin \phi - gt \quad 80 = 28 \cdot \frac{0.5005}{\cos \phi} \quad t = 4.04 \text{ s} \\ \frac{0.5005}{\cos \phi} &= t \quad 80 = 28 \cdot \frac{0.5005}{\cos \phi} \quad t = 4.04 \text{ s} \end{aligned}$$

- (c) The ball is now thrown at the same target, with the same initial speed, but at a lower angle. This time, it is aimed to bounce in front of the target, so that it hits the target on the second bounce. When the ball bounces the first time, it rebounds with the same angle as it came in, but it loses half its speed.

- (i) Calculate the time taken for the ball to reach the target.

$$\begin{aligned} 28 \cos \phi &= v_x \quad 14 \cos \phi = v_x \\ x_1 &= 28 \cos \phi t_1, \quad x_2 = 14 \cos \phi t_2, \quad x_1 + x_2 = 80 \\ x_1 &= \frac{28^2 \sin(2\phi)}{g}, \quad x_2 = \frac{14^2 \sin(2\phi)}{g}, \quad x_1 + x_2 = 80 \\ 28 \cos \phi t_1 &= \frac{-84 \sin(2\phi)}{g}, \quad 14 \cos \phi t_2 = \frac{196 \sin(2\phi)}{g}, \quad t_1 + t_2 = t \\ t_1 &= \frac{28 \cdot 2 \cdot \sin \phi}{g}, \quad t_2 = \frac{14 \cdot 2 \cdot \sin \phi}{g}, \quad t = \frac{56 \sin \phi}{g} + \frac{28 \sin \phi}{g} = \frac{84 \sin \phi}{g} \\ t_1 &= \frac{56 \sin \phi}{g}, \quad t_2 = \frac{28 \sin \phi}{g}, \quad t = \frac{x_1}{28 \cos \phi} + \frac{x_2}{14 \cos \phi} = \frac{x_1}{28 \cos \phi} + \frac{2x_2}{28 \cos \phi} \\ &= \frac{x_1 + 2x_2}{28 \cos \phi} = \frac{80 + x_2}{28 \cos \phi} = \frac{80 + \frac{196 \sin(2\phi)}{g}}{28 \cos \phi} = \frac{1176 \sin(2\phi)}{g} = 80 + x_2 \end{aligned}$$

- (ii) Discuss, with physical reasons, the difference in times between parts (b) and (c)(i).

Although in the second throw, the ball spends the second base with a lower ^{horizontal} ~~lower~~ ^{speed} velocity, the time is still lower for this throw, as the smaller angle means that a larger component of the overall velocity is horizontal. ~~It is the horizontal velocity that is relevant~~

cont page 15

- (d) Any real throw of a ball would be from approximately head height, rather than from ground level.

Show that the range achieved by a throw from a height of 2 m above the ground would be

$$v \cos \phi \left(\frac{v \sin \phi + \sqrt{v^2 \sin^2 \phi + 4g}}{g} \right)$$



$$R = v \cos \phi \cdot t_{\text{total}}$$

$$t_{\text{up}} + t_{\text{down}} = t_{\text{total}}$$

$$0 = v \sin \phi - gt_{\text{up}} \quad gt_{\text{up}} = v \sin \phi \quad t_{\text{up}} = \frac{v \sin \phi}{g}$$

$$ddown = \frac{v^2 \sin^2 \phi}{2g} + z \quad a = \frac{2g}{2g} = g \quad v_i = 0 \quad t_{\text{down}} = ?$$

$$t_{\text{down}} = \sqrt{\frac{2(v^2 \sin^2 \phi + 4g)}{g}} + z$$

$$d_{\text{up}} = \frac{v \sin \phi}{g} \cdot v \sin \phi = \frac{v^2 \sin^2 \phi}{g}$$

