Foundation logic and proof

propositional logic:- (i) negation ($\neg$)

(ii) Conjunction($\wedge$)

(iii) disconjunction($\vee$)

(iv) Exclusive OR ($\oplus$)

(v) Conditional statement ($\rightarrow$)

(vi) Biconditional statement ($\leftrightarrow$)

Negation:- p = "He eats rice"

The negation is "he does not eat rice"

p ≠ "what is your name"?

p = "Today is friday"

The negation is "Today is not friday"

p = "At least 10 inches of rain fell today in Miami"

The negation is "Less than 10 inches of rain fell today in Miami".

Truth table:-

p	$\neg p$
1	0
0	1

Conjunction:- Find the conjunction of the propositions p and q where p is the proposition "Today is Friday" and q is the proposition "It is raining today"

soln:- The conjunction of these proposition is $p \wedge q$, is the proposition "Today is Friday and it is raining today".

Truth table:-

p	q	$p \wedge q$
0	0	0
0	1	0
1	0	0
1	1	1

Disconjunction:- Find the disconjunction of the propositions p and q where p is the proposition "Today is friday" and q is the proposition "It is raining today"

soln:- The disconjunction of these proposition is $p \vee q$, is the proposition. "Today is Friday or it is raining today".

Truth table:

p	q	$p \vee q$
0	0	0
0	1	1
1	0	1
1	1	1

Exclusive OR:-

p	q	$p \oplus q$
F	F	F
F	T	T
T	F	T
T	T	F

Conditional statement:-

p	q	$p \rightarrow q$
F	F	T
F	T	T
T	F	F
T	T	T

* Let p be the statement " Maria learns discrete Mathematics" and q the statement " Maria will find a good job" express the statement $p \rightarrow q$ in English.

Soln: "If Maria learns discrete mathematics, then she will find a good job".

Biconditional statement:- let p and q be propositions, the biconditional statement is $p \leftrightarrow q$ is the proposition "p if and only if q" the biconditional statement $p \leftrightarrow q$ is true when p and q have the same truth values if false othe rwise.

* let p be the statement " You can take the flight" and q be the statement " You buy a ticket" then $p \leftrightarrow q$ is the statement.

" You can take flight if and only if you buy a bucket."

p	q	$p \leftrightarrow q$
F	F	T
F	T	F
T	F	F
T	T	T

* what are the contrapositive, the converse, and the inverse of the conditional statement?

" The home team wins whenever it is raining?"

Soln: Because "q whenever p" is one of the ways to express the conditional statement $p \rightarrow q$. the original statement can be written as,

" If it is raining. then the home team wins".

Contrapositive: "If the home team does not win, then it is not raining"

Converse: "If the home team wins, then it is raining".

inverse:- "If it is not raining, then the home team does not win"

Only the contrapositive is equivalent to the original statement.

<u>precedence of logical expression:-</u>

1. $\neg$
2. $\wedge$
3. $\vee$
4. $\rightarrow$
5. $\leftrightarrow$

※ $(p \vee \neg q) \rightarrow (p \wedge q)$

p	q	$\neg q$	$p \vee \neg q$	$p \wedge q$	$(p \vee \neg q) \rightarrow (p \wedge q)$
0	0	1	1	0	0
0	1	0	0	0	1
1	0	1	1	0	0
1	1	0	1	1	1

※ $(q \rightarrow \neg p) \vee (\neg p \rightarrow \neg q)$

p	q	$\neg p$	$(q \rightarrow \neg p)$	$\neg q$	$(\neg p \rightarrow \neg q)$	$(q \rightarrow \neg p) \vee (\neg p \rightarrow \neg q)$
0	0	1	1	1	1	1
0	1	1	1	0	0	1
1	0	0	1	1	1	1
1	1	0	0	0	1	1

✱ $p \oplus (p \wedge q) \to p$

p	q	$p \wedge q$	$(p \wedge q) \to p$	$p \oplus (p \wedge q) \to p$
0	0	0	1	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

✱ How can this English sentence be translated into logical expression?

" You can $\underbrace{\text{access the}}_{p}$ internet from campus $\underbrace{\text{if and only if}}_{\longleftrightarrow}$ you are $\underbrace{\text{a computer}}_{}$ science major $\underbrace{\text{or}}_{\vee}$ you $\underbrace{\text{are not a}}_{\neg t}$ freshman". q

$p \leftrightarrow (q \vee \neg t)$

✱ " $\underbrace{\text{You can not ride the roller coaster}}_{q}$ $\underbrace{\text{if}}_{\to}$ $\underbrace{\text{you are under 4feet tall}}_{r}$ $\underbrace{\text{unless}}_{\wedge}$ $\underbrace{\text{you are older then 16 years old}}_{s}$".

$(r \wedge \neg s) \to \neg q$

※ Knight $\rightarrow$ True
Kaves $\rightarrow$ liar

A say " B is knight "
B say " we are opposite".

(i) A = knight (ii) A = Kaves (iii) A = B = knight
B = Kaves B = knight

(iv) A = B = Kaves

Propositional equivalences

1. Tautology $\rightarrow$ all time true
2. Contradiction $\rightarrow$ all time false
3. Contigency — T/F

※ Tautology : $p \vee \neg p$

p	$\neg p$	$p \vee \neg p$
0	1	1
1	0	1

※ contradiction : $p \wedge \neg p$

p	$\neg p$	$p \wedge \neg p$
0	1	0
1	0	0

※ show that $\neg(p \vee q)$ are $\neg p \wedge \neg q$ are logically equivalent.

p	q	$p \vee q$	$\neg(p \vee q)$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

For $(\neg p \wedge \neg q)$

p	q	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	0	0	0

So they are logically equivalent.

$\therefore$ $\neg(p \vee q)$ and $(\neg p \wedge \neg q)$ are true.

※ show that the following logical expression are equivalent or not

(i) $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$

p	q	$p \leftrightarrow q$
0	0	1
0	1	0
1	0	0
1	1	1

p	q	$\neg p$	$\neg q$	$p \wedge q$	$\neg p \wedge \neg q$	$(p \wedge q) \vee (\neg p \wedge \neg q)$
0	0	1	1	0	1	1
0	1	1	0	0	0	0
1	0	0	1	0	0	0
1	1	0	0	1	0	1

they are equivalent.

※ $(p \rightarrow r) \wedge (q \rightarrow r)$ and $(p \vee q) \rightarrow r$

p	q	r	$p \rightarrow r$	$q \rightarrow r$	$(p \rightarrow r) \wedge (q \rightarrow r)$
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	1	0	0
0	1	1	1	1	1
1	0	0	0	1	0
1	0	1	1	1	1
1	1	0	0	0	0
1	1	1	1	1	1

p	q	r	p ∨ q	$(p \vee q) \to r$
0	0	0	0	1
0	0	1	0	1
0	1	0	1	0
0	1	1	1	1
1	0	0	1	0
1	0	1	1	1
1	1	0	1	0
1	1	1	1	1

So they are equivalent.

※ $(p \to r) \vee (q \to r)$ and $(p \wedge q) \to r$

p	q	r	$p \to r$	$q \to r$	$(p \to r) \vee (q \to r)$
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	1	0	1
0	1	1	1	1	1
1	0	0	0	1	1
1	0	1	1	1	1
1	1	0	0	0	0
1	1	1	1	1	1

p	q	r	$p \vee q$	$(p \vee q) \rightarrow r$
0	0	0	0	1
0	0	1	0	1
0	1	0	1	0
0	1	1	1	1
1	0	0	1	0
1	0	1	1	1
1	1	0	1	0
1	1	1	1	1

$\therefore$ They are not equivalent.

※ show that. $(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$ is a tautology.

p	q	r	$\neg p$	$p \vee q$	$\neg p \vee r$	$q \vee r$	$(p \vee q) \wedge (\neg p \vee r)$	main
0	0	0	1	0	1	0	0	1
0	0	1	1	0	1	1	0	1
0	1	0	1	1	1	1	1	1
0	1	1	1	1	1	1	1	1
1	0	0	0	1	0	0	0	1
1	0	1	0	1	1	1	1	1
1	1	0	0	1	0	1	0	1
1	1	1	0	1	1	1	1	1

※ If $a=2, b=3$

$a \wedge = b \wedge = a$

$a = ?$

$b = ?$

Sol^n:- $a \wedge = b \wedge = a \wedge = b$

$a = 2 = (10)_2$

$b = 3 = (11)_2$

$a = a \wedge b$

$= (10)_2 \wedge (11)_2$

$$= \begin{array}{r} 10 \\ 11 \\ \hline (01)_2 \end{array}$$

$\therefore a = 1$

$b = b \wedge (1)_2$

$= (11)_2 \wedge (1)_2$

$$= \begin{array}{r} 11 \\ 01 \\ \hline (10)_2 \end{array}$$

$= 2$

$a = (10)_2 \wedge (01)_2$

$$= \begin{array}{r} 10 \\ 01 \\ \hline (11)_2 \end{array}$$

$\therefore a = 3$ finally, $a = 3$, $b = 2$

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※ show that $\neg(p \to q)$ and $p \wedge \neg q$ are logically equivalent.

$\neg(p \to q) \equiv \neg(\neg p \vee q)$

$\equiv \neg(\neg p) \wedge \neg q$ by the second De Morgan Law

$\equiv p \wedge \neg q$ by the double negation law

※ show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent by developing a series of logical equivalences.

