S

93201Q

NZQA
NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

Scholarship 2010
Statistics and Modelling

2.00 pm Saturday 13 November 2010
Time allowed: Three hours
Total marks: 40

QUESTION BOOKLET

Pull out the Formulae and Tables Booklet S–STATF from the centre of this booklet.

There are FIVE questions in this booklet. Answer ALL questions.

Write your answers in the Answer Booklet 93201A.

Show ALL working.

Start your answer to each question on a new page. Number each question and part question carefully.

There is a grid provided on page 24 of the Answer Booklet for answering Question One (a), another grid on page 25 for answering Question Two (b), and a further grid on page 26 for answering Question Three (b)(i).

Check that this booklet has pages 2–9 in the correct order and that none of these pages is blank.

YOU MAY KEEP THIS BOOKLET AT THE END OF THE EXAMINATION.

You have three hours to complete this examination.

The theme of this paper involves the manufacture and sales of Statsmobile cars. Sales records are contained in a database.

QUESTION ONE (8 marks)

(a) Table 1 below gives a summary of the sales price data collected from a random sample of 1 000 sales records of second-hand Statsmobiles.

Table 1

Selling Price ($000)	Number of Sales	
	Air-conditioned	Non air-conditioned
0 and less than 9	52	71
9 and less than 12	132	138
12 and less than 15	190	102
15 and less than 18	130	83
18 and less than 21	61	31
21 and less than 24	0	10
Total Number	565	435

Draw a graph and calculate appropriate statistical measures that would enable the prices from the two types of car to be compared. Use your graph and statistical measures to compare the prices of the two types of car.

A grid is provided on page 24 of the Answer Booklet for you to answer this question.

(b) Second-hand Statsmobiles were randomly sampled a second time. The data is summarised in Table 2 below.

Table 2

Statistic	Selling Price	
	Air-conditioned	Non air-conditioned
Mean	$15 513	$12 581
Standard Deviation	$3 896	$4 074
Sample Size	200	300

(i) Give a full description of how the random sampling could have been carried out.

(ii) Construct a 95% confidence interval for the difference between the mean price of all second-hand air-conditioned Statsmobiles and the mean price of all second-hand non air-conditioned Statsmobiles.

Make a conclusion based on your calculated interval.

(iii) Suppose both sample sizes are increased by $p\%$ in order to reduce the margin of error in the estimate of the difference in the mean prices in (b) (ii) to $558.

Assuming that the sample standard deviations remain the same, find the value of p.

QUESTION TWO (8 marks)

(a)   Monthly demand for new Statsmobiles is modelled by a normal distribution with mean 600 and standard deviation 72.

Find the probability that the total demand for new Statsmobiles over a six-month period is at least 3 360.

What assumption is necessary in order to find this probability?

(b)   Two different models of Statsmobiles are manufactured, X and Y.

Let x be the number of model X manufactured annually and y be the number of model Y manufactured annually.

Annual demand for Statsmobiles is at least 6 000. Annual demand for model Y is between 95% and 170% of the annual demand for model X.

In each of the following three cases, find the optimal solution(s) that minimises the annual manufacturing cost, while meeting the above demand constraints.

(i)   The cost of producing each model Y is less than the cost of producing each model X.

(ii)   The cost of producing each model Y is equal to the cost of producing each model X.

(iii)   The cost of producing each model Y is greater than the cost of producing each model X.

A grid is provided on page 25 of the Answer Booklet to help you answer this question.

(c)   On average, 3% of new Statsmobiles have minor defects that occur randomly. In a random sample of n new Statsmobiles, find n so that there is a 0.2 chance of no more than one Statsmobile having a minor defect.

QUESTION THREE (8 marks)

(a) Statsmobiles may be classified as "two door" or "at least three door" models. Fifty-five percent of all two-door Statsmobiles are non air-conditioned and 30% of all Statsmobiles are both air-conditioned and have at least three doors.

