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SUPERVISOR'S USE ONLY

TOP SCHOLAR

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

Scholarship 2013
Statistics

9.30 am Monday 11 November 2013
Time allowed: Three hours
Total marks: 40

ANSWER BOOKLET

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

Write all your answers in this booklet.

Show ALL working. Start your answer to each question on a new page. Carefully number each question.

Check that this booklet has pages 2–24 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

QUESTION NUMBER

You have three hours to complete this examination.

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1a Ayrshire cows tend to produce milk for a greater number of days in a milking year than Jersey cows, throughout all ~~of their first ten~~ years ~~of life~~. from 2 until 10. Ayrshire cows produce milk for the greatest number of days (235) at age 2, and the least number of days per milking ~~season~~ year (216) at age 10, on average producing milk for 2.4 less days per year for each year older they grow. Jersey cows produce ~~the greatest~~ milk ~~for~~ in the greatest days / ~~breeding season~~ milking year at 2 years old also (227 days) and the least number of days at 10 years (209 days). From 3-5 years jersey cows plateau at 223 days of milk per milking year. Ayrshire cows also tend to produce greater quantities of milk than jersey cows ~~and~~ from 2-8 years of age. Both types of cows produce the most milk/year in their middle years, Ayshire producing approximately 4200 L/year from age 5-7 and Jersey peaking at approximately 3400L/year at age 5 and 6. Both types of cow decline after this, Jersey cows dropping to 2900L/year at age 10 and Ayrshire dropping to 3650L/year at age 10. Finally, although Jerseys tend to produce less milk/year, and for the least number of days, graph 3 shows Jersey cows have a much higher milk fat %. This peaks at 5.77% from age 3-5, then declining by 0.03% /year

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from age 5 - age 10, where there is 5.63% milk fat. Ayrshire cows have a gradual decline in milk fat from age 2 (4.48%) to 10 (4.29%) of a 0.02% drop ~~gs~~ per year.

b i As the amount of milk increases, the amount of milk protein also tends to increase. It is a fairly strong, positive, linear relationship, or possibly a non-linear relationship with very little curvature. There is a fair amount of scatter when milk amount and milk protein amount are small, but as they increase the amount of scatter decreases and the points tend to hug the regression line more closely. Following the linear model, the milk protein in grams increased from about 287g to 390g as the amount of milk increases from 6.0L to 9.0L. There is an average increase of 34.3g more milk protein per extra litre of milk, or as the amount of milk increases by 500mL, the milk protein increases by 17.2g

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ii for a cow producing 7.5L milk I would use the first equation $y = -7.2x^2 + 142x - 320$ to get milk protein = 340g. For a cow producing 9.2L milk I would average the two lines.

$y = -7.2x^2 + 142x - 320$ — $y = 34.3x + 80$

protein = 376.99g — protein = 395.56g

Actual milk protein = $\frac{376.99 + 395.56}{2} = 386.3$

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I would predict that for 7.5L milk there would be 340g milk protein, and for 9.2L milk there would be about 386g milk protein. I average the 2 values for 9.2L milk because the linear model was too high, while the non-linear was beginning to curve down so would be too low, so I found the middle point. I think that ~~my~~ prediction for 7.5L milk is ~~very p~~ fairly precise, as it falls in the middle of my data, and appears realistic, however there is a reasonable amount of scatter there so the actual milk protein value could be greater than or smaller than 340g. My prediction for 9.2L milk also has an element of less precision, as while there is less scatter there, and less variability, I had to ~~predict into the future~~ extrapolate to find the value, and I am unsure as to whether the points will continue to follow the trend at that value, or whether the actual value will be slightly different. Although both of my predictions are fairly precise, both have elements which reduce their precision.

For these predictions to be applied to the whole population, we ~~firstly~~ need to know whether the cows milked were representative of all the cows in New Zealand. This includes knowing if they were a special class of

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cow ie. one that may produce more milk/ better quality milk, as if this is different to the average acceross NZ then it cannot be applied to the whole population. We also need to know what they were fed, and how much - greater access to high quality foods would mean greater milk protein in their milk. If they have different diets to the ~~majority~~ average NZ Jersey cow, these predictions cannot be applied to Jersey cows acceross NZ as they would not be a fair representation.

