Heaven's Light is Our Guide

Computer Science & Engineering

Rajshahi University of Engineering & Technology

Lab Manual

Module- 04

Course Title: Sessional based on CSE 2101

Course No. : CSE 2102

Experiment No. 4

Name of the Experiment: Induction and Recursion

Duration: 1 Cycle

Background Study: Kenneth H. Rosen, "Discrete Mathematics and its Application", 6th Edition: Chapter 4 (Induction and Recursion)

- ALGORITHM 1. A recursive algorithm for computing n!
- ALGORITHM 2. A recursive algorithm for computing a^n
- ALGORITHM 3. Recursive modular exponentiation
- ALGORITHM 4. A recursive algorithm for computing gcd(a;b)
- ALGORITHM 5. A recursive linear search algorithm
- ALGORITHM 6. A recursive binary search algorithm
- ALGORITHM 7. A recursive algorithm for Fibonacci number
- ALGORITHM 8. An iterative algorithm for computing Fibonacci number
- ALGORITHM 9. A recursive merge sort
- ALGORITHM 10. Merging two list

Experiments/Problems:

[1] **Given a $2^n \times 2^n$ checkerboard with one square missing, construct a tiling of this checkerboard using right triominoes.

[2] **Generate all well-formed formulae for expressions involving the variables x, y, and z and the operators {+, ∗, /, −} with n orfewer symbols.

[3] **Generate al lwell-formed formulae for propositions with n or fewer symbols where each symbol is T, F, one of the propositional variables p and q, or an operator from {¬, ∨, ∧, →, ↔}.

[4] Given a string, find its reversal.

[5] Given a real number a and a non negative integer n, find a^n using recursion.

[6] Given a real number a and a non-negative integer n, find a^{2^n} using recursion.

[7] Given a real number a and a non negative integer n, find a^n using the binary expansion of n and a recursive algorithm for computing a^{2^k}

[8] Given two integers not both zero, find their greatest common divisor using recursion.

[9] Given a list of integers and an element x, locate x in this list using a recursive implementation of a linear search.

[10] Given a list of integers and an element x, locate x in this list using a recursive implementation of a binary search.

[11] Given a non negative integer n, find the nth Fibonacci number using iteration.

[12] Given a nonnegative integer n, find the nth Fibonacci number using recursion.

[13] Given a positive integer, find the number of partitions of this integer. (See Problem [13])

[14] Given positive integers m and n, find A(m,n), the value of Ackermann's function at the pair (m,n). (See Problem [14])

[15] Given a list of n integers, sort these integers using the merge sort.

Problem [13]:

A partition ofa positive integer n is away to write n as a sum of positive integers where the order of terms in the sum does not matter. For instance, 7 = 3 + 2 + 1 + 1 is a partition of 7.Let P_m equal the number of different partitions of m, and let $P_{m,n}$ be the number of different ways to express m as the sum of positive integers not exceeding n.

a) Show that $P_{m,m} = P_m$.

b) Show that the following recursive definition for $P_{m,n}$ is correct:

$$P_{m,n} = \begin{cases} 1 & \text{if } m = 1 \\ 1 & \text{if } n = 1 \\ P_{m,m} & \text{if } m < n \\ 1 + P_{m,m-1} & \text{if } m = n > 1 \\ P_{m,n-1} + P_{m-n,n} & \text{if } m > n > 1. \end{cases}$$

c) Find the number of partitions of 5 and of 6 using this recursive definition.

Answer:
a) $P_{m,m} = P_m$ because a number exceeding m cannot be used in a partition of m.

b) Because there is only one way to partition 1, namely, 1 = 1, it follows that $P_{1,n} = 1$. Because there is only one way to partition m into1s, $P_{m,1} = 1$.When n>m it follows that $P_{m,n} = P_{m,m}$ because a number exceeding m cannot be used. $P_{m,m} = 1+P_{m,m-1}$ because one extra partition, namely, m = m, arises when m is allowed in the partition. $P_{m,n} = P_{m,n-1}+P_{m-n}$, n if m>n because a partition of m into integers not exceeding n either does not use any ns and hence, is counted in $P_{m,n-1}$ or else uses an n and a partition of m−n, and hence, is counted in $P_{m-n,n}$.

c) P5 = 7,P6 = 11

Problem [14]:
Consider an inductive definition of a version of Ackermann's function. This function was named after Wilhelm Ackermann, German mathematician who was a student of the great mathematician David Hilbert. Ackermann's function plays an important role in the theory of recursive functions and in the study of the complexity of certain algorithms involving set unions. (There are several different variants of this function. All are called Ackermann's function and have similar properties even though their values do not always agree.)

$$A(m,n) = \begin{cases} 2n & \text{if } m = 0 \\ 0 & \text{if } m \geq 1 \text{ and } n = 0 \\ 2 & \text{if } m \geq 1 \text{ and } n = 1 \\ A(m-1, A(m, n-1)) & \text{if } m \geq 1 \text{ and } n \geq 2 \end{cases}$$

Question: Find these values of Ackermann's function (a) A(2,3) (b) A(3,3)

Answer: a) 16 b) 65,536

Question: Show that A(m, 2) = 4 whenever m ≥ 1.
Answer: Let P(n) be "A(n, 2) = 4."
Basis step: P(1) is true because A(1, 2) = A(0,A(1, 1)) = A(0, 2) = 2 · 2 = 4.
Inductive step: Assume that P(n) is true, that is, A(n, 2) = 4. Then A(n + 1, 2) = A(n,A(n + 1, 1)) = A(n, 2) = 4.

Report: Your completed work must be submitted through a LAB REPORT.
Read: Kenneth H. Rosen, "Discrete Mathematics and its Application", 7th Edition: Chapter 5 (Induction and Recursion).