

N.B:

Answer SIX questions taking THREE from each section.

The questions are of equal value.

Use separate answer script for each section.

SECTION A

- | | | <u>Marks</u> |
|---------------|---|--------------|
| Q.1(a) | Define statistics from the perspective of an investigation. What do you mean by anecdotal evidence and its generalization, discuss with a suitable example. | 04 |
| (b) | Discuss about different types of variables. For the following data matrix, determine the type of each variable: | 04 |

Country	Content-removal request	Content-removal comply	User-data request	hemisphere	HDI
Australia	21	100	134	Southern	High
USA	92	63	5950	Northern	Very high

(Google's transparency report)

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|---------------|---|-----------|
| (c) | Write down the differences between observational study and experimental study. | 04 |
| Q.2(a) | Explain the factors that need to be considered for evaluating the relationship between 2 variables. | 04 |
| (b) | What is <u>modality of a histogram</u> ? Given 3 histograms of images that are right skewed, left skewed and uniform. What can you guess about the images contents-Justify your answer. | 04 |
| (c) | Consider a statistic: $\frac{\text{mean}}{\text{median}}$. Given 3 distributions having this statistic >1 , <1 and $=1$. Discuss about the shapes of the 3 distributions. | 04 |

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|---------------|---|-----------|
| Q.3(a) | “Sample is used to visualize, understand the patterns and make quick statement about system’s behavior”, explain the statement. | 03 |
| (b) | Describe non sampling error by examples. | 03 |
| (c) | The following data set represents the number of new computer accounts registered during ten consecutive days.
43, 31, 50, 81, 58, 105, 52, 45, 45, 10
(i) Compute the mean, median, quartiles and standard deviation.
(ii) Check for outliers using the 1.5 (IQR) rule.
(iii) Delete the detected outliers and compute the mean, median, quartiles and standard deviation.
(iv) Make a conclusion about the effect of outliers on basic descriptive statistics. | 06 |

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|---------------|---|-----------|
| Q.4(a) | Consider the following data sets:
(i) 56, 52, 13, 34, 35, 18, 44, 41, 48, 75, 24, 19, 38, 21, 46, 62, 71, 24, 66, 40, 18, 15, 29, 53, 23, 41, 78, 18, 25
(ii) 19, 24, 15, 18, 24, 8, 3, 9, 26, 13, 1, 12, 11, 16, 22, 21, 7, 16, 15, 18, 26, 16, 18, 21, 24, 20, 19 | 08 |
| | For each data set, draw a histogram and determine whether the distribution is right skewed, left-skewed, or symmetric. Compute sample means and sample medians. Do they support your findings about skewness and symmetry? How? | |
| (b) | Prove that sum of the deviation of a group of numbers from their mean is equal to zero. | 04 |

SECTION B

- | | | |
|---------------|---|-----------|
| Q.5(a) | The distribution function of the random variable X is :
$F(x) = \begin{cases} 1 - e^{-2x}; & x > 0 \\ 0; & x \leq 0 \end{cases}$ | 04 |
| Find, | (i) the density function
(ii) The probability that $X > 2$, and
(iii) The probability that $-3 < X \leq 4$ | |

- (b) Suppose that in a certain region the daily rainfall (in inches) is a continuous random variable X with probability density function $f(x)$ given by

$$f(x) = \begin{cases} \frac{1}{2}(3x - 2x^2); & \text{if } 0 < x < 3 \\ 0 & \text{otherwise} \end{cases}$$

Find the probability that on a given day in this region the rainfall is,

- (i) not more than 2 inches.
- (ii) greater than 1 inch.
- (iii) between 1.5 and 2.0 inches
- (iv) equal to 1 inch, and
- (v) less than 2 inches.

- (c) A bag contains 4 red, 6 black and 7 white marbles. A marble is chosen at random from the bag. If the marble is not white what is the probability that it is red? 04

- Q.6(a) Write down the properties of standard deviation. 03

- (b) Explain the differences: 03

- (i) point estimate Vs interval estimate.
- (ii) Sample Vs population.
- (iii) Bi-modal Vs multi-modal.

- (c) Show that Geometric mean \leq Arithmetic mean. 03

- (d) Find the standard deviation of $1, 2, 3, \dots, n$ 03

- ~~Q.7(a)~~ For an M/M/1 queuing system with the average inter arrival time of 5 minutes and the average service time of 3 minutes, compute, 05

- (i) The expected response time.
- (ii) The fraction of time when there are fewer than 2 jobs in the system.
- (iii) The fraction of customers S who have to wait before their service starts.

- ~~(b)~~ What is memory less property? 2.5

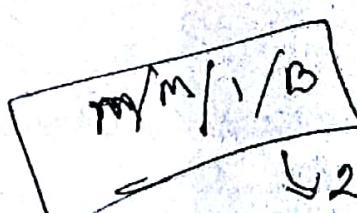
- ~~(c)~~ Explain Birth-death process. 2.5

- ~~(d)~~ Explain the relation between the binomial distribution and the normal distribution. 02

- ~~Q.8(a)~~ On a network gateway, measurements show that the packets arrive at a mean rate of 125 packets per seconds (PPS) and the gateway takes about 2 milliseconds to forward them. Using M/M/1 model, analyze the gateway, what is probability of buffer overflow if the gateway had only 13 buffers? How many buffers do we need to keep packet loss below one packet per million? 08

- (b) Find the probability that five tosses of a fair die a 3 appears

- (i) at no time,
- (ii) once,
- (iii) twice,
- (iv) 3 times.



