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SUPERVISOR'S USE ONLY

SCHOLARSHIP EXEMPLAR

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

QUALIFY FOR THE FUTURE WORLD
KIA NOHO TAKATŪ KI TŌ ĀMUA AO!

Tick this box if there is no writing in this booklet

Scholarship 2020
Calculus

9.30 a.m. Monday 16 November 2020
Time allowed: Three hours
Total score: 40

ANSWER BOOKLET

There are five questions in this examination. Answer ALL FIVE questions.

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

Write ALL your answers in this booklet.

Make sure that you have Formulae and Tables Booklet S–CALCF.

Show ALL working. Start your answer to each question on a new page. Carefully number each question.

Answers developed using a CAS calculator require **ALL commands to be shown**. Correct answers only will not be sufficient.

Check that this booklet has pages 2–27 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Question	Score
ONE	
TWO	
THREE	
FOUR	
FIVE	
TOTAL	/40

ASSESSOR'S USE ONLY

QUESTION NUMBER | ASSESSOR'S USE ONLY

Q1 a) $\lim_{x\to\infty} \frac{3x^2+2x-4}{5x^2+8x-1}$

$= \lim_{x\to\infty} \frac{3+\frac{2}{x}-\frac{4}{x^2}}{5+\frac{8}{x}-\frac{1}{x^2}}$

$= \frac{3}{5}$ //

Q1 b) $\int_0^a \frac{x^3}{\sqrt{a^4+x^4}}\,dx$

let: $u = a^4 + x^4$

$du = 4x^3\,dx$

$dx = \frac{1}{4x^3}\,du$

$\Rightarrow \int_0^a \frac{x^3}{\sqrt{u}} \cdot \frac{1}{4x^3}\,du$

$= \frac{1}{4}\int_0^a \frac{1}{\sqrt{u}}\,du$

$= \frac{1}{4}\int_0^a u^{-\frac{1}{2}}\,du$

$= \frac{1}{4}\left[2u^{\frac{1}{2}}\right]_0^a$

$= \frac{1}{4}\left[2\sqrt{a^4+x^4}\right]_0^a$

$= \frac{1}{4}\left(2\sqrt{a^4+a^4} - 2\sqrt{a^4+0}\right)$

$= \frac{1}{4}\left(2\sqrt{2a^4} - 2\sqrt{a^4}\right)$

$= \frac{1}{4}(2\sqrt{2}-2)a^2$

$= \left(\frac{\sqrt{2}-1}{2}\right)a^2$ //

QUESTION NUMBER | ASSESSOR'S USE ONLY

Q1 c) i) Form equation from roots:

$(x-\alpha)(x-\beta)(x-\gamma)(x-\delta) = 0$

$\Rightarrow (x^2 - \beta x - \alpha x + \alpha\beta)(x-\gamma)(x-\delta) = 0$

$\Rightarrow (x^2 + (-\beta-\alpha)x + \alpha\beta)(x-\gamma)(x-\delta) = 0$

$\Rightarrow (x^3 - \gamma x^2 + (-\beta-\alpha)x^2 - \gamma(-\beta-\alpha)x + \alpha\beta x - \alpha\beta\gamma)(x-\delta) = 0$

$\Rightarrow (x^3 + (-\alpha-\beta-\gamma)x^2 + (\alpha\beta+\alpha\gamma+\beta\gamma)x - \alpha\beta\gamma)(x-\delta) = 0$

$\Rightarrow x^4 - \delta x^3 + (-\alpha-\beta-\gamma)x^3 - \delta(-\alpha-\beta-\gamma)x^2 + (\alpha\beta+\alpha\gamma+\beta\gamma)x^2$

$- \delta(\alpha\beta+\alpha\gamma+\beta\gamma)x - \alpha\beta\gamma x + \alpha\beta\gamma\delta = 0$

$\Rightarrow x^4 + (-\alpha-\beta-\gamma-\delta)x^3 + (\alpha\delta+\beta\delta+\delta\gamma+\alpha\beta+\alpha\gamma+\beta\gamma)x^2$

$+ (-\alpha\beta\delta - \alpha\gamma\delta - \beta\gamma\delta - \alpha\beta\gamma)x + \alpha\beta\gamma\delta = 0$

