NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

# Scholarship, 2005

# Calculus

## 93202

## National Statistics

## Assessment Report

**Calculus, Scholarship, 2005**
**93202**

## National Statistics

| No. Scholarship Results | Results | | | |
|---|---|---|---|---|
| | **Outstanding Scholarship** | | **Scholarship** | |
| | No. Awards | % of L3 Cohort | No. Awards | % of L3 Cohort |
| 211 | 25 | 0.3% | 186 | 2.4% |

## Commentary

This examination proved challenging, and, unexpectedly, slightly harder than the 2004 paper.

Question 1 looked familiar and, therefore, most candidates attempted it. Some had difficulty with this question, as complex numbers were not part of their knowledge. The understanding that real numbers comprise a subset of complex numbers and, hence, that all real numbers are complex of the form $a + \mathrm{i}0$, was not understood by many candidates.

In Question 1(a) many candidates used DeMoivre's theorem rather than factorisation, and ended up resorting to decimal solutions that caused rounding errors and approximations. Many checked the given identity for each value of $\alpha$ rather than used a general approach. Candidates do not seem to understand that, when asked for an equation, an 'equals' sign must be part of the answer. Although this counted as a minor error, credit was lost. The meaning of the word 'hence' was not fully understood here - and elsewhere in the examination - and candidates were penalised accordingly.

In Question 1(b)(i) the Argand diagram was sketched well by the majority of candidates. The more able recognised the symmetry of the pentagon formed by these points and could give the answer to Question 2(b) with little further working. Those who chose to find the coordinates of each point in decimal form then tried to make the connection to get an expression in terms of $b$, which was time-consuming, and often produced a very complicated and often unacceptable expression.

Question 1(b)(ii) was understood and done well by a number of candidates. Others included 'i's in their calculation of distances. Confusion arose when some candidates thought to simplify the problem by incorrectly using the square of each distance before differentiating, failing to appreciate that the minimum of (AW+FW) is not generally the same as the minimum of ($AW^2 + FW^2$). A number of candidates used sound geometric reasoning to solve the problem.

Question 2 was attempted by most but, when taking the square root **of** a number, **many** did not allow for both the plus and the minus solution, or did not select the appropriate one to fit the problem. Candidates often seemed uncomfortable dealing with transformations in the plane. An unfamiliar situation initially seemed familiar, but created problems. The formula for the required area in Question 2(b) was not always found correctly.

Many struggled with the integration process in Question 2(b) but, with the answer given, some candidates managed the involved manipulation of the exponentials required to get the solution. There were one or two correct solutions integrating with respect to $y$.

A number of candidates did not see the similar structure of $x^2 + y^2 = r^2$ and $x^2 + 16(y - r)^2 = r^2$ so chose to use substitution, subsequently getting into problems with $x^2 + 16\left(\sqrt{(r^2 - x^2)} - r\right)^2 = r^2$.

Question 3(a) was done surprisingly well. In part (a)(i) some candidates did not know what to do with the % sign attached to the *p*. They either ignored it, left it there and carried it through their solution, or wrote it inappropriately as 0.0*p*.

With the answer given in part (a)(ii), candidates often managed to manipulate the exponentials to get the required solution. Those who recognised the difference of two squares in $\frac{e^{-2ma} - 1}{e^{-ma} - 1}$ and used it to simplify, were most likely to succeed. At this level it was disappointing to see how many candidates wrote versions of $\int_0^a \frac{k}{e^{my-mk}} dy = \left[\frac{k}{me^{my-mk}} \ln\left|e^{my-mk}\right|\right]_0^a$.

In Question 3(b) the first part was correctly done by most, but hardly any continued with the rest of the question. While this was unfamiliar to candidates there was a similarly styled question in 2004.

Question 4(a)(i) was, for the majority of the candidates, very difficult to do correctly. Many forgot that a function of *t* cannot be differentiated with respect to *x* without using the chain rule and, thus, they required a $\frac{dt}{dx}$. This error also occurred in part (a)(ii), but did not affect the final answer. Candidates wrote many different combinations of derivatives. However, some excellent fully correct solutions were presented. The major mistake within this question was that candidates did not understand that generally $\frac{d^2y}{dx^2} \neq \frac{d^2y}{dt^2}\frac{dt^2}{dx^2}$ or thought that $\frac{d^2y}{dt^2} = \frac{1}{\frac{dt^2}{dy^2}}$. The use of partial derivatives by some candidates produced errors.

