Unit 4

Image Restoration

Frequency Domain Filters
(Part III)

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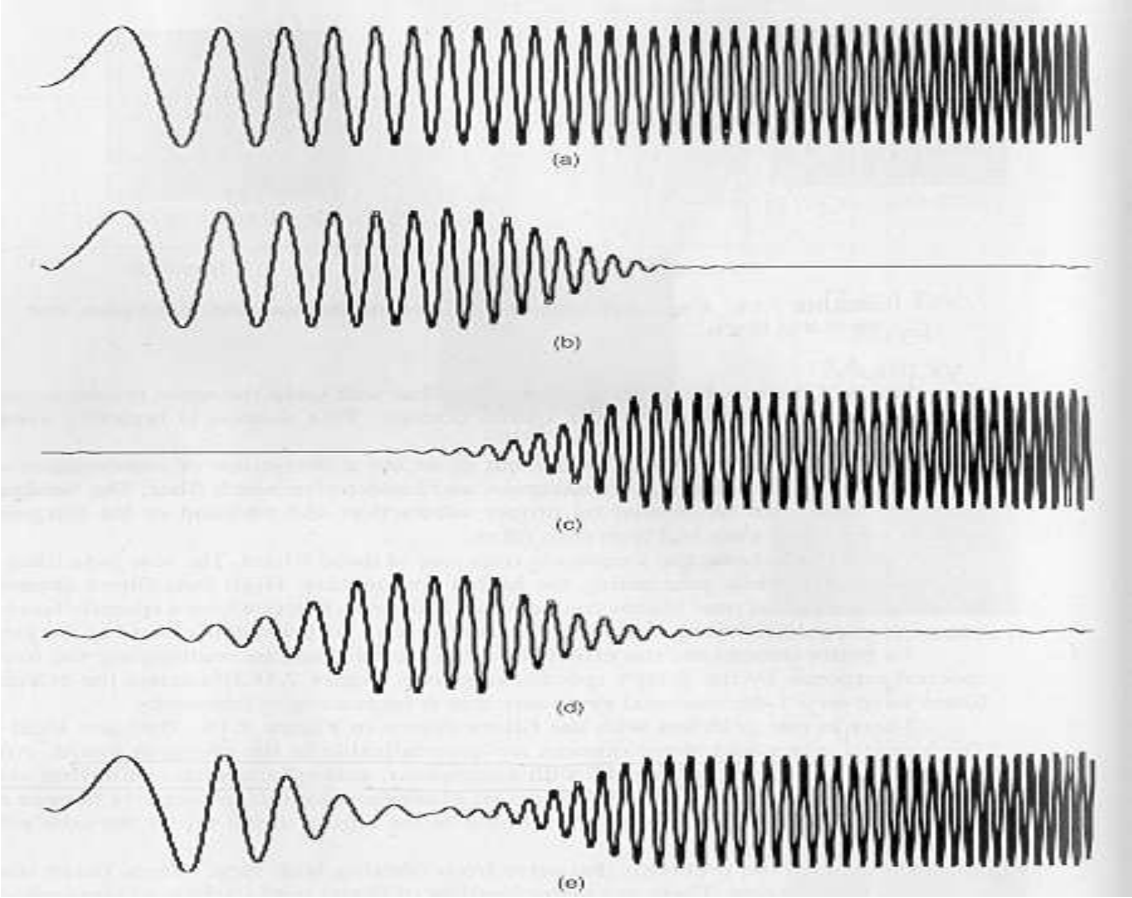
Filter

- Filter: A device or material for suppression or minimizing waves or oscillations of certain frequencies
- Frequency: The number of times that a periodic function repeats the same sequence of values during a unit variation of the independent variable.
- Filters are classified as (Frequency Domain):

  (1) Low-pass (2) High-pass

  (3) Band-pass (4) Band-stop ….many more

Filters Types



Original signal

Low-pass filtered

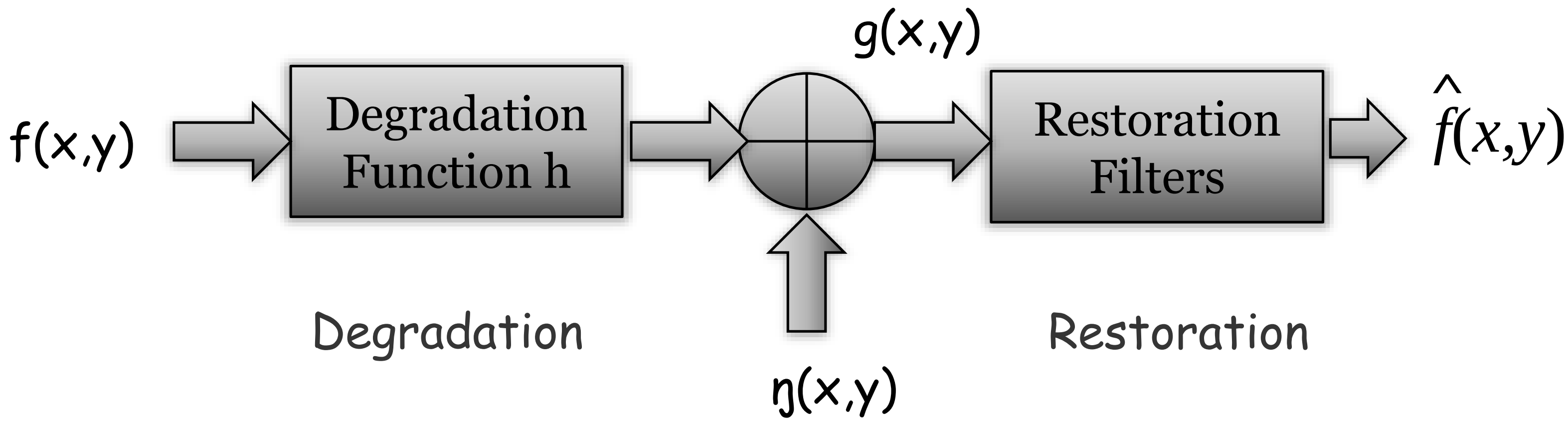
High-pass filtered

Band-pass filtered

Band-stop filtered

Image Restoration?

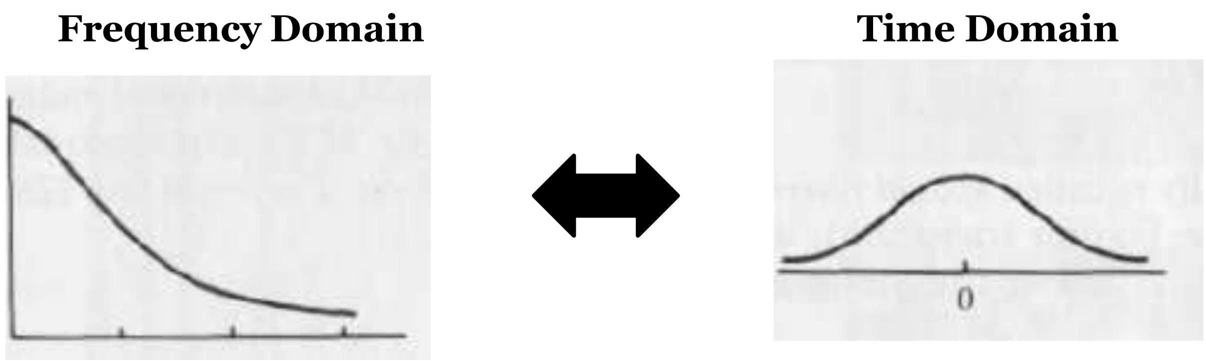
- Objective: To restore a degraded/distorted image to its original content and quality.



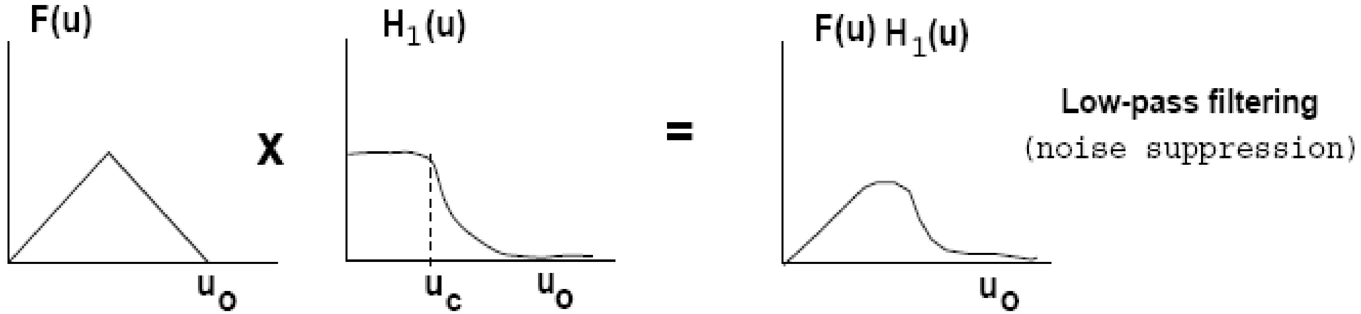
- Spatial Domain: $g(x,y)=h(x,y)*f(x,y)+\eta(x,y)$
- Frequency Domain: $G(u,v)=H(u,v)F(u,v)+\eta(u,v)$
- Matrix: $G=HF+\eta$

