- Analyze or design a square wave and a triangular wave generators.
- Draw the schematic diagram for and analyze the operation of a sawtooth wave generator.
- Draw the schematic diagram for and analyze the operation of a voltage-controlled oscillator and make necessary modifications in the circuit to satisfy the given requirements.

7-1 INTRODUCTION

In Chapter 6 you saw how op-amp circuits are used to provide ac/dc amplification, perform such mathematical operations as summing, averaging, differentiation, and integration, convert I-to-V and V-to-I signals, and provide very high input impedance. This chapter presents another important field of application using op-amps: filters and oscillators. The chapter begins with the analysis and design of basic and inexpensive filter types and then discusses the various oscillator circuits. At the end of the chapter, a voltage-controlled oscillator (VCO) using the NE/SE566 integrated circuit is presented.

7-2 ACTIVE FILTERS

An electric filter is often a *frequency-selective* circuit that passes a specified band of frequencies and blocks or attenuates signals of frequencies outside this band. Filters may be classified in a number of ways:

1. Analog or digital
2. Passive or active
3. Audio (AF) or radio frequency (RF)

Analog filters are designed to process analog signals, while *digital* filters process analog signals using digital techniques. Depending on the type of elements used in their construction, filters may be classified as passive or active. Elements used in *passive* filters are resistors, capacitors, and inductors. *Active* filters, on the other hand, employ transistors or op-amps in addition to resistors and capacitors. The type of element used dictates the operating frequency range of the filter. For example, RC filters are commonly used for audio or low-frequency operation, whereas LC or crystal filters are employed at RF or high frequencies. Especially because of their high Q value (figure of merit), the crystals provide more stable operation at higher frequencies.

First, this chapter presents the analysis and design of analog active-RC (audio-frequency) filters using op-amps. In the audio frequencies, inductors are often not used because they are very large, costly, and may dissipate more power. Inductors also emit magnetic fields.

An active filter offers the following advantages over a passive filter:

1. *Gain and frequency adjustment flexibility.* Since the op-amp is capable of providing a gain, the input signal is not attenuated as it is in a passive filter. In addition, the active filter is easier to tune or adjust.
2. *No loading problem.* Because of the high input resistance and low output resistance of the op-amp, the active filter does not cause loading of the source or load.
3. *Cost.* Typically, active filters are more economical than passive filters. This is because of the variety of cheaper op-amps and the absence of inductors.

Although active filters are most extensively used in the field of communications and signal processing, they are employed in one form or another in almost all sophisticated electronic systems. Radio, television, telephone, radar, space satellites, and biomedical equipment are but a few systems that employ active filters.

The most commonly used filters are these:

1. Low-pass filter
2. High-pass filter
3. Band-pass filter
4. Band-reject filter
5. All-pass filter

Each of these filters uses an op-amp as the active element and resistors and capacitors as the passive elements. Although the 741 type op-amp works satisfactorily in these filter circuits, high-speed op-amps such as the LM318 or ICL8017 improve the filter's performance through their increased slew rates and higher unity gain–bandwidths.

Figure 7–1 shows the frequency response characteristics of the five types of filters. The ideal response is shown by dashed curves, while the solid lines indicate the practical filter response. A low-pass filter has a constant gain from 0 Hz to a high cutoff frequency f_H. Therefore, the bandwidth is also f_H. At f_H the gain is down by 3 dB; after that ($f > f_H$) it decreases with the increase in input frequency. The frequencies between 0 Hz and f_H are known as the *passband* frequencies, whereas the range of frequencies, those beyond f_H, that are attenuated includes the *stopband* frequencies.

Figure 7–1(a) shows the frequency response of the low-pass filter. As indicated by the dashed line, an *ideal* filter has zero loss in its passband and infinite loss in its stopband. Unfortunately, ideal filter response is not practical because linear networks cannot produce the discontinuities. However, it is possible to obtain a practical response that approximates the ideal response by using special design techniques, as well as precision component values and high-speed op-amps.

Butterworth, Chebyshev, and Cauer filters are some of the most commonly used practical filters that approximate the ideal response. The key characteristic of the Butterworth filter is that it has a flat passband as well as stopband. For this

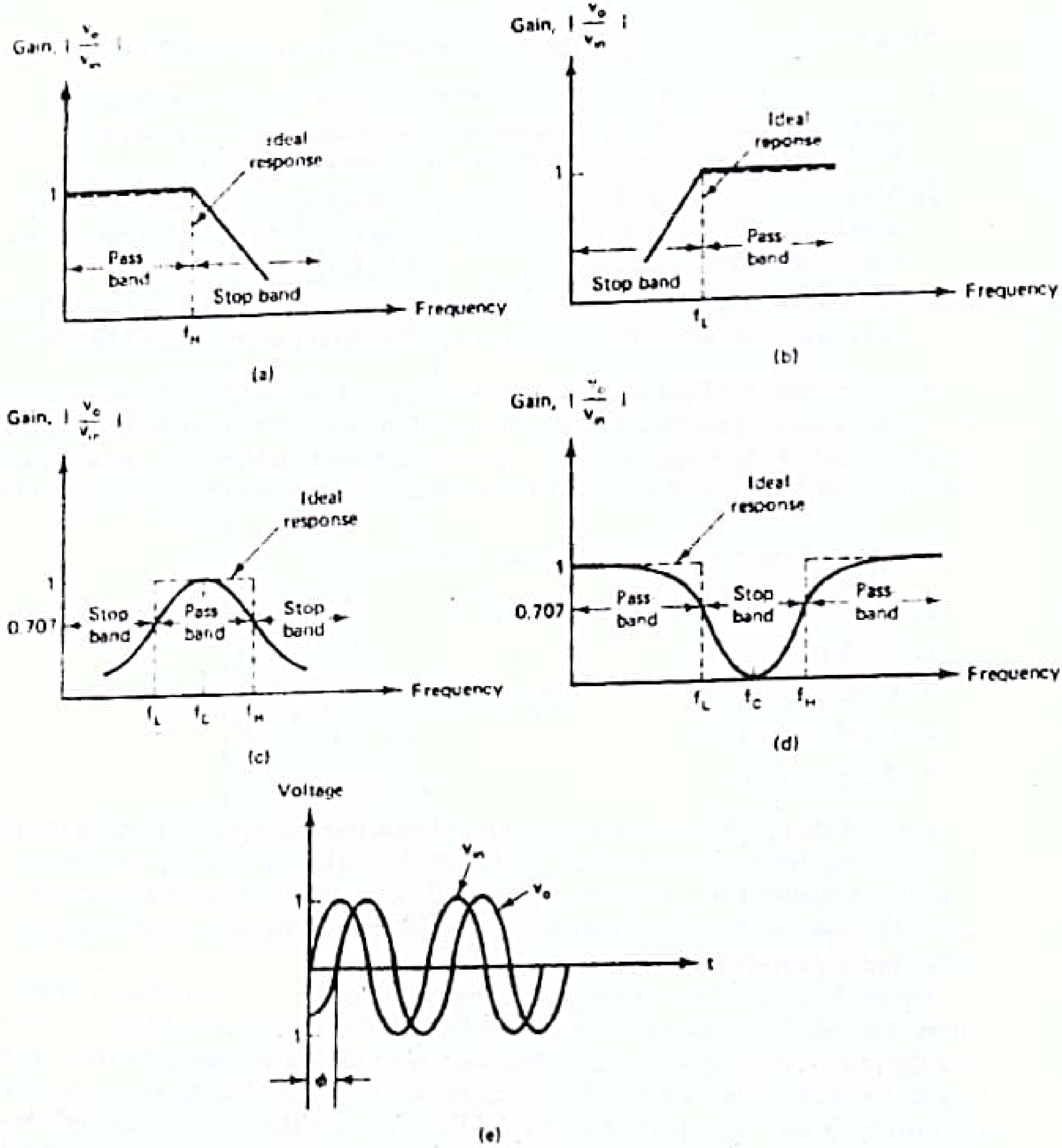


FIGURE 7-1 Frequency response of the major active filters. (a) Low pass. (b) High pass. (c) Band pass. (d) Band reject. (e) Phase shift between input and output voltages of an all pass filter.

reason, it is sometimes called a *flat-flat* filter. The Chebyshev filter has a ripple passband and flat stopband, while the Cauer filter has a ripple passband and a ripple stopband. Generally, the Cauer filter gives the best stopband response among the three. Because of their simplicity of design, the low-pass and high-pass Butterworth filters are discussed here.