$$d_{\text{down}} = \frac{1}{2} \cdot g \cdot t_{\text{down}}^2$$

$$t_{\text{down}} = \sqrt{\frac{2(v^2 \sin^2 \phi + 4g)}{g}} + z$$

$$t_{\text{down}} = \sqrt{\frac{2(v^2 \sin^2 \phi + 4g)}{g}} + z$$

$$R = v \cos \phi \cdot t_{\text{total}} = v \cos \phi \left(\frac{v \sin \phi + \sqrt{v^2 \sin^2 \phi + 4g}}{g} \right)$$

$$\frac{v^2 \sin^2 \phi}{g} + z = \frac{1}{2} \cdot g \cdot t_{\text{down}}^2$$

$$\frac{v^2 \sin^2 \phi + 4g}{2g} = \frac{gt_{\text{down}}^2}{2}$$

$$v^2 \sin^2 \phi + 4g = g^2 t_{\text{down}}^2$$

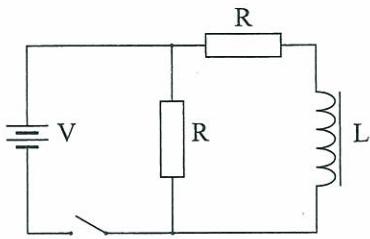
$$t_{\text{down}}^2 = \frac{v^2 \sin^2 \phi + 4g}{g^2}$$

$$t_{\text{down}} = \frac{\sqrt{v^2 \sin^2 \phi + 4g}}{g}$$

$$t_{\text{up}} + t_{\text{down}} = \frac{v \sin \phi + \sqrt{v^2 \sin^2 \phi + 4g}}{g}$$

8 (8)

QUESTION FOUR: CIRCUITS



- (a) In the electric circuit shown, the switch is closed at time $t = 0$.

- (i) Write an expression for the current immediately after the switch is closed.

Explain your reasoning.

$I = \frac{V}{R}$ Immediately after the switch is closed, there is more $\frac{dI}{dt}$, meaning that in the second branch, the inductor is making the new back EMF, so the current will flow through the other branch (of least resistance).

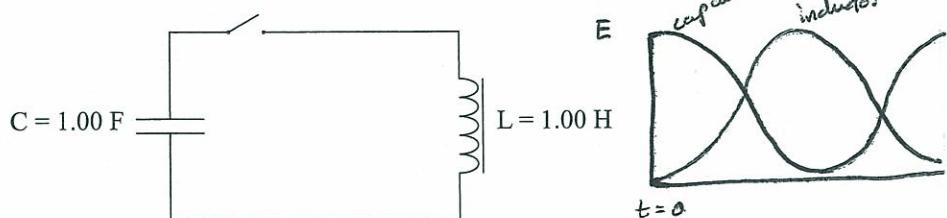
- (ii) Write an expression for the limiting value of the current a long time after the switch is closed.

Explain your reasoning.

$I = \frac{2V}{R}$. After a long time, $\frac{dI}{dt} = 0$, so there is no back EMF by the inductor. So $R_{\text{lim}} = \left(\frac{1}{R} + \frac{1}{R}\right)^{-1} = \left(\frac{2}{R}\right)^{-1} = \frac{R}{2}$. $I = \frac{V}{R_{\text{lim}}} \therefore I = \frac{2V}{R}$

- (b) (i) A charged capacitor (1.00 F) is connected to an inductor (1.00 H), as shown in the diagram below. When the switch is closed (at $t = 0$), the current in the circuit will oscillate sinusoidally with a period of 6.28 s.

Describe the energy changes that take place in the course of one complete cycle.



Before the switch is closed, all the energy is electrical, and stored in the electric field between the plates of the capacitor. When the switch is closed, the current increases, moving

- that the inductor opposes the change by generating a back EMF, storing energy in the magnetic field. The back EMF pushes charge back onto the plate, increasing the electrical energy again.
- (ii) The capacitor plates can be moved closer together so that the capacitance is increased to 4.00 F.