Low-Pass Filters (Smoothing filters)

- Preserve Low Frequencies-Useful For Noise Suppression



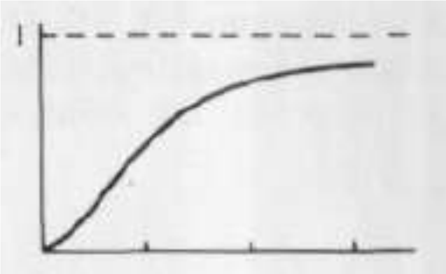
Example:

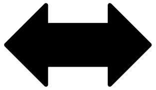


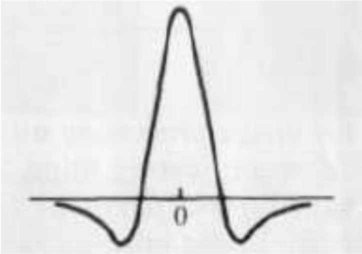
High-Pass Filters (Sharpening Filters)

- Preserves High Frequencies - Useful for Edge Detection

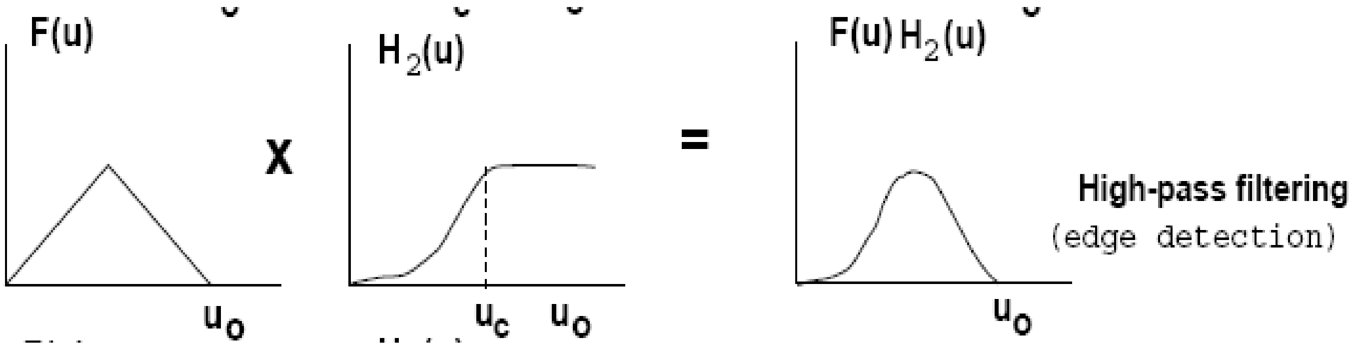
Frequency Domain **Time Domain**







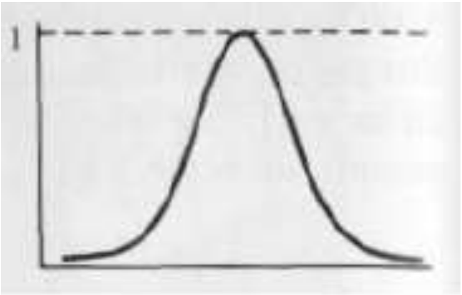
Example:

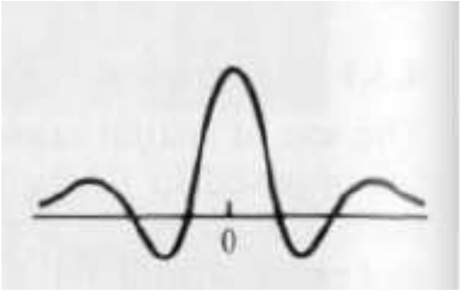


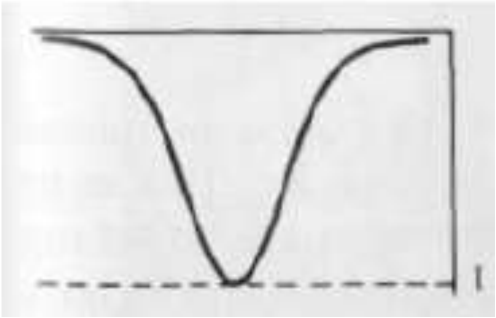
Band-Pass and Band Stop Filters

- Preserves Frequencies Within a Certain Band

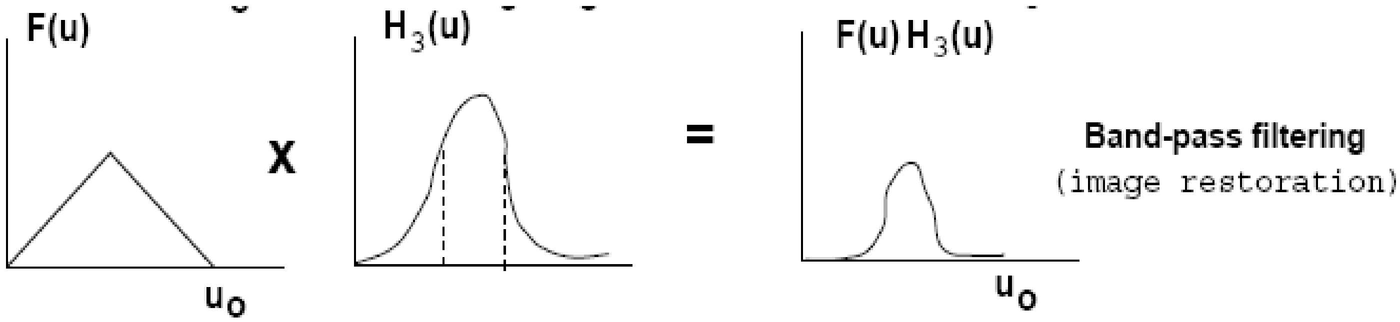
Frequency Domain **Time Domain** **Band Stop/ Reject**



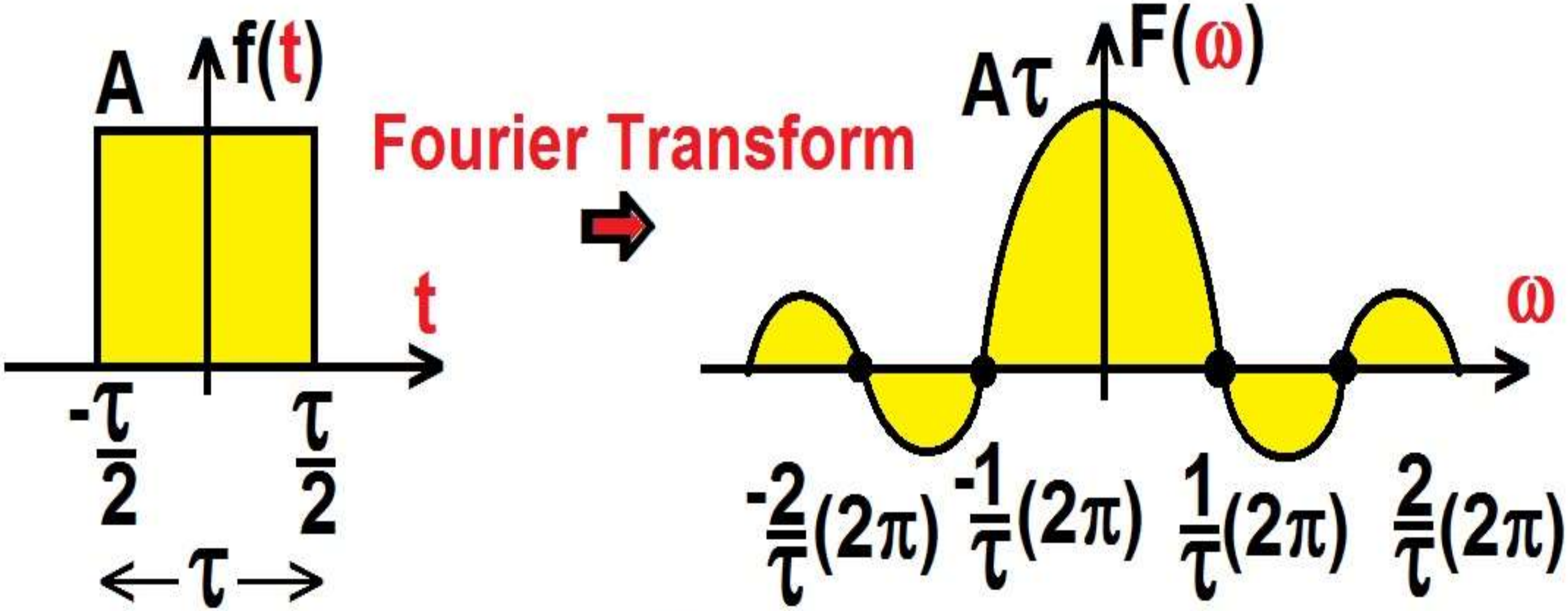




Example:



What is a Fourier Transform? Mathematical Def.



