NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

Scholarship, 2004

Statistics and Modelling

National Statistics

Assessment Report

Assessment Schedule

Statistics and Modelling, Scholarship, 2004

General Comments

Overall 37.1 % of the candidates achieved scholarship standard. Many scripts were below scholarship standard, largely due to many students being unable to get a sufficient number of questions correct. The initial question, intended as a start-up question, was either done extremely well or poorly. However in many other cases, candidates were able to carry out various calculating procedures and obtain correct answers

At various points in the paper there was the need for students to be able to communicate statistical ideas with clarity and efficiency. In addition, these ideas needed to be described “in context” to a real situation with the practicalities of an answer being clearly articulated. Many students achieved this; however, in five notable instances throughout the paper students had difficulties. These instances occurred when candidates were unable to discuss the practical implications of the answer in Question 2(ii), give realistic sales predictions and appreciate the limitations of sales forecasts in 3, locate both points where the profit was maximised in 4(a), deduce the conditional nature of the required probability in 5(a) 3, and discuss the features of a Time Series in context in 6(a) and 6(b).

The emphasis of this paper was on candidates being prepared to tackle a “thinking” type of paper involving interpretations and drawing conclusions when doing statistics, as opposed to a “calculating” type of paper.

Statistics and Modelling Scholarship (93201)

National Statistics

Number of	Percentage achieved		
Results	Not Achieved	Scholarship	Outstanding
868	62.9	35.1	2.0

Assessment Report

Candidates demonstrated an advanced level of statistical thinking by correctly interpreting confidence intervals, identifying and writing about correlation, outlier and scatter-plot sections, and identifying features of a Time Series “in context” to practical problems.

Candidates assessed as Not Achieved in the area of statistical thinking were not able to give enough structure for Question 3, with a lot of repetition and far too much waffle. Some wrote too much with 3 pages in some instances and in many of these cases undid their good work. The data in Question 3 were interpreted as a time series. Unrealistic sales predictions were calculated, and students were unable to recall and/or apply the basic features of a Time Series in Q6 (b).

Candidates demonstrated that they were able to apply probability theory and models to solve complex problems by constructing confidence intervals and calculating probabilities, and finding sample sizes for a given margin of error.

Candidates assessed as Not Achieved in the areas of probability modelling were not able to: use the correct $p = 0.17$ in Q1 (b), multiply the interval width in Q1 (d) by the correct value for a reduction of 40%, use the correct p value in Q1 (d), and consider both outcomes in the calculation of the probability in 5(a) 1.

Candidates demonstrated that they were able to apply techniques of mathematical modelling to solve complex problems by correctly formulating and solving 3×3 sets of linear equations, finding all feasible points by looking for other possible solutions that maximise profit in Q4 (a), and correctly calculating forecasts with a justifiable method in Q6 (c).

Candidates assessed as Not Achieved in the areas of mathematical modelling were unable to find a general solution in terms of *c* for both *a* and *b* in Q2(ii), deduce both solution points in 4(a), and find a forecast in 6(c) taking into account seasonal effects over both months and days.

Candidates gaining Scholarship with Outstanding Performance showed additional skills and knowledge. They were able to demonstrate sophistication of thinking across a range of concepts, display logical development and clarity in their solutions, and critically evaluate processes and solutions. Candidates achieved Outstanding Performance particularly by deducing a practical implication for the solution to the simultaneous equations in Q2, discussing the validity of predictions using a given Regression line, deducing and expressing multiple solutions in an LP problem. Also, they were able to identify and deduce conditional probabilities, discuss the validity of forecasts when using a Time Series, and work backwards when given a probability, in order to find a sample size.

Other candidates who failed to perform at an outstanding level were unable to deduce a practical implication for the solution to the simultaneous equations and, in most cases, just gave a geometrical interpretation. Also, they had difficulty in; discussing the validity of predictions, describing clearly the eight points as a feasible solution in Q4 (b) and obtaining an implicit equation in 4(c). In addition, they needed to recognise a conditional probability solution in Q5 (a) 3, proceed correctly in 5(b) and come up with limitations in 6(d). In many cases candidates just gave the same limitation expressed in two different ways.

Scholarship Statistics and Modelling 2004 — Assessment Schedule 93201

QUESTION ONE

Task: **Q1 (a)**

Criteria: **S2**

Evidence:
$n = 200$, $p = 46/200 = 0.23$, $z = 1.96$ (95% confidence)

Interval is given by: $0.23 - 1.96\sigma < \pi < 0.23 + 1.96\sigma$

Where $\sigma = \sqrt{\dfrac{0.23 \times (1 - 0.23)}{200}} = 0.0298$

So $0.17 < \pi < 0.29$

TI 83 calculator gives (0.17168,0.28832)

Judgement:
Confidence Interval correctly calculated – minor arithmetical error tolerated.