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SECTION A

- | | | |
|--------------|--|-----------|
| Q1(a) | How statistics is related to probability? Mention the necessity of studying applied statistics and queuing theory as a computer engineer. | 03 |
| (b) | Suppose X is a continuous random variable with probability density function
$f(x) = \begin{cases} K(x-1), & 1 < x < 2 \\ 0, & \text{otherwise} \end{cases}$ | 06 |
| (c) | Calculate i) K ii) $E(X)$ iii) $V(X)$ and iv) $F(x)$
Describe the independent and stationary properties of Poisson distribution by real world example. | 02% |
| Q2(a) | Define "Mutually exclusive" events and "Independent" events. Are mutually exclusive events independent? Explain. | 03 |
| (b) | It is conjectured that an impurity exists in 30% of all drinking wells in a certain rural community. In order to gain some insight on this problem, it is determined that some tests should be made. It is too expensive to test all of the many wells in the area, so 10 were randomly selected for testing.
i) What is the probability that exactly three wells have the impurity assuming that the conjecture is correct?
ii) What is the probability that more than three wells are impure? | 06 |
| | Why we dividing by $(n-1)$ instead of n when we are calculating the sample standard deviation? | Very Good |
| Q3(a) | Explain the memory less property and its effect on the exponential distribution. | 02% |
| (b) | An Engineer commutes daily from his home to his office. The average time for a one way trip is 24 minutes, with a standard deviation of 3.8 minutes. Assume the distribution of the trip times to be normally distributed.
i) What is the probability that a trip will take at least $\frac{1}{2}$ hour?
ii) If the office opens at 9.00AM and he leaves his house at 8.40AM, what percentage of time is he late for work?
iii) Find the length of time above which we find the slowest 15% of the trips.
iv) Find the probability that 2 of the next 3 trips will take at least $\frac{1}{2}$ hour. | 09 |
| Q4(a) | What is standard normal? Write some properties of normal distribution. | 03 |
| (b) | The probability that a patient recovers from a rare blood disease is 0.4. If 100 people are known to have contracted this disease, what is the probability that less than 30 survive? | 02% |
| (c) | The average zinc concentration recovered from a sample of zinc measurement in 36 different locations is found to be 2.6 grams per milliliter. Find the 95% and 99% confidence intervals for the mean zinc concentration in the river. Assume that the population standard deviation is 0.3. | 06 |

rex ^{do} at n.p SECTION B

- Q5(a)** What is the difference between correlation and covariance analysis? 02%

(b) The following table shows the heights to the nearest inch and the weight to the nearest pound of a sample of 10 male students drawn at random from the third year student at RUET. 09

Height	X(in)	62	65	67	70	68	61	64	69	64	65
Weight	Y(lb)	130	158	156	168	129	112	152	135	153	139

i) Find the equation of linear regression line to weight from height.
 ii) Estimate the weight when height is 66 inch
 iii) Estimate the correlation coefficient.

Q6(a) What is meant by Hypothesis testing? Why do you need to do this? 02

(b) Explain the Birth-Death process of queuing system. 03

A random sample of 64 RAM capacity, on average, 5.23 GB with a standard deviation of 0.24 GB. Test the hypothesis that $\mu=5.5$ GB against the alternative hypothesis, $\mu<5.5$ GB at the 0.05 level of confidence. 06%

Q7(a) Write the properties of stochastic process. Explain the application of stochastic process in engineering. 03

Derive the equation of steady state probability for M/M/1 queuing system. 04%

A secretary makes 2 errors per page, on average. What is the probability that on the next page he or she will make 4 or more errors. 04

Q8(a) What is Markov process? Explain with example. 03%

(b) An airline has 15 flights leaving a base per day, each with a hostess. The airlines keep three hostesses in reserve so that they may be called in case the scheduled hostesses for a flight reports sick. The probability distribution for daily number of sick hostesses is as follows: 09

No. of Sick	0	1	2	3	4	5
Probability	0.20	0.25	0.20	0.15	0.10	0.10

Estimate the utilization of reserve hostesses and also the probability that at least one flight will be canceled in a day because of non-availability of hostesses. A10

No. of Sick	0	1	2	3	4	5
Probability	0.29	0.33	0.20	0.15	0.10	0.10

Estimate the utilization of reserve hostesses and also the probability that at least one flight will be canceled in a day because of non-availability of hostesses.

A/CO

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SECTION AMarks0304

Q1(a) What is the geometric mean? Give an example in which this mean is used.

(b) Let $d_j = X_j - A$ denote the deviations of any class mark X_j in a frequency distribution from a given class mark A. Show that if all class intervals have equal size C, the arithmetic meancan be computed from $A + \frac{\sum f_j u_j}{N} \times C$, where $d_j = cu_j$; and $u_j = 0, \pm 1, \pm 2$.(c) Show that $\sum f_i (x_i - A)^2$ has a minimum value. Hence define the standard deviation (S.D.) 0402

Q2(a) The histogram derived from the pixel values of an image shows that it is bimodal in nature. what can be inferred from the curve.