$$\mathcal{F}\left[f(t)\right] = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt \qquad j = \sqrt{-1}$$

$$f(t) = \mathcal{F}^{-1}\left[F(\omega)\right] = \int_{-\infty}^{\infty} F(\omega)\, e^{j\omega t}\, dt$$

Correction:
Before Integration
1/2pi

Image Processing and Fourier Transform

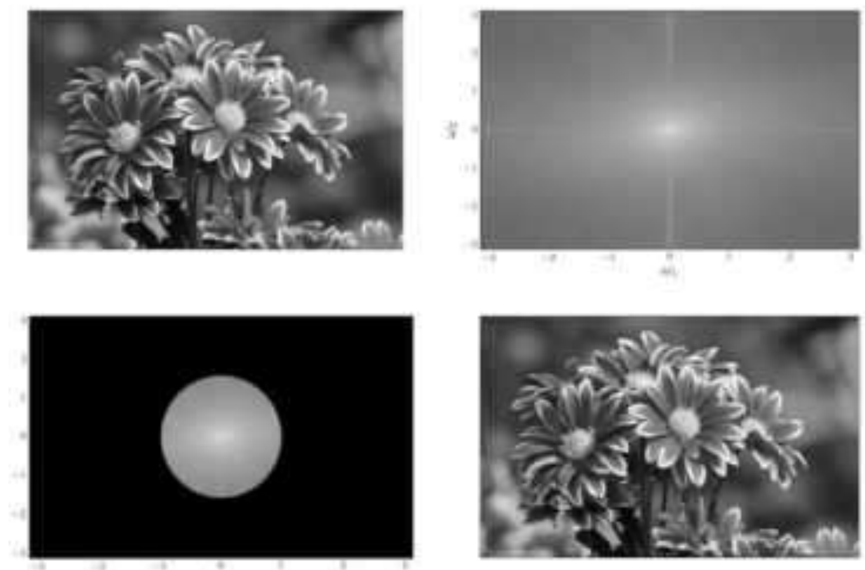
Input Image → **Fourier Transform** → **Do Operations** → **Inverse Fourier Transform**

Fourier Transform:

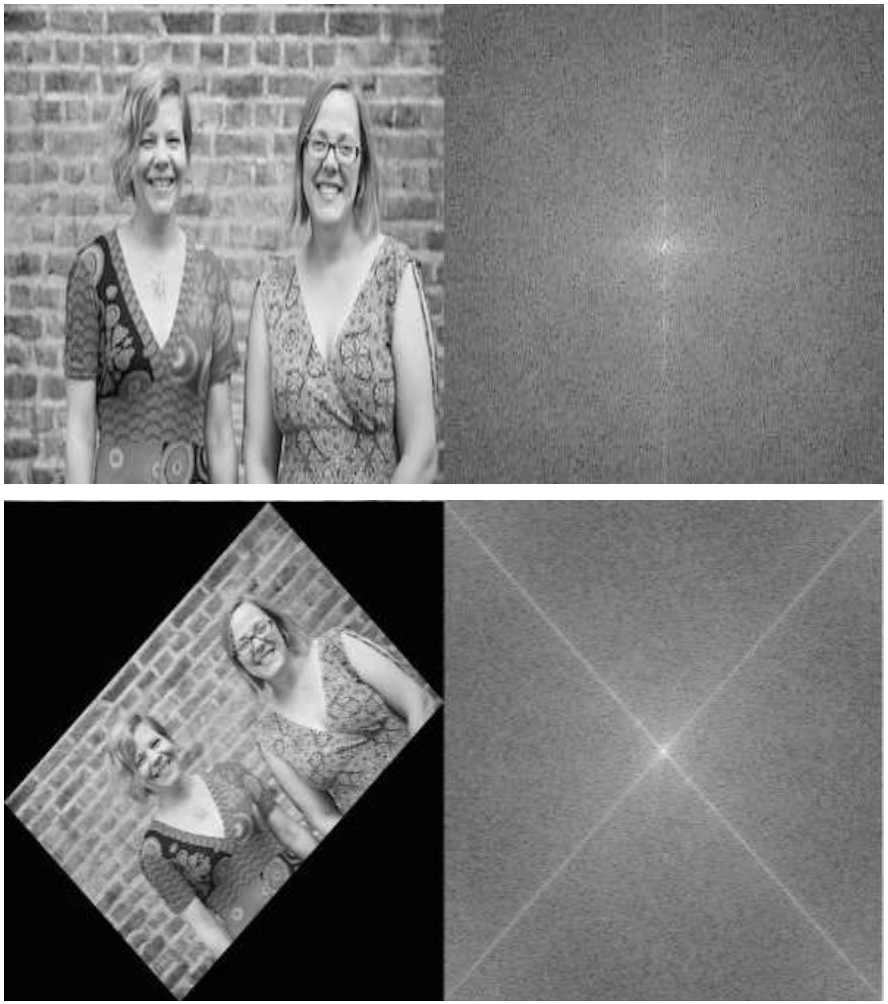
$$F(u,v) = \frac{1}{MN} \sum_{x=0}^{M} \sum_{y=0}^{N} f(x,y)\, e^{-j\,2\pi(ux/M + vy/N)}$$

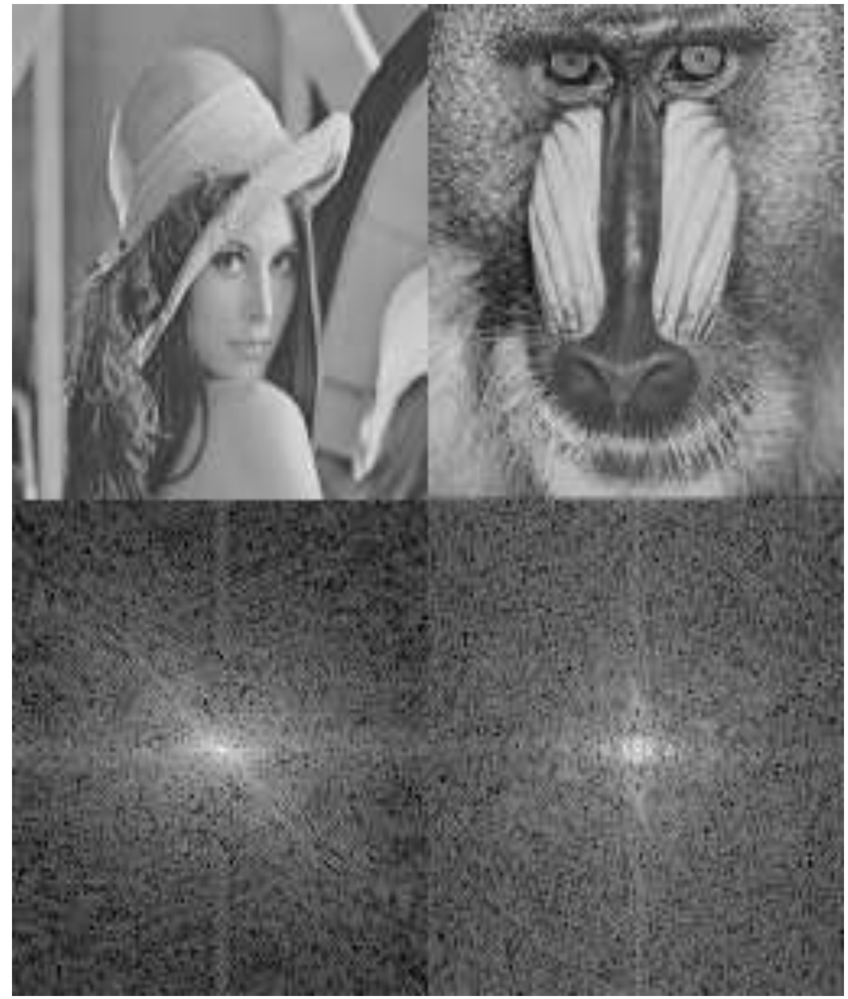
Inverse Fourier Transform

$$f(x,y) = \sum_{u=0}^{M} \sum_{v=0}^{N} F(u,v)\, e^{j\,2\pi(ux/M + vy/N)}$$

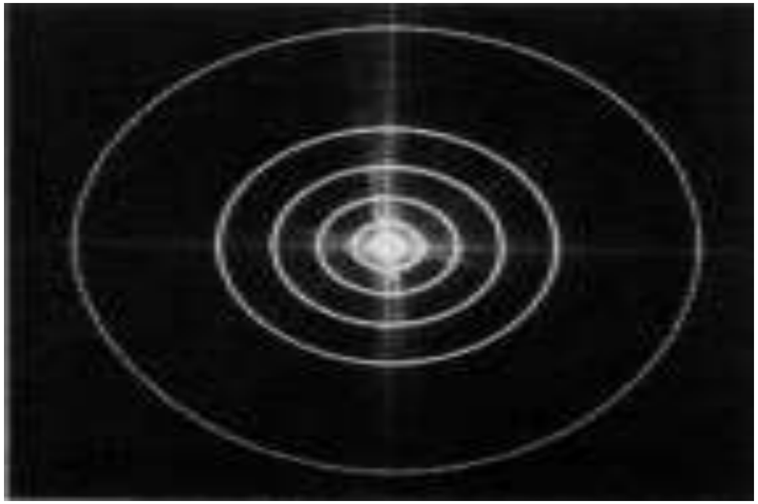


Fourier Transform





Fourier Spectrum



✓Percentage of image power enclosed in circles (Small to Large): 90, 95, 98, 99, 99.5, 99.9

