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# SCHOLARSHIP EXEMPLAR



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MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD  
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## Scholarship 2015 Physics

9.30 a.m. Monday 16 November 2015

Time allowed: Three hours

Total marks: 40

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should answer ALL the questions in this booklet.

For all 'describe' or 'explain' questions, the answers should be written or drawn clearly with all logic fully explained.

For all numerical answers, full working must be shown and the answer must be rounded to the correct number of significant figures and given with the correct SI unit.

**Formulae you may find useful are given on page 2.**

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–19 in the correct order and that none of these pages is blank.

You are advised to spend approximately 35 minutes on each question.

**YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.**

Question	Mark
ONE	
TWO	
THREE	
FOUR	
FIVE	
<b>TOTAL</b>	/40

ASSESSOR'S USE ONLY

## QUESTION ONE: PARTICLES AND WAVES

- (a) (i) Describe the photoelectric effect.

In your answer you should include a derivation of the relationship between the incident photon's frequency and the electron's kinetic energy, and how these relate to the work function of the metal.

*an electromagnetic wave particle (photon)*

Whenever ~~a~~ light with a frequency higher than the threshold frequency that is an  $E = hf$  energy greater than the work function, an electron will be instantaneously emitted from the surface of the metal. After ~~overcoming~~ the work function  $\Phi$ , the remaining energy turns into  $E_k$  of photoelectron, therefore  $E_k = hf - \Phi$ .

- (ii) The photoelectric effect was unable to be fully explained using classical physics.

Comment on this statement.

Classical physics ~~can~~ recognise EM ~~wave~~ visible light as a wave. There are several observations of photoelectric effect not able to be explained by wave theory, such as the wave theory suggests that any light can cause photoelectric effect provided that the metal is exposed <sup>for</sup> sufficiently long time, but in reality, only certain light or EM wave with ~~any~~ a frequency higher than a certain value can cause emission of electrons. The emission is instantaneous. Hence this effect is unable to be fully explained by classical physics.

- (b) Describe the similarities and the differences between the orbit of the Moon around the Earth and the orbit of an electron around a proton in a hydrogen atom.

The orbits are all nearly circular, all involve a <sup>smaller</sup> outer object ~~object~~ in circular motion around a <sup>larger</sup> centre object. All these orbits involve only one object ~~spinning~~ revolving around one centre object.

In the orbit of Moon around the Earth, centripetal force is provided by the gravitational attraction between Moon and the Earth. The Moon and the Earth are both ~~neutral~~ neutral. The orbit exist in only one energy state. In the orbit of Electron around proton, the centripetal force is provided by the electrostatic attraction force between the negatively charged electron and the positively charged proton. The orbit can have various energy states. The mass of the Moon and Earth are of orders  $(10^{27})$  kg and  $(10^{26})$  kg, and the mass of electron and proton are in order of  $(10^{-31})$  and  $(10^{-30})$ .

- (c) Sound from a small loudspeaker L reaches a point P by two paths, which differ in length by 1.2 m. When the frequency of the sound is gradually increased, the resultant intensity at P goes through a series of maxima and minima. A maximum occurs when the frequency is 1000 Hz, and the next maximum occurs at 1200 Hz.

- (i) Explain what causes the maxima and minima to occur.

The <sup>incident</sup> sound reflect from point P. The reflected sound have the same frequency and wavelength as the incoming sound. When the distance between L and P ~~is~~ is a integral multiple of half of the wavelength of the sound, the incident sound and reflected sound will be in phase. The result is a single standing wave. This will give a series of maxima and minima.

- (ii) Calculate the speed of sound in the medium between L and P.

Let the speed of sound be  $v$ . Then since  $v = f\lambda$ ,  $\lambda$  for the first and second maxima to occur is  $\frac{v}{1000}$  and  $\frac{v}{1200}$  respectively.

Half wavelength is  $\frac{v}{2000}$  and  $\frac{v}{2400}$ .

So 1.2 is a multiple of both  $\frac{v}{2000}$  and  $\frac{v}{2400}$ .

So  $v$  must be a factor of  $1.2 \times 1000 = 2400$  and  $1.2 \times 2400 = 2880$ .

Since  $2400 = 480 \times 5$ ,  $2880 = 480 \times 6$ , the maximum  $v$  satisfies both of these condition is  $480 \text{ m s}^{-1}$ .