Figure 7–1(b) shows a high-pass filter with a stopband $0 < f < f_L$ and a passband $f > f_L$. f_L is the low cutoff frequency, and f is the operating frequency. A band-pass filter has a passband between two cutoff frequencies f_H and f_L, where $f_H > f_L$, and two stop-bands: $0 < f < f_L$ and $f > f_H$. The bandwidth of the band-pass filter, therefore, is equal to $f_H - f_L$. The band-reject filter performs exactly opposite to the band-pass; that is, it has a bandstop between two cutoff frequencies f_H and f_L and two passbands: $0 < f < f_L$ and $f > f_H$. The band-reject is also called a *band-stop* or *band-elimination filter*. The frequency responses of band-pass and band-reject filters are shown in Figure 7–1(c) and (d), respectively. In these figures, f_c is called the center frequency since it is approximately at the center of the passband or stopband.

Figure 7–1(e) shows the phase shift between input and output voltages of an all-pass filter. This filter passes all frequencies equally well; that is, output and input voltages are equal in amplitude for all frequencies, with the phase shift between the two a function of frequency. The highest frequency up to which the input and output amplitudes remain equal is dependent on the unity gain–bandwidth of the op-amp. At this frequency, however, the phase shift between the input and output is maximum.

Before proceeding with specific filter types, let us reexamine the filter characteristics, especially in the stopband region. As shown in Figure 7–1(a)–(d), the actual response curves of the filters in the stopband either steadily decrease or increase or both with increase in frequency. The rate at which the gain of the filter changes in the stopband is determined by the order of the filter. For example, for the first-order low-pass filter the gain rolls off at the rate of 20 dB/decade in the stopband, that is, for $f > f_H$; on the other hand, for the second-order low-pass filter the roll-off rate is 40 dB/decade; and so on. By contrast, for the first-order high-pass filter the gain increases at the rate of 20 dB/decade in the stopband, that is, until $f = f_L$; the increase is 40 dB/decade for the second-order high-pass filter; and so on.

7–3 FIRST-ORDER LOW-PASS BUTTERWORTH FILTER

Figure 7–2 shows a first-order low-pass Butterworth filter that uses an *RC* network for filtering. Note that the op-amp is used in the noninverting configuration; hence it does not load down the *RC* network. Resistors R_1 and R_F determine the gain of the filter.

According to the voltage-divider rule, the voltage at the noninverting terminal (across capacitor C) is

$$v_1 = \frac{-jX_C}{R - jX_C} v_{\text{in}} \tag{7–1a}$$

where

$$j = \sqrt{-1} \quad \text{and} \quad -jX_c = \frac{1}{j2\pi fC}$$

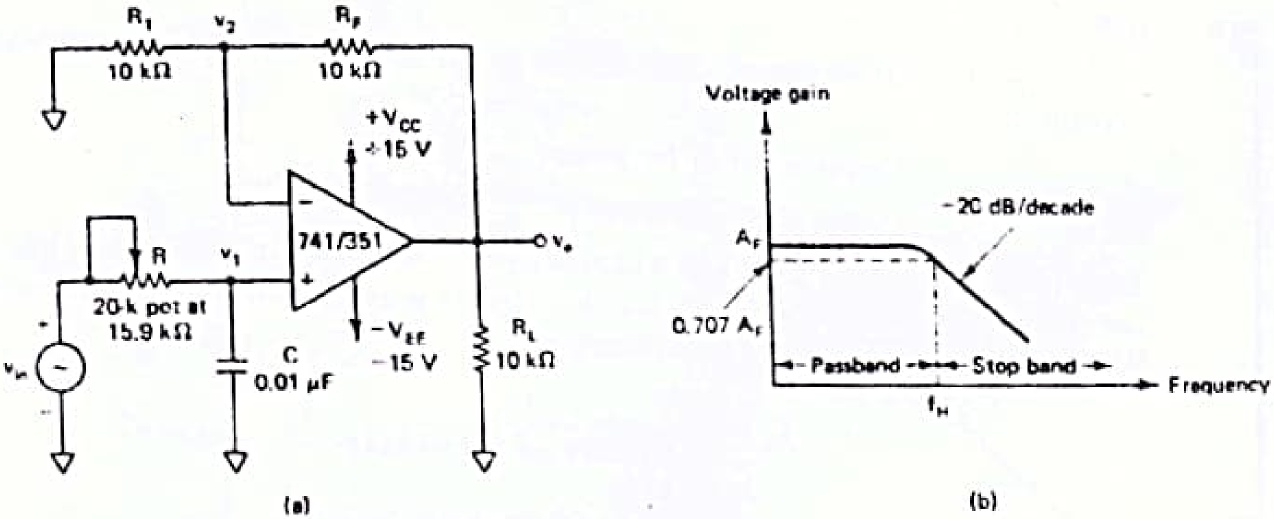


FIGURE 7–2 First-order low-pass Butterworth filter. (a) Circuit. (b) Frequency response.

Simplifying Equation (7–1a), we get

$$v_1 = \frac{v_{in}}{1 + j2\pi fRC}$$

and the output voltage

$$v_o = \left(1 + \frac{R_F}{R_1}\right) v_1$$

That is,

$$v_o = \left(1 + \frac{R_F}{R_1}\right) \frac{v_{in}}{1 + j2\pi fRC}$$

or

$$\frac{v_o}{v_{in}} = \frac{A_F}{1 + j(f/f_H)} \tag{7–1b}$$

where $\frac{v_o}{v_{in}}$ = gain of the filter as a function of frequency

$A_F = 1 + \frac{R_F}{R_1}$ = passband gain of the filter

f = frequency of the input signal

$f_H = \frac{1}{2\pi RC}$ = high cutoff frequency of the filter

The gain magnitude and phase angle equations of the low-pass filter can be obtained by converting Equation (7–1b) into its equivalent polar form, as follows:

$$\left|\frac{v_o}{v_{in}}\right| = \frac{A_F}{\sqrt{1 + (f/f_H)^2}} \qquad (7\text{–}2a)$$

$$\phi = -\tan^{-1}\left(\frac{f}{f_H}\right) \qquad (7\text{–}2b)$$

where ϕ is the phase angle in degrees.

The operation of the low-pass filter can be verified from the gain magnitude equation, (7–2a):

1. At very low frequencies, that is, $f < f_H$,

$$\left|\frac{v_o}{v_{in}}\right| \cong A_F$$

2. At $f = f_H$,

$$\left|\frac{v_o}{v_{in}}\right| = \frac{A_F}{\sqrt{2}} = 0.707A_F$$

3. At $f > f_H$,

$$\left|\frac{v_o}{v_{in}}\right| < A_F$$

Thus the low-pass filter has a constant gain A_F from 0 Hz to the high cutoff frequency f_H. At f_H the gain is $0.707A_F$, and after f_H it decreases at a constant rate with an increase in frequency [see Figure 7–2(b)]. That is, when the frequency is increased tenfold (one decade), the voltage gain is divided by 10. In other words, the gain decreases 20 dB (= 20 log 10) each time the frequency is increased by 10. Hence the rate at which the gain rolls off after f_H is 20 dB/decade or 6 dB/octave, where octave signifies a twofold increase in frequency. The frequency $f = f_H$ is called the *cutoff frequency* because the gain of the filter at this frequency is down by 3 dB (= 20 log 0.707) from 0 Hz. Other equivalent terms for cutoff frequency are *−3 dB frequency, break frequency,* or *corner frequency.*

7–3–1 Filter Design

A low-pass filter can be designed by implementing the following steps:

1. Choose a value of high cutoff frequency f_H.
2. Select a value of C less than or equal to 1 μF. Mylar or tantalum capacitors are recommended for better performance.

3. Calculate the value of R using

$$R = \frac{1}{2\pi f_H C}$$

4. Finally, select values of R_1 and R_F dependent on the desired passband gain A_F using

$$A_F = 1 + \frac{R_F}{R_1}$$

7-3-2 Frequency Scaling

Once a filter is designed, there may sometimes be a need to change its cutoff frequency. The procedure used to convert an original cutoff frequency f_H to a new cutoff frequency f'_H is called *frequency scaling*. Frequency scaling is accomplished as follows. To change a high cutoff frequency, multiply R or C, but not both, by the ratio of the original cutoff frequency to the new cutoff frequency. In filter design the needed values of R and C are often not standard. Besides, a variable capacitor C is not commonly used. Therefore, choose a standard value of capacitor, and then calculate the value of resistor for a desired cutoff frequency. This is because for a nonstandard value of resistor a potentiometer can be used (see Examples 7-1 and 7-2).

EXAMPLE 7-1

Design a low-pass filter at a cutoff frequency of 1 kHz with a passband gain of 2.

SOLUTION

Follow the preceding design steps.