Task: **Q1 (b)**

Criteria: **S2**

Evidence:
Pessimistic estimate from interval is 0.172.

Std Error $= \sqrt{\dfrac{0.172 \times (1 - 0.172)}{200}} = 0.027$

We require pr (proportion of support > 0.25) $= \text{pr}\left(z > \dfrac{0.250 - 0.172}{0.027}\right)$

$$= \text{pr}(z > 2.889)$$
$$= 0.5 - 0.4981$$
$$= 0.0019$$

The probability that the level of support is at least 25% is 0.2%.

Note: $\pi = 0.1717$ gives standard error = 0.02666 so probability = 0.0017
Exact answer is 0.0022303076
Pr($x > 49.5$) with continuity correction

Judgement:
Probability correctly calculated – minor arithmetical error tolerated.

Task: **Q1 (c)**

Criteria: **S1**

Evidence: Claim cannot be justified as 30% lies outside the upper limit of the confidence interval (0.3 > 0.29).

OR calculate P (p > 0.3) = 0.0094. This is very small and therefore unlikely to occur in a random sample.

Judgement:
Reason correctly stated as to why claim cannot be justified.

Task: **Q1 (d)**

Criteria: **S2**

Evidence:
Set prior estimate of the proportion to be 0.23 (point estimate from first survey).
Set new error = 60% of 0.058 = 0.0348

Set $0.0348 = 196 \times \sqrt{\dfrac{0.23 \times (1 - 0.23)}{n}}$

So $n = \left(\dfrac{1.96}{0.0348}\right)^2 \times 0.23 \times 0.77 = 562$

Could use another prior estimate of p = 0.29 (n = 653) with justification which gives the maximum possible sample size ***OR*** use p = 0.17 (n = 448).
Note: p = 0.5 not acceptable.

Judgement:
Sample size n correctly calculated – minor error like one out tolerated. Variation in rounding 0.0348 tolerated in answer.

QUESTION TWO

Task: **Q2 (i)**

Criteria: **S3**

Evidence:

$\frac{a}{3}+\frac{6b}{15}+\frac{7c}{18}=11.5$

Multiply through by 90 to get: $30a + 36b + 35c = 1035$

Judgement:

Third equation correctly deduced – can have differing multiples of the equation.

Task: **Q2 (ii)**

Criteria: **S3**

Evidence:

Set of equations reduced to two, so we have more than one solution.

Solve for $b = \dfrac{279-5c}{6}$ and $a = \dfrac{-5c-639}{30}$ or $a = -0.17c - 21.30$ and $b = 46.50 - 0.83c$

Judgement:

General solution is correctly found.

Task: **Q2 (iii)**

Criteria: **SOP3**

Evidence:

Note that a, b and c all are greater than 0. The general solution for b implies that $c < 55.8$. Also note that a can never be positive from Q2 (ii) and you cannot have negative quantities.
So you would have wastage of nitrogen, phosphorus or potassium to achieve compost production and/or find shortfall to make up difference by ordering more.

Judgement:

Correctly conclude that solution is unworkable with a suggested action.

QUESTION THREE

Task: **Q3**

Criteria: **S1/SOP 1 (S1 if required) /SOP 3**

Evidence:
The following points could be covered in a one-page summary:

1. Correlation.
2. Interpret at least two distinct sections of plot.
3. Interpreting the coefficients of the regression line.
4. Summary statistics of the values of expenditures and sales. (Could include ratio of sales to expenditure.)
5. Reference to outlier.
6. Sales predictions.
7. Validity of sales predictions.
8. Other factors affecting sales – like seasons, market share.

A complete example of a one-page summary is given below.

Overall there is a moderate positive correlation between expenditure and sales. This is evidenced by an overall correlation coefficient of 0.819. In fact, there are three distinct sections of the graph with weak positive correlation for expenditures below \$15 000 and almost constant sales for expenditures in excess of \$30 000. The value of the correlation improves markedly with the removal of the outlier at (\$23 000, \$210 000). The highest correlation of 0.972 is obtained when we consider the linear scatter between expenditures of \$15 000 and \$30 000. There is a non-linear scatter where the points tail off with expenditures greater than \$30 000. The graphs suggest that there is a saturation point for sales at approximately \$180 000, which means that no matter how much more is spent on promotion, sales will not increase appreciably beyond that value.

Sales prediction for say E = \$24 000 is given by:

$S = 53.2 + 3.67 \times 24 = 141.28$, so predicted sales = \$141 000.

This prediction would be valid as the E value lies within the linear section of the scatter that has the highest correlation coefficient.