The proportion of air-conditioned Statsmobiles that have at least three doors is the same as the overall proportion of Statsmobiles that have at least three doors.

Find the value of this proportion.

(b) The cost, in thousands of dollars ($000), of ordering and displaying Statsmobiles for sale in terms of the number ordered is modelled by the equation $C = aQ + \dfrac{b}{Q}$, where

Q is the number ordered, and a and b are positive constants.

This model is to be fitted to the data in Table 3 below.

Table 3

Q	50	70	100	150
C($000)	4.43	4.05	4.10	4.80

(i) Sketch the graph of $C = aQ + \dfrac{b}{Q}$.

Find values for a and b, to three significant figures, in order to fit this model to the data.

A grid is provided on page 26 of the Answer Booklet to help you answer this question.

(ii) When placing an order for Statsmobiles, how many should be ordered to minimise the cost of ordering and displaying the cars? State this minimum cost.

QUESTION FOUR (8 marks)

A random sample of 20 four-door, second-hand Statsmobiles sold this year was selected from the database.

Data was found for each car selected in the sample for the following variables:

Age, A: The age of the car in years. $A = 1$ for a 2009 car, $A = 2$ for a 2008 car, etc.
Odometer, M: The odometer reading (distance travelled) in thousands of kilometres.
Price, P: The selling price in thousands of dollars ($000).

The data is shown in Table 4 below.

Table 4

Age, A (years)	Odometer, M (thousands of km)	Price, P ($000)
14	184	5
12	136	5.5
11	124	7
10	90	6
9	61	9.5
9	69	8
8	98	7.5
8	92	8.5
8	64	9
5	84	13
5	54	14
4	110	9.5
4	74	16
4	119	13
4	38	19
3	90	17.5
3	47	21
3	88	18
2	38	23
1	63	23.5