02(b) Explain the correlation between voltage and current from the equation of a electrical network. 0207(c) The distribution function of the random variable X is $F(X) = \begin{cases} 1 - e^{-2x}, & x > 0 \\ 0, & x \leq 0 \end{cases}$ Find (i) the density function (ii) the probability that $X > 2$ and (iii) the probability that $-3 < X < 4$. 0506Q3(a) What is the Poisson's distribution? Show that its mean and variance have equal value. 06(b) If 10% of the bolts produced by a machine are defective, determine the probability that out of 50 bolts chosen at random, (i) 1 and (ii) at most 1 bolts will be defective. Use both binomial and Poisson's distribution and compare the results. 05Q4(a) Find the Kurtosis of the distribution $\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}$. 03(b) The mean grade on a final examination was 72 and the S.D. was 9. The top 10% of the students are to receive A+'s. What is the minimum grade that a student must get in order to receive an A+? 03(c) Assume that the heights of 3000 male students at a university are normally distributed with mean 64 inches and S.D. 3 inches. If 80 samples consisting of 25 students each are obtained. What would be the expected mean and S.D. of the resulting sampling distribution of means? 03SECTION B05Q5(a) What are the confidence limits? A random sample of 50 mathematics grades out of 200 showed a mean 70 and a S.D. of 10. What are the 95% confidence limits for estimates of the mean of the 200 grades? 04(b) Derive the equation of the first and second moment about the origin of the binomial distribution. 02(c) Differentiate between point estimates and interval estimates. 06Q6(a) To show that under the three conditions of a Poisson's process the number of arrivals in a fixed time follows the Poisson law i.e if the probability of an arrival in time interval t and $t + \Delta t$ is $\lambda \Delta t + O(\Delta t)^2$, then $P_n(t) = \frac{e^{-\lambda t} (\lambda t)^n}{n!}$, $n = 0, 1, \dots, \infty$. 0203(b) Why the exponential distribution lends itself well to model customer inter-arrival times or service times of a queuing system? 03(c) Model the following process with respect to the basic queuing model: (i) the input process (ii) the output process (iii) Birth-death process. 12Q7 On a network gateway, measurements show that the packets arrive at a mean rate of 125 packets per seconds (PPS) and the gateway takes about 2 milliseconds to forward them. Using an M/M/1 model, analyze the gateway, what is probability of buffer overflow if the gateway had only 13 buffers? How many buffers do we need to keep packet loss below one packet per million. 07Q8(a) A pumping station has two identical pumps connected in parallel, each capable of pumping 3000 gallons/hr. If the failure rate and repair rate of each is 1/50/hr and 1/45/hr respectively. Calculate the average throughput of the pumping station. 04(b) Derive the probability distribution function of the normal distribution. 04

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SECTION A

- | | <u>Marks</u> |
|---|-----------------|
| Q1(a) Describe the relationship between statistics and data. | 03 |
| (b) Differentiate between inductive and descriptive statistics. | 03 |
| (c) An image is combination of many pixel values; consider that the frequency curve of an image pixel is bimodal. What can be inferred from the curve? | 02 |
| (d) Prove that sum of the deviation of a group of numbers from their mean is equal to zero. | 03 ² |
| Q2(a) what do you mean by "Central tendency" of data? Explain some measures of central tendency. | 04 |
| (b) Assume that the height of 3000 students at a university is normally distributed with mean 157 cm and standard deviation 7 cm. If 80 samples consisting of 25 students each are obtained, what would be the expected mean and standard deviation of the resulting sampling distribution of means if the sampling were done i) with replacement, ii) without replacement. | 04 |
| (c) Write short note: i) Skewness, ii) Kurtosis, and iii) Dispersion. | 03 ² |
| Q3(a) Write down the significance of standard deviation as dispersion measure. What are the properties that it shows? | 04 |
| (b) What do you understand about biased estimator and un-biased estimator? Mention the limitations of those. | 03 ² |
| (c) Among point estimation and interval estimation, which one do you prefer and why? | 04 |
| Q4(a) Let X be a random variable with probability density function | 08 ² |
| $f(x) = \begin{cases} C(1-x^2), & -1 < x < 1 \\ 0, & \text{otherwise} \end{cases}$ | 22 |
| Calculate i) C, ii) E(x), iii) V(x), iv) F(x), and hence P(-1 < x < 0) | |
| (b) What is the probability? If A and B are any two events, show that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ | 03 |

SECTION B

- | | |
|---|-----------------|
| Q5(a) Derive the equation of the first and second order moment about the origin of the binomial distribution. | 04 ² |
| (b) Explain Birth-Death process of a queueing system. | 04 |
| (c) Determine the poison distribution from the binomial distribution. | 03 |
| Q6(a) What is hypothesis testing? Explain two types of errors involved in it. | 04 |
| (b) What is coefficient of correlation? If this coefficient is 0.8, hat it its physical interpretation. | 03 ² |
| (c) What is the physical interpretation of 95% confidence interval? Derive an expression for (1- α) confidence interval around the mean. | 04 |
| Q7(a) Derive the equation of steady state probability for M/M/1 Queueing system.) | 05 ² |
| (b) What is memory-less property? Explain with example. | 03 |
| (c) What do you mean by sampling distribution of means? As a CS engineer where can you apply this? | 03 |
| Q8(a) On a network gateway, measurements show that the packets arrive at a mean rate of 125 packets per second (pps) and the gateway takes about 2 milliseconds to forward them. Using M/M/1 model, analyze the gateway, what is the probability of buffer overflow if the gateway had only 13 buffers? How many buffers do we need to keep the packet loss one packet per million? | 07 ² |
| (b) What is Markov Chain? In what situation Markov Chain is useful for analysis purpose? Explain. | 04 |