$\neg(p \vee (\neg p \wedge q) \equiv \neg p \wedge \neg(\neg p \wedge q)$ second De Morgan law

$\equiv \neg p \wedge [\neg(\neg p) \vee \neg q]$ first De " "

$\equiv \neg p \wedge (p \vee \neg q)$ Double negation law

$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q)$ second distributive "

$\equiv F \vee (\neg p \wedge \neg q)$

$\equiv (\neg p \wedge \neg q)$ ※ F communicative law for disjunction

$\equiv \neg p \wedge \neg q$ identity Law for F

Identity Law:- $p \wedge T = p$, $p \vee F = p$

Domination " :- $p \wedge F = F$, $p \vee T = T$

※ Show that $(p \wedge q) \to (p \vee q)$ is a tautology.

$(p \wedge q) \to (p \vee q) \equiv \neg(p \wedge q) \vee (p \vee q)$

$\equiv (\neg p \vee \neg q) \vee (p \vee q)$ first De Morgan law

$\equiv (\neg p \vee p) \vee (\neg q \vee q)$ associative and communicative law for disjunction

$\equiv T \vee T$ communicative law for ,,

$\equiv T$ Domination Law

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Quantifier

Quantifier :- 1. Universal $\rightarrow \forall$

2. Existential $\rightarrow \exists$

Statement	when true	when false
$\forall x\ P(x)$	$P(x)$ is true for every x	There is an x for which $P(x)$ is false.
$\exists x\ P(x)$	There is an x for which $P(x)$ is true	$P(x)$ is false for every x.

※ Let $P(x)$ be the statement "$x+1>x$" what is the truth value of the quantification $\forall x\ P(x)$, where the domain consists of all real number?

Solⁿ :- For all values of x $P(x)$ is true.

of $x \in \mathbb{R}$,

$\forall x\ P(x)$ is true.

※ Let $Q(x)$ be the statement "$x<2$" what is the truth value of the quantification $\forall x\ Q(x)$. where the domain consists of all real numbers?

Solⁿ :- For $x=1$, $x \in \mathbb{R}$, $Q(x)$ is "$1<2$" $\therefore Q(x) \equiv 1$

For $x = 3$, $x \in \mathbb{R}$ $Q(x)$ is "$3 < 2$".

$$Q(x) = 0$$

So, $\forall x\, Q(x)$ is false.

※ let $P(x)$ denote the statement "$x > 3$" what is the truth value of the quantification $\exists x\, P(x)$ where the domain consists of all real numbers?

Sol^n:- For $x = 0$, $x \in \mathbb{R}$, $P(x)$ is "$0 > 3$"

So $P(x) = 0$

For $x = 4$; $x \in \mathbb{R}$ $P(x)$ is "$4 > 3$"

So $P(x) = 1$

So $\exists x\, P(x)$ is true.

※ let $P(x)$ denote the statement "$x = x + 1$" what is the truth value of the quantification $\exists x P(x)$ where the domain consists of all real numbers?

Sol^n:- For $x = 0$, $x \in \mathbb{R}$ $P(x)$ is "$0 = 1$"

So $P(x) = 0$

For $x = 1$, $x \in \mathbb{R}$ $P(x)$. is "$1 = 2$".

$P(x) = 0$

$\therefore$ $\exists x\, P(x)$ is false.

※ Suppose that $p(x)$ is "$x^2>0$" To show that the statement $\forall x\, P(x)$ is false where the universe of discourse cosists of all integers.

Solⁿ:- Here for $x=0$ so $p(x)$ is "$0>0$"

$\therefore P(x)=0$

so $\forall x\, P(x)$ is false.

※ What is the truth value of $\forall x\, P(x)$ where $P(x)$ is the statement "$x^2<10$" and the domain of consists of the positive integers not exceeding 4?

Solⁿ:- Here the $\forall x\, P(x)$ is as same as the conjunction $P(1) \wedge P(2) \wedge P(3) \wedge P(4)$

Here the domain $\{1,2,3,4\}$

for $x=1$, $P(x)$ is "$1<10$" $\therefore P(x)=1$

" $x=2$, $P(x)$ " "$4<10$" $\therefore P(x)=1$

" $x=3$, $P(x)$ " "$9<10$" $\therefore P(x)=1$

" $x=4$, $P(x)$ " "$16<10$" $\therefore P(x)=0$

Because of $P(4)$ statement $\forall x\, P(x)$ is false.

※ what is the truth value of $\forall x\ (x^2 \geq x)$ if the domain consists of all real numbers?

soln: The universal quantification of $\forall x\ (x^2 \geq x)$ is false for the domain consists of all real numbers.

For $x = 1/2$, $\frac{1}{4} \geq 1/2$ so that the statement is false.

Note that $x^2 \geq x$ if and only if $x^2 - x = x(x-1) \geq 0$

Consequently if and only if $x \leq 0$ or $x \geq 1$; so that $\forall x\ (x^2 \geq x)$ is false. $(0 \geq x \geq 1)$

However if the domain consists of all integers, $\forall x\ (x^2 \geq x)$ is true because there are no integer x with $0 < x < 1$

predicates

※ Let $p(x)$ denote the statement "$x > 3$" what are truth values of $p(4)$ and $p(2)$?

sol^n:- Here the statement $p(4)$ setting $x=4$ in the statement "$x>3$", Hence $p(4)$ which is the statement "$4>3$" is true. However $p(2)$ which is the statement "$2>3$" is false.

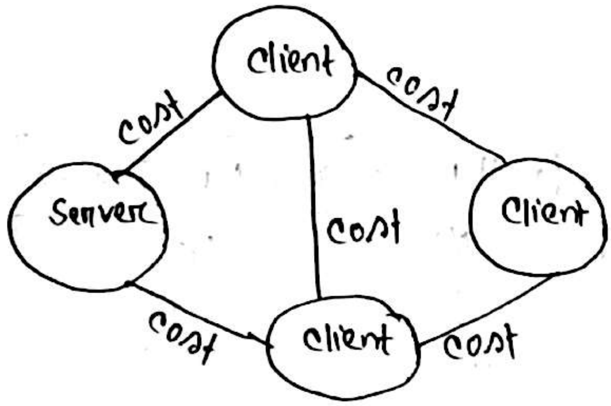
Graph

Graph:- A graph $G = (V, E)$ consists of V, a non-empty set of vertices or nodes and E a set of edges. Each edge has either one or two vertices associated with it called end points.

Infinite graph:- The vertices of a graph G may be infinite. A graph with an infinite vertex set is called an infinite graph.

Finite graph:- A graph with a finite vertex set is called a finite graph.

computer network:-



Simple graph:- A graph in which each edge connects two different vertices is called simple graph.

Multiple graph:- Graphs that may have multiple edges connecting the same vertices are called multiple graph.

Pseudograph :- Graphs may include loops and possibly multiple edges connecting the same pair of vertices are sometimes called pseudograph.

Directed graph :- A directed graph (V, E) consists of a non-empty set of vertices V and a set of directed edges E. Each directed edge is associated with an ordered pair of vertices. The directed edge associated with the ordered pair (u,v) is said to start at u and end at v.

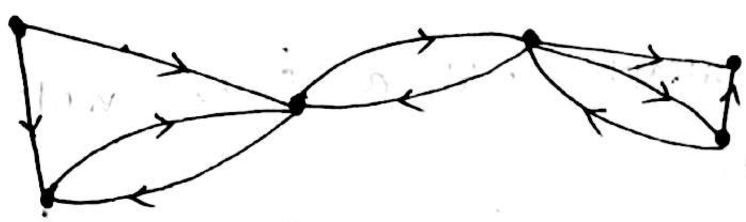


Fig: Directed graph.

Graph terminology :-

Type	Edges	multiple edges allowed?	Loops allowed?
Simple graph	undirected	No	No
Multigraph	"	yes	No
pseudograph	"	yes	yes
Simple directed	Directed	No	No
Directed multig-	"	yes	yes
Mixed graph	Directed & undirected	yes	yes

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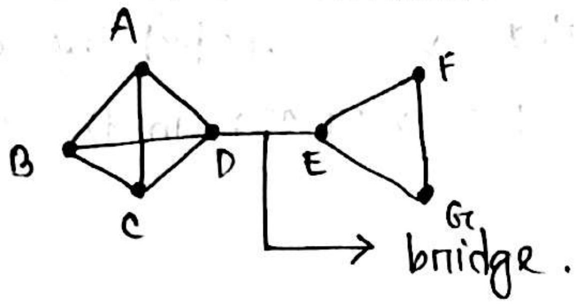
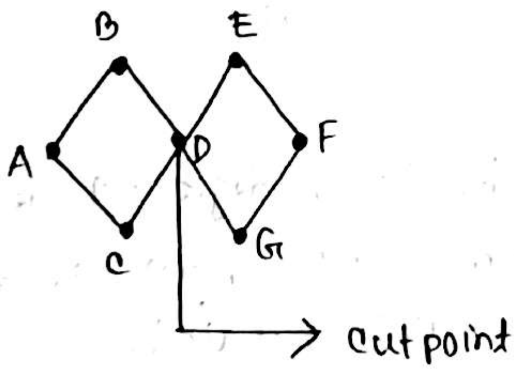
Trivial graph:- A finite graph with one vertex and no edge i.e single point is called trivial graph.

