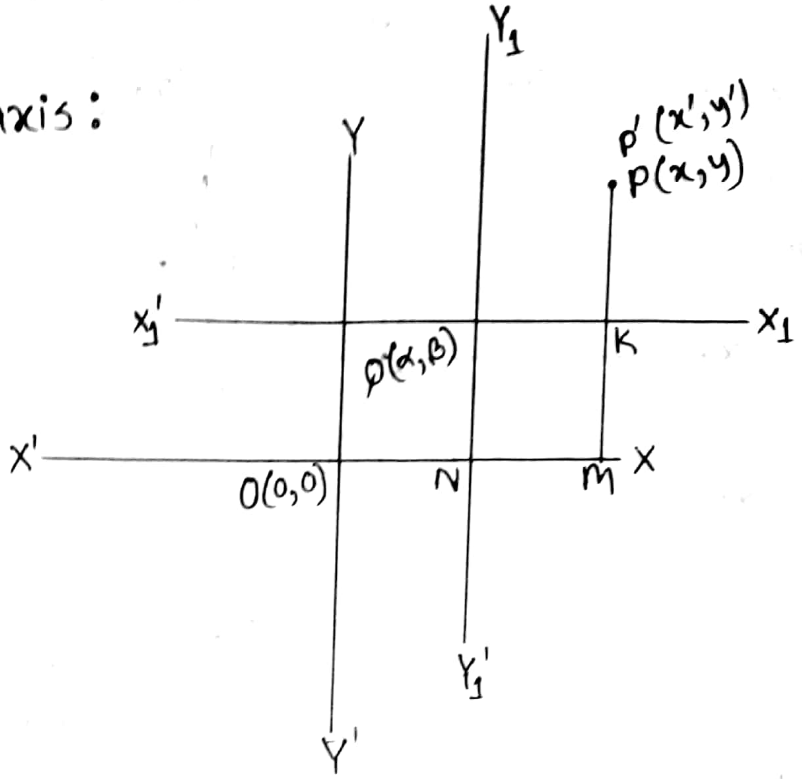
13-08-2017 : Sunday : 2A

Change of axis:



$OM = x$

$PM = y$

$ON = \alpha$

$O'N = \beta$

$PK = y'$

$O'K = x'$

from figure

$OM = ON + NM$

$\Rightarrow x = \alpha + O'K$

$\Rightarrow x = \alpha + x'$

$\boxed{\therefore x' = x - \alpha}$

$PM = PK + KM$

$\Rightarrow y = y' + O'N$

$\Rightarrow y = y' + \beta$

$\boxed{\Rightarrow y' = y - \beta}$

$x^2 + y^2 + 6x - 8y + 5 = 0$

centre $= (-3, 4)$

radius $= \sqrt{9 + 16 - 5}$

$= \sqrt{20}$

$= 2\sqrt{5}$

Let, the origin be transferred to the point $(-3, 4)$

if $(\alpha, \beta) = (-3, 4)$

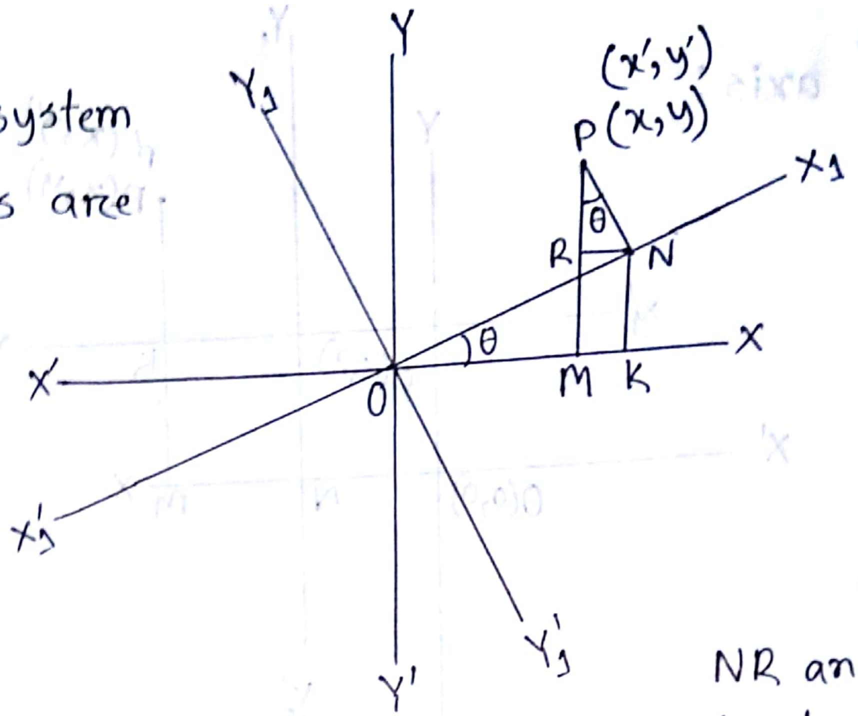
$\therefore x = x' - 3$

& $y = y' + 4$

$\therefore x'^2 + y'^2 = 20$

Oblique axis system

→ when the axis are not at 90° angle



Let, (x', y') be the positions w.r.to new set of axis

$PM = y$

$OM = x'$

$PN = y'$

$ON = x'$

NR and NK perpendiculars are drawn to OX and PM line respectively.

from figure

$OM = OK - MK$

$$\boxed{\Rightarrow x = x'\cos\theta - y'\sin\theta}$$

Now, from the triangle ONK

$$\cos\theta = \frac{OK}{ON}$$

$$\Rightarrow OK = x'\cos\theta$$

Now, from the triangle PRN

$$\sin\theta = \frac{RN}{PN}$$

$$\Rightarrow RN = y'\sin\theta$$

$$\Rightarrow MK = y'\sin\theta$$

Now, $PM = PR + RM$

$$\Rightarrow y = PR + NK$$

$$\boxed{\Rightarrow y = y'\cos\theta + x'\sin\theta}$$

Now, from the triangle ONK

$$\sin\theta = \frac{NK}{ON}$$

$$\Rightarrow NK = x'\sin\theta$$

Now, from the triangle PRN

$$\cos\theta = \frac{PR}{PN}$$

$$\Rightarrow PR = y'\cos\theta$$

19-08-2017 : 2C : Saturday

$$x = x'\cos\theta - y'\sin\theta$$

$$y = x'\sin\theta + y'\cos\theta$$

$\rightarrow ax^2 + 2hxy + by^2$ -

after how much angle rotation, the 'xy' term will be removed? $\theta = ?$

Expression in new axis after rotating $\theta°$.

$$a(x'\cos\theta - y'\sin\theta)^2 + 2h(x'\cos\theta - y'\sin\theta)(x'\sin\theta + y'\cos\theta) + b(x'\sin\theta + y'\cos\theta)^2$$

$$= x'^2(a\cos^2\theta + 2h\cos\theta\sin\theta + b\sin^2\theta) + x'y'(-2a\cos\theta\sin\theta + 2h\cos^2\theta - 2h\sin^2\theta + 2b\cos\theta\sin\theta) + y'^2(a\sin^2\theta - 2h\sin\theta\cos\theta + b\cos^2\theta)$$

$x'y'$ will be removed if

$$-2a\cos\theta\sin\theta + 2h\cos^2\theta - 2h\sin^2\theta + 2b\sin\theta\cos\theta = 0$$

$$\Rightarrow (b-a)\sin 2\theta + 2h(\cos^2\theta - \sin^2\theta) = 0$$

$$\Rightarrow (b-a)\sin 2\theta + 2h\cos 2\theta = 0$$

$$\Rightarrow 2h\cos 2\theta = (a-b)\sin 2\theta$$

$$\Rightarrow \frac{\sin 2\theta}{\cos 2\theta} = \frac{2h}{a-b}$$

$$\Rightarrow \tan 2\theta = \frac{2h}{a-b}$$

$$\boxed{\Rightarrow \theta = \frac{1}{2}\tan^{-1}\frac{2h}{a-b}}$$

$$\rightarrow 3x^2 + 2xy + 3y^2$$

$$\theta = \frac{1}{2}\tan^{-1}\left(\frac{2}{0}\right)$$

$$= \frac{1}{2} \times \frac{\pi}{2}$$

$$= \frac{\pi}{4}$$

$$x = x'\cos\frac{\pi}{4} - y'\sin\frac{\pi}{4}$$

$$= \frac{x'}{\sqrt{2}} - \frac{y'}{\sqrt{2}}$$

$$y = x'\sin\frac{\pi}{4} + y'\cos\frac{\pi}{4}$$

$$= \frac{x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}$$

putting the value of x, y in the expression,

$$3\left(\frac{x'}{\sqrt{2}}-\frac{y'}{\sqrt{2}}\right)^2+2\left(\frac{x'}{\sqrt{2}}-\frac{y'}{\sqrt{2}}\right)\left(\frac{x'}{\sqrt{2}}+\frac{y'}{\sqrt{2}}\right)+3\left(\frac{x'}{\sqrt{2}}+\frac{y'}{\sqrt{2}}\right)^2$$

$$=3\left(\frac{x'^2}{2}-x'y'+\frac{y'^2}{2}\right)+2\left(\frac{x'^2}{2}-\frac{y'^2}{2}\right)+3\left(\frac{x'^2}{2}+x'y'+\frac{y'^2}{2}\right)$$

$$=3x'^2+x'^2+3y'^2-y'^2$$

$$=4x'^2+2y'^2$$

* Invariants: If any expression does not change, when transformation of axis system applied then that expression is called Invariants.

$\rightarrow ax^2+2hxy+by^2$

for angle transformation:

$a+b$
$ab-h^2$ will always be same.

hence, $\boxed{a+b=a'+b'}$

$\boxed{ab-h^2=a'b'-h'^2}$

$\rightarrow 3x^2+2xy+3y^2$

$a+b=6$

$ab-h^2=9-1=8$

$\rightarrow 4x^2+2y'^2$

$a'+b'=4+2=6$

$a'b'-h'^2=8-0=8$

$\rightarrow \underline{a'}x'^2 + \underline{2h'}x'y' + \underline{b'}y'^2$

where,

$a' = a\cos^2\theta + 2h\cos\theta\sin\theta + b\sin^2\theta$

$2h' = -2a\cos\theta\sin\theta + 2h\cos^2\theta - 2h\sin^2\theta + 2b\sin\theta\cos\theta$

$b' = a\sin^2\theta - 2h\cos\theta\sin\theta + b\cos^2\theta$

$\rightarrow$

$a'+b' = a(\cos^2\theta + \sin^2\theta) + 0 + b(\sin^2\theta + \cos^2\theta)$

$\Rightarrow a'+b' = a+b$

$\rightarrow$

$2a' = 2a\cos^2\theta + 2h\sin 2\theta + 2b\sin^2\theta$

$= a(1+\cos 2\theta) + 2h\sin 2\theta + b(1-\cos 2\theta)$

$\therefore 2a' = (a+b) - (b-a)\cos 2\theta + 2h\sin 2\theta$

and $2b' = (a+b) + (b-a)\cos 2\theta - 2h\sin 2\theta$

$2h' = 2(b-a)\cos\theta\sin\theta + 2h(\cos^2\theta - \sin^2\theta)$

$\Rightarrow 2h' = (b-a)\sin 2\theta + 2h\cos 2\theta$

$2a' \times 2b' = (a+b)^2 - \{(b-a)\cos 2\theta - 2h\sin 2\theta\}^2$

$\therefore 4(a'b' - h'^2)$

$= (a+b)^2 - \{(b-a)\cos 2\theta - 2h\sin 2\theta\}^2 - \{(b-a)\sin 2\theta + 2h\cos 2\theta\}^2$

$= (a+b)^2 - \{(b-a)^2 + 4h^2\}$

$= (a+b)^2 - (a-b)^2 - 4h^2 = 4ab - 4h^2 = 4(ab-h^2)$

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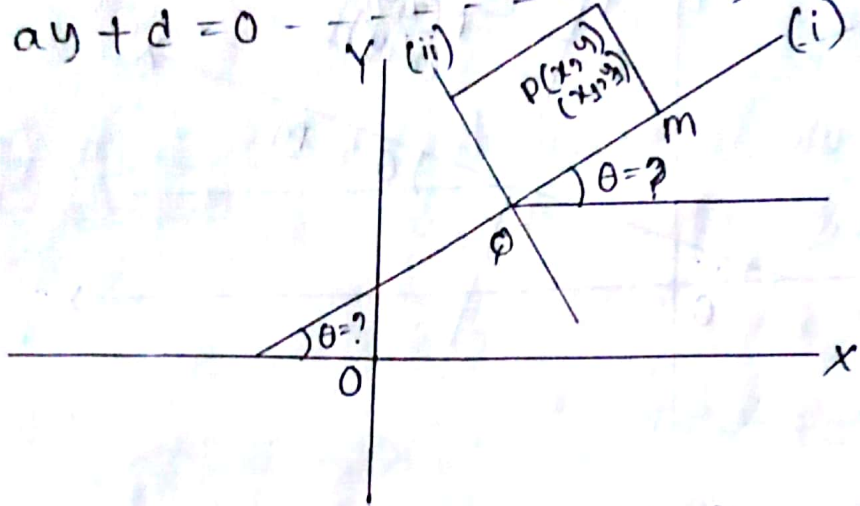
22-08-2017 : 3A : Tuesday

$ax + by + c = 0$ ------------ (i)

$bx - ay + d = 0$ ------------ (ii)



slope of (i) equation, $m = \tan\theta = -\frac{a}{b} \Rightarrow \theta = \tan^{-1}\left(-\frac{a}{b}\right)$

$$PM = y' \ \& \ \phi M = x'$$

Here,

distance from p to equation (i), $y' = \frac{ax+by+c}{\sqrt{a^2+b^2}}$ ----- (iii)

distance from p to equation (ii), $x' = \frac{bx-ay+d}{\sqrt{a^2+b^2}}$ ----- (iv)

$ax+by+c-y'\sqrt{a^2+b^2} = 0$ ------ (v)

$bx-ay+d-x'\sqrt{a^2+b^2} = 0$ ------ (vi)

solving (v) & (vi) we get,

$$x = \frac{\sqrt{a^2+b^2}\,(ay'+bx') - ca - bd}{a^2+b^2}$$

$$y = \frac{\sqrt{a^2+b^2}\,(by'-ax') - bc + ad}{a^2+b^2}$$

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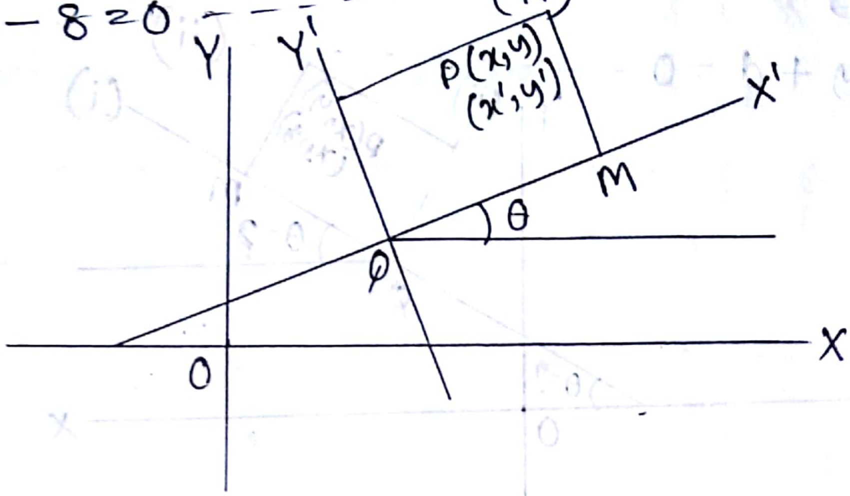
→

