2nd cycle (C Day) JK

Ref.

$\boxed{\text{Boolean}}$

Propositions

P → Today is friday
¬P → Today is not friday

* Disjunction ($p \vee q$)
* Conjuction ($p \wedge q$)
* Exclusive or ($p \oplus q$) → $\bar{p}q + \bar{q}p$

$\underline{F \cdot T} \quad \underline{F \cdot T}$

$F + F = F$

$F + T \to T$

p	q	$p \oplus q$
T	T	F
F	T	T
T	F	T
T	T	F

Implication

Premise
hypothesis

$P \to q$ → Cosequence
Conclusion

if in T → F out then only F

otherwise always T.

If it is sunny today, then we will go to beach

p   q

T → F then its F otherwise its true.

You can access the internet from campus only if you are a computer science major or you are not a fresh man.

$$a \longrightarrow (c \vee \neg f_{\text{fresh man}})$$

Lab JK   2nd cycle (D day)   4.4.10

Propositional Equivalences.

(i) * tautology → যে কোন Statement এর জন্য alltime True.
(ii) * Contradiction
(iii) * Contigency

(i)

P	$\neg P$	(i) $P \vee \neg P$	(ii) $P \wedge \neg P$
T	F	T	F
F	T	T	F

Biconditional

P	q	$P \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Logical Equivalences

$$(P \to q) == (\neg P \vee q)$$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

p	q	$\neg p$	$\neg p \vee q$
T	T	F	F
F	F	F	F
F	T	T	T
F	F	T	T

$$p \vee (q \wedge r) \Longleftrightarrow (p \vee q) \wedge (p \vee r)$$

p	q	r	$p \vee (q \wedge r)$	$(p \vee q) \wedge (p \vee r)$

$$\neg (p \vee (\neg p \wedge q)) \Longleftrightarrow (\neg p \wedge \neg q)$$

$\neg p \wedge \neg (\neg p \wedge q)$

$\neg p \wedge (p \vee \neg q)$

$\Rightarrow (\neg p \wedge p) \vee (\neg p \wedge \neg q)$

$\Rightarrow F \vee (\neg p \wedge \neg q)$

$= \neg p \wedge \neg q$

Predicates & Quantifiers

$P(x)$: x is greater than three

$P(x)$: $x > 3$

Universe of discorsoes

Domain

$P(x, y)$, $x = y + 3$

$P(1, 2)$

$P(5, 2)$

* Universal Quantification:
* Existential "

$P(x) \to$ time (for all $\to \forall x$) int the U of D.

$x < x + 1$ $\quad$ $x < 0$

Every student in ~~dis~~ this class has studied

Calculus.

$P(x) \to x$ has studied calculus.

U of D $\to$ All the studs in this class

$\forall x \quad P(x)$

U of $D = \{1, 2, 3, 4\}$

$\forall x \quad P(x)$

$\exists x \quad P(x)$

$\forall \quad P(1) \wedge (P(2) \wedge P(3) \wedge P(4)$ $\quad P(x)$

$\exists \quad P(1) \vee (P(2) \vee P(3) \vee (P(4)$ $\quad x^2 > 10$

$\forall x \ (C(x) \vee \exists y \ (C(y) \wedge F(x, y)))$

$C(x) \rightarrow$ x has a computer

$F(x, y) \rightarrow$ x & y are friend

U of D $\rightarrow (x, y) \in$ {all student in the class}

---

1. All lions are fierce
2. Some lions do not drink coffee.
3. Some fierce creatures do not drink coffee.

$P(x) \rightarrow x$ is a linon

$Q(x) \rightarrow x$ is fierce

$R(x) \rightarrow x$ drink coffee

$$\forall x\,(P(x) \longrightarrow Q(x))$$

$$\exists x\,(P(x) \wedge \neg R(x))$$

3rd cycle A day 6.4.10

Sets

$A = \underbrace{\{1, 2, 3, 4, 5\}}_{\text{elements}}$

number.

$R = \{x \mid x \text{ is a real number}\}$

$A \subseteq B$

$\forall x \{x \in A \rightarrow x \in B\}$

$S \rightarrow$ finite set

$n \rightarrow$ district element

$\neq$ – ve

cardinality of S

$A = \underbrace{\{1, 3, 5, 7, 9\}}$ n district element

< 10 $= 5$

$|A| = 5$

Identity law, deviation law.

