

93201Q



932012

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

Scholarship 2011 Statistics and Modelling

9.30 am Saturday 12 November 2011

Time allowed: Three hours

Total marks: 40

QUESTION BOOKLET

There are FIVE questions over THREE sections in this booklet. Section A contains Questions One and Two, Section B contains Questions Three and Four, and Section C contains Question Five.

You should attempt ALL the questions in this booklet.

Pull out Formulae and Tables Booklet S–STATF from the centre of this booklet.

Write your answers in Answer Booklet 93201A.

Show ALL working. Start your answer to each question on a new page. Carefully number each question.

There is a grid provided on page 26 of the Answer Booklet for answering Question Five (a) (i).

Check that this booklet has pages 2–9 in the correct order and that none of these pages is blank.

YOU MAY KEEP THIS BOOKLET AT THE END OF THE EXAMINATION.

You have three hours to complete this examination.

SECTION A: HOUSEHOLD LABOUR FORCE SURVEY

Questions One and Two refer to the Household Labour Force Survey, which is undertaken by Statistics New Zealand. Each quarter a sample of 15 000 private households (about 30 000 New Zealand residents aged 15 years or older) is surveyed. Each person in the working age population is classified as employed, unemployed, or not in the labour force.

Employed means the person, during the survey week, worked for one hour or more for pay or profit, or had a job, but was not at work due to illness, injury, or holiday leave.

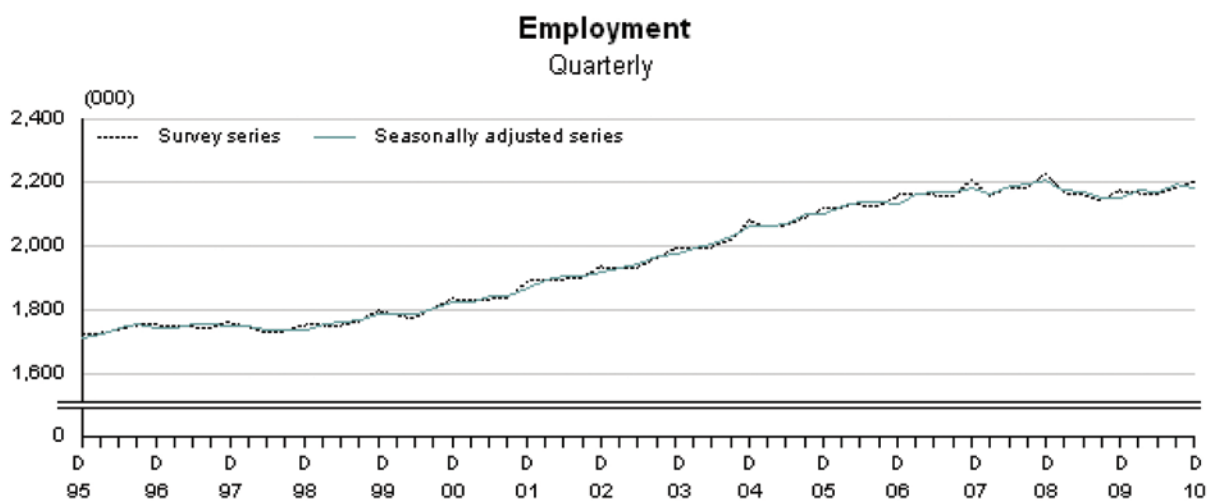
Unemployed means the person, during the survey week, was without a paid job, was available for work and had either actively sought work in the past four weeks, or had a new job to start within the next four weeks.

Not in the labour force includes people who are retired, attending educational institutions, or who are home-makers.

QUESTION ONE (8 marks)

Figure 1 below shows the actual and seasonally adjusted numbers of people employed, from the December 1995 quarter to the December 2010 quarter. The scale on the vertical axis is in thousands (000).