$x - 2y + 1 = 0$ ------- (i)

$2x + y - 8 = 0$ ------- (ii)



We get,

$$y' = \frac{x-2y+1}{\sqrt{1^2+2^2}} \Rightarrow x - 2y + 1 - y'\sqrt{5} = 0 \text{ ---- (iii)}$$

$$x' = \frac{2x+y-8}{\sqrt{2^2+1^2}} \Rightarrow 2x + y - 8 - x'\sqrt{5} = 0 \text{ ---- (iv)}$$

solving (iii) & (iv) we get,

$$x = \frac{2\sqrt{5}\,x' + \sqrt{5}\,y' + 15}{5}$$

$$y = \frac{\sqrt{5}\,x' - 2\sqrt{5}\,y' + 10}{5}$$

$$11x^2 - 4xy + 14y^2 - 58x - 44y + 126 = 0 \text{ ------ (v)}$$

Now, putting the value of x & y in equation (v),

$$11\left(\frac{2\sqrt{5}x'+\sqrt{5}y'+15}{5}\right)^2-4\left(\frac{2\sqrt{5}x'+\sqrt{5}y'+15}{5}\right)\left(\frac{\sqrt{5}x'-2\sqrt{5}y'+10}{5}\right)$$

$$+14\left(\frac{\sqrt{5}x'-2\sqrt{5}y'+10}{5}\right)^2-58\left(\frac{2\sqrt{5}x'+\sqrt{5}y'+15}{5}\right)-44\left(\frac{\sqrt{5}x'-2\sqrt{5}y'+10}{5}\right)$$

$$+126=0$$

$$\Rightarrow \frac{11}{25}\left(20x'^2+5y'^2+225+20x'y'+30\sqrt{5}y'+60\sqrt{5}x'\right)$$

$$-\frac{4}{25}\left(10x'^2-20x'y'+20\sqrt{5}x'+5x'y'-10y'^2+10\sqrt{5}y'\right.$$

$$\left.+15\sqrt{5}x'-30\sqrt{5}y'+150\right)+\frac{14}{25}\left(5x'^2+20y'^2+100-20x'y'\right.$$

$$\left.-40\sqrt{5}y'+20\sqrt{5}x'\right)-\frac{58}{5}\left(2\sqrt{5}x'+\sqrt{5}y'+15\right)-\frac{44}{5}\left(\sqrt{5}x'\right.$$

$$\left.-2\sqrt{5}y'+10\right)+126=0$$

$$\Rightarrow x'^2\left(\frac{44}{5}-\frac{8}{5}+\frac{14}{5}\right)+y'^2\left(\frac{11}{5}+\frac{8}{5}+\frac{56}{5}\right)+x'y'\left(\frac{44}{5}\right.$$

$$\left.+\frac{12}{5}-\frac{56}{5}\right)+x'\left(\frac{132\sqrt{5}}{5}-\frac{28\sqrt{5}}{5}+\frac{56\sqrt{5}}{5}-\frac{116\sqrt{5}}{5}\right.$$

$$\left.-\frac{44\sqrt{5}}{5}\right)+y'\left(\frac{66\sqrt{5}}{5}+\frac{16\sqrt{5}}{5}-\frac{112\sqrt{5}}{5}-\frac{58\sqrt{5}}{5}+\frac{88\sqrt{5}}{5}\right)$$

$$+99-24+56-174-88+126=0$$

$$\Rightarrow 10x'^2+15y'^2-5=0$$

$$\Rightarrow 2x'^2+3y'^2=1$$

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8. $17x^2+18xy-7y^2-16x-32y-18=0$

remove the terms involving x, y and xy.

Solution:

If we transfer the origin from $(0,0)$ to (α,β) then we get,

where,
$$x = x'+\alpha$$
$$y = y'+\beta$$

$17x^2+18xy-7y^2-16x-32y-18=0$ ------- (i)

putting $x=x'+\alpha$ and $y=y'+\beta$ in equation (i) we get,

$17(x'+\alpha)^2+18(x'+\alpha)(y'+\beta)-7(y'+\beta)^2-16(x'+\alpha)$
$-32(y'+\beta)-18=0$

$\Rightarrow 17x'^2+18x'y'-7y'^2+x'(34\alpha+18\beta-16)$
$+y'(18\alpha-14\beta-32)+17\alpha^2+18\alpha\beta-7\beta^2-16\alpha-3$
$-18=0$ ------- (ii)

If x' & y' are not present in equation (ii) we g

$34\alpha+18\beta-16=0$ --------- (iii)

$18\alpha-14\beta-32=0$ -------- (iv)

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solving (iii) & (iv) we get,

$\alpha = 1, \beta = -1$

putting the value of α, β in equation (ii) we get

$17x'^2 + 18x'y' - 7y'^2 - 10 = 0$ ------- (v)

from (v) we get,

$a = 17, h = 9, b = -7$

$\tan 2\theta = \frac{2h}{a-b}$

$\Rightarrow \tan 2\theta = \frac{18}{17+7}$

$\Rightarrow \tan 2\theta = \frac{3}{4}$

$\Rightarrow \sqrt{\sec^2 2\theta - 1} = \frac{3}{4}$

$\Rightarrow \sec^2 2\theta = \frac{9}{16} + 1$

$\Rightarrow \cos 2\theta = \frac{1}{\frac{5}{4}}$

$\Rightarrow 2\cos^2\theta - 1 = \frac{4}{5}$

$\Rightarrow \cos^2\theta = \frac{9}{10}$

$\therefore \cos\theta = \frac{3}{\sqrt{10}}$ $\Rightarrow \sin^2\theta = 1 - \frac{9}{10}$

$\therefore \sin\theta = \frac{1}{\sqrt{10}}$

$x' = x''\cos\theta - y''\sin\theta$ & $y' = x''\sin\theta + y''\cos\theta$

$\Rightarrow x' = x''\frac{3}{\sqrt{10}} - y''\frac{1}{\sqrt{10}}$ $\Rightarrow y' = x''\frac{1}{\sqrt{10}} + y''\frac{3}{\sqrt{10}}$

$\Rightarrow x' = \frac{3x''-y''}{\sqrt{10}}$ $\Rightarrow y' = \frac{x''+3y''}{\sqrt{10}}$

putting the value of x' & y' in equation (v) we get,

$$17\left(\frac{3x''-y''}{\sqrt{10}}\right)^2 + 18\left(\frac{3x''-y''}{\sqrt{10}}\right)\left(\frac{x''+3y''}{\sqrt{10}}\right) - 7\left(\frac{x''+3y''}{\sqrt{10}}\right)^2 - 10 = 0$$

$$\Rightarrow 17(9x''^2 - 6x''y'' + y''^2) + 18(3x''^2 + 8x''y'' - 3y''^2) - 7(x''^2 + 6x''y'' + 9y''^2) - 100 = 0$$

$$\Rightarrow x''^2(153 + 54 - 7) - x''y''(102 - 144 + 42) + y''^2(17 - 54 - 63) = 100$$

$$\Rightarrow 200x''^2 - 100y''^2 = 100$$

$$\Rightarrow 2x''^2 - y''^2 = 1.$$

26-08-2017 : 3C : Saturday

$$3x^2 + 5xy - 2y^2 = 0$$

$$\Rightarrow (x+2y)(3x-y) = 0$$

$$x + 2y = 0 \qquad 3x - y = 0$$

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* $\longrightarrow ax^2 + 2hxy + by^2 = 0$ ---------- (i)

Homogeneous equation of 2^{nd} degree represent a pair of straight lines passing through the origin.

Let,

$$y = m_1 x$$

$$y = m_2 x$$

then,

$$(y - m_1 x)(y - m_2 x) = 0$$

$$\Rightarrow y^2 - (m_1 + m_2) xy + m_1 m_2 x^2 = 0 \text{ ---------- (ii)}$$

from (i)

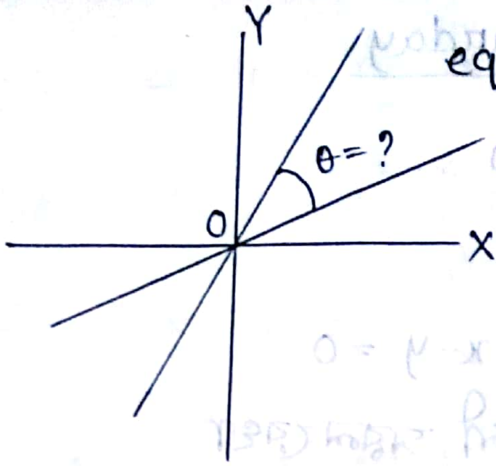
$$y^2 + \frac{2h}{b} xy + \frac{a}{b} x^2 = 0 \text{ ---------- (iii)}$$

from (ii) & (iii)

$$m_1 + m_2 = -\frac{2h}{b}$$

$$m_1 m_2 = \frac{a}{b}$$

equation (i)



$$\theta = \tan^{-1} \frac{m_1 - m_2}{1 + m_1 m_2}$$

$$= \tan^{-1} \frac{\sqrt{(m_1+m_2)^2 - 4m_1 m_2}}{1 + \frac{a}{b}}$$

$$= \tan^{-1} \frac{\sqrt{\frac{4h^2}{b^2} - \frac{4a}{b}}}{1 + \frac{a}{b}}$$

$$= \tan^{-1} \frac{\sqrt{4h^2 - 4ab}}{a+b}$$

$$\boxed{\therefore \theta = \tan^{-1} \frac{2\sqrt{h^2 - ab}}{a+b}}$$

$$\theta = \tan^{-1} \frac{2\sqrt{\frac{25}{4} + 6}}{1}$$

$$= \tan^{-1} 7$$

$$= 81.870^\circ$$

$x + 2y = 0$

$3y - y = 0$

$3x^2 + 5xy - 2y^2 = 0$

⊛ → If $\theta = 90°$ (perpendicular)

$$\tan 90° = \frac{2\sqrt{h^2-ab}}{a+b}$$

$$\Rightarrow \frac{1}{0} = \frac{2\sqrt{h^2-ab}}{a+b}$$

$$\boxed{\Rightarrow a+b = 0}$$

If equation (i)'s $a+b=0$, then the two equation will be perpendicular.

⊛ → If $\theta = 0°$ (paralled)

$$\tan 0° = \frac{2\sqrt{h^2-ab}}{a+b}$$

$$\Rightarrow 0 = \frac{2\sqrt{h^2-ab}}{a+b}$$

$$\Rightarrow 2\sqrt{h^2-ab} = 0$$

$$\Rightarrow h^2-ab = 0$$

$$\boxed{\Rightarrow h^2 = ab}$$

If equation (i)'s $h^2=ab$, then one straight line will be on other straight line.

$$\circledast \longrightarrow \quad ax^2+2hxy+by^2=0 \text{ ------ (i)}$$

$$\begin{cases} y-m_1x=0 \text{ ---------- (ii)} \\ y-m_2x=0 \text{ ---------- (iii)} \end{cases}$$

$$m_1+m_2=-\frac{2h}{b}$$

$$m_1m_2=\frac{a}{b}$$

equation of bisector of angle between (ii) & (iii)

$$\frac{y-m_1x}{\sqrt{1+m_1^2}} = \pm\frac{y-m_2x}{\sqrt{1+m_2^2}}$$

$$\Rightarrow (y-m_1x)^2(1+m_2^2) = (y-m_2x)^2(1+m_1^2)$$

$$\Rightarrow (y^2-2m_1xy+m_1^2x^2)(1+m_2^2) = (y^2-2m_2xy+m_2^2x^2)(1+m_1^2)$$

$$\Rightarrow y^2+y^2m_2^2-2m_1xy-2m_1m_2^2xy+m_1^2x^2+m_1^2m_2^2x^2$$
$$= y^2+y^2m_1^2-2m_2xy-2m_2m_1^2xy+m_2^2x^2+m_1^2m_2^2x^2$$

$$\Rightarrow y^2m_1^2+m_2^2x^2-y^2m_2^2-m_1^2x^2 = 2m_2xy+$$
$$2m_2m_1^2xy-2m_1xy-2m_1m_2^2xy$$

$$\Rightarrow x^2(m_2^2-m_1^2)-y^2(m_2^2-m_1^2) = 2xy\{(m_2-m_1)+m_2m_1^2-m_1m_2^2\}$$

$$\Rightarrow (x^2-y^2)(m_2^2-m_1^2) = 2xy\{(m_2-m_1)-m_1m_2(m_2-m_1)\}$$

$$\Rightarrow (x^2-y^2)(m_2+m_1)(m_2-m_1) = 2xy\{(m_2-m_1)(1-m_1m_2)\}$$

$$\Rightarrow (x^2-y^2)(m_1+m_2) = 2xy(1-m_1m_2)$$

$$\Rightarrow (x^2-y^2)\left(-\frac{2h}{b}\right) = 2xy\left(1-\frac{a}{b}\right)$$

$$\Rightarrow (x^2-y^2)\,h = xy\,(a-b)$$

$$\boxed{\Rightarrow \frac{x^2-y^2}{xy} = \frac{a-b}{h}}$$

29-08-2017 : 4A : Tuesday

General equation of 2^{nd} degree :-

$$ax^2+2hxy+by^2+2gx+2fy+c=0 \text{ ------ (i)}$$

It is the general equation of conic or circle if this equation ~~is~~ abide by $h=0$ and $a=b$.

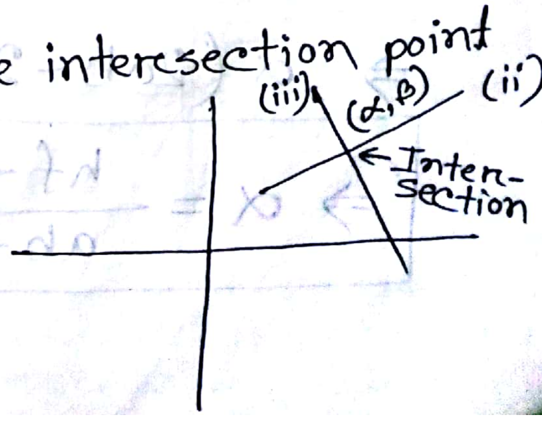
equation (i) represents two straight lines,

$$a_1x+b_1y+c_1=0 \text{ ---------- (ii)}$$

$$a_2x+b_2y+c_2=0 \text{ ---------- (iii)}$$

But their are not intersection in origin.

