

Heaven's Light is Our Guide
RAJSHAHI UNIVERSITY OF ENGINEERING & TECHNOLOGY
DEPARTMENT OF COMPUTER SCIENCE & ENGINEERING
4th Year Odd Semester Examination 2019
COURSE NO: CSE 4103 COURSE TITLE: Digital Signal Processing
FULL MARKS: 72 TIME: 3 HRS

- N.B. (i) Answer any SIX questions taking any THREE from each section.
(ii) Figures in the right margin indicate full marks.
(iii) Use separate answer script for each section.

	<u>Marks</u>
SECTION : A	
Q.1. (a) What is signal? What are the advantages of digital signal processing over analog signal processing? 3	
(b) Explain multichannel and multidimensional signal with proper example. 3	
(c) What is sampling? Mathematically show how aliasing occurs? 3	
(d) Prove that- "A discrete time sinusoid is periodic only if its frequency f is rational number" 3	
Q.2. (a) Consider the following signals $x(n)=\{ \dots, 0, 0, 3, -6, 2, 0, -2, 5, -3, 0, 0, \dots \}$ $h(n)=\{ \dots, 0, 0, 1, 2, 3, 0, 1, 2, 3, 0, 0, \dots \}$ Find out the followings: (i) Evaluate the even part of $x(n)$ 2 (ii) For the signal $x(n)$ and $h(n)$ given above draw the graphical representation of the signal $x(-n+3)$ and $h(-2n-3)$ 4 (iii) Suppose $x(n)$ denotes the impulse response of a LTI system. Now determine the response of the system to the input signal $x(n)$ given above 4	
(b) Consider the system $y(n)=x[x(n)] = x(n^2)$. Determine whether the system is time invariant or not. 2	
Q.3. (a) Why is convolution important in DSP? 1	
(b) Compute the circular convolution and linear convolution of the following sequences $x_1(n) = \{1, 2, 2, 1\}$, $x_2(n) = \{1, 2, 3, 4\}$ Compare the result of both convolutions. If they are not same then how can you make both of their result equal? 3	
(c) Determine the step response of the system with impulse response $h(n) = \begin{cases} 3^n, & n < 0 \\ \left(\frac{2}{5}\right)^n, & n \geq 0 \end{cases}$	
(2) 2	
Q.4. (a) What is z-transform? Why is z-transform used in DSP? Explain with example. 4	
(b) Define region of convergence (ROC). Determine the z-transform of the signal, $x(n) = 3^n u(n) + 2^n u(-n-1)$ Draw ROC of z-transform. 5	
(c) Determine the causal signal $x(n)$ if its z-transform $X(z) = \frac{1+2z^{-1}+z^{-2}}{1+4z^{-1}+4z^{-2}}$ 3	

• 3

$$y(n) = ay(n-1) + bx(n)$$

SECTION : B

o c a c l

- Q.5. (a) Determine the spectra of the signal if $x(n)$ is periodic with $N=5$ and $x(n)=\{1, 1, 1, 0, 0\}$ 5

- (b) Suppose a linear time-invariant system is described by the following difference equation, $y(n)=ay(n-1)$

- (i) Determine the magnitude and phase of the frequency response $H(\omega)$ of

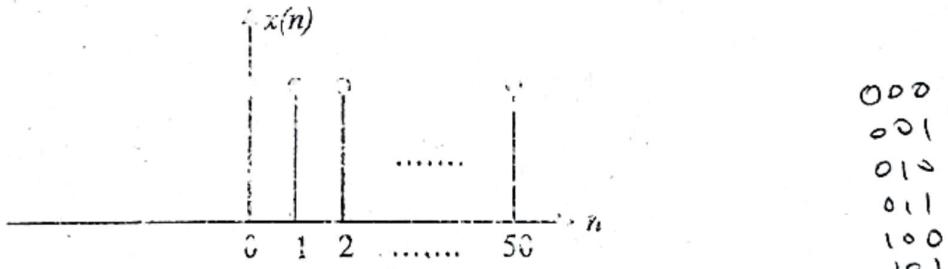
- (ii) Consider $a=0.9$ and choose the parameter b so that the maximum value of $|H(\omega)|$ is unity. Finally determine the output of the system to the input signal,

$$x(n) = 5 + 12 \sin \frac{\pi}{2} n - 20 \cos(\pi n + \frac{\pi}{4})$$

- Q.6. (a) Write down following properties of Fourier transform:

- (i) Linearity
 - (ii) Frequency shifting
 - (iii) Time reversal
 - (iv) convolution

- (b) Consider the following signal,



Calculate (i) Fourier transform (ii) Magnitude and phase spectra of $f(n)$.

- Given $X(z) = \frac{1}{2}z^{-1} + 1 + z$ and $H(\omega) = 1 + 2\cos\omega$, calculate $y(n)$. (11) 3

- Q.7. (a) Define frequency response $H(\omega)$ of LTI system. Explain role of $H(\omega)$ in filtering.

- (d) Calculate output of a system using $h(n) = \left(\frac{1}{3}\right)^n u(n)$ and $x(n) = e^{j\pi n/2}$

- (c) Considering $N=8$, derive mathematical equation and show computation in time for radix-2 FFT algorithm.

- Q.8. (a) Consider the sequence $x(n) = \{4, 5, 6, 7\}$. Calculate

- (i) Discrete Fourier Transform (DFT)
 - (ii) Inverse Discrete Fourier Transform

- (b) Determine input-output equation of filter where cutoff frequencies are ω_c

- (c) Why is Discrete Fourier Transform (DFT) used in real life instead of Fourier transform?

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$$\sum_{k=0} h(n) x(n-k)$$

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$\sqrt{2}x$

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N.B. (i) Answer any SIX questions taking any THREE from each section.

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SECTION : A

- Q.1. (a) What is digital signal processing? Write down the advantages of digital signal processing over analog signal processing. 3
(b) What is aliasing? Mathematically demonstrate aliasing effect. 5
(c) Determine which of the following sinusoids are periodic and compute their fundamental period. 4
- | | |
|---|--|
| (i) $\cos\left(\frac{30\pi n}{105}\right)$ | (ii) $\cos\left(\frac{\pi}{8}\right) \cos\left(\frac{\pi n}{8}\right)$ |
| (iii) $\sin\left(\frac{62\pi n}{10}\right)$ | (iv) $\cos\left(\frac{\pi n}{2}\right) + \sin\left(\frac{\pi n}{8}\right)$ |
- Q.2. (a) Consider the signal $x(n)=\dots, 0, -1, -2, -3, -2, -1, 0, 1, 2, 3, 2, 1, 0, \dots$. Now, i) Determine and sketch even part of $x(n)$. ii) Determine and sketch odd part of $x(n)$. 4
(b) Find out whether the following systems described by the equations are linear/nonlinear and time invariant/time variant? 4
(i) $y(n) = x(n^2)$ (ii) $y(n) = nx^2(n)$
(c) Prove that an energy signal has zero power while power signal has infinite energy. 4
- Q.3. (a) Compute the convolution $y(n)=x(n)*h(n)$ of the following signals: 6
 $x(n)=u(n)-u(n-5)$
 $h(n)=u(n-2)-u(n-8)+\delta(n-1)+u(n-11)$
(b) The discrete time system $y(n)=ny(n-1)+x(n)$, $n \geq 0$ is at rest [i.e., $y(-1)=0$]. Check if the system is linear time invariant and BIBO stable. 4
(c) Sketch and label carefully of the following signal: 2
 $x(n)=[u(n+2)-u(n-3)].(3-|n|)$
- Q.4. (a) The impulse response of linear time-invariant system is: 5
 \downarrow
 $h(n)=\{1, 1, 2, -2, 3\}$. Now determine the response of the system to the input signal
 \downarrow
 $x(n)=\{5, 4, 3, 2, 1\}$. Also sketch the response.
(b) Determine whether the following nonlinear system is stable or unstable 2
 $y(n) = y^2(n-1) + x(n)$
(c) Determine the response $y(n)$, $n \geq 0$ of the system described by the second order difference equation: $y(n)-3y(n-1)-4y(n-2)=x(n)+2x(n-1)$, when the input sequence is $x(n)=4^n u(n)$. 5

SECTION : B

- Q.5. (a) What is Z-transform? What are the advantages of using Z-transform? 3
(b) Determine the Z-transform of the following signals: 6
- (i) $x(n)=\begin{cases} \left(\frac{1}{2}\right)^n & n \geq 5 \\ 0 & n \leq 4 \end{cases}$
- (ii) $x(n)=na^n (\sin \omega_0 n)v(n)$
- (c) Determine the region of convergence of right-sided, left-sided and finite duration two sided sequence. 3
- Q.6. (a) Explain the Dirichlet conditions that guarantees the existence of Fourier transform 3
(b) Derive the Fourier transform equations for continuous time aperiodic signals. 5
(c) Determine the magnitude and phase spectra of the signal $x(n)=\{1, 0, 2, 3\}$, where $x(n)$ is periodic with $N=4$. 4

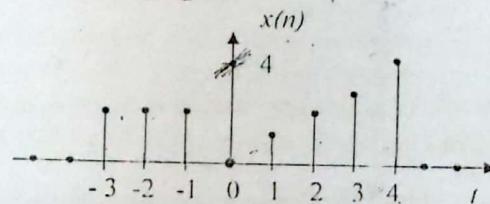
7. (a) Define symmetric properties of DFT. Why windowing is required in DFT process?
(b) Compute 4 point DFT of the sequence $x(n)=(2, 6, 8, 10)$.
(c) Write down the constraints that must be imposed during pole-zero placements in filter.
8. (a) What are the advantages and disadvantages of digital filter over analog filter?
(b) Design a two pole band pass filter that has the center of its pass-band at $\omega = \pi/2$ zero in its frequency response characteristic at $\omega=0$ and $\omega=\pi$ and a magnitude response of $\frac{1}{\sqrt{2}}$ at $\omega=\frac{4\pi}{9}$.
(c) Compute the DFT of the four point sequence
 $x(n)=\{3, 2, 1, 0\}$

N.B. Answer six questions, taking three from each section.
 The questions are of equal value.
 Use separate answer script for each section.