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2ai	There is a mean difference between the two groups of 1.69L, with Group B producing 1.69L more milk. We are unable to say that the new product caused an increase in milk production as this could be a random difference, owing to the random cows selected, ~~and has~~ as it is a small difference therefore could occur by many random factors. However, as the proportion produced by re-randomisation is only 0.004 (0.4%), ~~th~~ and lies in a tail, we are able to say that there is a significant difference between the mean amount of milk produced by cows in each group. This is because a ~~mean~~ difference ~~of~~ in means of 1.69 only occurs by chance in 0.4% of all samples, so it is unlikely to occur by chance, and it is more likely that this difference in means is due to an actual difference in means. Therefore, we are able to say that cows in Group B produce on average 1.69L more than cows in Group A, and therefore the new product is more successful.	
ii	The sample size should be bigger - 27 cows in each group is not large enough to claim that there is a difference in the whole population, with sample size so small there is high variability so a difference could	

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occur by chance. Also, cows of all ages should be used, to see if they all react to the new product equally well. Was the experiment carried out on different types of cows? ~~They~~ Some could react better than others to different treatments, so in order to apply the results ~~$~~ to all cows, we need to be assure that all types of cows will react the same. Finally, was the experiment carried out in identical environments for both groups? Did one have better access to food/water, or shade, so they were better rested? These are all lurking variables which could also affect the amount of milk produced, so we need to ensure the environmental ~~&~~ conditions are the same for both groups in order to accredit the change purely to the treatment, and thus claim the new treatment to be better. Also, were the treatments applied to both groups by the same person? One person could be more effective at applying it, thus giving those cows an advantage.

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b1. The ~~mean~~ median weight of Jersey cows, at 456kg, is 96kg less than the median weight of Ayrshire cows. The mean weight is 90.3kg less, 457.6 compared to 547.9 of Ayrshire cows. Jersey cows have a range of 132kg,

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an interquartile range of 31kg and a standard deviation of 26.6kg. In comparison Ayrshire cows have a range of 140kg, an interquartile range of 51kg and a standard deviation of 38.3kg. These 3 numbers are measures of spread, showing there is far more variability in Ayrshire cow weights than Jersey weights, even with the smaller sample size. The minimum point of the Ayrshire cows overlaps with the upper quartile of Jersey cow weights, and the max. Jersey cow weight overlaps with the lower quartile of Ayrshire weights. However the UQ of Jersey weights does not overlap with the LQ of Ayrshire weights, which shows there is a significant difference between the weights of the two breeds. The bootstrap confidence intervals also do not overlap, which supports the idea that the two breeds have significantly different weights. Looking at the box and whisker plots, Jersey cow weights taken were bimodal, with a right skew, while Ayrshire cow mean weights were fairly evenly distributed with a slight left skew.

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bii

$\mu(A-J)$

$E(\bar{x}) = E(A) - E(J)$

$= 547.9 \times 0.03 - 457.6 \times 0.03$

$= 2.709$ kg

$\cancel{VAR(\bar{x}) = 0.03^2 \times 547.9^2 + 0.03^2 \times 457.6^2}$

$\cancel{= 270.175 + 188.458}$

$\cancel{= 458.633}$

$\cancel{\sigma(\bar{x}) = \sqrt{458.633}}$

$\cancel{= 21.416 \text{ kg}}$

$E(A) = 16.437$ kg $E(J) = 13.728$ kg

$\cancel{VAR J = 0.0}$

$SD(\bar{x}) = 0.03 \times 16.437 + 0.03 \times 13.728$

$= 0.905$

$E(\bar{x}) = 2.709$ kg $SD(\bar{x}) = 0.905$ kg

95% CI :