✓

Explain at what point in the cycle, could the plates of the capacitor be moved closer to each other so that no energy is transferred to the circuit.

The plates could be moved closer together when the capacitor has completely discharged. (When all the energy is being stored in the magnetic field). This is because at this time, there is no charge on the plates, and because $E = \frac{1}{2}QV$, then the energy stored in the electric field will remain 0 ($Q=0 \therefore E=0$) no matter what.

✓

- (c) A slab of copper falls freely under the influence of gravity before entering the region between the poles of a strong magnet. As it enters the magnetic field, the copper slab slows considerably.

Explain why this occurs, and state what has happened to the kinetic energy of the copper slab.

As the slab enters the field, a voltage is induced (due to the motion of a conductor in a magnetic field). Because there is now effectively a current of + charge $\begin{smallmatrix} \text{going to} \\ \text{the + end} \end{smallmatrix}$, a force will act on the current carrying conductor in an upwards direction. This will act in the opposite direction to gravity, decreasing the downwards acceleration and hence speed. The kinetic energy that the block originally had is now being stored in the magnetic field, as well as being used to separate the charges (electrical potential energy).



QUESTION FIVE: WAVES ON STRINGS

The speed v of a wave on a string is given by, $v = \sqrt{\frac{T}{\mu}}$, where T is the tension in the string, and μ is the mass per unit length, measured in kg m^{-1} .

- (a) Show that the above equation is dimensionally correct.

$$V : \text{ms}^{-1}$$

$$T : N : \text{kg ms}^{-2} \quad \frac{T}{\mu} : \frac{\text{kg ms}^{-2}}{\text{kg m}^{-1}} = \text{m}^2 \text{s}^{-2} \quad \sqrt{\frac{T}{\mu}} = \sqrt{\text{m}^2 \text{s}^{-2}} = \text{ms}^{-1}$$

$$\mu : \text{kg m}^{-1} \quad \therefore V$$

- (b) One end of a string of mass per unit length μ is attached to a solid wall, while the other end passes over a pulley, and is attached to a hanging mass, m , as shown in Figure 1.

A second string of the same length and made of the same material, but with twice the diameter, is mounted in a similar fashion with an identical mass, m , as shown in Figure 2.

The first string oscillates in its first harmonic when it is driven at a frequency of 200 Hz.

Calculate the frequency that will cause the second string to oscillate in its third harmonic.

Figure 1

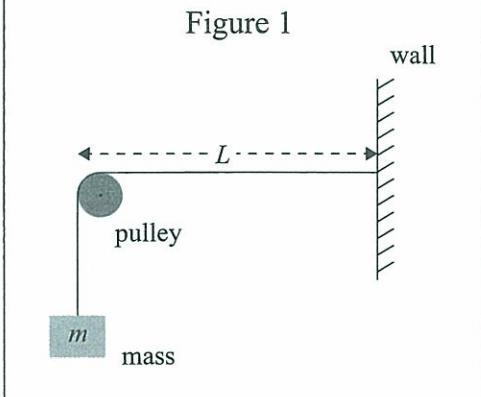
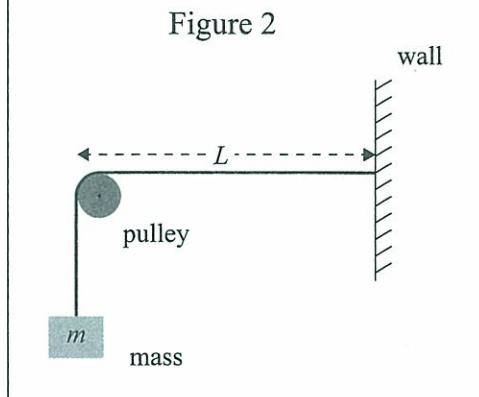


