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SUPERVISOR'S USE ONLY

OUTSTANDING SCHOLARSHIP EXEMPLAR

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Scholarship 2015
Physics

9.30 a.m. Monday 16 November 2015
Time allowed: Three hours
Total marks: 40

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should answer ALL the questions in this booklet.

For all 'describe' or 'explain' questions, the answers should be written or drawn clearly with all logic fully explained.

For all numerical answers, full working must be shown and the answer must be rounded to the correct number of significant figures and given with the correct SI unit.

Formulae you may find useful are given on page 2.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–19 in the correct order and that none of these pages is blank.

You are advised to spend approximately 35 minutes on each question.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Question	Mark
ONE	
TWO	
THREE	
FOUR	
FIVE	
TOTAL	/40

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QUESTION ONE: PARTICLES AND WAVES

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(a) (i) Describe the photoelectric effect.

In your answer you should include a derivation of the relationship between the incident photon's frequency and the electron's kinetic energy, and how these relate to the work function of the metal.

Photoelectric effect: when high energy (e.g infrared) light is shone onto a metal, the metal will release electrons. ~~The relationship between the work function of the metal (amount of~~ All energy provided to the electron by the photons of light is first used to release the electron from the atom's orbit, and any remaining energy (pg 14)

✓ ✓

(ii) The photoelectric effect was unable to be fully explained using classical physics.

Comment on this statement.

Classical physics says that light is a wave. However, the photoelectric effect does not happen for low frequency photons, contradicting the wave model which implies any frequency should be able to eventually emit electrons, due to the energy of the wave building up over time. This led Einstein to suggest that light was quantised, and that it could display behaviour of both waves and particles.

✓

(b) Describe the similarities and the differences between the orbit of the Moon around the Earth and the orbit of an electron around a proton in a hydrogen atom.

The orbit of the moon around the Earth and of the electron around a photon are both made possible by centripetal force, which pulls the electron/Moon into the Earth/proton. For the Earth/Moon system, this ~~or~~ Fc is provided by the gravitational attraction between the Moon and the Earth. ~~This for the~~ But gravity does not behave the same on a quantum scale, so the orbit of the electron around the proton is provided by the electromagnetic attraction between the oppositely charged bodies. Also, while the Moon maintains a fixed radius around the Earth, the radius of the electron changes as it gains and loses energy, making its orbit like a wave shape.

✓ 0 //

(c) Sound from a small loudspeaker L reaches a point P by two paths, which differ in length by 1.2 m. When the frequency of the sound is gradually increased, the resultant intensity at P goes through a series of maxima and minima. A maximum occurs when the frequency is 1000 Hz, and the next maximum occurs at 1200 Hz.

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(i) Explain what causes the maxima and minima to occur.

The maxima and minima are caused by interference between the two paths. At maxima, the path difference is an integer multiple of the λ, and the troughs collide with troughs to intensify the signal. At minima, the path difference is a half multiple of the λ, and troughs collide with crests to cancel out and reduce the wave's intensity. //

O

(ii) Calculate the speed of sound in the medium between L and P.

$v = f\lambda$, $\lambda = v/f$, $\lambda_1 = v/1000$, $\lambda_2 = v/1200$

p.d. $= n\lambda$, $1.2 = n_1\lambda_1 = n_1 v/1000$, $1200 = n_1 v$

$1.2 = n_2\lambda_2 = n_2 v/1200$, $1440 = n_2 v$

Given that there is no maximum between 1000 Hz and 1200 Hz: $n_1 + 1 = n_2$

$1440 = (n_1 + 1)v = n_1 v + v$

$1440 = 1200 + v$, $v = 240\ ms^{-1}$

~~So~~ Speed of sound in medium $= 240\ ms^{-1}$

✓

✓

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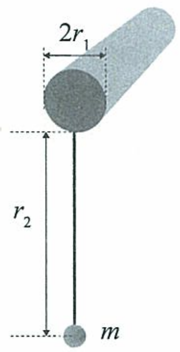
(8)

QUESTION TWO: THE VERTICAL CIRCLE

ASSESSOR'S USE ONLY

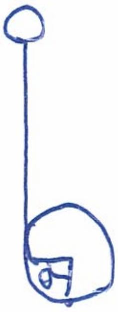
A small ball of mass m, hangs from a light, inextensible string attached to a fixed horizontal post of radius r_1, as shown.

The ball is hit horizontally with a large bat so that the ball wraps the string around the post.