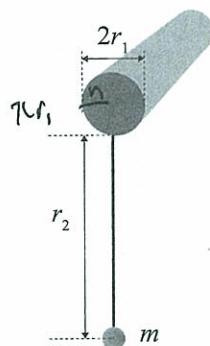
## QUESTION TWO: THE VERTICAL CIRCLE

A small ball of mass  $m$ , hangs from a light, inextensible string attached to a fixed horizontal post of radius  $r_1$ , as shown.

The ball is hit horizontally with a large bat so that the ball wraps the string around the post.

- (a) Show that the ball's speed at the top of its first swing must be at least

$$v_{\text{top}} = \sqrt{g \left( r_2 - \frac{\pi r_1}{2} \right)} \text{ so that the string remains taut.}$$



Length of string after  $\frac{1}{2}$  wrap  $r = r_2 - \frac{1}{2} \times (\pi r_1) = r_2 - \frac{\pi r_1}{2}$

Tension and weight of the ball provide centripetal force, so

$$mg + T = m \frac{v_{\text{top}}^2}{r} \Rightarrow v_{\text{top}} = \sqrt{r(g + \frac{T}{m})} = \sqrt{(r_2 - \frac{\pi r_1}{2})(g + \frac{T}{m})}$$

For string remain taut,  $T \geq 0$

$$\text{so } v_{\text{top}} \geq \sqrt{g(r_2 - \frac{\pi r_1}{2})}.$$

- (b) For the speed of the ball in (a), show that the initial speed must be at least

$$v_{\text{initial}} = \sqrt{g \left( 5r_2 - \left( \frac{3\pi}{2} - 2 \right) r_1 \right)}.$$

Change in kinetic energy = change in gravitational potential energy

$$\frac{1}{2}mv_{\text{initial}}^2 - \frac{1}{2}mv_{\text{top}}^2 = mgh, \text{ where } \Delta h = r_2 + r_1 + (r_2 - \frac{\pi r_1}{2}) = 2r_2 + r_1 - \frac{\pi r_1}{2}$$

$$\text{so } v_{\text{initial}}^2 - v_{\text{top}}^2 = 2g(2r_2 + r_1 - \frac{\pi r_1}{2}) = g(4r_2 + 2r_1 - \pi r_1)$$

$$v_{\text{initial}} = \sqrt{g(4r_2 + 2r_1 - \pi r_1)} + v_{\text{top}} = \sqrt{g(4r_2 + 2r_1 - \pi r_1)} + \sqrt{g(2 - \frac{\pi r_1}{2})} = \sqrt{g(5r_2 + r_1 - \frac{3\pi r_1}{2})}$$

$$v_{\text{initial}} = \sqrt{g(5r_2 + 2r_1 - \frac{3\pi}{2} r_1)}$$

$$= \sqrt{g(5r_2 - (\frac{3\pi}{2} - 2)r_1)}$$

- (c) Assuming an elastic collision, show that the speed of the bat is approximately half that of the ball's initial speed.

State any other assumptions made, and the reasons for them.

Assume that the weight of the ball bat ~~consists of~~ is a point mass at the point of impact. This is because the bat can be thought to have a uniform distribution of mass and the mass of the grip is negligible. These assumptions are made so that also assume that there is no force acting on the ~~the~~ bat at the point of impact. All these are made so that conservation of <sup>linear</sup> momentum is applied.

If the bat has a mass  $M$  and a speed  $v$ , then by conservation of momentum,  $Mv = mV_{\text{initial}} - MV_1$ , where  $V_1$  is the speed of bat after impact.

$$\cancel{Mv} - \cancel{V_1} = V_{\text{initial}} + V \quad \text{Since impact is elastic, } V = V_{\text{initial}} - V_1 \Rightarrow V_1 = V_{\text{initial}} - V.$$

$$\therefore Mv = mV_{\text{initial}} - M(V_{\text{initial}} - V)$$

$$\Rightarrow 2Mv = (m+M)V_{\text{initial}}$$

$$\Rightarrow v = \left( \frac{m}{2M} + \frac{1}{2} \right) V_{\text{initial}}$$

Since the bat is large,  $M$  is significantly larger than  $m$ , so  $\frac{m}{2M} \approx 0$ .

$$\therefore v \approx \frac{1}{2} V_{\text{initial}}$$

- (d) As the ball completes its first orbit around the post, explain why the ball appears to be travelling at a speed greater than its initial value.