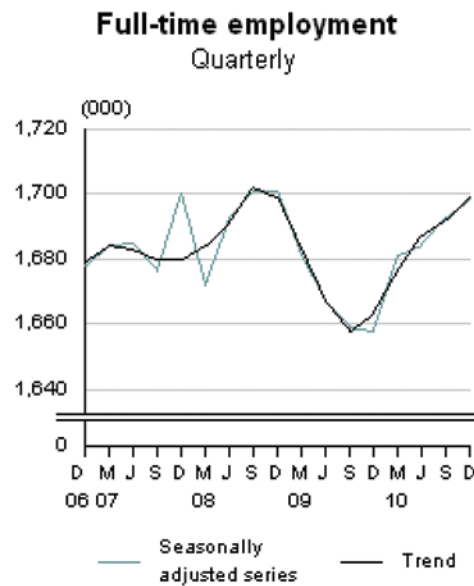
Figure 1



Source: Statistics New Zealand

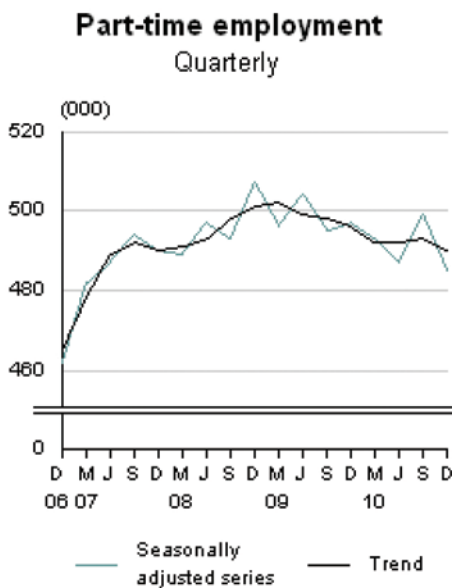
Figures 2 and 3 below show the seasonally adjusted numbers of people employed full-time and part-time respectively, from the December 2006 quarter to the December 2010 quarter. A trend has been fitted to each graph. The scale on the vertical axis is in thousands (000).

Figure 2



Source: Statistics New Zealand

Figure 3



Source: Statistics New Zealand

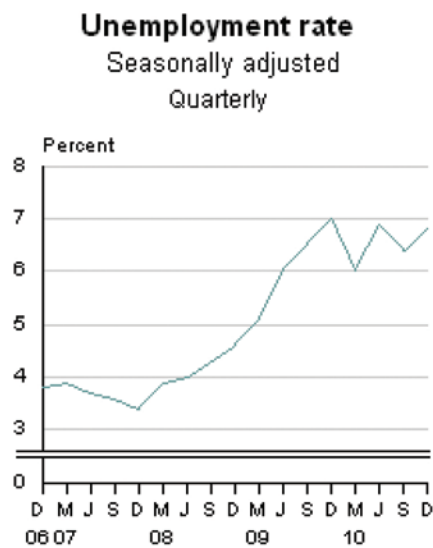
- (a) Using Figures 1, 2 and 3, list SIX distinct observations that can be made about employment in New Zealand.
- (b) A trend line $y = 1\,670 + 40t$ was fitted to the data of Figure 1, where y (000) represents the average quarterly employment over the year, and t is the number of years since 1995.

Given that the seasonal effects for each quarter are (in 000): -20 , -90 , $+40$ and $+70$ respectively, obtain a forecast for each of the last two quarters in 2012.

Give TWO reasons why these forecasts may be invalid.

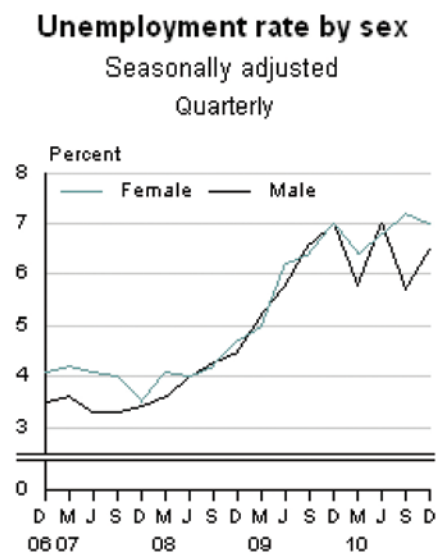
- (c) Figure 4 below shows the seasonally adjusted unemployment rate from the December 2006 quarter to the December 2010 quarter. Figure 5 below shows the unemployment rate by sex for the same period.