SECTION-A

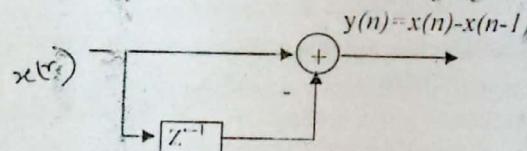
- Q1. (a) Prove that: 6
 i) A discrete-time sinusoid is periodic only if its frequency f is a rational number.
 ii) The highest rate of oscillation in a discrete-time sinusoid is attained where $\omega = \pi$ (or $\omega = -\pi$) or equivalently $f = 1/2$ (or $f = -1/2$).
 (b) What are the advantages of digital over analog signal processing? 3
 (c) What is aliasing? What should be done to avoid aliasing? 3

- Q2. (a) Prove that an energy signal has zero power while a power signal has infinite energy. 3
 (b) Show the graphical representation of the signals $x(-n)$ and $x(-n+2)$, where $x(n)$ is the signal illustrated in the following figure. 3



- (c) Using the basic building block, sketch the block diagram representation of the discrete-time system from the following input-output relation. 3
 $y(n) = \frac{1}{4}y(n-1) + \frac{1}{2}x(n) + \frac{1}{2}x(n-1)$, where $x(n)$ is the input and $y(n)$ is the output of the system.

- (d) Determine if the system shown in the following figure is time invariant or time variant. 3



- Q3. (a) Sketch and define: i) Symmetric signal, and ii) anti-symmetric signal. 2
 (b) Determine and sketch the convolution $y(n) = x(n) * h(n)$ of the following signals 5

$$x(n) = \begin{cases} 1 & ; \text{ for } n = -2, 2 \\ 2 & ; \text{ for } n = -1, 1 \\ 3 & ; \text{ for } n = 0 \\ 0 & ; \text{ elsewhere} \end{cases}$$

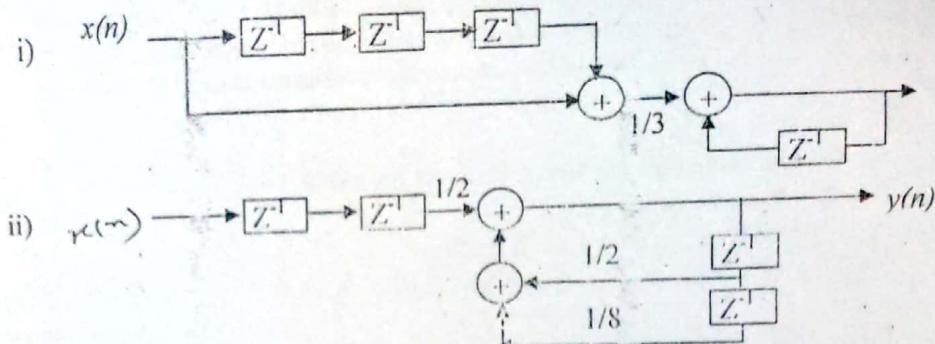
$$h(n) = 3\delta(n+2) + 2\delta(n+1) + \delta(n) + 2\delta(n-1) + 3\delta(n-2)$$

- (c) Determine the response $y(n)$, $n \geq 0$ of the system described by second order difference equation 5

$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1) \text{ when the input sequence is } x(n) = (-1)^n u(n)$$

- Q4. (a) Three systems with impulse responses $h_1 = \delta(n) - \delta(n-1)$, $h_2 = h(n)$ and $h_3 = u(n)$ are connected in cascade. 6
 i) What is the impulse response $h_c(n)$ of the overall system?
 ii) Does the order of the interconnection affect the overall system?

- (b) Determine and sketch the impulse response of the following systems for 6
 $n=0,1,\dots,9$



SECTION-B

- Q5. (a) Show the relationship between Fourier transform and Z-transform. 3

- (b) Determine the z-transform of the following signals: 4

i) $x(n)=n a^n u(n)$

ii) $x(n)=\begin{cases} 1 & ; 0 \leq n \leq N-1 \\ 0 & ; \text{elsewhere} \end{cases}$

- (c) Determine the signal $x(n)$ whose z-transform is 5

i) $X(z) = \log(1+az^{-1})$ ROC: $|z| > |a|$

ii) $X(z) = \frac{1}{1 - \frac{3}{2}z^{-1} + \frac{1}{2}z^{-2}}$ ROC: $|z| < \frac{1}{2}$

- Q6. (a) For a signal $x(n)=X_R(n)+jX_I(n)$ show that if $x(n)$ is real then 4

$$X_R(-\omega) = X_R(\omega)$$

$$X_I(-\omega) = -X_I(\omega)$$

- (b) Using wiener khinchine theorem determine the energy density spectrum $S_{xx}(\omega)$ of the signal: $x(n)=e^{jn\pi/4}u(n)$ 4

- (c) List up the dirichlet conditions that guarantee the existence of Fourier transform. 4

- Q7. (a) Determine whether the statements are true or false. Your answer must be equipped with explanation. 6

i) The signal $x(t)=1.5 \cos(4\pi t)$ is an even signal.

ii) The signal $x(t)=e^{-|t|}$ is a power signal.

- (b) Determine the causal signal $x(n)$ whose z-transform is given by 3

$$X(z) = \frac{1+z^{-1}}{1-z^{-1}+0.5z^{-2}}$$

- (c) Determine the spectra of the signal $x(n)=\cos(\pi n/3)$ 3

- Q8. (a) Compute 4 point DFT of the sequence, $x(n)=(2,4,2,4)$. 4

- (b) A two pole low pass filter has system function $H(z) = \frac{b_0}{(1-pz^{-1})^2}$, determine the

value of b_0 such that it satisfies the condition, $H(0)=1$ and $\left|H\left(\frac{\pi}{4}\right)\right|^2 = \frac{1}{2}$

- (c) Perform circular convolution of the following sequence, 4

$$x_1(n)=(1,2,1,2)$$

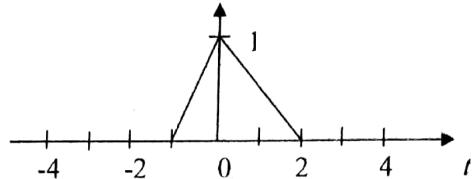
$$x_2(n)=(1,2,3,4)$$

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SECTION-A

- Q1. (a) Define signal and system. Discuss about the different types of signals. 3
- (b) Given a signal, $x[n] = \{1, 5, 1, 2, 2.5, 1, 0.75, 3, 4, 3.5\}$, show examples of down sampling (decimation) and up-sampling (interpolation) over $x[n]$. 4
- (c) Given a signal $x(t)$ $4^{2/3}$
 Find
 i. $x(-t/2)$
 ii. $x(2(t+2))$
 iii. $x(-t+1)$
- 
- Q2. (a) Give example of i) even signal, ii) odd signal, iii) causal signal, iv) anti-causal signal, v) non-causal signal. 3
- (b) Prove that an energy signal has zero power while a power signal has infinite energy. 4
- (c) Discuss the cases when a sinusoid $\cos(\omega t + \theta)$ is multiplied by an exponential signal $e^{\alpha t}$ (α can be >0 or <0) and $e^{\alpha t}$ gets multiplied by a unit step signal. $4^{2/3}$

- Q3 (a) Determine the response $y(n)$; $n \geq 0$ of the system described by the second order difference equation 4

$y(n)-4y(n-1)+4y(n-2)=x(n) - x(n-1)$
 when the input is $x(n)=(-1)^n u(n)$ and the initial conditions are $y(-1)=y(-2)=0$.

- (b) Write down Parseval's relation for discrete time periodic signal with finite energy. 3
- (c) Write down the following properties of Discrete Fourier Transform (DFT) with examples: $4^{2/3}$
 i) Periodicity, ii) Linearity, iii) Symmetry.

- Q4. (a) What is the Gibbs phenomenon? Explain the reason behind it with suitable example. $4^{2/3}$

- (b) How multipath echo can be cancelled by using an invertible system? 2

- (c) Determine the output sequence of the system with impulse response 5
 $h(n)=(1/2)^n u(n)$ when the input is

$$x(n)=10-5\sin\frac{\pi}{2}n+20\cos\pi n, \quad -\infty < n < \infty.$$

SECTION-B

- Q5. (a) Determine the causal signal $x(n)$ whose z-transform is given by 4

$$X(z) = \frac{1+z^{-1}}{1-z^{-1}+0.5z^{-2}}$$

- (b) Determine the convolution of the following pairs of signals by means of z-transform: 4

$$x_1(n)=(1/4)^n u(n-1) \quad x_2(n)=[1+(1/2)^n] u(n) \quad 3^{2/3}$$

- (c) State and proof time shifting and time reversal property of z-transform. 3

- Q6. (a) Determine the spectra of the signal if $x(n)$ is periodic with period $N=4$ and $x(n)=\{1, 1, 0, 0\}$. 4

- (b) A two-pole lowpass filter has the system function $H(z)=\frac{b_0}{(1-Pz^{-1})^2}$. Determine $4^{2/3}$

the values of b_0 and P , such that the frequency response $H(w)$ satisfies the

$$\text{conditions } H(0)=1 \text{ and } |H(\pi/4)|=\frac{1}{2}.$$

(c) Prove that if $x(n) \xleftrightarrow{z^+} x^+(Z)$ then $x(n-k) \xleftrightarrow{z^+} Z^{-k} [x^+(Z) + \sum_{n=1}^k x(-n)Z^n]$

- Q7. (a) Determine whether the system $H(z) = \frac{z^2}{z^2 + 2z + 3}$ and $H(z) = \frac{1 + 4z^2}{3z^2 - 1}$ are stable 5^{2/3}
 (b) Prove that an all pass filter satisfies: $H(z) = \frac{1}{H(z^{-1})}$. 4
 (c) Differentiate between a FIR and IIR filter. 2
- Q8. (a) What is aliasing? What should be done to avoid aliasing? 4
 (b) Complete the cross-correlation and auto-correlation of the signals $f = \{1, 3, -2, 1\}$ ↑
3
 and $h = \{1, 1\}$ i.e., $\text{xcorr}(f, h)$, $\text{xcorr}(f, f)$ and $\text{xcorr}(h, h)$.
 (c) What is impulse response, step response and frequency response of a filter?
 Given $y = x * h$ a filtering system, determine y if x is a complex sinusoid signal.

