Assessment Schedule – 2011

Scholarship Statistics and Modelling (93201)

Evidence Statement

General Principles:

1. Ignore incorrect answers if alongside correct answers. The exception is contradictory statements.
2. Ignore minor copying errors.

QUESTION ONE

Tasks Q1 (a)

Evidence:

- Employment was reasonably steady until mid to late 1998.
- Employment has been increasing from 1998 to 2007 at 40,000 per year on average overall.
- Employment peaked at 2,200,000 in the years 2007 and 2008.
- Full -time employment reached a low at 1,660,000 in the September quarter 2009.
- Part-time employment rose sharply between the December quarter 2006 and the September quarter 2007.
- Part-time employment has been steadily decreasing since the March quarter 2009.
- Full-time employment peaked at 1,700,000 in the September quarter 2008 then again at the same level in the December Quarter 2010. (At least one peak required).
- Seasonal peaks in December with respect to employment overall.
- NZ has significantly more full-time employees than part-time employees.

Note:

1. All observations have to be distinct.
2. All observations need to have relevant context.
3. Maximum of three observations from each graph.
4. Comment about peaks and troughs count as one observation per graph – maximum of two observations.

Judgement:

S: At least four distinct observations.
P: One, two, or three distinct observations.

Tasks Q1 (b)

Evidence:
The year 2012 implies that $t = 17$ so $y = 1670 + 40 \times 17 = 2350(000)$
For the September quarter forecast = 2350 + 40 = 2390(000)
For the December quarter forecast = 2350 + 70 = 2420(000)

Reservations:

1. The trend line assumes a constant rise in average quarterly employment over the years when the current trend appears to be levelling out. There is a steeper increase in the central part and the trend line appears to reflect that more.
2. Forecast is two years beyond the finish of the data so there is doubt that the trend would continue.
3. The prediction doesn't take into account factors such as the Canterbury earthquake or the global economic recession which happened since the data was collected. This may influence unemployment figures.

Note:

1. Accept answer of $t = 16.75$ so $y = 2340(000)$ so September quarter forecast = 2380(000) however December quarter forecast = 2420(000).
2. Can use $t = 16.75$ for September and i = 17 for December.
3. Forecast is INCORRECT if no (000) units.

Judgement:
O: Both forecasts and both reservations correct
S: One forecast correct and corresponding reservation correct
P: One forecast correct or one reservation correct

Tasks Q1 (c)

Evidence:

- Unemployment rose from 3.8% to 7.0% in four years.
- The unemployment rate peaked in the December quarter 2009.
- Between December 2009 and December 2010, the unemployment rate fluctuated between 6 and 7%.
- The unemployment rate was often higher for females.
- The gap between the genders was the greatest in the September quarter 2010.
- Both the male and female unemployment rate reached 7% in the December quarter 2009.
- The fluctuations in unemployment between December 2009 and December 2010 were greater for males than females.

Note:

1. The increase in unemployment could be described between any two time periods.
2. Comment about peaks and troughs count as one observation per graph.

Judgement:

S: Three distinct observations with at least one from each graph about unemployment.
P: One or two distinct observations about unemployment.

Tasks Q2 (a)

Evidence:
Point estimate for the employment rate = 2 182 000 / 2 341 000 = 0.932, ie 93.2%
99% confidence interval is given by 0.932 +/– 0.004

A 99% confidence interval for the true percentage employed in the population is between 92.8% and 93.6%.

Note:

1. No penalty if not a percentage.
2. Score P if percent given to nearest integer.

Judgement

S: Correct point estimate and confidence interval.
P: Either point estimate or confidence interval correct.

Tasks Q2 (b)

Evidence:
Error in population proportion estimate E = 8000 / 2 341 000 = 0.0034
Now $n = (1.96 / 0.0034)^2 \times 0.928 \times 0.072 = 22\,204$
No more people need to be surveyed (22 204 < 29 456).

Note:

1. If $p = 0.932$ get $n = 21\,061$
2. If $p = 0.633$ get $n = 165\,808$
3. Carried error from (a) marked correct.
4. For O must choose lower end of confidence interval for prior estimate of p.
5. For O the prior estimate for p can have the value 0.633 (2 182 000/3 448 000) with E = 8000 / 3448000 = 0.00232 (n more = 136 355) or 0.928 from the confidence interval with E = 0.0034.
6. If $p = 0.5$ as a prior estimate answer is not accepted.
7. Score S if subtraction isn't made at the end when appropriate.
8. Score S if $n = 20\,820$ when p = 0.932.

Judgement

O: Correct answer with some working.
S: Correct value for n calculated from either 0.0034 or 0.00232 (8000/3 448 000) for the error in the estimate.
P: Error calculation correctly indicated.

Tasks Q2(c)

Evidence:

- Divide New Zealand into two strata according to rural and urban towns/areas classifications. Split 15 000 into proportion according to total size of the rural and urban populations.
- Randomly select proportionally according to strata size an urban and a rural town/area. (So each town/area has the same chance of being chosen).
- Randomly select the required number from the chosen towns/areas to make up a total sample of 15 000 private households.