Figure 4



Source: Statistics New Zealand

Figure 5

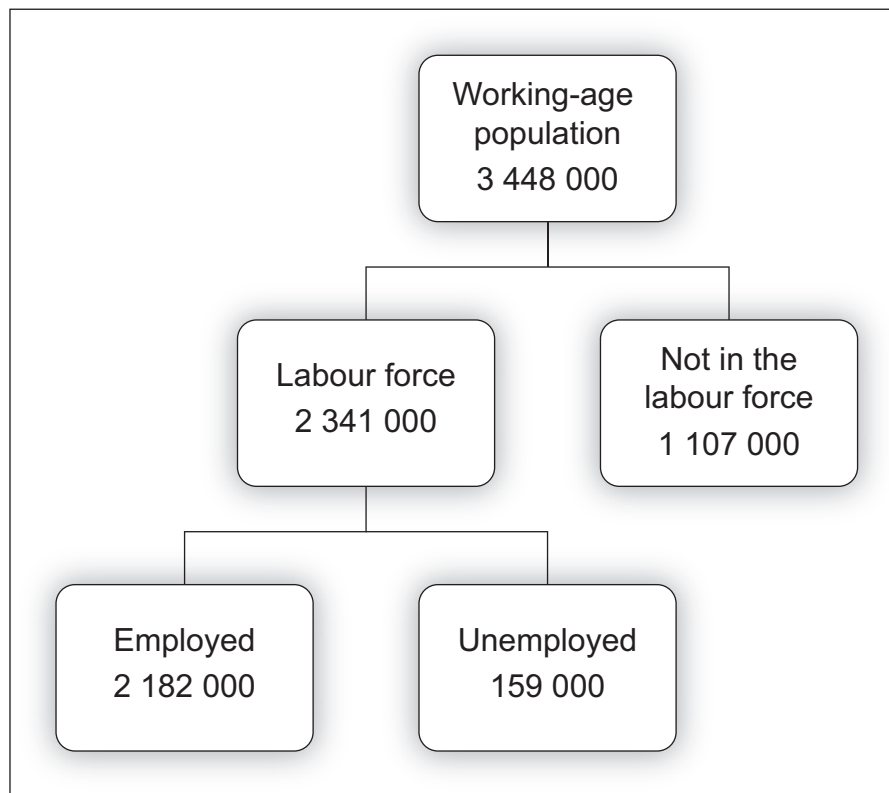


Source: Statistics New Zealand

Using Figures 4 and 5, write a brief paragraph describing the unemployment rate over the period December 2006 to December 2010.

QUESTION TWO (8 marks)

Figure 6 below summarises the labour market in the December 2010 quarter.

Figure 6

Source: Statistics New Zealand

The results of the Household Labour Force Survey given in Figure 6 were based on a sample of 29 456 people selected from the New Zealand working-age population in the December 2010 quarter.

- (a) Using Figure 6 and this sample, find a point estimate of the percentage of the **labour force** in the December 2010 quarter that was employed.
Use this value to construct a 99% confidence interval for the true percentage employed.
- (b) To ensure, with 95% confidence, that the error in the estimate of the number of employed people in the December 2010 quarter is no more than 8 000, how many more people from the New Zealand working age population should be surveyed?
- (c) Give a full description of a random sampling method that could be used to sample 15 000 private households. The people to be sampled from these households must be from both rural and urban areas throughout New Zealand.

SECTION B: EMPLOYMENT AT XYZ BANK

Questions Three and Four refer to employment at the XYZ Bank.

QUESTION THREE (8 marks)

In a study of the effect of work experience on the efficiency of tellers, a random sample of 36 tellers was selected. Data was collected for the following variables:

Experience, E . The time employed as a teller in months.

Service Rate, S . The average number of customers served per hour.

The data is shown in Table 1 below.

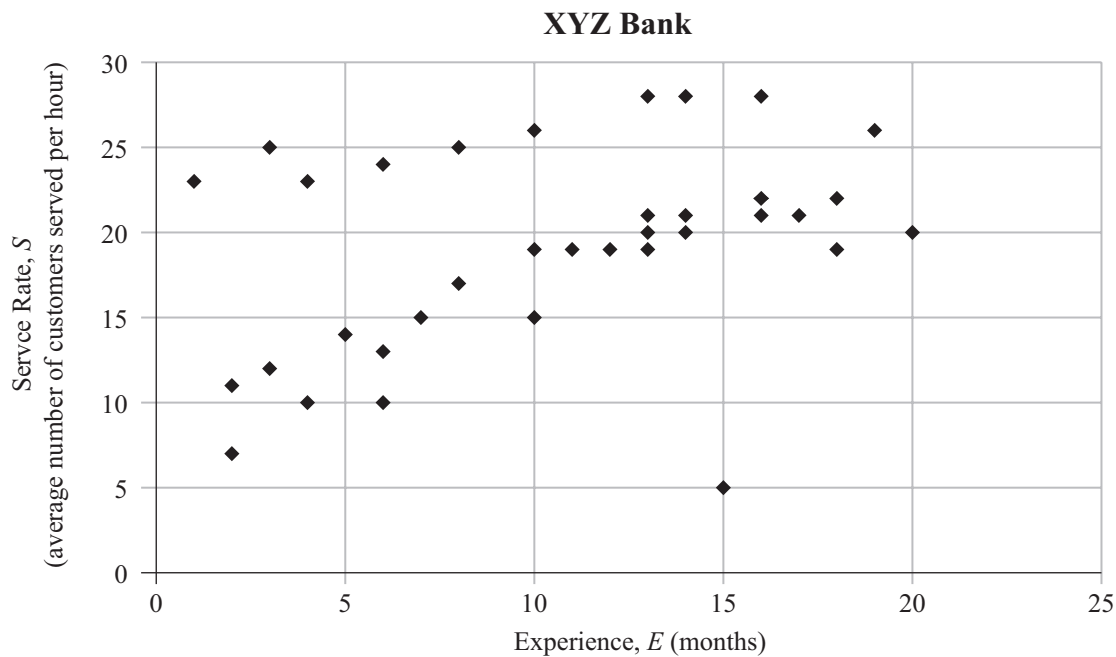
Table 1

Experience, E (Months)	Service Rate, S
15	5
2	11
4	10
3	12
6	13
5	14
7	15
8	17
6	10
10	15
13	20
14	21
16	22
13	19
20	20
16	21
18	19
13	21