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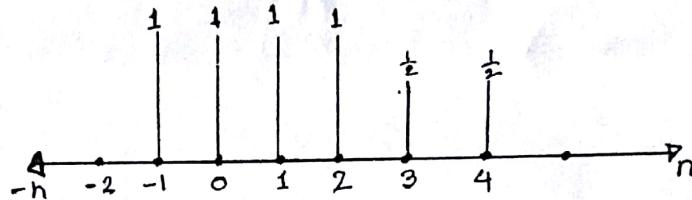
SECTION A

- Q.1(a)** An analog signal $x_a(t) = \sin(480\pi t) + 3 \sin(720\pi t)$ is sampled 600 times per second. Marks
04
 (i) Determine the nyquist sampling rate for $x_a(t)$, and (ii) What are the frequencies in radians, in the resulting discrete time signals $x(n)$?
- (b)** Consider the system $y(n) = T\{x(n)\} = x(n^2)$. (i) Determine if the system is time-variant. To clarify the result in (i), assume that the signal $x(n) = \{1, 1, 1, 1\}$ is applied to the system. Determine $y(n) = T\{x(n)\}$, $x_2(n) = x(n-2)$, $y_2(n) = T\{x_2(n)\}$, $\hat{y}_2(n) = y(n-2)$. Compare $y_2(n)$ and $\hat{y}_2(n)$. 05
 (ii) Repeat part (i) for the system, $y(n) = T\{x(n)\} = n \cdot x(n)$.
- (c)** State the advantages of digital signal processing over analog signal processing. 02
- Q.2(a)** Compute the convolution $y(n) = x(n) * h(n)$ of the following signals. 06

$$x_n = \begin{cases} 1, & n = -2, 0, 1 \\ 2, & n = -1 \\ 0, & \text{elsewhere} \end{cases}$$

$$h(n) = \delta(n) - \delta(n-1) + \delta(n-4) + \delta(n-5).$$

- (b)** A discrete time signal $x(n)$ is shown in the following figure. Sketch and label carefully each of the following: 05



- (i) $x(-n-2)$ (ii) $x(n)u(2-n)$ and (iii) $x(n-1)\delta(n-2)$. 02

- Q.3(a)** Compute the correlation sequences $r_{xx}(l)$ and $r_{xy}(l)$ for the following signal sequence. 06

$$x(n) = \begin{cases} 1, & n_0 - N \leq n \leq n_0 + N \\ 0, & \text{otherwise} \end{cases}$$

$$y(n) = \begin{cases} 1, & -N \leq n \leq N \\ 0, & \text{otherwise} \end{cases}$$

- (b)** Show that the necessary and sufficient condition for a relaxed LTI system to be BIBO stable is 03

$$\sum_{n=-\infty}^{\infty} |h(n)| \leq M_h < \infty$$

For some constant M_h .

- (c)** List the Dirichlet conditions that guarantee the existence of Fourier Transform. 02

- Q.4(a)** Define z-transformation. Write the applications of it. 02

- (b)** Determine the z-transforms of the following signals and sketch the corresponding pole-zero patterns. 06

- (i) $x(n) = (n a^n \sin \omega_0 n) u(n)$ and (ii) $x(n) = (1/2)^n [u(n) - u(n-10)]$.

- (c)** If $x(n) = X(z)$, ROC: $r_1 < |z| < r_2$ then proof that time interval property, $x(-n) \leftrightarrow X(z^{-1})$, ROC: $1/r_2 < |z| < 1/r_1$. 03

SECTION B

- Q.5(a)** Determine the causal signal $x(n)$ having the z-transform, $X(z) = 1/(1 - 2z^{-1})(1 - z^{-1})^2$. 05
(b) Consider the signal $x(n) = \{-1, 2, -3, 2, -1\}$, with Fourier Transform $X(\omega)$. Compute the following quantities, without explicitly computing $X(\omega)$. 06
- (i) $x(0)$ (ii) $\angle X(\omega)$ (iii) $\int_{-\pi}^{\pi} x(\omega) d\omega$ (iv) $X(\pi)$.
- Q.6(a)** What do you mean by Gibbs phenomenon? Describe it. 02
(b) What are advantages of circular convolution over conventional convolution? 02
(c) Convert the analog filter with system function $H_a(s) = (s + 0.1)/((s + 0.1)^2 + 16)$ into a digital IIR filter by means of the bilinear transformation. The digital filter is to have resonant frequency of $\omega_r = \pi/2$. 07
- Q.7(a)** Determine the eight-point DFT of the signal $x(n) = \{1, 1, -1, 1, 1, 1, 0, 0\}$ and sketch its magnitude and phase. 05
(b) What is the mechanism of Bilinear Transformation? 02
(c) A two-pole low pass filter has the system function $H(z) = b_0/(1 - pz^{-1})^2$. Determine the values of b_0 and p such that the frequency response $H(\omega)$ satisfies the conditions $H(0) = 1$ and $|H(j\omega/4)|^2 = 1/2$. 04
- Q.8(a)** What are the applications of FFT algorithm? Calculate the number of multiplication needed in the calculation of DFT and FFT with 64-point sequence. 04
(b) What are the differences between FIR and IIR Filters? How do you choose any one of them? 04
(c) Determine z-transform and the ROC of the signal: 03

$$X(n) = a^n u(n) = \begin{cases} a^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

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SECTION A

- Q1** A digital communication link carries binary-coded words representing samples of an input signal. **1.10** Marks 05²

$$x_a(t) = 3\cos 600\pi t + 2\cos 1800\pi t$$

The link is operated at 10000 bits/s and each input sample is quantized into 1024 different voltage levels.

- (i) What are the sampling frequency and folding frequency?
- (ii) What is the Nyquist rate for the signal $x_a(t)$?

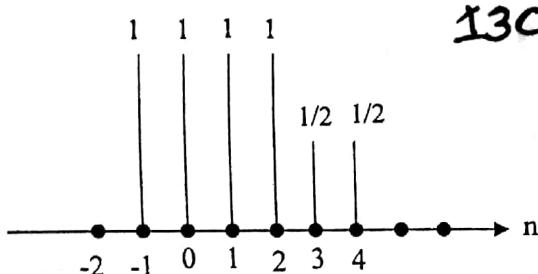
- (b)** Determine and sketch the convolution $y(n)$ of the signals, **2.19** 06

$$x(n) = \begin{cases} a^n, & -3 \leq n \leq 5 \\ 0, & \text{elsewhere} \end{cases}$$

$$h(n) = \begin{cases} 1, & 0 \leq n \leq 4 \\ 0, & \text{elsewhere} \end{cases}$$

- Q2(a)** A discrete time signal $x(n)$ is shown in the following figure. Sketch and label carefully each of the following: 04

- (i) $x(n)u(2-n)$
- (ii) $x(n-1)\delta(n-3)$
- (iii) $x(4-n)$



130 (2.2)

- (b)** Determine the impulse response $h(n)$ for the system described by the second-order differential equation $y(n)-3y(n-1)-4y(n-2)=x(n)+2x(n-1)$ 04

- (c)** Show that, the energy (power) of a real valued energy (power) signal is equal to the sum of energies (power) of its even and odd components. **130 (2.5)** 03²

- Q3(a)** Determine the eight-point DFT of the signal $x(n) = \{1, 1, 1, 1, 1, 1, 0, 0\}$ and sketch its magnitude and phase. **P- 505 (7.17)** 05²

(b) Write the following properties of Discrete Fourier Transform (DFT) with examples: P- 465 (7.21) 06

- (i) Periodicity
- (ii) Linearity
- (iii) Symmetry

- Q4(a)** Determine the z-transform of the following signals and sketch the corresponding pole-zero patterns: 06

(i) $x(n) = (-1)^n 2^{-n} u(n)$ **P- 214 (3.2(c))**

~~(ii)~~ $x(n) = \begin{cases} \left(\frac{1}{3}\right)^n & n \geq 0 \\ \left(\frac{1}{2}\right)^{-n} & n < 0 \end{cases}$ **P- 215 (3.3(a))**

~~(iii)~~ $x(n) = (-1)^n \left(\cos \frac{\pi}{3} n\right) u(n)$ **P- 215 (3.4(d))**

- Convert the analog filter with system function $H_a(S) = \frac{s+0.1}{(s+0.1)^2 + 16}$ into a digital IIR filter 05²

by means of bilinear transformation. The digital filter is to have a resonant frequency $\omega_r = \pi/2$.

P-715

SECTION B

- Q5(a)** What are the applications of FFT? 02
(b) What do you mean by Gibbs phenomenon? Describe with an example. 03
(c) Consider the signal $x(n) = a^n u(n)$, $0 < a < 1$ 06
 The spectrum of this signal is sampled at frequencies $\omega_k = 2\pi k/N$, $k=0, 1, \dots, N-1$.
 Determine the reconstructed spectra for $a=0.8$ when $N=5$ and $N=50$.