The apparent speed we see is the angular speed. The faster the angular speed, the quicker it completes an orbit and so the faster it appears to be. Despite losing ~~a~~ kinetic energy of  $mgh = mg \cdot 2\pi r$ , after an orbit, the ~~loss~~ of speed is not significant as the ~~is~~ in orbital radius since the initial speed must be large. Since  $V = \omega r \Rightarrow \omega = \frac{V}{r}$ ,  $\omega$  = angular velocity,  $V$  = linear velocity,  $r$  = radius, a more significant reduction in  $r$  than reduction in  $V$  makes  $\omega$  larger, so the circular speed increases after the first orbit, ~~so~~ so it appears to be traveling faster.

### QUESTION THREE: CRICKET – THROW IN FROM THE BOUNDARY

Acceleration due to gravity =  $9.81 \text{ m s}^{-2}$

- (a) Show that the range,  $R$ , of a projectile thrown from ground level at angle,  $\phi$ , to the horizontal

with starting velocity,  $v$ , is  $\frac{v^2 \sin 2\phi}{g}$ .



(Note that  $2 \sin \phi \cos \phi = \sin 2\phi$ .)

$$\text{Horizontal velocity} = v \cos \phi, \text{ vertical velocity} = v \sin \phi$$

In the vertical component, after landing, the final velocity is  $-v \sin \phi$ .

~~Since~~ since  $v = u + at$ ,  $-v \sin \phi = v \sin \phi - gt$ ,  $t = \frac{2v \sin \phi}{g}$ .

Hence after time  $t$ , the object has travelled  $v \cos \phi \cdot \frac{2v \sin \phi}{g}$  horizontally.

This gives  $\frac{v^2 (2 \sin \phi \cos \phi)}{g} = \frac{v^2 \sin 2\phi}{g}$ .

- (b) A cricket ball is thrown from ground level with a velocity  $28.0 \text{ m s}^{-1}$ , and hits a target on the ground  $80.0 \text{ m}$  away.

Show that the time of flight of the ball is  $4.04 \text{ s}$ .

The effects of air resistance can be ignored.

$$\text{Since range} = \frac{v^2 \sin 2\phi}{g}, 80.0 = \frac{28^2 \sin 2\phi}{9.81} \Rightarrow \sin 2\phi = 1, 2\phi = 90^\circ, \phi = 45^\circ$$

$$\text{So from (a), } t = \frac{2v \sin \phi}{g} = \frac{2 \times 28 \times \sin 45^\circ}{9.81} = 4.03649131 \dots \approx 4.04 \text{ s}$$

- (c) The ball is now thrown at the same target, with the same initial speed, but at a lower angle. This time, it is aimed to bounce in front of the target, so that it hits the target on the second bounce. When the ball bounces the first time, it rebounds with the same angle as it came in, but it loses half its speed.

- (i) Calculate the time taken for the ball to reach the target.

Let the range of first ~~bounce~~ throw be  $R_1$  and of ~~second~~ after first bounce be  $R_2$ . So  $R_1 = \frac{v^2 \sin 2\phi_1}{g}$  and  $R_2 = \frac{v^2 \sin 2\phi_2}{g}$ .

Since the ball reaches the ground with the same angle  $\alpha$  it is thrown and is reflected at the same angle,  $\phi_1 = \phi_2$ . Also  $v_2 = \frac{1}{2} v_1 = 14.0 \text{ m s}^{-1}$ , so from  $R_1 + R_2 = 80$ ,

$$\frac{28^2}{9.81} \cdot \sin 2\phi + \frac{14^2}{9.81} \cdot \sin 2\phi = 80 \Rightarrow 99.9 \sin 2\phi = 80 \Rightarrow \sin 2\phi = 0.801, \phi = 26.6^\circ$$

$$t = \frac{2v \sin \phi}{g} + \frac{2v_2 \sin \phi}{g} = \frac{2 \times 28 \times \sin 26.6^\circ}{9.81} + \frac{2 \times 14 \times \sin 26.6^\circ}{9.81}$$

$$t = 3.83 \text{ s}$$

- (ii) Discuss, with physical reasons, the difference in times between parts (b) and (c)(i).