$E(A) = 16.437$ $E(J) = 13.728$

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Q2

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3a. ~~P(correct) = ?~~

~~$n = 15 \quad x = 11$~~

~~$P = \frac{11}{15} = 0.733$~~

~~$0.733 = {}^{15}C_{11} \times \pi^{11} \times (1-\pi)^4$~~

~~$0.0008372 = \pi^{11}(1-\pi)^4$~~

3b $\mu = 513g \quad \sigma = 3.5g$

~~Using a 95% confidence interval~~

$P(x < 500) = 0.0001012$

$P(x > 520) = 0.02275$

$\therefore P(x < 500 \cup x > 520)$

$= 0.02285$

The probability of an ice cream tub having a weight outside the limits is 0.0229. Therefore, in 97.71% of samples, the mean weight of the tubs will be inside the limits. As the sample size is very large, it is safe to assume the margin of error will be small. Therefore, I believe I am justified in saying the production is meeting acceptable requirements, as the mean weight of a sample will be inside the limits in more than 95% of all samples (97.7%).

c. $\lambda = 6/\text{hour}$ op $\Rightarrow \lambda = 2/\text{hour}$

In one hour; for bags to be left unfilled:

4 workers will keep none underfilled 84.72% of the time

5 workers " " " " 95.74% of the time

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3c

0.137677

0.0688

0
1
2
3
4
5
6
7

Therefore 5 workers are needed to ensure none are left underfilled over 90% of the time.

I used the Poisson model as there is a fixed rate of underfilling (6/hours), each underfilling ~~is~~ is independent of others, number Bags ~~unfilled~~ underfilled is proportional to the amount of time, data is discrete

0
0

0.062	0, 1, 2	1 worker
0.22306	3, 4	2
~~0.1606 + 0.1606~~ ~~0.3212~~ 0.2409	5, 6	3
0.1101	7, 8	4
0.0426	9, 10	5
	11+	6+

5 workers $\Rightarrow P(B \leq 10) = 0.9574$

$P(B \leq 9) = 0.9161$

$P(B \leq 8) = 0.8472$

Q3

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4a. From year 1 to year 5 milking production remained approximately steady at 14500 millions of litres produced, dropping only to 14000 millions of litres in Year 4, but otherwise staying constant. It rose from 15000 - 18000 million from year 5-6, where it peaked, before dropping back to just below 16000 million L in year 7, and rising again to 17000 million L in year 8. It fluctuates a lot from Year 5 - Year 8, but there is an overall increase over those 3 years of 800 million L per year (2400 million L increase over all. The milk production per season fluctuates greatly. with highest milk production present in seasons 2, 6, 10 etc (spring seasons) and lowest milk production in seasons 4, 8, 12 etc (autumn seasons). The greatest amount of milk produced was in season 6 - spring ~~2005~~ 2004, with almost 6500 million litres of milk produced. The least was in Autumn 2005, season 8, with only 1500 million L produced. Overall, the amount of milk produced increases by 54.8 million L / season, from 3000 million to 4700 million in season 32.

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b. Autumn 2013

~~M = 4284.75~~

Increase / year = $\frac{17139 - 14746}{3}$ = 797.67

$M = 797.67x + 14746$

where $x = 0$ in year 5, 1 in year 6 etc.

$M = 797.67(5) + 14746$

$= \$18734.33$

In year 8, autumn = $\frac{4661}{17139}$ = 0.2720

∴ for year 10, autumn = 0.2720 × 18734.33

Autumn 2013, M = 5094.86 million L

= 5095 million

This forecast is not very valid, as there is a high amount of fluctuating from year 5-year 8, so it is hard to understand the trend. Also, this forecast assumes the production will continue to increase, as was the overall pattern, and does not drop again due to unforseen events eg. drought. Also, it is likely that the amount produced is slightly less as the autumn quarter in Year 8 produced more milk than in other years, when cows produced the least milk in that season, and that is what my predictions are based on. Any unforseen events affecting milk production would make my prediction invalid.