Q6(a) Show that the necessary and sufficient condition for a relaxed LTI system to be BIBO stable is $\sum_{n=-\infty}^{\infty} |h(n)| \leq M_h < \infty$ for some constant M_h . 03

- (b)** Determine the convolution $y(n) = x(n) * h(n)$ of the signals. 05

$$x(n) = u(n+1) - u(n-4) - \delta(n-5)$$

$$135 \quad (2-21(2))$$

$$h(n) = [u(n+2) - u(n-3)].(3 - |n|)$$

- Determine the range of values of a and b for which the linear time invariant system with impulse response.

$$h(n) = \begin{cases} a^n, & n \geq 0 \\ b^n, & n < 0 \end{cases}$$

$$\text{is stable.} \quad P-88(2-3-7)$$

- Q7(a)** Determine a closed-form expression for the n -th term of the Fibonacci sequence. 04

- (b)** Determine the unit sample response of the system characterized by the following difference equation $y(n) = 2.5y(n-1) - y(n-2) + x(n) - 5x(n-1) + 6x(n-2)$ 05

- (c)** List the Dirichlet conditions that guarantee the existence of Fourier transform. 02

- Q8(a)** What are the advantages and disadvantages of digital filter over analog filter? 03

- (b)** Define (i) Nyquist rate (ii) Quantization error. 02

- (c)** Design an FIR linear phase digital filter approximating the ideal frequency response 06

$$H_d(\omega) = \begin{cases} 1, & \text{for } |\omega| \leq \pi/6 \\ 0, & \text{for } \pi/6 < |\omega| < \pi/3 \\ 1, & \text{for } \pi/3 \leq |\omega| \leq \pi \end{cases}$$

$\delta(\omega)$

Determine the coefficient of 25-tap filter based on the window method with

- (i) Rectangular window
 (ii) Hamming window.

Heaven's Light is our Guide
 Rajshahi University of Engineering and Technology
 B.Sc. Engineering 4th Year 7th Semester Examination, 2013
 Department of Computer Science and Engineering
 Course No. CSE 713 Course Title: Digital Signal Processing
 Full Marks: 70 Time: THREE (03) hours

N.B.

Answer SIX questions taking THREE from each section.
 The questions are of equal value.
 Use separate answer script for each section.

SECTION A

- | Q1(a) | Describe the steps of analog-to-digital conversion process. | P-19 01 1 | <u>Marks</u>
03 ² |
|------------------|---|-------------------|---------------------------------|
| (b) | Consider the following analog signal,
$x_a(t) = 3\cos 50\pi t + 10\sin 300\pi t - \cos 100\pi t$. What is the Nyquist rate for this signal? | P-19 (1.4.3) | 03 |
| (c) | Examine the following systems whether those systems are stable or unstable. | P-131 (2.7 (i,j)) | 05 |
| (i) | $y(n) = x(n) + nx(n+1)$ | P-131 (2.7 (i,j)) | |
| (ii) | $y(n) = x(2n)$ | P-131 (2.7 (i,j)) | |
| Q2(a) | Determine and sketch the convolution $y(n)$ of the signals | 2.18 (P-134) | 06 ² |
| | $x(n) = \begin{cases} \frac{1}{3}n, & 0 \leq n \leq 6 \\ 0, & \text{elsewhere} \end{cases}$
$h(n) = \begin{cases} 1, & -2 \leq n \leq 2 \\ 0, & \text{elsewhere} \end{cases}$ | | |
| | (i) Graphically
(ii) Analytically | | |
| (b) | Determine the impulse response and unit step response of the systems described by the difference equation. | 2.33(b) (P-137) | 05 |
| | $y(n) = 0.7y(n-1) - 0.1y(n-2) + 2x(n) - x(n-2)$ | 2.33(b) (P-137) | |
| Q3(a) | Determine the response $y(n)$, $n \geq 0$ of the system described by the second order difference equation $y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$ when the input is $x(n) = (-1)^n u(n)$ and the initial conditions are $y(-1) = y(-2) = 0$. | 2.57 (P-143) | 05 ² |
| (b) | Compute the correlation sequences $r_{xx}(l)$ and $r_{xy}(l)$ for the following signal sequences | P-143 (2.61) | 06 |
| | $x(n) = \begin{cases} 1, & n_0 - N \leq n \leq n_0 + N \\ 0, & \text{otherwise} \end{cases}$
$y(n) = \begin{cases} 1, & -N \leq n \leq N \\ 0, & \text{otherwise} \end{cases}$ | P-143 (2.61) | |
| Q4(a) | Why z-transform is necessary in DSP? | | 02 ² |
| (b) | Determine the causal signal $x(n)$ having the z-transform | | 04 |
| | $X(Z) = \frac{1}{(1+z^{-1})(1-z^{-1})^2}$ | P-216 | |
| (c) | If $X(Z)$ is the z-transform of $x(n)$, show that | P-217 (3,18) | 05 |
| | (i) $Z\{x^*(n)\} = X^*(Z^*)$
(ii) $Z\{Re[x(n)]\} = \frac{1}{2}[X(Z) + X^*(Z^*)]$
(iii) $Z\{e^{j\omega n}x(n)\} = X(Ze^{j\omega})$ | | |

SECTION B

~~Q5(a)~~ Determine the z-transform of the following sequence and sketch the ROC:

$$(i) x(n) = \begin{cases} \left(\frac{1}{3}\right)^n - 2^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

P-115 (3.3(b))

04

$$(ii) u(n) = \left(\frac{1}{2}\right)^n [u(n) - u(n-1)]$$

P-245 (3.2(c))

(b) Determine the order and the poles of a type I low pass Chebyshev filter that has a 1dB ripple in the pass band, a cutoff frequency $\Omega_p = 100\pi$, a stop band frequency of 2000π and an attenuation of 40dB or more for $\Omega \geq \Omega_s$.

04^{2s}

(c) What is zero padding? What are its uses?

03

~~Q6(a)~~ Write the following properties of the Fourier transform with example:

- (i) Linearity and time shifts.
- (ii) Convolution.
- (iii) Differentiation.

06

~~(b)~~ Perform the circular convolution of the following two sequences:

$$x_1(n) = \{1, 2, 3, 1\}$$

P-503 (7.8)

05^{2s}

~~Q7(a)~~ What are the differences between FIR and IIR filter? How do you choose any one of them?

04

~~(b)~~ Determine the autocorrelation of the signal $x(n) = \alpha^n u(n)$, $0 < \alpha < 1$

P-124 (2.6.2)

03^{2s}

~~(c)~~ Prove that if $x(n) \xrightarrow{Z^+} X^+(Z)$ then $x(n-k) \xrightarrow{Z^+} Z^{-k}[X^+(Z) + \sum_{n=1}^k x(-n)Z^n]$, $k > 0$

04

Q8(a) What are the applications of FFT algorithm?

02

(b) Compute the eight point DFT of the sequence, $x(n) = \left\{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 0, 0, 0, 0\right\}$ using in place radix-2 decimation-in-time algorithm. Follow exactly the corresponding signal flow graph and keep track of all the intermediate quantities by putting them on the diagram.

06^{2s}

~~(c)~~ Distinguish between linear convolution and circular convolution. 474

03

$$3z^2 + 6z + 4 + 8z^{-1}$$

$$3z^2 + 10 + 8z^{-1}$$

$$6 + 3z + 2z^{-1} + 1 + 2z^{-2}$$

$$3z^2 + 7 + 2z^{-1}$$

$$(z_1 \pm z_2) - (z_1 - z_2) \times 2$$

$$\frac{z_1 - z}{z_1 - z} - \frac{z_1 - z}{z_1 - z}$$

N.B. Answer six questions, taking three from each section.

The questions are of equal value.

Use separate answer script for each section.

SECTION-A

Q1. (a) A discrete time system can be static or dynamic, causal or non-causal, linear or non-linear, time-variant or time invariant, stable or unstable. Examine the following systems with respect to the properties above. 131

- (i) $y(n) = x(n) \cos(\omega_0 n)$
- (ii) $y(n) = x(n)u(n)$
- (iii) $y(n) = x(-n + 2)$
- (iv) $y(n) = \sin(x(n))$
- (v) $y(n) = \cos(x(n))$

(b) Write the following properties of the Fourier transform with some examples:

- (i) Linearity and time shifts 290 25279
- (ii) Differentiation 289
- (iii) Convolution 283

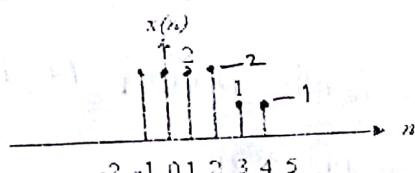
Q2. (a) State the disadvantages of digital signal processing over analog signal processing. 6 2

(b) What is Gibbs phenomenon? Explain with example. 669 3

(c) Explain the significance of Nyquist rate and aliasing during the sampling of continuous time signal. 1/2

(d) Distinguish between linear convolution and circular convolution. 474 3

Q3. (a) A discrete time signal $x(n)$ is shown below. 130 6



Sketch and label each of the following signals:

- (i) $x_c(n) = x(4 - n)$
- (ii) $x_d(n) = x(n)u(2 - n)$
- (iii) $x_f(n) = \frac{1}{2}x(n - 1) + x(n)$

(b) Consider the system $y(n) = T\{x(n)\} = v(n^2)$ (ii)

(i) Determine if the system is time-variant. To clarify the result in (i), assume that the signal $v(n) = \{1,1,1,1\}$ is applied to the system. Determine $y(n) = T\{x(n)\}$, 1/2

$x_2(n) = x(n - 2)$, $y_2(n) = T\{x_2(n)\}$, $y'_2(n) = v(n - 2)$. Compare $y_2(n)$ and $y'_2(n)$.

(ii) Repeat part (i) for the system $y(n) = T\{x(n)\} = n \cdot x(n)$. 130 6

Q4. (a) Determine the unit step response of the system whose difference equation is $y(n) - 0.7y(n-1) + 0.12y(n-2) = x(n-1) + x(n-2)$ if $y(-1) = y(-2) = 1$ 2.33 4

(b) Determine the convolution sum of two sequences $x(n) = \{3,2,1,2\}$ and $h(n) = \{1,2,1,2\}$ ch-2 4

(c) What is SISO system and MIMO system?