(a) Show that the ball's speed at the top of its first swing must be at least

$v_{\text{top}} = \sqrt{g\left(r_2 - \frac{\pi r_1}{2}\right)}$ so that the string remains taut.



For string to be taut: $F_c \geq mg$, $\frac{mv^2}{r} \geq mg$

At top of swing: $\theta = \pi/2$, $s = r\theta = \pi r_1/2$

$\therefore$ length of string at top: $r_2 - \pi r_1/2$

$mv^2/(r_2 - \pi r_1/2) \geq mg$, $v^2 \geq g(r_2 - \pi r_1/2)$

$v_{top} \geq \sqrt{g\left(r_2 - \frac{\pi r_1}{2}\right)}$, min is $v_{top} = \sqrt{g\left(r_2 - \frac{\pi r_1}{2}\right)}$

(b) For the speed of the ball in (a), show that the initial speed must be at least

$v_{\text{initial}} = \sqrt{g\left(5r_2 - \left(\frac{3\pi}{2} - 2\right)r_1\right)}$.

given $E_p = 0$ at start

E_K lost = gravitational E_p gained

$(E_K + E_p)_{initial} = (E_K + E_p)_{final}$, $E_{K\,initial} = (E_K + E_p)_{final}$

$v^2 = g(r2 - \pi r_1/2)$, $E_K = \frac{1}{2}mv^2 = \frac{1}{2}mg(r2 - \pi r_1/2)$

$\Delta E_p = E_{pf} - E_{pi} = mgh_f - mgh_i = mg\Delta h$

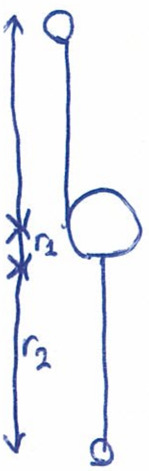
$\Delta h = r_2 + r_1 + (r_2 - \pi r_1/2) = 2r_2 + r_1 - \pi r_1/2$

$E_p = mg(2r_2 + r_1 - \pi r_1/2)$

$E_{K\,initial} = mg\left(\frac{1}{2}(r2 - \pi r_1/2) + (2r_2 + r_1 - \pi r_1/2)\right)$

$E_K = mg(5r_2/2 + r_1 - 3\pi r_1/4) = \frac{1}{2}mg\left(5r_2 + \left(\frac{4-3\pi}{2}\right)r_1\right)$

$E_K = \frac{1}{2}mg\left(5r_2 - \left(\frac{3\pi}{2} - 2\right)r_1\right) = \frac{1}{2}mv^2$



$\rightarrow v^2 = g\left(5r_2 - \left(\frac{3\pi}{2} - 2\right)r_1\right)$

$\rightarrow v_i = \sqrt{g\left(5r_2 - \left(\frac{3\pi}{2} - 2\right)r_1\right)}$

(c) Assuming an elastic collision, show that the speed of the bat is approximately half that of the ball's initial speed.

State any other assumptions made, and the reasons for them.

Elastic collision: Both energy and momentum conserved

~~$\frac{1}{2} m_B v_B^2 = \frac{1}{2} m_B v_B^2 = \frac{1}{2} mg\left(5r_2 - \left(\frac{3\pi}{2} - 2\right)r_1\right)$~~

~~$v_B^2 = \frac{m}{m_B} g\left(5r_2 - \left(\frac{3\pi}{2} - 2\right)r_1\right)$~~

~~p conserved: $m_B v_B = mv$, $m_B = \frac{mv}{v_B}$~~

~~$v_B^2 = m/(mv/v_B)\, \left(g\left(5r_2 - \left(\frac{3\pi}{2} - 2\right)r_1\right)\right)$~~

~~$v_B^2 = \left(\frac{v_B}{v}\right) g\left(5r_2 - \left(\frac{3\pi}{2} - 2\right)r_1\right) = v_B$~~

~~$\frac{1}{2} v_B (mv) = \frac{1}{2} mv^2$~~ v = speed of ball

M = mass of bat

v_i = initial vel of bat

v_f = final v of bat

~~$\frac{1}{2} m_i v_i = \frac{1}{2}$~~ $\frac{1}{2} M v_i^2 = \frac{1}{2} M v_f^2 + \frac{1}{2} mv^2$

p conserved: $Mv_i = Mv_f + mv$, $v_f = v_i - \frac{mv}{M}$

$\frac{1}{2} M v_i^2 = \frac{1}{2} M\left(v_i - \frac{mv}{M}\right)^2 + \frac{1}{2} mv^2$

