Assessment Report

Scholarship, 2007

Mathematics with Calculus

Mathematics with Calculus, Scholarship, 2007

Commentary

The overall standard of performance improved in 2007 with fewer very poor scripts. However, there were also relatively fewer outstanding ones. Some candidates are still using pencil.

The structure and difficulty level of the paper this year were such that the vast majority of candidates were able to engage with several question parts and there were fewer scores of zero. However, a significant minority of students continue to be poorly prepared for an examination at this level. Most candidates appear to have had sufficient time to attempt all the questions they felt able to attempt. There was a good spread of marks produced by the examination, enabling clear identification of the top scholars, with some really excellent scripts.

The best performing candidates most commonly demonstrated the following skills and / or knowledge:

- excellent algebraic manipulation
- a high level of differentiation and integration skills, with attention to detail such as limits
- the application of familiar mathematics knowledge to unfamiliar contexts
- the ability to understand new mathematical constructs described in the question
- analysis of an unfamiliar application with extraction of mathematical models
- work with translations of graphs and link these to algebra
- a good understanding of trigonometric relationships and solving trigonometric equations
- the ability to work with surds (square roots) and with exact answers rather than decimals
- the ability to reason geometrically, to apply geometric principles and relate these to algebra
- knowledge of, and ability to work with, parametric forms
- the insight to generate novel or original solution methods
- very good logical thinking skills
- writing of clear and correct mathematical statements within a sound argument
- recognition of the relevance of earlier parts of questions to those following
- displayed a thorough knowledge of NCEA Level 3 Mathematics.

Candidates who did NOT achieve scholarship lacked some or all of the skills and knowledge above and in addition they:

- had limited algebraic and trigonometric manipulation skills
- could not deal with related rates of change
- did not make effective use of graphic calculators to assist with thinking and working
- resorted to the use of decimals prematurely
- did not find all required solutions to trigonometric equations
- did not read questions carefully enough
- had very limited understanding of complex numbers.

Comments on individual questions:

Q1(a) This question was answered correctly more often than any other in the paper. The most common cause of error was reading the question as $y = 4ax$. The value of k was not required but only proved a distraction to a few candidates.

Q1(b) Here some could not handle the surds and, for example, could not cope with $\int_0^h \sqrt{4ax}\sqrt{1+\frac{a}{x}}dx$ or similar because they did not know that $\sqrt{p}\sqrt{q}=\sqrt{pq}$. Others tried results such as $\int_0^h 2\sqrt{ax+a^2}dx \Rightarrow \int_0^h \left(2\sqrt{ax}+2a\right)dx$. The missing 2π in the formula caused no problems for candidates.

Q1(c) A good knowledge of the chain rule for related rates of change was required, but often lacking. Good algebra skills were required in what proved to be a difficult part.

Q2(a) Most candidates recognised the correct solution method but were sometimes let down in the detail, for example not realising there would be 4 solutions in the range $0 \le x \le 2\pi$, failing to divided the angle sums by 2, or not knowing results such as $\cos(-y) = \cos y$.

Q2(b) (i) and (ii) This was well done by many candidates, with correct answers for the function $h(t)$ and the maximum common, although some were determined to use the model $h = A\sin(Bx+C) + D$. Some did not show that their answer was a maximum, as required. Very, very few considered the need for a domain of $0 \le t \le 12$, from the periodic nature of the function.

Q3(a) Proved a relatively straightforward question and was well answered by most candidates who attempted it. Many did not show any working and presumably used a graphic calculator to solve the trigonometric equation in the required interval, which was an acceptable method. Some again failed to give all the solutions.

Q3(b) Those candidates who had the insight that the intersection of $p(x)$ and $g(x)$ had to be at the vertex of $p(x)$ and a turning point of $g(x)$ to avoid three points of intersection, and thus $(b,-c)=(\pi,-2)$, were most likely to complete this part. It was more common to equate the function values than the gradients.

Q3(c) Very few students could solve this part of the question, which proved to be an indicator of outstanding performance. A good degree of visualisation was required, and this was a question for which drawing a good, clear diagram was very helpful. A very common error was to assume that the turning points $x = \frac{\pi}{4}, \frac{5\pi}{4}$ were the intersections. Some candidates started with $f(x) = h(x)$ rather than considering where the gradients were equal, ie $f'(x) = h'(x)$.

Q4(a) Was well answered by many candidates, showing the ability to simplify algebraically, work with surds or indices and recognise a somewhat complicated difference of squares. However, it was disappointing to see $\left(\sqrt{\frac{1}{2}(a+\sqrt{(a^2+b^2)})}+i\sqrt{\frac{1}{2}(-a+\sqrt{a^2+b^2})}\right)^2$

$=\left(\sqrt{\frac{1}{2}(a+\sqrt{(a^2+b^2)})}\right)^2+\left(i\sqrt{\frac{1}{2}(-a+\sqrt{a^2+b^2})}\right)^2$ at this level. Those who chose to use polar or exponential form often did not succeed in getting back to the a + ib form asked for.

Q4(b) (i) This question required candidates to understand and work with an unfamiliar recursive definition. Successful candidates often demonstrated flair and insight in their solutions. Some, surprisingly, failed to remember that the modulus of a complex number, $z = x + iy$ $|z| = \sqrt{x^2+y^2}$, is real and used the i's in their attempt to calculate them. A number of candidates over-generalised in their sequence to get $c^4 + c^3 + c^2 + c$, etc.

Q4 (b) (ii) Early substitution of *b*, and later of *a*, and recognition of the relevance of part (a) was important in helping many students to the result.