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SECTION-A

Q1. (a) Prove that sum of the deviation of a group of numbers from their mean is equal to zero. 3 $\frac{2}{3}$

(b) What is the difference between leptokurtic and mesokurtic nature of a data set? 2

(c) Suppose a set of data gave you a positively skewed bimodal shaped frequency curve, draw some conclusions about the data set. 2

(d) What is the difference between relative frequency distribution and cumulative frequency distribution? 4

Q2. (a) What is coefficient of correlation? If this coefficient is 0.8, what is its physical interpretation? 4

(b) Measurement of the diameter of a random sample of 200 ball bearings made by a certain machine during one week showed a mean of 0.824 mm and standard deviation is 0.042 mm-Find 95% confidence interval for the mean diameter. 5 $\frac{2}{3}$

(c) With examples, explain the difference between mutually exclusiveness and dependence. 3

Q3. (a) The distribution function for a random variable X is, 4 $\frac{2}{3}$

$$F(x) = \begin{cases} 1 - e^{-x^2}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

Find (i) the density function, (ii) the probability that $X > 2$, and (iii) the probability that $-3 < X \leq 4$.

(b) The density function of a random variable X is given by, 4

$$f(x) = \begin{cases} \frac{1}{2}x, & 0 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

Find (i) $E(X)$, (ii) $E(X^2)$, and (iii) $\text{Var}(X)$

(c) Explain the characteristics of the normal distribution. 3

Q4. (a) In a company of 80 employees, 60 earn \$10.00 per hour and 20 earn \$ 13.00 per hour. 4 $\frac{2}{3}$

i) Determine the mean earning per hour. ~~avg 10.75~~

ii) Would the answer in part (i) be the same if the 60 employees earn a mean hourly wage of \$ 10.00 per hour? Prove your answer.

(b) Show that Geometric mean \leq Arithmetic mean 3 $\frac{1}{2}$

(c) Find the standard deviation of $1, 2, 3, \dots, n$ 3 $\frac{1}{2}$

SECTION-B

Q5. (a) Why the exponential distribution lends itself well to model customer inter-arrival times or service times of a queuing system? 2 $\frac{2}{3}$

(b) Model the following process with respect to the basic queuing model: 5

- i) The input process,
- ii) The output process,
- iii) Birth-death process

Derive the steady-state probability of a Birth-death process? 4

Q6. (a) Explain the basic characteristics of an M/M/1 Queuing model. 3 $\frac{2}{3}$

(b) The average response time on a database system is 5 seconds. During 1 minute observation interval the idle time on the system was measured to be 12 seconds. Using an M/M/1 model for the system determine the followings:

- i) System utilization.
- ii) Average service time per query.
- iii) Number of queries completed during the observation.
- iv) Average number of jobs in the system.
- v) Probabilities of number of jobs in the system being greater than 20.
- vi) 90- Percentile response time.
- vii) 90- Percentile waiting time.

Q7. (a) Derive the equation of the first and second moment about the origin of the binomial distribution. 4 $\frac{2}{3}$

(b) Derive the probability distribution function of the Poisson distribution. 4

(c) What do you mean by sampling distribution of means? Define the condition of asymptotically normal sampling distribution. 3

Q8. (a) A pumping station has two identical pumps connected in parallel, each capable of pumping 3000 gallons/hr. If the failure rate and repair rate of each is 0.5 f/hr and 0.4 f/hr respectively, calculate the average throughput of the pumping station. 7

(b) A box contains 2 red and 5 blue balls. Find the probability that 2 balls are drawn at random (without replacement) both are red. 4 $\frac{2}{3}$

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SECTION-A

- Q1. (a) The following distances for 19 Sugar Maple trees:
6.00, 4.00, 6.00, 6.45, 5.00, 5.00, 5.50, 2.35, 2.35, 3.00, 3.90, 5.35, 3.15, 2.10, 4.80, 3.10, 5.15, 3.10, 6.25. 4 $\frac{2}{3}$

(i) Find the five-number summary for each set of distances.

(ii) Construct box plots on the same figure.

(iii) Construct a back-to-back stem-and-leaf diagram.

- (b) What is meant by central tendency of data? What are the various measures of central tendency? Explain with examples. 4

- (c) Given below are the monthly household incomes (in Tk.) for ten families. 3

10648	17416	6517	13555	14821
9226	152936	11800	18527	12222

Compute the range, inter-quartile range and standard deviation as measure of variability.

- Q2. (a) Define mathematical expectation. Find $\mu = E(X)$, $\sigma^2 = E(X^2) - \mu^2$ for the following distributions: $f(x) = \frac{4-x}{6}$, $x=1,2,3$. 3

- (b) What is Poisson distribution? Formulate Poisson distribution. 4 $\frac{2}{3}$

- (c) Let X equal the number of telephone calls per hour that are received by 911 between midnight and noon. The following number of calls were reported: 4

0	1	1	1	0	1	2	1	4	1	2	3
0	3	0	1	0	1	1	2	3	0	2	2

Calculate the same mean and sample variance for these data.