$\frac{1}{2} M v_i^2 = \frac{1}{2} M\left(v_i^2 - \frac{2mvv_i}{M} + \frac{m^2v^2}{M^2}\right) + \frac{1}{2} mv^2$

$\frac{1}{2} M v_i^2 = \frac{1}{2} M v_i^2 - mvv_i + \frac{m^2v^2}{2M} + \frac{1}{2} mv^2$

$-mvv_i + \frac{m^2v^2}{2M} + \frac{1}{2} mv^2 = 0$

$-v_i + \frac{mv}{2M} + \frac{1}{2}v = 0$, $\frac{mv}{2M} + \frac{v}{2} = v_i$ (pg 14).

(d) As the ball completes its first orbit around the post, explain why the ball appears to be travelling at a speed greater than its initial value.

As the ball travels around the post, ~~it will lose some of~~ the length of the string will become shorter. This means that it will lose some of its kinetic energy and instead gain gravitational Ep. However, the period of a pendulum is $T = 2\pi\sqrt{L/g}$, (the ball can be roughly assumed to be a pendulum at this point in time), and so the period of the pendulum will be shortened by a much greater value, resulting in a net increase in angular velocity. ~~and therefore linear speed as ... the increase in~~ Shortening the period makes the ball appear to travel faster even though its linear velocity has decreased, because of the increase in angular velocity (it travels a larger angle in the same time).

ASSESSOR'S USE ONLY

✓ ✓ ~~✓~~ BTN ✓ O 7 (8)

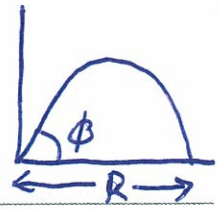
ASSESSOR'S USE ONLY

QUESTION THREE: CRICKET – THROW IN FROM THE BOUNDARY

Acceleration due to gravity = 9.81 m s^{-2}

(a) Show that the range, R, of a projectile thrown from ground level at angle, ϕ, to the horizontal with starting velocity, v, is $\frac{v^2 \sin 2\phi}{g}$.

(Note that $2\sin\phi\cos\phi = \sin 2\phi$.)



Vertical speed: $v_f = v_i + at$

$v\sin\phi = -v\sin\phi + gt$, $2v\sin\phi = gt$,

Time spent in air: $t = 2v\sin\phi/g$

Horizontal velocity is constant, so $v = d/t$, $d = vt$

$R = vt = v\cos\phi \times 2v\sin\phi/g = v2(2\sin\phi\cos\phi)/g$

$R = \frac{v^2 \sin 2\phi}{g}$.

(b) A cricket ball is thrown from ground level with a velocity 28.0 m s^{-1}, and hits a target on the ground 80.0 m away.

Show that the time of flight of the ball is 4.04 s.

The effects of air resistance can be ignored.

~~$80.0 = 28^2 \sin 2\phi/9.81$, $\sin 2\phi = 80.0\times9.81/28.0^2$~~

$80.0 = 28^2 \sin 2\phi/9.81$, $\sin 2\phi = 80.0\times 9.81/28.0^2 =$

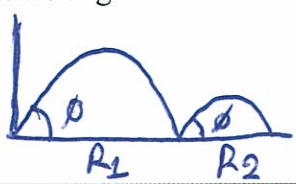
$\sin 2\phi = 1.00$ (3sf), $2\phi = 90°$, $\phi = 45°$

Horizontal velocity is constant: $v = d/t$, $t = d/v$

$t = 80.0/28\cos 45° = 4.04$ s (3sf)

(c) The ball is now thrown at the same target, with the same initial speed, but at a lower angle. This time, it is aimed to bounce in front of the target, so that it hits the target on the second bounce. When the ball bounces the first time, it rebounds with the same angle as it came in, but it loses half its speed.

(i) Calculate the time taken for the ball to reach the target.



$R_1 = 28^2 \sin 2\phi/9.81$, $R_2 = 14^2 \sin 2\phi/9.81$

$R_1 + R_2 = 80$, $28^2 \sin 2\phi/9.81 + 14^2 \sin 2\phi/9.81 = 80.0$

$\sin 2\phi(28^2 + 14^2) = 784.8$, $\sin 2\phi = 784.8/980$

$2\phi = \sin^{-1}(784.8/980) = 53.2°$, $\phi = 26.6°$ (3sf)

Horizontal speed constant between bounces: $t = d/v$

$R_1 = 28^2 \sin 2\phi/9.81 = 64$ m, $R_2 = 80 - 64 = 16$ m

$t = 64/28\cos 26.6 + 16/14\cos 26.6$

$= 3.83$ s (3sf)

(ii) Discuss, with physical reasons, the difference in times between parts (b) and (c)(i).