$\overline{A \cap B} = \bar{A} \cup \bar{B}$

$\overline{A \cap B} = \{x \mid x \notin A \cap B\}$

$= \{x \mid \neg(x \in A \wedge x \in B)\}$

$= \{x \mid x \notin A \vee x \notin B\}$

$= \{x \mid \bar{A} \cup \bar{B}\}$

— o —

$\overline{A \cup (B \cap C)} = (\bar{C} \cup \bar{B}) \cap \bar{A}$

$\overline{A \cap B}$

$= \bar{A} \cup \bar{B}$, $\bar{A} \cup \bar{B} = \bar{B} \cup \bar{A}$

$\bar{A} \cap (\overline{B \cap C})$

$= \bar{A} \cap (\bar{B} \cup \bar{C})$

$= (\bar{C} \cup \bar{B}) \cap \bar{A}$

JK

Functions

3rd cycle C day

One to One

Onto

One to one correspondence

1 to 1 plus onto, bijection.

* Inverse function

*Invertible $\longrightarrow$ 1-to-1 corres

* Not " $\longrightarrow$ not 1-to-1 "

Composition of function:

$$(f \circ g)a = f(g(a))$$

$$f(x) = 2x + 3$$
$$g(x) = 3x + 7$$

$$\begin{aligned}(f \circ g)x &= f(g(x)) \\ &= f(3x+7) \\ &= 2(3x+7)+3 \\ &= 6x+17.\end{aligned}$$

$$(g \circ f)(x) = g(f(x))$$
$$= g(2x+3)$$
$$= 3(2x+3)+7$$
$$= 6x+16.$$

Ex 17.