1. $f_H = 1$ kHz.
2. Let $C = 0.01\ \mu$F.
3. Then $R = 1/(2\pi)(10^3)(10^{-8}) = 15.9$ kΩ. (Use a 20-kΩ potentiometer.)
4. Since the passband gain is 2, R_1 and R_F must be equal. Therefore, let $R_1 = R_F = 10$ kΩ. The complete circuit with component values is shown in Figure 7-2(a).

EXAMPLE 7-2

Using the frequency scaling technique, convert the 1-kHz cutoff frequency of the low-pass filter of Example 7-1 to a cutoff frequency of 1.6 kHz.

SOLUTION

To change a cutoff frequency from 1 kHz to 1.6 kHz, we multiply the 15.9 kΩ resistor by

$$\frac{\text{original cutoff frequency}}{\text{new cutoff frequency}} = \frac{1\ \text{kHz}}{1.6\ \text{kHz}} = 0.625$$

Therefore, new resistor $R = (15.9\ \text{k}\Omega)(0.625) = 9.94\ \text{k}\Omega$. However, 9.94 kΩ is not a standard value. Therefore, use $R = 10\ \text{k}\Omega$ potentiometer and adjust it to 9.94 kΩ. Thus the new cutoff frequency is

$$f_H = \frac{1}{(2\pi)(0.01\ \mu\text{F})(9.94\ \text{k}\Omega)} = 1.6\ \text{kHz}$$

EXAMPLE 7–3

Plot the frequency response of the low-pass filter of Example 7–1.

SOLUTION

To plot the frequency response, we have to use Equation (7–2a). The data of Table 7–1 are, therefore, obtained by substituting various values for f in this equation. Equation (7–2a) will be repeated here for convenience:

$$\left|\frac{v_o}{v_{in}}\right| = \frac{A_F}{\sqrt{1 + (f/f_H)^2}}$$

where $A_F = 2$ and $f_H = 1$ kHz. The data of Table 7–1 are plotted as shown in Figure 7–3.

TABLE 7–1 Frequency Response Data for Example 7–3.

Input frequency, f (Hz)	*Gain magnitude, $\lvert v_o/v_{in}\rvert$*	*Magnitude (dB) = 20 log $\lvert v_o/v_{in}\rvert$*
10	2	6.02
100	1.99	5.98
200	1.96	5.85
700	1.64	4.29
1,000	1.41	3.01
3,000	0.63	−3.98
7,000	0.28	−10.97
10,000	0.20	−14.02
30,000	0.07	−23.53
100,000	0.02	−33.98

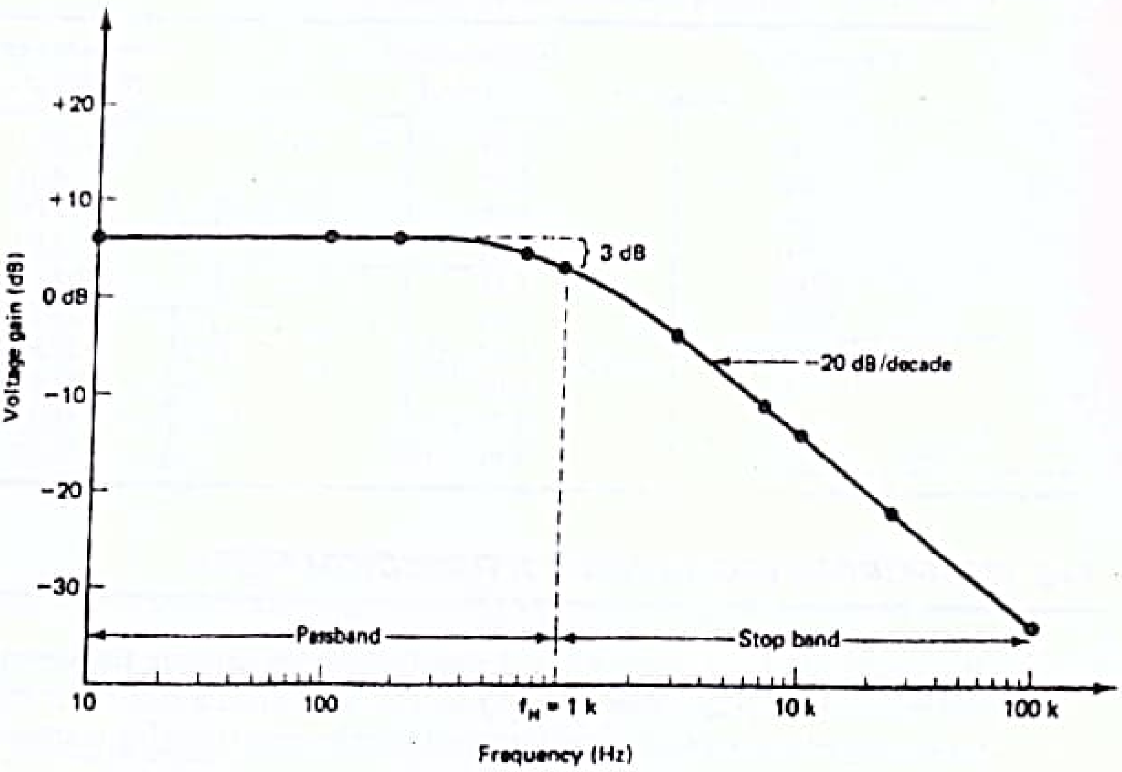


FIGURE 7–3 Frequency response for Example 7–3.

7–4 SECOND-ORDER LOW-PASS BUTTERWORTH FILTER

A stop-band response having a 40-dB/decade roll-off is obtained with the second-order low-pass filter. A first-order low-pass filter can be converted into a second-order type simply by using an additional *RC* network, as shown in Figure 7–4.

Second-order filters are important because higher-order filters can be designed using them. The gain of the second-order filter is set by R_1 and R_F, while the high cutoff frequency f_H is determined by R_2, C_2, R_3, and C_3, as follows:

$$f_H = \frac{1}{2\pi\sqrt{R_2R_3C_2C_3}} \tag{7–3}$$

For the derivation of f_H, refer to Appendix C.

Furthermore, for a second-order low-pass Butterworth response, the voltage gain magnitude equation is

$$\left|\frac{v_o}{v_{in}}\right| = \frac{A_F}{\sqrt{1 + (f/f_H)^4}} \tag{7–4}$$

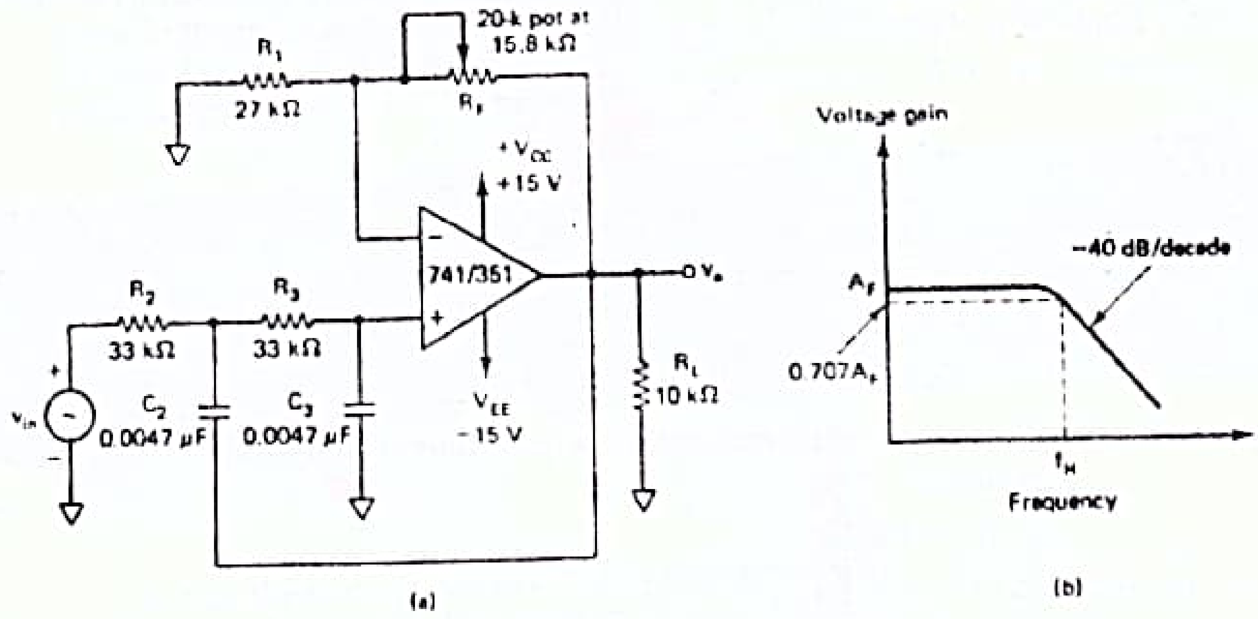


FIGURE 7-4 Second-order low-pass Butterworth filter. (a) Circuit. (b) Frequency response.