Let, we shift the origin to the intersection point of (ii) & (iii)



$$\boxed{\begin{aligned} x &= x'+\alpha \\ y &= y'+\beta \end{aligned}}$$

$\therefore$ New form of (i)

$$a(x'+\alpha)^2+2h(x'+\alpha)(y'+\beta)+b(y'+\beta)^2+2g(x'+\alpha)$$
$$+2f(y'+\beta)+c=0$$

$$\Rightarrow ax'^2+2hx'y'+by'^2+x'(2a\alpha+2h\beta+2g)+y'(2h\alpha+2b\beta+2f)$$
$$+a\alpha^2+2h\alpha\beta+b\beta^2+2g\alpha+2f\beta+c=0 \text{ ------ (iv)}$$

Equation (iv) must reprent a point of straight lines passing through the origin, so we get,

$$2a\alpha+2h\beta+2g=0 \text{ ---------- (v)}$$
$$2h\alpha+2\beta b+2f=0 \text{ ---------- (vi)}$$
$$a\alpha^2+2h\alpha\beta+b\beta^2+2g\alpha+2f\beta+c=0 \text{ ------ (vii)}$$

from (vii) we get,

$$\alpha(a\alpha+h\beta+g)+\beta(h\alpha+b\beta+f)+g\alpha+f\beta+c=0$$
$$\Rightarrow g\alpha+f\beta+c=0 \text{ ------- (viii)} \quad \left[\text{using (v) \& (vi)}\right]$$

~~slove~~ solve (v) & (vi),

$$2ab\alpha+2bh\beta+2bg-2h^2\alpha-2bh\beta-2hf=0$$
$$\Rightarrow \alpha(2ab-2h^2)=2hf-2bg$$

$$\boxed{\Rightarrow \alpha=\frac{hf-bg}{ab-h^2}} \qquad \boxed{\therefore \beta=\frac{gh-af}{ab-h^2}}$$

$$g\left(\frac{hf-gb}{ab-h^2}\right)+f\left(\frac{gh-af}{ab-h^2}\right)+c=0$$

$$\Rightarrow ghf-g^2b+fgh-af^2+abc-h^2c=0$$

$$\Rightarrow abc+2ghf-(af^2+bg^2+h^2c)=0$$

other way from (v), (vi) & (viii)

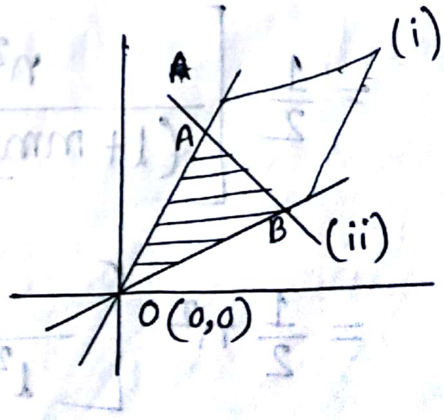
$$\boxed{\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}=0}$$

If equation (i) abide it, then the equation represents two straight line.

<u>10-09-2017 : 4C : Sunday :</u>

\# $ax^2+2hxy+by^2=0$ ---------- (i)

$lx+my+n=0$ ---------- (ii)



Let, $y=m_1x$ ---------- (iii)

and $y=m_2x$ ---------- (iv)

be two straigh lines reprented by (i) where,

$$m_1+m_2=-\frac{2h}{b}$$

$$m_1m_2=\frac{a}{b}$$

$$m_1 - m_2 = \sqrt{\frac{4h^2}{b^2} - \frac{4a}{b}} \qquad m_2 - m_1 = \sqrt{\frac{4h^2}{b^2} - \frac{4a}{b}}$$

$$= \frac{2\sqrt{h^2-ab}}{b} \qquad = \frac{2\sqrt{h^2-ab}}{b}$$

solving (ii) and (iii) we get

$$A\left(-\frac{n}{l+mm_1}, -\frac{m_1 n}{l+mm_1}\right)$$

solving (ii) and (iv) we get,

$$B\left(-\frac{n}{l+mm_2}, -\frac{m_2 n}{l+mm_2}\right)$$

$\therefore$ Area of triangle OAB

$$= \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ -\frac{n}{l+mm_1} & -\frac{m_1 n}{l+mm_1} & 1 \\ -\frac{n}{l+mm_2} & -\frac{m_2 n}{l+mm_2} & 1 \end{vmatrix}$$

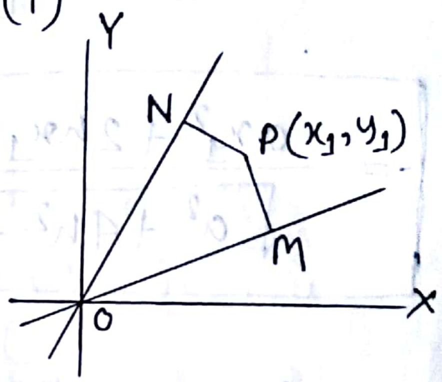
$$= \frac{1}{2}\left[\frac{n^2 m_2}{(l+mm_1)(l+mm_2)} - \frac{n^2 m_1}{(l+mm_1)(l+mm_2)}\right]$$

$$= \frac{1}{2}n^2\left[\frac{m_2 - m_1}{l^2 + lm(m_1+m_2) + m^2(m_1 m_2)}\right]$$

$$= \frac{1}{2} n^2 \left[\frac{\frac{2\sqrt{h^2-ab}}{b}}{l^2 - lm\frac{2h}{b} + m_2 \frac{a}{b}} \right]$$

$$\boxed{= \frac{1}{2} n^2 \left[\frac{2\sqrt{h^2-ab}}{l^2 b - 2hlm + am^2} \right]}$$

$ax^2 + 2hxy + by^2 = 0$ ---------- (i)



$y = m_1 x$ -------- (ii)

$y = m_2 x$ -------- (iii)

$m_1 + m_2 = -\frac{2h}{b}$

$m_1 m_2 = \frac{a}{b}$

$$\therefore PN = \frac{m_1 x_1 - y_1}{\sqrt{m_1^2 + 1^2}}$$

$$\& \; PM = \frac{m_2 x_1 - y_1}{\sqrt{m_2^2 + 1^2}}$$

$\therefore$ Product of the perpendiculars

$PN \cdot PM$

$$= \frac{(m_1 x_1 - y_1)(m_2 x_1 - y_1)}{\sqrt{(m_1^2+1)(m_2^2+1)}}$$

$$= \frac{x_1^2 m_1 m_2 - m_2 x_1 y_1 - m_1 y_1 x_1 + y_1^2}{\sqrt{m_1^2 m_2^2 + m_1^2 + m_2^2 + 1}}$$

$$= \frac{x_1^2 m_1 m_2 - x_1 y_1 (m_2+m_1) + y_1^2}{\sqrt{(m_1 m_2)^2 + (m_1+m_2)^2 - 2m_1 m_2 + 1}}$$

$$= \frac{\frac{a x_1^2}{b} + \frac{2h x_1 y_1}{b} + y_1^2}{\sqrt{\frac{a^2}{b^2} + \frac{4h^2}{b^2} - \frac{2a}{b} + 1}}$$

$$\boxed{= \frac{a x_1^2 + 2h x_1 y_1 + y_1^2 b}{\sqrt{a^2 + 4h^2 - 2ab + b^2}}}$$

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ ---------- (i)

Let, equation (i) represents

$l_1 x + m_1 y + n_1 = 0$ ---------- (ii)

and $l_2 x + m_2 y + n_2 = 0$ ---------- (iii)

multiplying (ii) and (iii) we get,

$l_1 l_2 x^2 + xy(l_1 m_2 + m_1 l_2) + m_1 m_2 y^2 + x(l_1 n_2 + n_1 l_2)$

$+ y(m_1 n_2 + m_2 n_1) + n_1 n_2 = 0$ ---------- (iv)

Since (i) and (iv) are identical.

$a = l_1 l_2$	$2h = l_1 m_2 + m_1 l_2$	$c = n_1 n_2$
$b = m_1 m_2$	$2g = l_1 n_2 + l_2 n_1$	
	$2f = m_1 n_2 + m_2 n_1$	

~~so~~ slope of equation (ii) is $-\frac{l_1}{m_1}$

slope " " (iii) is $-\frac{l_2}{m_2}$

$\therefore$ Angle between (ii) and (iii)

$$\tan\theta = \frac{-\frac{l_1}{m_1} + \frac{l_2}{m_2}}{1+\left(-\frac{l_1}{m_1}\right)\left(-\frac{l_2}{m_2}\right)}$$

$$= \frac{m_1 l_2 - m_2 l_1}{m_1 m_2 + l_1 l_2}$$

$$= \frac{\sqrt{(m_1 l_2 + m_2 l_1)^2 - 4 m_1 m_2 l_1 l_2}}{m_1 m_2 + l_1 l_2}$$

$$= \frac{\sqrt{4h^2 - 4ab}}{a+b}$$

$$\therefore \tan\theta = \frac{2\sqrt{h^2-ab}}{a+b}$$

$$\boxed{\therefore \theta = \tan^{-1}\frac{2\sqrt{h^2-ab}}{a+b}}$$

17-09-2017 : 5C : Sunday:

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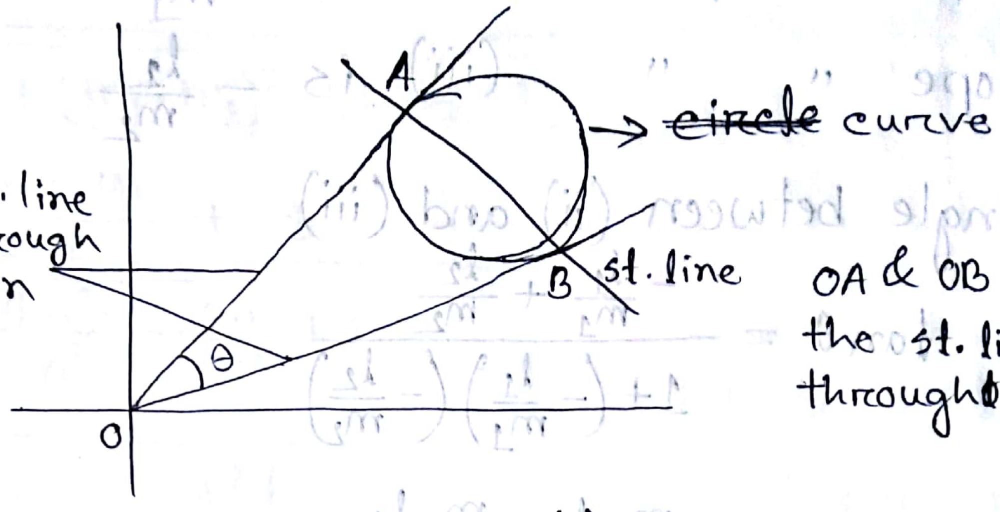
34 - Ex:V



OA & OB are the st. line through origin

$y = 3x + 2$ ------------- (i)

$x^2 + 3y^2 + 2xy + 4x + 8y - 11 = 0$ ---------- (ii)

From equation (ii) we get,

$x^2 + 3y^2 + 2xy + (4x + 8y) \cdot 1 - 11 \cdot 1^2 = 0$

$\Rightarrow x^2 + 3y^2 + 2xy + (4x + 8y)\left(\frac{y - 3x}{2}\right) - 11\left(\frac{y - 3x}{2}\right)^2 = 0$

$\Rightarrow x^2 + 3y^2 + 2xy + 2xy + 4y^2 - 6x^2 - 12xy - \frac{11}{4}(y^2 - 6xy + 9x^2)$

$\Rightarrow 4x^2 + 12y^2 + 8xy + 8xy + 16y^2 - 24x^2 - 48xy - 11y^2 + 66xy - 99x^2 = 0$

$\Rightarrow x^2(4 - 24 - 99) + xy(8 + 8 - 48 + 66) + y^2(12 + 16 - 11) = 0$

$\Rightarrow -119x^2 + 34xy + 17y^2 = 0$

$\Rightarrow 119x^2 - 34xy - 17y^2 = 0$

we know,

$$\theta = \tan^{-1} \frac{2\sqrt{h^2-ab}}{a+b}$$

$$= \tan^{-1} \frac{2\sqrt{17^2+(119\times17)}}{119 - 17}$$

$$= \cancel{\tan}\ 43.31^\circ$$

Page-53
43-Ex: V

$$kx+hy = 2hk \quad \text{-------------} (i)$$

$$(x-h)^2+(y-k)^2 = c^2 \quad \text{-------------} (ii)$$

prove if $\theta = 90^\circ$ then $h^2+k^2 = c^2$

from equation (ii)

$$x^2-2hx+h^2+y^2-2yk+k^2-c^2=0$$

$$\Rightarrow x^2+y^2-2(hx+yk)+(h^2+k^2-c^2)=0$$

$$\Rightarrow x^2+y^2-\left(\frac{kx+hy}{hk}\right)(hx+yk)+(h^2+k^2-c^2)\left(\frac{kx+hy}{2hk}\right)^2=0$$

$$\Rightarrow x^2+y^2-\frac{1}{hk}(khx^2+khy^2+h^2xy+k^2xy)+\frac{h^2+k^2-c^2}{4h^2k^2}(k^2x^2+h^2y^2+2hkxy)=0$$

$$\Rightarrow x^2\left(1-1+\frac{h^2+k^2-c^2}{4h^2}\right)+xy\left(-\frac{h^2+k^2}{hk}+\frac{h^2+k^2-c^2}{2hk}\right)$$
$$+y^2\left(1-1+\frac{h^2+k^2-c^2}{4k^2}\right)=0$$

$$\Rightarrow \frac{h^2+k^2-c^2}{4h^2}x^2+\left(\frac{h^2+k^2-c^2}{2hk}-\frac{h^2+k^2}{hk}\right)xy+\frac{h^2+k^2-c^2}{4k^2}y^2=0$$

If $\theta = 90°$ then,

$$a + b = 0$$

$$\Rightarrow \frac{h^2+k^2-c^2}{4h^2} + \frac{h^2+k^2-c^2}{4k^2} = 0$$

$$\Rightarrow h^2+k^2-c^2\left(\frac{1}{4h^2} + \frac{1}{4k^2}\right) = 0$$

$$\Rightarrow h^2+k^2-c^2 = 0$$

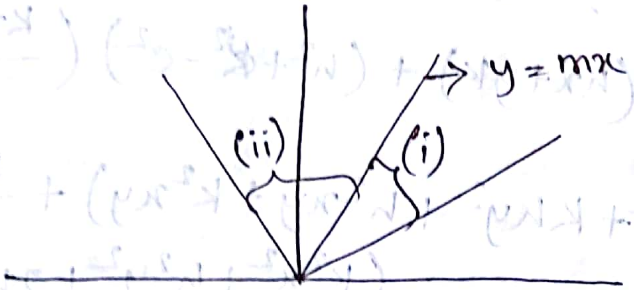
$$\Rightarrow h^2+k^2 = c^2$$

$\rightarrow$

$$ax^2 + 2hxy + by^2 = 0 \quad \text{------- (i)}$$

$$a_1x^2 + 2h_1xy + b_1y^2 = 0 \quad \text{------- (ii)}$$

one st. line common between both



Let, common st. line,

$$y = mx$$

~~ax^2+2hxm~~

~~put the~~ putting the value of y in equation (i)

$$ax^2 + 2hx\,mx + b(mx)^2 = 0$$

$\Rightarrow ax^2 + 2hmx^2 + bm^2x^2 = 0$

$\Rightarrow bm^2 + 2hm + a = 0 \quad [\because x^2 \neq 0]$ ------ (iii)