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In part B, the angle is greater than for part C(i). Although they have the same total speed, the horizontal speed is much greater in C, because it is given by $v\cos\theta$, and $\cos\theta$ becomes greater as θ becomes smaller (for $0° \le \theta \le 90°$). Ignoring air resistance, the horizontal speed will be constant, and therefore a lower angle will cover the required distance quicker.

(φ15)

(d) Any real throw of a ball would be from approximately head height, rather than from ground level.

Show that the range achieved by a throw from a height of 2 m above the ground would be

$$v\cos\phi\left(\frac{v\sin\phi+\sqrt{v^2\sin^2\phi+4g}}{g}\right)$$

Vertical: $d = 2\text{m}$, $v_i = -v\sin\phi$, $v_f = ?$, $a = g$

$v_f^2 = v^2\sin^2\phi + 4g$, ~~$v_f = \sqrt{v^2\sin^2\phi+4g}$~~

$v_f^2 = v_i^2 + 2ad$, $v_f = \sqrt{v^2\sin^2\phi + 4g}$

~~Horizontal speed $= (v_i + v_f)/2$, $t = 2d/(v_i+v_f)$~~

~~Flight time~~ $v_f = v_i + at$, $t = v_f - v_i / a$

Flight time: $t = (\sqrt{v^2\sin^2\phi + 4g} + v\sin\phi)/g$

Horizontal speed constant: $v = d/t$, $d = vt$

$$R = vt = v\cos\phi\left(\frac{v\sin\phi + \sqrt{v^2\sin^2\phi+4g}}{g}\right)$$

8

(8)

QUESTION FOUR: CIRCUITS

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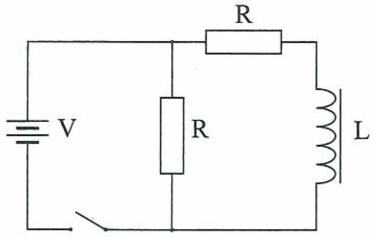


(a) In the electric circuit shown, the switch is closed at time $t = 0$.

(i) Write an expression for the current immediately after the switch is closed.

Explain your reasoning.

Initially the inductor will block the build of current in ~~the~~ the far loop, so this loop essentially has no current, and it all goes into the other loop.

By Ohm's Law = $I = V/R$

(ii) Write an expression for the limiting value of the current a long time after the switch is closed.

Explain your reasoning.

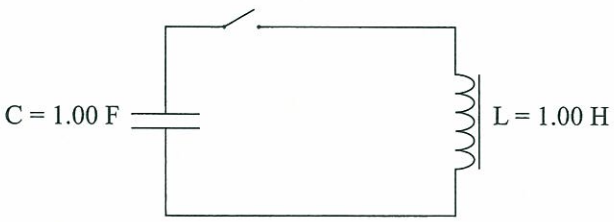
After a long time the inductor no longer opposes the current, so it is equivalent to a wire (provides no R).

~~[illegible]~~ $1/R_T = 1/R + 1/R = 2R/R^2$, $R_T = R/2$

~~[illegible]~~, ~~[illegible]~~ $I = 2V/R$

(b) (i) A charged capacitor (1.00 F) is connected to an inductor (1.00 H), as shown in the diagram below. When the switch is closed (at $t = 0$), the current in the circuit will oscillate sinusoidally with a period of 6.28 s.

Describe the energy changes that take place in the course of one complete cycle.



The capacitor will discharge, ~~the batteries energy moving into the magnetic that~~ creating a current which induces a magnetic field in the inductor. All the electrical energy stored in the capacitor is then converted to magnetic potential stored in the magnetic field of the inductor. Once the capacitor is fully ~~disc~~ discharged, the magnetic field in the inductor induces a current which recharges the capacitor, converting all magnetic potential to electric energy. ~~This is a resonant circuit.~~

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(ii) The capacitor plates can be moved closer together so that the capacitance is increased to 4.00 F.

Explain at what point in the cycle, could the plates of the capacitor be moved closer to each other so that no energy is transferred to the circuit.