Assume that $\lambda=1.3$. Draw a probability histogram for the Poisson distribution and a relative frequency histogram of the data on the same graph.

- Q3. (a) What do you mean by moment-generating function? Find out mean, variance of Binomial distribution using moment-generating function. 5 $\frac{2}{3}$

- (b) Let X have an exponential distribution with a mean of $\theta = 20$. Compute $P(10 < X < 30)$ (ii) $P(X > 30)$ (iii) $P(X > 40 | X > 10)$. 3

- (c) Define joint probability mass function. Let the joint p.m.f. of X and Y be 3

$$f(x, y) = \frac{xy^2}{30}, x = 1, 2, 3 \quad y = 1, 2$$

Find (i) $f_1(x)$ for all values of x (ii) $f_1(y)$ for all values of y.

- Q4. (a) Suppose a survey of 100 computers installations in a certain city shows that 75 of them at least one brand X computer. If three of these installations are chosen at random without replacement, what is the probability that each of them has at least one brand X machine? 7

- (b) What are the functions of random variables? 2

- (c) Define some properties of distribution function. 1 $\frac{1}{3}$

SECTION-B

- Q5. (a) What do you mean by sampling distribution? If X_1, X_2, \dots, X_n are independent random variables and for $i=1, 2, \dots, n$ $E[U_i(X_i)]$ exists, then prove 4 $\frac{2}{3}$

$$E[U_1(X_1)U_2(X_2)\dots U_n(X_n)] = E[U_1(X_1)]E[U_2(X_2)]\dots E[U_n(X_n)]$$

- (b) Explain point estimation with example. 3

- (c) Define Type I error and Type II error. How can you perform single mean test? Explain with an example. 4

- Q6. (a) Classify stochastic process. Explain each kind of stochastic process with example. 4

- (b) Define Markov process and Markov chain. Give an example which Markov process and give an example which is not Markov process. 3

- (c) Explain Poisson counting process. 4 $\frac{2}{3}$

- Q7. Jobs are sent to a mainframe computer at a rate of 2 jobs per minute. Arrivals are modeled by a Binomial counting process. 11 $\frac{2}{3}$

(i) Choose such a frame size that makes the probability of a new job during each frame equal to 0.1.

(ii) Using the chosen frames, compute the probability of more than 3 jobs received during one minute.

(iii) What is the average inter arrival time and what is the variance?

(iv) Compute the probability that the next job does not arrive during the next 30 seconds.

- Q8. (a) Taxis pass a certain corner with an average inter arrival time at 20 seconds. What is the average time that one would expect to wait for a taxi? 6

- (b) Describe the M/M/1 Queuing system? 3 $\frac{2}{3}$

- (c) Define some applications of Queuing theory. 2

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SECTION-A

Q1. (a) Define H.M. Mention at least two physical problems where it is used. $\frac{3}{3}$

(b) For two unequal numbers, show that $A.M.X.H.M = G.M^2$ $\frac{3}{3}$

(c) An observer measured the frequency of a periodic force 11.05 and 0.95 in several times. What is the average period? $\frac{5}{5}$

Q2. (a) What is S.D.? Show that $\sum f_i(x_i - \bar{x})^2$ has a minimum value. $\frac{3}{3}$

(b) Find the S.D. and M.D. of the numbers 1, 2, 3, ..., n. $\frac{4}{4}$

(c) What is the skewness? Find the coefficient of skewness of 1, 2, 3, 5, 6, 7, 9, 10. $\frac{4}{4}$

Q3. (a) What are the characteristics of Binomial distribution? How does it differ from Hypergeometric distribution? $\frac{4}{4}$

(b) Computer lab is equipped with 50 computers. The class size for a certain batch is 52 and the probability of absent in the class is 0.05. What is the probability that the students will get a computer for a certain class? $\frac{3}{3}$

(c) Average no. of researchers connecting the server per day is 12. The server has a 15 user license of a certain software. What is the probability that on a certain day the researchers will have to be sent back due to non-availability of the user license? Assume the researchers arrival process follows poison distribution. $\frac{4}{4}$

Q4. (a) What is the normal distribution? Find the coefficient of Kurtosis of this distribution. $\frac{6}{6}$

(b) The probability density for bullets hitting a target is given by $\frac{2}{3}$

$$P(x, y) = \frac{1}{2\pi\sigma_x\sigma_y} e^{-\frac{1}{2}(\frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2})}$$
 where σ_x and σ_y are constants. Sketch the curves of constant density in the xy-plane when $\sigma_x = \sigma_y = \sigma$. $\frac{1}{3}$

SECTION-B

Q5. (a) What is the transition probability? A and B are two transition probability matrices of order 2 x 2. Show that the sum of the rows of the product of A and B is 1. $\frac{2}{3}$

(b) Let S be the system consisting of two players A and B who begin with 1 k. 2 apiece and match coins until one or the other of them has no money. If the states of the system are defined by the number of Taka in A's possession, especially if the system is in the state S_{ii} whenever A has i Taka ($i = 0, 1, 2, 3, 4$), find the matrix of one step transition probabilities. $\frac{6}{6}$

What is the probability that A will be ruined at most three turns?

Q6. (a) What do you understand about Markov Chain and the associated transitional probabilities?