where $A_F = 1 + \dfrac{R_F}{R_1}$ = passband gain of the filter

f = frequency of the input signal (Hz)

$f_H = \dfrac{1}{2\pi\sqrt{R_2R_3C_2C_3}}$ = high cutoff frequency (Hz)

7-4-1 Filter Design

Except for having twice the roll-off rate in the stopband, the frequency response of the second-order low-pass filter is identical to that of the first-order type. Therefore, the design steps of the second-order filter are identical to those of the first-order filter, as follows:

1. Choose a value for the high cutoff frequency f_H.
2. To simplify the design calculations, set $R_2 = R_3 = R$ and $C_2 = C_3 = C$. Then choose a value of $C \leq 1\ \mu F$.
3. Calculate the value of R using Equation (7-3):

$$R = \frac{1}{2\pi f_H C}$$

4. Finally, because of the equal resistor ($R_2 = R_3$) and capacitor ($C_2 = C_3$) values, the passband voltage gain $A_F = (1 + R_F/R_1)$ of the second-order low-pass filter has to be equal to 1.586. That is, $R_F = 0.586R_1$. This gain is necessary to guarantee Butterworth response. Hence choose a value of $R_1 \leq 100\ k\Omega$ and calculate the value of R_F.

As outlined in Section 7–3–2, the frequency scaling method of the first-order filter is also applicable to the second-order low-pass filter.

EXAMPLE 7–4

a. Design a second-order low-pass filter at a high cutoff frequency of 1 kHz.
b. Draw the frequency response of the network in part (a).

SOLUTION

(a) To design the second-order low-pass filter, simply follow the steps just presented:

1. $f_H = 1$ kHz.
2. Let $C_2 = C_3 = 0.0047\ \mu F$.
3. Then

$$R_2 = R_3 = \frac{1}{(2\pi)(10^3)(47)(10^{-10})} = 33.86\ \text{k}\Omega$$

(Use $R_2 = R_3 = 33\ \text{k}\Omega$.)

4. Since R_F must be equal to $0.586R_1$, let R_1 equal 27 kΩ. Therefore,

$$R_F = (0.586)(27\ \text{k}\Omega) = 15.82\ \text{k}\Omega$$

(Use $R_F = 20\ \text{k}\Omega$ pot.) Thus the required components are

$$R_2 = R_3 = 33\ \text{k}\Omega$$

$$C_2 = C_3 = 0.0047\ \mu F$$

$$R_1 = 27\ \text{k}\Omega \quad \text{and} \quad R_F = 15.8\ \text{k}\Omega\ (20\ \text{k}\Omega\ \text{pot})$$

Another method to design the second-order low-pass filter is to use the same values of resistor and capacitor obtained for the first-order filter in Example 7–1. This is because the cutoff frequency of both the second-order and first-order filters is 1 kHz. Therefore, we may use $R_2 = R_3 = 15.9\ \text{k}\Omega$ and $C_2 = C_3 = 0.01\ \mu F$ However, the values of R_1 and R_F must be chosen such that $R_F = 0.586R_1$. Therefore, use $R_1 = 27\ \text{k}\Omega$ and $R_F = 15.8\ \text{k}\Omega$.

(b) The frequency response data shown in Table 7–2 are obtained from the magnitude Equation, (7–4), by substituting various values from 10 Hz to 100 kHz for f. Equation (7–4) is repeated here for convenience:

$$\left|\frac{v_o}{v_{in}}\right| = \frac{A_F}{\sqrt{1 + (f/f_H)^4}}$$

where $A_F = 1.586$ and $f_H = 1$ kHz. The frequency response of the second-order low-pass filter of Example 7–4 is shown in Figure 7–5.

TABLE 7-2 Frequency Response Data for Example 7-4.

Frequency, f (Hz)	Gain magnitude, $\lvert v_o/v_{in}\rvert$	Magnitude (dB) = $20 \log \lvert v_o/v_{in}\rvert$
10	1.59	4.01
100	1.59	4.01
200	1.58	4.00
700	1.42	3.07
1,000	1.12	1.00
3,000	0.18	−15.13
7,000	0.03	−29.80
10,000	0.02	−35.99
30,000	1.76×10^{-3}	−55.08
100,000	1.59×10^{-4}	−75.99

7-5 FIRST-ORDER HIGH-PASS BUTTERWORTH FILTER

High-pass filters are often formed simply by interchanging frequency-determining resistors and capacitors in low-pass filters. That is, a first-order high-pass filter is formed from a first-order low-pass type by interchanging components R and

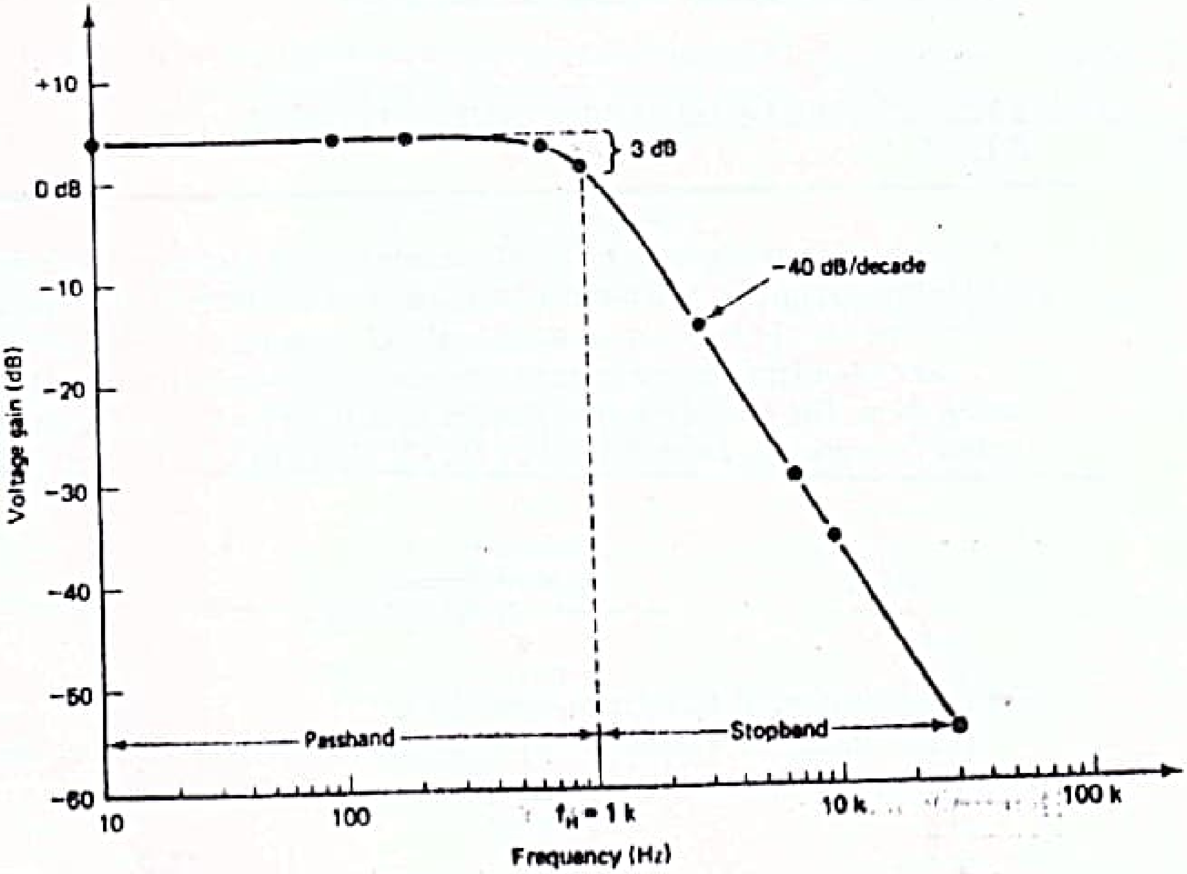


FIGURE 7-5 Frequency response for Example 7-4.