Since  $t = \frac{2v \sin \phi}{g}$ , the smaller the angle, the quicker it reaches the target.

This is because the vertical component of velocity is reduced, so it under the effect of acceleration of free fall, the faster it gets to the opposite value.

Since the answer in part (i) consists a much smaller angle than part (b),

although speed for the second bounce is reduced, it reaches the target slightly faster. //

- (d) Any real throw of a ball would be from approximately head height, rather than from ground level.

Show that the range achieved by a throw from a height of 2 m above the ground would be

$$v \cos \phi \left( \frac{v \sin \phi + \sqrt{v^2 \sin^2 \phi + 4g}}{g} \right)$$

When the ball reaches 2m above the ground the second time, its vertical component of velocity is  $-v \sin \phi$ , and it would have travelled  $\frac{2v^2 \sin^2 \phi}{g}$ .

since  $s = ut + \frac{1}{2}gt^2$ , for a fall of 2m,  $-2 = -v \sin \phi t - \frac{1}{2}g t^2$

$\Rightarrow gt^2 + 2v \sin \phi t - 4 = 0$  where  $t$  is the time taken for it to reach the ground from 2m high. so  $t = \frac{-2v \sin \phi \pm \sqrt{4v^2 \sin^2 \phi + 16g}}{2g}$ . The range achieved after this is  $\Rightarrow v \cos \phi t = -v \cos \phi \sin \phi + \frac{\sqrt{4v^2 \sin^2 \phi + 16g}}{2g} \cdot v \cos \phi = -v \cos \phi \sin \phi + \frac{\sqrt{v^2 \sin^2 \phi + 4g}}{2g}$

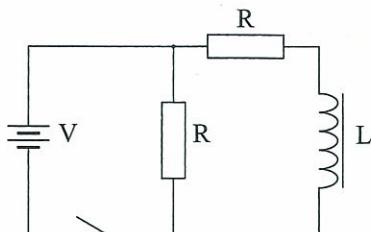
$$\text{so range } R = \frac{2v^2 \sin \phi \cos \phi}{g} - \frac{v^2 \cos^2 \phi}{g} + \frac{\sqrt{v^2 \sin^2 \phi + 4g}}{g} \cdot v \cos \phi$$

$$= \frac{\sqrt{v^2 \sin^2 \phi + 4g}}{g} + v \cos \phi \sqrt{\frac{v^2 \sin^2 \phi + 4g}{g}}$$

$$= v \cos \phi \left( \frac{v \sin \phi}{g} + \frac{\sqrt{v^2 \sin^2 \phi + 4g}}{g} \right)$$

$$= v \cos \phi \left( \frac{v \sin \phi + \sqrt{v^2 \sin^2 \phi + 4g}}{g} \right)$$

## QUESTION FOUR: CIRCUITS



- (a) In the electric circuit shown, the switch is closed at time  $t = 0$ .

- (i) Write an expression for the current immediately after the switch is closed.

Explain your reasoning.

~~$I = \frac{V}{R}$~~   $I = \frac{V}{2R}$ . This is because a back EMF is induced in the inductor  $L$ , which opposes  $V$  and prevents current from going in this branch.

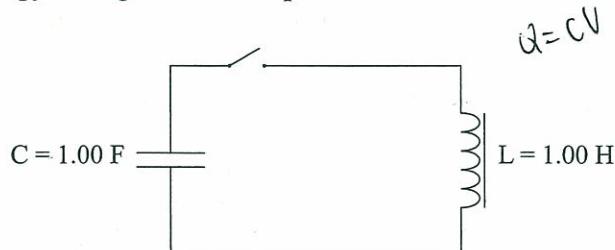
- (ii) Write an expression for the limiting value of the current a long time after the switch is closed.

Explain your reasoning.

Assume that  $L$  is ideal and have no resistance, then ~~total~~ after switch for a long time,  $L$  has no effect and is just a long wire. So total resistance is  $\frac{R}{2}$ ,  
 $\therefore$  Limiting current  $\Rightarrow \frac{V}{\frac{R}{2}} = \frac{2V}{R}$ .

- (b) (i) A charged capacitor ( $1.00 \text{ F}$ ) is connected to an inductor ( $1.00 \text{ H}$ ), as shown in the diagram below. When the switch is closed (at  $t = 0$ ), the current in the circuit will oscillate sinusoidally with a period of  $6.28 \text{ s}$ .