Q5(a) Quite well done. A clear understanding of how to move between parametric and Cartesian equation forms was required here, with virtually no candidate choosing to work with parametric forms throughout. However, many candidates were not comfortable working with parameters, with a common cause of errors being difficulty with elimination of parameters, for example when substituting $t = \frac{y}{2a}$ and $s = \frac{y}{\sqrt{2a}}$ into $x = at^2 + b$ and $x = 3b - as^2$, respectively. A number of candidates were unable to break out of a circular passage between parametric and Cartesian equation forms.

Q5(b) Succesful candidates were able to identify the correct integral forms, and the correct limits for their integrals, especially following substitution of variables. This question produced a huge variety of different strategies and working, all of which required careful attention to detail and simplification to succeed. Some candidates displayed a high level of flair and insight, producing novel and elegant solutions, often including translating the graphs of the functions. The chord AB was often wrongly assumed to be the *y*-axis, and this, coupled with the relatively large number of students who attempted to use versions of $\int xdy$, led to 11:23 as a common wrong answer. Only a very small number were successful with this approach. A number of candidates treated the ratio as an equation, for example squaring both sides, leading to wrong answers.

Q5(c) Though some found difficulties with parametric differentiation, finding the gradient of the tangent was often well done and many candidates found the gradient of the normal correctly. Those who worked with $y = mx + c$ were less likely to succeed than those who chose $y - y_1 = m\left(x - x_1\right)$, and put $y = 0$ early on. A common error was to leave the gradient in terms of *x* and / or *y*, leading to confused working. There was some evidence that the formula for the mid-point of a chord was not well known, with quite a number of candidates subtracting the co-ordinates of the end points.

Assessment Report

Scholarship, 2007

Statistics and Modelling

Statistics and Modelling, Scholarship, 2007

Commentary:

On the whole questions 1(b), 2(a) (i), Q2 (b) (ii), 3(a), 4(a), 5(a) were well done. The questions found hardest by the candidates were Q3(c) and Q6 (b).
Some candidates were ill-prepared to answer questions that had content from level 1 and 2 achievement standards. Full evidence of steps taken to reach answers like probability statements were not given, especially when candidates used graphics calculators. Candidates must write legibly. Some handwriting was almost too difficult to read.

The best-performing candidates most commonly demonstrated the following skills and / or knowledge:

- effective communication of statistical ideas with clarity and efficiency of observations. Their answers were set out in a clear and logical way with very good layout, presentation, and writing. They were able to write well-structured paragraphs and essays
- a high level of advanced statistical thinking
- ability to plan statistical answers and draw appropriate conclusions in unfamiliar situations
- ability to apply probability theory and mathematical knowledge to solve complex problems in the construction of confidence intervals and the calculation of probabilities
- identification of steps required for a statistical test along with the assumptions required to carry out the test in the context of the problem
- ability to write equations and constraints to model situations
- ability to read the questions and interpret them with well-written and nonrepetitive answers identifying the key features of the data and graphs provided
- being able to focus on the requirements of questions and correctly interpret them
- correctly justify the choice of model and discuss the validity of the resulting predictions.

Candidates who did not achieve Scholarship lacked some or all of the skills and knowledge above and, in addition, they:

- made vague, superficial, and irrelevant comments, often repeating the same feature in different words
- could not describe information contained in a graph
- had difficulty calculating the standard deviation
- calculated the confidence interval for six trays by taking six times the confidence interval for one tray
- could not relate their answer to the context of the question
- regurgitated the same point several times
- ignored in their discussions statistics given in the question
- were unable to correctly identify and discuss features of a residual plot
- incorrectly stated that a larger sample will improve the model
- outlined improvements to the model rather than improvements to the investigation
- were unable to select and apply knowledge and skills from separate standards to solve problems showing a limited appreciation of the complexity involved
- could not use correct terminology, eg stating that the size of the mark is proportional to the weight of the fruit.

Specific comments about the questions:

Question One
This question was well done by those who could define the variables and interpret the ratio type constraints when written in a sentence. Some candidates had difficulty with "the ratio must be no more than 5:2", and in understanding the objective function so the optimal point was not found. Some candidates:

- failed to show all the constraints.
- did not really understand the difference between gradient (sliding line) and optimal solution.

Question Two
Cluster sampling was not clearly described. The Binomial distribution was set up correctly; however, some candidates could not proceed from there by just looking in the tables.

Question Three
Common errors by candidates were:

- limits were given for the total rather than the mean
- standard error for the total was calculated incorrectly
- conditional probability was not recognised in (c)
- not getting the standard deviation correct
- not identifying "independence of the weights of the kiwifruit" as an assumption.

Very few got question 3(c) correct; in fact, most candidates had no idea where to start.

Question Four
Many candidates gave improvements to the model rather to than the investigation.
Key words positive and linear were required to describe the correlation. The pattern in residual plot was missed by most. Candidates were worried by the negative answer to line A so incorrectly switched to Line B.
"Take more samples" was a common answer for part (c). Also, many said they wanted to exclude the outliers rather than investigate the reason for those outliers.

Question Five
Some candidates with graphical calculators were unable to change the scale in order to see the required domain and range.
The word "compare" was not appreciated in the answers ie both shifts were being compared. Three pages were written instead of the one page asked for. Statistics were simply listed for both day and night shifts with no actual comparison being made. Very few candidates made a comparison of trend of constant versus increasing.
Details were missing in the description of the use of the confidence interval for differences in the mean for the comparison of shifts.

Question Six
The values of n and d were often not given as whole numbers and that n progressed in fives.
Many candidates recognised that confidence intervals would be a good approach; however, they were unable to write a complete answer.
In (b), $n = 25$ and $\pi = 0.04$ were used and very few candidates went through the $n = 20$ process correctly.
A lot of candidates compared 2.5% to 10% and took no account of variation and didn't realise that a confidence interval was required.