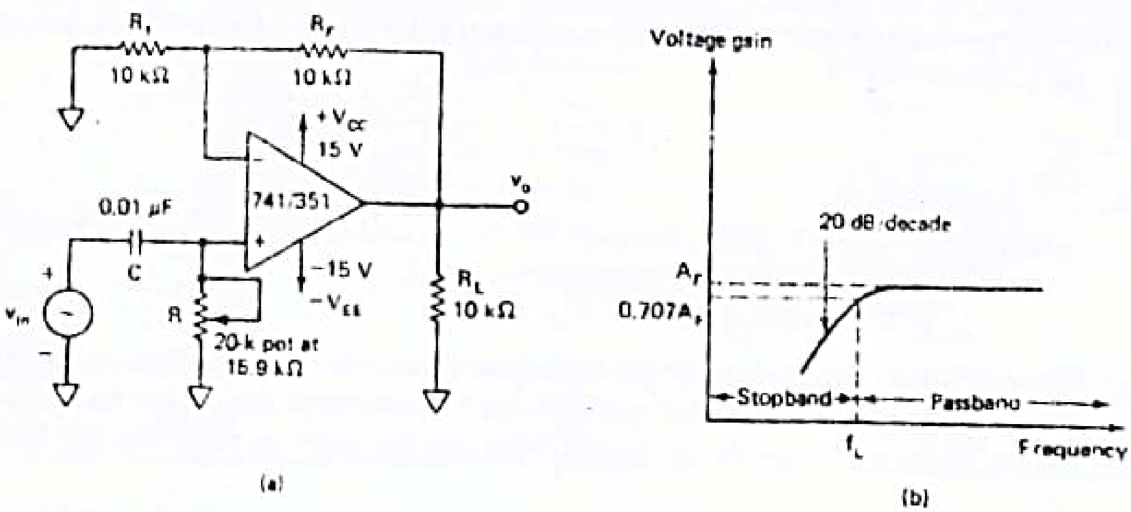


FIGURE 7–6 (a) First-order high-pass Butterworth filter. (b) Its frequency response.

C. Similarly, a second-order high-pass filter is obtained from a second-order low-pass filter if R and C are interchanged, and so on. Figure 7–6 shows a first-order high-pass Butterworth filter with a low cutoff frequency of f_L. This is the frequency at which the magnitude of the gain is 0.707 times its passband value. Obviously, all frequencies higher than f_L are passband frequencies, with the highest frequency determined by the closed-loop bandwidth of the op-amp.

Note that the high-pass filter of Figure 7–6(a) and the low-pass filter of Figure 7–2(a) are the same circuits, except that the frequency-determining components (R and C) are interchanged.

For the first-order high-pass filter of Figure 7–6(a), the output voltage is

$$v_o = \left(1 + \frac{R_F}{R_1}\right)\frac{j2\pi fRC}{1 + j2\pi fRC}v_{in}$$

or

$$\frac{v_o}{v_{in}} = A_F\left[\frac{j(f/f_L)}{1 + j(f/f_L)}\right] \tag{7–5}$$

where $A_F = 1 + \dfrac{R_F}{R_1}$ = passband gain of the filter

f = frequency of the input signal (Hz)

$f_L = \dfrac{1}{2\pi RC}$ = low cutoff frequency (Hz)

Hence the magnitude of the voltage gain is

$$\left|\frac{v_o}{v_{in}}\right| = \frac{A_F(f/f_L)}{\sqrt{1 + (f/f_L)^2}} \tag{7–6}$$

Since high-pass filters are formed from low-pass filters simply by interchanging R's and C's, the design and frequency scaling procedures of the low-pass filters are also applicable to the high-pass filters (see Sections 7–3–1 and 7–3–2).

EXAMPLE 7-5

a. Design a high-pass filter at a cutoff frequency of 1 kHz with a passband gain of 2.

b. Plot the frequency response of the filter in part (a).

SOLUTION

a. Use the same values of R and C that were determined for the low-pass filter of Example 7-1, since $f_L = f_H = 1$ kHz. That is, $C = 0.01\ \mu$F and $R = 15.9$ kΩ. Similarly, use $R_1 = R_f = 10$ kΩ, since $A_F = 2$.

b. The data for the frequency response plot can be obtained by substituting for the input frequency f values from 100 Hz to 100 kHz in Equation (7-6). These data are included in Table 7–3. Equation (7–6) is repeated here for convenience:

$$\left|\frac{v_o}{v_{in}}\right| = \frac{A_F(f/f_L)}{\sqrt{1 + (f/f_L)^2}}$$

where $A_F = 2$ and $f_L = 1$ kHz. The frequency response data of Table 7–3 are plotted in Figure 7–7. In the stopband (from 100 Hz to 1 kHz) the gain increases at the rate of 20 dB/decade. However, in the passband (after $f = f_L = 1$ kHz) the gain remains constant at 6.02 dB. Moreover, the upper-frequency limit of the passband is set by the closed-loop bandwidth of the op-amp.

TABLE 7–3 Frequency Response Data for the First-Order High-Pass Filter of Example 7–5.

Frequency, f (Hz)	*Gain magnitude,* $\lvert v_o/v_{in}\rvert$	*Magnitude (dB)* = $20 \log\lvert v_o/v_{in}\rvert$
100	0.20	−14.02
200	0.39	−8.13
400	0.74	−2.58
700	1.15	1.19
1,000	1.41	3.01
3,000	1.90	5.56
7,000	1.98	5.93
10,000	1.99	5.98
30,000	2	6.02
100,000	2	6.02

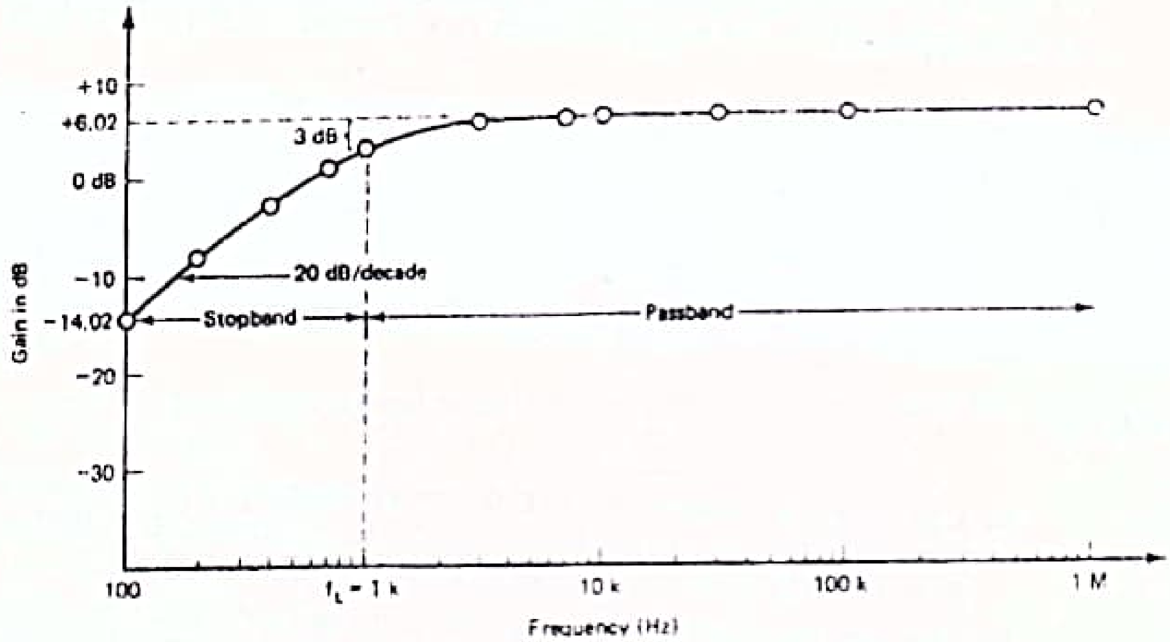


FIGURE 7–7 Frequency response for Example 7–5.

7–6 SECOND-ORDER HIGH-PASS BUTTERWORTH FILTER

As in the case of the first-order filter, a second-order high-pass filter can be formed from a second-order low-pass filter simply by interchanging the frequency-determining resistors and capacitors. Figure 7–8(a) shows the second-order high-pass filter.