Note:

1. Answer should be understood by the reader so they could apply the random sampling method.
2. Key word random needs to be in answer.
3. Keywords that should be in answer are random, proportional allocation and both strata.

Judgement

S: Three key points.
P: One or two key points.

Tasks Q3 (a)

Evidence:

- There are two distinct subgroups.
- One subgroup has a strong positive correlation while the other shows a constant slightly increasing trend.
- After 15 months of experience the service rate stops increasing and both subgroups come together.
- There is a distinct outlier at (15, 5).
- The work experience ranges from 1 to 20 months while (the service rate per hour ranges from 5 to 28 customers served per hour).
- As the work experience increases, the overall range of the service rates decreases from large to small while the data overall shows a weak positive relationship.
- For low values of experience the scatter of service rate values is high. As the length of experience increases the scatter of service rate values gets less.

Note:

1. A rough calculation and interpretation of the gradient of the subgroup having the positive correlation is acceptable.

Judgement

S: Three points.
P: One or two points.

Tasks Q3 (b)

Evidence:

(i) $S = 0.4234 \times 9 + 14.662 = 18.5$ customers served per hour (Can have 18 or 19).
(ii) $S = 0.4234 \times 27 + 14.662 = 26.1$ customers served per hour (Can have 26).

Prediction of (i) appears to fit the upward trend of points (on the higher side) of the subgroup that shows a strong positive correlation. The scatter in the service rate values is still high at this point. Although this prediction occurs in a small gap it could be still valid. When E = 27 in (ii), we are seven months outside the data range of the fitted model so it's unlikely the fitted model will apply. In fact we would probably have reached a saturation point where the average service rate has levelled out.

Note:

1. Vague validity comments aren't acceptable.

Judgement

O: Both predictions and validity comments reasonable.
S: One prediction and its corresponding validity comment reasonable.
P: One or both predictions correct.

Tasks Q3 (c)

Evidence:

Part	Direction	Strength	Justification
(i)	positive	weak/moderate	As age increases, service rate increases
(ii)	none	none	Height has no bearing on teller's service rate
(iii)	negative	strong	As the time increases, the service rate decreases

Note:

1. A part is correct if all three components are correct.
2. Accept other reasonable answers with justification.
3. Don't accept negative direction for (i) unless strongly argued.
4. For part (ii) a comment of no correlation with justification is counted as a part correct.

Judgement

S +P: Three parts correct.
S: Two parts correct.
P: One part correct.

Task Q4 (a)

Evidence:

Using a Poisson distribution with mean 1.4 we get the following probabilities for the number sick and whether Chris is chosen:

Number Sick x	Probability	Probability Chris Works	Probability Chris Works with x sick
0	0.2466	0.000	$0.000 \times 0.2466 = 0.000$
1	0.3452	0.333	$0.333 \times 0.3452 = 0.115$
2	0.2417	0.667	$0.667 \times 0.2417 = 0.161$
At least 3	0.1665	1.000	$1.000 \times 0.1665 = 0.167$

Prob (Two "on call" working given that Chris is working) = Prob (Chris is working with two sick) / Prob (Chris is working)
= 0.161 / (0.115 + 0.161 + 0.167)
= 0.161 / 0.443
= **0.363**

Note:

1. Some evidence of correct method would be the use of a probability tree with at least two probabilities calculated correctly.
2. If the Poisson probability of $x = 3$ is calculated instead of "at least 3" then score S with MEI. Answer is 0.414 in this case.

Judgement

S: Answer correct
P: Some evidence of correct method like the use of the Poisson to get the probabilities (at least two) of the number sick.

Task Q4 (b)

Evidence:

Let p = probability that A's weekly earnings exceeds B's weekly earnings in a given week.

So ${}^4C_2\, p^2 (1-p)^2 = 0.24$ with p between 0 and 1.
$p^2 (1 - p^2) = 0.04$ so $p^2 - p + 0.2 = 0$

So $p = 0.7236$ or 0.2764.

Now the Normal distribution of the difference in weekly earnings (A – B) has
mean $x - 1120$ and standard deviation $(84^2 + 112^2)^{0.5}$

Now prob ((A – B) > 0) = 0.7236 so using Z scores gives x = **$1,203.02** ($1200) or
when prob ((A – B) > 0) = 0.2764 using Z scores gives **$1,036.98** ($1040)

Note:

1. If tables are used to get p by interpolation, score S if answer is correct based on that p value. Eg, if $p = 0.275$ then answer is $1,014.

Judgement

S + P: At least one answer and corresponding p value is correct.
S: A value of p is correct **AND** the Normal distribution of differences is correctly described in terms of mean and standard deviation.
P: At least one value of p is correct **OR** the Normal distribution of differences is correctly described in terms of mean and standard deviation.

