S–CALCMF

S

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Karahipi 2017
Te Tuanaki

9.30 i te ata Rāmere 10 Whiringa-ā-rangi 2017

TE PUKAPUKA O NGĀ TIKANGA TĀTAI ME NGĀ TŪTOHI

Tirohia tēnei pukapuka hei whakatutuki i ngā tūmahi mō te whakamātautau Karahipi Te Tuanaki 93202MQ.

Tirohia mēnā e tika ana te raupapatanga o ngā whārangi 2–7 kei roto i tēnei pukapuka, ka mutu, kāore tētahi o aua whārangi i te takoto kau.

KA TAEA TĒNEI PUKAPUKA TE PUPURI HEI TE MUTUNGA O TE WHAKAMĀTAUTAU.

TE TUANAKI – ĒTAHI TURE WHAI HUA

TE TAURANGI

Ngā Whārite Pūrua

Mēnā $ax^2 + bx + c = 0$

kāti $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Ngā Taupū Kōaro

$y = \log_b x \Leftrightarrow x = b^y$

$\log_b(xy) = \log_b x + \log_b y$

$\log_b\left(\dfrac{x}{y}\right) = \log_b x - \log_b y$

$\log_b(x^n) = n\log_b x$

$\log_b x = \dfrac{\log_a x}{\log_a b}$

Ngā Tau Matatini

$z = x + \mathrm{i}y$

$\quad = r\,\mathrm{cis}\,\theta$

$\quad = r(\cos\theta + \mathrm{i}\sin\theta)$

$\bar{z} = x - \mathrm{i}y$

$\quad = r\,\mathrm{cis}\,(-\theta)$

$\quad = r(\cos\theta - \mathrm{i}\sin\theta)$

$r = |z| = \sqrt{z\bar{z}} = \sqrt{(x^2 + y^2)}$

$\theta = \arg z$

ina $\cos\theta = \dfrac{x}{r}$

ā, $\sin\theta = \dfrac{y}{r}$

Te Ture a De Moivre

Mēnā he tau tōpū a n, kāti,

$(r\,\mathrm{cis}\,\theta)^n = r^n\,\mathrm{cis}\,(n\theta)$

Ture Tohurua

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + \binom{n}{n}b^n$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{(n-r)!\,r!}$$

Kua hōmai ētahi uara o $\binom{n}{r}$ i te tūtohi i raro nei.

n \ r	0	1	2	3	4	5	6	7	8	9	10
0	1										
1	1	1									
2	1	2	1								
3	1	3	3	1							
4	1	4	6	4	1						
5	1	5	10	10	5	1					
6	1	6	15	20	15	6	1				
7	1	7	21	35	35	21	7	1			
8	1	8	28	56	70	56	28	8	1		
9	1	9	36	84	126	126	84	36	9	1	
10	1	10	45	120	210	252	210	120	45	10	1
11	1	11	55	165	330	462	462	330	165	55	11
12	1	12	66	220	495	792	924	792	495	220	66

TE ĀHUAHANGA TAUNGA

Te Rārangi Torotika

Whārite $y - y_1 = m(x - x_1)$

Te Porohita

$(x - a)^2 + (y - b)^2 = r^2$

ko te (a,b) te pū, ko te r te pūtoro

Te Unahi

$y^2 = 4ax$, $(at^2, 2at)$ rānei

Arotahi $(a,0)$

Rārangi whakarite $x = -a$

Te Pororapa

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, $(a\cos\theta, b\sin\theta)$ rānei

Arotahi $(c,0)$ $(-c,0)$ ina ko $b^2 = a^2 - c^2$

Ōwehenga tawhiti: $e = \dfrac{c}{a}$

Te Pūwerewere

$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$, $(a\sec\theta, b\tan\theta)$ rānei

ngā rārangi pātata $y = \pm\dfrac{b}{a}x$

Arotahi $(c,0)$ $(-c,0)$ ina ko $b^2 = c^2 - a^2$

Ōwehenga tawhiti: $e = \dfrac{c}{a}$

CALCULUS – USEFUL FORMULAE

ALGEBRA

Quadratics

If $ax^2 + bx + c = 0$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Logarithms