Connected graph:- If in a graph all vertices are connected then the graph is called connected graph, if there is a path between two vertices.



Bridge: If two graphs are connected by an edge than that edge is called bridge.

cutpoint: If a graph is divided into two individual graph from one point or vertices then the point is called cutpoint.



Def^n.:- Two vertices u and v in an undirected graph G are called adjacent in G if u and v are endpoints of an edge G. If e is associated with [u,v], the edge is called incident with the vertices u and v. The edge e is also said

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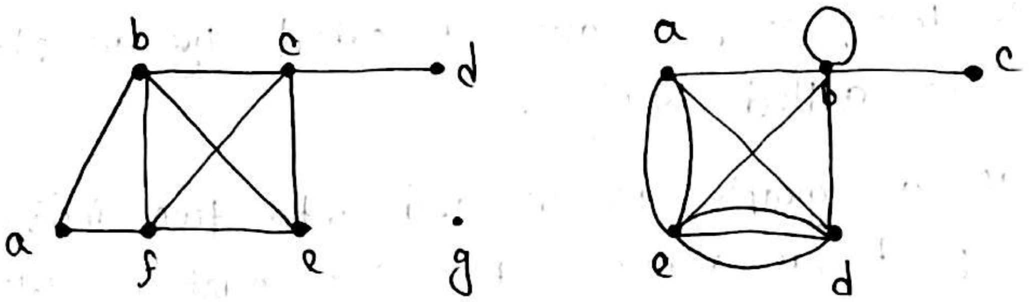
to connect u and v. The vertices u and v are called end-points of an edge associated with [u,v].

Defn: The degree of a vertex in an undirected graph is the number of edges incident with it, except that a loop at a vertex contributes twice to the degree of that vertex. The degree of the vertex v is denoted by degree(v).

What are the degree of the vertices in the G and H displayed in figure 1?

Soln:-



G H

In G, deg(a) = 2, deg(b) = 4, deg(c) = 4, deg(e) = 3, deg(d) = 1, deg(g) = 0
and in H, deg(a) = 4, deg(b) = deg(e) = 6, deg(c) = 1, deg(d) = 5

A vertex of degree zero is called isolated.

A vertex is pendant if and only if it has degree one.

The handshaking theorem: Let $G = (V, E)$ be an undirected graph with e edges. then.

$$2e = \sum_{v \in V} \deg(v)$$

[this applies if Multiple loops and edges are present]

※ How many edges are there in a graph with 10 vertices each of degree six?

Soln:- Because the sum of the degrees of the vertices $6 \cdot 10 = 60$ it follows that $2e = 60$ $\therefore e = 30$.

Theorem:- An undirected graph has an even number of vertices of odd degree.

Proof:- Let V_1 and V_2 be the set of vertices of even degree and the set of vertices of odd degree, respectively in an undirected graph. $G = (V, E)$ then

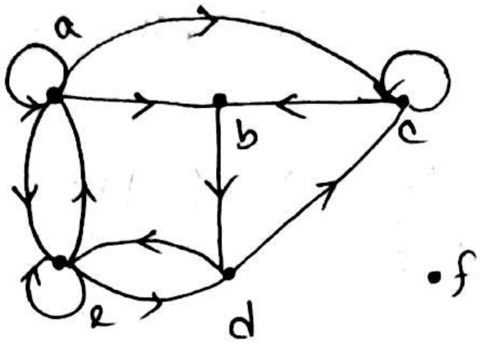
$$2e = \sum_{v \in V} \deg(v) = \sum_{v \in V_1} \deg(v) + \sum_{v \in V_2} \deg(v)$$

Defn:- In a graph with directed edges the in-degree of a vertex v, denoted by $\deg^-(v)$, is the number of edges with v as their terminal vertex. The outdegree v denoted by $\deg^+(v)$. is the number of edges with v as their initial vertex.

※ Theorem:- Let $G = (V, E)$ be a graph with directed edges, then.

$$\sum_{v \in V} \deg^-(v) = \sum_{v \in V} \deg^+(v) = |E|$$

※ Find the in-degree and out-degree of vertex in the graph G with the directed edges shown in figure?

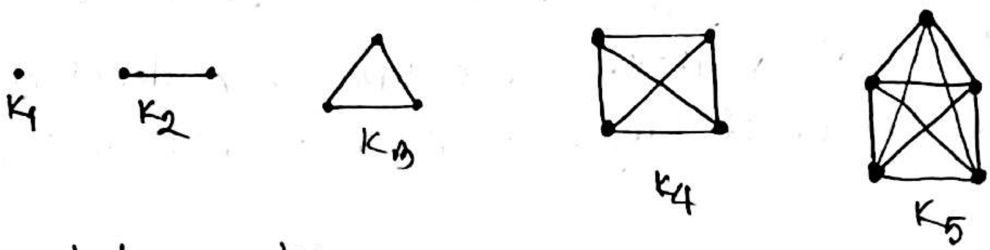


soln: The in-degrees are $deg^-(a) = 2$, $deg^-(b) = 2$, $deg^-(c) = 3$, $deg^-(d) = 2$, $deg^-(e) = 3$ and $deg^-(f) = 0$

The out degrees are $deg^+(a) = 4$, $deg^+(b) = 1$, $deg^+(c) = 2$, $deg^+(d) = 2$, $deg^+(e) = 3$, $deg^+(f) = 0$

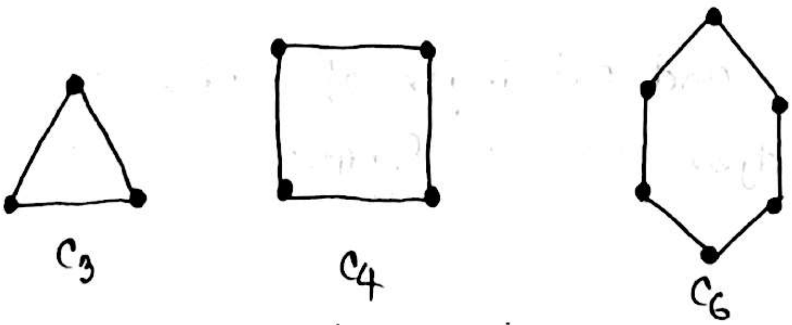
Complete graph:- The complete graph on n vertices, denoted by K_n, is the simple graph that contains exactly one edge between each pair of distinct vertices.

For, K_n for $1 \leq n \leq 6$



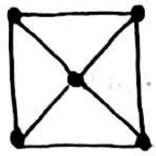
~~Complete graph:- The complete graph on n. vertices. denoted~~

cycles:- The cycle C_n. $n \geq 3$. consists of n vertices $v_1, v_2 \cdots v_n$ and edges $\{v_1, v_2\}$, $\{v_2, v_3\}$, $\cdots$ $\{v_{n-1}, v_n\}$ and $\{v_n, v_1\}$



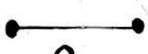
※ **wheel:-** we obtain the wheel W_n when we add an additional vertex to the cycle C_n for $n \geq 3$. and connect this new vertex to each of the n. vertices in C_n. by new - edges.

W_3



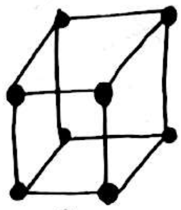
W_4

n- cube graph:-



Q_1

Q_2



Q_3

Bipartite graph:- A simple graph $G(V, E)$ is called bipartite if it is vertex V can be partitioned by two disjoints sets V_1 and V_2 such that every edges are in the graph connects a veritex V_1 and a vertex V_2 if the condition hold. then we can call the pair of (V_1, V_2) are the bipartion of vertex set V of G.

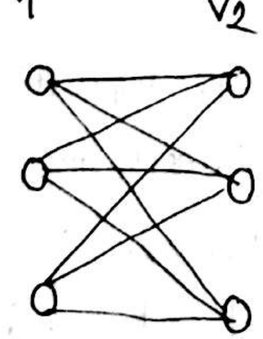
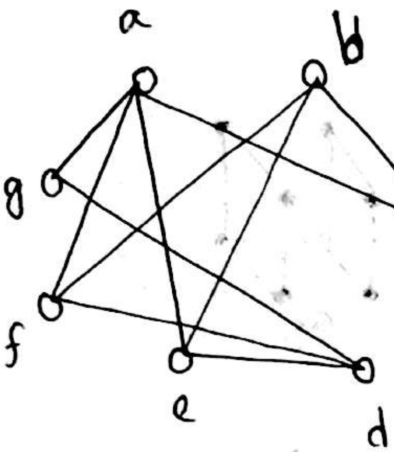


Fig:- Bipartite graph.

Theorems. A simple graph G(V, E) is called bipartite if and only if it is possible to assign one of two different colours of each vertex such that no adjacent vertices are assigned to the same colour.