Figure 2



$$\text{Area}_2 = 4 \cdot \text{Area}_1, \therefore \mu_2 = 4 \cdot \mu_1$$



$$\text{For string } 2, mg = mg \therefore T_1 = T_2$$



$$V_1 = \sqrt{\frac{T}{\mu_1}}, \quad V_2 = \sqrt{\frac{T}{\mu_2}} = \sqrt{\frac{T}{4\mu_1}} = \frac{1}{2}\sqrt{\frac{T}{\mu_1}}$$



$$\therefore V_1 = 2 \cdot V_2 \quad V_2 = \frac{V_1}{2}$$

$$3^{\text{rd}} \text{ harmonic } \lambda = \frac{1}{3} 1^{\text{st}} \text{ harmonic } \lambda$$

Dividing v by λ harmonic

$$3^{\text{rd}} \text{ harmonic } f = 3 \cdot 1^{\text{st}} \text{ harmonic } f$$

2 means dividing f by 2

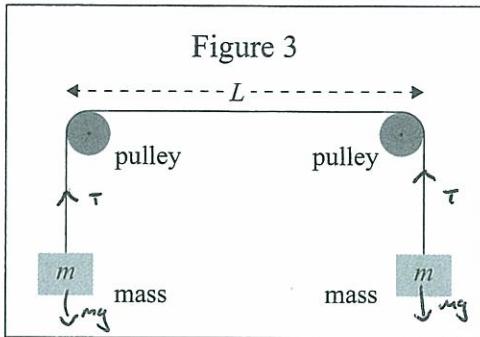
(for the first harmonic)

$$f_2^{3^{\text{rd}}} = 300 \text{ Hz}$$

$$\therefore f_2^{1^{\text{st}}} = 100 \text{ Hz}$$

- (c) Now the first string is hung so that both ends go over pulleys, with the masses suspended at each end, as shown in Figure 3.

Calculate the frequency of the fifth harmonic.



Tension is constant

throughout the string $\therefore T_3 = T_1$ (Tension

is still the same) $\therefore v_3 = v_1 =$

$$f_3^{1st} = f_1^{1st} = 200 \text{ Hz} \therefore f_3^{5th} = 5 \cdot f_3^{1st}$$

$$= 1000 \text{ Hz}$$

✓

(d)

- Two strings made from the same material are both fixed at each end, and both are under the same tension. The first string has a length L_1 ($= 1.00 \text{ m}$), and is being driven so that it oscillates in a transverse standing wave mode with a frequency of 400 Hz. The second string, with length L_2 ($= 1.18 \text{ m}$), is also oscillating in a standing wave mode, but with a slightly lower frequency. An observer notices that the standing wave on the second string has one more node than that on the first string. The observer hears a 4.5 Hz beat, as a result of the combined sound coming from the two standing waves.

1 m
 ∞
 400 Hz

1.18 m
 ∞
 $< 400 \text{ Hz}$

Calculate the number of nodes present in the first standing wave.

$$f_{beat} = |f_1 - f_2| \quad 4.5 = |400 - f_2| \quad f_2 < f_1$$

$$\therefore f_2 = 395.5 \text{ Hz}$$

Same string and tension \therefore same velocity

$$v_1 = f_1 L_1 \quad v_2 = f_2 L_2 \quad v_1 = v_2 \quad \therefore$$

$$400 L_1 = 395.5 L_2$$

$$0.98875 = \frac{L_1}{L_2}$$

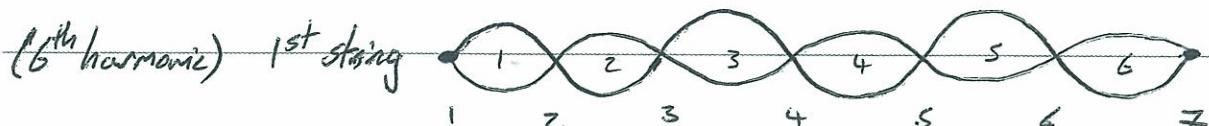
$$L_1 = 1 \cdot \frac{2}{3} = \frac{1}{3} \quad \text{The } 6^{\text{th}} \text{ harmonic}$$

$$L_2 = 1.18 \cdot \frac{2}{3} = 0.337 \quad \text{The } 1^{\text{st}} \text{ string } 6^{\text{th}} \text{ harmonic}$$

$$\frac{0.333}{0.337} = 0.98875 \quad \text{The } 7^{\text{th}} \text{ harmonic}$$

