Heaven's Light is Our Guide
Computer Science & Engineering
Rajshahi University of Engineering & Technology

Lab Manual

Module- 6

Course Title: Sessional based on CSE 2101
Course No. : CSE 2102

Experiment No. 6

Name of the Experiment: Advanced Counting Techniques

Duration: 1 Cycle

Background Study: Kenneth H. Rosen, "Discrete Mathematics and its Application", 6th Edition: Chapter 6 (Advanced Counting Techniques)

- Problem 1. Rabbits and Fibonacci Number
- Problem 2. The Tower of Hanoi
- Problem 3. Divide and Conquer Recurrence Relations
  - Example.1. Binary search
  - Example.2. Finding the maximum and minimum of a sequence
  - Example.3. Merge sort
  - Example.4. Fast Multiplication
  - Example.5. Fast Matrix multiplication
  - Example.6. The closest-pair problem
- Problem 4. Application of Inclusion-Exclusion
  - Example.1. The Sieve of Erotosthenes

Experiments/Problems:

[1] Given a positive integer n, list all the moves required in the Tower of Hanoi puzzle to move n disks from one peg to another according to the rules of the puzzle.

[2] Given a positive integer n and an integer k with1 $\leq$ k $\leq$ n, list all the moves used by the Frame–Stewart algorithm (see problem [2]) to move n disks from one peg to another using four pegs according to the rules of the puzzle.

[3] Given a positive integer n, list all the bit sequences of length n that donot have a pair of consecutive 0s.

[4] Given an integer n greater than 1, write out all ways to parenthesize the product of n + 1 variables.

[5] Given a recurrence relation $a_n = c_1a_{n-1} + c_2a_{n-2}$, where c_1 and c_2 are real numbers, initial conditions $a_0 = c_0$ and $a_1 = c_1$, and a positive integer k, find a_k using iteration.

[6] Given a recurrence relation $a_n = c_1a_{n-1} + c_2a_{n-2}$ and initial conditions $a_0 = c_0$ and $a_1 = c_1$, determine the unique solution.

[7] Given a recurrence relation of the form f(n) = af(n/b) + c, where a is a real number, b is a positive integer, and c is a real number, and a positive integer k, find $f(b^k)$ using iteration.

[8] Given the number of elements in the intersection of three sets, the number of elements in each pairwise intersection of these sets, and the number of elements in each set, find the number of elements in their union.

[9] Given a positive integer n, produce the formula for the number of elements in the union of n sets.

[10] Given positive integer's m and n, find the number of onto functions from a set with m elements to a set with n elements.

[11] Given a positive integer n, list all the derangements of the set {1, 2, 3,...,n}.

Problem [2]:

Frame–Stewart algorithm: Frame–Stewart algorithm, given the number of disks n as input, depends on a choice of an integer k with 1 $\leq$ k $\leq$ n. When there is only one disk, move it from peg1 to peg 4 and stop. For n> 1, the algorithm proceeds recursively, using these three steps. Recursively move the stack of the n − k smallest disks from peg1 to peg2, using all four pegs. Next move the stack of the k largest disks from peg1 to peg 4, using the three-peg algorithm from the Tower of Hanoi puzzle without using the peg holding the $n - k$ smallest disks. Finally, recursively move the smallest $n - k$ disks to peg 4, using all four pegs. Frame and Stewart showed that to produce the fewest moves using their algorithm, k should be chosen to be the smallest integer such that n does not exceed $t_k = k(k + 1)/2$,the kth triangular number, that is, $t_{k-1} < n \leq t_k$. The unsettled conjecture, known as Frame's conjecture, is that this algorithm uses the fewest number of moves required to solve the puzzle, no matter how the disks are moved.

Report:

Your completed work must be submitted through a LAB REPORT.

Read:

[1] Kenneth H. Rosen, "Discrete Mathematics and its Application", 7th Edition: Chapter 8 (Advanced Counting Techniques).