S–CALCF

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Scholarship 2011
Mathematics with Calculus

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FORMULAE AND TABLES BOOKLET

Refer to this booklet to answer the questions for Scholarship Mathematics with Calculus 93202Q.

Check that this booklet has pages 2–4 in the correct order and that none of these pages is blank.

YOU MAY KEEP THIS BOOKLET AT THE END OF THE EXAMINATION.

CALCULUS – USEFUL FORMULAE

ALGEBRA

Quadratics

If $ax^2 + bx + c = 0$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Logarithms

$y = \log_b x \Leftrightarrow x = b^y$

$\log_b(xy) = \log_b x + \log_b y$

$\log_b\left(\dfrac{x}{y}\right) = \log_b x - \log_b y$

$\log_b\left(x^n\right) = n\log_b x$

$\log_b x = \dfrac{\log_a x}{\log_a b}$

Complex numbers

$z = x + \mathrm{i}y$

$\quad = r\,\mathrm{cis}\,\theta$

$\quad = r(\cos\theta + \mathrm{i}\sin\theta)$

$\bar{z} = x - \mathrm{i}y$

$\quad = r\,\mathrm{cis}\,(-\theta)$

$\quad = r(\cos\theta - \mathrm{i}\sin\theta)$

$r = |z| = \sqrt{z\bar{z}} = \sqrt{(x^2 + y^2)}$

$\theta = \arg z$

where $\cos\theta = \dfrac{x}{r}$

and $\sin\theta = \dfrac{y}{r}$

De Moivre's Theorem

If n is any integer, then

$(r\,\mathrm{cis}\,\theta)^n = r^n\,\mathrm{cis}\,(n\theta)$

Binomial Theorem

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + \binom{n}{n}b^n$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{(n-r)!\,r!}$$

Some values of $\binom{n}{r}$ are given in the table below.

n \ r	0	1	2	3	4	5	6	7	8	9	10
0	1										
1	1	1									
2	1	2	1								
3	1	3	3	1							
4	1	4	6	4	1						
5	1	5	10	10	5	1					
6	1	6	15	20	15	6	1				
7	1	7	21	35	35	21	7	1			
8	1	8	28	56	70	56	28	8	1		
9	1	9	36	84	126	126	84	36	9	1	
10	1	10	45	120	210	252	210	120	45	10	1
11	1	11	55	165	330	462	462	330	165	55	11
12	1	12	66	220	495	792	924	792	495	220	66

COORDINATE GEOMETRY

Straight Line

Equation $y - y_1 = m(x - x_1)$

Circle

$(x-a)^2 + (y-b)^2 = r^2$

has a centre (a, b) and radius r

Parabola

$y^2 = 4ax$ or $(at^2, 2at)$

Focus $(a, 0)$ Directrix $x = -a$

Ellipse

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ or $(a\cos\theta, b\sin\theta)$

Foci $(c,0)$ $(-c,0)$ where $b^2 = a^2 - c^2$

Eccentricity: $e = \dfrac{c}{a}$

Hyperbola

$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ or $(a\sec\theta, b\tan\theta)$

asymptotes $y = \pm\dfrac{b}{a}x$

Foci $(c,0)$ $(-c,0)$ where $b^2 = c^2 - a^2$

Eccentricity: $e = \dfrac{c}{a}$

CALCULUS

Differentiation

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
$\ln x$	$\frac{1}{x}$
e^{ax}	ae^{ax}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\cot x$	$-\operatorname{cosec}^2 x$

Integration

$f(x)$	$\int f(x)\mathrm{d}x$
x^n	$\frac{x^{n+1}}{n+1} + c$ $(n \neq -1)$
$\frac{1}{x}$	$\ln\lvert x\rvert + c$
$\frac{f'(x)}{f(x)}$	$\ln\lvert f(x)\rvert + c$

First principles

$$f'(x) = \lim_{h\to 0}\frac{f(x+h) - f(x)}{h}$$

Parametric Function

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t}\cdot\frac{\mathrm{d}t}{\mathrm{d}x}$$