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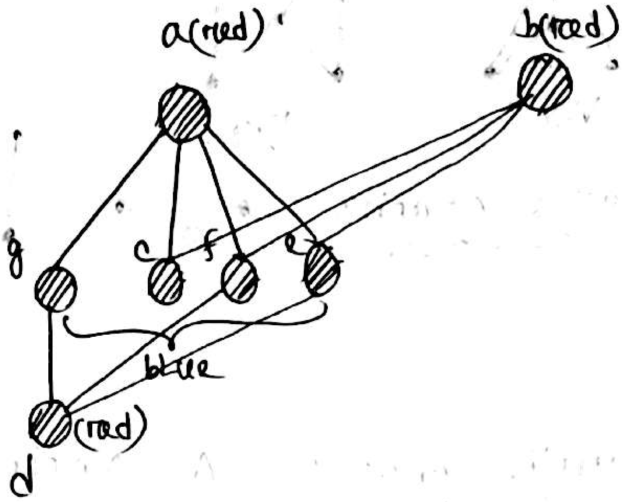

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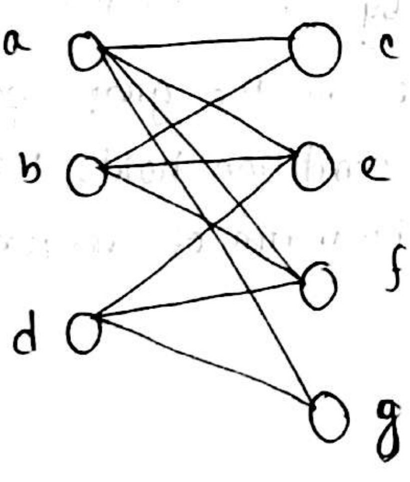

Complete bipartite graph:- The complete bipartite graph $K_{m,n}$ -> is the graph that it's vertex is partioned into two subsets m, n respectively. There is an edge between the vertices if and only if one vertex is from the first set and another from the second set.

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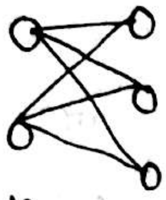
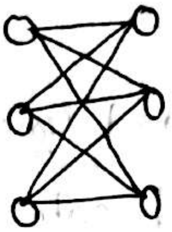
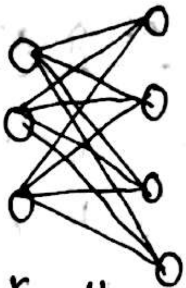


$K_{2,3}$



$K_{3,3}$



$K_{3,4}$

New graphs from old

Def^n:- A subgraph of a graph $G = (V, E)$ is a graph $H = (W, F)$ where, $W \subseteq V$ and $F \subseteq E$, a subgraph H of G is the proper subgraph of graph G if $G \neq H$.

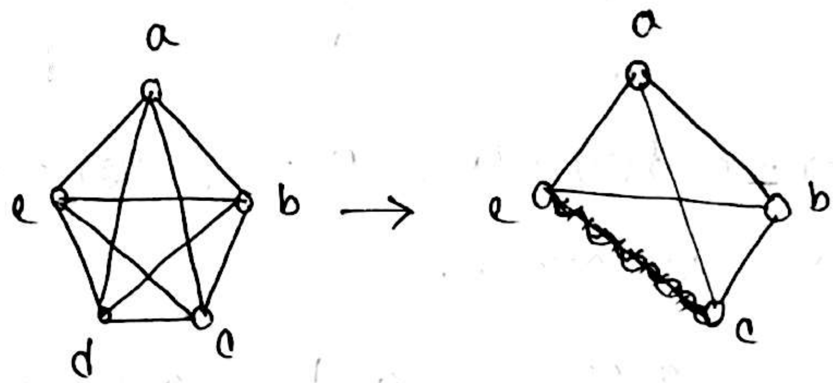
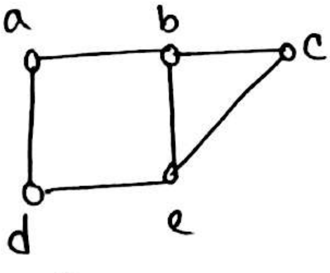
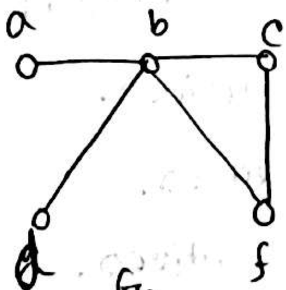


Fig:- A subgraph of K_5

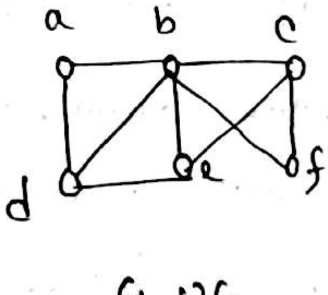
Def^n:- The union of two simple graph $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is the simple graph where the vertex set $V_1 \cup V_2$ and the edge set $E_1 \cup E_2$.



G_1



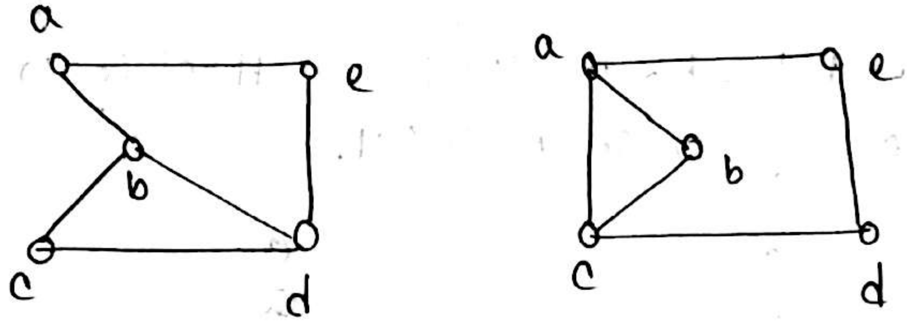
G_2



$G_1 \cup G_2$

Isomorphism of graph

Def^n:- The simple graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are isomorphic if there is one to one and onto function f from V_1 and V_2 the property that a and b are adjacent in G_1 if and only if $f(a)$ and $f(b)$ are adjacent in G_2 for all a and b in G_1. The function is called isomorphism.



Here, $f(a) = e$, $f(b) = a$, $f(c) = e$, $f(e) = d$, $f(d) = a$

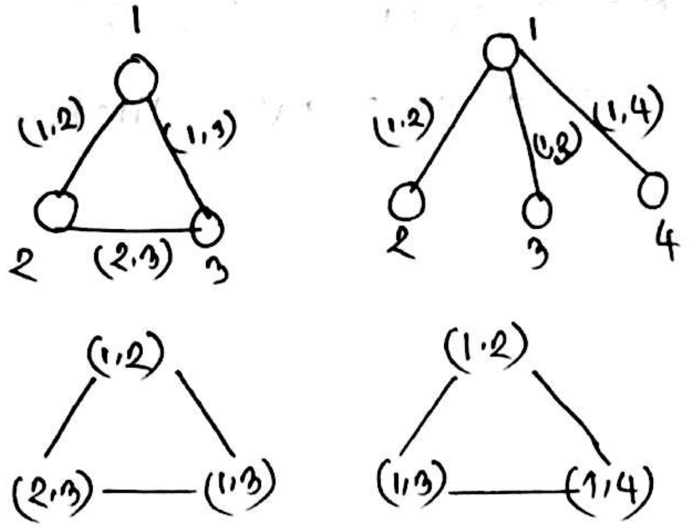
so they are isomorphic.

Theorem:- The graphs G_1 and G_2 are isomorphic is the ~~adjac~~ vertices of the graphs are ordered in such a way that the adjacent matrices MG_1 and MG_2 are identical.

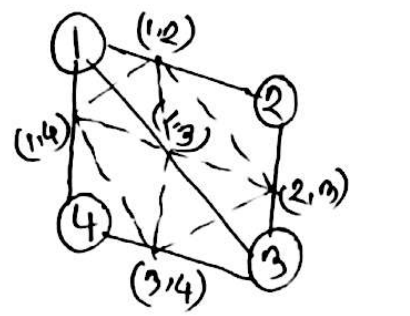
Theorem:- We can check invarients that the two isomorphic graphs must have the properties:

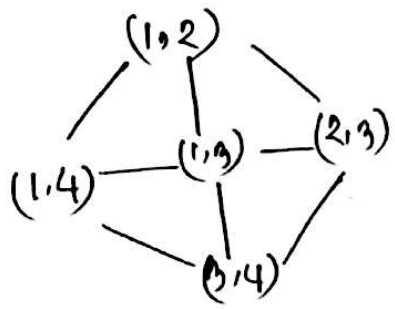
1. The same number of vertices.
2. The same number of edges.
3. The same degree of vertices.

Whitney theorem:- Two connected graphs are isomorphic if and only if their line graphs are isomorphic, with a single exception. K_3 is a complete graph with three vertices, and the complete bipartite graph $K_{1,3}$ are not isomorphic but their line graph both same K_3.



Line graph:-





Euler circuit and path :- An euler circuit in a graph G is a simple circuit containing every edge of G. An euler path in a graph G is a path containing every edge of G.

Hamilton path and circuit :- A hamilton circuit in a graph G is a simple circuit that passes every vertex only once. A hamilton path in a graph G, is a simple path that passes every vertex only once.

Karnaugh Maps

A karnaugh map is a unique graphical representation of a boolean functions. K-Map representing a booLean function of N variables consists of 2^N cells. The K- map representation is useful for booLean functions of up to 5 variables.

Condition:- All 1's should be adjacent.

