Fourier Series

$$\delta(n) = \begin{cases} 1 & n=0 \\ 0 & \text{otherwise} \end{cases}$$

delta function $\longrightarrow$ only one output.

Even Signal

$$x(n) = x(-n)$$

Odd signal

$$x(n) = -x(-n)$$

Correlation: Similarity between two signals.

Cross-Correlation:

Correlation between two different signals till a certain range.

Auto correlation:

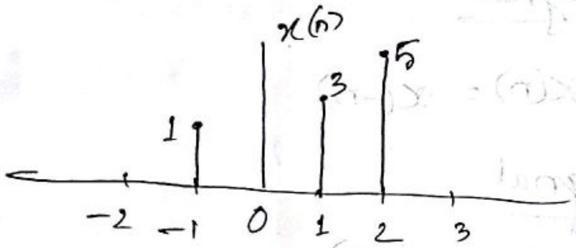
Similarity between the sending and receiving signals.

Shifting

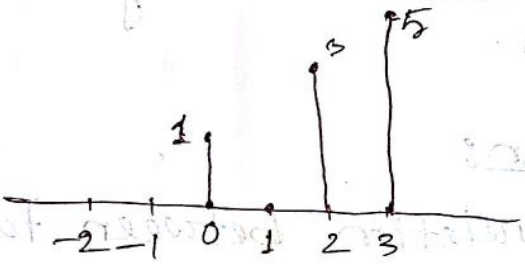
→ Delay signal

→ Advance signal

$x(n) = \{1, \underset{\uparrow}{0}, 3, 5\}$

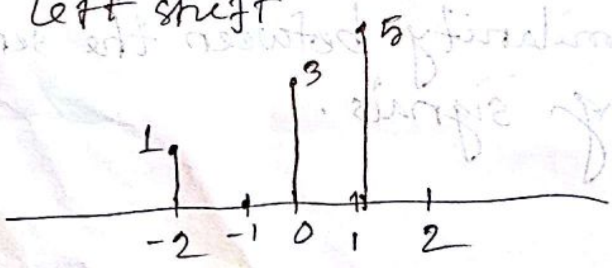


$x(n-1)$ → [illegible] delay → Right shift



$K = 1, 2, 3, \ldots$ $x(n-k)$ → k delay

$x(n+1)$ → left shift



Scalling factor

→ Scalling factor is a constant on the basis of which, the signal will be changed.

$t=0 \quad f(t)=0$

$t=1 \quad f(t)=2$

$t=2 \quad f(t)=4$

Here, $f(t)=2t$
↳ scalling factor

scalling factor >0 → signal amplified
" " <0 → " reduced

$$\boxed{\cancel{X(f)=} \int_{-\infty}^{+\infty} x(t)\, e^{-i2\pi f t}\, dt}$$

$$\boxed{F(s)=\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\, e^{isx}\, dx}$$

Scanned by CamScanner

Properties

→ Linear property

$$F[a f_1(x) + b f_2(x)] = a F_1(s) + b F_1(s)$$

where a and b are constants

→ Scalling property

$$F\{f(ax)\} = \frac{1}{|a|} F(s/a)$$

where a is a constant

→ shifting property

$$F\{f(x-a)\} = e^{isa} F(s)$$

where a is a constant

$$\boxed{F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\, e^{isx}\, dx}$$

Proof

→ Linear Property

$$F\left[a f_1(x) + b f_2(x)\right] = \frac{1}{\sqrt{2\pi}} \int_{-\alpha}^{\alpha} \left[a f_1(x) + b f_2(x)\right] e^{isx}\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\alpha}^{\alpha} a f_1(x) e^{isx}\, dx + \frac{1}{\sqrt{2\pi}} \int_{-\alpha}^{\alpha} b f_2(x) e^{isx}\, dx$$

$$= a F_1(s) + b F_1(s)$$

[Proved]

→ Scalling property

$$F\{f(ax)\} = \frac{1}{\sqrt{2\pi}} \int_{-\alpha}^{\alpha} f(ax)\, e^{isx}\, dx$$