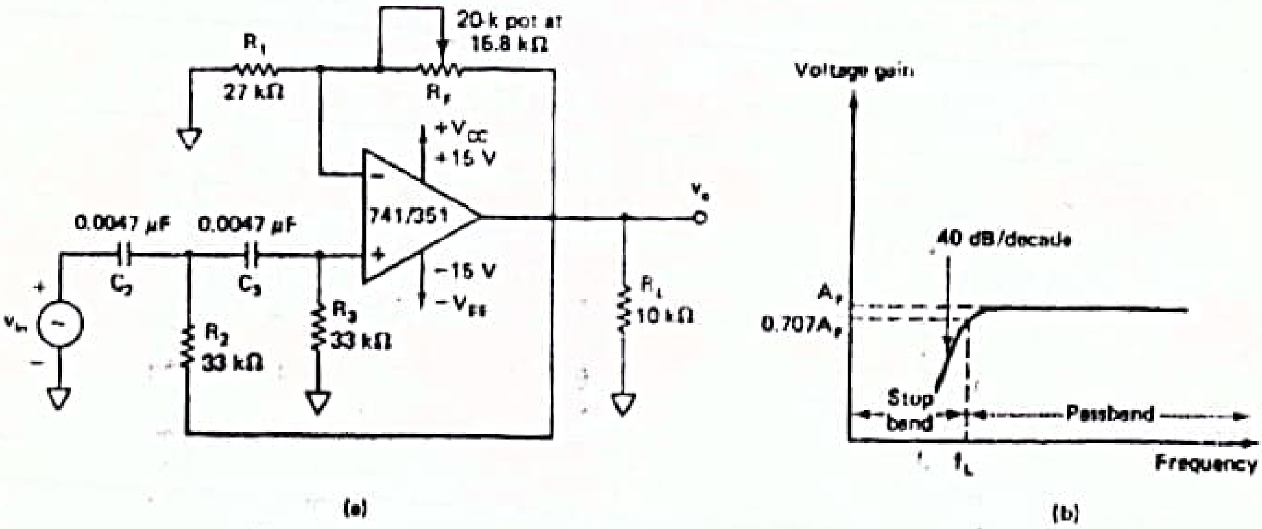


FIGURE 7–8 (a) Second-order high-pass Butterworth filter. (b) Its frequency response.

The voltage gain magnitude equation of the second-order high-pass filter is as follows:

$$\left|\frac{v_o}{v_{in}}\right| = \frac{A_F}{\sqrt{1 + (f_L/f)^4}} \tag{7-7}$$

where A_F = 1.586 = passband gain for the second-order Butterworth response
f = frequency of the input signal (Hz)
f_L = low cutoff frequency (Hz)

Since second-order low-pass and high-pass filters are the same circuits except that the positions of resistors and capacitors are interchanged, the design and frequency scaling procedures for the high-pass filter are the same as those for the low-pass filter.

EXAMPLE 7-6

a. Determine the low cutoff frequency f_L of the filter shown in Figure 7-8(a).
b. Draw the frequency response plot of the filter.

SOLUTION

a.

$$f_L = \frac{1}{2\pi\sqrt{R_2R_3C_2C_3}}$$

$$= \frac{1}{2\pi\sqrt{(33\text{ k}\Omega)^2(0.0047\ \mu\text{F})^2}} = 1\text{ kHz}$$

b. The frequency response data in Table 7-4 are obtained from the voltage gain magnitude equation, (7-7), which is repeated here for convenience.

$$\left|\frac{v_o}{v_{in}}\right| = \frac{A_F}{\sqrt{1 + (f_L/f)^4}}$$

where A_F = 1.586 and f_L = 1 kHz. The resulting frequency response plot is shown in Figure 7-9.

7-7 HIGHER-ORDER FILTERS

From the preceding discussions of filters we can conclude that in the stopband the gain of the filter changes at the rate of 20 dB/decade for first-order filters and at 40 dB/decade for second-order filters. This means that, as the order of the filter is increased, the actual stopband response of the filter approaches its ideal stopband characteristic.

TABLE 7–4 Frequency Response Data for Second-Order High-Pass Filter of Example 7–6

Input frequency (Hz)	Gain magnitude $\lvert v_o/v_{in}\rvert$	Magnitude (dB) = $20\log\lvert v_o/v_{in}\rvert$
100	0.01586	−35.99
200	0.0634	−23.96
700	0.6979	−3.124
1,000	1.1215	0.9966
3,000	1.5763	3.953
7,000	1.5857	4.004
10,000	1.5859	4.006
30,000	1.5860	4.006
100,000	1.5860	4.006

Higher-order filters, such as third, fourth, fifth, and so on, are formed simply by using the first- and second-order filters. For example, a third-order low-pass filter is formed by connecting in series or cascading first- and second-order low-pass filters; a fourth-order low-pass filter is composed of two cascaded second-order low-pass sections, and so on. Although there is no limit to the order of the filter that can be formed, as the order of the filter increases, so does its size. Also, its accuracy declines, in that the difference between the actual stopband response and the theoretical stopband response increases with an increase in the order of the filter. Figure 7–10 shows third- and fourth-order low-pass Butterworth filters. Note that in the third-order filter the voltage gain of the first-order section is *one*, and that of the second-order section is *two*. On the other hand, in the fourth-

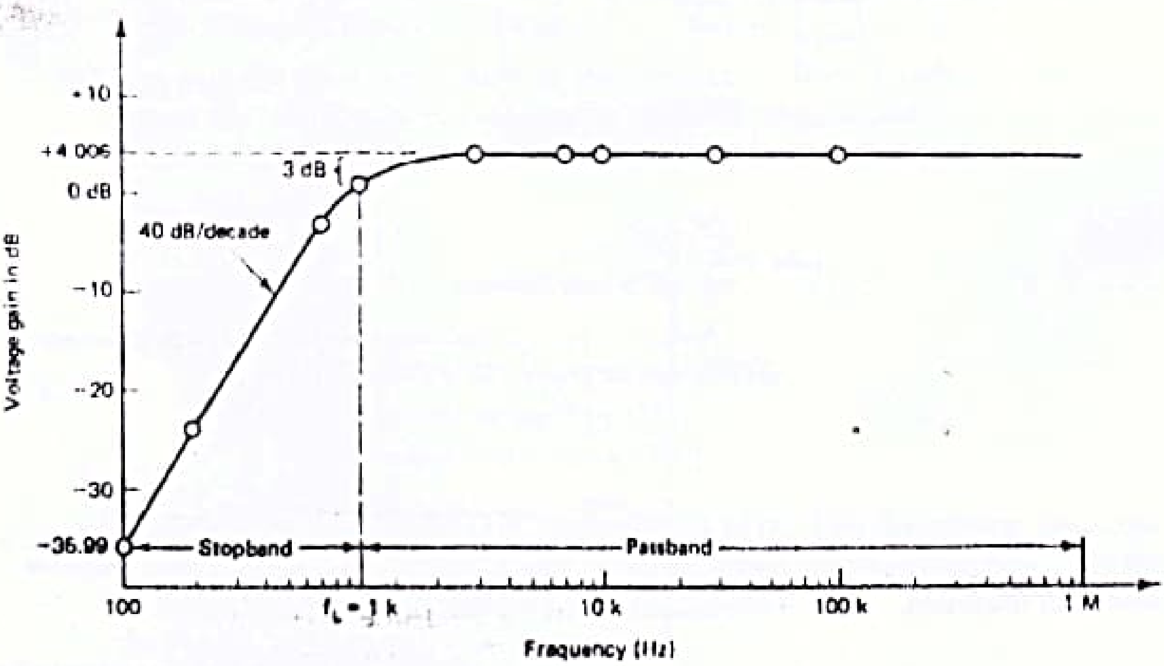


FIGURE 7–9 Frequency response for Example 7–6.