If we obtain a sales prediction for E = \$45 000, we get:
\$176 000 using the scatter. This occurs in the flat part of the graph where market saturation could have occurred and may be subjected to a greater error.

For the equation: $S = 53.2 + 3.67\,E$, for every \$1 000 increase in expenditure, the model suggests sales increase by \$3 670 on average. It also suggests that the base line sales with no advertising are approximately \$53 000.

Summary statistics are as follows:

	Expenditure ($000)	**Sales** ($000)
Mean	26.4	144.9
Median	24.0	149.5
Std Deviation	17.5	32.2
Range	0–60	98–210

Judgement:

S1: 1, 2, 3, 4, 5 – 3 out of 5

SOP 1: 6 (At least two predictions with no more than one graphically)

SOP 3: 7, 8 – 1 out of 2.

QUESTION FOUR

Task: **Q4 (a)**

Criteria: **S3 – both answer pairs required**

Evidence:
Let x = number of bottles of POW and y = number of bottles of ZAP.
Deduce constraints as follows:

1. $30x + 20y \le 25 \times 60$ which simplifies to $3x + 2y \le 150$
2. $x \le 35$ and $x + y \le 65$
3. $x \ge 15$ and $y \ge 15$

Profit function is $P = 20x + 10y$

So, sketch the feasible region with corner points as follows:

Going clockwise: (15,15), (35,15), (35,22.5), (20,45) and (15,50):

(a) P is maximised at (35,22) and (34,24); these are either side of (35,22.5), which you cannot have due to x and y needing to be integers.
Thus the recommendation is that either 34 bottles of POW and 24 bottles of ZAP or 35 bottles of POW and 22 bottles of ZAP will maximise profit.

Judgement:
1. Correct answers obtained with an acceptable method. Profit not required.
2. Both correct combinations specified.

Task: Q4 (b)

Criteria: SOP 1 (S3 if required) – every possible combination pair identified

Evidence:

P becomes $P = 15x + 10y$
Recommendation becomes any of the production combinations:
(20,45), (22,42), (24,39), (26,36), (28,33), (30,30), (32,27) and (34,24).

Judgement:
1. Correct answers obtained with an acceptable method. Profit not required.
2. All correct (implied integer) combinations specified in any appropriate format including line description.

Task: **Q4 (c)**

Criteria: **SOP1 (S3 if required)**

Evidence:

(c) The graph $y = 9\ln(x)$ intersects the line $3x + 2y = 150$ on the edge of the feasible region.
So: $x + 6\ln(x) - 50 = 0$

Judgement:

Equation correctly deduced. Can have equivalent equations.

Task Q4 (d)

Criteria: S3

Evidence:

(d) To solve this equation, several methods can be employed. Some students have graphical calculators. Get 29 bottles of POW and 31 bottles of ZAP.

Eg, graphical technique by plotting $y = 9\ln(x)$ on graph constructed previously plus trial by error involving substitution of values.

Judgement:
Correct answers obtained. Allow (29,31), (29,30) and (30,30) as solutions. If wrong equations in (c) no credit if used to get answer. Un-rounded answers not accepted.

QUESTION FIVE

Task: **Q5 (a)**

Criteria: **1. and 2. S2 in each / 3. SOP2 (S2 if required)**

Evidence:

1. Binomial with $n = 20$ and $\pi = 0.02$.
   So probability of truckload being accepted is $0.98^{20} + 20 \times 0.02 \times 0.98^{19} = 0.940$
   Probability of one truckload being accepted = $2 \times 0.94 \times 0.06 = 0.113$

   OR
   Use Poisson approximation to the Binomial with $\lambda = n\pi$.
   This gives probability of truckload as being accepted as 0.9384 and probability of one truckload being accepted as 0.116.

2. Target acceptance rate is ≥ 0.96 so use Poisson approximation with $\lambda = n\pi$.
   From tables, closest λ value to $\text{pr}(x = 1) + \text{pr}(x = 0)$ being closest to 0.96 is 0.3.
   So $\pi = 0.3/20 = 0.015$, ie 1.5%.

3. Probability of no defectives = $0.98^{20} = 0.668$
   Probability of acceptance on second check = $0.06 \times 0.668 = 0.040$.
   So proportion of accepted truckloads that would be accepted on the second
   $$\text{sample} = \frac{0.040}{0.94 + 0.04} = 0.041,$$
   ie P (truckload accepted from second sample/accepted) = 0.041.