SECTION-B

(a) $y(n) = x(n) * h(n)$. Show that $\sum_{n=-\infty}^{\infty} y(n) = \sum_{n=-\infty}^{\infty} x(n) \cdot \sum_{n=-\infty}^{\infty} h(n)$. 133 1/2

(b) Compute the convolution $y(n) = x(n) * h(n)$ of the following sequences and check the correctness of the results by using the test in (a)

- (i) $x(n) = \{1, 0, 1, 1\}$, $h(n) = \{1, -2, -3, 4\}$ 33

- (ii) $x(n) = \alpha^n u(n)$, $h(n) = b^n u(n)$, $|\alpha| < 1$, $|b| < 1$

RP35 (2)(a)

zero padding consists of appending zeros to a signal. It maps a length N signal to a length $M > N$ signal but M need not be an integer multiple of N . $\text{zeropad}_M[n] \triangleq \begin{cases} n & 0 \leq n \leq N-1 \\ 0 & N \leq n \leq M-1 \end{cases}$

Q6. (a) Determine the Z-transform of the following sequences and sketch the ROC.

$$(i) x(n) = \begin{cases} (-\frac{1}{2})^n & n \geq 5 \\ 0 & n \leq 4 \end{cases} \quad 214$$

$$(ii) x(n) = \begin{cases} (\frac{1}{3})^n - 2^n & n \geq 0 \\ 0 & n < 0 \end{cases} \quad 215$$

$$(iii) x(n) = (\frac{1}{2})^n [u(n) - u(n-10)]$$

(b) If $X(Z)$ is the Z-transform of $x(n)$, show that (i) $Z[\operatorname{Re}\{x(n)\}] = \frac{1}{2}[X(Z) + X^*(Z^*)]$ (ii) If $y(n) = \begin{cases} x(\frac{n}{k}) & \text{if } \frac{n}{k} \text{ integer} \\ 0 & \text{otherwise} \end{cases}$ then $Y(Z) = X(Z^k)$ 217

Q7. (a) What is the mechanism of Bilinear Transformation? 712

(b) Find the Fourier transform $X(\omega)$ of the following signal. Write $X(\omega)$ in terms of its real and imaging parts. Find the magnitude of $X(\omega)$.

$$x(t) = 2 \sin(2\pi t - 3) + \sin(6\pi t)$$

(c) What is wrapping effect?

Q8. (a) What is zero padding? What are its uses? 456

(b) What are the applications of FFT algorithm? Calculate the number of multiplication needed in the calculation of DFT and FFT with 64-point sequence. 64-DFT radix-2 FFT $= 32 \times 6 = 192 \approx \frac{2}{3}$

(c) Which of the methods do you prefer for designing IIR filter? Why? 712

(d) A two-pole low pass filter has the system function $H(Z) = \frac{b_0}{(1-PZ^{-1})^2}$. Determine the values of b_0 and P such that the frequency response $H(\omega)$ satisfies the conditions $H(0) = 1$ and $|H(\pi/4)|^2 = \frac{1}{2}$. 333

Let, $x(n) = \sum_{l=-\alpha}^{\alpha} \delta(n+lN)$ *****

Hence, $x(n)$ is periodic with period N , i.e.

$$x(n) = 1, \quad n = 0, \pm N, \pm 2N, \dots$$

$$= 0, \quad \text{otherwise}$$

$$\text{Then } X(k) = \sum_{n=0}^{N-1} x(n) e^{-j \frac{2\pi}{N} nk} = 1, \quad 0 \leq k \leq N-1$$

$$\text{and } x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j \frac{2\pi}{N} nk}$$

$$\therefore \text{ Hence, } \sum_{l=-\alpha}^{\alpha} \delta(n+lN) = \frac{1}{N} \sum_{k=0}^{N-1} e^{j \frac{2\pi}{N} nk}$$

8(a)

* Zero padding a discrete time signal increases the period of its periodicity in the time domain, which is equivalent to performing a finer sampling in the frequency domain.

* It allows to increase the resolution of the DFT.

* It allows one to increase the length of the signal without adulterating its spectrum.

* It is also useful to make the length of a discrete-time signal a power of 2 , which is useful when using FFT algorithm to compute the DFT.

N.B. Answer six questions, taking three from each section.

The questions are of equal value.

Use separate answer script for each section.

SECTION-A

Q1. (a) Define which of the following sinusoidal are periodic and compute their fundamental period. 04

37 (i) $\cos 0.01\pi n$

(ii) $\sin\left(\pi \frac{60n}{10}\right)$

(iii) $\cos\left(\pi \frac{30n}{105}\right)$

(b) An analog signal $x_a(t) = \sin(480\pi t) + 3\sin(720\pi t)$ is sampled 600 times per second. 04

39 (i) Determine the nyquist sampling rate for $x_a(t)$.

(ii) What are the frequencies, in radians, in the resulting discrete time signal $x(n)$?

(c) A digital communication link carries binary coded words representing samples of an input signal $x_a(t) = 3\cos 600\pi t + 2\cos 1800\pi t$. The link is operated at 1000 bits/s and each input sample is quantized into 1024 different voltage levels. 3 2/3

39 (i) What is the resolution?

(ii) What is the Nyquist rate for the signal $x_a(t)$?

Q2. Sampling of sinusoidal signals: Consider the following continuous-time sinusoidal signal $x_a(t) = \sin 2\pi F_0 t$, $-\infty < t < \infty$. 11 2/3

Since $x_a(t)$ is described mathematically. Its sampled version can be described by values every T seconds. The sampled signal is described by the formula

$$x(n) = x_a(nT) = \sin 2\pi \frac{F_0}{T} n, -\infty < n < \infty, \text{ where } F_s = 1/T \text{ is the sampling frequency.}$$

40 (i) Plot the signal $x(n)$, $0 \leq n \leq 99$ for $F_s = 5\text{ KHz}$ and $F_0 = 0.5, 2, 3$ and 4.5 KHz . Explain the similarities and differences among the various plots. 03

Q3. (a) Prove the identity, $\sum_{l=-\infty}^{\infty} \delta(n+lN) = \frac{1}{N} \sum_{k=0}^{N-1} e^{j(2\pi/N)Kn}$. 06

51 (b) Write down the advantages of Divide-and-Conquer approach to computation of DFT. 1 2/3

(c) Consider an IIR system describe by the difference equation,

$$y(n) = -\sum_{k=1}^{N-1} a_k y(n-K) + \sum_{k=0}^M b_k x(n-K). \text{ Describe a procedure that compute the frequency response } H\left(\frac{2\pi}{N} K\right), K=0, 1, \dots, (N-1) \text{ using the FFT algorithm.}$$

55 (d) Determine the eight-point DFT of the signal, $x(n) = \{1, 1, 1, 1, 1, 1, 0, 0\}$. 05

Q4. (a) Write down the ideal filter characteristics 327 02

(b) Define with real time example 3 2/3

(i) Low pass filter

(ii) Band pass filter

(iii) All pass filter

(c) Consider the FIR filter, $y(n) = x(n) - x(n-4)$ 06

365 (i) Compute and sketch its magnitude and phase response.

(ii) Compute its response to the input $x(n) = \cos \frac{\pi}{2} n + \cos \frac{\pi}{4} n, -\infty < n < \infty$

SECTION-B

Q5. (a) Why Z-transform is necessary in DSP? 1 2/3

(b) Define ROC. Determine the Z-transform of the signal, $x(n) = a^n u(n) + b^n u(n-1)$ 1 5/3

(c) Prove that the following signal in Z-domain follows linearity property. 05

$$X(n) = [3(2^n) - 4(3^n)]u(n) \quad \checkmark \text{ 5/4}$$

Q6. (a) Consider the signal signal $y(n) = \frac{3}{4}y(n-1) - \frac{1}{8}y(n-2) + x(n)$, Determine 04

- (i) The impulse response 2/1
- (ii) Plot the pole-zero patterns. 04

(b) Determine the $x(n)$ for the expression $x(z) = \frac{1}{(1+z^{-1})(1-z^{-1})^2}$ 1 9/1

(c) If a signal contains both causal and anticausal parts, then its two-sided z-transform. Explain it with an example. 3 2/3

Q7. (a) Compute the furrier transform of the following signals: 06

(i) $x(n) = (\alpha^n \sin \omega_0 n)u(n), |\alpha| < 1$

(ii) $x(n) = \{-2, -1, 0, 1, 2\}$

(iii) $x(n) = \begin{cases} 2 - \left(\frac{1}{2}\right)^n, & |n| \leq 4 \\ 0, & \text{elsewhere} \end{cases}$

(b) Consider the signal $x(n) = 2 + 2\cos \frac{\pi n}{4} + \cos \frac{\pi n}{2} + \frac{1}{2} \cos \frac{3\pi n}{4}$ 5 2/3

- (i) Determine and sketch its power density spectrum
- (ii) Evaluate the power of the signal

Q8. (a) A signal $x(n)$ has the following furrier transform. $X(\omega) = \frac{1}{1 - ae^{-j\omega}}$ 08

Determine the furrier transform of the following signals.

(i) $x(2n+1)$

(ii) $x(n) * x(-n)$

(iii) $x(n) \cos(0.3\pi n)$

(iv) $e^{jn\pi/2} x(n+2)$

(b) Show that the energy of a real-valued energy signal is equal to the sum of energies of its even and odd components. 3 2/3

8(b): first we prove that,

$$\sum_{n=-\alpha}^{\alpha} x_e(n) x_o(n) = 0 \quad \text{*****}$$

$$\sum_{n=-\alpha}^{\alpha} x_e(n) x_o(n) = \sum_{m=-\alpha}^{\alpha} x_e(-m) x_o(-m) = - \sum_{m=\alpha}^{\alpha} x_e(m) x_o(m)$$

$$= - \sum_{n=-\alpha}^{\alpha} x_e(n) x_o(n)$$

$$\text{Then, } \sum_{n=-\alpha}^{\alpha} x(n) = \sum_{n=-\alpha}^{\alpha} [x_e(n) + x_o(n)]$$

$$= \sum_{n=-\alpha}^{\alpha} x_e(n) + \sum_{n=-\alpha}^{\alpha} x_o(n) + \sum_{n=-\alpha}^{\alpha} 2x_e(n)x_o(n)$$

$$= E_e + E_o$$

N.B. Answer six questions, taking three from each section.
 The questions are of equal value.
 Use separate answer script for each section.