$y = \log_b x \Leftrightarrow x = b^y$

$\log_b(xy) = \log_b x + \log_b y$

$\log_b\left(\dfrac{x}{y}\right) = \log_b x - \log_b y$

$\log_b\left(x^n\right) = n\log_b x$

$\log_b x = \dfrac{\log_a x}{\log_a b}$

Complex numbers

$z = x + \mathrm{i}y$

$\quad = r\,\mathrm{cis}\,\theta$

$\quad = r(\cos\theta + \mathrm{i}\sin\theta)$

$\overline{z} = x - \mathrm{i}y$

$\quad = r\,\mathrm{cis}\,(-\theta)$

$\quad = r(\cos\theta - \mathrm{i}\sin\theta)$

$r = |z| = \sqrt{z\overline{z}} = \sqrt{(x^2 + y^2)}$

$\theta = \arg z$

where $\cos\theta = \dfrac{x}{r}$

and $\sin\theta = \dfrac{y}{r}$

De Moivre's Theorem

If n is any integer, then

$(r\,\mathrm{cis}\,\theta)^n = r^n\,\mathrm{cis}\,(n\theta)$

Binomial Theorem

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + \binom{n}{n}b^n$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{(n-r)!\,r!}$$

Some values of $\binom{n}{r}$ are given in the table below.

n \ r	0	1	2	3	4	5	6	7	8	9	10
0	1										
1	1	1									
2	1	2	1								
3	1	3	3	1							
4	1	4	6	4	1						
5	1	5	10	10	5	1					
6	1	6	15	20	15	6	1				
7	1	7	21	35	35	21	7	1			
8	1	8	28	56	70	56	28	8	1		
9	1	9	36	84	126	126	84	36	9	1	
10	1	10	45	120	210	252	210	120	45	10	1
11	1	11	55	165	330	462	462	330	165	55	11
12	1	12	66	220	495	792	924	792	495	220	66

COORDINATE GEOMETRY

Straight Line

Equation $y - y_1 = m(x - x_1)$

Circle

$(x-a)^2 + (y-b)^2 = r^2$

has a centre (a,b) and radius r

Parabola

$y^2 = 4ax$ or $(at^2, 2at)$

Focus $(a,0)$ Directrix $x = -a$

Ellipse

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ or $(a\cos\theta, b\sin\theta)$

Foci $(c,0)$ $(-c,0)$ where $b^2 = a^2 - c^2$

Eccentricity: $e = \dfrac{c}{a}$

Hyperbola

$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ or $(a\sec\theta, b\tan\theta)$

asymptotes $y = \pm\dfrac{b}{a}x$

Foci $(c,0)$ $(-c,0)$ where $b^2 = c^2 - a^2$

Eccentricity: $e = \dfrac{c}{a}$

TE TUANAKI

Kimi Pārōnaki

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
$\ln x$	$\frac{1}{x}$
e^{ax}	ae^{ax}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\cot x$	$-\operatorname{cosec}^2 x$

Ngā Tikanga Pāwhaitua

$f(x)$	$\int f(x)\,\mathrm{d}x$
x^n	$\frac{x^{n+1}}{n+1} + c$ $(n \neq -1)$
$\frac{1}{x}$	$\ln\lvert x\rvert + c$
$\frac{f'(x)}{f(x)}$	$\ln\lvert f(x)\rvert + c$

Ngā mātāpono tuatahi

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Te Pānga Tawhā

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \cdot \frac{\mathrm{d}t}{\mathrm{d}x}$$

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) \cdot \frac{\mathrm{d}t}{\mathrm{d}x}$$

Te Ture mō te Otinga Whakarau

$(f.g)' = f.g' + g.f'$ mēnā rānei $y = uv$ kāti $\frac{\mathrm{d}y}{\mathrm{d}x} = u\frac{\mathrm{d}v}{\mathrm{d}x} + v\frac{\mathrm{d}u}{\mathrm{d}x}$