$a = 1$

$b = -\alpha-\beta-\gamma-\delta$

$c = \alpha\delta + \beta\delta + \delta\gamma + \alpha\beta + \alpha\gamma + \beta\gamma$

$d = -\alpha\beta\delta - \alpha\gamma\delta - \beta\gamma\delta - \alpha\beta\gamma$ $\quad \boxed{e = \alpha\beta\gamma\delta}$

$\therefore -\frac{b}{a} = -\left(\frac{-\alpha-\beta-\gamma-\delta}{1}\right) = \alpha+\beta+\gamma+\delta$

$\therefore \frac{c}{a} = \beta\gamma + \gamma\alpha + \alpha\beta + \alpha\delta + \beta\delta + \gamma\delta$

$\therefore -\frac{d}{a} = -(-\alpha\beta\delta - \alpha\gamma\delta - \beta\gamma\delta - \alpha\beta\gamma) = \beta\gamma\delta + \gamma\alpha\delta + \alpha\beta\delta + \alpha\beta\gamma$

$\therefore \frac{e}{a} = \alpha\beta\gamma\delta$ //

QUESTION NUMBER

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Q1c)(ii) $x^4 - 8x^3 + 19x^2 + px + 2 = 0$

~~[illegible]~~ let roots be: $\alpha, \beta, \gamma, \delta$

$\alpha + \beta = \gamma + \delta$ ①

$\alpha + \beta + \gamma + \delta = -\frac{-8}{1} = 8$ ②

So: $8 - \gamma - \delta = \gamma + \delta$

$\gamma + \delta = 4$ and $\alpha + \beta = 4$

$\beta\gamma\delta + \gamma\alpha\delta + \alpha\beta\delta + \alpha\beta\gamma = -\frac{p}{1} = -p$

$\beta\gamma + \gamma\alpha + \alpha\beta + \alpha\delta + \beta\delta + \gamma\delta = 19$

$\alpha\beta\gamma\delta = 2$

$\alpha(4-\alpha)(4-\gamma)\gamma = 2$

$\gamma(\beta + \alpha + \delta) + \alpha\beta + \alpha\delta + \beta\delta = 19$

$\gamma(8 - \gamma) + \alpha\beta + \alpha\delta + \beta\delta = 19$

$\gamma(\beta\delta + \alpha\delta + \beta\alpha) + \beta\alpha\delta = -p$

~~[illegible]~~ $\gamma(\beta\delta + \alpha\delta + \beta\alpha) =$ ~~[illegible]~~ $-p - \alpha\beta\delta$

$\alpha\beta\delta = \frac{2}{\gamma}$

So $\gamma(\beta\delta + \alpha\delta + \beta\alpha) = -p - \frac{2}{\gamma}$

$\gamma(8-\gamma) + -\frac{p}{\gamma} - \frac{2}{\gamma^2} = 19$

Q1

6

QUESTION NUMBER

ASSESSOR'S USE ONLY

Q2a) $x^4 + \frac{1}{x^4} = 7$

$\left(x^4 + \frac{1}{x^4}\right)^2 = 49$

$x^8 + 2 + \frac{1}{x^8} = 49$

$x^8 + \frac{1}{x^8} = 47$

Q2 b)(i) $f(x) = \frac{\cos x}{2 + \sin x}$

$$f'(x) = \frac{(2+\sin x)\cdot\sin x - \cos x \cdot \cos x}{(2+\sin x)^2}$$

$$= \frac{-2\sin x - \sin^2 x - \cos^2 x}{(2+\sin x)^2}$$

$$= \frac{-2\sin x - 1}{(2+\sin x)^2} \quad \text{as } \sin^2 x + \cos^2 x = 1$$

$f'(x) = 0$ for the turning points

$-2\sin x - 1 = 0$

$2\sin x = -1$

$\sin x = -\frac{1}{2}$

$x_1 = \frac{7\pi}{6}$, $x_2 = \frac{11\pi}{6}$ for solutions in the domain $0 \leq x \leq 2\pi$

$$y_1 = \frac{\cos\frac{7\pi}{6}}{2 + \sin\frac{7\pi}{6}} = \frac{-\frac{\sqrt{3}}{2}}{2 + -0.5} = \frac{-\frac{\sqrt{3}}{2}}{\frac{3}{2}} = -\frac{\sqrt{3}}{3}$$

$$y_2 = \frac{\cos\left(\frac{11\pi}{6}\right)}{2 + \sin\left(\frac{11\pi}{6}\right)} = \frac{\frac{\sqrt{3}}{2}}{2 + -\frac{1}{2}} = \frac{\frac{\sqrt{3}}{2}}{\frac{3}{2}} = \frac{\sqrt{3}}{3}$$