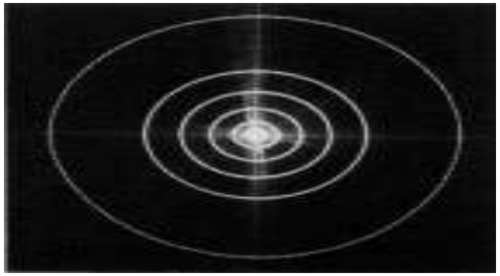
Fourier Transform

$g(x,y) = f(x,y) * h(x,y)$

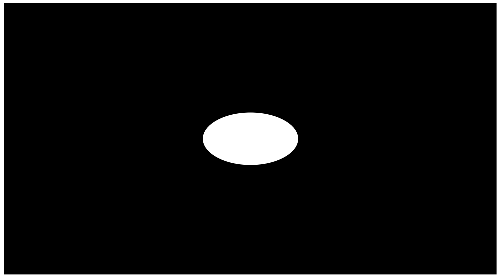
$G(u,v) = F(u,v) \cdot H(u,v)$

f(x,y)

F(u,v)



g(x,y)



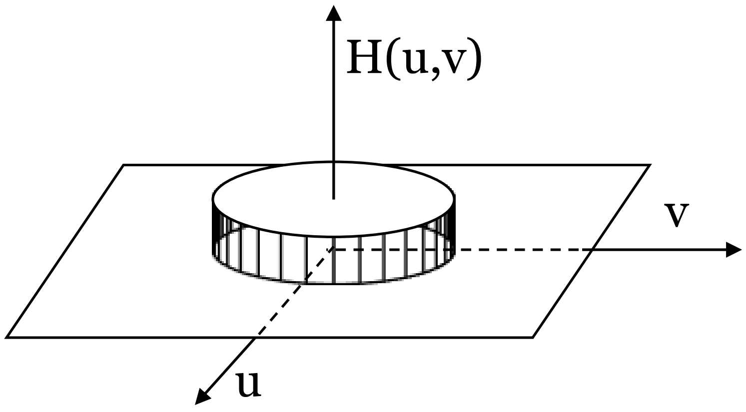
H(u,v)

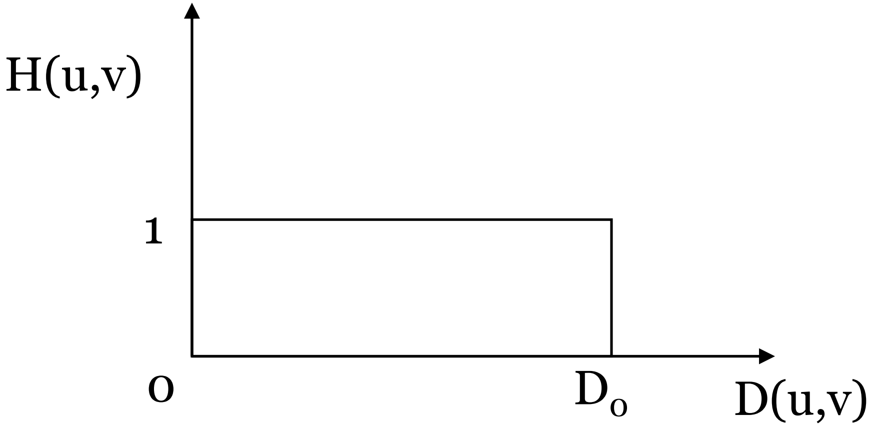
Ideal Low Pass Filters

$$H(u,v) = \begin{cases} 1 & D(u,v) \le D_o \\ 0 & D(u,v) > D_o \end{cases}$$

$$D(u,v) = \sqrt{u^2 + v^2}$$

D_o = cut off frequency





Blurring-Ideal Low Pass Filter

99.7%

99.37%

98.65%

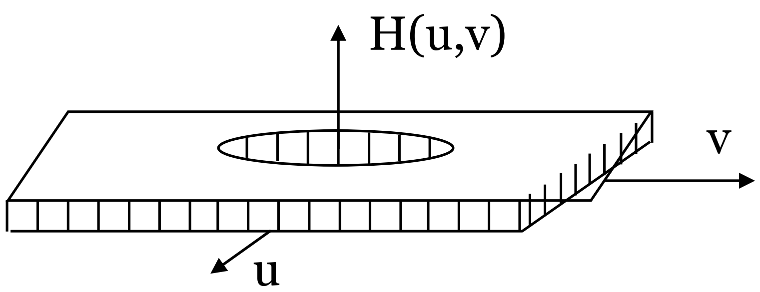
Image Sharpening - High Pass Filter

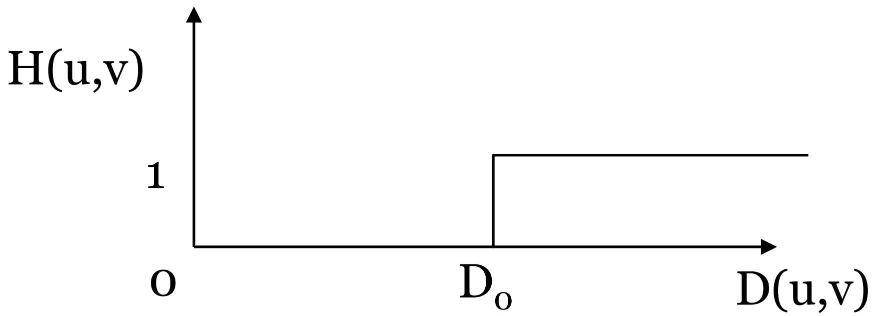
H(u,v) - Ideal Filter

$$H(u,v) = \begin{cases} 0 & D(u,v) \le D_0 \\ 1 & D(u,v) > D_0 \end{cases}$$

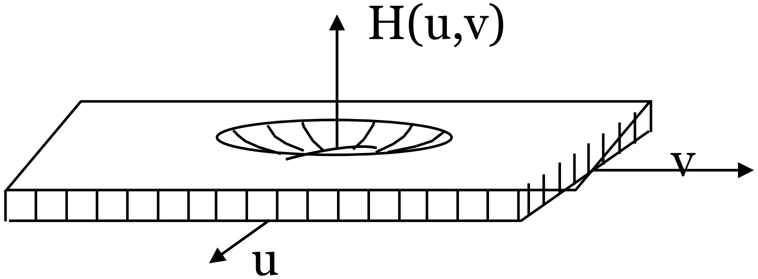
$$D(u,v) = \sqrt{u^2 + v^2}$$

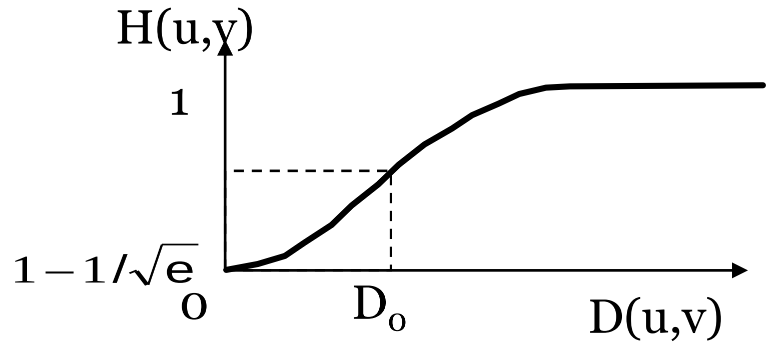
D_0 = cut off frequency





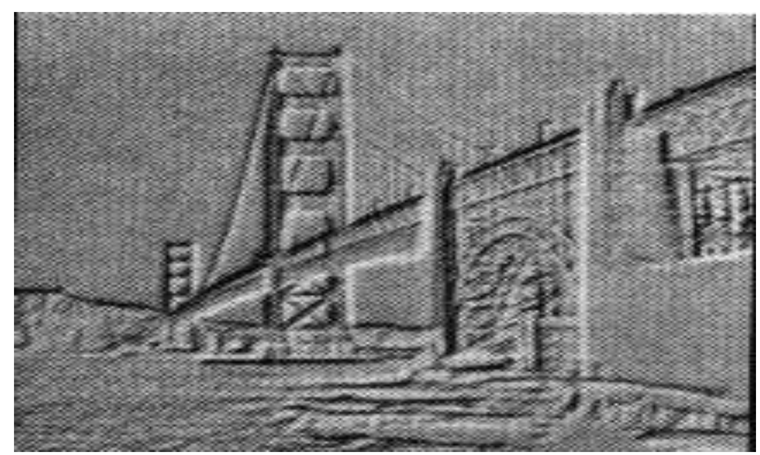
High Pass Gaussian Filter





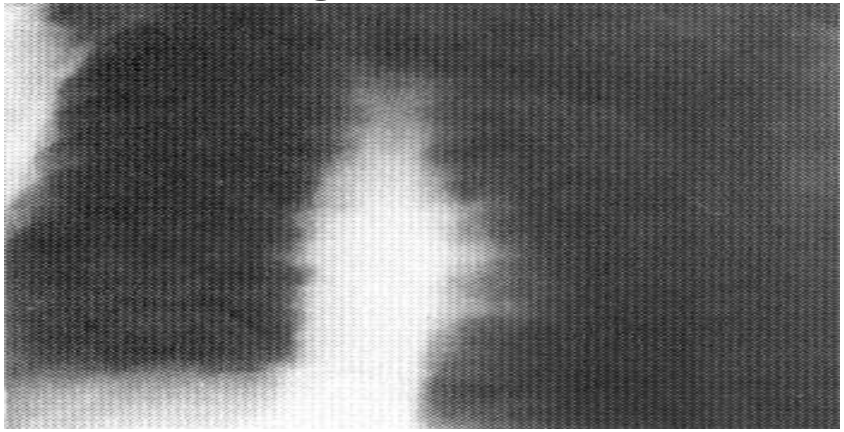
$$H(u,v) = 1 - e^{-D^2(u,v)/(2D^2{}_0)}$$