* floor $\lfloor 3.4 \rfloor = 3$
* Ceiling $\lceil 3.7 \rceil = 4$

$$\lceil 3.4 \rceil = 4$$

A. 7. Sequences & Sum Self.

let,

$y = \sqrt{3-x}$ $\quad x = \{$ 1, 2, 3

$y = ax^2 + bx + c$

Read, a, b, c

put value of x

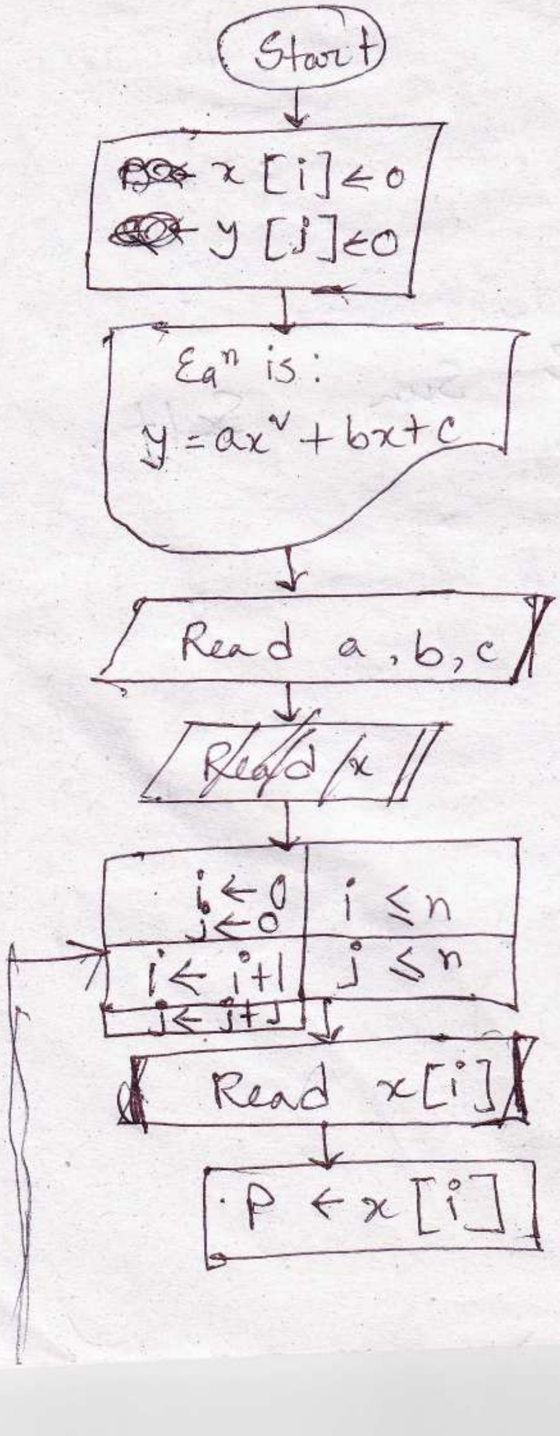
$y = \boxed{x^2 + 1}$

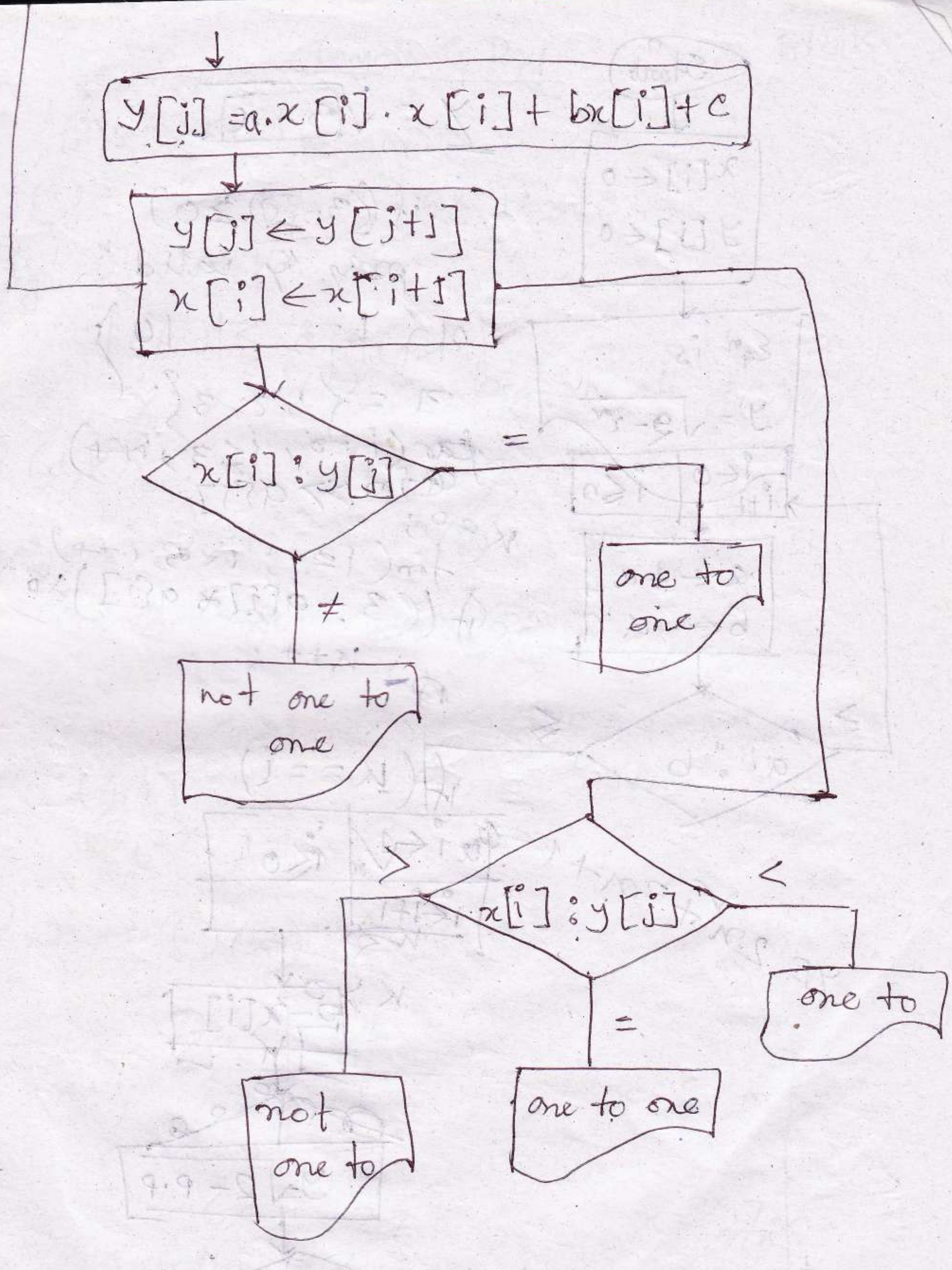
$y =$

$x[2] \rightarrow 2$

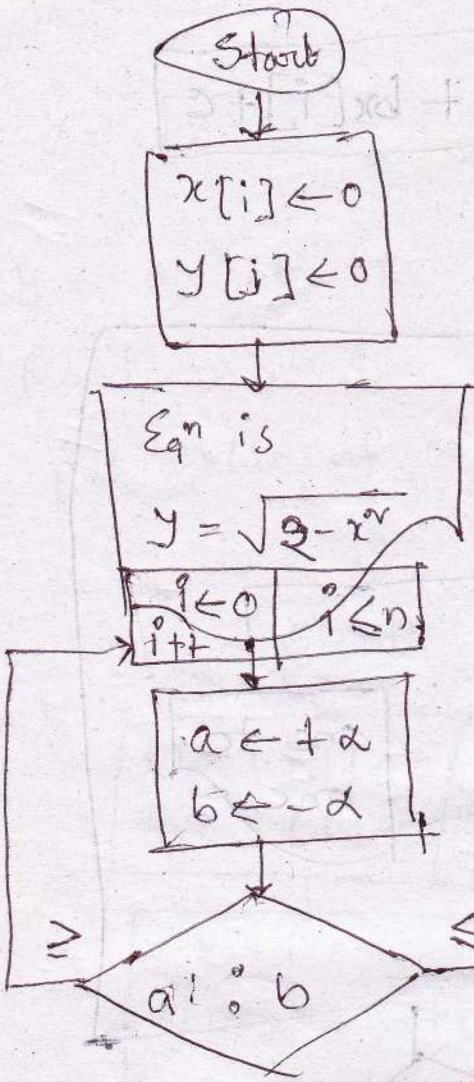