(b) Arrival of service engineer at your lab may be assumed to depend on the arrival at his previous job/office. If he arrived early at his previous job then the probability of arrival early at your lab is 0.5 and on time 0.4 and late 0.1 respectively. For his on time arrival at the previous job, the probabilities are 0.1, 0.6, and 0.3 respectively. The probabilities are 0.1, 0.2 and 0.7 respectively while he was late at his previous job. What will be his expected percentage of early arrival, on time arrival and late arrival at your lab? If the service engineer is late at his previous job today, then what is the probability of late at your lab after two days?

Q7. (a) What are the characteristics or components of a Queuing system?

(b) Arrival of call at a telephone switching server are considered to be following poisson process with an average time of 10 minutes between one call and the next. Length of a phone call is assumed to be distributed exponentially with mean 3 minutes. $\frac{4}{4}$

Q8. (a) Identify the customers and the servers in the queuing system representation of each of the following situations (i) Toll booth for a bridge (ii) Fire station (iii) IT help desk and (iv) Banking systems.

(b) Consider a viva board consisting of two interviewers. Suppose that an entering candidate first will go to 1st interviewer. When the interview is completed, he will go either to 2nd interviewer if the 2nd interviewer is free or else wait in front of 1st interviewer until 2nd interviewer becomes free. Potential candidate will enter this viva board as long as 1st interviewer is free. The interview times for the two interviewers are independent and have different interview time length.

(i) What proportion of potential candidates enters the viva board?

(ii) What is the mean number of candidates in the system?

(iii) What is the average amount of time that an entering candidate spends in the system? $\frac{4}{4}$

(a) What is the probability that a person making a phone call will have to wait for network?

(b) What is the average length of queue that from time to time?

(c) The telephone company will install a second switching server when advised that a person would expect to wait at least three min for a phone

N.B: Answer six questions, taking three from each section.

Figures in the margin indicate full marks.

The questions are of equal value.

Use separate answer script for each section.

SECTION-A

- Q.1. (a) What is the probability? If A and B are any two events, show that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Marks 03 2 / 3
(b) If $P(X)$ is a random variable with probability density function $P(X) = \begin{cases} Cx^{-1/2} e^{-x}, & x > 0 \\ 0, & x \leq 0 \end{cases}$. 08
- Find the value of C. Find also its standard deviation. Is it possible to calculate $P(x) \leq 1$ by an analytical approach? Draw approximately $P(X)$ from $x = 0$ to $x = 5$. 08 2 / 3
- Q.2. (a) What is the binomial distribution? From the binomial theory, formulate this distribution. Then find its variance and the coefficient of skewness. 08 2 / 3
(b) If 10% of the bolts produced by a machine are defective, determine the probability that out of 10 bolts chosen at random (i) 1 (ii) less than 1 bolts will be defective. 03
- Q.3. (a) What is sampling? A population consists of the five numbers 2, 3, 6, 8, 11. Consider all possible samples of size two which can be drawn without replacement from the population. Find (i) the standard deviation of the population (ii) the standard deviation of the sampling distribution of means. What is the standard error of means? 07 2 / 3
(b) Assume that the height of 3000 students at a university is normally distributed with mean 157 cm and standard deviation 7 cm. If 80 samples consisting of 25 students each are obtained, what would be the expected mean and standard deviation of the resulting sampling distribution of means if sampling were done (i) with replacement (ii) without replacement? 04
- Q.4. (a) What do you understand about biased estimator and un-biased estimator? 03
(b) Among point estimation and interval estimation, which one do you prefer and why? 03
(c) In measuring CPU time, you estimate that the standard deviation is 0.05 sec. How large a sample of replication must you take in order to be (i) 95% and (ii) 99% confident that the error in your estimation of mean CPU time will not exceed 0.1 sec? 05 2 / 3

SECTION-B

- Q.5. (a) What is the transition probability? If Λ represents a transition probability matrix of order 2 and its all elements are nonzero, verify that the sum of the elements of each row of Λ^2 is equal to 1. 06
(b) Consider the system S consisting of three Boxes B_1 , B_2 , and B_3 and a single ball, and let the system be in the state S_i ($i = 1, 2, 3$) if the ball is in the Box B_i . Transition from one state to another take place in the following manner. Three coins are tossed. If no heads turn up, the ball is not moved; if one or more heads turn up, the ball is taken from its Box and placed in the Box corresponding to the number of heads showing. Find the matrix of one and two-steps transition probabilities for the system. 05 2 / 3
- Q.6. (a) What is Markov Chain? Give example of any system or situation where this can be applied. 03 2 / 3
(b) Suppose your computer can be in three states "Good operation", "Operating with virus" and "No operation". Failure rate from Good operation to operating with virus and operating with virus to no operation states are 0.5 f/hr and 0.7 f/hr respectively. Repair rates from the operating with virus and no operation states to Good operation state are 0.6 r/hr and 0.4 r/hr respectively. It is assumed that full failure does not occur from the Good operation state and once failed the repair process returns to only Good operation state. Draw the state space diagram. What average number of programs would be run if the system has the capacity of 300 programs/hr at Good operation state and 200 programs/hr at operating with virus state? Consider a printing lab consisting of computer and printer. Suppose an entering student first go to computer. When his work is completed in computer he will go either to printer if the printer is empty or wait in front of computer until the printer becomes empty. Suppose that a student will enter this lab as long as computer is empty. If students arrive as Poisson process and service times for computer and printer are independent then (i) What proportion of students enter the system? (ii) What is the mean number of students in the system? (iii) What is the average amount of time that an entering student spends in the system? 11 2 / 3
- A self service store employee's one customer care executive at its counter. Nine customers arrive on an average every 5 minutes while the executive can serve 10 customers in 5 minutes. Assuming Poisson distribution for arrival rate and exponential distribution for service rate find (i) Average number of customers in the system (ii) Average number of customers in the queue (iii) Mean time a customer spends in the system and (iv) Mean time a customer waits before being served. 11 2 / 3

N.B. Answer six questions, taking three from each section.

