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Scholarship 2018
Calculus

9.30 a.m. Friday 9 November 2018

FORMULAE AND TABLES BOOKLET

Refer to this booklet to answer the questions for Scholarship Calculus 93202Q.

Check that this booklet has pages 2−4 in the correct order and that none of these pages is blank.

YOU MAY KEEP THIS BOOKLET AT THE END OF THE EXAMINATION.

CALCULUS – USEFUL FORMULAE

ALGEBRA

Quadratics

If $ax^2 + bx + c = 0$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Logarithms

$y = \log_b x \Leftrightarrow x = b^y$

$\log_b(xy) = \log_b x + \log_b y$

$\log_b\left(\dfrac{x}{y}\right) = \log_b x - \log_b y$

$\log_b(x^n) = n\log_b x$

$\log_b x = \dfrac{\log_a x}{\log_a b}$

Binomial Theorem

$(a+b)^n = \begin{pmatrix} n \\ 0 \end{pmatrix} a^n + \begin{pmatrix} n \\ 1 \end{pmatrix} a^{n-1}b^1 + \begin{pmatrix} n \\ 2 \end{pmatrix} a^{n-2}b^2 + \dots + \begin{pmatrix} n \\ r \end{pmatrix} a^{n-r}b^r + \dots + \begin{pmatrix} n \\ n \end{pmatrix} b^n$

$\begin{pmatrix} n \\ r \end{pmatrix} = {}^nC_r = \dfrac{n!}{(n-r)!\,r!}$

Some values of $\begin{pmatrix} n \\ r \end{pmatrix}$ are given in the table below.

Complex numbers

$z = x + iy$

$= r\operatorname{cis}\theta$

$= r(\cos\theta + i\sin\theta)$

$\bar{z} = x - iy$

$= r\operatorname{cis}(-\theta)$

$= r(\cos\theta - i\sin\theta)$

$r = |z| = \sqrt{z\bar{z}} = \sqrt{(x^2 + y^2)}$

$\theta = \arg z$

where $\cos\theta = \dfrac{x}{r}$

and $\sin\theta = \dfrac{y}{r}$

De Moivre's Theorem

If n is any integer, then

$(r\operatorname{cis}\theta)^n = r^n \operatorname{cis}(n\theta)$

COORDINATE GEOMETRY

Straight Line

Equation $y - y_1 = m(x - x_1)$

Circle

$(x - a)^2 + (y - b)^2 = r^2$

has a centre (a,b) and radius r

Parabola

$y^2 = 4ax$ or $(at^2, 2at)$

Focus $(a,0)$ Directrix $x = -a$

Ellipse

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ or $(a\cos\theta, b\sin\theta)$

Foci $(c,0)$ $(-c,0)$ where $b^2 = a^2 - c^2$

Eccentricity: $e = \dfrac{c}{a}$

Hyperbola

$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ or $(a\sec\theta, b\tan\theta)$

asymptotes $y = \pm\dfrac{b}{a}x$

Foci $(c,0)$ $(-c,0)$ where $b^2 = c^2 - a^2$

Eccentricity: $e = \dfrac{c}{a}$

n \ r	0	1	2	3	4	5	6	7	8	9	10
0	1										
1	1	1									
2	1	2	1								
3	1	3	3	1							
4	1	4	6	4	1						
5	1	5	10	10	5	1					
6	1	6	15	20	15	6	1				
7	1	7	21	35	35	21	7	1			
8	1	8	28	56	70	56	28	8	1		
9	1	9	36	84	126	126	84	36	9	1	
10	1	10	45	120	210	252	210	120	45	10	1
11	1	11	55	165	330	462	462	330	165	55	11
12	1	12	66	220	495	792	924	792	495	220	66

CALCULUS
Differentiation

$y = f(x)$	$\dfrac{dy}{dx} = f'(x)$
$\ln x$	$\dfrac{1}{x}$
e^{ax}	ae^{ax}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\cot x$	$-\operatorname{cosec}^2 x$

Integration

$f(x)$	$\int f(x)dx$
x^n	$\dfrac{x^{n+1}}{n+1} + c$ $(n \neq -1)$
$\dfrac{1}{x}$	$\ln|x| + c$
$\dfrac{f'(x)}{f(x)}$	$\ln|f(x)| + c$

First principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Parametric Function

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$$

$$\frac{d^2 y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \cdot \frac{dt}{dx}$$

Product Rule

$$(f \cdot g)' = f \cdot g' + g \cdot f' \quad \text{or if } y = uv \text{ then } \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

Quotient Rule

$$\left(\frac{f}{g}\right)' = \frac{g \cdot f' - f \cdot g'}{g^2} \quad \text{or if } y = \frac{u}{v} \text{ then } \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Composite Function or Chain Rule

$$\left(f(g)\right)' = f'(g) \cdot g'$$

or if $y = f(u)$ and $u = g(x)$ then $\dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx}$

NUMERICAL METHODS
Trapezium Rule

$$\int_a^b f(x)\ dx \approx \frac{1}{2}h\left[y_0 + y_n + 2(y_1 + y_2 + \ldots + y_{n-1})\right]$$

where $h = \dfrac{b-a}{n}$ and $y_r = f(x_r)$

Simpson's Rule

$$\int_a^b f(x)\ dx \approx \frac{1}{3}h\left[y_0 + y_n + 4(y_1 + y_3 + \ldots + y_{n-1}) + 2(y_2 + y_4 + \ldots + y_{n-2})\right]$$

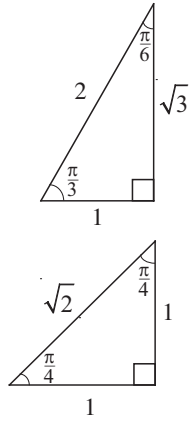where $h = \dfrac{b-a}{n}$, $y_r = f(x_r)$ and n is even.

S–CALCF

TRIGONOMETRY

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine Rule

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Identities

$$\cos^2\theta + \sin^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\cot^2\theta + 1 = \operatorname{cosec}^2\theta$$

General Solutions

If $\sin\theta = \sin\alpha$ then $\theta = n\pi + (-1)^n\alpha$

If $\cos\theta = \cos\alpha$ then $\theta = 2n\pi \pm \alpha$

If $\tan\theta = \tan\alpha$ then $\theta = n\pi + \alpha$

where n is any integer

Compound Angles

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

Double Angles

$$\sin 2A = 2\sin A\cos A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2\cos^2 A - 1$$

$$= 1 - 2\sin^2 A$$

Products

$$2\sin A\cos B = \sin(A+B) + \sin(A-B)$$

$$2\cos A\sin B = \sin(A+B) - \sin(A-B)$$

$$2\cos A\cos B = \cos(A+B) + \cos(A-B)$$

$$2\sin A\sin B = \cos(A-B) - \cos(A+B)$$

Sums

$$\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$$

$$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$$

$$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$$

$$\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$$

MEASUREMENT

Triangle

$$\text{Area} = \frac{1}{2}ab\sin C$$

Trapezium

$$\text{Area} = \frac{1}{2}(a+b)h$$

Sector

$$\text{Area} = \frac{1}{2}r^2\theta$$

$$\text{Arc length} = r\theta$$

Cylinder

$$\text{Volume} = \pi r^2 h$$

$$\text{Curved surface area} = 2\pi rh$$

Cone

$$\text{Volume} = \frac{1}{3}\pi r^2 h$$

$$\text{Curved surface area} = \pi rl \text{ where } l = \text{ slant height}$$

Sphere

$$\text{Volume} = \frac{4}{3}\pi r^3$$

$$\text{Surface area} = 4\pi r^2$$