2- variable K- map

01: Truth table:-

x	y	F
0	0	0
0	1	1
1	0	1
1	1	1

x \ y	0	1
0	0	1
1	2	3

x \ y	0	1
0	0	1
1	1	1

Here, $F = x + y$

solution from the truth table, $F = x'y + xy' + xy$

$= y(x + x') + xy' + xy$

$= y(x + x') + x(y + y')$

$= x + y$

02.

x	y	F
0	0	1
0	1	1
1	0	1
1	1	1

x \ y		
0	1	1
1	1	1

$\therefore F = 1$

03.

x	y	F
0	0	0
0	1	1
1	0	1
1	1	0

$x \backslash y$	0	1
0	0	1
1	1	0

$\therefore F(x,y) = x'y + xy'$

$= x \oplus y$

04.

x	y	F
0	0	0
0	1	0
1	0	0
1	1	1

$x \backslash y$	0	1
0	0	0
1	0	1

$\therefore F(x,y) = xy$

05.

x	y	F
0	0	0
0	1	0
1	0	1
1	1	1

$x \backslash y$	0	1
0	0	0
1	1	1

$\therefore F(x,y) = x$

06.

x	y	F
0	0	1
0	1	1
1	0	0
1	1	1

x \ y	0	1
0	1	1
1	0	1

$$F(x, y) = x' + y$$

3- Variable K-map

01. Truth table:-

	x	y	z	F
0	0	0	0	
1	0	0	1	
2	0	1	0	
3	0	1	1	
4	1	0	0	
5	1	0	1	
6	1	1	0	
7	1	1	1	

yz \ x	0	1
00	(0,0,0) 0	(100) 4
01	(001) 1	(101) 5
11	(011) 2	(111) 7
10	(010) 3	(110) 6

02.

x	y	z	F
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

yz \ x	0	1
00	1	1
01	1	1
11	0	0
10	1	1

$\therefore F(x, y, z) = y' + z'$

03.

x	y	z	F
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

K-map:-

yz \ x	0	1
00	1	1
01	1	1
11	1	0
10	0	0

$\therefore F(x, y, z) = y' + x'z$

04. K-map:

yz \ x	0	1
00	1	1
01	1	0
11	1	0
10	1	1

$\therefore F(x, y, z) = x' + z'$

05.

yz \ x	0	1
00	1	0
01	0	1
11	1	0
10	0	1

$F(x, y, z) = x'y'z' + xy'z + x'yz + xyz'$

06.

x	y	z	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

K-map:

yz \ x	00	01
00	1	0
01	0	0
11	1	1
10	1	1

$\therefore F(x, y, z) = y + x'z'$

07.

x \ yz	00	01	11	10
0	0	0	1	1
1	1	1	0	0

$\therefore F(x, y, z) = x'y + xy'$

08.

x \ yz	00	01	11	10
00	0	0	1	
01	1		1	1

$\therefore F(x, y, z) = yz + xz'$

09.

x \ yz	00	01	10	10
0	1			1
1	1	1		1

$\therefore F(x, y, z) = xy' + z'$

4-Variable K-map

01.

ab \ cd	00	01	11	10
00	1	1	1	1
01	0	1	1	0
11	0	1	1	0
10	1	1	1	1

$\therefore F(a,b,c,d) = d + \bar{b}$

02.

ab \ cd	00	01	11	10
00	1	0	0	1
01	0	1	1	0
11	0	1	1	0
10	1	0	0	1

$\therefore F(a,b,c,d) = bd + \bar{b}\bar{d}$

$= \overline{b \oplus d}$

03.

	$c'd'$	$c'd$	cd	cd'
$a'b'$	0	0	1	1
$a'b$	0	0	0	0
ab	0	0	0	0
ab'	1	0	0	1

$\therefore \; F(a,b,c,d) = a'b'c + ab'd'$

04.

	$c'd'$	$c'd$	cd	cd'
$a'b'$	0	0	0	0
$a'b$	0	0	0	0
ab	1	1	1	1
ab'	0	0	0	0

$\therefore \; F(a,b,c,d) = ab$

05.

	$c'd'$	$c'd$	cd	cd'
$a'b'$	0	0	0	0
$a'b$	0	0	0	0
ab	1	0	0	1
ab'	1	0	0	1

$\therefore F(a,b,c,d) = ad'$

06.

	$c'd'$	$c'd$	cd	cd'
$a'b'$	0	1	0	0
$a'b$	0	1	1	1
ab	0	0	0	1
ab'	1	1	0	1

$\therefore F(a,b,c,d) = a'bc + ab'c' + a'c'd + acd'$

07.

	$c'd'$	$c'd$	cd	cd'
$a'b'$	0	1		
$a'b$	0	1	1	1
ab	0			1
ab'	1	1		1

$\therefore F(a,b,c,d) = a'bd + bcd' + b'c'd + ab'd'$

08.

	$c'd'$	$c'd$	cd	cd'
$a'b'$	1	1	0	1
$a'b$	1	1	0	1
ab	1	1	0	1
ab'	1	1	1	1

$\therefore F(a,b,c,d) = c' + ab' + d'$

09.

	$c'd'$	$c'd$	cd	cd'
$a'b'$	0	1	1	1
$a'b$	1	1	1	1
ab	0	0	0	0
ab'	0	0	0	0

$\therefore\ F(a,b,c,d) = a'b + a'd + a'c'$

Complementation of a boolean function

01: The complement representation for a function F is denoted as F' and obtained by interchanging 1's to 0's and 0's to 1's in the truth table column showing F.

02. The complement of a function can be derived algebrically by applying de-Morgan's theorem

03. The complement of a function can be derived by taking the dual of the equation (interchanging

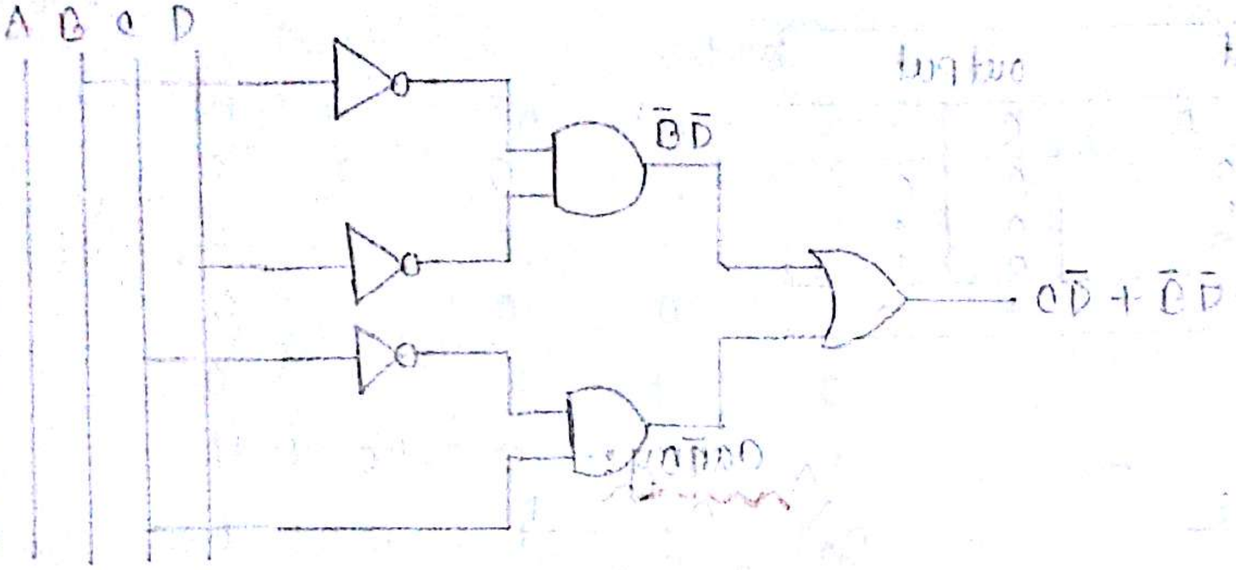
"." $\leftrightarrow$ "+" and 1's to 0's) and complementing each literal.