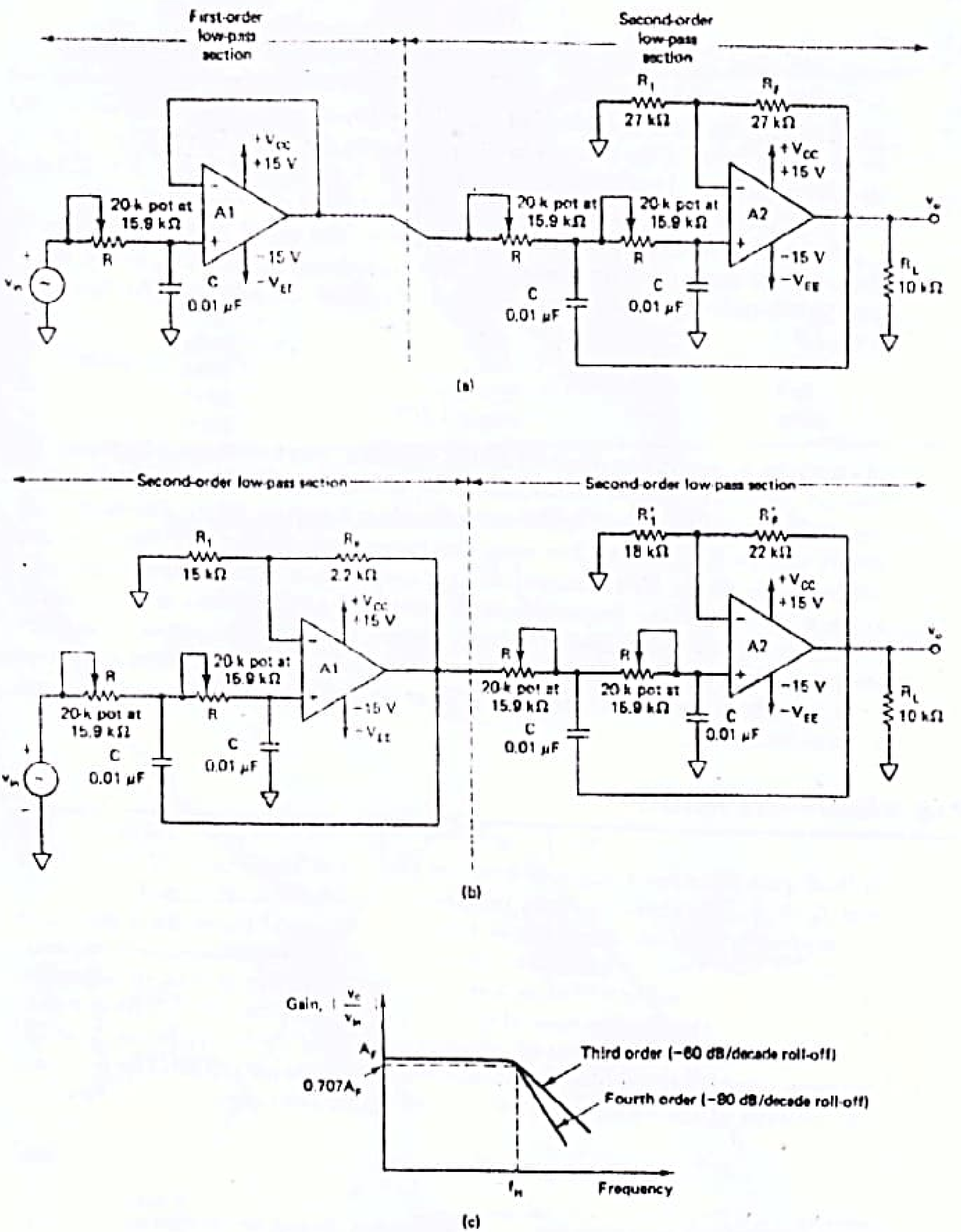


FIGURE 7–10 (a) Third-order and (b) fourth-order low-pass Butterworth filters. (c) Their frequency responses. A_1 and A_2 dual op-amp: 1458/353.

order filter the gain of the first section is 1.152, while that of the second section is 2.235. These gain values are necessary to guarantee Butterworth response and must remain the same regardless of the filter's cutoff frequency. Furthermore, the overall gain of the filter is equal to the product of the individual voltage gains of the filter sections. Thus the overall gain of the third-order filters is 2.0, and that of the fourth order filters is (1.152)(2.235) = 2.57.

Since the frequency-determining resistors are equal and the frequency-determining capacitors are also equal, the high cutoff frequencies of the third and fourth-order low-pass filters in Figure 7–10(a) and (b) must also be equal. That is,

$$f_H = \frac{1}{2\pi RC} \tag{7–8}$$

As with the first- and second-order filters, the third- and fourth-order high-pass filters are formed by simply interchanging the positions of the frequency-determining resistors and capacitors in the corresponding low-pass filters. The high-order filters can be designed by following the procedures outlined for the first- and second-order filters. However, note that the overall gain of the higher-order filters is *fixed* because all the frequency-determining resistors and capacitors are equal.

Generally, the minimum-order filter required depends on the application specifications. Although a higher-order filter than necessary gives a better stop-band response, the higher-order type filter is more complex, occupies more space, and is more expensive.

7–8 BAND-PASS FILTERS

A band-pass filter has a passband between two cutoff frequencies f_H and f_L such that $f_H > f_L$. Any input frequency outside this passband is attenuated.

Basically, there are two types of band-pass filters: (1) wide band pass, and (2) narrow band pass. Unfortunately, there is no set dividing line between the two. However, we will define a filter as wide band pass if its *figure of merit* or *quality factor* $Q < 10$. On the other hand, if $Q > 10$, we will call the filter a narrow band-pass filter. Thus Q is a measure of selectivity, meaning the higher the value of Q, the more selective is the filter or the narrower its bandwidth (BW). The relationship between Q, the 3-dB bandwidth, and the center frequency f_c is given by

$$Q = \frac{f_C}{\mathrm{BW}} = \frac{f_C}{f_H - f_L} \tag{7–9a}$$

For the wide band-pass filter the center frequency f_C can be defined as

$$f_C = \sqrt{f_H f_L} \tag{7–9b}$$

where f_H = high cutoff frequency (Hz)

f_L = low cutoff frequency of the wide band-pass filter (Hz)

In a narrow band-pass filter, the output voltage peaks at the center frequency.

7-8-1 Wide Band-Pass Filter

A wide band-pass filter can be formed by simply cascading high-pass and low-pass sections and is generally the choice for simplicity of design and performance. To obtain a ± 20 dB/decade band-pass, first-order high-pass and first order low-pass sections are cascaded; for a ± 40-dB/decade band-pass filter, second-order high-pass and second-order low-pass sections are connected in series, and so on. In other words, the order of the band-pass filter depends on the order of the high-pass and low-pass filter sections.

Figure 7–11 shows the ± 20-dB/decade wide band-pass filter, which is composed of first-order high-pass and first-order low-pass filters. To realize a band-pass response, however, f_H must be larger than f_L, as illustrated in Example 7–7.

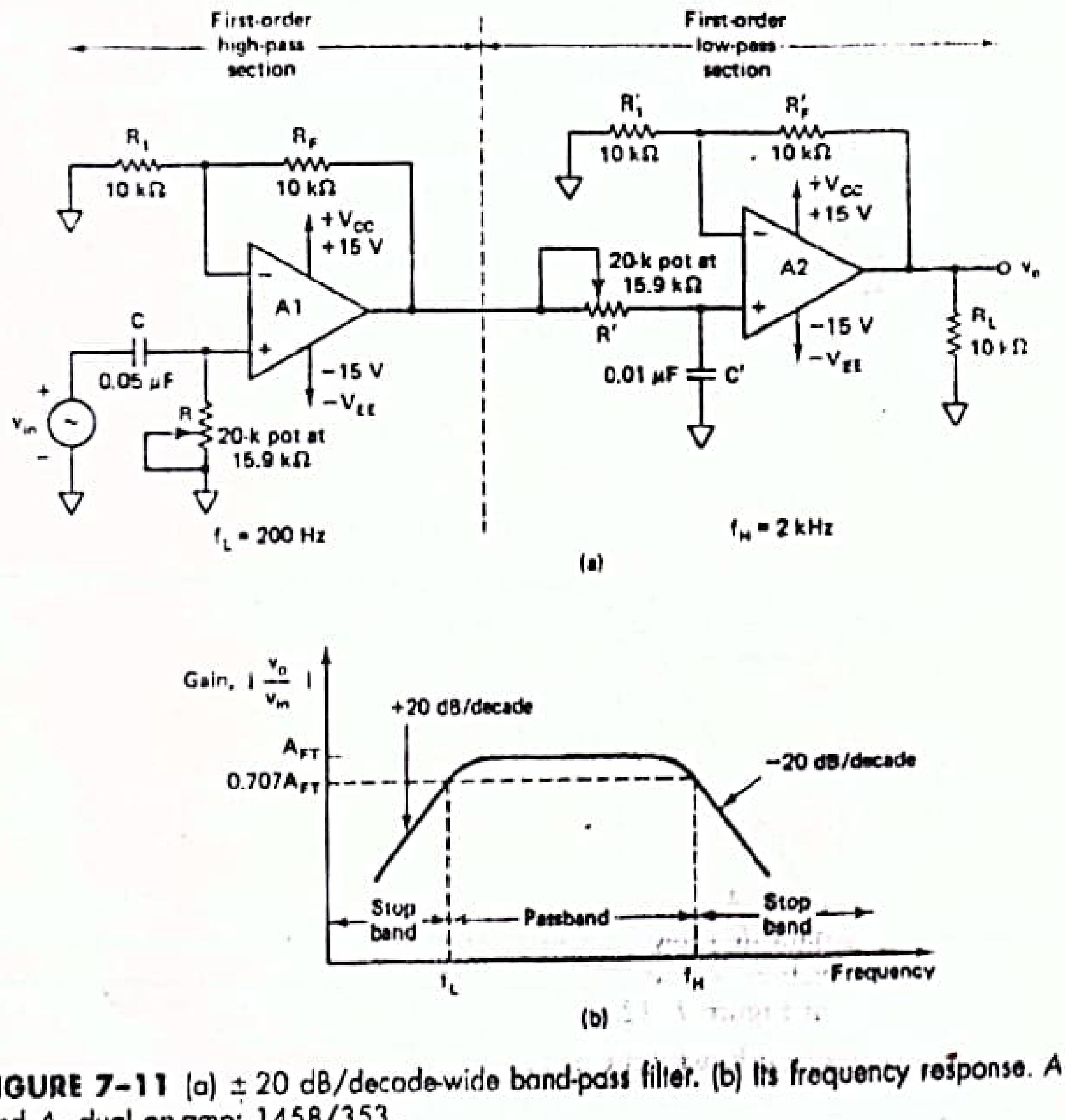


FIGURE 7–11 (a) ± 20 dB/decade-wide band-pass filter. (b) Its frequency response. A_1 and A_2 dual op-amp: 1458/353.