SECTION-A

- Q1.** (a) Classify the following signals according to whether they are
- (i) One dimensional or multi dimensional 2
 - (ii) Single or multi channel
 - (iii) Continuous time or discrete time
 - (iv) Continuous valued or discrete valued 10

Give a brief explanation

(i) the trajectory of an airplane M S C C

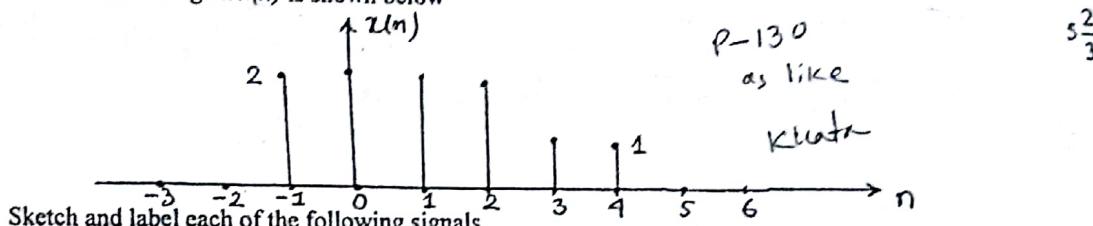
(ii) a color movie P-37 M M/S C C

(iii) number of boys and girls born in the world everyday. 0 M DD

Marks
6

P-6

- ✓ (b) A discrete time signal $x(n)$ is shown below



$5\frac{2}{3}$

Sketch and label each of the following signals.

- (i) $x_a(n) = x(n+2)$
- (ii) $x_b(n) = x(4-n)$

- Q2.** (a) Consider the analog signal:

$$x_a(t) = 3\cos 2000\pi t + 5\sin 6000\pi t + 10\cos 12000\pi t$$

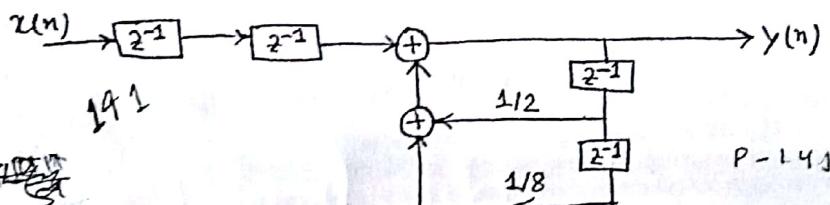
$09-26$

$5\frac{2}{3}$

- (i) What is the Nyquist rate for this signal?
- (ii) Assume now that we sample this signal using a sampling rate $F_s = 5000$ samples/S. What is the discrete time signal obtained after sampling?
- (iii) What is the analog signal $y_a(t)$. We can reconstruct from the samples if we use ideal interpolation?

- ✓ (b) Determine and sketch the impulse response of the following systems for $n=0, 1, 2, \dots, 9$.

4



What is the advantage of digital signal over analog signal? 08-1 ~ 5

2
6

- Q3.** (a) Consider the system $y(n) = T\{x(n)\} = x(n^2)$

- (i) Determine if the system is time-variant.
- (ii) To clarify the result in (a), assume that the signal $x(n) = \{1, 1, 1, 1\}$ is applied to the system.

Determine,

$$\begin{aligned} y(n) &= T\{x(n)\}, \quad x_1(n) = x(n-2) \\ y_2(n) &= T\{x_2(n)\}, \quad y_2(n) = y(n-2) \end{aligned}$$

$Y30$
Ch-2 (Exercise)

Compare $y_2(n)$ and $y_1(n)$

A discrete time system can be → static or dynamic, → causal or non causal, → linear or non linear, → time-invariant or time variant, → stable or unstable. Examine the following system w.r.t. the properties above

$$(i) \quad y(n) = \cos(x(n)) \quad Y31$$

$$(ii) \quad y(n) = \text{Sign}(x(n))$$

- ✓ (b) What is the difference between systems and signals? P-2 3

$1\frac{2}{3}$
5

- Q4.** (a) Consider the following three operations:

- (i) Multiply the integer numbers 131 and 122.
- (ii) Multiply the polynomials $x^2 + 3x + 1$ and $x^2 + 2x + 2$
- (iii) Compute the convolution $\{1, 3, 1\} * \{1, 2, 2\}$

Comment on your results.

P-134

4

- (b) If $y(n) = x(n) * h(n)$, Show that

$$\sum_{n=-\infty}^{\infty} y(n) = \sum_{n=-\infty}^{\infty} x(n) \cdot \sum_{n=-\infty}^{\infty} h(n) \quad Y33$$

✓ (c) Show that folding and time delaying (or advancing) a signal are not commutative P-52

$2\frac{2}{3}$

SECTION-B

Q5. (a) Compute the convolution $y(n)=x(n)*h(n)$ of the following sequences and check the correctness of the results by using the test in 4(b)

- 133*
 (i) $x(n)=\{1, 1, 1, 1\}$, $h(n)=\{1, 1, 1\}$
 (ii) $x(n)=a^n u(n)$, $h(n)=b^n u(n)$, $|a|<1$, $|b|<1$

(b) Consider the first order discrete time system described by the difference equation
 $y(n)+ay(n-1)=x(n)$

- (i) Find the causal impulse response $h(n)$ of the system
 (ii) Find general formulas for the zero-state and zero input response of the given system depending on the initial value $y(-1)$ and the input signal $x(n)$

Q6. (a) A linear time-invariant system is characterized by the system function $H(z)=\frac{3-4z^{-1}}{1-3.5z^{-1}+1.5z^{-2}}$

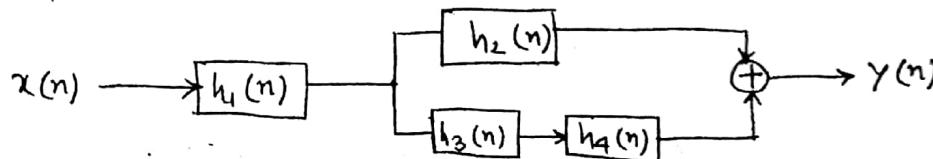
Specify the ROC of $H(z)$ and determine $h(n)$ for the following conditions:

- (i) the system is stable
 (ii) the system is causal
 (iii) the system is anticausal

(b) Determine the z-transform of the signal $x(n)=(\cos \omega_0 n)u(n)$ P-158
 (c) Consider the special case of a finite-duration sequence given as $x(n)=\{2, 4, 0, 3\}$. Reserve the sequence $x(n)$ into a sum of weighted impulse response.

Q7. (a) Consider the interconnection of LTI system as shown below

P-137



- (i) Express the overall impulse response $h(n)$ in terms of $h_i(n)$, $i=1, 2, 3, 4$
 (ii) Determine $h(n)$ when

$$h_1(n)=\left\{\frac{1}{2}, \frac{1}{4}, \frac{1}{2}\right\}, h_2(n)=h_3(n)=(n+1)u(n), h_4(n)=\delta(n-2)$$

(b) Determine the z-transforms of the following signals and sketch ROC

(i) $x(n)=\{2, 0, 0, 0, 0, 6, 1, -4\}$

(ii) $x(n)=\begin{cases} \left(\frac{1}{2}\right)^2 & n \geq 5 \\ 0 & n \leq 4 \end{cases}$ P-214

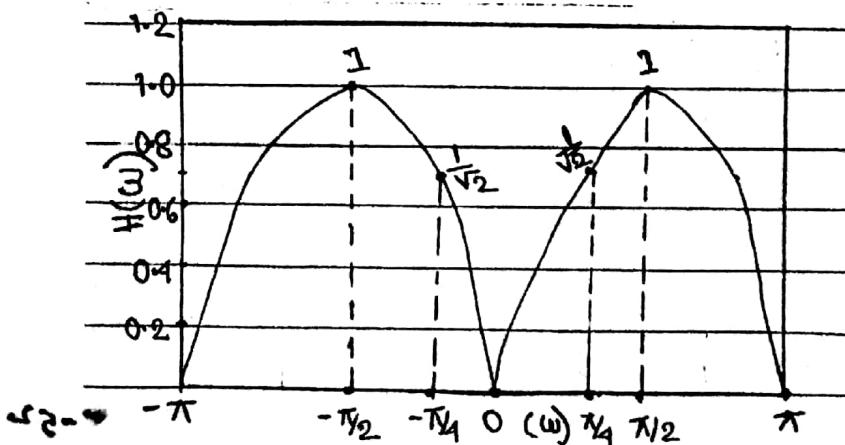
Q8. (a) Determine the causal signals $x(n)$ corresponding to the following z-transforms

(i) $X(z)=\frac{1+3z^{-1}}{1+3z^{-1}+2z^{-2}}$

(ii) $X(z)=\frac{1+2z^{-1}+z^{-2}}{1-3z^{-1}+2z^{-2}}$ P-216, 217

(iii) $X(z)=\frac{z^{-6}+z^{-7}}{1+z^{-1}}$

(b) Design a two-pole band pass filter for the following magnitude versus phase diagram.