<u>De-Morgan method:-</u> $F(x, y, z) = x'yz' + xy'z'$

$$F' = (x'yz' + xy'z')'$$
$$= (x'yz')' \cdot (xy'z')'$$
$$= (x + y' + z) \cdot (x' + y + z)$$

<u>Dual method:-</u> $F(x, y, z) = x'yz' + xy'z'$

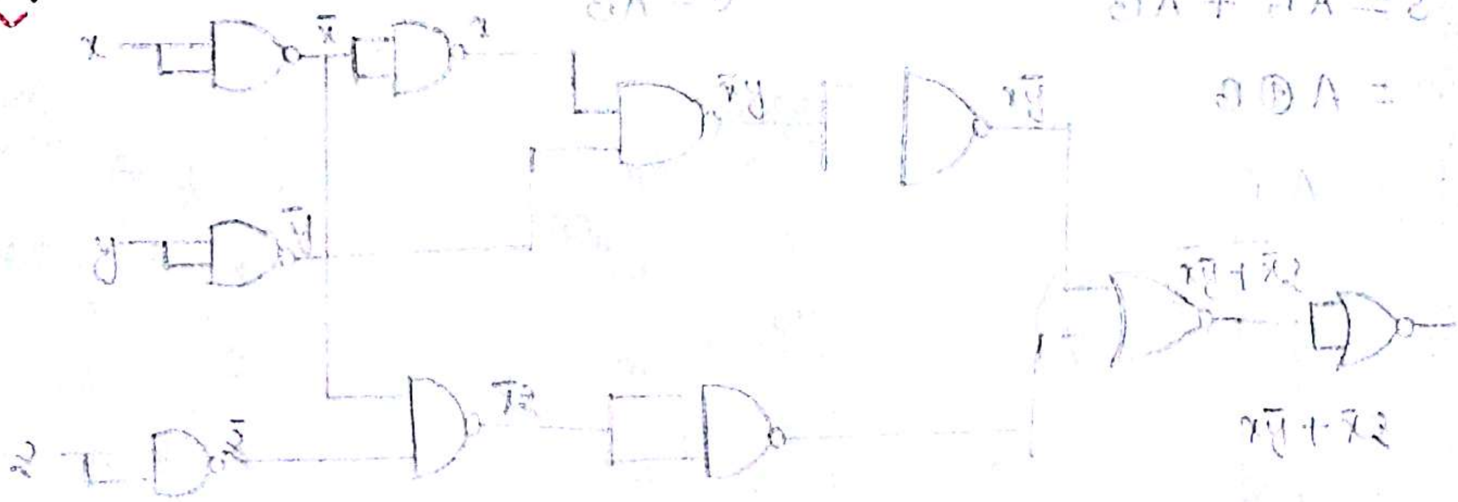
$$F = x'yz' + xy'z'$$
$$= (x' + y + z') \cdot (x + y' + z')$$
$$= (x + y' + z) \cdot (x' + y + z)$$



Design a Logic circuit using universal gate only?

$F = x\bar{y} + \bar{x}z$

sol^n:-



** Design a half adder circuit ?

Soln :-

input		output	
A	B	C	S
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

Sum :-

B \ A	0	1
0	0	1
1	1	0

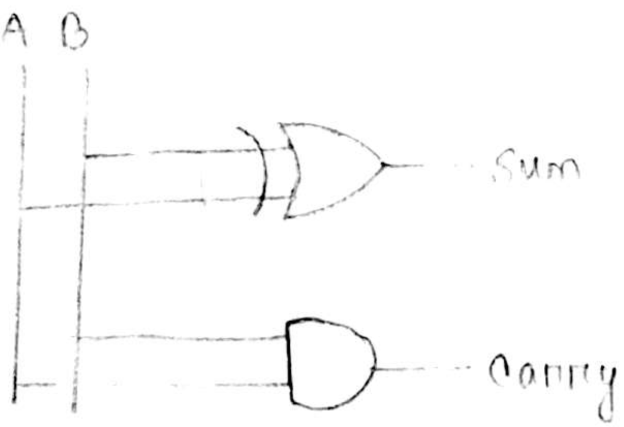
Carry :-

B \ A	0	1
0	0	0
1	0	1

$S = A\bar{B} + \bar{A}B$

$= A \oplus B$

$C = AB$



* Design a full adder circuit?

solⁿ:-

input			output	
A	B	C_{in}	C_{out}	Sum
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

[md1]

sum:-

BC_{in} \ A	0	1
00	0	1
01	1	0
11	0	1
10	1	0

carry:-

BC_{in} \ A	0	0
00	0	0
01	0	1
11	1	1
10	0	1

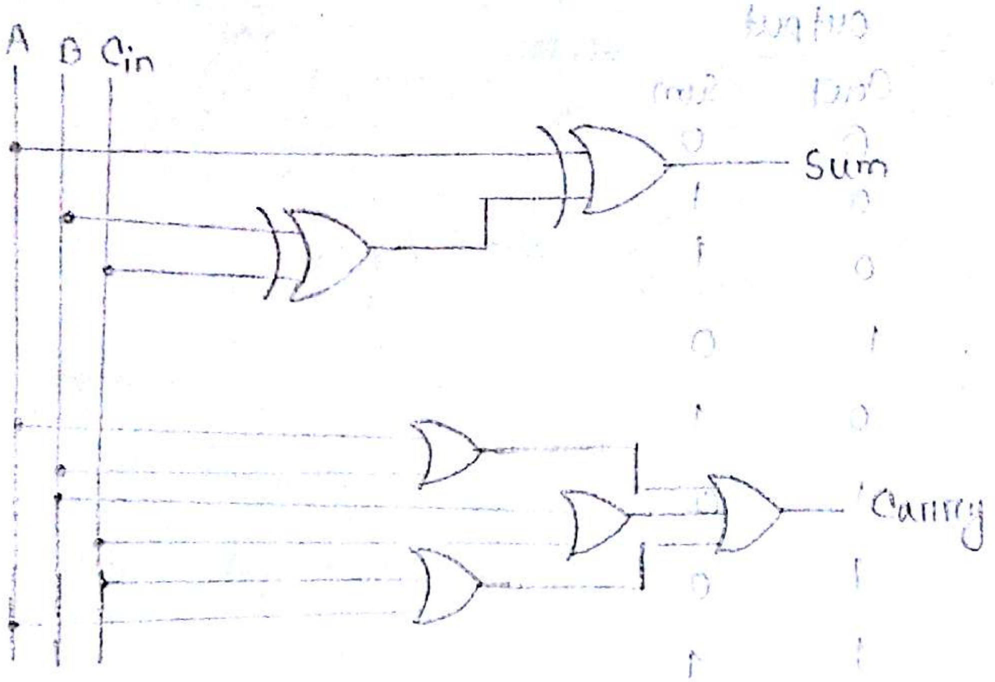
$$C_{out} = AB + BC_{in} + C_{in}A$$

$$Sum = A\bar{B}\bar{C}_{in} + \bar{A}\bar{B}C_{in} + ABC_{in} + \bar{A}B\bar{C}_{in}$$

$$= A(\bar{B}\bar{C}_{in} + BC_{in}) + \bar{A}(\bar{B}C_{in} + B\bar{C}_{in})$$

$$= A(\overline{B \oplus C_{in}}) + \bar{A}(B \oplus C_{in})$$

$= A \oplus (B \oplus C_{in})$



Design a full substractor circuit? [8bm]

Sequential Logic design

⋇ what if $S=0$ and $R=0$?

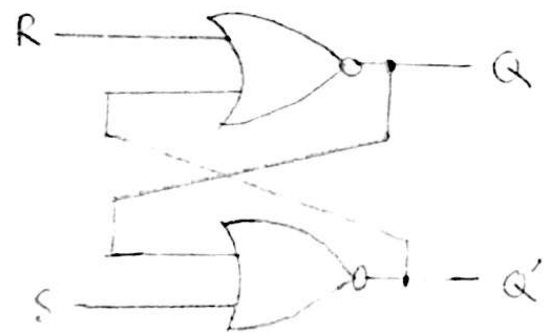
$Q_{next} = (0 + Q'_{curr})' = Q_{curr}$

$Q'_{next} = (0 + Q_{curr})' = Q'_{curr}$

when $SR = 00$ then $Q_{next} = Q_{curr}$

Here, $Q_{next} = (R + Q'_{curr})'$

$Q'_{next} = (S + Q_{curr})'$



⋇ what if $S=1$ and $R=0$?

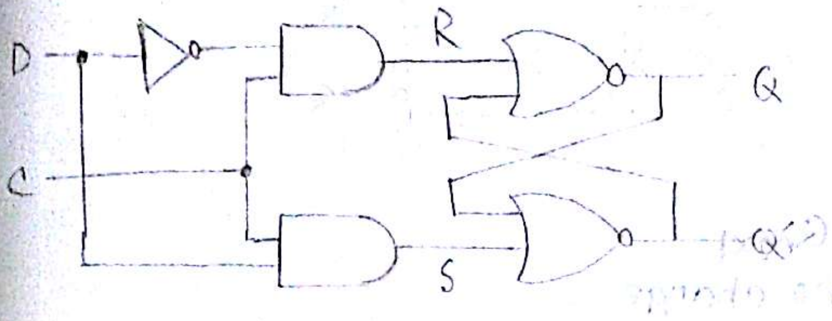
$Q'_{next} = (1 + Q'_{curr})' = 0$

$Q_{next} = (0+0)' = 1$

when $SR = 10$ then, $Q'_{next} = 0$ and $Q_{next} = 1$

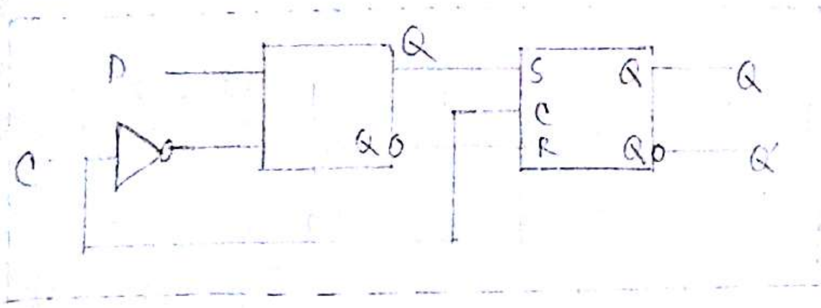
S	R	Q
0	0	no change
0	1	0 (reset)
1	0	1 (set)
1	1	Avoid!

※ D- Latch:- when C = 0 the state Q does not change. when C = 1 the Latch output Q will equal the input D.