$\therefore$ Turning points at: $\left(\frac{7\pi}{6}, -\frac{\sqrt{3}}{3}\right)$ (a local minima)

and $\left(\frac{11\pi}{6}, \frac{\sqrt{3}}{3}\right)$ (a local maxima)

QUESTION NUMBER

ASSESSOR'S USE ONLY

Q2b(ii) $f''(x) = \frac{d}{dx} \frac{-2\sin x - 1}{(2+\sin x)^2}$

$$= \frac{(2+\sin x)^2 \cdot (-2\cos x) - (-2\sin x - 1)\cdot 2(2+\sin x)\cos x}{(2+\sin x)^4}$$

$$= \frac{(2+\sin x)^2 \cdot -2\cos x + 2\cos x(2\sin x + 1)(2+\sin x)}{(2+\sin x)^4}$$

$f''(x) = 0$ for points of inflection.

So $-2\cos x(4 + 4\sin x + \sin^2 x) + 2\cos x(4\sin x + 2\sin^2 x + 2 + \sin x) = 0$

$-8\cos x - 8\sin x\cos x - 2\sin^2 x\cos x + 10\sin x\cos x + 4\sin^2 x\cos x + 4\cos x = 0$

$2\sin^2 x\cos x - 4\sin x\cos x - 4\cos x = 0$

$2\cos x(\sin^2 x - 2\sin x - 2) = 0$

So $\cos x = 0$ gives $x = 2n\pi \pm \frac{\pi}{2}$ where n is any integer for an inflection point.

~~From graphics calculator, it seems~~

$$f''(x) = \frac{2\cos x(\sin^2 x - 2\sin x - 2)}{(2+\sin x)^4}$$

From $f''(x)$, note that $\sin^2 x - 2\sin x - 2$ is always negative so $f''(x) > 0$ for concave up for when $\cos x > 0$. Similarly, $f''(x) < 0$ for concave down when $\cos x < 0$.

~~This shows $f(x)$ is concave up on $k\frac{\pi}{2} < x < k\frac{3\pi}{2}$~~

This shows $f(x)$ alternates from concave up to concave down on intervals of 2π with the first concave up interval as $\frac{\pi}{2} < x < \frac{3\pi}{2}$, then concave down for $\frac{3\pi}{2} < x < \frac{5\pi}{2}$ and concave up for $\frac{5\pi}{2} < x < \frac{7\pi}{2}$ and so on...

QUESTION NUMBER

ASSESSOR'S USE ONLY

Q2c)(i) Because AC is the diameter of the circle, $\angle ABC = \angle ADC = 90°$.

$\tan\left(\frac{\theta}{2}\right) = \frac{b}{a}$ follows from this

$\frac{a}{b} = \frac{1}{\tan\left(\frac{\theta}{2}\right)}$

$$= \frac{\cos\left(\frac{\theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right)}$$

$$= \frac{\cos^2\left(\frac{\theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}$$

$$= \frac{\frac{1}{2}\cos\theta + \frac{1}{2}}{\frac{1}{2}\sin\left(\frac{\theta}{2}\cdot 2\right)}$$

$$= \frac{\frac{1}{2}(\cos\theta + 1)}{\frac{1}{2}\sin\theta}$$

$$= \frac{\cos\theta + 1}{\sin\theta}$$

$$= \frac{1 + \cos\theta}{\sin\theta}$$ as required

QUESTION NUMBER

2c)(ii) $\tan\left(\frac{\theta}{2}\right) = \frac{b}{a}$

$b = \frac{a}{\sqrt{3}}$

$\frac{b}{a} = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan\left(\frac{\theta}{2}\right) = \frac{1}{\sqrt{3}}$

~~$\frac{\theta}{2} = \frac{\pi}{6}$~~

~~$\theta = \frac{\pi}{3}$ for $60°$~~

~~$\frac{\theta}{2} = \frac{7\pi}{6}$~~

~~$\theta = \frac{7\pi}{3}$~~

$\frac{\theta}{2} = \frac{\pi}{6}$

$\theta = \frac{\pi}{3}$ (or $60°$)