$y[\ ]$

$y = ax^2 + bx + c$





y[j] = a.x[i] . x[i] + bx[i] + c
y[j] ← y[j+1]
x[i] ← x[i+1]
x[i] : y[j]
=
≠
one to one
not one to one
>
<
x[i] : y[j]
=
one to
one to one
not one to



$y = \sqrt{\boxed{3 - x}}$

if ((3 - x) ≥ 0)
print "y valid"

x { 1 ...... 10 }

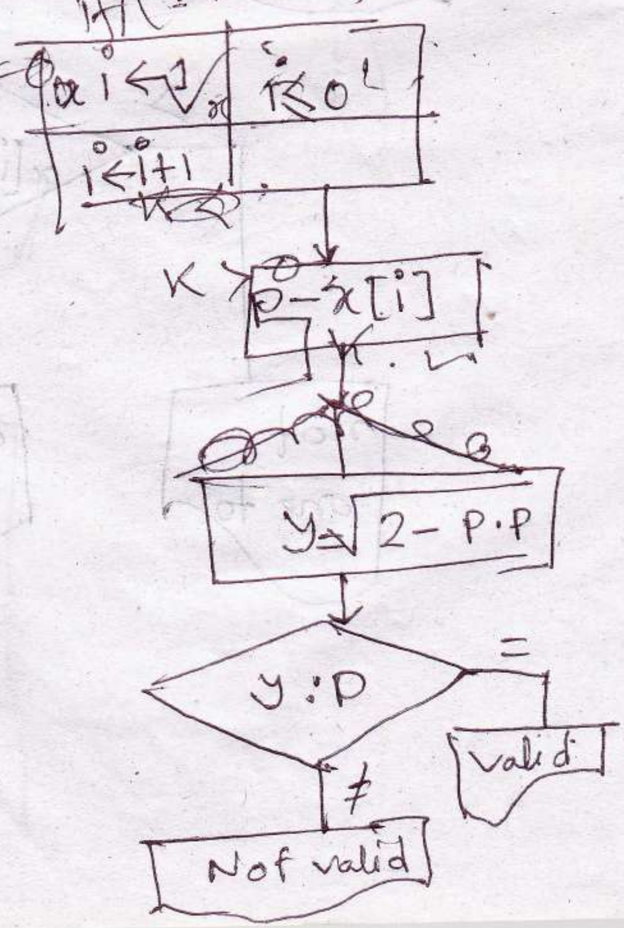
x = { 1, 2, 3 }

for (i = 0 , i ≤ 3 , i++)
cin >> a[i]