$$D(u,v) = \sqrt{u_2 + v_2}$$

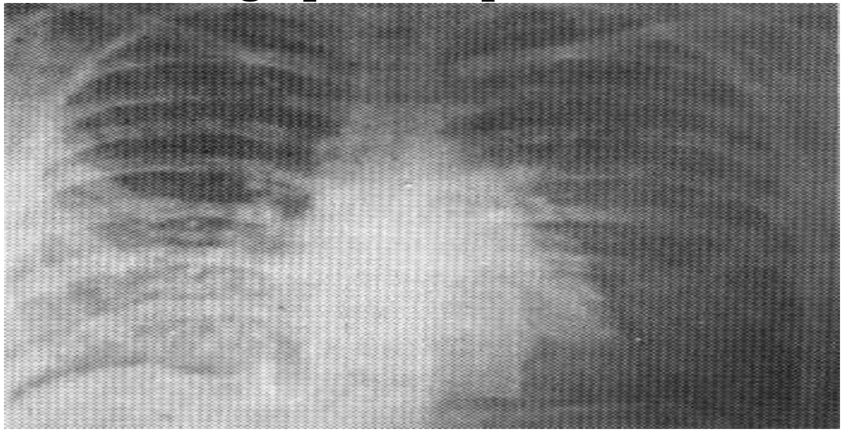


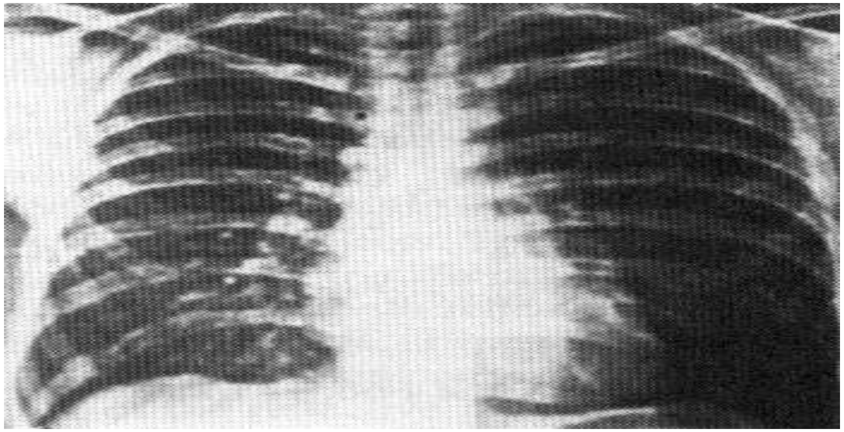
High Pass Filtering - Example

Original



High pass Emphasis





High Frequency Emphasis + Histogram Equalization

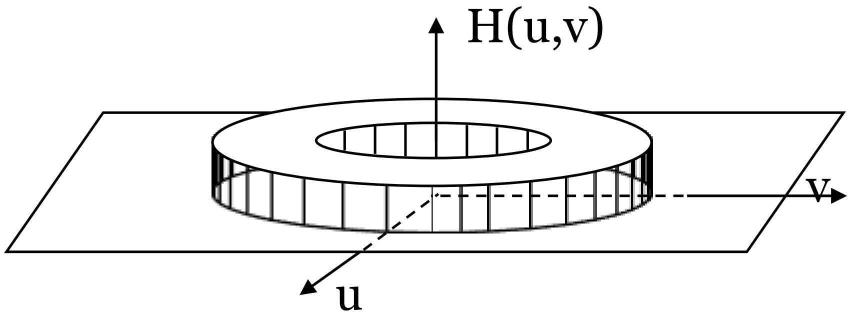
Band Pass Filtering

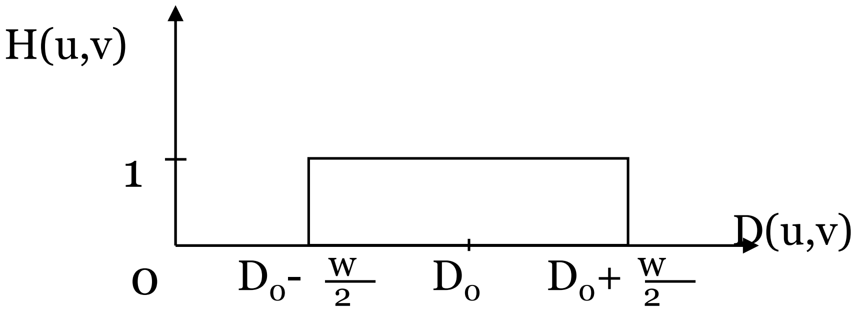
$$H(u,v) = \begin{cases} 0 & D(u,v) \le D_0 - \frac{w}{2} \\ 1 & D_0 - \frac{w}{2} \le D(u,v) \le D_0 + \frac{w}{2} \\ 0 & D(u,v) > D_0 + \frac{w}{2} \end{cases}$$

$D(u,v) = \sqrt{u^2 + v^2}$

D_0 = cut off frequency

w = band width





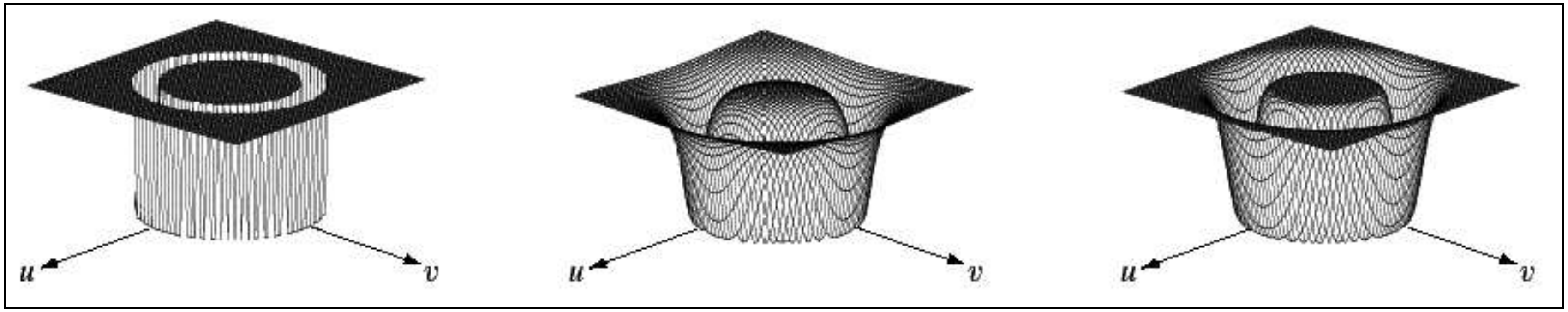
Band Reject Filters

- Removing periodic noise form an image involves removing a particular range of frequencies from that image.
- Band reject filters can be used for this purpose.
- An ideal band reject filter is given as follows:

$$H(u,v)=\begin{cases}1 & if\ D(u,v)<D_0-\dfrac{W}{2}\\ 0 & if\ D_0-\dfrac{W}{2}\leq D(u,v)\leq D_0+\dfrac{W}{2}\\ 1 & if\ D(u,v)>D_0+\dfrac{W}{2}\end{cases}$$

Band Reject Filters contd..

- The ideal band reject filter is shown below, along with Butterworth and Gaussian versions of the filter.