Scatter plots of Price against Age and Price against Odometer are shown below in Figures 1 and 2 respectively.
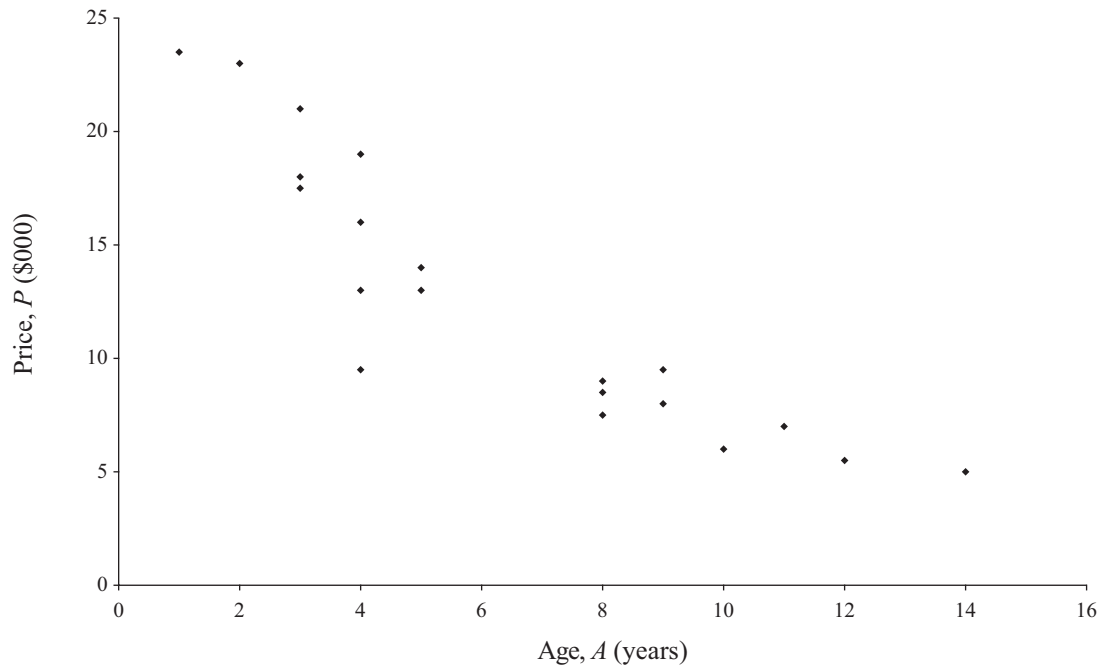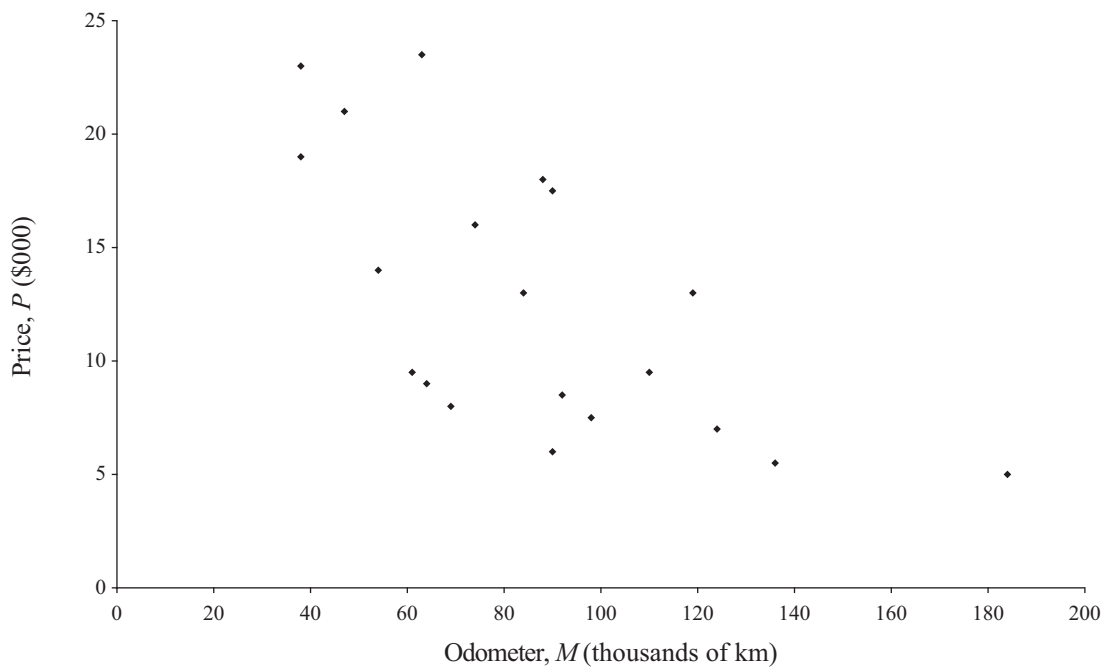
Figure 1: Price against Age

Figure 2: Price against Odometer

(a) Write a short paragraph describing the relationships between the variable pairs; (Age, Price) and (Odometer, Price).

(b) Using regression analysis, both a line and a curve were fitted to each scatter plot. The regression equations are shown below.

Price against Age:
1. $P = {}^-1.47A + 22$
2. $P = 25.4e^{-0.13A}$

Price against Odometer:
1. $P = {}^-0.11M + 22.1$
2. $P = 443M^{-0.84}$

Obtain a prediction for the selling price of a seven-year-old, four-door Statsmobile, sold this year, with an odometer reading of 80 000 km.

Show full working and justify your method. Comment on the validity of your prediction.

(c) State three other factors that could be considered in order to make this analysis more extensive.

For each factor, state its likely impact on the selling price of four-door, second-hand Statsmobiles.

QUESTION FIVE (8 marks)

Statsmobiles are also available as five-door wagons. For each year from 1999 to 2009, the mean price of new five-door wagons was calculated. The values were then adjusted for inflation and expressed in terms of 1999 dollars.