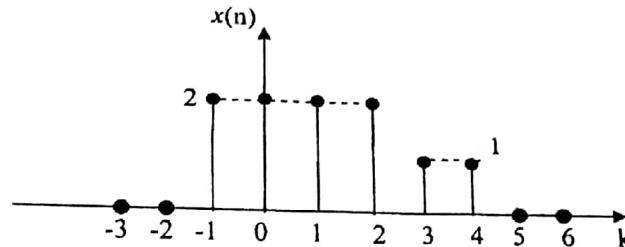
3/3

N.B.

1. Answer any SIX questions, taking THREE from each section.
2. Figures in the margin indicate full marks.
3. Use separate answer script for each section.

SECTION-A

Q1(a) ~~(b)~~ Discuss some advantages of Digital Signal Processing over Analog Signal Processing? → 08-10 → 03
04



Sketch and label each of the following signals

- i) $x_a(n) = x(n-2)$
- ii) $x_b(n) = x(n) + \frac{1}{2}x(n-1)$
- iii) $x_c(n) = x(n).u(2-n)$

✓(c) Examine whether the following signals are (i) causal or non-causal, (ii) LTI or non-LTI → 13 → 04%
 and (iii) stable or unstable → 13

- i) $y(n) = x(n).\cos(\omega_0 n)$
- ii) $y(n) = x(-n+2)$
- iii) $y(n) = x(n).u(n)$

Q2(a) ~~(b)~~ Define Nyquist sampling rate. → 2 → 02%

Consider the analog signal → 2 → 05

$$x_a(t) = 3\cos 2000\pi t + 5\sin 6000\pi t + 10\cos 12000\pi t$$

- (i). What is the Nyquist sampling rate for this signal?
- (ii). Assume now that we sample this signal using a sampling rate, $F_s=5000$ samples/s. What is the discrete-time signal obtained after sampling?

✓(c) Draw graphical representation of

- i) Unit step signal P-43.44
- ii) Unit ramp signal
- iii) Unit sample sequence

Q3(a) Determine the response of the following systems to the input signal 04

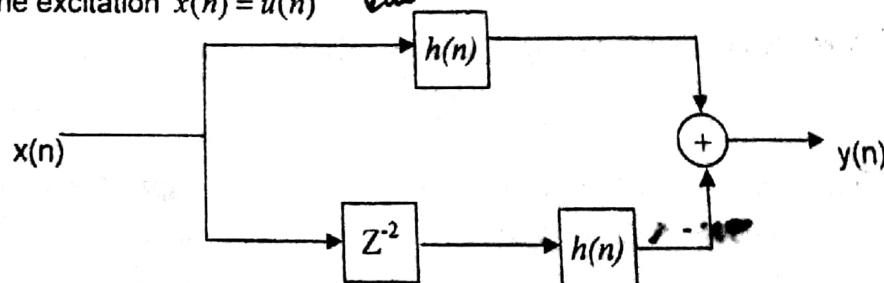
~~5~~ $x(n) = \begin{cases} |n|, & -3 \leq n \leq 3 \\ 0, & \text{otherwise} \end{cases}$ ← note

- i) $y(n) = x(n)$
- ii) $y(n) = \max\{x(n+1), x(n), x(n-1)\}$

✓(b) Show that any signal can be decomposed into an even and odd part. Derive the expression for the even and odd parts. → 03 → 49

✓(c) Consider the system below with $h(n) = a^n u(n)$ $|a|<1$. Determine the response $y(n)$ of the system for the excitation $x(n) = u(n)$ 04

P-137



Q4(a) Show that the response of an LTI system with an input $x[n]$ can be expressed as
 $y(n) = x(n) * h(n)$ [P-177]

(b) The impulse response of a linear time-invariant system is $h(n) = \{1, 2, 1, -1\}$. Determine the response of the system to the input signal $x(n) = \{1, 2, 3, 1\}$. ↑ 08-36

(c) What is meant by correlation? Determine the cross-correlation sequence $r_{xy}(L)$ of the sequences

$$x(n) = \{\dots, 0, 0, 2, -1, 3, 7, 1, 2, -3, 0, 0, \dots\}$$

$$y(n) = \{\dots, 0, 0, 1, -1, 2, -2, 4, 1, -2, 5, 0, 0, \dots\}$$

SECTION-B

Page - 18

Q5(a) Determine the response $y(n)$, $n \geq 0$ if the system

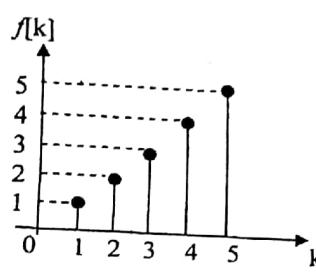
$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$$

to the input $x(n) = a^4 u(n)$ and the initial conditions $y(-1) = 0$, $y(-2) = 1$.

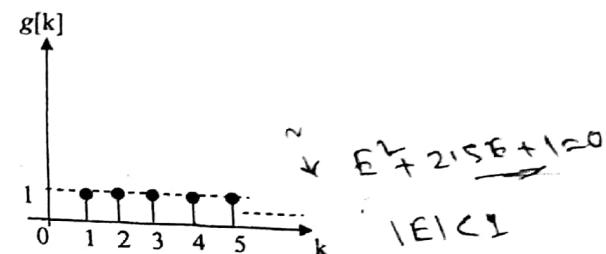
Determine the zero-input response and zero-state response

(b) Using the sliding tape method, convolve the following two sequences

06



P-134



|E| < 1

Q6(a) Find out natural and forced response from the following total response of a LTID system. Total response = $0.2(-0.2)^k + 0.8(0.8)^k - 1.26(4)^{-k} + 0.444(-0.2)^k + 5.81(0.8)^k$ P-96 02%

(b) Determine whether the systems specified by the following equations are asymptotically stable, marginally stable or unstable.

$$\text{i)} y[k+2] + 2.5y[k+1] - y[k] = f[k+1] - 2f[k] \quad E^{-k} + 2.5E^{-k+1} < 0$$

$$\text{ii)} y[k] - y[k-1] + 0.21y[k-2] = 2f[k-1] + 3f[k-2] \quad |E| < 1$$

(c) Find the Discrete-time Fourier Series (DTFS) for $f[k] = \sin 0.1\pi k$. Sketch the amplitude and phase spectra.

05

Q7(a) A signal $x(n)$ has the Fourier transform $X(e^{j\omega})$. Determine the Fourier transforms of the following signals i) $x(n) * x(n-1)$ ii) $x(n)\cos(0.3\pi n)$ P-298 04

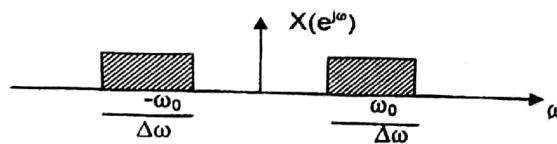
(b) Draw the butterfly figure of eight-point decimation-in-time FFT algorithm. P-523 04

(c) Determine the signals corresponding to the following Fourier transforms

$$X(e^{j\omega}) = \begin{cases} 1 & \omega_0 - \frac{\Delta\omega}{2} \leq |\omega| \leq \omega_0 + \frac{\Delta\omega}{2} \\ 0 & \text{otherwise} \end{cases}$$

P-298 - 294

4.10 294 page



Q8(a) Determine the z-transforms of the following signals and sketch the ROC

$$\text{i). } x(n) = \{3, 0, 0, 0, 0, 6, 1, -4\}$$

$$\text{ii) } x(n) = \begin{cases} (\frac{1}{2})^n - 2^n & n \geq 0 \\ 0 & n < 0 \end{cases}$$

P-294

04

(b) Draw the canonical realization of a third-order transfer function

$$H[z] = \frac{b_3z^3 + b_2z^2 + b_1z + b_0}{z^3 + a_2z^2 + a_1z + a_0}$$

04%

(c) "The performance of recursive filters to be superior than non-recursive filters" – Discuss briefly. 03

N.B.

1. Answer any SIX questions, taking THREE from each section.
2. Figures in the margin indicate full marks.
3. Use separate answer script for each section.

SECTION-A

Q1. What are the advantages of Digital over Analog signal processing? *khata* 03%
04

(i) An analog signal contains frequencies up to 10KHz.

(ii) What range of sampling frequencies allows exact reconstruction of this signal from its samples?

(iii) Suppose that we sample this signal with a sampling frequency $F_s = 8\text{ KHz}$. Examine what happens to the frequency $F_1 = 5\text{ KHz}$ and $F_2 = 9\text{ KHz}$. *p-38*

Q2(a) A digital communication link carries binary coded words representing samples of an input signal 04

$$x_a(t) = 3\cos 600\pi t + 2\cos 1800\pi t$$

— 39 —

The link is operated at 10,000 bits/sec and each input sample is quantized into 2048 different voltage levels.

(i). What is the sampling frequency and the folding frequency?

(ii). What is the Nyquist rate for the signal $x_a(t)$?

(iii). What are the frequencies in the resulting discrete-time signal $x(n)$?

(iv). What is the resolution, Δ .

Q2(b) Determine whether the following systems are causal or noncausal and describe their reasons: 03%
04

(i). $y(n) = x(n) + 3x(n+4)$

(ii). $y(n) = x(n^2) + x(2n)$

(iii). $y(n) = x(-n)$

— 67 —

67

Distinguish between static and dynamic system with proper example. *khata* 60 — 62 04

What are the purpose of a unit delay element and a unit advance element of discrete time system? *khata* 58 04

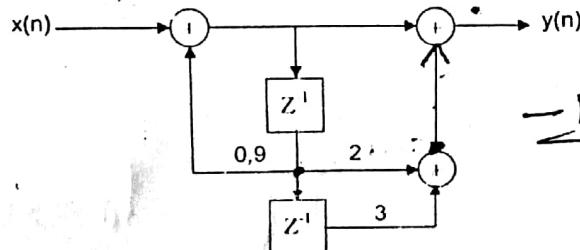
Q3. What is convolution? What role does it play in determining system response? *khata* 03%
04

The impulse response of a linear time-invariant system is $h(n) = \{1, 2, 1, -1\}$. Determine the response of the system to the input signal $x(n) = \{1, 2, 3, 1\}$. *p-75*

Q4. Determine the local solution $y(n)$, $n \geq 0$, to the difference equation $y(n) + a_1y(n-1) = x(n)$ where $x(n)$ is a initial unit step sequence. *khata* 103 04

Q4. State and prove the time reversal property of the Z-transform. *khata* 162 03%
04

Consider the following discrete-time system.