ASSESSOR'S USE ONLY

Q2

7

QUESTION NUMBER

Q3 a) ~~Find $f'(0)$ using x^2+ax+b~~

$f''(0)$ only exists if function is continuous at $x=0$ so $x^2+ax+b = e^x+\sin x$ and the derivatives of both will be equal at $x=0$

$y_1 = x^2+ax+b$ $\quad$ $y_2 = e^x+\sin x$

$y_1' = 2x+a$ $\quad$ $y_2' = e^x+\cos x$

$y_1'' = 2$ $\quad$ $y_2'' = e^x - \sin x$

$y_1 = y_2$ at $x=0$

$0 + a\cdot 0 + b = e^0 + \sin 0$

$b = 1$

$y_1' = y_2'$ at $x=0$

$2\cdot 0 + a = e^0 + \cos 0$

$a = 1+1$

$a = 2$

So $a=2$ and $b=1$

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QUESTION NUMBER

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Q3 b)

$y - y_0 = m(x - x_0)$

$y - y_0 =$

QUESTION NUMBER

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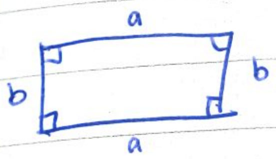
Q3 c)



a is length $\frac{da}{dt} = 2$

b is width $\frac{db}{dt} = 3$

L is diagonal length

$L = \sqrt{a^2 + b^2}$

$\frac{dL}{dt} = \frac{2a\frac{da}{dt} + 2b\frac{db}{dt}}{2\sqrt{a^2+b^2}}$ at $a = 12$, at $b = 9$

$= \frac{2 \cdot 12 \cdot 2 + 2 \cdot 9 \cdot 3}{2\sqrt{12^2 + 9^2}}$

$= \frac{102}{2 \cdot 15}$

$= \frac{102}{30}$

$= \frac{17}{5}$

Length increases at $\frac{17}{5}$ cm ~~length~~ per second.

QUESTION NUMBER

Q3 d) let L be distance from some point (x_0, y_0) to $(1,0)$

$L = \sqrt{(x_0-1)^2 + (y_0)^2}$

$L = \sqrt{(x_0-1)^2 + x_0}$ (x_0, y_0) is a point on $x = y^2$

$= \sqrt{x_0^2 - 2x_0 + 1 + x_0}$

$= \sqrt{x_0^2 - x_0 + 1}$

$L^2 = x_0^2 - x_0 + 1$

$2L \frac{dL}{dx_0} = 2x_0 - 1$

$\frac{dL}{dx_0} = 0$ for minimum distance

$2x_0 - 1 = 0$

$x_0 = \frac{1}{2}$

now $y_0 = \pm\sqrt{\frac{1}{2}}$

So points on $x = y^2$ shortest distance to $(1,0)$ ~~are~~ are $\left(\frac{1}{2}, \frac{\sqrt{2}}{2}\right)$ and $\left(\frac{1}{2}, -\frac{\sqrt{2}}{2}\right)$

$2L \frac{dL}{dx_0} = 2x_0 - 1$

$2 \frac{d^2L}{dx_0^2} = 2$

$\frac{d^2L}{dx_0^2} = 1$

Because $\frac{d^2L}{dx_0^2} = 1 > 0$, via the second derivative test, we conclude the found points indeed have the minimum distance.

ASSESSOR'S USE ONLY

Q3

6

QUESTION NUMBER

Q4a) let $p(x) = f(x) \cdot g(x)$

$p'(x) = \lim_{h \to 0} \frac{p(x+h) - p(x)}{h}$

$= \lim_{h \to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$

$(f(x+h) - f(x))(g(x+h) - g(x))$

$= f(x+h)g(x+h) - g(x)f(x+h) - f(x)g(x+h) + f(x)g(x)$

Prove this ↓

$\frac{d\,f(x)}{dx} \cdot g(x) + f(x) \frac{d\,g(x)}{dx} = \lim_{h \to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$

$LHS = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} g(x) + f(x) \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$