Judgement:

1. Method correctly applied. Minor variation in answer accepted.

2. Method correctly applied. Minor variation in answer accepted like 1.6% or 0.015 (or equivalent).

3. Method correctly applied. Minor variation in answer accepted. Must show conditional aspect in answer.

Task: **Q5 (b)**

Criteria: **SOP2 (S2 if required)**

Evidence:

Use Poisson approximation and look for closest $\text{pr}(x = 0) + \text{pr}(x = 1)$ to 0.91.
Observe that $\lambda = 0.5$.

$$\text{So } \lambda = n\,\pi = n \times \frac{1}{150} = 0.5, \text{ so } n = 75.$$

So a maximum of **75 punnets** should be sampled by NAILS.

Judgement:

Correct answer obtained. $n = 74$ or $n = 77$ is acceptable. Variations in method accepted but must be expressed as integers.

QUESTION SIX

Task: **Q6 (a)**

Criteria: **S1**

Evidence:

1.
   - The long-term trend, which shows a gradual increase in sales over time.
   - The seasonal fluctuations in the monthly retail sales, which show a high in December and lows in July/August.
   - The daily fluctuations over a week, like peaks in the weekend and troughs mid-week.

2.
   - Calculate a seasonal effect for each month and each day using the appropriate data.
   - Obtain a trend forecast for a particular quarter, then for the month under consideration.
   - 'Add' the seasonal to get a forecast for that month.
   - Divide to get an average daily forecast for the days in that month.
   - Adjust this forecast depending on the day of the week, eg if Saturday is 30% above average we multiply by 1.3
   - To get a daily sales target we then multiply by 1.05

Judgement:

An implied comment about **Trend** plus mention all of the following at least once: **seasonal**, **forecast**, **adjust for day** and **inflate by 5%**.

Task: **Q6 (b)**

Criteria: **S1**

Evidence:

1. The high sales for Labour Day have the effect of increasing 7 values of the centred moving mean either side of the Monday, ie Friday before to Thursday after. This irregular variation in the time series would provide an inflated trend in this section of the time series for daily sales.

2. To calculate a seasonal effect for Mondays, we could ignore the Labour Day sales figure and work on the other Monday sales. A trend value would be obtained for the Monday under consideration, and then the seasonal effect is 'added' into the trend to make a forecast. A special adjustment could be made for Labour Day based on previous data, ie if sales are generally 30% above the daily average for Labour Day, then multiply by 1.3

Judgement:

1. Key point of increasing the moving average for a week of values must be mentioned, **AND**
2. mentions using other Mondays, then adjust specially for Labour Day. Accept an answer that doesn't specifically identify a Labour Day forecast.

Task: Q6 (c)

Criteria: S3

Evidence (either solution accepted):

Trend Value for December 2004 = 225.21 + 0.91 × 42 = 263.43

Trend Value for each day in December 2004 = $\dfrac{263.43}{29}$ = 9.084

Seasonal effect for Tuesdays = –2.6 (–1.9 accepted as alternative)

Forecast for 7th December 2004 = 9.084 – 2.6 = 6.484, ie 6.5 to 1 d.p., ie $6 500.

OR

An adjustment for the month of December is included, so 129.95 is added to the trend value before division into 31 days. This gives 12.69 leading to a December 7th forecast of $10 100.

Judgement:

Correct method in either case, with minor variation in answer tolerated. Accept rounding but not missing $000. Forecasts change if 30 days: $6 200, or if 31 days: $5 900. If number of months within 2 of 42, accept resulting calculations if correct. Daily trend line approach not accepted, however multiplicative approach to “adding” seasonal accepted.

Task: Q6 (d)

Criteria: SOP3

Evidence:

1. No account is taken of the increasing trend over a particular month. It would tend to be lower than average at the start and higher at the end.
2. The gradual upwards trend is assumed to continue into late 2004.
3. The calculated seasonal effects are approximate and have been calculated over a small number of Tuesdays and/or Decembers.
4. No sales data are used between 1st May and present day, so recent sales have no effect on calculated forecast.
5. The calculated seasonal effect is unreliable due to the Labour Day increased sales.

Judgement:

A minimum of two of these limitations required in answer. Statements, where relevant, must be consistent with the answer in part (c).

Scholarship Criteria	Minimum Requirements
S1 Statistics	1
S2 Probability	1
S3 Modelling	1
SOP 1 Sophisticated Thinking	1
SOP 2 Mathematical Expression	1
SOP 3 Critically Evaluate	1

Overall requirements for Scholarship standard including conversions when applicable:

At least <u>seven</u> S's overall with a minimum of <u>one S</u> in each category.

Overall requirements for Scholarship Outstanding standard:

Achieved S overall AND at least <u>five</u> SOP's overall with a minimum of <u>one SOP</u> in each category.

Note: Conversion of SOP's towards S's cannot be counted twice ie towards both S and SOP overall.