Similarly from (ii)

$b_1m^2 + 2h_1m + a_1 = 0$ ---------- (iv)

From (iii) & (iv) we get,

$$\frac{m^2}{2ha_1 - 2h_1a} = \frac{m}{ab_1 - a_1b} = \frac{1}{2h_1b - 2b_1h}$$

$$\therefore m = \frac{ab_1 - a_1b}{2h_1b - 2b_1h}$$

$$\text{and, } m^2 = \frac{2ha_1 - 2h_1a}{2h_1b - 2hb_1}$$

$$\therefore \left(\frac{ab_1 - a_1b}{2h_1b - 2b_1h}\right)^2 = \left(\frac{2ha_1 - 2h_1a}{2h_1b - 2hb_1}\right)$$

$$\Rightarrow \frac{(ab_1 - a_1b)^2}{2h_1b - 2hb_1} = 2ha_1 - 2h_1a$$

$$\Rightarrow a^2b_1^2 - 2aa_1bb_1 + a_1^2b^2 = 4hh_1a_1b - 4h^2a_1b_1 - 4h_1^2ab + 4hh_1ab_1$$

$\Rightarrow$

→ $ax^2 + 2hxy + by^2 = 0$ ------------- (i)

$a_1x^2 + 2h_1xy + b_1y^2 = 0$ ------------- (ii)

one st. line of (i) is perpendicular to one st. line of (ii),

Let,

perpendicular st. lines,

$$y = mx$$

$$y = -\frac{x}{m}$$

$y = mx$ satisfies (i)

$\therefore ax^2 + 2hxmx + b(mx)^2 = 0$

$\Rightarrow ax^2 + 2hmx^2 + bm^2x^2 = 0$

$\Rightarrow bm^2 + 2hm + a = 0 \quad [\because x^2 \neq 0]$ ------ (iii)

$y = -\frac{x}{m}$ satisfies (ii)

$\therefore a_1x^2 + 2h_1x\left(-\frac{x}{m}\right) + b_1\left(-\frac{x}{m}\right)^2 = 0$

$\Rightarrow m^2a_1x^2 - 2h_1x^2m + b_1x^2 = 0$

$\Rightarrow a_1m^2 - 2h_1m + b_1 = 0 \quad [\because x^2 \neq 0]$ ---- (iv) solving (iii) & (iv) we get,

$$\frac{m^2}{2hb_1+2h_1a} = \frac{m}{aa_1-bb_1} = \frac{1}{-2h_1b-2ha_1}$$

$$\therefore m = \frac{bb_1-aa_1}{2h_1b+2ha_1}$$

$$\& \quad m^2 = -\frac{2hb_1+2h_1a}{2h_1b+2ha_1}$$

$$\therefore \left(\frac{bb_1-aa_1}{2h_1b+2ha_1}\right)^2 = -\frac{2hb_1+2h_1a}{2h_1b+2ha_1}$$

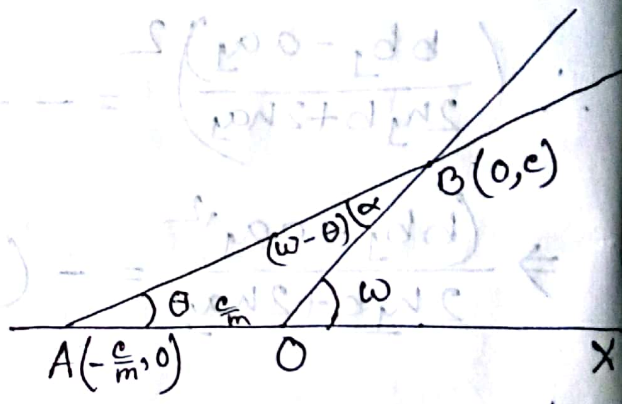
$$\Rightarrow \frac{(bb_1-aa_1)^2}{2h_1b+2ha_1} = -(2hb_1+2h_1a)$$

20-09-2017: 6A: Wednesday

→ Problem: If the axes are inclined at an angle ω, prove that the slope of the line $y = mx + c$ is $\left(\frac{m \sin\omega}{1 + m\cos\omega}\right)$. Hence, prove that the lines $y = m_1 x + c$ and $y = m_2 x + c$ are at right angle if
$$m_1 m_2 + (m_1 + m_2)\cos\omega + 1 = 0$$

Solution:

Let, θ be the angle made by the line $y = mx + c$ and x-axis and it intersects with x and y-axis in A and B where $A\left(-\frac{c}{m}, 0\right)$ and $B(0, c)$. The angle between y-axis and the line $y = mx + c$ is $(\omega - \theta)$



$\theta + \pi - \omega + \alpha = \pi$

$\Rightarrow \alpha = \omega - \theta$

$y = mx + c$

$\Rightarrow 0 = mx + c$

$\Rightarrow x = -\frac{c}{m}$

A at x axis

B at y axis

From fig:

we have

$$\frac{c}{\sin\theta} = \frac{\frac{c}{m}}{\sin(\omega-\theta)}$$

$$\Rightarrow \frac{m}{\sin\theta} = \frac{1}{\sin(\omega-\theta)}$$

$$\Rightarrow m(\sin\omega\cos\theta - \cos\omega\sin\theta) = \sin\theta$$

$$\Rightarrow m\sin\omega\cos\theta = \sin\theta(1+m\cos\omega)$$

$$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{m\sin\omega}{1+m\cos\omega}$$

$$\Rightarrow \tan\theta = \frac{m\sin\omega}{1+m\cos\omega}$$

$$y = m_1 x + c$$

$$y = m_2 x + c$$

$$\tan\beta_1 = \frac{m_1\sin\omega}{1+m_1\cos\omega}$$

$$\tan\beta_2 = \frac{m_2\sin\omega}{1+m_2\cos\omega}$$

$$\tan\beta_1 \cdot \tan\beta_2 = -1$$

$$\Rightarrow \frac{m_1\sin\omega}{1+m_1\cos\omega}\cdot\frac{m_2\sin\omega}{1+m_2\cos\omega} = -1$$

$$\Rightarrow m_1 m_2 \sin^2\omega = -(1+m_1\cos\omega)(1+m_2\cos\omega)$$

$\Rightarrow m_1 m_2 \sin^2\omega = -(1 + m_1 \cos\omega + m_2 \cos\omega + m_1 m_2\} \cos$

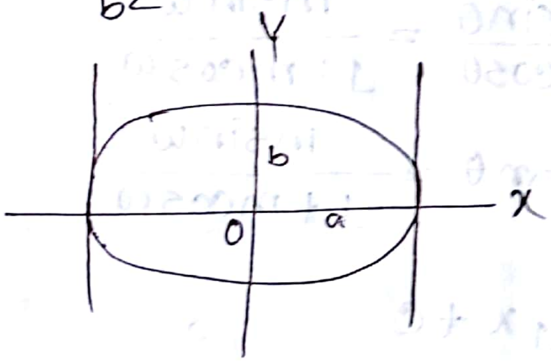
$\Rightarrow m_1 m_2 (\sin^2\omega + \cos^2\omega) + 1 + \cos\omega\,(m_1 + m_2) = 0$

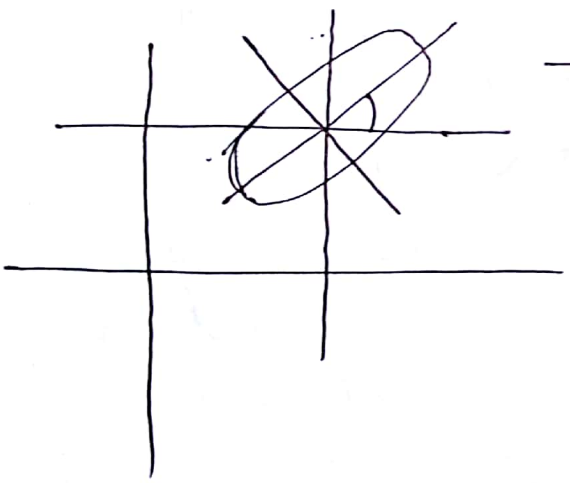
$\Rightarrow m_1 m_2 + \cos\omega\,(m_1 + m_2) + 1 = 0$

24-09-2017 : 6C : Sunday :

$\rightarrow$ standard form of elipse :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$





$ax^2 + 2hxy + bx^2 + 2gx + 2fy + c = 0$

General equation of second degree represents

(i) a circle if $a = b$ and $h = 0$

(ii) a pair of st. lines if

$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

(iii) a parabola if $\Delta \neq 0$ and $ab = h^2$

(iv) a hyperbola if $\Delta \neq 0$ and $ab < h^2$

(v) an elipse if $\Delta \neq 0$ and $ab > h^2$

$\rightarrow$ $8x^2 + 4xy + 5y^2 - 24x - 24y = 0$ ---- (i)

Page-68 Ex

$$\Delta = \begin{vmatrix} 8 & 2 & -12 \\ 2 & 5 & -12 \\ -12 & -12 & 0 \end{vmatrix} = -1296 \neq 0$$

And $ab - h^2 = 8\times5 - 2^2 = 40 - 4 = 36 > 0$

$\therefore$ Given equation is an elipse.

~~insection poi~~

intersection point for st. lines, center for elipse, $\boxed{\alpha = \dfrac{hf - gb}{ab - h^2}}$, $\boxed{\beta = \dfrac{gh - af}{ab - h^2}}$

Here, centre is,

i.e. $\left(\dfrac{2\times(-12) - (-12)\times5}{36}, \dfrac{(-12)\times2 - 8\times(-12)}{36} \right)$

$= (1, 2)$

we get,

$$x = x' + 1$$
$$y = y' + 2$$

From (i) we get,

$$8(x'+1)^2 + 4(x'+1)(y'+2) + 5(y'+2)^2 - 24(x'+1) - 24(y'+2) = 0$$

$$\Rightarrow 8x'^2 + 4x'y' + 5y'^2 + x'(16+8-24) + y'(4+20-24) + 8+8+20-24-24 = 0$$

$$\Rightarrow 8x'^2 + 4x'y' + 5y'^2 - 36 = 0$$

After rotating the equation (ii) be

$$Ax''^2 + By''^2 - 36 = 0 \quad \text{------ (iii)}$$

By the law of invariants,

$$A + B = 8 + 5 \cancel{= 13}$$
$$\Rightarrow A + B = 13 \quad \text{------ (iv)}$$

$$AB - H^2 = 8 \times 5 - 2^2$$
$$\Rightarrow AB = 36 \quad \text{------ (v)}$$

solving (iv) & (v) we get,

$$A = 9, \; B = 4$$

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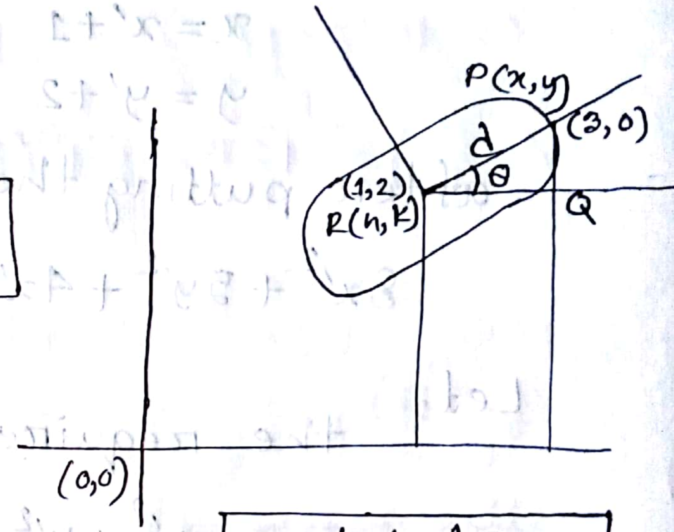
putting the value of A & B in equation (iii),

$$9x''^2 + 4y''^2 - 36 = 0$$

$$\Rightarrow \frac{x''^2}{\frac{36}{9}} + \frac{y''}{\frac{36}{4}} = 1$$

$$\Rightarrow \frac{x''^2}{4} + \frac{y''^2}{9} = 1$$

03-10-2017 : 7A : Tuesday



$$\frac{x''^2}{9} + \frac{y''^2}{4} = 1 \qquad \boxed{\begin{matrix} a=3 \\ b=2 \end{matrix}}$$

$$\cos\theta = \frac{RQ}{PR}$$

$$\Rightarrow RQ = d\cos\theta \quad [PR = d]$$

$$\boxed{\begin{matrix} x = h + d\cos\theta \\ y = k + d\,\cancel{\cos\theta}\sin\theta \end{matrix}}$$

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$= \sqrt{\frac{9-4}{9}} = \frac{\sqrt{5}}{3}$$

→ equation of directrices of ellipse

$$x = \pm \frac{a}{e}$$

→ equation of major-axis of ellipse

$$y = 0$$

→ focus of ellipse

$$(\pm ae, 0)$$

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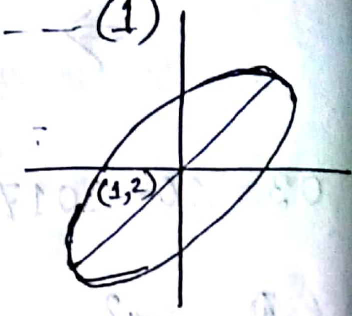
→ equation of minor axis of ellipse

$x=0$

→ General equation:

$$ax^2+2hxy+by^2+2gx+2fy+c=0$$

$$\Rightarrow 8x^2+4xy+5y^2-24x-24y=0 \quad \text{------ (1)}$$



$$x=x'+1$$
$$y=y'+2$$

after putting these value

$$8x'^2+5y'^2+4x'y'-36=0$$

Let, the required equation,

$$x'^2+y'^2=r^2$$

$$\Rightarrow \left(\frac{x'}{r}\right)^2+\left(\frac{y'}{r}\right)^2=1 \quad \text{------ (ii)}$$

$$ax'^2+2hx'y'+by'^2=R \quad (\text{const. term})$$

$$\Rightarrow \frac{a}{R}x'^2+\frac{2h}{R}x'y'+\frac{b}{R}y'^2=1$$

$$\Rightarrow Ax'^2+2Hx'y'+By'^2=1 \quad \text{------ (iii)}$$

from equation (ii) & (iii) we get,

$$Ax'^2+2Hx'y'+By'^2=\frac{x'^2}{r^2}+\frac{y'^2}{r^2}$$

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$$\Rightarrow \left(A-\frac{1}{r^2}\right)x'^2+2Hx'y'+\left(B-\frac{1}{r^2}\right)y'^2=0 \quad \text{------ (iii)}$$

since equation (iii) represents two parallel st. lines passing through the origin

$$\left(A-\frac{1}{r^2}\right)\left(B-\frac{1}{r^2}\right)=H^2$$

$$\Rightarrow AB-\frac{1}{r^2}(A+B)+\frac{1}{r^4}=H^2$$

$$\Rightarrow \frac{1}{r^4}-\frac{1}{r^2}(A+B)+AB-H^2=0 \quad \text{-------- (iv)}$$

$$8x'^2+4x'y'+5y'^2=36$$

$$\Rightarrow \frac{8}{36}x'^2+\frac{4}{36}x'y'+\frac{5}{36}y'^2=1 \quad \text{-------- (2)}$$