Task Q4(c)

Evidence:
Let customers be denoted by a, b, c, d.....
The following table gives all the possibilities where d will be waiting for less than two minutes:

Cases	Teller 1	Teller 2	d goes to
1	a + c > 3	b < 3	Teller 2
2	a + c < 3	b > 3	Teller 1
3	a < 3	b + c > 3	Teller 1
4	a > 3	b + c < 3	Teller 2
5	a + c < 3	b < 3	Either
6	a < 3	b + c < 3	Either

Now the probability that the time taken to serve a customer is at least three minutes
= 1 – prob (Customer is served in less than 3 minutes)
= 1 – prob ($z < (3 - 2)/0.6$) = 0.0478.

The probability that the time taken to serve two customers is at least three minutes
= 1 – prob (Two customers are served in less than three minutes)
= 1 – prob ($z < (3 - 4)/ (0.6^2 + 0.6^2)^{0.5}$) = 0.8807.

Method 1:
Given that the probability that initially a chose either Teller 1 or Teller 2 with probability 0.5:

Cases	Probability that d waits for less than two minutes
1	$0.5 \times 0.8807 \times 0.9522 = 0.4193$
2	$0.5 \times 0.1193 \times 0.0478 = 0.0029$
3	$0.5 \times 0.9522 \times 0.8807 = 0.4193$
4	$0.5 \times 0.0478 \times 0.1193 = 0.0029$
5	$0.5 \times 0.1193 \times 0.9522 = 0.0568$
6	$0.5 \times 0.9522 \times 0.1193 = 0.0568$

So the probability that d waits for less than two minutes = 0.4193 + 0.0029 + 0.4193 + 0.0029 + 0.0568 + 0.0568 = **0.958**

Method 2:
Prob (d waits for less than two minutes) = 1 – Prob (Both tellers take longer than 3 minutes to serve a, b and c)
$= 1 - 0.25(4 \times 0.0478 \times 0.8807) =$ **0.958**

Note:

1. There are 4 combinations for teller 1 and teller 2 respectively which are given in the table below:

Teller 1	a & c	a	b & c	b
Teller 2	b	b & c	a	a & c

2. All are equally likely to occur with probability 0.25.
3. The probability for each combination is the same at 0.8807 x 0.0478

Judgement

O: Correct answer.
P: Evidence that various scenarios have been described.

Tasks Q5 (a) (i)

Evidence:
Constraints are:

$x \le 50$, $y \le 70$, $x + y \le 80$ and $70 - y + 50 - x \le 45$ so $x + y \ge 75$
Also $x \ge 0$ and $y \ge 0$.

Overall delivery cost $C = 0.5x + 0.6y + 0.4\,(50 - x) + 0.55\,(70 - y) = 0.1x + 0.05y + 58.5$
From graph optimal point is **(5, 70).**

Delivery plan will be 5 sheets of plywood from eastern warehouse to customer A, 45 sheets of plywood from the western warehouse to A and 70 sheets of plywood from the eastern warehouse to B.

Judgement

S: Correct delivery plan.
P: First four constraints are correct or optimal point (5, 70) is correct.

Tasks Q5 (a) (ii)

Evidence:
Cost line becomes parallel to $x + y = 75$
So $C = 0.1x + (K – 0.55)\, y + 58.5$ would be such that $K – 0.55 = 0.1$ so **K = 0.65**.
Minimum overall delivery cost = $0.1 \times 75 + 58.5$ = **$66**

Judgement

O: Correct K value and minimum cost.
S: Correct K value only.
P: Recognition that cost line is parallel to one of the constraints.

Tasks Q5 (b) (i)

Evidence:
One constraint: $x + y \geq 75$ changes to $70 – y + 50 – x \leq 40$ so $x + y \geq 80$.
Feasible region is reduced to a line segment.
Cost is minimised at **(10, 70).**

Tasks Q5 (b) (ii)
Evidence:
$C = 0.5x + 0.7y + 0.4\,(48 – x) + 0.55\,(70 – y)$
$C = 0.1x + 0.15y + 57.7$

$x \leq 48$, $y \leq 70$, $x + y \leq 80$ and $70 – y + 48 – x \leq 45$ implies $x + y \geq 73$

Cost is minimised at **(48, 25)**.

Judgement

S: Either (i) or (ii) correct
P: Changed constraint correct in either case.

Scoring for each Question

Each question part within a question is scored as:

N = No meaningful work, insufficient or incorrect answer.
P = partially correct to a predetermined level.
S = totally correct to a scholarship level.
O = totally correct to an outstanding level.

The codes are put together for each question and then converted to a mark out of eight according to the following table:

Mark	Codes
8	O + 2S, O + S + P, O + 2S + P, O + S + 2P
7	O, O + P, O + 2P, O + S, 3S, 3S + P
6	2S, 2S + P, 2S + 2P
5	S + P, S + 2P, S + 3P
4	S
3	3P
2	2P
1	P
0	N

The marks for each question are totalled to give an overall mark. Best possible overall mark is 40.