Ideal Band Reject Filter

Butterworth Band Reject Filter (of order 1)

Gaussian Band Reject Filter

Result of Band Reject Filter

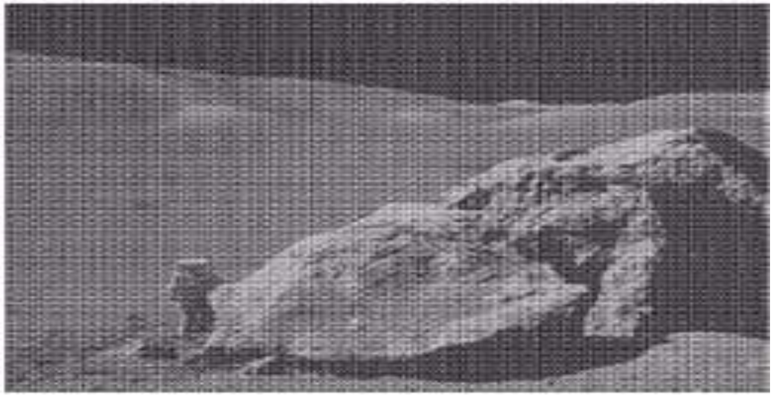


Fig: Corrupted by Sinusoidal Noise

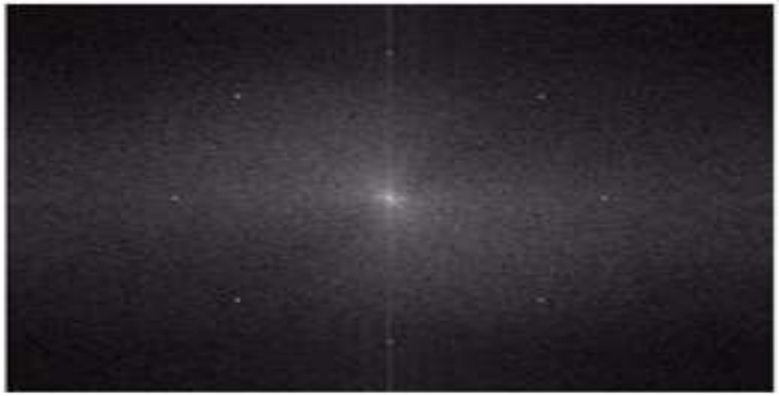


Fig: Fourier spectrum of Corrupted Image

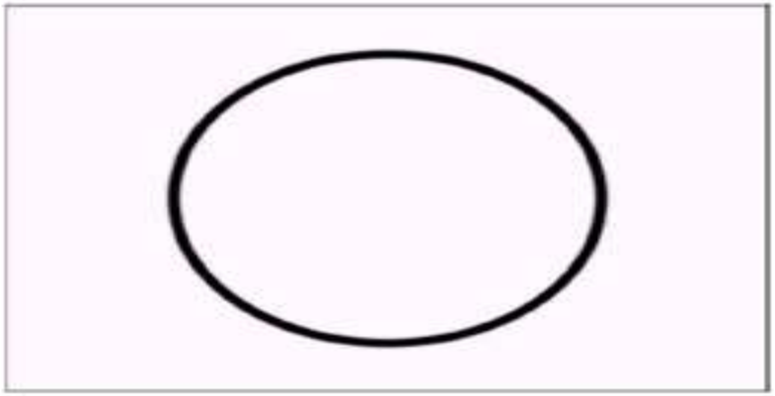


Fig: Butterworth Band Reject Filter

Fig :Filtered image

Adaptive Filters

- Adaptive, local noise reduction filter
  - If σ_η^2 is zero, return simply the value of $g(x,y)$
  - If $\sigma_\eta^2 < \sigma_L^2$, return a value close to $g(x,y)$
  - If $\sigma_\eta^2 = \sigma_L^2$, return the arithmetic mean value m_L

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_L^2}\left[g(x,y) - m_L\right]$$

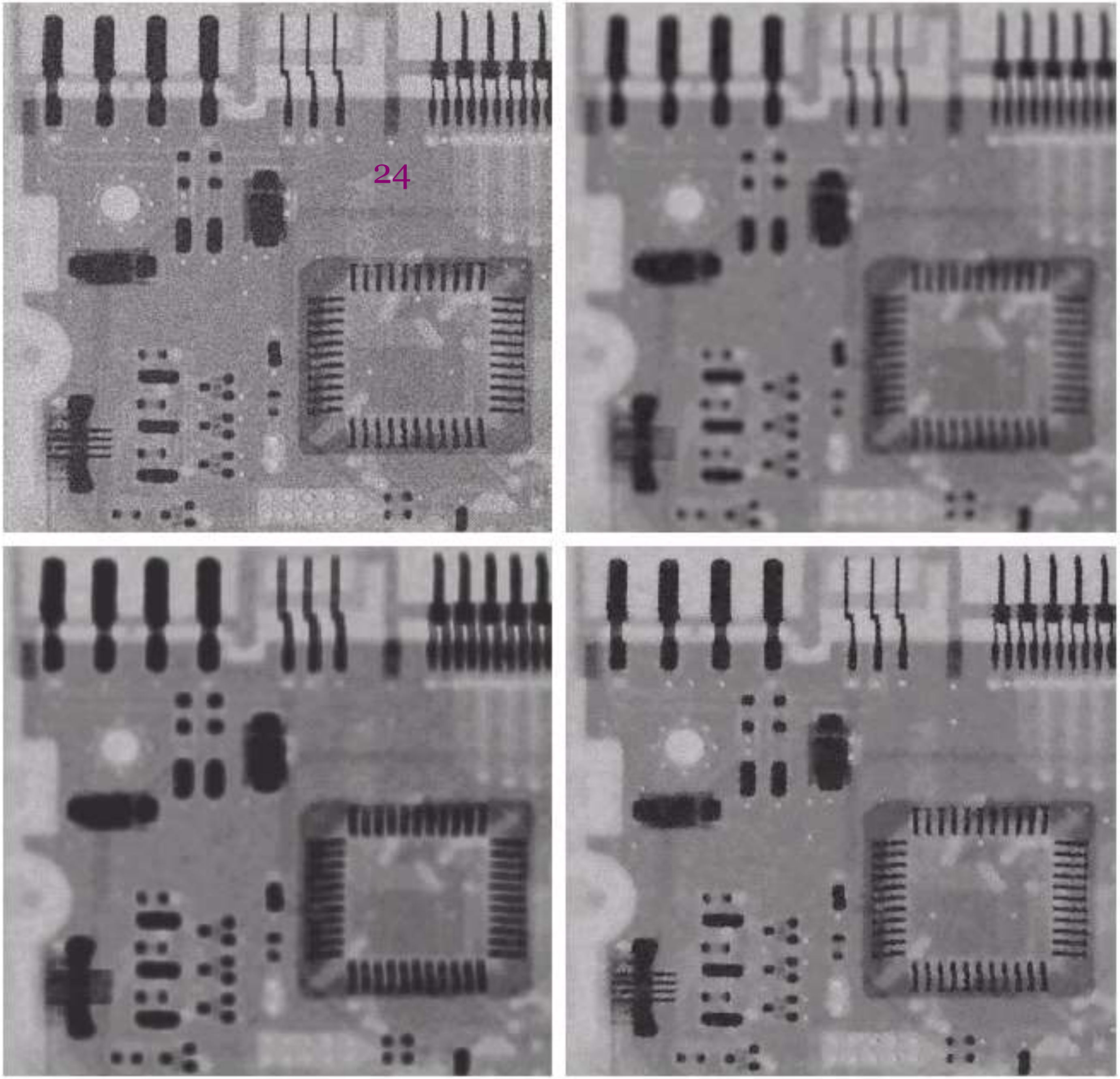
where σ_L^2 - Local variance of the local region

m_L - Local Mean

σ_η^2 -Variance of overall noise

g(x,y)- Pixel value at the position (x,y)

$$\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 3 & 0 & 4 \\ 0 & 0 & 5 & 0 \\ 6 & 0 & 0 & 7 \end{bmatrix}$$



a b
c d

FIGURE 5.13 (a) Image corrupted by additive Gaussian noise of zero mean and variance 1000. (b) Result of arithmetic mean filtering. (c) Result of geometric mean filtering. (d) Result of adaptive noise reduction filtering. All filters were of size 7×7.

Adaptive Median Filter

- Adaptive median filter
  - $z_{\min}$ = minimum gray level value in S_{xy}
  - $z_{\max}$ = maximum gray level value in S_{xy}
  - z_{med} = median of gray levels in S_{xy}
  - z_{xy} = gray level at coordinates (x, y)
  - $S_{\max}$ = maximum allowed size of S_{xy}

Algorithm

Algorithm:

Level A: A1= $Z_{med} - Z_{\min}$

A2= $Z_{med} - Z_{\max}$

If A1>0 AND A2<0, Go to level B

Else increase the window size

If window size $\leq S_{\max}$ repeat level A

Else output Z_{med}

Level B: B1= $Z_{xy} - Z_{\min}$

B2= $Z_{xy} - Z_{\max}$

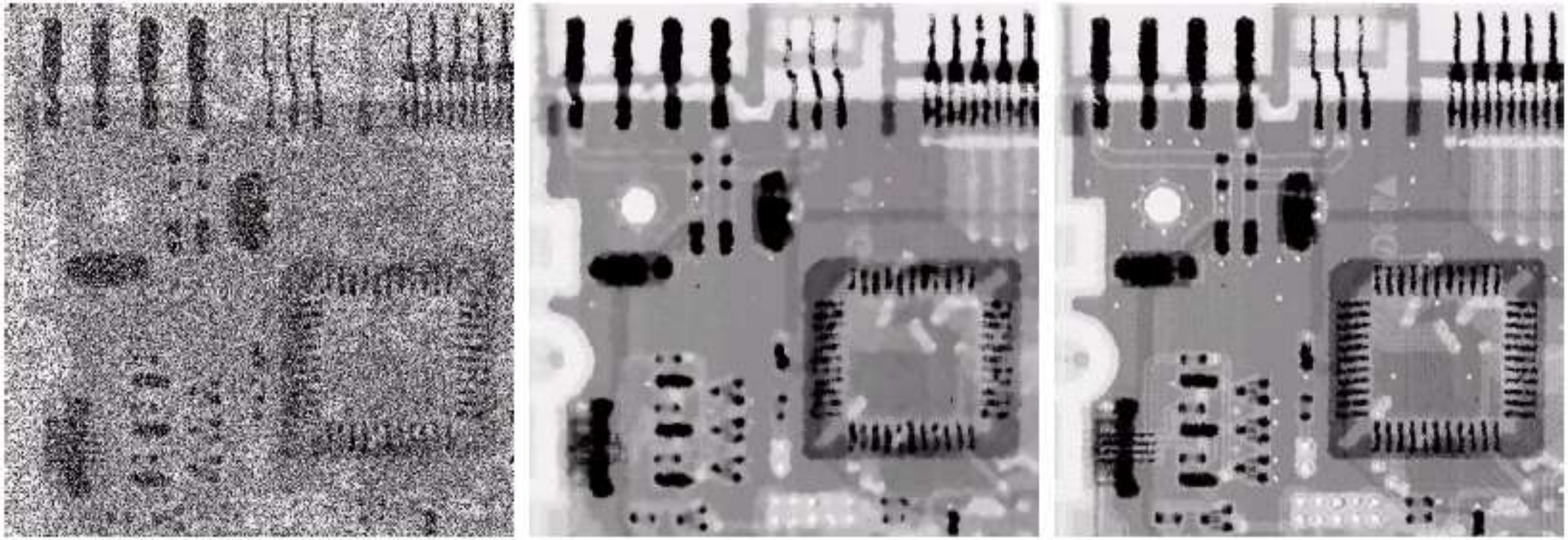
If B1>0 AND B2<0, output Z_{xy}

Else output Z_{med}

Objectives:

- ✓Remove salt and pepper (Impulse) noise
- ✓Provide smoothing
- ✓Reduce distortion, such as excessive thinning or thickening of object boundaries

Results



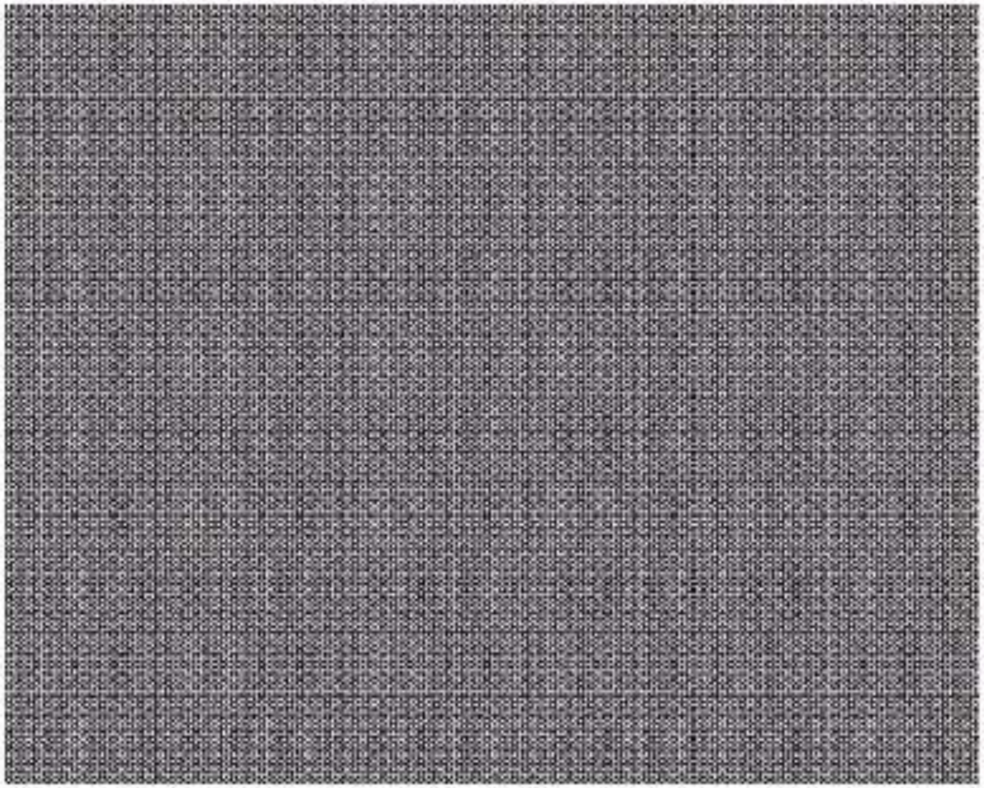
a b c

FIGURE 5.14 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.25$. (b) Result of filtering with a 7×7 median filter. (c) Result of adaptive median filtering with $S_{\max} = 7$.

Band Pass Filters

$$H_{bp}(u,v) = 1 - H_{br}(u,v)$$

FIGURE 5.17 Noise pattern of the image in Fig. 5.16(a) obtained by bandpass filtering.



Notch Filter

- Are used to remove repetitive "Spectral" noise from an image.
- Are like a narrow High pass filter, but they "notch" out frequencies other than the dc component.
- Attenuate a selected frequency (and some of its neighbors) and leave other frequencies of the Fourier transform relatively unchanged.

Notch Filters

- Notch filters
  - **Ideal Notch Reject Filter**

$$H(u,v) = \begin{cases} 0 & \text{if } \mathrm{D}_1(\mathrm{u},\mathrm{v}) \le \mathrm{D}_0 \text{ or } \mathrm{D}_2(\mathrm{u},\mathrm{v}) \le \mathrm{D}_0 \\ 1 & \text{otherwise} \end{cases}$$

