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Department of Mathematics

Some Important Theorems on Matrix

Th-1.1: Every square matrix can be uniquely expressed as the sum of a symmetric and a skew-symmetric matrix.

Th-1.2: The transpose of the product of two matrices is the product in reverse order of their transpose.

Th-1.3: If a given square matrix A has an inverse, then it is unique.

Th-1.4: If A and B are two non-singular matrices of the same order, then AB is also non-singular and $(AB)^{-1} = B^{-1}A^{-1}$.

Th-1.5: If A is a square matrix of order $n \times n$, then

$$A * (adjA) = (adjA) * A = |A| * I$$

Th-1.6: A square matrix has an inverse if and only of it is non-singular.

Th-1.7: If A and B are two square matrices of order $n \times n$, then

$$adj(AB) = (adjB) * (adjA)$$

Th-1.8: Every square matrix satisfies its characteristic equation.

Th-1.9: If A and B are two square matrices of order $n \times n$ such that AB=A and BA=B, then A and B are idempotent matrices.

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Q-02: Find the rank of the following matrices using Minor Test Procedure/Normal Form Technique/Echelon Form Technique:

(i) $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 & -1 & 3 & 6 \\ 1 & 3 & -3 & -4 \\ 5 & 3 & 3 & 11 \end{bmatrix}$

(iv) $\begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$ (v) $\begin{bmatrix} 1 & 1 & 2 & -3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 0 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}$ (vi) $\begin{bmatrix} 1 & 1 & 1 & -1 \\ 1 & 2 & 3 & 4 \\ 3 & 4 & 5 & 2 \end{bmatrix}$

(vii) $\begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ (viii) $\begin{bmatrix} 1 & -2 & 1 & -1 \\ 1 & 1 & -2 & 3 \\ 4 & 1 & -5 & 8 \\ 5 & -7 & 2 & -1 \end{bmatrix}$ (ix) $\begin{bmatrix} 1 & -1 & 2 & -1 \\ 4 & 2 & -1 & 2 \\ 2 & 2 & -2 & 0 \end{bmatrix}$

(x) $\begin{bmatrix} 1 & 1 & -3 & 2 \\ 2 & -1 & 2 & -3 \\ 3 & -2 & 1 & -4 \\ -4 & 1 & -3 & 1 \end{bmatrix}$ (xi) $\begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & -8 \end{bmatrix}$ (xii) $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

Q-03: Solve the following system of linear equations:

(i)
$$\begin{aligned} 2x - 2y + 5z + 3w &= 0 \\ 4x - y + z + w &= 0 \\ 3x - 2y + 3z + 4w &= 0 \\ x - 3y + 7z + 6w &= 0 \end{aligned}$$

(ii)
$$\begin{aligned} x - 2y + z - w &= 0 \\ x + y - 2z + 3w &= 0 \\ 4x + y - 5z + 8w &= 0 \\ 5x - 7y + 2z - w &= 0 \end{aligned}$$

(iii)
$$\begin{aligned} 2x + 3y - z - w &= 0 \\ x - y - 2z - 4w &= 0 \\ 3x + y + 3z - 2w &= 0 \\ 6x + 2y + 9z - 7w &= 0 \end{aligned}$$

(iv)
$$\begin{aligned} x + y - 3z + 2w &= 0 \\ 2x - y + 2z - 3w &= 0 \\ 3x - 2y + z - 4w &= 0 \\ -4x + y - 3z + w &= 0 \end{aligned}$$

(v)
$$\begin{aligned} x + y + 4z &= 6 \\ 3x + 2y - 2z &= 9 \\ 5x + y + 2z &= 13 \end{aligned}$$

(vi)
$$\begin{aligned} 2x + 4y - z &= 9 \\ 3x - y + 5z &= 5 \\ 8x + 2y + 9z &= 19 \end{aligned}$$

(vii) $\begin{aligned} x + y + z &= 6 \\ x + 2y + 3z &= 14 \\ x + 4y + 7z &= 30 \end{aligned}$ (viii) $\begin{aligned} 2x - y + 3z &= 8 \\ -x + 2y + z &= 4 \\ 3x + y - 4z &= 0 \end{aligned}$

Q-04: Find all the eigen values and eigen vectors of the following matrices:

(i) $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ (ii) $\begin{bmatrix} 3 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 3 \end{bmatrix}$

(iii) $\begin{bmatrix} 3 & -5 & -4 \\ -5 & -6 & -5 \\ -4 & -5 & 3 \end{bmatrix}$ (iv) $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

(v) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix}$ (vi) $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

(vii) $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ (viii) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{bmatrix}$

Diagonalization

Q-5.1: Consider the matrix $A = \begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix}$.

a) Find all eigenvalues and corresponding eigenvectors.
b) Find a non-singular matrix P such that $D = P^{-1}AP$ is diagonal, and P^{-1}.
c) Find a positive square root of A, that is a matrix B such that $B^2 = A$ and B has positive eigenvalues.
d) Compute A^8 using diagonal factorization.

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Q-5.2: Consider the matrix $A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$.

a) Find all eigenvalues and corresponding eigenvectors.
b) Find a non-singular matrix P such that $D = P^{-1}AP$ is diagonal, and P^{-1}.
c) Find a positive square root of A, that is a matrix B such that $B^2 = A$ and B has
positive eigenvalues.
d) Compute A^9 using diagonal factorization.

Q-5.3: Consider the matrix $A = \begin{bmatrix} 3 & -4 \\ 2 & -6 \end{bmatrix}$.

a) Find all eigenvalues and corresponding eigenvectors.
b) Find a non-singular matrix P such that $D = P^{-1}AP$ is diagonal, and P^{-1}.
c) Find a positive square root of A, that is a matrix B such that $B^2 = A$ and B has
positive eigenvalues.
d) Compute A^{12} using diagonal factorization.

Q-5.4: Consider the matrix $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$.

a) Find all eigenvalues and corresponding eigenvectors.
b) Find a non-singular matrix P such that $D = P^{-1}AP$ is diagonal, and P^{-1}.
c) Find a positive square root of A, that is a matrix B such that $B^2 = A$ and B has
positive eigenvalues.
d) Compute A^7 using diagonal factorization.