The energy stored in a capacitor is given by $E_p = \frac{1}{2}QV$ or $E_p = \frac{1}{2}(Q^2/C)$. ~~Therefore energy is proportional~~ If the plates are pushed ~~to~~ together when the capacitor has no charge, ~~they~~ ~~will~~ the capacitance will change, but as shown by $E_p = \frac{1}{2}(Q^2/C)$, this will not affect the energy stored by the capacitor as the charge is zero.

(pg 14)

(c) A slab of copper falls freely under the influence of gravity before entering the region between the poles of a strong magnet. As it enters the magnetic field, the copper slab slows considerably.

Explain why this occurs, and state what has happened to the kinetic energy of the copper slab.

The magnetic field exerts a force on the copper slab which opposes its direction of motion, as per Lenz's Law.* This decreases the speed of the copper slab. The kinetic energy of the slab is converted into a potential difference across the ~~two~~ poles of the magnetic field.

*The motion of the copper slab induces a voltage across the poles, increasing the strength of the magnetic field and providing a force against the slab.

(8)

QUESTION FIVE: WAVES ON STRINGS

ASSESSOR'S USE ONLY

The speed v of a wave on a string is given by, $v = \sqrt{\frac{T}{\mu}}$, where T is the tension in the string, and μ is the mass per unit length, measured in kg m^{-1}.

(a) Show that the above equation is dimensionally correct.

$T = N = kgms^{-2}$, $\mu = kgm^{-1}$, $v = ms^{-1}$

$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{kgms^{-2}}{kgm^{-1}}} = \sqrt{\frac{m^2}{s^2}} = \frac{m}{s} = ms^{-1}$. ✓

✓

(b) One end of a string of mass per unit length μ is attached to a solid wall, while the other end passes over a pulley, and is attached to a hanging mass, m, as shown in Figure 1.

A second string of the same length and made of the same material, but with twice the diameter, is mounted in a similar fashion with an identical mass, m, as shown in Figure 2.

The first string oscillates in its first harmonic when it is driven at a frequency of 200 Hz.

Calculate the frequency that will cause the second string to oscillate in its third harmonic.

Figure 1

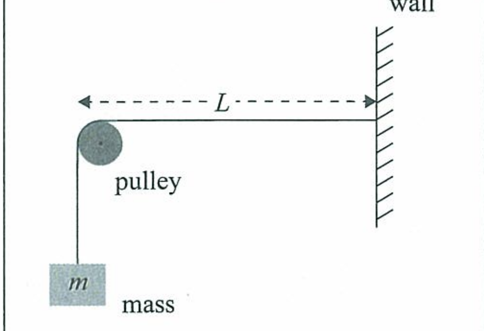
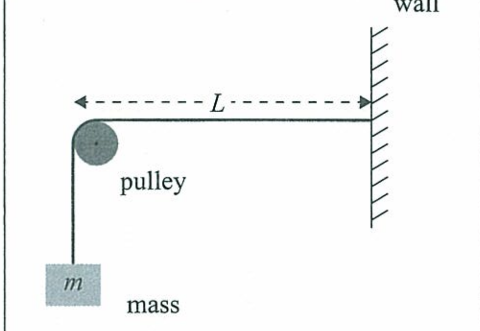


Figure 2



$v = \sqrt{T/\mu} = f\lambda$, $f = \frac{1}{\lambda}\sqrt{\frac{T}{\mu}} = 200$ (first string)

Tension = mg = same for both strings (m, g constant)

$\mu = m/L$. μ is proportional to the cross sectional area of the string, thus if r is doubled, A multiplied by 4, μ is multiplied by 4

$f = \frac{1}{\lambda}\sqrt{\frac{T}{4\mu}} = \frac{1}{2\lambda}\sqrt{\frac{T}{\mu}} = \frac{1}{2} \times 200 = 100$ Hz

100 Hz is the fundamental of the second string.

3rd harmonic: $f_3 = 3 \times 100 = \underline{300 \text{ Hz}}$.

✓
✓

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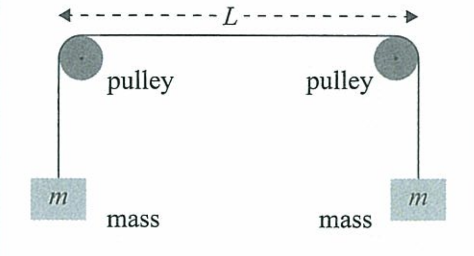
(c) Now the first string is hung so that both ends go over pulleys, with the masses suspended at each end, as shown in Figure 3.