Let, $ax = t$, $dx = \frac{1}{a} dt$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\alpha}^{\alpha} f(t)\, e^{is\frac{t}{a}}\, \frac{dt}{a}$$

$$= \frac{1}{a\sqrt{2\pi}} \int_{-\alpha}^{\alpha} f(t)\, e^{i\frac{s}{a}t}\, dt$$

$$= \frac{1}{a} F\left(\frac{s}{a}\right)$$

[Proved]

Scanned by CamScanner

→ shifting Property

$$F\{f(x-a)\} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x-a)\, e^{isx}\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t)\, e^{is(a+t)}\, dt$$

Let, $x-a=t$
$dx = dt$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t)\, e^{ist} \cdot e^{isa}\, dt$$

$$= e^{isa} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t)\, e^{ist}\, dt$$

$$= e^{isa} F(s)$$ [Proved]

$F\{f(t-5)\} = e^{is5} F(s)$

$F\{f(t+5)\} = e^{is(-5)} F(s)$

$x(t) = 4x^2 + 5x \longrightarrow$ Fourier transform?

$$F(s) = \frac{1}{\sqrt{2\pi}} \int_{-\alpha}^{\alpha} f(x)\, e^{isx}\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\alpha}^{\alpha} (4x^2 + 5x)\, e^{isx}\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\alpha}^{\alpha} 4x^2 e^{isx}\, dx + \frac{1}{\sqrt{2\pi}} \int_{-\alpha}^{\alpha} 5x\, e^{isx}\, dx$$

$=$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{L}$$

$$a_0 = \frac{1}{L} \int_{-L}^{L} f(x)\, dx$$

$$a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos \frac{n\pi x}{L}\, dx$$

$$b_n = \frac{1}{L} \int_{-L}^{L} f(x) \sin \frac{n\pi x}{L}\, dx$$

Prob 1

$f(x) = x \qquad 0 < x < 2\pi$

$$a_0 = \frac{1}{\pi} \int_{0}^{2\pi} x\, dx$$

$$= \frac{1}{2\pi} \left[\pi^2 + \pi^2\right]$$

$$= \frac{2\pi^2}{2\pi}$$

$$= \pi$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} x \cos \frac{n\pi x}{\pi} \, dx$$

$$= \frac{1}{\pi} \int_{-\pi}^{\pi} x \cos nx \, dx$$

$$= \frac{1}{\pi} \left[\frac{x \sin nx}{n} + \frac{\cos nx}{n^2} \right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi} \left[\frac{\pi \sin n\pi}{n} + \frac{\cos n\pi}{n^2} - \frac{-\pi \sin(-n\pi)}{n} - \frac{\cos(-n\pi)}{n^2} \right]$$

$$= \frac{1}{\pi} \left[0 + \frac{(-1)^n}{n^2} - 0 - \frac{(-1)^n}{n^2} \right]$$

$$= \frac{1}{\pi} [0]$$

$$= 0$$

$$b_n = \frac{1}{4}\int_{-4}^{4} f(t)\, \sin\frac{n\pi t}{4}\, dt$$

$$= \frac{1}{4}\int_{-4}^{0} (0)\, \sin\frac{n\pi t}{4}\, dt + \frac{1}{4}\int_{0}^{4} 5 \sin\frac{n\pi t}{4}\, dt$$

$$= 0 + \frac{5}{4}\left[\frac{-\cos\frac{n\pi t}{4}}{\frac{n\pi}{4}}\right]_0^4$$

$$= 0 - \frac{5}{n\pi}\left(\cos n\pi - \cos 0\right)$$

$$= -\frac{5}{n\pi}\left[(-1)^n - 1\right]$$

when $n = 1, 2, 3, \ldots$

$$b_n = \frac{10}{\pi}, 0, \frac{10}{3\pi}, 0, \frac{10}{5\pi}, \ldots$$

$$\therefore f(t) = \frac{5}{2} + \frac{10}{\pi}\sin\frac{\pi x}{4} + \frac{10}{3\pi}\sin\frac{3\pi x}{4} + \frac{10}{5\pi}\sin\frac{5\pi x}{4} + \cdots$$

Dirichlet Condition: $P = 2L$

→ It has a finite number of discontinuities within the period 2L

→ It has a finite average value in period 2L

→ It has a finite number of positive and negative maxima and minima.

For odd function → a_n term $= 0$

" even " → b_n " $= 0$