Te Ture mō te Otinga Wehe

$\left(\frac{f}{g}\right)' = \frac{g.f' - f.g'}{g^2}$ mēnā rānei $y = \frac{u}{v}$ kāti $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Te Ture Pānga Hiato, te Ture Mekameka rānei

$$\big(f(g)\big)' = f'(g).g'$$

mēnā rānei $y = f(u)$ ā, $u = g(x)$ kāti $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \cdot \frac{\mathrm{d}u}{\mathrm{d}x}$

NGĀ TIKANGA TAU

Te Ture Taparara

$$\int_a^b f(x)\ \mathrm{d}x \approx \frac{1}{2}h\left[y_0 + y_n + 2(y_1 + y_2 + \ldots + y_{n-1})\right]$$

ina $h = \frac{b-a}{n}$ ā, $y_r = f(x_r)$

Te Ture a Simpson

$$\int_a^b f(x)\ \mathrm{d}x \approx \frac{1}{3}h\left[y_0 + y_n + 4(y_1 + y_3 + \ldots + y_{n-1}) + 2(y_2 + y_4 + \ldots + y_{n-2})\right]$$

ina $h = \frac{b-a}{n}$, $y_r = f(x_r)$, ā, he taurua te n.

CALCULUS

Differentiation

$y = f(x)$	$\dfrac{dy}{dx} = f'(x)$
$\ln x$	$\dfrac{1}{x}$
e^{ax}	ae^{ax}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\cot x$	$-\operatorname{cosec}^2 x$

Integration

$f(x)$	$\int f(x)dx$
x^n	$\dfrac{x^{n+1}}{n+1} + c$ $(n \neq -1)$
$\dfrac{1}{x}$	$\ln\lvert x \rvert + c$
$\dfrac{f'(x)}{f(x)}$	$\ln\lvert f(x) \rvert + c$

First principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Parametric Function

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

$$\frac{d^2 y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \cdot \frac{dt}{dx}$$

Product Rule

$(f.g)' = f.g' + g.f'$ or if $y = uv$ then $\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

Quotient Rule

$\left(\dfrac{f}{g}\right)' = \dfrac{g.f' - f.g'}{g^2}$ or if $y = \dfrac{u}{v}$ then $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

Composite Function or Chain Rule

$$\left(f(g)\right)' = f'(g).g'$$

or if $y = f(u)$ and $u = g(x)$ then $\dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx}$

NUMERICAL METHODS

Trapezium Rule

$$\int_a^b f(x)\ dx \approx \frac{1}{2}h\left[y_0 + y_n + 2(y_1 + y_2 + ... + y_{n-1})\right]$$

where $h = \dfrac{b-a}{n}$ and $y_r = f(x_r)$

Simpson's Rule

$$\int_a^b f(x)\ dx \approx \frac{1}{3}h\left[y_0 + y_n + 4(y_1 + y_3 + ... + y_{n-1}) + 2(y_2 + y_4 + ... + y_{n-2})\right]$$

where $h = \dfrac{b-a}{n}$, $y_r = f(x_r)$ and n is even.

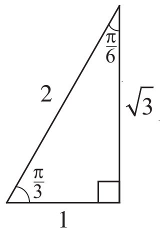
TE PĀKOKI

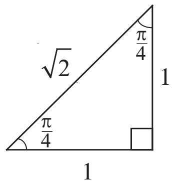
$\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$

$\sec\theta = \dfrac{1}{\cos\theta}$

$\cot\theta = \dfrac{1}{\tan\theta}$

$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$





Te Ture Aho

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Te Ture Whenu

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Ngā Whārite ka Pono Ahakoa ngā Uara Ka Whakaurua Atu

$\cos^2\theta + \sin^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$\cot^2\theta + 1 = \operatorname{cosec}^2\theta$

Ngā Otinga Whānui

Mēnā $\sin\theta = \sin\alpha$ kāti $\theta = n\pi + (-1)^n\alpha$

Mēnā $\cos\theta = \cos\alpha$ kāti $\theta = 2n\pi \pm \alpha$

Mēnā $\tan\theta = \tan\alpha$ kāti $\theta = n\pi + \alpha$

ko te n, he tau tōpū ahakoa

Ngā Koki Hiato

$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$

Ngā Koki Rearua

$\sin 2A = 2\sin A\cos A$

$\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

$$\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A\\ &= 2\cos^2 A - 1\\ &= 1 - 2\sin^2 A\end{aligned}$$