— 149 —
p-140

- (i). Compute the first six values of the impulse response of the system
- (ii). Compute the first six values of the zero-state response of the system.

Q5. Compute the correlation sequences $r_{xx}(l)$ and $r_{xy}(l)$ for the following signal sequences. 04

$$x(n) = \begin{cases} 1, & n_0 - N \leq n \leq n_0 + N \\ 0 & \text{otherwise} \end{cases}$$

p-147

04

$$y(n) = \begin{cases} 1, & -N \leq n \leq N \\ 0 & \text{otherwise} \end{cases}$$

p-143

2^{N+1}-1
n_0 - 2^N \leq n \leq n_0

SECTION-B

Q5(a) Show that the ROC for an infinity duration two sided signal is an annular region in the Z-plane. 158 mark 03%

Determine the Z-transform and the ROC of the signal

$$x(n) = a^n u(n) = \begin{cases} a^n & n \geq 0 \\ 0 & n < 0 \end{cases} \quad -151$$

$$(z-z_1) \cdots (z-z_M)$$

Determine the pole-zero plot for the signal

$$\sum_{n=0}^{\infty} z(a^{-1})^n = \frac{1}{1-az^{-1}}$$

$$x(n) = \begin{cases} a^n & 0 \leq n \leq 8 \\ 0 & \text{otherwise} \end{cases}$$

$$-175 \quad \text{mark}$$

$$z = r + i\omega \quad M-1 = 8$$

$$M=9 \quad z = e^{j\omega T}$$

Q6(a) What is matched Z-transformation? -681

b) Determine the response of the system $y(n) = \frac{1}{6}y(n-1) - \frac{1}{6}y(n-2) + x(n)$ to the input signal $x(n) = \delta(n) - \frac{1}{3}\delta(n-1)$. -198

c) Design a two-pole band-pass filter that has the center of its pass-band at $\omega = \pi/2$, zero in its frequency response characteristic at $\omega = 0$ and $\omega = \pi$, and its magnitude response is $1/\sqrt{2}$ at $\omega = 4\pi/9$. -332

Q7(a) Briefly explain the transformation process of low-pass filter to high-pass filter.

A linear time invariant system is characterized by the system function

$$H(z) = \frac{3-4z^{-1}}{1-3.5z^{-1}+1.5z^{-2}} \quad 197$$

has poles at $z = \frac{1}{2} \pm j\frac{\sqrt{3}}{2}$ 209

Specify the ROC of $H(z)$ and determine $h(n)$ for the following conditions:

ROC must be inside the unit circle

- (i). The system is stable
- (ii). The system is causal
- (iii). The system is anticausal.

Determine and sketch the energy density spectrum $s_x(\omega)$ of the signal $x(n) = a^n u(n)$, $-1 < a < 1$ -256

Q8(a) Write short note about Notch filters and Comb filters.

b) What is Gibbs phenomenon? Describe with necessary example. -339, 345

c) Convert the analog band-pass filter with system function

$$H_a(s) = \frac{1}{(s+0.1)^2 + 9}$$

$$P = \frac{1}{705}, \frac{1}{705}$$

into a digital IIR filter by use of the mapping $s = \frac{1}{T}(z - z^{-1})$

$$H(z) = \frac{1}{\left(\frac{z-z'}{T} + 0.1\right)^2 + 9}$$

$$= \frac{1}{\left(\frac{z-z'}{T}\right)^2 + 2\left(\frac{z-z'}{T}\right)0.1 + 0.01 + 9}$$

$$= \frac{z^2 - 2z'z + z'^2 + 0.02(z-z')^2 + 9}{T^2}$$

$$= \frac{T^2}{z^2 - 2z'z + 0.02(z-z')^2} \quad n < 0$$

0

SECTION A

Q. 1 Briefly describe the Analog-to-Digital and Digital-to-Analog conversion process. Marks 17

Ans: (P-17)

Q. 2 Consider the analog signal $x_a(t) = 3\cos(200\pi t) + \sin(100\pi t) + 100\cos(2000\pi t)$

(i) What is the Nyquist rate of the signal? 09-16

(ii) Assuming a sampling rate $f_s = 5000$ samples/sec of this signal. Determine the discrete time signal obtained after sampling.

Ans: (P-17)

Q. 3 Define Unit sample sequence and Unit step signal with example. P-13

Ans: (P-13)

Q. 4 Define Energy signals and Power signals. Q5

Ans: (P-5)

Q. 5 The accumulator $y(n) = \sum_{k=0}^{n-1} x(k) - 3x(n) + x(n-1) - x(n-2)$ is excited by

the sequence $x(n) = u(n)$. Determine its output under the condition

(i) It is initially relaxed. 57

(ii) Initially $x(-1) = 1$ P-57

Q. 6 What is the necessary and sufficient condition of a linear-time invariant system to be causal. P-66

Q. 7 Express the Z-transform of $x(n) = \sum_{k=0}^n u(k)$ in terms of $X(z)$. 215

Ans: (P-215)

Q. 8 Distinguish between

(i) Static and Dynamic systems 60-08-26

(ii) Time-invariant and Time-variant systems 60

(iii) Linear and Non-linear systems 63

Q. 9 Determine the range of values of a and b for which the linear time invariant system with impulse response $h(n) = \begin{cases} a^n & n \geq 0 \\ b^n & n < 0 \end{cases}$ is stable. P-88

Q. 10 Determine whether the following systems are static or dynamic, linear or Non-linear, Time-variant or Time-invariant, Causal or Non-causal.

(i) $y(n) = \text{Cov}[x(n)]$ 130

(ii) $y(n) = x(n)\text{Cov}(x(n))$

(iii) $y(n) = x(n+2)$

(iv) $y(n) = n(x)$

(v) $y(n) = e^{nx}$

Q. 11 What is convolution? What role does it play in determining system response?

The impulse response of a linear time invariant system is $h(n) = \{1, 2, 1, -1\}$.

Determine the response of the system to the input signal $x(n) = \{1, 2, 3, 1\}$. 08-36 75

Q. 12 What is the normalized autocorrelation sequence of the signal $x(n)$ given by

$$x(n) = \begin{cases} 1, & n \leq N \\ 0, & \text{otherwise} \end{cases}$$

Ans: (P-14)

~~Ans: (P-37)~~

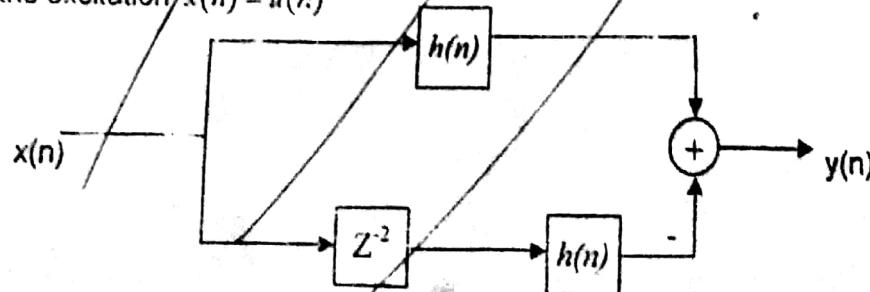
~~08-36 75~~

~~(2N+1-121)~~

~~2N+1~~

Page 1 of 2

(C) Consider the system below with $h(n) = a^n u(n)$ $|a| < 1$. Determine the response $y(n)$ of the system for the excitation $x(n) = u(n)$ U4



Page 1 of 2

SECTION- B

Q.5 (a) Find the DFT of the following sequences $x(n) = e^{(2\pi/N)n^2}$, $n = 0, 1, \dots, N-1$. 556

(b) Develop a radix-3 decimation-in-time FFT algorithm for $N = 3^k$ and draw the corresponding flow graph for $N = 9$. 557 559

(c) Compute the zero state response of the system described by the difference equation $y(n) + \frac{1}{2}y(n-1) = x(n) + 2x(n-2)$ to the input $x(n) = \{1, 2, 3, 4, 2, 1\}$ by solving the difference equation recursively. 139 (2.42)

Q.6 (a) Define ROC. Show that ROC for an infinity duration two-sided signal is an annular region in the Z-plane. 154

(b) Determine the causal signal $x(n)$ whose z transform is given by $X(z) = \frac{1+z^{-1}}{1-z^{-1}+0.5z^{-2}}$. 191 191

(c) Determine the system function and the unit sample response of the system described by the difference equation, $y(n) = \frac{1}{2}y(n-1) + 2x(n)$. 179

Q.7 (a) State and prove the Time shifting and Time reversal property of Z-transform. 159, 168 3

A linear time-invariant system is characterized by the system function

$H(z) = \frac{3-4z^{-1}}{1-3.5z^{-1}+1.5z^{-2}}$. Specify the ROC of $H(z)$ and determine $h(n)$ for the following conditions

(i) The system is stable

(ii) The system is causal

(iii) The system is anticausal.

(c) Determine and sketch the energy density spectra $S_{xx}(n)$ of the signal $x(n) = a^n u(-n-1)$, $-1 < a < 1$. 08-7c

Q.8 (a) How an IIR filter can be designed from an analog filter using bilinear transformation.

Show the mapping between the variables. 701

(b) On which conditions a stable analog filter will be converted to a stable digital filter? v- 3

(c) Convert the analog band pass filter with system function $H_a(s) = \frac{1}{(s+0.1)^2 + 9}$ into a digital IIR filter by use of the backward difference for the derivative. 08-8c

705

51

***** The End *****