Describe the energy changes that take place in the course of one complete cycle.



When switch is closed, the capacitor discharges, creating an e.m.f. (capacitor loss) energy. This sudden increase in (capacitor loss) energy induces a back e.m.f. from the inductor, so energy is transferred to the inductor. Then the e.m.f. in the circuit starts to drop. This induces a e.m.f. in the inductor which runs in the

opposite direction. This will charge C again so L loses energy and C gains energy. The process then repeats itself.

6

- (ii) The capacitor plates can be moved closer together so that the capacitance is increased to 4.00 F.

Explain at what point in the cycle, could the plates of the capacitor be moved closer to each other so that no energy is transferred to the circuit.

When C reaches zero energy, this is because  $E = \frac{1}{2}CV^2$   
 $E = \frac{1}{2} \frac{Q^2}{C}$ . If Q is not zero, then any change in C will result in a change in E. If Q is zero, then any change in C will not change E since E is at zero regardless of the value of C.

✓



- (c) A slab of copper falls freely under the influence of gravity before entering the region between the poles of a strong magnet. As it enters the magnetic field, the copper slab slows considerably.

Explain why this occurs, and state what has happened to the kinetic energy of the copper slab.

Copper is a current-carrying conductor. As copper moves in between the magnetic fields, it cuts the field lines. This induces an e.m.f. inside copper. This e.m.f. has an effect of slowing copper down (Len's law) as copper is the cause of the e.m.f., or otherwise kinetic energy is gained from nothing, violate conservation of energy. This e.m.f. induced in copper will cause a current, and since copper has resistance, this current will dissipate heat. This heat must be coming from the kinetic energy of copper as this is the only source of energy directly related to the current.

6

✓

6
(8)

## QUESTION FIVE: WAVES ON STRINGS

The speed  $v$  of a wave on a string is given by,  $v = \sqrt{\frac{T}{\mu}}$ , where  $T$  is the tension in the string, and  $\mu$  is the mass per unit length, measured in  $\text{kg m}^{-1}$ .

- (a) Show that the above equation is dimensionally correct.

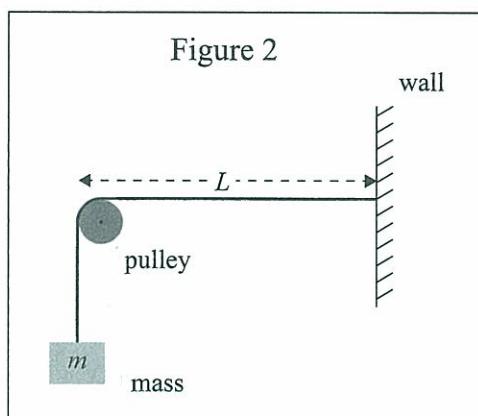
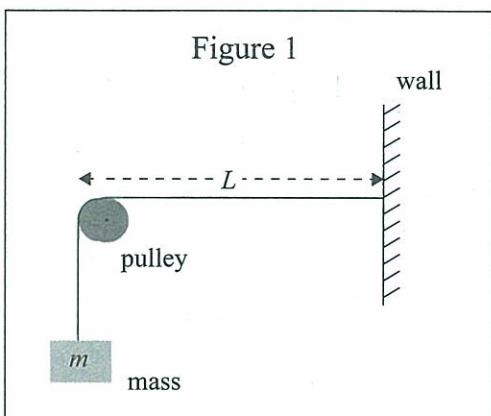
Tension is measured in Newtons,  $F = ma$ ,  $N = \text{kg m s}^{-2}$ . Since  $M$  is measured in  $\text{kg m}^3$ ,  $\mu$  is measured in  $\text{kg}^{-1}\text{m}$ . So  $\sqrt{\frac{T}{\mu}}$  is measured in  $\sqrt{\text{kg m s}^2 \cdot \text{kg}^{-1}\text{m}} = \sqrt{\text{m}^2\text{s}^2} = \text{m s}^{-1}$ , which measures  $v$ .

- (b) One end of a string of mass per unit length  $\mu$  is attached to a solid wall, while the other end passes over a pulley, and is attached to a hanging mass,  $m$ , as shown in Figure 1.

A second string of the same length and made of the same material, but with twice the diameter, is mounted in a similar fashion with an identical mass,  $m$ , as shown in Figure 2.