C	D	Q
0	x	no change
1	0	0
1	1	1

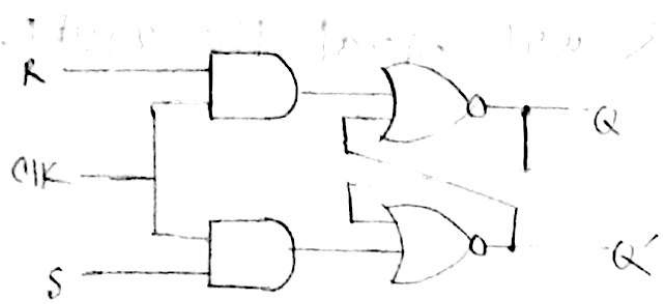
※ positive edge triggering:.



This is called positive edge triggered flip. flop.

C	D	Q
0	x	no change
1	0	0(reset)
1	1	1 (set)

SR Flip-flop:-



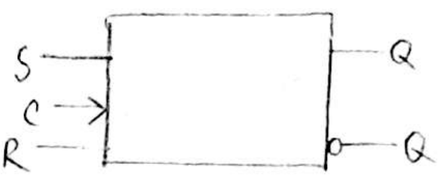
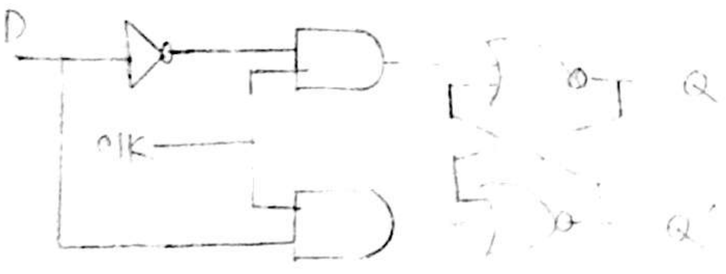


table:-

C	S	R	Qnext
0	x	x	no change
1	0	0	no change
1	0	1	0 (reset)
1	1	0	1 (set)
1	1	1	unpredictable (ambiguis)

D- flip flop:-



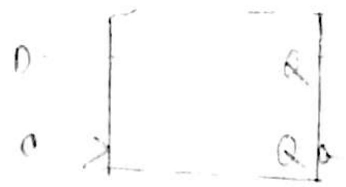
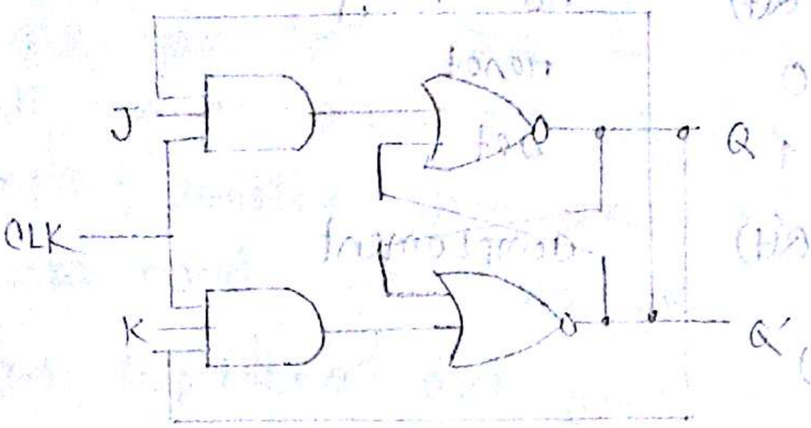


table:-

C	D	Qnext
0	x	no change
1	0	0
1	1	1

※ J-K flip flop:-



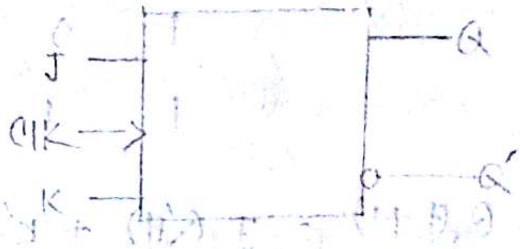


table:-

C	J	K	Q_{next}
0	x	x	no change
1	0	0	no change
1	0	1	0 (reset)
1	1	0	1 (set)
1	1	1	Q'_{curr}

T- Flip flop:-

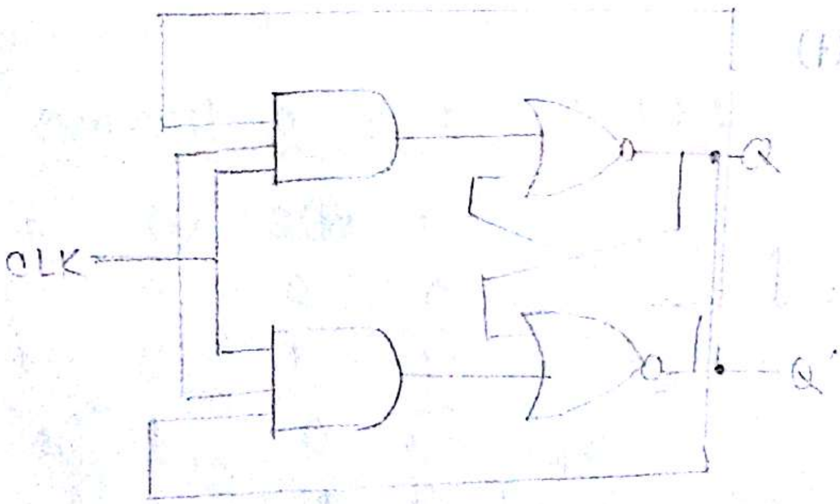


table:-

C	T	Q_{next}
0	x	no change
1	0	"
1	1	Q'_{curr}

※ characteristics table and equations:-

JK flip flop:-

J	K	Q(t+1)	operation
0	0	Q(t)	no change
0	1	0	reset
1	0	1	set
1	1	Q'(t)	complement

$\therefore \quad Q(t+1) = J\,Q'(t) + K'\,Q(t)$

D- flip flop:-

D	Q(t+1)	operation
0	0	reset
1	1	set

$\therefore Q(t+1) = D$

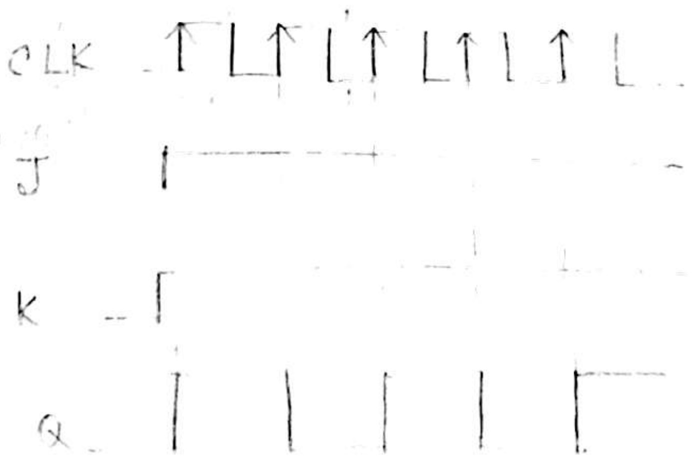
T- flipflop:-

T	Q(t+1)	operation
0	Q(t)	no change
1	Q'(t)	Complement

$$Q(t+1) = T'\,Q(t) + T\,Q'(t)$$

$$= T \oplus Q(t)$$

※



This is called toggled - T flip flip.

※ Flip flop comparison:- D flip flop have the advantage that you don't have to set up the flip flops inputs at all, since $Q(t+1) = D$. D flip flop input equations are more complex then JK. D can be implemented with less hardware then JK flip flops.

J-K flip flops are good because they are many don't care values which can lead simpler circuit.

※ Flip flop conversion excitation table:-

Q(t)	Q(t+1)	S	R	J	K	D	T
0	0	0	x	0	x	0	0
0	1	1	0	1	x	1	1
1	0	0	1	x	0	0	1
1	1	x	0	x	1	1	0

※ Convert a D-FF to T-FF:-

T	Q(t)	Q(t+1)	D
0	0	0	0
1	0	1	1
1	1	0	0
0	1	1	1

$\therefore D = TQ' + QT' = T \oplus Q$

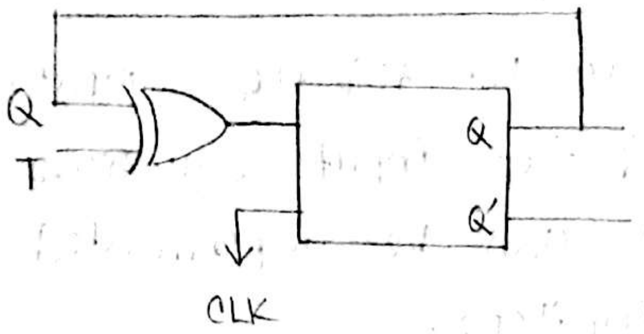


Fig: D-FF to T-FF

Convert a RS-FF to D-FF:

D	Q(t)	Q(t+1)	S	R
0	0	0	0	x
1	0	1	1	0
0	1	0	0	1
1	1	1	x	0

D \ Q	0	1
0	0	0
1	1	x

D \ Q	0	1
0	x	1
1	0	0

$\therefore S = D \qquad R = D'$

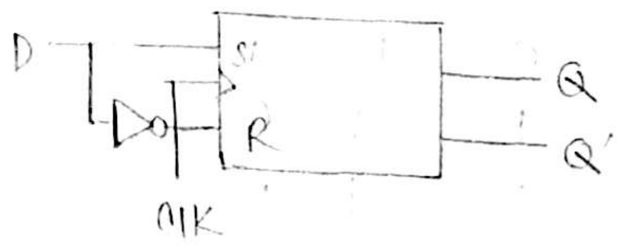


Fig: RS-FF to D-FF

** Convert a SR-FF to J-K FF:-

J	K	Q(t)	Q(t+1)	S	R
0	0	0	0	0	x
0	0	1	1	x	0
0	1	0	0	0	x
0	1	1	0	0	1
1	0	0	1	1	0
1	0	1	1	x	0
1	1	0	1	1	0
1	1	1	0	0	1

J \ KQ(t)	00	01	11	10
0	0	x	0	0
1	1	x	0	1

J \ KQ(t)	00	01	11	10
0	x	0	1	x
1	0	0	1	0

$S = JQ'_{(t)}$

$R = KQ_{(t)}$

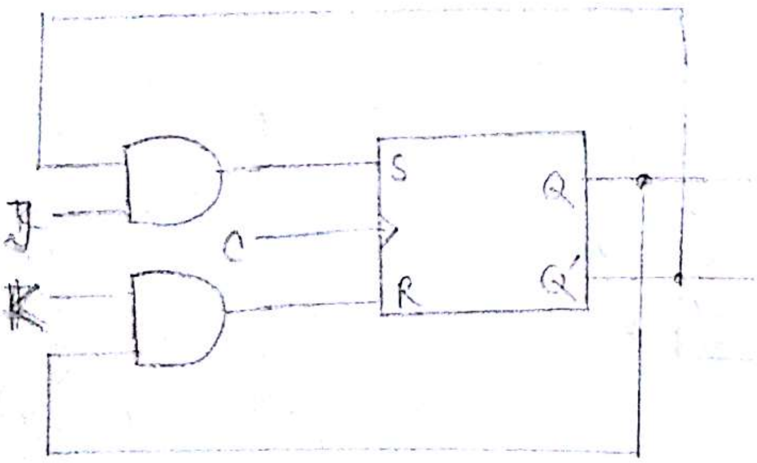


Fig: SR-FF to J-K FF

※ Convert a D-FF to J-K FF:-

J	K	$Q_{(t)}$	$Q_{(t+1)}$	D
0	0	0	0	0
0	0	1	1	1
0	1	0	0	0
0	1	1	0	0
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	0	0

J \ $KQ_{(t)}$	00	01	11	10
0	0	1	0	0
1	1	1	0	1

$\therefore\ D = K'Q_{(t)} + JQ'_{(t)}$

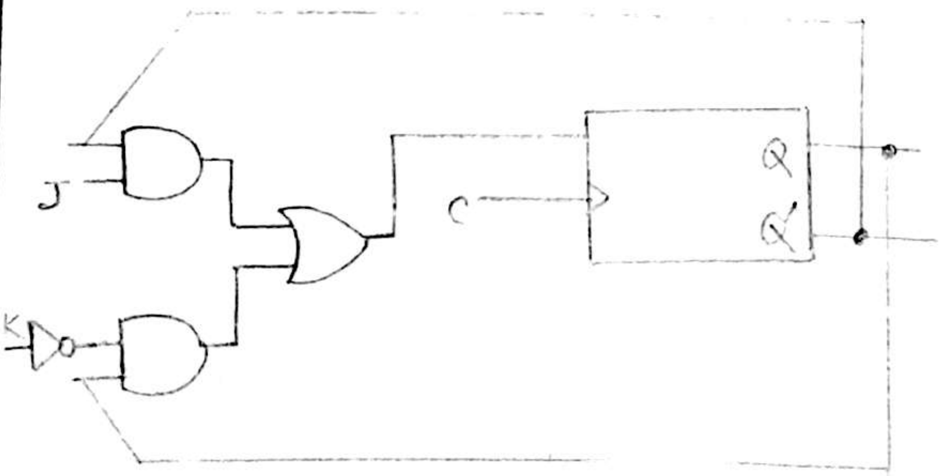


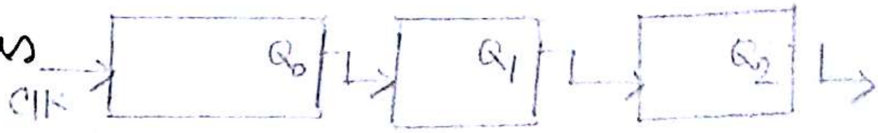
Fig: D-FF to J-K FF

Counter

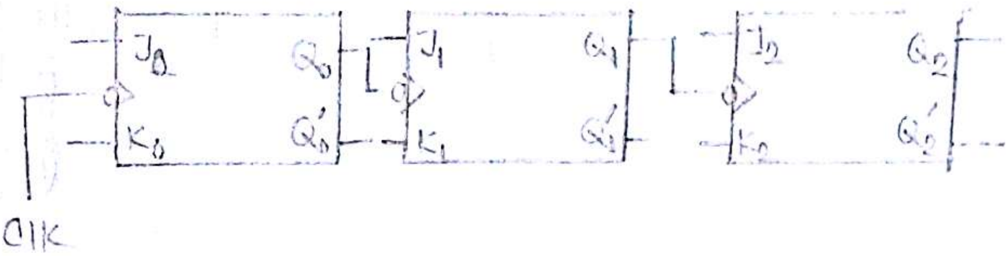
Counter:-

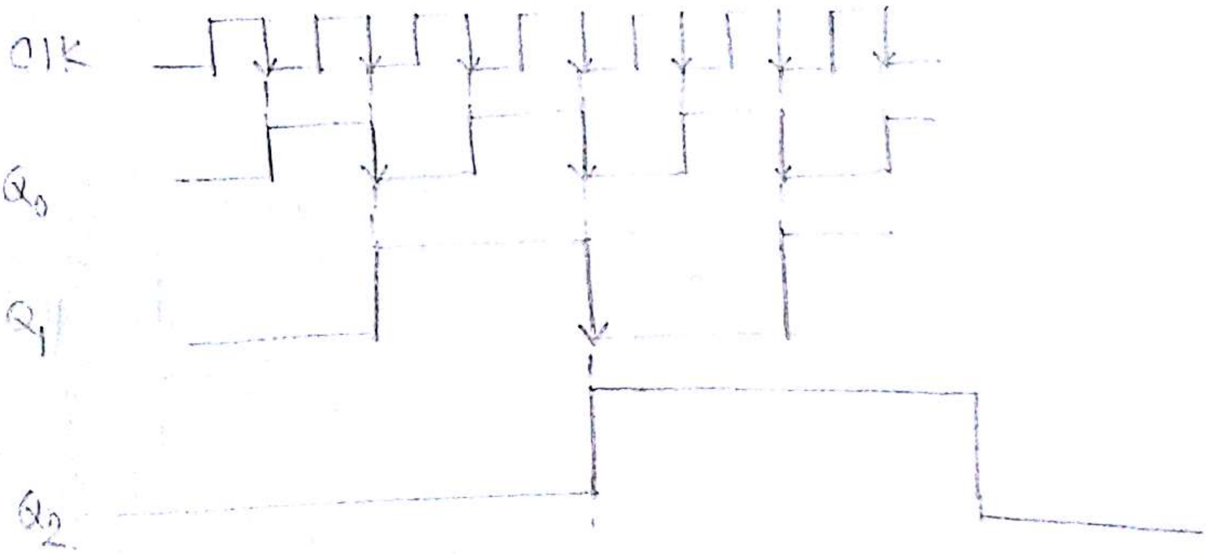
→ Synchronous

→ Asynchronous



※ Design a 3 bit binary counter: (0–7) ↔ (000 - 111)





Design a Logic circuit from the following functions:-

$F = \bar{B}\bar{C}\bar{D} + BC\bar{D} + ABC\bar{D}$

$d = \bar{B}C\bar{D} + \bar{A}B\bar{C}D$

Solⁿ: $F = \bar{B}\bar{C}\bar{D} + BC\bar{D} + ABC\bar{D}$

$= (A+\bar{A})\bar{B}\bar{C}\bar{D} + (A+\bar{A})BC\bar{D} + ABC\bar{D}$

$= A\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}\bar{D} + ABC\bar{D} + \bar{A}BC\bar{D} + ABC\bar{D}$

$\therefore F(A,B,C,D) = \sum(8,0,14,6)$

$d = \bar{B}C\bar{D} + \bar{A}B\bar{C}D$

$= (A+\bar{A})\bar{B}C\bar{D} + \bar{A}B\bar{C}D$

$= A\bar{B}C\bar{D} + \bar{A}\bar{B}C\bar{D} + \bar{A}B\bar{C}D$

$\therefore d(A,B,C,D) = \sum(10,2,5)$

CD \ AB	00	01	11	10
00	1			1
01		d		
11				
10	d	1	1	d

$\longleftrightarrow$

CD \ AB	00	01	11	10
00	1			1
01				
11				
10	1	1	1	1

$\therefore F(A,B,C,D) = C\bar{D} + \bar{B}\bar{D}$