Experience, E (Months)	Service Rate, S
11	19
2	7
18	22
14	20
19	26
16	22
1	23
17	21
4	23
12	19
10	19
3	25
13	28
6	24
8	25
10	26
14	28
16	28

A scatter plot of the data is shown in Figure 7 below.

Figure 7



- (a) Write a short paragraph describing the relationship between the variables Experience and Service Rate.

- (b) The regression line $S = 0.4234E + 14.662$ was fitted to this data.

Predict, to one decimal place, the average number of customers served per hour in the case of a teller having:

- (i) 9 months' experience
- (ii) 27 months' experience.

Comment on the validity of each prediction.

- (c) Other variables may influence the teller's service rate.

Comment, with justification, on the direction (positive, negative or zero) and strength of the correlation that you would expect between a teller's service rate and each of the following variables:

- (i) age of a teller
- (ii) average height of a teller's customers
- (iii) average time taken for a teller to serve a customer.

QUESTION FOUR (8 marks)

- (a) Chris is one of three people who work on an “on call” basis as replacements when regular staff call in sick. Each of the three people who work “on call” is used equally. The number of staff who call in sick each day can be modelled by a Poisson distribution having a mean of 1.4.

If Chris is working on a particular day, what is the probability that exactly one other “on call” person is working that day?

- (b) The weekly earnings of two part-time employees A and B are independent, and are each modelled by a normal distribution with parameters given in Table 2 below.

Table 2

Employee	Mean	Standard deviation
A	x	\$84
B	\$1 120	\$112

In a four-week period the probability that A’s weekly earnings are greater than B’s in exactly two of the weeks is 0.24.

Calculate x , the mean weekly earnings for employee A.

- (c) The time (in minutes) taken by a teller to serve a customer is modelled by a normal distribution with mean 2 and standard deviation 0.6. The time taken to serve one customer is assumed to be independent of the time taken to serve another.

When the bank opens at 9 am, three customers who want to be served by a teller enter and find that there are two tellers operating. Each of the first two customers is served by a different teller and the third waits in the queue for tellers. There is only one queue; the first customer in the queue is served by the first available teller and new customers join the end of the queue.

At 9.01 am a fourth customer joins the queue.

What is the probability that the fourth customer will be waiting in the queue for less than two minutes?

SECTION C: SUPPLIER OF BUILDING MATERIALS

Question Five refers to a supplier of building materials.

QUESTION FIVE (8 marks)

This supplier operates out of two warehouses: eastern and western. The supplier receives orders from two customers, A and B, who require sheets of plywood.

Delivery costs per sheet are:

- \$0.50 from the eastern warehouse to customer A
- \$K (K is a constant) from the eastern warehouse to customer B
- \$0.40 from the western warehouse to customer A
- \$0.55 from the western warehouse to customer B.

Let x represent the number of sheets delivered from the eastern warehouse to customer A

and y represent the number of sheets delivered from the eastern warehouse to customer B.

- (a) Customer A needs 50 sheets and customer B needs 70 sheets.

The eastern warehouse has 80 sheets in stock and the western warehouse has 45 sheets in stock.

- (i) Use $K = 0.60$.

Identify all the constraints in terms of x and y .

Draw the feasible region on the graph paper provided on page 26 of the Answer Booklet.

Write the overall delivery cost in terms of x and y .

Describe the delivery plan from each warehouse to each customer that minimises the overall delivery cost.

- (ii) Find the value of K that gives multiple delivery plans and also minimises the overall delivery cost. State this minimum overall delivery cost.

- (b) This part investigates the effect of changes made to the situation in part (a). Customer B still needs 70 sheets and the eastern warehouse still has 80 sheets in stock.

For each of the following find the values of x and y that minimise the overall delivery cost.

- (i) Use $K = 0.60$, customer A still needs 50 sheets, and the western warehouse has 40 sheets in stock (instead of 45).
- (ii) Use $K = 0.70$, customer A needs 48 sheets (instead of 50), and the western warehouse has 45 sheets in stock.