$= \lim_{h \to 0} \frac{g(x)f(x+h) - f(x)g(x) + f(x)g(x+h) - f(x)g(x)}{h}$

$= \lim_{h \to 0} \frac{f(x+h)g(x) + f(x)g(x+h) - 2f(x)g(x)}{h}$

$= \lim_{h \to 0} \frac{f(x+h)g(x) - f(x)g(x) + g(x+h)f(x) - g(x)f(x)}{h}$

~~$p'(x) = \lim_{h \to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$~~

~~$\lim_{h \to 0} \frac{(f(x+h) - f(x))g(x)}{h} + \lim_{h \to 0}$~~

Continued page 21 →

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QUESTION NUMBER

Q4b(i)

$$\int e^{-x}\cos x\,dx = e^{-x}\sin x - \int -e^{-x}\sin x\,dx$$
$$= e^{-x}\sin x + \int e^{-x}\sin x\,dx$$
$$\cancel{= e^{-x}\sin x + e^{-x}\sin x - \int -e^{-x}\cos x\,dx}$$
$$= e^{-x}\sin x + e^{-x}\cdot -\cos x - \int -e^{-x}\cdot -\cos x\,dx$$
$$= e^{-x}\sin x - e^{-x}\cos x - \int e^{-x}\cos x\,dx$$
$$\Rightarrow 2\int e^{-x}\cos x\,dx = e^{-x}\sin x - e^{-x}\cos x$$
$$\int e^{-x}\cos x\,dx = \tfrac{1}{2}e^{-x}(\sin x - \cos x)$$

mei

mei

$$\frac{d}{dx}\tfrac{1}{2}(e^{-x}\sin x - e^{-x}\cos x)$$
$$= \tfrac{1}{2}(e^{-x}\cos x - e^{-x}\sin x - e^{-x}\cos x + e^{-x}\sin x)$$
$$= \tfrac{1}{2}(2e^{-x}\cos x)$$
$$= e^{-x}\cos x$$

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QUESTION NUMBER

Q4b(ii)

$$\text{Area} = \int_0^{2\pi} e^{-x}\cos(x)\,dx$$
$$= \left[\tfrac{1}{2}e^{-x}(\sin x - \cos x)\right]_0^{2\pi}$$
$$= \tfrac{1}{2}e^{-2\pi}(0-1) - \tfrac{1}{2}e^{0}(0-1)$$
$$\cancel{= \tfrac{1}{2}\cdot -1 - \tfrac{1}{2}\cdot -1}$$
$$\cancel{= -\tfrac{1}{2} + \tfrac{1}{2}}$$
$$= -\tfrac{1}{2}e^{-2\pi} - \tfrac{1}{2}\cdot -1$$
$$= -\tfrac{1}{2}e^{-2\pi} + \tfrac{1}{2}$$
$$= \frac{1-e^{-2\pi}}{2} \text{ units}^2$$

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QUESTION NUMBER

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Q4 c)

$xy + e^y = 2x + 1$

$y + x\frac{dy}{dx} + e^y\frac{dy}{dx} = 2$

$\frac{dy}{dx}(x + e^y) = 2 - y$

$\frac{dy}{dx} = \frac{2-y}{x+e^y}$

$\frac{2-0}{0+1} = 2$

$\frac{d^2y}{dx^2} = \frac{(x+e^y)\cdot -\frac{dy}{dx} - (1+e^y\frac{dy}{dx})(2-y)}{(x+e^y)^2}$

$\left.\frac{dy}{dx}\right|_{x=0} = \frac{2-y}{e^y} = 2$ for y at $x=0$

~~$\left.\frac{d^2y}{dx^2}\right|_{x=0} = \frac{e^y \cdot -\frac{2-y}{e^y} - (1+e^y\frac{2-y}{e^y})(2-y)}{e^{2y}}$~~

~~$= -2+y-(2-y+2(2-y)-y(2-y))$~~

~~at $x=0$, $0\cdot y + e^y = 2e^{2y}$~~

at $x=0$, $0\cdot y + e^y = 2\cdot 0 + 1$

$e^y = 1$

$y = 0$

$\left.\frac{d^2y}{dx^2}\right|_{x=0} = \frac{(0+e^0)\cdot -2 - (1+1\cdot 2)(2-0)}{(0+e^0)^2}$