~~⇒ since equation (2) represents two parallel st. lines p~~

putting the value of A, B & H in equation (iv)

$$\frac{1}{r^4}-\frac{1}{r^2}\left(\frac{8}{36}+\frac{5}{36}\right)+\left(\frac{8}{36}\times\frac{5}{36}\right)-\left(\frac{2}{36}\right)^2=0$$

$$\Rightarrow \frac{1}{r^4}-\frac{13}{36r^2}+\frac{36}{1296}=0$$

$$\Rightarrow r^4-13r^2+36=0$$

$$\Rightarrow (r^2-9)(r^2-4)=0$$

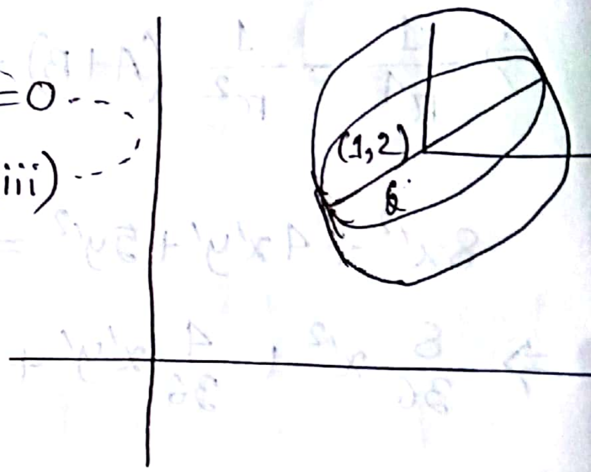
• if $r^2-9=0$ $\Rightarrow r=3$ | else $r^2-4=0$ $\Rightarrow r=2$

putting the value of r in (i)

$$\frac{x'^2}{9}+\frac{y'^2}{4}=1$$

07-10-2017 : 7C : Saturday

$$\left(A-\frac{1}{r^2}\right)x^2+2Hx'y'+\left(B-\frac{1}{r^2}\right)y'^2=0 \quad \text{(iii)}$$



$$\left(A-\frac{1}{r^2}\right)\left(B-\frac{1}{r^2}\right)=H^2$$

from this equation, we get,

$$r^2=9,4$$

$$\left(A-\frac{1}{r^2}\right)x'^2+2Hx'y'+\left(B-\frac{1}{r^2}\right)y'^2=0$$

$$\Rightarrow \left(A-\frac{1}{r^2}\right)^2x'^2+2H\left(A-\frac{1}{r^2}\right)x'y'+\left(A-\frac{1}{r^2}\right)\left(B-\frac{1}{r^2}\right)y'^2$$

$$\Rightarrow \left(A-\frac{1}{r^2}\right)^2x'^2+2H\left(A-\frac{1}{r^2}\right)x'y'+H^2y'^2=0$$

$$\Rightarrow \left[\left(A-\frac{1}{r^2}\right)x'+Hy'\right]^2=0$$

$$\Rightarrow \left(A-\frac{1}{r_1^2}\right)x'+Hy'=0$$

Equation of major axis

$$\left(A-\frac{1}{r_2^2}\right)x'+Hy'=0$$

Equation of minor axis

where $r_1 > r_2$

Here,

$$A=\frac{8}{36},\ H=\frac{2}{36} \quad \& \quad r_1^2=9 \text{ or } r_2^2=4$$

Equation of major axis

$$\left(\frac{8}{36}-\frac{1}{9}\right)x'+\frac{2}{36}y'=0$$

$$\Rightarrow \left(\frac{8-4}{36}\right)x'+\frac{2}{36}y'=0$$

$$\Rightarrow 2x'+y'=0$$

$$\Rightarrow 2(x-1)+(y-2)=0 \quad [\because x=x'+1,\ y=y'+2]$$

$$\Rightarrow 2x+y-4=0$$

Equation of minor axis

$$\left(\frac{8}{36}-\frac{1}{4}\right)x'+\frac{2}{36}y'=0$$

$$\Rightarrow -\frac{1}{36}x'+\frac{2}{36}y'=0$$

$$\Rightarrow -x'+2y'=0$$

$$\Rightarrow x'-2y'=0$$

$\Rightarrow (x-1) - 2(y-2) = 0 \quad [\because x = x'+1, \; y = y'+2]$

$\Rightarrow x - 2y + 3 = 0$

$(h,k) = (1,2)$ $\boxed{\begin{array}{l} x = h \pm d\cos\theta \\ y = k \pm d\sin\theta \end{array}}$

slope of major axis,

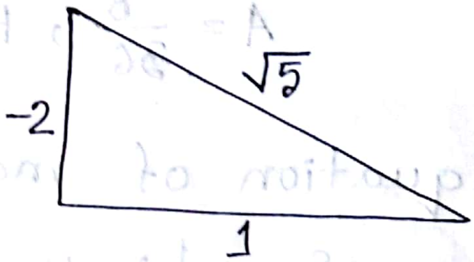
$$2x + y - 4 = 0$$

$$\therefore y = -2x + 4$$

$$\tan\theta = -2$$

$$\cos\theta = \frac{1}{\sqrt{5}}$$

$$\sin\theta = \frac{-2}{\sqrt{5}}$$



vertex,

$$x = 1 \pm 3 \times \frac{1}{\sqrt{5}}$$

$$= 1 \pm \frac{3}{\sqrt{5}}$$

$$y = 2 \pm \frac{-6}{\sqrt{5}}$$

$$= 2 \mp \frac{6}{\sqrt{5}}$$

Equation of tangent at the vertex is always perpendicular to the major axis,

$$\cancel{2} \; x - 2y + k = 0 \quad \text{---------} \quad (iv)$$

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(iv) passes through $\left(1+\frac{3}{\sqrt{5}}, 2-\frac{6}{\sqrt{5}}\right)$

$$\left(1+\frac{3}{\sqrt{5}}\right)-2\left(2-\frac{6}{\sqrt{5}}\right)+k=0$$

$$\Rightarrow 1+\frac{3}{\sqrt{5}}-4+\frac{12}{\sqrt{5}}+k=0$$

$$\Rightarrow \frac{15-3\sqrt{5}}{\sqrt{5}}=-k$$

$$\Rightarrow k=\frac{3\sqrt{5}-15}{\sqrt{5}}=3-3\sqrt{5}=3(1-\sqrt{5})$$

putting the value of k in equation (iv)

$$x-2y+3(1-\sqrt{5})=0$$

focus $=(\pm\sqrt{5},0)$ $[\because d=\sqrt{5}]$

$(h,k)=(1,2)$

$x=h\pm d\cos\theta=1\pm\left(\sqrt{5}\times\frac{1}{\sqrt{5}}\right)=(2,0)$

$y=k\pm d\sin\theta=2\pm\left\{\sqrt{5}\times\left(-\frac{2}{\sqrt{5}}\right)\right\}=(0,4)$

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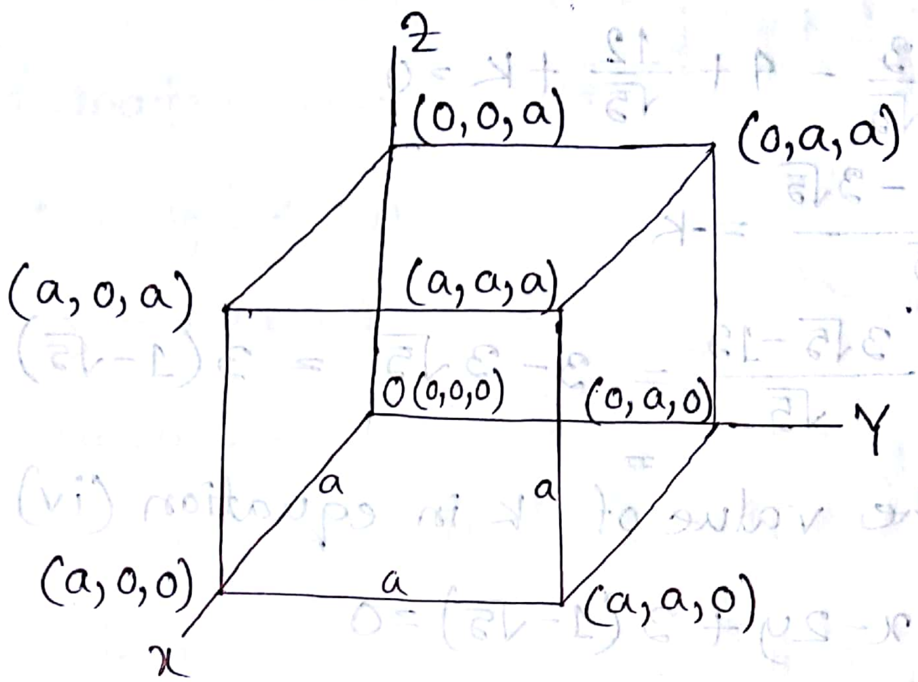
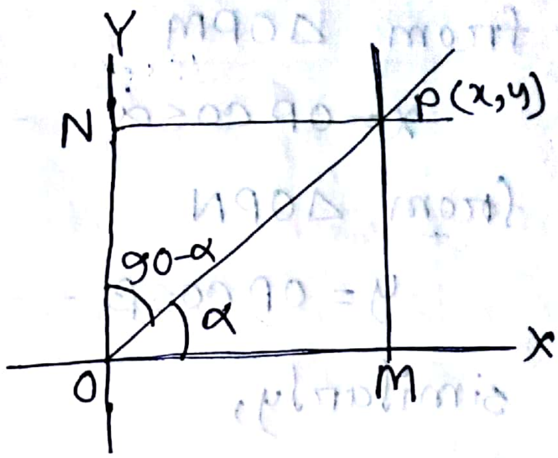
3D



$$\text{Distance} = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$$

$$\left.\begin{aligned} x &= \frac{m_1x_2+m_2x_1}{m_1+m_2} \\ y &= \frac{m_1y_2+m_2y_1}{m_1+m_2} \\ z &= \frac{m_1z_2+m_2z_1}{m_1+m_2} \end{aligned}\right\} \text{Intersection laws}$$

10-10-2017 : 8A : Tuesday



ΔOPM

$$\cos\alpha = \frac{OM}{OP}$$

$$\Rightarrow \cos\alpha = \frac{x}{OP}$$

$$\Rightarrow x = OP\cos\alpha$$

ΔOPN

$$\cos(90^\circ-\alpha) = \frac{ON}{OP}$$

$$\Rightarrow \sin\alpha = \frac{y}{OP}$$

$$\Rightarrow y = OP\sin\alpha$$

$$\frac{y}{x} = \frac{\cos(90^\circ-\alpha)}{\cos\alpha}$$

$$\Rightarrow \frac{y}{x} = \frac{\sin\alpha}{\cos\alpha}$$

$$\Rightarrow \frac{y}{x} = \tan\alpha$$

$$\Rightarrow \frac{y}{x} = m$$

(2D) slope $\rightarrow$ (3D) Direction Cosine

If a st. line makes angles α, β, γ with the positive direction of x, y, z axis respectively then $(\cos\alpha, \cos\beta, \cos\gamma)$ are called direction cosine of that line.

The direction cosine of x ~~axs~~ axis ~~is~~ is $(\cos 0^\circ, \cos 90^\circ, \cos 90^\circ) = (1, 0, 0)$

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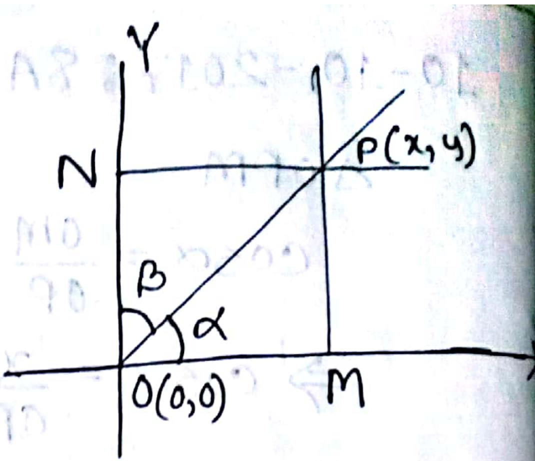
from ΔOPM

$$x = OP\cos\alpha \text{ ------- (i)}$$



from ΔOPN

$$y = OP\cos\beta \text{ ------- (ii)}$$

similarly,

$$z = OP\cos\gamma \text{ ------- (iii)}$$

$(i)^2 + (ii)^2 + (iii)^2$

$$x^2+y^2+z^2 = OP^2(\cos^2\alpha+\cos^2\beta+\cos^2\gamma) \text{ ----- (iv)}$$

we know,

$$OP^2 = x^2+y^2+z^2$$

from (iv) we get,

$$\boxed{\cos^2\alpha+\cos^2\beta+\cos^2\gamma = 1}$$

similarly,

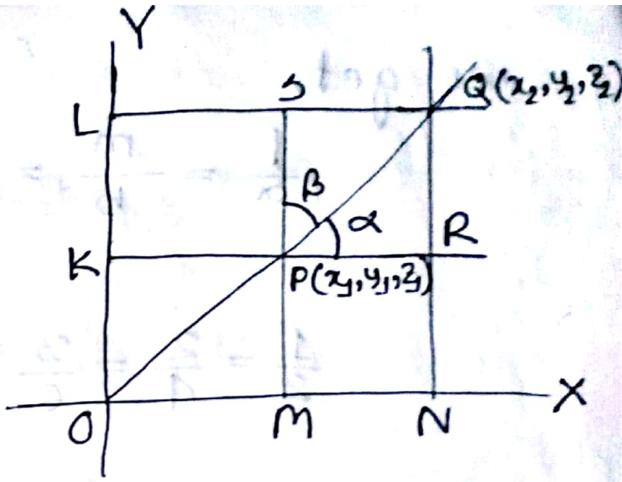
$$\sin^2\alpha+\sin^2\beta+\sin^2\gamma = 2$$

Let, $\cos\alpha = l$, $\cos\beta = m$, $\cos\gamma = n$

$$\therefore l^2+m^2+n^2 = 1$$

(x_1, y_1, z_1) & (x_2, y_2, z_2)

$$\frac{x-x_1}{x_1-x_2} = \frac{y-y_1}{y_1-y_2} = \frac{z-z_1}{z_1-z_2}$$



ΔPQR,

$$\cos\alpha = \frac{PR}{PQ}$$

$$\Rightarrow \cos\alpha = \frac{x_2-x_1}{PQ}$$

$$\Rightarrow \frac{\cos\alpha}{x_2-x_1} = \frac{1}{PQ} \quad \text{---- (i)}$$

ΔPQS

$$\cos\beta = \frac{SP}{PQ}$$

$$\Rightarrow \cos\beta = \frac{y_2-y_1}{PQ}$$

$$\Rightarrow \frac{\cos\beta}{y_2-y_1} = \frac{1}{PQ} \quad \text{------ (ii)}$$

similarly,

$$\frac{\cos\gamma}{z_2-z_1} = \frac{1}{PQ} \quad \text{------- (iii)}$$

From equation (i), (ii) & (iii), we get

$$\frac{\cos\alpha}{x_2-x_1} = \frac{\cos\beta}{y_2-y_1} = \frac{\cos\gamma}{z_2-z_1} = \frac{1}{PQ}$$

→ Direction Ratio:

If any three number a, b, c are at same ratio with the direction of a line then a, b, c are called direction ratio of that line.

i.e. $x_2-x_1 = a$, $y_2-y_1 = b$, $z_2-z_1 = c$

we get,

$$\frac{l}{a} = \frac{m}{b} = \frac{n}{c} = \frac{\sqrt{l^2+m^2+n^2}}{\sqrt{a^2+b^2+c^2}}$$

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{\sqrt{1^2+2^2+3^2}}{\sqrt{2^2+4^2+6^2}} = \frac{\sqrt{14}}{2\sqrt{14}} = \frac{1}{2}$$

$$\frac{l}{a} = \frac{m}{b} = \frac{n}{c} = \frac{1}{\sqrt{a^2+b^2+c^2}}$$

$$\therefore l = \frac{a}{\sqrt{a^2+b^2+c^2}}, m = \frac{b}{\sqrt{a^2+b^2+c^2}}, n = \frac{c}{\sqrt{a^2+b^2+c^2}}$$

→ If (0,0,0) and (1,5,3) be the two points on a st. lines then find the angle which the st. line is making with the positive direction of X, Y & Z axis.