$$D_1(u,v) = \left[(u - M/2 - u_0)^2 + (v - N/2 - v_0)^2\right]^{1/2}$$

$$D_2(u,v) = \left[(u - M/2 + u_0)^2 + (v - N/2 + v_0)^2\right]^{1/2}$$

Notch Filters

- Butterworth Notch Reject Filter of order n

$$H(u,v) = \frac{1}{1 + \left[\frac{D_0^2}{D_1(u,v) D_2(u,v)} \right]^n}$$

- Gaussian notch reject filter

$$H(u,v) = 1 - e^{-\frac{1}{2}\left[\frac{D_1(u,v) D_2(u,v)}{D_0^2} \right]}$$

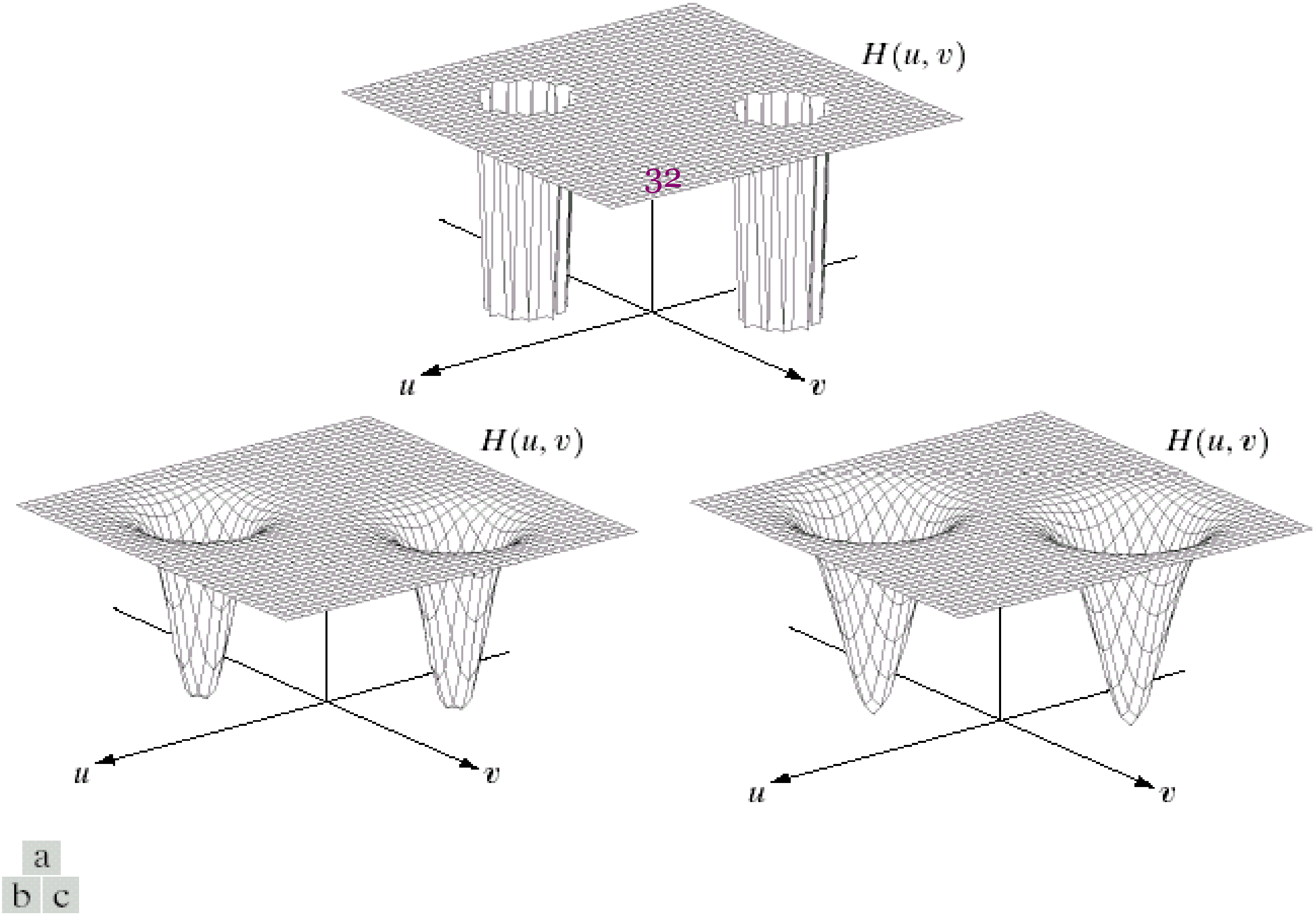
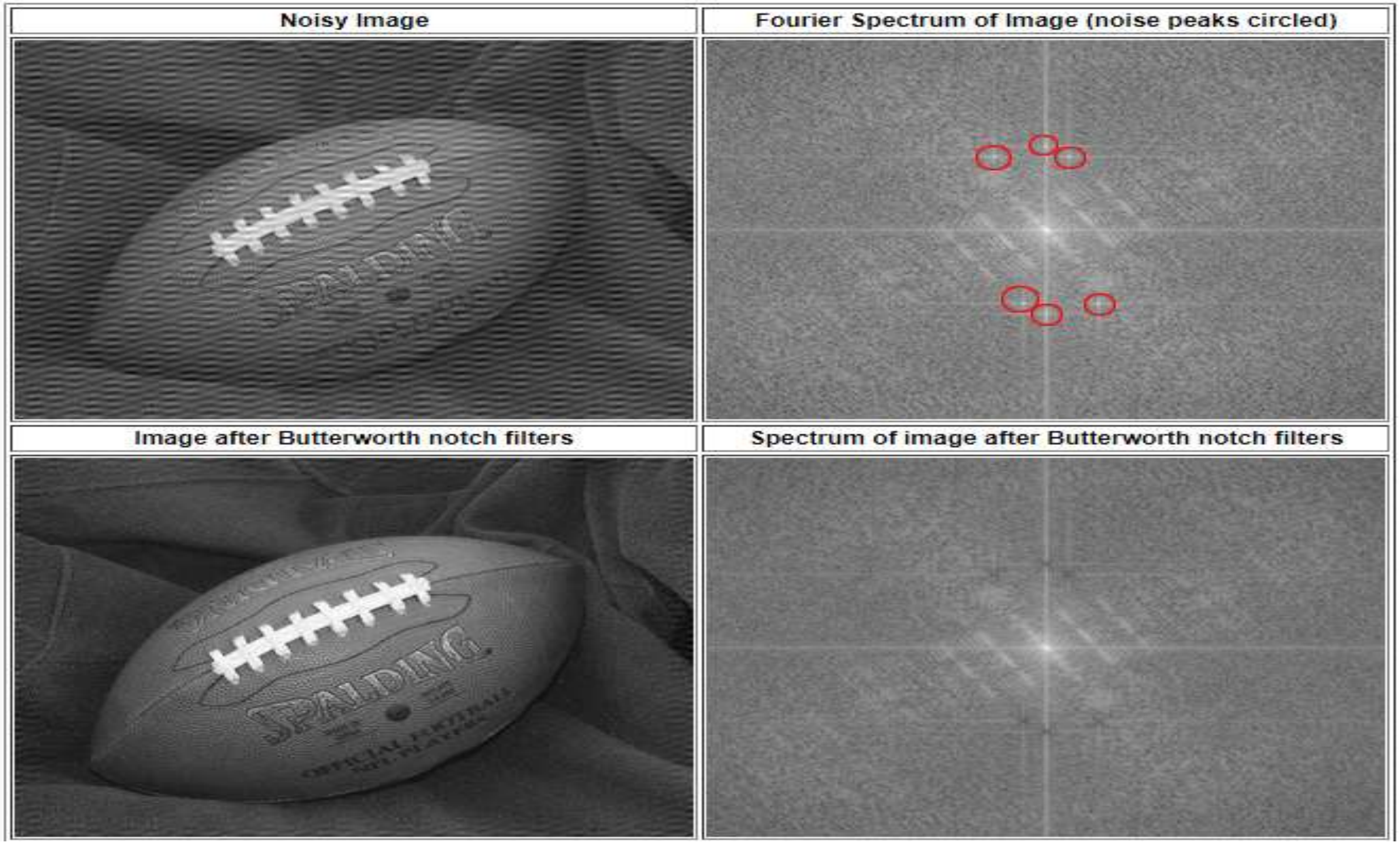
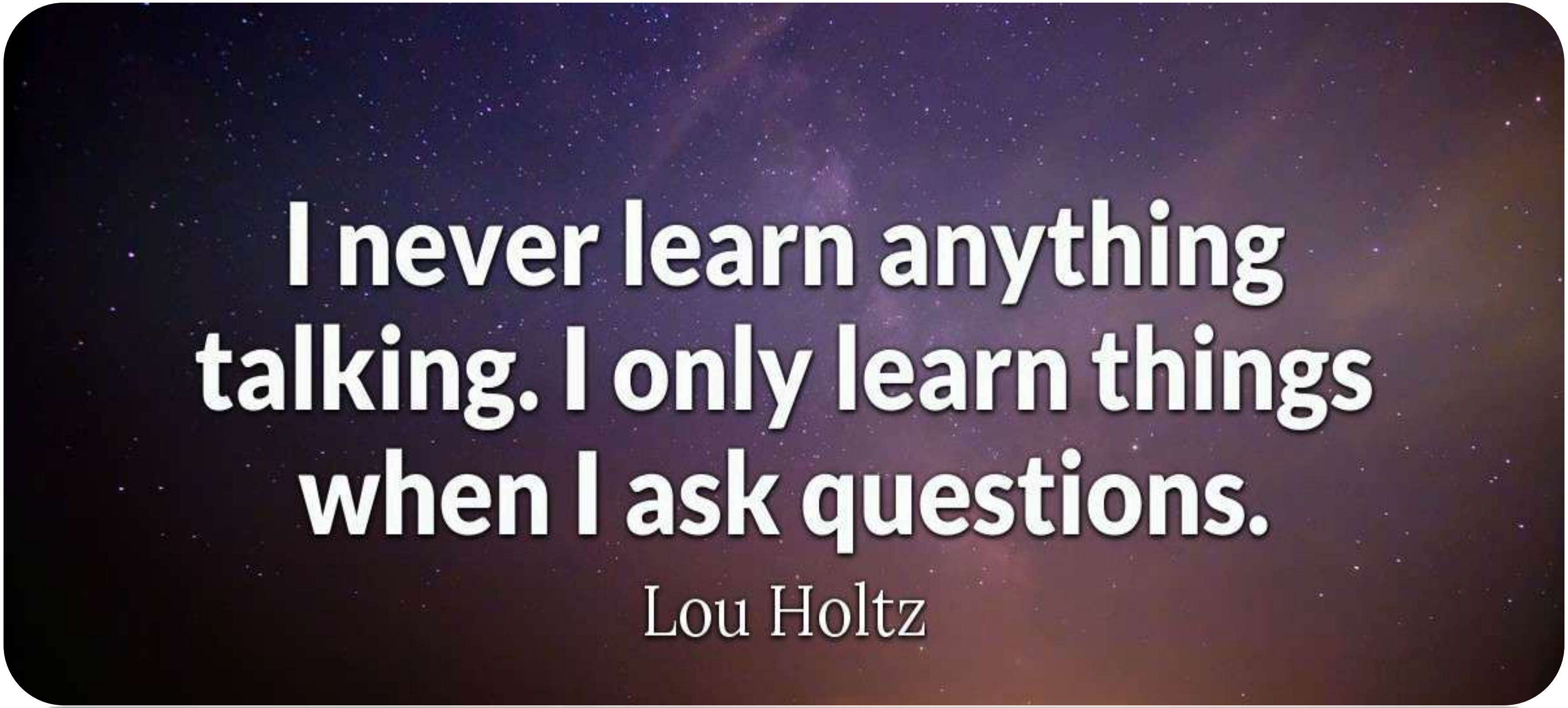


FIGURE 5.18 Perspective plots of (a) ideal, (b) Butterworth (of order 2), and (c) Gaussian notch (reject) filters.

Notch Filter Result





Thank You!
Any Question Please?

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