$= \frac{1\cdot -2 - (+3)(2)}{1}$

$\frac{-2-6}{1} = -8$

~~$= 0$~~

Q4

7

QUESTION NUMBER

ASSESSOR'S USE ONLY

Q5a)(i) $\cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2}$

$RHS = \frac{\cos\theta + i\sin\theta + \cos(-\theta) + i\sin(-\theta)}{2}$

$= \frac{\cos\theta + i\sin\theta + \cos\theta - i\sin\theta}{2}$

$= \frac{2\cos\theta}{2}$

$= \cos\theta = RHS$

Note: $\cos(-\theta) = \cos\theta$ since $\cos\theta$ is even function

$\sin(-\theta) = -\sin\theta$ since $\sin\theta$ is odd function

Q5a)(ii) $\sin^3\theta = \frac{3}{4}\sin\theta - \frac{1}{4}\sin(3\theta)$

$e^{i3\theta} = \cos(3\theta) + i\sin(3\theta)$

$e^{i3\theta} = (\cos\theta + i\sin\theta)^3$

$= \cos^3\theta + 3i\cos^2\theta\sin\theta - 3\cos\theta\sin^2\theta - i\sin^3\theta$

so $\sin(3\theta) = 3\cos^2\theta\sin\theta - \sin^3\theta$ by equating imaginary

$\sin(3\theta) = 3(1-\sin^2\theta)\sin\theta - \sin^3\theta$

$\sin(3\theta) = 3\sin\theta - 3\sin^3\theta - \sin^3\theta$

$\sin(3\theta) = 3\sin\theta - 4\sin^3\theta$

$4\sin^3\theta = 3\sin\theta - \sin 3\theta$

$\sin^3\theta = \frac{3}{4}\sin\theta - \frac{1}{4}\sin 3\theta$ as required

QUESTION NUMBER

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Q5 b) $\int \cos x \cdot e^x \, dx$ $\qquad \cos x = e^{ix} - i\sin x$

$= \int (e^{ix} - i\sin x)\, e^x \, dx$

$= \int e^x e^{ix} - ie^x \sin x \, dx$

$= \int e^{ix} e^x - ie^x \sin x \, dx$

$\int \cos x \cdot e^x \, dx = \left(\int e^{ix} e^x \, dx\right) - i\left(e^x \sin x - \int e^x \cos x \, dx\right)$

$\int \cos x \, e^x \, dx = \left(\int e^{ix} e^x \, dx\right) - ie^x \sin x + i\int e^x \cos x \, dx$

$\int \cos x \, e^x \, dx \,(1-i) = \left(\int e^{ix} e^x \, dx\right) - ie^x \sin x$

$\int \cos x \, e^x \, dx = \dfrac{\frac{1}{i+1} e^{(i+1)x} - ie^x \sin x}{1-i}$

$= \dfrac{\frac{e^{(i+1)x} - ie^x \sin x \,(i+1)}{i+1}}{1-i}$

$= \dfrac{e^{(i+1)x} + e^x \sin x - ie^x \sin x}{(i+1)(1-i)}$

$= \dfrac{e^{(i+1)x} + e^x \sin x - ie^x \sin x}{2}$

QUESTION NUMBER

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Q5 c) $z = x + iy$

$|z| = 1$ so $\sqrt{x^2 + y^2} = 1 \Rightarrow x^2 + y^2 = 1$

$w = \dfrac{i+z}{i-z} = \dfrac{x+iy+i}{-x-iy+i}$

$\Rightarrow \dfrac{x+i(y+1)}{-x-i(y-1)} = -\dfrac{x+i(y+1)}{x+i(y-1)}$

$= \dfrac{-(x+i(y+1))(x-i(y-1))}{(x+i(y-1))(x-i(y-1))}$

$= -\dfrac{(x^2 - ix(y-1) + ix(y+1) + (y+1)(y-1))}{x^2 + (y-1)^2}$

$= -\dfrac{(x^2 - ixy + ix + ixy + ix + y^2 - 1)}{x^2 + (y-1)^2}$

$= \dfrac{-(x^2 + 2ix + y^2 - 1)}{x^2 + (y-1)^2}$

$= \dfrac{-(2ix)}{x^2 + y^2 - 2y + 1}$

$= \dfrac{-2ix}{2-2y}$

$= \dfrac{-ix}{1-y}$

$= \dfrac{ix}{y+1}$ #

Purely imaginary NS

QUESTION NUMBER

(Q5d)