k = 0 ;
for (i = 0 ; i < 3 ; i++)
if ((3 - a[i] * a[i]) ≥ 0)
k++ ;

if (k = = i)

y = 2x² + 3x + 1



9 cycle C Day

MJK

1'8 growth of f^n

$f(x) = x^2 + \ldots = ax^2 + bx + c$.

Big O Notation

$|f(x)| \leq c\,|g(x)|$

$x > k$ $f(x)$ is big-oh of $g(x)$

pair (c, k)

$f(x) = x^2 + 2x + 1$ $\quad O(x^2)$

$0 \leq x^2 + 2x + 1 \rightarrow x^2 + 2x^2 + x^2 = 4x^2$

$x > 1$

$c = 4$

$k = 1$

$(4, 1)$

$(3, 2)$

$7x^2 \rightarrow O(x^3)$

	$7x^2$		x^3
$x = 1$	7	>	1
2	28		8
3	63		27
4	112		64
5	175		125
6	252		198
7	343	=	343

$x = 8 = k =$

$x^3 > 7x^2$

$x > 7$

Why $O(7x^2) \rightarrow x^3$

$f(x) = |a_n x^n + a_{n-1}^1 x^n + \cdots a_1 + a_0|$

$\leq |a_n x^n| + |a_{n-1} x^{n-1}| + \cdots + a_1 1$

$= x^n \left|a_n + |a_{n-1}/x^n| + \cdots + \left|\frac{a_0}{x^n}\right|\right|$

$\leq x^n |a_n + a_{n-1}'|$

$6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$n! = 1 \cdot 2 \cdot 3 \cdots n$

$\downarrow \ \downarrow \ \downarrow \qquad \downarrow$

$n \ \ n \ \ n \ \cdots \ n$

$= n^n$

$6! = 6^6$

JK

5 cycle C Day

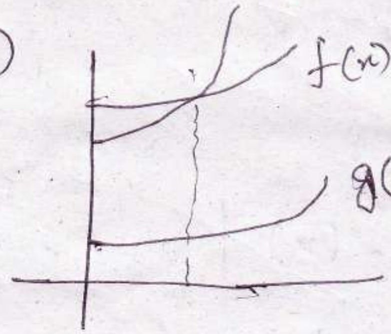
$f(x) \leq c(g(x))$ $\quad O(g(x))$

$x > k$

$(c, k) \to O(x^2)$ $\quad f_1(x) + f_2(x) \to O(\ )$

$f_1(x) f_2(x) \to O(\ )$.

(c, k)



$x^2 + 3 \to O(2x^2)$

$c = 1$	4	2
$c = 2$	7	8
$c = 3$	12	18

$1 + 2 = 3 < 4$ $\quad 1 + 2 + 3$, $6 < 9$

$1 + 2 + 3 + 4 <$

$10 < 16$

$1 + 2 + \cdots n \leq n^2$

$1 \leq 1^2$

$x^3 + 3$ $\quad O(x^2)$

$$f_1(x) + f_2(x) \leqslant c_1 |g_1(x)| + c_2 |g_2(x)|$$
$$\leqslant c_1 |g(x)| + c_2 |g(x)|$$
$$= g|x| (c_1 + c_2)$$
$$= c |g(x)|$$

$$g(x) = \max(g_1(x), g_2(x))$$

$$|f_1(x) \; f_2(x)| = |f_1(x)| \, |f_2(x)|$$
$$\leqslant |c_1 \; g_1(x)| \, |c_2 \; g_2(x)|$$
$$\leqslant c_1 c_2 |g_1(x)| |g_2(x)|$$
$$\leqslant c |g(x)|$$