Question 4(b) was not often recognised as a differential equation of the form $\frac{dz}{dx} = kz^2$. This whole question required understanding of the first and second derivatives.

Question 5(a) was attempted by nearly all candidates. Although many could not prove the result successfully, they often managed to solve the equation. While the successful use of radian measure in terms of $\pi$ to get exact solutions was well done, and general solutions were accepted, the connection from the equation to the final product was beyond the majority.

Question 5(b) was not attempted by many candidates. The start was done well, particularly by candidates who used partial fraction techniques (although these were NOT required to do the question) to show the equivalent expression for $f(n)$. Very few continued to complete part (b) in which an arithmetic simplification cancelled out to the given answer.

Question 6 clearly looked familiar and all candidates attempted it, sometimes as their first question. Parts (a) and (c) were done well by many, but the “completing of the square’ required in part (a) caused some errors. Once again, the transformations caused difficulty for many. Very,

very few met the challenge of part (d), and those attempting it often wrongly thought that the point $\left(\frac{2a}{3}+2h, h\right)$ lay on the hyperbola.

**The best-performing candidates most commonly demonstrated the following skills and / or knowledge:**

- very high levels of both algebraic manipulation and conceptual thinking, together with excellent problem-solving skills
- ability to produce an extended argument, often combining thinking from more than one domain of mathematics, such as algebra and geometry. They were also able to apply these abilities to unfamiliar contexts or ideas
- insight to find the best solutions to problems, as well as to produce novel approaches and solutions.

**Some candidates demonstrated the following skills and / or knowledge:**

- excellent algebraic skills and ability to manipulate expressions, including those with trigonometric functions, with clear and correct mathematical statements that could be followed
- understanding of the ideas in the questions, and ability to inter-relate concepts and see how to approach solutions, using appropriate techniques rather than relying on learned procedures. They showed the ability to think and understand what they were doing
- ability to make connections between different strands of mathematics, showing they are creative thinkers ie finding ways to solve a problem new to them by applying appropriate problem-solving techniques. For example, they had sound knowledge of geometric concepts and were able to think geometrically as well as algebraically about problems
- sound understanding of the concepts of differentiation and integration and their applications
- accuracy in their work, seldom making more than a minor error
- ability to handle proofs and persist with working to reach a conclusion. While they may have used them, they did not rely on calculators and were able to work without decimals, preferring surds and other exact forms.

**Other candidates commonly lacked the following skills and / or knowledge:**

- familiarity with questions of a scholarship standard. This made them seem poorly prepared for the examination
- ability / insight to know where to start solving a problem at this level. It seems that they need to practise problems *without the answer*, so they can get experience in selecting an appropriate approach
- algebraic skills, often making errors when manipulating or transferring terms, such as squaring a binomial or being able to “complete the square” correctly when the coefficient of the quadratic term was not equal to one. They also seemed uncomfortable using trigonometric, exponential and logarithmic expressions within a calculus problem
- ability to appreciate when an approach was not going to be successful, with the result that they spent much time on unfruitful work, such as expanding $\cos(2A+B)$ in as many different ways as they could
- understanding that ‘hence’ means they must use the information gained in the previous part of the question
- ability to use general forms, preferring to use specific examples, such as the individual solutions to equations, or they wrongly simplified a general problem by substitution of values

- understanding of some basic mathematical concepts, such as that an equation has an 'equals' sign
- ability to use factorisation to simplify expressions as much as possible before using them and so ended up with very complex algebraic forms involving eg $\frac{e^{2mk-2ma}}{e^{ma-mk}}$
- understanding of complex numbers
- ability to write or make use of correct mathematical statements and symbols eg that the *dx* (or similar) needs to be included at the end of an integral sign
- understanding of the relationship between connected notations eg that in Question 4b(ii) you can go from $\frac{dx}{dy}\frac{d^2y}{dx^2} = k\frac{dy}{dx}$ to $\frac{d^2y}{dx^2} = k\left(\frac{dy}{dx}\right)^2$
- geometric thinking eg not recognising the symmetry of a pentagon or **not** making the connection from geometry to coordinate geometry.