The first string oscillates in its first harmonic when it is driven at a frequency of 200 Hz.

Calculate the frequency that will cause the second string to oscillate in its third harmonic.



twice diameter = 8 times the volume, since  $m = \rho V$ , it means 8 times of mass per unit length. If the first string has a wave speed  $v$ , the second string will have a wave speed  $v' = \sqrt{\frac{T}{\mu}} = \frac{1}{\sqrt{8}} \sqrt{\frac{T}{\mu}} = \frac{1}{2\sqrt{2}} v$ .

At the first harmonic,  $\lambda = \frac{L}{2} \Rightarrow \lambda = 2L$ . So  $f_1 = \frac{v}{\lambda} = \frac{v}{2L}$ .

At the third harmonic,  $\lambda = \frac{3L}{2} \Rightarrow \lambda = \frac{2L}{3}$ . So  $f_2 = \frac{v'}{\lambda} = \frac{v}{\lambda} = \frac{3}{2} f_1$ .

$$f_2 = \frac{v'}{\lambda} = \frac{\frac{1}{2\sqrt{2}} v}{\frac{3}{2} L} = \frac{3}{2\sqrt{2}} \cdot \frac{v}{2L} = \frac{3}{2\sqrt{2}} f_1$$

Since  $f_1 = 200 \text{ Hz}$ ,  $f_2 = \frac{3}{2\sqrt{2}} \times 200 = 212 \text{ Hz}$ .

- (c) Now the first string is hung so that both ends go over pulleys, with the masses suspended at each end, as shown in Figure 3.

Calculate the frequency of the fifth harmonic.

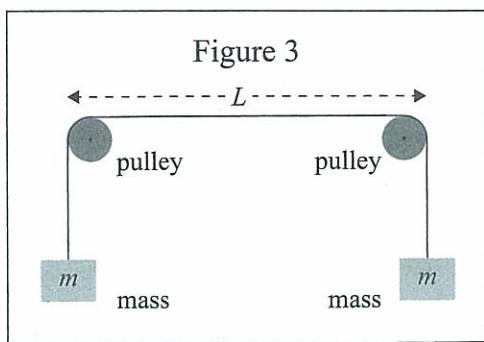
Tension in the string is twice that

$$\text{in figure 1, so } V_2 = \sqrt{\frac{2T}{\mu}} = \sqrt{2}V.$$

$$\text{At the fifth harmonic, } L = \frac{5\lambda}{2}, \text{ so } \lambda = \frac{2L}{5}.$$

$$f_5 = \frac{V_2}{\lambda} = \frac{\sqrt{2}V}{\frac{2L}{5}} = 5\sqrt{5} \cdot \frac{V}{2L} = 5\sqrt{2} f_1.$$

$$\text{Since } f_1 = 200 \text{ Hz } f_5 = 5\sqrt{2} \times 200 = 1410 \text{ Hz.}$$



- (d) Two strings made from the same material are both fixed at each end, and both are under the same tension. The first string has a length  $L_1$  ( $= 1.00 \text{ m}$ ), and is being driven so that it oscillates in a transverse standing wave mode with a frequency of 400 Hz. The second string, with length  $L_2$  ( $= 1.18 \text{ m}$ ), is also oscillating in a standing wave mode, but with a slightly lower frequency. An observer notices that the standing wave on the second string has one more node than that on the first string. The observer hears a 4.5 Hz beat, as a result of the combined sound coming from the two standing waves.

Calculate the number of nodes present in the first standing wave.

Suppose that the first standing wave has  $n$  nodes, so second string has  $n+1$  nodes. Since number of nodes = number of harmonics + 1, first standing wave has  $n-1$ <sup>th</sup> harmonic and second standing wave is  $n$ th harmonic. Since  $L = \frac{n}{2}\lambda$  for  $n$ th harmonic,  $L_1 = \frac{(n-1)}{2}\lambda \Rightarrow \lambda = \frac{2L_1}{n-1}$ ,  $f_1 = \frac{V}{\lambda} = \frac{V(n-1)}{2L_1}$ . Similarly  $f_2 = \frac{V(n)}{2L_2}$ .