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c. If we adjust all values so that CPI = 1000, therefore removing the effects of inflation, the table looks like this:

Year	Milk fat price ($/kg)	Dairy landsale value ($/hectare)
1998	38.32 ✓	13265 ✓
2002	47.60 ✓	15301 ✓
2006	60.17 ✓	26937 ✓
2010	55.25 ✓	25634 ✓

Both milk fat price and dairy land sale value increased in real dollars from 1998 to 2006, where they both peaked - milk fat price at $60.17 /kg in real dollars and dairy landsale value at $26,937/ hectare in real dollars. Although they both increased in this time, the increase is not as great in real dollars as it is when not adjusted for inflation. However, from 2006 to 2010 both prices dropped, milk fat from $60.17 to $55.25, or $1.23 ✓ real dollars /year, and dairy land sale value from $26937 to $25634 / hectare, or $326 per hectare per year in real dollars. When not adjusted ~~$~~ for inflation these prices appear to rise between 2006 and 2010, but inflation adjusted values ✓ prove differently. The greatest change for both occurs between 2002 and 2006

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real dollars

where milk fat prices rise at $3.14 per year and dairy land sale value increases by $2909 real dollars per hectare per year. As these changes all occur at the same time, it shows there is a correlation between milk fat price and dairy land sale value, but this is likely to be due to lurking variables and does not imply causation.

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F2

5 firstly, cows that produce greater amounts of milk are more likely to become lame. It was shown that lame cows tended to produce greater quantities of milk than non-lame cows, with a mean of 342 kilograms more milk per 305 DIM. However, there is a large amount of variability, and at the 95% confidence level the amount more than non-lame cows ranges from 135 kg more to 549 kg more per ~~year~~ 305 DIM. As this confidence interval does not include zero, we are able to conclude that lame cows do produce more milk than non lame cows. However once they reach the 5th ~~mo~~ month of lactation the mean reduction in milk is 357 kg. As this is greater than the mean increase in milk production of 342 kg, despite this, there is an overall reduction in milk produced, therefore ~~there will be~~ lame cows reduce milk production in the long run. This is because while those with a high potential to be lame produce a large amount of milk, once they have been lame for a long amount of time milk production decreases to the extent that milk production is reduced. Thus, lameness has a significant effect on milk production by reducing it after the 5th month of lactation.

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For every cow that goes lame there is a predicted loss of milk produced of between 160 and 550 kg at the 95% confidence level. This means that in 95% of cases, a cow going lame will result in the reduction of between 160 and 550kg of milk. This means that for every cow that goes lame there is likely to be a significant reduction in the amount of milk produced. ~~With the mean loss as a result of~~ This is the milk lost after diagnosis, but the farmers can often be slow to diagnose lame cows, so there is often a reduction in milk ~~of~~ upto 5 months before the diagnosis, with it almost certain between 4 and 1 months before diagnosis (these are the months where the 95% confidence interval for cumulative milk loss do not include zero, thus milk loss is likely to occur in 95% of all cases. Therefore, lameness has a significant impact on milk production by decreasing milk production in lame cows upto 5 months before diagnosis, and for over 5 months after diagnosis.

Finally, lameness has a great impact on milk production as it occurs so often yet is often detected late, therefore

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F3

is not treated properly. The extract states that over 70% of cows become lame once, and the most common 4 diagnoses had an ~~[illegible]~~ incidence of 9 - 11 cases per 100 cows each year. This is a very large proportion, therefore we are able to see that a large proportion of the stock is affected by lameness. However, in the study of 53 herds farmers underestimated the prevalence of lame cows greatly, with a mean underestimate of 20%. ~~[illegible]~~ This is a great worry, as with lameness affecting their milk production so greatly, farmers need to be able to detect it and treat it so as to minimise the effects of lameness on their milk production. Cows are often showing symptoms of reduced milk production 4 or 5 months before diagnosis - farmers need to learn to idenfify these symptoms and treat them as quickly as possible, so as to minimise the effect of lame cows on their milk production. Therefore, lameness is finally having a significant impact on milk production as it goes for so long without being noticed, that the loss of milk cumulates ~~of~~ over many more years than it would if diagnosis occured sooner and treatment could be applied.

Q5

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