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\cdot\frac{\mathrm{d}t}{\mathrm{d}x}$$

Product Rule

$(f.g)' = f.g' + g.f'$ or if $y = uv$ then $\frac{\mathrm{d}y}{\mathrm{d}x} = u\frac{\mathrm{d}v}{\mathrm{d}x} + v\frac{\mathrm{d}u}{\mathrm{d}x}$

Quotient Rule

$\left(\frac{f}{g}\right)' = \frac{g.f' - f.g'}{g^2}$ or if $y = \frac{u}{v}$ then $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Composite Function or Chain Rule

$$\left(f(g)\right)' = f'(g).g'$$

or if $y = f(u)$ and $u = g(x)$ then $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u}\cdot\frac{\mathrm{d}u}{\mathrm{d}x}$

Volume of Revolution

$y = f(x)$ between $x = a$ and $x = b$

rotated about the x-axis

$$\text{Volume} = \int_a^b \pi y^2\,\mathrm{d}x$$

NUMERICAL METHODS

Trapezium Rule

$$\int_a^b f(x)\,\mathrm{d}x \approx \frac{1}{2}h\left[y_0 + y_n + 2(y_1 + y_2 + \ldots + y_{n-1})\right]$$

where $h = \frac{b-a}{n}$ and $y_r = f(x_r)$

Simpson's Rule

$$\int_a^b f(x)\,\mathrm{d}x \approx \frac{1}{3}h\left[y_0 + y_n + 4(y_1 + y_3 + \ldots + y_{n-1}) + 2(y_2 + y_4 + \ldots + y_{n-2})\right]$$

where $h = \frac{b-a}{n}$, $y_r = f(x_r)$ and n is even.

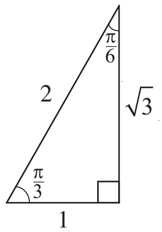
TRIGONOMETRY

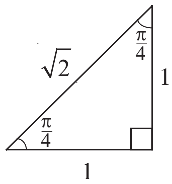
$$\operatorname{cosec}\,\theta = \frac{1}{\sin\theta}$$

$$\sec\,\theta = \frac{1}{\cos\theta}$$

$$\cot\,\theta = \frac{1}{\tan\theta}$$

$$\cot\,\theta = \frac{\cos\theta}{\sin\theta}$$





Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine Rule

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Identities

$$\cos^2\theta + \sin^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\cot^2\theta + 1 = \operatorname{cosec}^2\theta$$

General Solutions

If $\sin\,\theta = \sin\,\alpha$ then $\theta = n\pi + (-1)^n\alpha$

If $\cos\,\theta = \cos\,\alpha$ then $\theta = 2n\pi \pm \alpha$

If $\tan\,\theta = \tan\,\alpha$ then $\theta = n\pi + \alpha$

where n is any integer

Compound Angles

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

Double Angles

$$\sin 2A = 2\sin A\cos A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

Products

$$2\sin A\cos B = \sin(A+B) + \sin(A-B)$$

$$2\cos A\sin B = \sin(A+B) - \sin(A-B)$$

$$2\cos A\cos B = \cos(A+B) + \cos(A-B)$$

$$2\sin A\sin B = \cos(A-B) - \cos(A+B)$$

Sums

$$\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$$

$$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$$

$$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$$

$$\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$$

MEASUREMENT

Triangle

$$\text{Area} = \frac{1}{2}ab\sin C$$

Trapezium

$$\text{Area} = \frac{1}{2}(a+b)h$$

Sector

$$\text{Area} = \frac{1}{2}r^2\theta$$

$$\text{Arc length} = r\theta$$

Cylinder

$$\text{Volume} = \pi r^2 h$$

$$\text{Curved surface area} = 2\pi rh$$

Cone

$$\text{Volume} = \frac{1}{3}\pi r^2 h$$

Curved surface area $= \pi rl$ where $l =$ slant height

Sphere

$$\text{Volume} = \frac{4}{3}\pi r^3$$

$$\text{Surface area} = 4\pi r^2$$

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