Figure 3



Calculate the frequency of the fifth harmonic.

~~Mass~~ $f = \frac{1}{\lambda}\sqrt{\frac{T}{\mu}} = 200$

λ is the same, as it relies on L.

$T = ma = 2mg$, μ constant

$f = \frac{1}{\lambda}\sqrt{\frac{2T}{\mu}} = \frac{\sqrt{2}}{\lambda}\sqrt{\frac{T}{\mu}} = \sqrt{2} \times 200 = 283$ Hz (3sf)

283 Hz is the fundamental of the string.

5th harmonic: $f_5 = 5 \times 283 = \underline{1410}$ Hz (3sf)

0

0

(d) Two strings made from the same material are both fixed at each end, and both are under the same tension. The first string has a length L_1 (= 1.00 m), and is being driven so that it oscillates in a transverse standing wave mode with a frequency of 400 Hz. The second string, with length L_2 (= 1.18 m), is also oscillating in a standing wave mode, but with a slightly lower frequency. An observer notices that the standing wave on the second string has one more node than that on the first string. The observer hears a 4.5 Hz beat, as a result of the combined sound coming from the two standing waves.

Calculate the number of nodes present in the first standing wave.

v (speed of waves) constant over both strings, given they are same material and same tension

$v = v$, $f_1\lambda_1 = f_2\lambda_2$, $400\lambda_1 = f_2\lambda_2$

~~$\lambda_1 = n(L_1/2) = 0.5n$, $200n = f_2\lambda_2$~~

(pg. 15)

~~$\lambda_2 = (n+1)(L_2/2) = (n+1)(1.18/2) = 0.59(n+1)$~~

~~$f_B = |f_1 - f_2|$, $4.5 = |400 - f_2|$,~~

~~$f_2 = 395.5$ Hz, given $f_2 < f_1$~~

~~$200n = 0.59(n+1) \times 395.5 = 233.345n + 233.345$~~

~~$33.345n = 233.345$, $n = 6.99 = 7$ (0dp)~~

~~First string oscillating at seventh harmonic.~~

~~∴ 8 nodes~~

6

(8)

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER

ASSESSOR'S USE ONLY

1ai is ~~...~~ converted into kinetic energy of the electron. The relationship is therefore $hf = \phi + E_K$, where hf is the energy of the photon, ϕ is the work function of the metal (amount of energy required to release electrons), and E_K is the kinetic energy of the electron.

seen

2c $\frac{mv}{2M} + \frac{v}{2} = v_i$

seen

If $M >> m$ then the first term will be negligible. Hence, assuming that the mass of the bat is much greater than that of the ball (which could be expected), the ~~mass~~ initial speed of the bat is twice that of the ball (approx.)

Assumed mass of bat >> mass of ball

Assumed all of ~~the~~ bat's E_K is linear.

4bii And therefore E_p is zero. In a charged capacitor, ~~the~~ the plates are oppositely charged and therefore attracted to each other, but this is not the case in a discharged capacitor as the plates have no charge.

seen

Therefore no work will be done ~~...~~ as there is no ~~for~~ electromagnetic force ~~on~~ the plates ($W = Fd$), and no energy is given to the circuit. So the plates could be moved halfway through the cycle, when the capacitor is fully discharged, to avoid providing any energy to the circuit.

Extra space if required.
Write the question number(s) if applicable.

QUESTION NUMBER		ASSESSOR'S USE ONLY
~~2d~~	~~Angular momentum is generally conserved by the~~	

seen

5d

$\lambda_1 = 2L_1/n = 2\times1.00/n = 2/n$

$\lambda_2 = 2L_2/(n+2) = 2\times1.28/(n+1) = 2.36/(n+1)$

$f_B = |f_1 - f_2|,\ 4.5 = |400 - f_2|$

$f_2 = 395.5\ Hz$, given $f_2 < f_1$

$f_1\lambda_1 = f_2\lambda_2,\ 400\times 2/n = 395.5\times 2.36/(n+1)$

$800(n+1) = 933.38n$

seen

$800n + 800 = 933.38n,\ 133.38n = 800$

$n = 800/133.38 = 5.997 = 6\ (0dp)$

First string oscillating at 6th harmonic

 $\therefore$ 7 nodes

3cii This outweighs the loss in speed caused by the collision of the ball and the ground, thus the average horizontal speed is greater in part c i than part b, and as time $= d/v$, it will cover the same 80 m distance in a shorter time. seen