L_1 is $\frac{1}{3}$ of the string (6^{th} harmonic)

✓
✓
✓



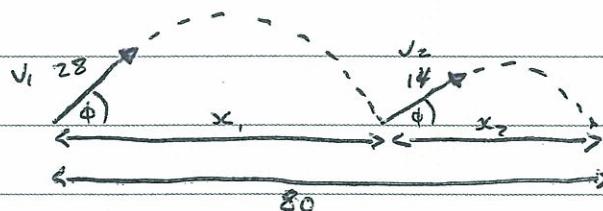
The first string has 7 nodes (including the terminal nodes)

8 (8)

QUESTION
NUMBER

Extra space if required.
Write the question number(s) if applicable.

3(c)i)



$$R = \frac{v^2 \sin(2\phi)}{g}$$

$$2 \sin \phi \cos \phi = \sin(2\phi)$$

$$t_{\text{total}} = ?$$

~~$v_1 = 28 \cos \phi$~~

~~$v_1 = 28 \sin \phi$~~

$x_1 = 28 \cos \phi t_1, \quad x_2 = 14 \cos \phi t_2 \quad v_2 = 14 \cos \phi$

$x_1 = \frac{v_1^2 \sin(2\phi)}{g} \quad x_2 = \frac{v_2^2 \sin(2\phi)}{g} \quad v_2 = 14 \sin \phi$

$v_1 \cos \phi t_1 = \frac{v_1^2 \sin(2\phi)}{g}$

$t_1 = \frac{2v_1^2 \sin(\phi) \cos(\phi)}{v_1 g \cos(\phi)}$

$= \frac{2v_1 \sin \phi}{g} \quad \frac{1}{x_2} \frac{2v_2 \sin \phi}{g}$

$t_{\text{total}} = \frac{2v_1 \sin \phi}{g} + \frac{2v_2 \sin \phi}{g} = \frac{2 \sin \phi}{g} (v_1 + v_2)$

$$\boxed{t_{\text{total}} = \frac{84 \sin \phi}{g}}$$

$x_1 + x_2 = 80 \quad \frac{v_1^2 \sin(2\phi)}{g} + \frac{v_2^2 \sin(2\phi)}{g} = 80$

~~$\frac{784 \sin(2\phi)}{g} + \frac{196 \sin(2\phi)}{g} = 80$~~

$980 \sin(2\phi) = 80g \quad \sin(2\phi) = 0.8008$

$2\phi = \sin^{-1}(0.8008)$

$= 53.2^\circ \quad \therefore$

$\phi = 26.6^\circ$

$t_{\text{total}} = \frac{84 \sin \phi}{g}$

$= \frac{84 \sin 26.6}{9.81} = 3.838$

Ae

Extra space if required.
Write the question number(s) if applicable.

3(c)ii) to the time it takes for the ball to reach the target.
 i.e. For the first throw the horizontal velocity was $28 \cos 45 = 19.8 \text{ ms}^{-1}$
 For the first bounce of the second throw, the horizontal velocity was $28 \cos(26.6) = 25.0 \text{ ms}^{-1}$, and for the second bounce, it was $24 \cos(26.6) = 17.5 \text{ ms}^{-1}$. Because the first bounce was a larger component, $\frac{\text{more distance}}{\text{+ time}}$ the increased velocity in the first bounce had a greater effect than the decreased bounce velocity in the second bounce.

Ale