The questions are of equal value.

Figures in the margin indicate full marks.

Use separate answer script for each section.

SECTION-A

Marks

Q1. (a) What is the conditional probability? Find the probability that a single toss of die results in a number less than 4 if (i) no other information is given (ii) it is given that the toss result is an odd number. $\frac{2}{3}$

(b) If x is a random variable with probability density function $p(x) = Ce^{-kx}$, $x > 0$ 06
 $= 0$, $x \leq 0$.

It is known that both average and variance of this distribution are 1.

(i) Calculate C and k (ii) $p(x \leq 1)$ (iii) Draw $p(x)$.

(c) What are the moments and moment generating functions? 02

Q2. (a) Establish Poisson distribution from Binomial distribution. $\frac{2}{3}$

(b) A can hit a target 4 times in 7 shots, B 3 times in 5 shots and C 4 times in 6 shots. 04

All of them fire one shot each simultaneously at the target. What is the probability that, at least two shots hit?

(c) An insurance company found that only 0.01% of the population is involved in a certain type of accident each year. If its 1000 policy holders were randomly selected from the population. What is the probability that that not more than two of its clients are involved in such an accident next year? 05

Q3. (a) What are Skewness and Kurtosis? Find the coefficient of Kurtosis of the normal distribution. $\frac{2}{3}$

(b) Calculate the coefficient of Skewness of the binomial distribution. 05

Q4. (a) Explain the followings: 03

(i) Stochastic process (ii) Steady-state probability

(b) What is meant by 99% confidence interval? Measurements of the diameters of a random sample of 200 ball bearings made by a certain machine during one week shows a mean of 0.824 units and a standard deviation of 0.042 units. Find 99% confidence limits for the mean diameter of all the ball bearings. $\frac{2}{3}$

(c) Find the mean and variance of the Normal distribution. 03

σ

\sqrt{N}

SECTION-B

Q5. (a) Explain the stationary increment and independent increment related to Poisson's distribution. $\frac{2}{3}$

(b) Suppose a system fails according to Poisson's distribution by two methods A and B. 08 40% by A. The failure occurs on an average of 15 in three months.

(i) What is the expected numbers of failure such that the last failure occurs by B for the first time

(ii) In one month, what is the probability that there will be at least four failures?

(iii) If there are 60 failures in five months, what is the probability that there will be at most 40 failures by B? Write the expression only.

(iv) What is the probability that there will be 6 failures in two months of a year given that there have been 12 failures in the early months?

(v) Given that first failure occurs on the 4th day, what is the probability that the next failure will not occur before 8th day?

(vi) What is the expected time of 15 failures?

Q6. (a) Explain the relationship among Poisson, exponential and Gamma distributions. 03

(b) Consider a typical grocery shop. Demonstrate that it is a queuing system describing its components. 04

(c) Describe aspiration level model of a queuing system. What is its purpose? P(604) 2

Q7. (a) Consider a single server queuing model ($s=1$), with inter arrival times and service times, having exponential distribution with mean arrival rate β_n and mean service rate μ_n . Also consider $\beta_n = \beta$ for $n = 0, 1, 2, \dots$ and $\mu_n = \mu$ for $n = 1, 2, 3, \dots$ with rate in = rate out principle, develop the balance equation and hence deduce $P_n = (1-\xi) \xi^n$ for $n = 0, 1, 2, \dots$ when ξ is the utilization factor.

- (b) Consider a shoeshine shop consisting of two chairs. Suppose that an entering customer will first go to chair 1. When his work is completed in chair 1, he will either go to chair 2 if that chair is empty or else wait in chair 1 until chair 2 becomes empty. Suppose that a potential customer will enter this shop as long as chair 1 is empty. The potential customer arrive in accordance with a Poisson Process at rate 1/min, and that the service times for the two chairs are independent and have respectively exponential rates of 2/min and 1/min,
- Define the states and calculate the corresponding probabilities
 - What proportion of potential customers enters the system?
 - What is the mean numbers of customers in the system?
 - What is the average amount of time that an entering customer spends in the system?

Q8. (a) It is necessary to determine how much process storage space to allocate to a particular work center in a new factory. Jobs would arrive at this work center according to Poisson process with a mean rate of three per hour and the time required to perform the necessary work has an exponential distribution with a mean of 0.3 hours. If each job would require one square feet floor space while in process how much space allocate to accommodate all waiting jobs 95% of the time?

(b) At a small store of readymade garments, there is one clerk at the counter who is to check bill, receive payments and place the packed garments into fancy bags etc. The customer's arrival at the check counter is a random phenomenon and the time between arrival varies from 1 minute to 6 minutes, the frequency distribution of which is given below:

Time between arrived minutes : 1 2 3 4 5 6

Frequency in % : 30 20 15 15 10 10

The service time varies from 1 minute to 3 minutes with the following frequency distribution:

Service time in minute : 1.0 1.5 2.0 2.5 3.0

Frequency in % : 20 30 20 10 15

The manager of the store feels that the counter clerk is not sufficiently loaded with work and wants to assign him some additional work. Calculate what percentage of time the counter clerk is idle?