– Direction ratio of the st. line is

$$(x_2-x_1, y_2-y_1, z_2-z_1)$$

$$= (1-0, 5-0, 3-0)$$

$$= (1,5,3) = (a,b,c)$$

→ Direction Cosine :

$$l = \frac{1}{\sqrt{1^2+5^2+3^2}} = \frac{1}{\sqrt{35}} \Rightarrow \cos\alpha = \frac{1}{\sqrt{35}} \therefore \alpha = \cos^{-1}\left(\frac{1}{\sqrt{35}}\right)$$

$$m = \frac{5}{\sqrt{1^2+5^2+3^2}} = \frac{5}{\sqrt{35}} \Rightarrow \cos\beta = \frac{5}{\sqrt{35}} \therefore \beta = \cos^{-1}\left(\frac{5}{\sqrt{35}}\right)$$

$$n = \frac{3}{\sqrt{1^2+5^2+3^2}} = \frac{3}{\sqrt{35}} \Rightarrow \cos\gamma = \frac{3}{\sqrt{35}} \therefore \gamma = \cos^{-1}\left(\frac{3}{\sqrt{35}}\right)$$

21-10-2017 : 8C : Saturday :

→ If m_1 and m_2 are the slopes of two straight lines,
∴ the angle between two straight line,

$$\theta = \tan^{-1}\frac{m_1 - m_2}{1 + m_1 m_2}$$

→ If (l_1, m_1, n_1) and (l_2, m_2, n_2) are the Direction Cosine of two straight lines,
∴ the angle between two straight line in 3D, when Direction Cosine of those straight lines are given,

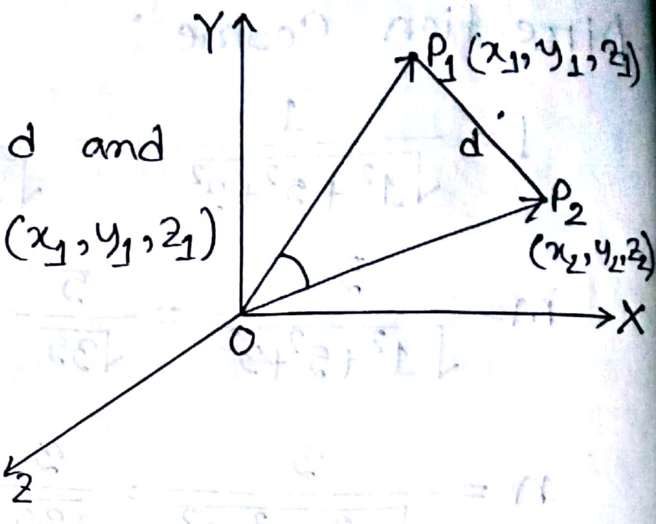
$$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$$

Proof:

Let,

$OP_1 = r_1$, $OP_2 = r_2$; $P_1P_2 = d$ and

the co-ordinates of P_1 ; P_2 be (x_1, y_1, z_1) and (x_2, y_2, z_2) respectively.



Then, $x_1 = r_1 l_1$; $y_1 = r_1 m_1$; $z_1 = r_1 n_1$

and $x_2 = r_2 l_2$; $y_2 = r_2 m_2$; $z_2 = r_2 n_2$

then by geometry, we have

$$P_1P_2^2 = OP_1^2 + OP_2^2 - 2OP_1OP_2\cos\theta$$

$$\Rightarrow \cos\theta = \frac{OP_1^2 + OP_2^2 - P_1P_2^2}{2OP_1OP_2}$$

$$\Rightarrow \cos\theta = \frac{r_1^2 + r_2^2 - d^2}{2r_1r_2}$$

Now, $r_1^2 = x_1^2 + y_1^2 + z_1^2$

$r_2^2 = x_2^2 + y_2^2 + z_2^2$

and, $P_1P_2^2 = d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$

$= x_1^2 + y_1^2 + z_1^2 + x_2^2 + y_2^2 + z_2^2 - 2(x_1x_2 + y_1y_2 + z_1$

$= r_1^2 + r_2^2 - 2r_1r_2(l_1l_2 + m_1m_2 + n_1n_2)$

$$\therefore \cos\theta = \frac{r_1^2 + r_2^2 - r_1^2 - r_2^2 + 2r_1r_2(l_1l_2 + m_1m_2 + n_1n_2)}{2r_1r_2}$$

$$\Rightarrow \cos\theta = l_1l_2 + m_1m_2 + n_1n_2.$$

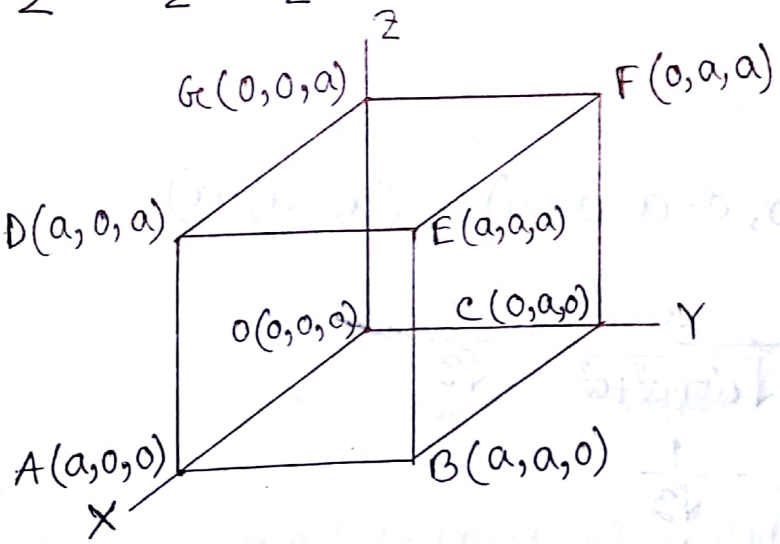
If $\theta = 90°$

$$l_1l_2 + m_1m_2 + n_1n_2 = 0$$

if $\theta = 0°$

$$l_1l_2 + m_1m_2 + n_1n_2 = 1$$

$$\Rightarrow \frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$$



Diagonal of the cube OE, AF, BG, CD.

D.R. of OE: $(a-0, a-0, a-0) = (a, a, a)$

D.C. of OE: $l = \frac{a}{\sqrt{a^2+a^2+a^2}} = \frac{1}{\sqrt{3}}$

$m = \frac{1}{\sqrt{3}}$

$n = \frac{1}{\sqrt{3}}$

D.R. of BG: $(a-0, a-0, 0-a) = (a, a, -a)$

D.C. of BG:

$$l = \frac{a}{\sqrt{a^2+a^2+a^2}} = \frac{1}{\sqrt{3}}$$

$$m = \frac{1}{\sqrt{3}}$$

$$n = -\frac{1}{\sqrt{3}}$$

Angle between OE and BG

$$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$$

$$= \frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} \cdot \left(-\frac{1}{\sqrt{3}}\right)$$

$$= \frac{1}{3}$$

$$\therefore \theta = \cos^{-1}\frac{1}{3}$$

D.R. of AF: $(a-0, 0-a, 0-a) = (a, -a, -a)$

D.C. of AF:

$$l = \frac{a}{\sqrt{a^2+a^2+a^2}} = \frac{1}{\sqrt{3}}$$

$$m = -\frac{1}{\sqrt{3}}$$

$$n = -\frac{1}{\sqrt{3}}$$

Angle between OE and AF

$$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$$

$$= \frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} \cdot \left(-\frac{1}{\sqrt{3}}\right) + \frac{1}{\sqrt{3}} \cdot \left(-\frac{1}{\sqrt{3}}\right)$$

$$= -\frac{1}{3} \qquad \therefore \theta = \cos^{-1}\left(-\frac{1}{3}\right)$$

D.R. of CD: $(0-a, a-0, 0-a) = (-a, a, -a)$

D.C. of CD:

$$l = \frac{-a}{\sqrt{a^2+a^2+a^2}} = -\frac{1}{\sqrt{3}}$$

$$m = \frac{1}{\sqrt{3}}$$

$$n = -\frac{1}{\sqrt{3}}$$

24-10-2017: 9A; Tuesday:

$$l+m+n=0 \quad \text{------------ (i)}$$

$$2lm+2ln-mn=0 \quad \text{------------ (ii)}$$

This two equation's represents D.C. of two straight lines. Find the angle between the straight lines.

From equation (i),

$$l=-(m+n) \quad \text{------------ (iii)}$$

using equation (iii) in equation (ii) we get,

$$2\{-(m+n)\}m+2\{-(m+n)\}n-mn=0$$

$$\Rightarrow -2m^2-2mn-2mn-2n^2-mn=0$$

$$\Rightarrow 2m^2+5mn+2n^2=0$$

$$\Rightarrow (2m+n)(m+2n)=0$$

$2m+n=0$

$\Rightarrow n=-2m$ ------- (iv)

$\Rightarrow \frac{n}{-2}=\frac{m}{1}$ ------- (vi)

$m+2n=0$

$\Rightarrow m=-2n$ ------- (v)

$\Rightarrow \frac{m}{-2}=\frac{n}{1}$ ------- (vii) when,

$n = -2m$ then from equation (iii)

$l = -(m+n) = -(m-2m) = m$

$\Rightarrow \frac{l}{1} = \frac{m}{1}$ ------------ (viii)

From equation (vi) & equation (viii) we get,

$$\frac{l}{1} = \frac{m}{1} = \frac{n}{-2}$$

$$l = \frac{1}{\sqrt{1^2+1^2+(-2)^2}} = \frac{1}{\sqrt{6}}$$

$$m = \frac{1}{\sqrt{6}}$$

$$n = \frac{-2}{\sqrt{6}}$$

similarly,

$m = -2n$ then from equation (iii)

$l = -(m+n) = -(-2n+n) = n$

$\Rightarrow \frac{l}{1} = \frac{n}{1}$ ------------ (ix)

From equation (ix) & equation (vii)

$$\frac{l}{1} = -\frac{m}{2} = \frac{n}{1}$$

$$l = \frac{1}{\sqrt{1^2+(-2)^2+1^2}} = \frac{1}{\sqrt{6}}$$

$$m = -\frac{2}{\sqrt{6}}, \quad n = \frac{1}{\sqrt{6}}$$

If θ is the angle between the straight lines,

$$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$$

$$= \frac{1}{\sqrt{6}} \cdot \frac{1}{\sqrt{6}} + \frac{1}{\sqrt{6}} \cdot \left(-\frac{2}{\sqrt{6}}\right) + \frac{1}{\sqrt{6}}\left(-\frac{2}{\sqrt{6}}\right)$$

$$= \frac{1}{6} - \frac{2}{6} - \frac{2}{6}$$

$$= -\frac{3}{6}$$

$$= -\frac{1}{2}$$

$$\therefore \theta = \cos^{-1}\left(-\frac{1}{2}\right) = 120^\circ$$

$\longrightarrow$

$$al + bm + cn = 0 \quad \text{-----------} (i)$$

$$ul^2 + vm^2 + wn^2 = 0 \quad \text{-----------} (ii)$$

From equation (i) we get,

$$l = -\left(\frac{bm + cn}{a}\right) \quad \text{-----------} (iii)$$

using equation (iii) in equation (ii),

$$u\left\{-\left(\frac{bm+cn}{a}\right)\right\}^2 + vm^2 + wn^2 = 0$$

$$\Rightarrow ub^2m^2 + uc^2n^2 + 2bcmnu + a^2vm^2 + wa^2n^2 = 0$$

$$\Rightarrow m^2(ub^2 + a^2v) + 2bcmnu + n^2(uc^2 + wa^2) = 0$$

$$\Rightarrow (ub^2 + a^2v)\left(\frac{m}{n}\right)^2 + 2bcu\left(\frac{m}{n}\right) + (uc^2 + wa^2) = 0$$

$$\Rightarrow (ub^2+a^2v)x^2 + 2bcux + (uc^2+wa^2) = 0 \text{ - - - - - - (i}$$

Let, $x_1 = \frac{m_1}{n_1}$ and $x_2 = \frac{m_2}{n_2}$ be the two roots of equation (iv),

$\therefore$ we get $$x_1 x_2 = \frac{uc^2+wa^2}{ub^2+va^2}$$

$$\Rightarrow \frac{m_1 m_2}{n_1 n_2} = \frac{uc^2+wa^2}{ub^2+va^2}$$

$$\Rightarrow \frac{m_1 m_2}{uc^2+wa^2} = \frac{n_1 n_2}{ub^2+va^2} \text{ - - - - - - - - (v)}$$

If we eliminate m from equation (i) and equation (ii) then similarly we get,

$$\frac{l_1 l_2}{wb^2+vc^2} = \frac{n_1 n_2}{ub^2+va^2} \text{ - - - - - - - - (vi}$$

From equation (v) and equation (vi)

$$\frac{l_1 l_2}{b^2w+c^2v} = \frac{m_1 m_2}{c^2u+a^2w} = \frac{n_1 n_2}{b^2u+a^2v} = K$$

we get,

$$\frac{l_1 l_2}{b^2 w + c^2 v} = k$$

$$\Rightarrow l_1 l_2 = k\left(b^2 w + c^2 v\right)$$

$$\therefore m_1 m_2 = k\left(\cancel{a}c^2 u + a^2 w\right)$$

$$\text{and } n_1 n_2 = k\left(b^2 u + \cancel{w}a^2 v\right)$$

we know if two straight lines are perpendicular then,

$$l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$$

$$\Rightarrow k\left(b^2 w + c^2 v\right) + k\left(c^2 u + a^2 w\right) + k\left(b^2 u + a^2 v\right) = 0$$

$$\Rightarrow b^2 w + c^2 v + c^2 u + a^2 w + b^2 u + a^2 v = 0$$

$$\Rightarrow a^2(v+w) + b^2(u+w) + c^2(u+v) = 0$$

If two straight lines are parallel then

$$\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$$

$$\cancel{\Rightarrow} \therefore \frac{m_1}{m_2} = \frac{n_1}{n_2}$$

$$\Rightarrow \frac{m_1}{n_1} = \frac{m_2}{n_2}$$

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Hence we see that two roots of equation (iv) are equal, so we get,

$$(2bcu)^2 - 4(ub^2 + va^2)(uc^2 + wa^2) = 0$$

$$\Rightarrow b^2c^2u^2 \mp b^2c^2u^2 \mp a^2b^2uw = a^2c^2uv + a^4vw$$

$$\cancel{\Rightarrow a^2b^2uw + a^2c^2uv}$$

$$\Rightarrow a^4wv + a^2b^2uw + a^2c^2uv = 0$$

$$\Rightarrow \frac{a^2}{u} + \frac{b^2}{v} + \frac{c^2}{w} = 0$$

31-10-2017 : 10A : Tuesday

→ surface – upper portion of any 3D object is called surface

→ plane – a flat surface on which a straight line joining any two points on it would wholly lie.