The mean price, y, in thousand of dollars, adjusted for inflation, is shown in Table 5 below, along with x, which represents the number of years since 1998. A graph of these mean (inflation-adjusted) prices is displayed in Figure 3 together with two fitted trend models.

Table 5

Year	Years since 1998, x	Mean price, y ($000)
1999	1	23.6
2000	2	27.8
2001	3	26.4
2002	4	42.7
2003	5	47.4
2004	6	48.1
2005	7	51.7
2006	8	50.1
2007	9	52.5
2008	10	42.5
2009	11	46.8

Figure 3: Mean price of new five-door wagons
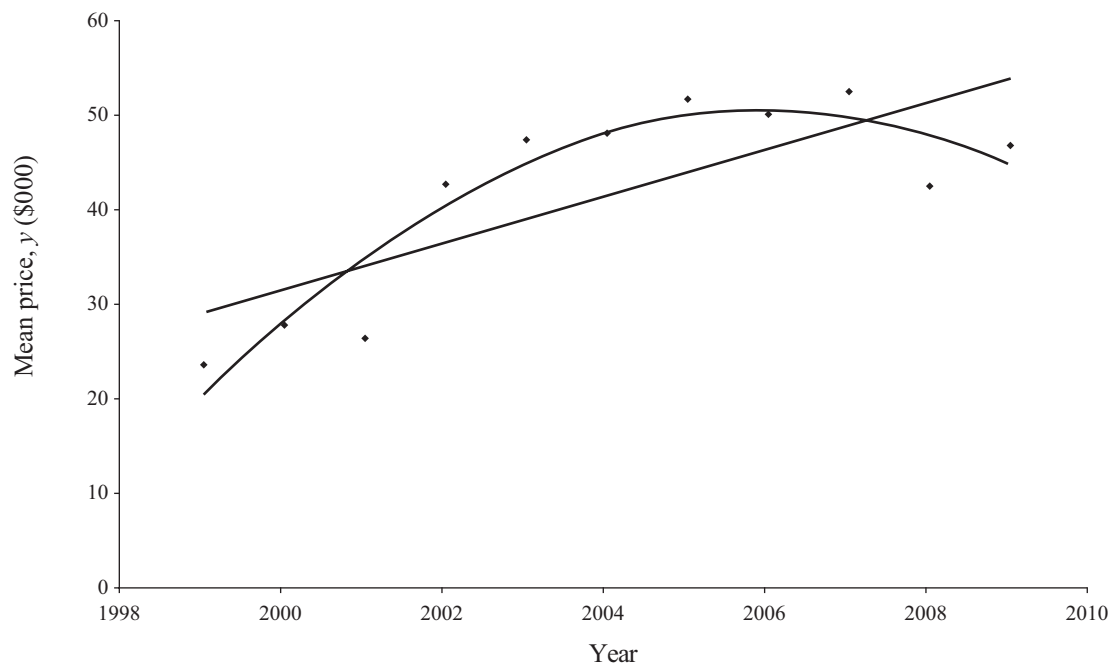
(a)   Describe the change in the mean (inflation-adjusted) price of new five-door Statsmobiles from 1999 to 2009.

(b)   The equations of the fitted trend models are:

   1. $y = {}^-0.6186x^2 + 9.8983x + 10.85$

   2. $y = 2.4745x + 26.935$

Using no more than these two models, calculate a forecast for the mean (inflation-adjusted) prices of new five-door Statsmobiles in 2012.

Show full working and justify your method.

(c)   The mean price per barrel of crude oil for four different years is shown in Table 6 below. These prices have been adjusted for inflation in the same way as in part (a).

Table 6

Year	1999	2003	2007	2009
Crude oil price per barrel (inflation-adjusted)	$21.50	$32.60	$91.70	$64.10

Compare the changes in the mean (inflation-adjusted) price of new, five-door Statsmobiles with the changes in the mean (inflation-adjusted) price of crude oil over the period from 1999 to 2009.

Include relevant calculations and explanations.