Use the following Random numbers for solving the above problem.

For arrival : 49 16 36 76 68 91 97

85 56 84 39 78 78 01

For servicing : 05 94 59 66 25 24 95

93 01 29 18 63 52 85

The End

Input source \Rightarrow customer (population) having wanted.
 calling unit \rightarrow customer wanting things,
 Queue \rightarrow customers waiting for a shop keeper
 Service discipline \rightarrow Usually, First in First out.
 Service mechanism \rightarrow Shop keeper & things

your own NP.

Course No.: CSE-505
 Full Marks: 70

Course Title: Applied Statistics and Queuing Theory

Time: 3 hours

- N.B.:
- Answer SIX questions, taking THREE from each section.
 - Figures in the margin indicate full marks.
 - Use separate answer script for each section.

$$P(A \cap B) = P(A)P(B|A)$$

SECTION-A

Q1(a) A and B are mutually exclusive. $P(A)=0.4$, $P(B)=0.5$. Find $P(A \cup B)$, $P(A')$ and $P(A' \cap B)$. Are A and B independent? 04

(b) Probability of Passing in mathematics is $\frac{1}{3}$ and in Biology is $\frac{1}{2}$. Probability of passing at least in one subject is $\frac{1}{3}$. What is the probability of passing in both courses? 03

(c) One bag contains 5 white and 3 red balls and the second bag contains 4 white and 5 red balls. From one of them, chosen at random, two balls are drawn. Find the chance that they are of different colors. 04

Q2 If x is a random variable with probability density function

$$f(x) = \begin{cases} c(1-x^2), & -1 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Given success save the calculate
 (a) trial indep. (b) outcome
 (c) key Q3(a) What are the similarities and dissimilarities between Binomial and Geometric distribution? 03

Q3(b) Failure of a system occurs by either motherboard or CPU. If probability of failure by motherboard is 0.6, what is the probability that out of next 20 failures at least 2 failures will occur by CPU? 05

Q4(a) What is standard normal Z? For any normal distribution, calculate $P(-2 < Z < 2)$. 03

Q4(b) How do you recognize an exponential distribution? How are they related to Poisson distribution? 03

Q4(c) Explain the independent and stationary increment of an exponential distribution. 02

Q4(d) Suppose that the amount of time a person spends in a bank is exponentially distributed with mean 10 minutes. What is the probability that a customer will spend more than 12 minutes? What is the probability that he spends more than 12 minutes given that he is still in the bank after 10 minutes? 06

SECTION-B

Q5(a) What do you mean by 95% confidence interval? Explain. 02

(b) The average power failure is found to be 2.6 times a day counted from 36 samples. Find the 95% confidence interval for the mean power failure. Assume variance as 0.09. How large a sample is required if you need to be 95% confident and error to be less than 0.04? 05

(c) A sample of 100 motherboards showed average life span 71.8 months. Assume population standard deviation = 8.9 months. Does it seem to indicate that life span is greater than 70 months while $\alpha = 0.05$? 04

Q6(a) Define the notations P , P^2 , $P(2)$ used in Markov chain models. 02

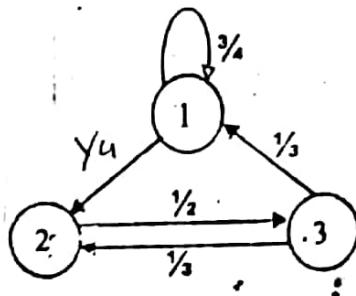
(b) A computer system is in one of three states: busy, idle, or undergoing repair. The transition probability matrix is as: 04

$$P = \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.6 & 0.0 & 0.4 \end{bmatrix}$$

What percentage of time the computer system will be in good condition?

Consider the 3-stage system shown below and the transition probabilities indicated. Calculate the steady-state probabilities.

05



Q7(a) Give a real life example of a queuing system and explain Why do you need to study queuing model as a CSE graduate?

04%

(b) A potential customer enters the system with two servers as long as the first server is idle. The servers are in series and both servers need to be used. If the second server is busy then the customer needs to wait at first server even after getting service from the first server. If the arrival rate is λ and service rates are μ_1 and μ_2 . What proportion of the customers enters the system and the average time that a customer spends in the system? Draw the state space diagram when the customer does not wait at the first server after completion his service when the second server is not freed.

07

Q8(a) Average repair time (exponential) for a computer is 20 minutes. The arrival rate of computers in the service station is 12 for an 8 hour-day. How many computers are ahead of the average set just brought in? What percentage of time will the operator be idle?

05%

(b) It is necessary to determine how much storage space to allocate to a particular work centre in a new factory. Jobs would arrive at this work centre according to a poisson process with a mean rate of three per hour and the time required to perform the necessary work has an exponential distribution with a mean rate of 0.3 hours. If each job would require 2 square feet of floor space while in the process storage at the work centre, how much space must be provided to accommodate all waiting jobs 50% of the time?

06

$$\frac{1}{\mu} = \frac{1}{\lambda} - \frac{1}{\lambda}$$