General equation of plane :

$$ax + by + cz + d = 0 \quad \text{--------- (i)}$$

Let, $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ be two points on the plane (i), we get,

$$ax_1 + by_1 + cz_1 + d = 0 \quad \text{--------- (ii)}$$

$$ax_2 + by_2 + cz_2 + d = 0 \quad \text{--------- (iii)}$$

$$x_3 = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \quad y_3 = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}, \quad z_3 = \frac{m_1 z_2 + m_2 z_1}{m_1 + m_2}$$

we get, when (x_3, y_3, z_3) on the plane (i)

$$L.H.S. = a\left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}\right) + b\left(\frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right) + c\left(\frac{m_1 z_2 + m_2 z_1}{m_1 + m_2}\right) + d$$

$$= m_2 (ax_1 + by_1 + cz_1 + d) + m_1 (ax_2 + by_2 + cz_2 + d)$$

$$= m_2 \times 0 + m_1 \times 0$$

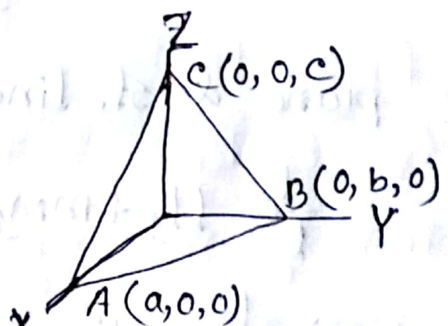
$$= 0$$

$= R.H.S.$ $\quad\because L.H.S. = R.H.S.$

$\therefore$ so given equation is the general equation of the plane according to the definition.

→ Intercept form of a plane

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$



→ Normal form of a plane

$$x\cos\alpha + y\cos\beta + z\cos\gamma = p$$

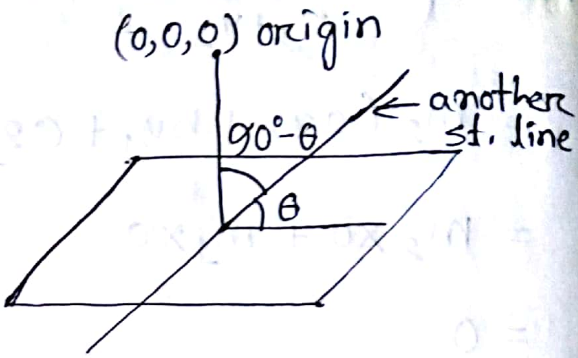
$$\Rightarrow lx + my + nz = p$$

14.

Here, l, m, n is the direction cosine of the st. line drawn perpendicularly from the origin to the plane and p is the perpendicular distance.

$\rightarrow$

$$lx + my + nz = p$$



$$\cos(90^\circ - \theta) = ll_1 + mm_1 + nn_1$$

$$\Rightarrow \sin\theta = ll_1 + mm_1 + nn_1$$

Angle beteween the st. line and the plane $= \theta$

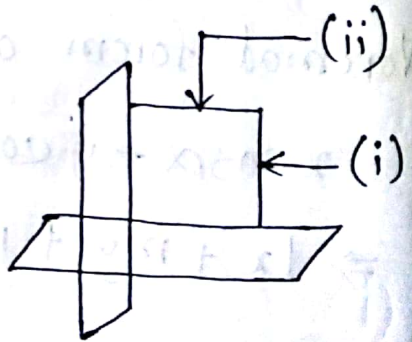
if, plane & st. line are parallel, $(\theta = 0^\circ)$

$$ll_1 + mm_1 + nn_1 = 0$$

if, plane & st. line are perpendicular, $(\theta = 90^\circ)$

$$ll_1 + mm_1 + nn_1 = 1$$

$$\Rightarrow \frac{l}{l_1} = \frac{m}{m_1} = \frac{n}{n_1}$$



$$l_1 x + m_1 y + n_1 z = P_1 \text{ ---------- (i)}$$

$$l_2 x + m_2 y + n_2 z = P_2 \text{ ---------- (ii)}$$

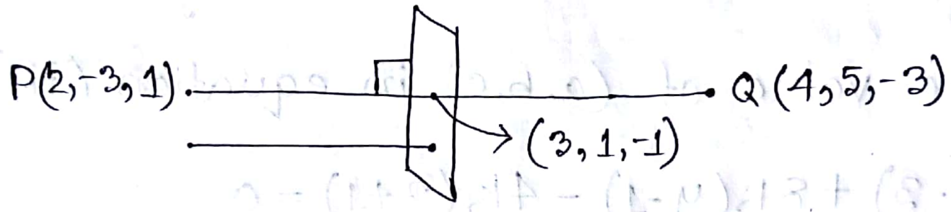
Angle between two plane,

$$\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$$

07-11-2017 : 11A : Tuesday :

→ Find the equation of a plane passing through the middle point of the st. line joining the points (2,-3,1) and (4,5,-3) and the plane is also perpendicular to the st. line.

Solution:



Let, the equation of plane be,

$$ax+by+cz+d=0 \quad \text{------------ (i)}$$

middle point of st. line PQ

$$x_1 = \frac{2+4}{2} = 3, \quad y_1 = \frac{-3+5}{2} = 1, \quad z = \frac{1-3}{2} = -1$$

Since (i) passes through the middle point,

$$3a+b-c+d=0 \quad \text{------------ (ii)}$$

(i) − (ii), we get,

$$a(x-3)+b(y-1)+c(z+1)=0$$

Direction ratio of PQ

$$(4-2,\ 5+3,\ -3-1) = (2,8,-4)$$

According to condition (As plane is perpendicular to the st. line and (a,b,c) are the Direction ratio of a st. line ~~and~~ then this new ~~line~~ st. line is parallel to the given st. line.)

$$\frac{a}{2} = \frac{b}{8} = \frac{c}{-4} = K$$

$\therefore a = 2K,\ b = 8K,\ c = -4K$

putting the value of (a, b, c) in equation (iii)

$$2K(x-3) + 8K(y-1) - 4K(z+1) = 0$$

$$\Rightarrow K(2x - 6 + 8y - 8 - 4z - 4) = 0$$

$$\therefore 2x + 8y - 4z - 18 = 0 \quad [\because K \neq 0]$$

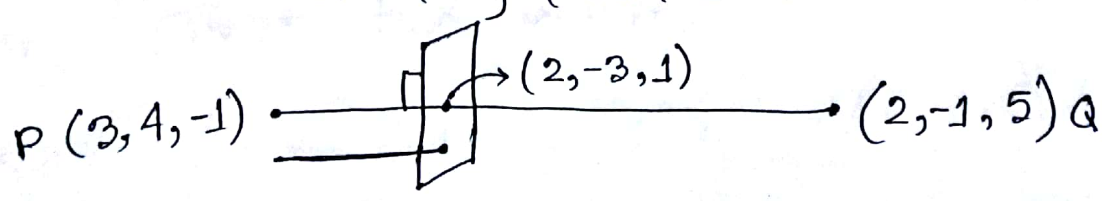
It is the required plane equation

→ Find the equation of a plane passing through the ~~middle~~ (2,−3,1) point which is on the st. line joining the points (3,4,−1) and (2,−1,5) and the plane is also perpendicular to the st. line.

Solution:

Let, the equation of plane be,

$$ax + by + cz + d = 0 \quad \text{-----} \quad (i)$$



Since the plane passes through $(2,-3,1)$

$$2a-3b+c+d=0 \quad \text{------ (ii)}$$

(i) − (ii) we get.

$$a(x-2)+b(y+3)+c(z-1)=0 \quad \text{------ (iii)}$$

D.R. of PQ

$$(3-2,\ 4+1,\ -1-5) = (1,5,-6)$$

~~According to~~

As plane is perpendicular to the st. line and a, b, c are the D.R. of a st. line and this new st. line is parallel to the given st. line.

$$\frac{a}{1}=\frac{b}{5}=\frac{c}{-6}=k$$

$\therefore\ a=k,\ b=5k,\ c=-6k$

putting the value of a, b, c in equation (iii)

$$k(x-2)+5k(y+3)-6k(z-1)=0$$

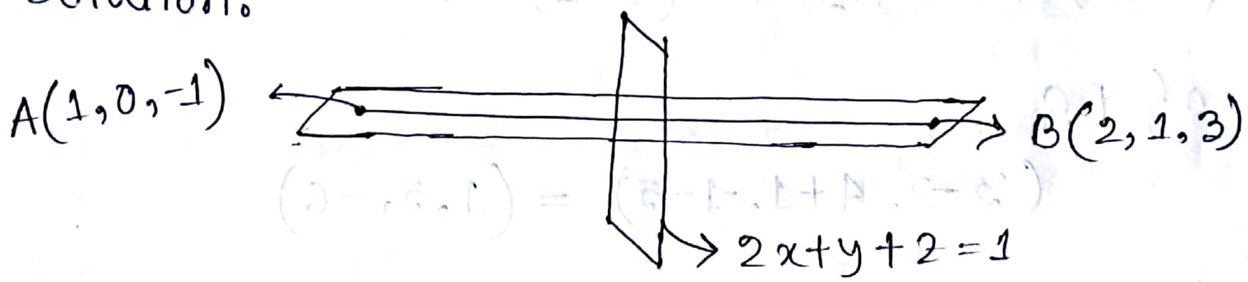
$$\Rightarrow k(x-2+5y+15-6z+6)=0$$

$$\Rightarrow x+5y-6z+19=0 \quad [\because k\neq 0]$$

It is the required equation.

→ Find a plane passing through $(1,0,-1)$ and $(2,1,3)$ and perpendicular to the plane $2x+y+z=1$.

Solution:



Let, the equation of plane be

$ax+by+cz+d=0$ ------------ (i)

A passes through plane (i)

$a+0-c+d=0$

$\Rightarrow a-c+d=0$ ------------ (ii)

B passes through plane (ii)

$2a+b+3c+d=0$ ------------ (iii)

As given plane is perpendicular to required plane.

$\therefore l_1l_2+m_1m_2+n_1n_2=0$

$\Rightarrow 2a+b+c=0$ ------------ (iv)

(i) - (ii), we get,

$a(x-1) + by + c(z+1) = 0$ ---------- (v)

(ii) - (iii), we get,

$-a - b - 4c = 0$ ---------------- (vi)

Eliminating a, b, c from equation (iv), (v) and (vi),

$$\begin{vmatrix} 2 & 1 & 1 \\ x-1 & y & z+1 \\ -1 & -1 & -4 \end{vmatrix} = 0$$

$\Rightarrow 2(-4y + z + 1) - 1(-4x + 4 + z + 1) + 1(-x + 1 + y) = 0$

$\Rightarrow -8y + 2z + 2 + 4x - 4 - 1 - z - x + 1 + y = 0$

$\Rightarrow 3x - 7y + z - 2 = 0$

→ Find an equation of a plane passing through (-1, 3, 2) point and perpendicular to the planes $x + 2y + 2z = 5$ and $3x + 3y + 2z = 8$

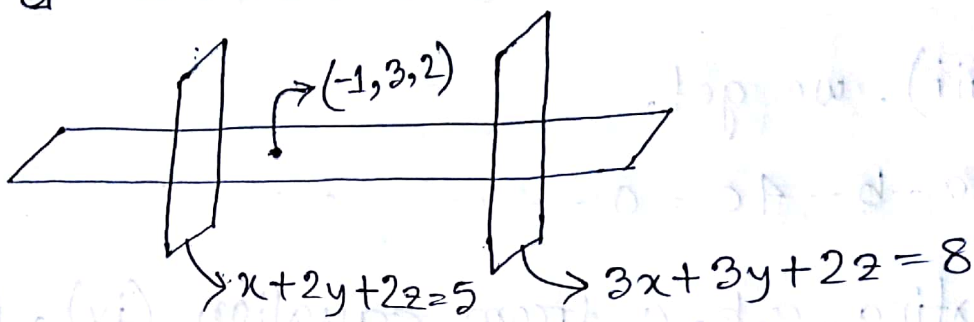
Solution:

Let, the required equation of plane,

$ax + by + cz + d = 0$ ---------- (i)

plane (i) passing through $(-1, 3, 2)$ point.

$$-a + 3b + 2c + d = 0 \quad \text{--------} \quad (ii)$$



plane (i) is perpendicular to plane $x+2y+2z=5$

$$a + 2b + 2c = 0 \quad \text{--------} \quad (iii)$$

plane (i) is perpendicular to plane $3x+3y+2z=8$

$$3a + 3b + 2c = 0 \quad \text{--------} \quad (iv)$$

(i) - (ii), we get,

$$a(x+1) + b(y-3) + c(z-2) = 0 \quad \text{--------} \quad (v)$$

Eliminating a, b, c from equation (iii), (iv) & (v).

$$\begin{vmatrix} 1 & 2 & 2 \\ 3 & 3 & 2 \\ x+1 & y-3 & z-2 \end{vmatrix} = 0$$

$$\Rightarrow 1(3z-6-2y+6) - 2(3z-6-2x-2) + 2(3y-9-3x-3) =$$

$$\Rightarrow 3z - 2y - 6z + 16 + 4x + 6y - 24 - 6x = 0$$

$$\Rightarrow 2x - 4y + 3z + 8 = 0$$

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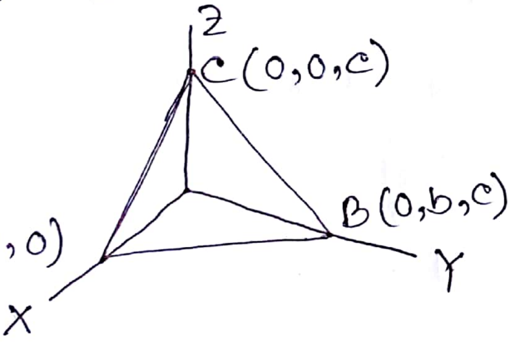
→ A plane meets the Co-ordinate axes in A, B, C such that the centroid of the triangle ABC is the point (p, q, r); show that the equation of the plane is

$$\frac{x}{p} + \frac{y}{q} + \frac{z}{r} = 3.$$

Solution:

Let, the equation of the plane be

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \quad \text{-------- ----} \quad (1)$$



Since, (p, q, r) is the centroid of ABC

$$p = \frac{a+0+0}{3} = \frac{a}{3} \Rightarrow a = 3p$$

$$q = \frac{0+b+0}{3} = \frac{b}{3} \Rightarrow b = 3q$$

$$r = \frac{0+0+c}{3} = \frac{c}{3} \Rightarrow c = 3r$$

From equation (i)

$$\frac{x}{3p} + \frac{y}{3q} + \frac{z}{3r} = 1$$

$$\Rightarrow \frac{x}{p} + \frac{y}{p} + \frac{z}{r} = 3. \quad \text{(Proved)}$$

14-11-2017 : 12A : Tuesday :

Symmetric form of a st. line :

$$\frac{x-x_1}{l}=\frac{y-y_1}{m}=\frac{z-z_1}{n}=r \quad \text{------------ (i)}$$

General equation of ~~plane,~~ first degree,

$$\left.\begin{array}{l} a_1x+b_1y+c_1z+d_1=0 \\ a_2x+b_2y+c_2z+d_2=0 \end{array}\right\}$$ Combination of these two create a ~~•~~ general equation of a st. line

$$\frac{x-x_1}{x_1-x_2}=\frac{y-y_1}{y_1-y_2}=\frac{z-z_1}{z_1-z_2}$$

Distance ~~of~~ between these two equation,

$$=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$$

From (i)

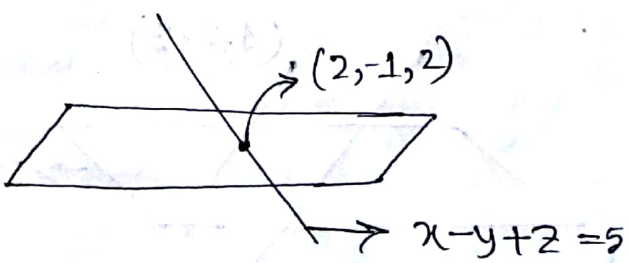
$$x=x_1+lr$$

$$y=y_1+mr$$

$$z=z_1+nr$$

→ Find the intersect point ~~of~~ between st. line $x-y+z=5$ and plane $\frac{x-2}{2}=\frac{y+1}{4}=\frac{z-2}{12}$.

Solution:



Let

$$\frac{x-2}{2} = \frac{y+1}{4} = \frac{z-2}{12} = r \text{ ------ (i)}$$

$$x - y + z = 5 \text{ ---------- (ii)}$$

Any point on equation (i)

$$(2+2r, -1+4r, 2+12r)$$

when ~~a~~ any point will be boiled by the st. line, then any point will be the intersection point.

$$2+2r+1-4r+2+12r = 5$$

$$\Rightarrow r = 0$$

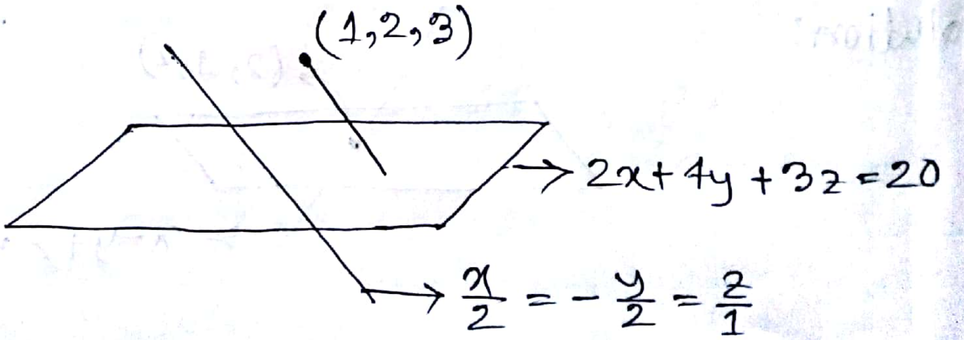
$\therefore$ The intersection point

$$(2+2\times0, -1+4\times0, 2+12\times0)$$

$$= (2, -1, 2)$$

→ Find the distance of ~~between~~ the point (1,2,3) from ~~and~~ the plane $2x+4y+3z=20$ ~~and~~ measured ~~the distance line is~~ parallel to the st. line $\frac{x}{2} = -\frac{y}{2} = \frac{z}{1}$

Solution:



$2x+4y+3z=20$ ---------- (i)

$\frac{x}{2}=-\frac{y}{2}=\frac{z}{1}$ ---------- (ii)

From (ii)

$\frac{2}{a}=-\frac{2}{b}=\frac{1}{c}=1$

$\therefore a=2, b=-2, c=1$

The line which is going through the point (1,2,3)

$\frac{x-1}{a}=\frac{y-2}{b}=\frac{z-3}{c}$ ~~(i)~~

$\Rightarrow \frac{x-1}{2}=\frac{y-2}{-2}=\frac{z-3}{1}=r$ ---------- (iii)

The intersection point of (i) & (iii)

$x=2r+1$

$y=-2r+2$

$z=r+3$

(x, y, z) goes through (i)

$2(2r+1)+4(-2r+2)+3(r+3)=20$

$\Rightarrow r=-1$

$\therefore$ The intersection point

$$(2(-1)+1, -2(-1)+2, (-1)+3) = (-1, 4, 2)$$

$\therefore$ The required distance between (1,2,3) and plane $2x+4y+3z=20$

$$= \sqrt{(1+1)^2+(2-4)^2+(3-2)^2} = 3$$

→ Find in symmetrical form of the equation of a line $5x-y-3z+12=0=x-7y+5z-6$

Solution:

Let, the symmetrical form of the line,

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} \quad \text{------ (i)}$$

If l, m, n be the D.C. ~~of~~ then

$$\cancel{l+m+n=0} \quad 5l-m-3n=0$$

$$l-7m+5n=0$$

$$\therefore \frac{l}{-5-21} = \frac{-m}{25+3} = \frac{n}{-35+1}$$

$$\Rightarrow \frac{l}{13} = \frac{m}{14} = \frac{n}{17} = \frac{\sqrt{l^2+m^2+n^2}}{\sqrt{169+196+289}} = \frac{1}{\sqrt{654}}$$

$\therefore$ D.C.'s are $\left(\frac{13}{\sqrt{654}}, \frac{14}{\sqrt{654}}, \frac{17}{\sqrt{654}}\right)$

$\therefore$ D.R's are $(13, 14, 17)$

putting D.C.'s in equation (i)

$$\frac{x-x_1}{13} = \frac{y-y_1}{14} = \frac{z-z_1}{17} \quad \text{------- (ii)}$$

~~We can rewrite the equation as~~

$$\cancel{5x-y-32=0}$$
$$\cancel{x-7y+52=0}$$

Any point of the st. line $(x, y, 0)$

we get,

$$5x-y+12=0 \quad \text{------- (iii)}$$
$$x-7y-6=0 \quad \text{------- (iv)}$$

solving (iii) & (iv) we get

$$x=-\frac{45}{17} \quad \& \quad y=-\frac{21}{17}$$

$\therefore \left(-\frac{45}{17}, -\frac{21}{17}, 0\right)$ point is on the st. line (ii)

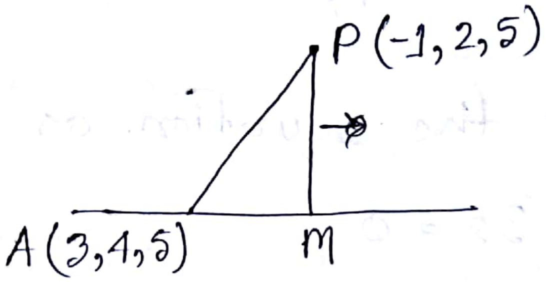
$$\frac{x+\frac{45}{17}}{13} = \frac{y+\frac{21}{17}}{14} = \frac{z-0}{17}$$

$$\Rightarrow \frac{17x+45}{221} = \frac{14y+21}{238} = \frac{z}{17}$$

27-11-2017 : 13A : Monday

→ Find the distance from $(-1, 2, 5)$ to $\frac{x-3}{2} = \frac{y-4}{-3} = \frac{z-5}{6}$.

Solution:



AM = Projection of AP on the st. line.

Projection on a st. line with direction cosine (l, m, n) of a st. line passing through (x_1, y_1, z_1) and (x_2, y_2, z_2) $= l(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1)$

D.R. of AM $(2, -3, 6)$

D.C. of AM $\left(\frac{2}{\sqrt{2^2+(-3)^2+6^2}}, \frac{-3}{\sqrt{2^2+(-3)^2+6^2}}, \frac{6}{\sqrt{2^2+(-3)^2+6^2}}\right)$

$= \left(\frac{2}{7}, -\frac{3}{7}, \frac{6}{7}\right)$

$\therefore$ length of AM $= \frac{2}{7}(-1-3) - \frac{3}{7}(2-4) + \frac{6}{7}(5-5)$

$= -\frac{8}{7} + \frac{6}{7}$

$= -\frac{2}{7}$

$AP^2 = (-1-3)^2 + (2-4)^2 + (5-5)^2$

$= 16 + 4$

$= 20$

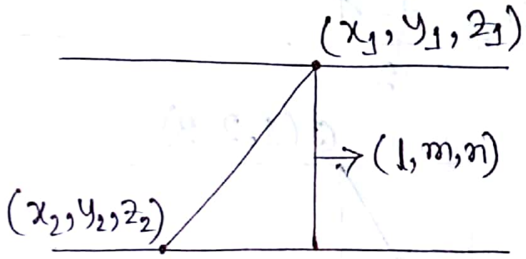
From ΔADM

$$AP^2 = AM^2 + PM^2$$

$$\Rightarrow PM = \sqrt{AP^2 - AM^2}$$

$$\Rightarrow PM = \sqrt{20 - \frac{4}{49}}$$

$$\Rightarrow PM = 4.46$$



$$\frac{x - x_1}{l_1} = \frac{y - y_1}{m_1} = \frac{z - z_1}{n_1}$$

$$\frac{x - x_2}{l_2} = \frac{y - y_2}{m_2} = \frac{z - z_2}{n_2}$$

$$ll_1 + mm_1 + nn_1 = 0 \text{ ------------ (i)}$$

$$ll_2 + mm_2 + nn_2 = 0 \text{ ------------ (ii)}$$

solving (i) & (ii)

$$\frac{l}{m_1 n_2 - m_2 n_1} = \frac{m}{n_1 l_2 - n_2 l_1} = \frac{n}{l_1 m_2 - l_2 m_1}$$

Denominators are D.R.

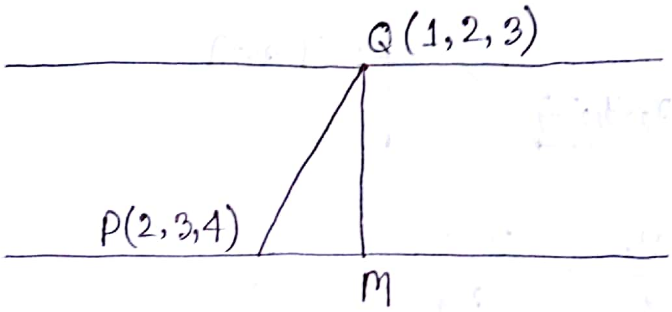
From D.R. we get D.C.

→ Find the shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$; $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$. state wheather the lines are coplanar or not.

Solution:

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4} \quad \text{-------------- (i)}$$

$$\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5} \quad \text{-------------- (ii)}$$



Let, (l,m,n) be the D.C.'s of the S.D. line.

S.D. line is perpendicular to both the lines (i) & (ii), we get,

$$2l+3m+4n=0 \quad \text{----------- (iii)}$$

$$3l+4m+5n=0 \quad \text{----------- (iv)}$$

solving (iii) & (iv)

$$\frac{l}{15-16}=\frac{m}{12-10}=\frac{n}{8-9}$$

$$\Rightarrow \frac{l}{-1}=\frac{m}{2}=\frac{n}{-1}$$

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$$l = \frac{-1}{\sqrt{(-1)^2+2^2+(-1)^2}} = -\frac{1}{\sqrt{6}}, \quad m = \frac{2}{\sqrt{6}}, \quad n = -\frac{1}{\sqrt{6}}$$

length of S.D. = QM = Projection of PQ on the line QM

$$= (1-2)\left(-\frac{1}{\sqrt{6}}\right) + (2-3)\frac{2}{\sqrt{6}} + (3-4)\left(-\frac{1}{\sqrt{6}}\right)$$

$$= 0$$

Since the S.D. is zero

so the given lines (i) and (ii) are coplaner lines.

– For non-zero S.D.:

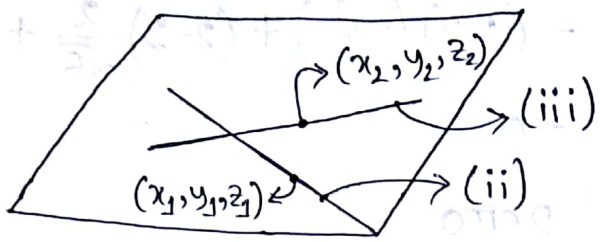
equation of plane containing S.D. line and first line:

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l & m & n \end{vmatrix} = 0$$

equation of plane containing S.D. line and second line:

$$\begin{vmatrix} x-x_2 & y-y_2 & z-z_2 \\ l_2 & m_2 & n_2 \\ l & m & n \end{vmatrix} = 0$$

the intersection line of the both plane is the S.D. line.



$$ax + by + cz + d = 0 \text{ ---------- (i)}$$

(a, b, c) is D.C. or D.R. of the st. line of the perpendicular plane of the current plane.

$$\frac{x - x_1}{l_1} = \frac{y - y_1}{m_1} = \frac{z - z_1}{n_1} \text{ ------- (ii)}$$

$$\frac{x - x_2}{l_2} = \frac{y - y_2}{m_2} = \frac{z - z_2}{n_2} \text{ -------- (iii)}$$

we get,

$$ax_1 + by_1 + cz_1 + d = 0 \text{ ---------- (iv)}$$

$$ax_2 + by_2 + cz_2 + d = 0 \text{ ---------- (v)}$$

$$al_1 + bm_1 + cn_1 = 0 \text{ ---------- (vi)}$$

$$al_2 + bm_2 + cn_2 = 0 \text{ ---------- (vii)}$$

(iv) − (v), we get,

$$a(x_1-x_2)+b(y_1-y_2)+c(z_1-z_2)=0 \quad \text{-----------} (viii)$$

Eliminating a,b,c from (vi), (vii) & (viii)

$$\begin{vmatrix} x_1-x_2 & y_1-y_2 & z_1-z_2 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

equation of plane

(iii) − (iv) we get,

$$a(x-x_1)+b(y-y_1)+c(z-z_1)=0 \quad \text{------} (ix)$$

Eliminating a,b,c from (vi), (vii), (ix)

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$