$V$  is constant since  $T$  and  $\mu$  are constant for both. Since  $f_1 = 200$ ,  $L_1 = 1.00 \text{ m}$ ,  $V(n-1) = 200 \times 2 \times 1.00 = 400 \Rightarrow V = \frac{400}{n-1}$ . Substitute into  $f_2$  gives  $f_2 = \frac{400n}{2(n-1)L_2}$

$$\Rightarrow f_2 = \frac{200n}{(n-1) \times 1.18} = 169.5 \times \frac{n}{n-1} \text{ Hz. Since string 1 completes a cycle in } \frac{1}{400} \text{ s.}$$

String 2 completes a cycle in  $f_2$  s, they first have a common maxima when  $t = \frac{1}{400} - \frac{1}{f_2}$

Angular velocity of string 1 is  $2\pi f_1 = 800\pi$ , angular velocity of string 2 is  $2\pi f_2$ . The time when they have a common maxima occurs

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QUESTION  
NUMBER

Extra space if required.  
Write the question number(s) if applicable.

Five

(d)

~~When the phase difference is  $2k\lambda$  for some integer  $k$ , so~~

~~$$169.5 \times \frac{n}{n-1} = 379.5$$~~

~~$$(f_0\lambda - 2\pi f_2)t = 2k\lambda$$~~

Since the combined wave have a frequency of  $4.5 \text{ Hz}$ ,  $t = \frac{1}{4.5} \text{ s}$ .

~~$$so (f_0\lambda - 2\pi f_2) \cdot \frac{1}{4.5} = 2k\lambda$$~~

~~$$f_0\lambda - 2\pi f_2 = 9k\lambda$$~~

~~$$f_2 = \frac{79\lambda}{2\pi} = 395.5 \text{ Hz}$$~~

Hence

~~$$(169.5 \times \frac{n}{n-1}) = 395.5$$~~

~~$$\frac{n}{n-1} = 2.33345$$~~

~~$$1.33345 = 2.33345$$~~

~~$$n = 1.75$$~~

Nearest integer multiple of  $n$  is 7, so first string has 7 nodes.

~~When the phase difference is  $2k\lambda$  for some integer  $k$ , so~~

~~$$(f_0\lambda - 2\pi f_2)t = 2k\lambda$$~~

~~Since combined wave have a frequency of  $4.5 \text{ Hz}$ ,  $t = \frac{1}{4.5} \text{ s}$ .~~

~~$$(f_0\lambda - 2\pi f_2) \cdot \frac{1}{4.5} = 2k\lambda$$~~

~~$$f_0\lambda - 2\pi f_2 = 9k\lambda$$~~

~~$$f_2 = \frac{400\lambda - 9k\lambda}{2\pi}$$~~

~~$$= (400 - 4.5k) \text{ Hz}$$~~

so

~~$$400 - 4.5k = 169.5 \times \frac{n}{n-1}$$~~

~~$$\frac{n}{n-1} = \frac{400 - 4.5k}{169.5}$$~~

~~$$(169.5)n = \frac{400 - 4.5k}{169.5} n - (400 - 4.5k)$$~~

~~$$400 - 4.5k = (230.5 - 4.5k)n$$~~

~~$$n = \frac{400 - 4.5k}{230.5 - 4.5k} = 1 + \frac{169.5}{230.5 - 4.5k} = 1 + \frac{339}{461 - 9k}$$~~

QUESTION  
NUMBER

Extra space if required.  
 Write the question number(s) if applicable.

~~Since  $f_2$  is only slightly less than  $f_1$ ,  $k$  must be as small as possible.~~

~~n has an integer value of close to 2 when  $k=14$ . Therefore~~

~~string 1 has 2 nodes. and string 2 has~~

When the phase difference is exactly  $2\pi$ . Since the observer hears a ~~few~~ beat at maximum displacement or negative maximum displacement, the actual frequency of the resultant wave is  $4.5 \div 2 = 2.25$  Hz. So

$$(n\lambda - 2\pi f_2)t = 2\pi \quad \text{where } t = \frac{1}{2.25} \text{ s.}$$

*See*

$$n\lambda - 2\pi f_2 = 2.25 \times 2\pi = 4.5\lambda$$

$$f_2 = \frac{n\lambda - 4.5\lambda}{2\pi}$$

$$= 397.75 \text{ Hz}$$

$$\text{so } 397.75 \text{ Hz} = \frac{200}{1.18} \times \frac{n}{n-1}$$

$$\frac{n}{n-1} = 3.57975$$

$$n = 1.39$$