$x^2 - yz = 1$ $\quad x^2$

$y^2 - zx = 2$

$z^2 - xy = 3$

$x^2 + y^2 + z^2 - xy - xz - yz = 6$

$y^2 - zx = 2x^2 - 2yz$

$x^2 - yz = 1$

$x^4 - 2x^2yz + y^2z^2 = 1$

$x^4 - 2x^2(x^2-1) + (2+zx)(3+xy) = 1$

$x^4 - 2x^4 + 2x^2 + 6 + 2xy + 3xz + x^2yz = 1$

$-x^4 + 2x^2 + x^2(x^2-1) = -5 - 2xy - 3xz$

$-x^4 + 2x^2 + x^4 - x^2 = -5 - 2xy - 3xz$

$x^2 + 2xy + 3xz + 5 = 0$

$y^4 - 2y^2xz + z^2x^2 = 4$

$z^4 - 2z^2xy + x^2y^2 = 9$

$x^4 - 2x^2(x^2-1) + (x^2-1)^2 = 1$

$x^4 - 2x^4 + 2x^2 + x^4 - 2x^2 + 1 = 1$

ASSESSOR'S USE ONLY

Q5

6

QUESTION NUMBER

(Q4a) $\frac{d}{dx}(f(x)\cdot g(x)) = \lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$

$= \lim_{h\to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h} \cdot \frac{g(x+h)\,f(x+h)}{g(x+h)\,f(x+h)}$

$= \lim_{h\to 0} \frac{f^2(x+h)g^2(x+h) - f(x)g(x)\,g(x+h)\,f(x+h)}{h\,g(x+h)\,f(x+h)}$

$\frac{d}{dx}f(x)\,(g(x)) + f(x)\cdot\frac{dg(x)}{dx}$

$= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}g(x) + \lim_{h\to 0}\frac{g(x+h) - g(x)}{h}f(x)$

$= \lim_{h\to 0} \frac{f(x+h)g(x) + g(x+h)f(x) - 2f(x)g(x)}{h}$

$\cancel{= \lim_{h\to 0} \frac{f(x+h)g(x) + g(x+h)f(x) - 2f(x)g(x) + f(x)+g(x) - f(x)g(x)}{h}}$

$= \lim_{h\to 0} \frac{f(x+h)g(x+h)g(x) + (g(x+h))^2 f(x) - 2f(x)g(x)g(x+h)}{h\;g(x+h)}$

$= \lim_{h\to 0} \frac{f(x+h)g(x+h)}{h}\cdot\lim_{h\to 0}\frac{g(x)}{g(x+h)} - \lim_{h\to 0}\frac{(g(x+h))^2 f(x) - 2f(x)g(x)g(x+h)}{h\;g(x+h)}$

$= \lim_{h\to 0} \frac{f(x+h)g(x+h)}{h} - \lim_{h\to 0}\frac{g(x+h)f(x) - 2f(x)g(x)}{h}$

$= \lim_{h\to 0}$

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Q4a) $= \lim_{h \to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$

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Question	Mark	Annotation
1	**6**	The candidate successfully integrated the function by selecting a suitable substitution in **1b**. Although the limits were left unchanged, it was treated as a minor error as they managed to obtain the correct answer by changing the integral back using original variable. The candidate also showed understanding of strategies in finding limit of a rational function in **1a**.
2	**7**	The candidate accurately used 'quotient rule' while differentiating the function involving trigs in **2bi** and subsequently solved the simple trig equation in the given domain, and gave exact x and y coordinates of the turning points as required. The candidate also successfully connected the ratio of *b* over *a* with the trig (cotangent) of the half angle in **2ci**, and applied double angle formulae of 'sine' and 'cosine' to complete the proof.
3	**6**	The candidate could apply related rates of change rule correctly and concisely, therefore simplified the working process effectively in **3c.** The candidate constructed a correct function in **3d,** and successfully used differentiation to optimise it. Unfortunately, they made an error in finding the second derivative implicitly.
4	**7**	The candidate managed to correctly finding the second derivative by using implicit differentiation in **4c.** They could apply 'integrate by parts' rule consistently in **4bi**, and recognise the need for the rearrangement of the integral. Unfortunately, they failed to acknowledge the connection of the 'area under the curve and the integral' in **4bii**.
5	**6**	The candidate worked with clear setting out and explained the steps taken in **5aii.** The candidate correctly rationalised the complex number using conjugate, then successfully simplified it to an imaginary expression in **5b.** They attempted **5d** with little success.