Ngā Otinga Whakarau

$2\sin A\cos B = \sin(A+B) + \sin(A-B)$

$2\cos A\sin B = \sin(A+B) - \sin(A-B)$

$2\cos A\cos B = \cos(A+B) + \cos(A-B)$

$2\sin A\sin B = \cos(A-B) - \cos(A+B)$

Ngā Otinga Tāpiri

$\sin C + \sin D = 2\sin\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

$\sin C - \sin D = 2\cos\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$

$\cos C + \cos D = 2\cos\dfrac{C+D}{2}\cos\dfrac{C-D}{2}$

$\cos C - \cos D = -2\sin\dfrac{C+D}{2}\sin\dfrac{C-D}{2}$

TE INE

Te Tapatoru

Horahanga $= \dfrac{1}{2}ab\sin \mathrm{C}$

Te Taparara

Horahanga $= \dfrac{1}{2}(a+b)h$

Te Pewanga

Horahanga $= \dfrac{1}{2}r^2\theta$

Te roa o te pewa $= r\theta$

Te Rango

Rōrahi $= \pi r^2 h$

Horahanga mata kōpiko $= 2\pi rh$

Te Koeko

Rōrahi $= \dfrac{1}{3}\pi r^2 h$

Horahanga mata kōpiko $= \pi rl$ ina ko te l te teitei o te tītaha

Te Poi

Rōrahi $= \dfrac{4}{3}\pi r^3$

Horahanga mata $= 4\pi r^2$

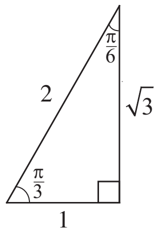
TRIGONOMETRY

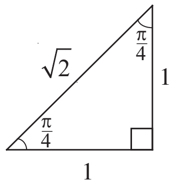
$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$





Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine Rule

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Identities

$$\cos^2\theta + \sin^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\cot^2\theta + 1 = \operatorname{cosec}^2\theta$$

General Solutions

If $\sin\theta = \sin\alpha$ then $\theta = n\pi + (-1)^n\alpha$

If $\cos\theta = \cos\alpha$ then $\theta = 2n\pi \pm \alpha$

If $\tan\theta = \tan\alpha$ then $\theta = n\pi + \alpha$

where n is any integer

Compound Angles

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

Double Angles

$$\sin 2A = 2\sin A\cos A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

$$\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A\\ &= 2\cos^2 A - 1\\ &= 1 - 2\sin^2 A\end{aligned}$$

Products

$$2\sin A\cos B = \sin(A+B) + \sin(A-B)$$

$$2\cos A\sin B = \sin(A+B) - \sin(A-B)$$

$$2\cos A\cos B = \cos(A+B) + \cos(A-B)$$

$$2\sin A\sin B = \cos(A-B) - \cos(A+B)$$

Sums

$$\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$$

$$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$$

$$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$$

$$\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$$

MEASUREMENT

Triangle

Area $= \frac{1}{2}ab\sin C$

Trapezium

Area $= \frac{1}{2}(a+b)h$

Sector

Area $= \frac{1}{2}r^2\theta$

Arc length $= r\theta$

Cylinder

Volume $= \pi r^2 h$

Curved surface area $= 2\pi rh$

Cone

Volume $= \frac{1}{3}\pi r^2 h$

Curved surface area $= \pi rl$ where $l =$ slant height

Sphere

Volume $= \frac{4}{3}\pi r^3$

Surface area $= 4\pi r^2$

English translation of the wording on the front cover

S–CALCMF

Scholarship 2017
Calculus

9.30 a.m. Friday 10 November 2017

FORMULAE AND TABLES BOOKLET

Refer to this booklet to answer the questions for Scholarship Calculus 93202Q.

Check that this booklet has pages 2–7 in the correct order and that none of these pages is blank.

YOU MAY KEEP THIS BOOKLET AT THE END OF THE EXAMINATION.