$$f(x) = 3x \log x! + (x^2+3) \log x$$

$(x^2 + 3) \log x$

$3x \log x!$

$x^2 \log x$

$3x \log x^x$

$g(x) = x^2 \log x$

$3x^2 \log x$

$f(x) = (x+1)\log(x^2+1) + 3x^2$

$x^2+1 \leq 2x^2$

$\log(2x^2)$

$= \log 2 + \log x^2$

$= \log 2 + 2\log x \leq 3\log$

$O(x) \rightarrow f(x) \leq c|g(x)|$ upper Limit

$\Omega(x) \rightarrow f(x) \geq c|g(x)|$ Lower Limit

$\theta(x) \rightarrow \; = \; \leq f(x) \leq$

$x^2 + x$

$3x + 7 < x$

$-13x < \cancel{x}$

$3 < 1$

$6 < 2$

$9 < 3$

$12 < 4$

1 (b) $\cancel{f(x) = 3x+7}$

$\cancel{3x+7 \leq x}$

$\cancel{x=1}$ $\cancel{3x+7 \leq x}$ = $\cancel{10 \leq 1}$

$\cancel{x=2}$ $\cancel{13 \leq 1}$

$\cancel{x=3}$ $\cancel{16 \leq 1}$

~~So f(x) is not O(x)~~

1. (c) $x^2+x+1 = f(x)$

$x^2+x+1 \leq x$

Not $O(x)$

(a) $f(x) = 10$

$10 \leq x$

$x = 11$, $k = 10$, $c = 1$

(b) $f(x) = 3x+7$

$0 \leq 3x+7 \leq 3x+x = 4x \rightarrow c = 4$

$3x+7 \leq x$

$k = 7$

1(d) $f(x) = 5\log x$

$0 \leq 5\log x$

$x = 1$, $\cancel{5\log x \leq x}$ $x \leq 5\log x$

$K = 1$.

1 (e) $f(x) = \text{floor}\, x$

$c = 1,\ K = 0$

1 f) $f(x) = \left[\frac{x}{2}\right]$

$= 1,$

2 b) $f(x) = x^2 + 1000$

$0 \leq x^2 + 1000$

$0 \leq x^2 + x^2 = 2x^2$

$c = 2$

$x^2 + 1000 \leq x^2$

~~Not $O(x^2)$~~

$K = 0$.

c) $f(x) = x\log x$

$= \log x^x$

$c = 1$

$\log x^{x} < x$

$x = 1 \quad 0 \leq 1$

$x = 2 \quad 0.60 \leq 2$

$x = 3 \quad 1.43 \leq 3$

$c = 1, \; k = 3$

2(d) $f(x) = 2^{x}$

$c = 1$

$2^{x} \leq x^{2}$

$x = 1 \quad 2 \leq 1$

$x = 2 \quad 4 \leq 4$

$x = 3 \quad 8 \leq 9$

$c = 1, \; k = 3.$

3. $f(x) = x^{4} + 9x^{2} + 4x + 7$

$x^{4} + 9x^{3} + 4x + 7 \leq 4x^{4}$

$c = 4$

$K = 9$

4. $f(x) = 2^x + 17$

$$2^x + 17 \leq 3^x$$

$k = 3,\ C = 1$

5. $f(x) = \dfrac{x^2+1}{x+1}$ $\quad C = 1$

$$\frac{x^2+1}{x+1} \leq x$$

$x = 1,\quad x^2+1 \leq x^2+x$

$k = 1$

6. $f(x) = \dfrac{x^3+2x}{2x+1}$

$$\frac{x^3+2x}{2x+1} \leq x^2$$

$C = 1.\quad k = 3$

6 cycle C Day

Seq Search (L, r, x)

L is an Array, n entries (indexed 1 .. n), $x \rightarrow$ key to be Searched

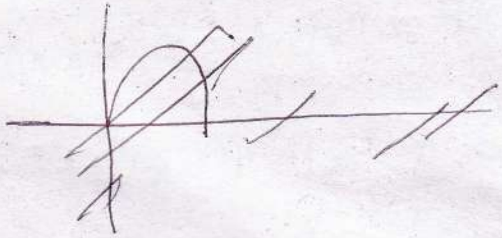
O/P: if x is in L, then O/P is its index else O/P 0.

