Home Work 4 - Problem 5

Algorithms

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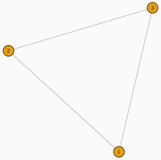
Home Work 4: Problem 5: Thickness

Let G be a simple undirected graph with n vertices labeled $1, 2, ..., n$. The graph $G[r_1, r_2, ..., r_n]$ is the graph obtained from G by replacing vertex i with K_{r_i} and connecting all possible vertices in neighboring complete graphs. So, for example, $G[1, 1, ..., 1] = G$, $K_2[2, 2] = K_4$, $K_2[m, n] = K_{m+n}$, and $K_1[n] = K_n$.

Using R and igraph:

```
g ← make_empty_graph(directed=F ) +
  vertices(1:3) +
  edge( c(1:3 ,1:3))
  plot(g)
```



Part a

Problem

The independence number $\alpha(G)$ of a graph G is the size of the largest set of independent (mutually nonadjacent) vertices in G. Prove that $\chi(G) \geq \frac{|V(G)|}{\alpha(G)}$.

Proof.

Let a_1 be the largest set of independent (mutually nonadjacent) vertices in G.
Then there is a set $!a_1$ the members of which are by definition not in a_1 but are agent to a member of a_1.
The set $!a_1$ can be sub divided in to set of independent vertices none of wich can be lager than a_1 or smaller than 1.
It fallows that the maximum number of subsets is then $\frac{|V(G)|}{\alpha(G)}$.
Note that for a correct coloring all members of an independent set can share one color(they are not adjacent), no two sets can share a color however (their nodes are adjacent).
Thus the chromatic number must be greater or equal the number of independent sub sets.
Therefore $\chi(G) \geq \frac{|V(G)|}{\alpha(G)}$. □

Part b

If G is a graph with n vertices, prove that $\alpha(G) = \alpha(G[r_1, r_2, ..., r_n])$, where each $r_i \in \mathbb{N}$.

Proof. $\square$

Part c

Find both the chromatic number and thickness of $C_3[2, 2, 2]$ and prove that your answers are correct.

Part d

Find both the chromatic number and thickness of $C_5[2,2,2,2,2]$ and prove that your answers are correct.

Part e

Find both the chromatic number and thickness of $C_n[2, 2, ..., 2]$ and prove that your answers are correct.

Part f

Find both the chromatic number and thickness of $C_5[3, 3, 3, 3, 3]$ and prove that your answers are correct.

Part g

Find both the chromatic number and thickness of $C_5[4, 4, 4, 4, 3]$ and prove that your answers are correct.

Part h

Find both the chromatic number and thickness of $C_7[4, 4, 4, 4, 4, 4, 4]$ and prove that your answers are correct.