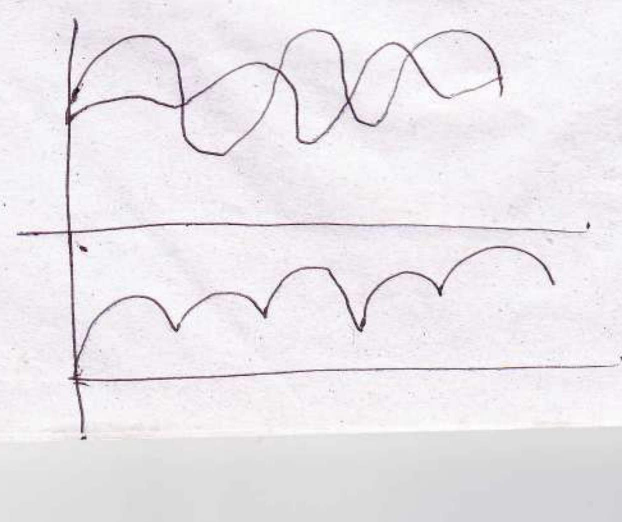
Pseudo code:

```
index = 1
while (index <= n && L[index] != x)
    index = index + 1;
if (index > n), then index = 0
return Index;
```

* Time
* Space

big-Oh → worst case





Ω

$$f(x) \geq c\, g(x)$$

$$x > k$$

$$f(x) = 3x^2 - 3x - 5 \qquad \Omega(x^2)$$

$$3x^2 - 3x - 5 > x^2$$

	$f(x)$	$\Omega(x^2)$
0 →	−5	0
1 →	−5	1
2 →	1	4
3 →	13	9
4 →	31	16

Ω   O

$$0 \leq c_1\, g(x) \leq f(x) \leq c_2\, g(x)$$

* Algorithm

* Computational Complexity
  → Time (Matrix)
  → Space (Database)

Logic 1·1

Propositional Equiv 1·2

Predicales 1·3

Set 1·4

Set Op (1·5

Funtions (1·6)

Seq & Ser (1·7)

Growth of f^n (1·8)

7cycle C Day 30.05.10

2.1 Algorithms + 2.2 complexity of alg.

LSA (Linear Search Alg.)

Procedure : linear search (x : int, a_1, a_2, a_n : distinct integer)

$i = 1$

while ($i \leq n$ and $x \neq a$)

$i = i+1$

if ($i \leq n$) then location $= i$

else location $= 0$.

i/p

o/p

correctness

finiteness

$i = 2i + 1$

খুঁজে না পেলে $2n+2$

Binary Search পড়তে হবে + Complexity.

Avarage Case

2.3

Prime $P > 1$

$P \to P, 1$.

a

b

a/b → $b = ac$ + r reminder

a/b → factor

a/b → multiple of a

a, b, c → integers

1. a/b && a/c then $a/(b+c)$

2. a/b the a/bc $\forall c$

3. a/b && b/c then a/c.

* Fundamental theorem of Arith
* integer → product of primes

8cycle β Day

Primes :

* $P > 1$

Positive factor of P are 1 & P

* Composite [Product of 2 Prime num]

↳ $n \rightarrow$ Prime division $\leq \sqrt{n}$

* Mersenne primes

$a = d\,q + r$

a → dividend, d → division, q → quotien, r → remainder

$2^P - 1 = (\ ) \rightarrow$ Prime হয় যদি

Otherwise not

অনুশীলনী

GCD

LCM

$(24, 34)$

$24 = 2^3 \cdot 3 = 12 \cdot 2$

$36 = 2^2 \cdot 3^2 = 12 \cdot 3$

$GCD(17, 22) = 1$ — relatively Prime

$GCD(17, 34)$ $\neq$ R.P.

$GCD(10, 17, 21) = 1$ R.P

$120 = 2^3 \cdot 3 \cdot 5$

$500 = 2^2 \cdot 5^3$ $\quad P_1 = 2$

$P_2 = 5$

$P_1^{\min(3,2)} \; P_2^{\min(1,3)}$

$2^2 \cdot 5^{(1)}$

$= 20$

$GCD(500, 120) = 20$

LCM

Modular Arithmetic:

$r = a \bmod m$

Congruent $\quad m \mid a - b$

$a \equiv b \pmod m$

$a \bmod m = b \bmod m$

Ex. $17 \equiv 5 \pmod 6$

$24 \equiv 14 \pmod 6$

$$\boxed{17 - 5 = 12/6}$$

9cycle B Day

$C = (P+K) \bmod 26$

$= \bmod (26 + (\ ))$

$0 \to 9$

$r = (??) \bmod 10$

$= (??) \bmod 11$

$a \equiv b \bmod (m)$

$m \mid a-b$

$m = (+)$ ve

If & only if $\to k$

such that $a = b + km$

$a \equiv b \pmod m$

$c = d \pmod m$

then, $a+c \equiv b+d$ (mod

& $ac = bd \pmod m$

$a = b + km$

$c = d + tm$

$\underbrace{(a+c)}_{x} = \underbrace{(b+d)}_{y} + m \underbrace{(k+t)}_{z}$

$x = y + zm$.

The Euclidean Alg.

$\gcd(91, 287)$

$\overset{a}{287} = \overset{q}{3} \cdot \overset{b}{91} + \overset{r}{\underline{\underline{14}}}$

$\gcd(91, 14)$

$91 = 14 \cdot 6 + \underline{\underline{7}}$

$\gcd(7, 14)$

$14 = 7 \cdot 2$

Calculati $\gcd(91, 287) = \gcd(91, 14) = \gcd(7, 14)$
$= 7$

$a = bq + r$

$a, b, q, r = \text{int}$

$\gcd(a, b) = \gcd(b, r)$

$a \geqslant b$

$a = bq + r_2 \quad 0 \leqslant r_2 < b$

$b = r_2 q_2 + r_3 \quad 0 \leqslant r_3 \leqslant r_2$

$r_2 = r_3 q_3 + r_4 \quad 0 \leqslant r_4 \leqslant r_3$

$0 \leqslant r_2 < r_{n-1}$

$r_{n-2} = r_{n-1} q_{n-1} + r_n$

$r_{n-1} = r_n q_n$

10^2 10^1 10^0
2 8 7

$n = a_k b^n + a_{k-1} b^{k-1} + \cdots + a_0 b^0$

b = bass of no.

Chapter 2.5

$a, b = (+)$ve

gcd (a, b)
$sa + tb$

gcd $(252, 198) = 18$

Linear Combination of 252 & 198

$252 = 1 \cdot 198 + 54$

$198 = 3 \cdot 54 + 36$

$54 = 1 \cdot 36 + 18$

$36 = 2 \cdot 18$

→ next to last

$18 = 54 - 36 \cdot 1$

$36 = 198 - 3 \cdot 54$

$18 = 54 - 1(198 - 3 \cdot 54) = 4 \cdot 54 = 1 \cdot 198$

$54 = 252 - 1 \cdot 198$

Finally $18 = 4(252 - 1 \cdot 198)$

$$= \underset{s}{4} \cdot \underset{a}{252} - \underset{t}{4} \cdot \underset{b}{198}$$

$a, b, c = (+)$ ve int

$\gcd(a, b) = 1$ & a/bc then a/c

Theory $m = (+)$ ve int, $(a, b, c) =$ int

if $ac \equiv bc \pmod{m}$ & $\underbrace{\gcd(c, m) = 1}_{R.P}$

then $a \equiv b \pmod{m} \rightarrow m/(a-b)$

$\rightarrow m/c(a-b)$

9cycle C Day

Linear Conjruences

$ax \equiv b \pmod{m}$ | * $m = (+)$ve int,
$a, b =$ int
$x =$ Variable

$\bar{a} a \equiv 1 \pmod{m}$ → $\bar{a}$ is an inverse of a module m

Relatively prime

a & $m \Rightarrow RP$ & $(m>1)$
i.e $\gcd(a, m) = 1$ → Inverse of a mod m exist

$sa + tm = 1 \Rightarrow sa + tm \equiv 1 \pmod{m}$
& $tm \equiv 0 \pmod{m}$
$sa = 1 \pmod{m}$

So, $\bar{a} = s$ that means inverse of a.

3 module 7. $\gcd(3,7) = 1$

$\overset{3}{a}$ & $\overset{7}{m} \Rightarrow RP$ & $(\overset{7>1}{m>1})$ হলে inverse বের করতে পারবো।

$3(-2) + 1 \cdot 7 \equiv 1 \pmod{7}$

$3(-2) + 7 \cdot 1 = 1$

$s \quad a + t m$