EXAMPLE 7-7

a. Design a wide band-pass filter with $f_L = 200$ Hz, $f_H = 1$ kHz, and a passband gain = 4.

b. Draw the frequency response plot of this filter.

c. Calculate the value of Q for the filter.

SOLUTION

(a) A low-pass filter with $f_H = 1$ kHz was designed in Example 7-1; therefore, the same values of resistors and capacitors can be used here, that is, $R' = 15.9$ kΩ and $C' = 0.01$ μF. As in the case of the high-pass filter, it can be designed by following the steps of section 7-3-1:

1. $f_L = 200$ Hz.
2. Let $C = 0.05$ μF.
3. Then

$$R = \frac{1}{2\pi f_L C} = \frac{1}{(2\pi)(200)(5)(10^{-8})}$$
$$= 15.9 \text{ k}\Omega$$

Since the band-pass gain is 4, the gain of the high-pass as well as low-pass sections could be set equal to 2. That is, input and feedback resistors must be equal in value, say 10 kΩ each. The complete band-pass filter is shown in Figure 7-11(a).

(b) The voltage gain magnitude of the band-pass filter is equal to the product of the voltage gain magnitudes of the high-pass and low-pass filters. Therefore, from Equations (7-2a) and (7-6),

$$\left|\frac{v_o}{v_{in}}\right| = \frac{A_{FT}(f/f_L)}{\sqrt{[1 + (f/f_L)^2][1 + (f/f_H)^2]}} \qquad (7\text{-}10)$$

where A_{FT} = total passband gain
f = frequency of the input signal (Hz)
f_L = low cutoff frequency (Hz)
f_H = high cutoff frequency (Hz)

Here $A_{FT} = 4$, $f_L = 200$ Hz, and $f_H = 1$ kHz. The frequency response data in Table 7-5 are obtained by substituting into Equation (7-10) the values of f from 10 Hz to 10 kHz. The frequency response plot is shown in Figure 7-12.

(c) From Equation (7-9b),

$$f_C = \sqrt{(1000)(200)} = 447.2 \text{ Hz}$$

TABLE 7–5 Frequency Response Data for the Band-Pass Filter of Example 7–7.

Input frequency, f (Hz)	*Gain magnitude,* $\lvert v_o/v_{in}\rvert$	*Magnitude* (dB) = $20\log\lvert v_o/v_{in}\rvert$
10	0.1997	−13.99
30	0.5931	−4.54
100	1.780	5.01
200	2.774	8.861
447.2	3.33	10.46
700	3.151	9.969
1,000	2.774	8.861
2,000	1.780	5.001
7,000	0.5655	−4.95
10,000	0.3979	−8.004

Substituting this value in Equation (7–9a),

$$Q = \frac{447.2}{100 - 200} = 0.56$$

Thus Q is less than 10, as expected for the wide band-pass filter.

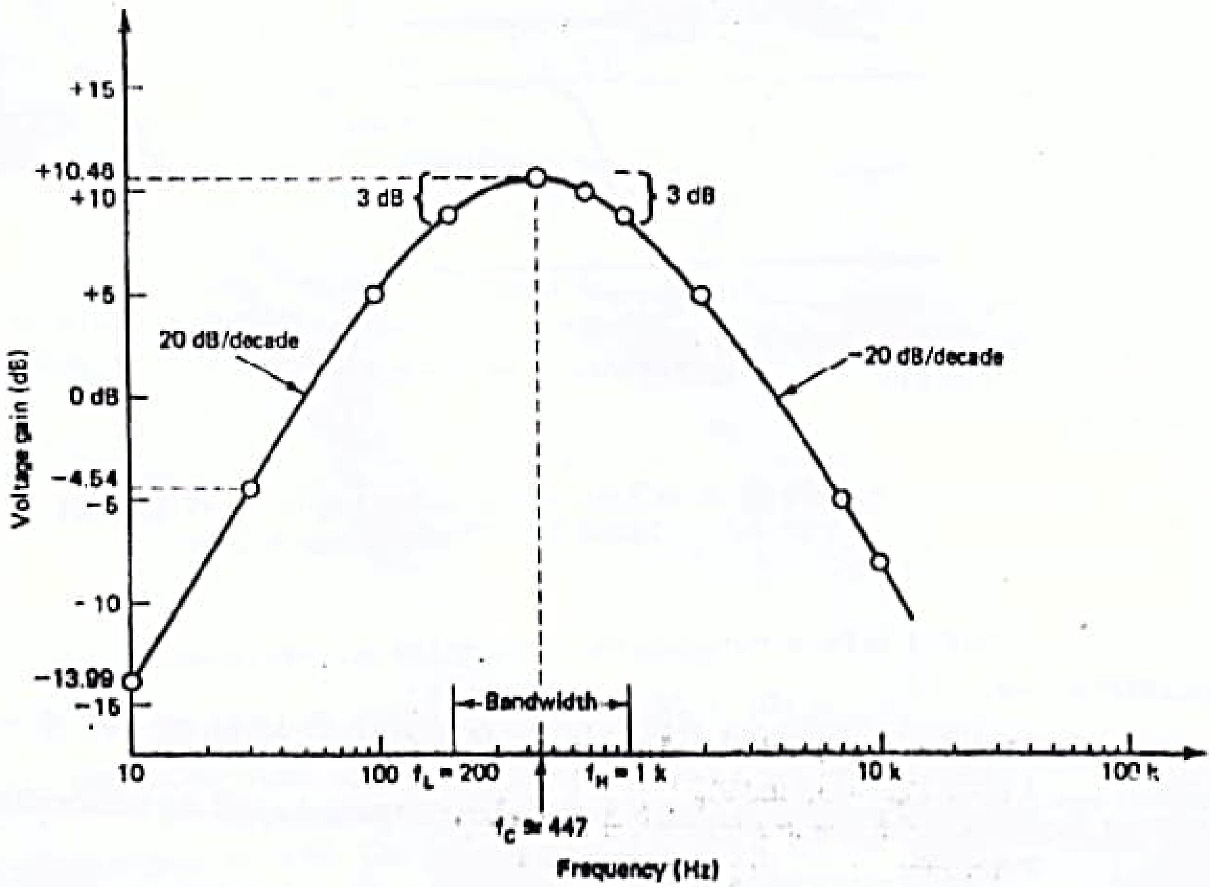


FIGURE 7–12 Frequency response for Example 7–7.

7-8-2 Narrow Band-Pass Filter

The narrow band-pass filter using multiple feedback is shown in Figure 7-13. As shown in this figure, the filter uses only one op-amp. Compared to all the filters discussed so far, this filter is unique in the following respects:

1. It has two feedback paths, hence the name *multiple-feedback filter*.
2. The op-amp is used in the *inverting* mode.

Generally, the narrow band-pass filter is designed for specific values of center frequency f_C and Q or f_C and bandwidth [see Equation (7-9a)]. The circuit components are determined from the following relationships.

To simplify the design calculations, choose $C_1 = C_2 = C$.

$$R_1 = \frac{Q}{2\pi f_C C A_F} \tag{7-11}$$

$$R_2 = \frac{Q}{2\pi f_C C(2Q^2 - A_F)} \tag{7-12}$$

$$R_3 = \frac{Q}{\pi f_C C} \tag{7-13}$$

where A_F is the gain at f_C, given by

$$A_F = \frac{R_3}{2R_1} \tag{7-14a}$$

The gain A_F, however, must satisfy the condition

$$A_F < 2Q^2 \tag{7-14b}$$

Another advantage of the multiple feedback filter of Figure 7-13 is that its center frequency f_C can be changed to a new frequency f_C' without changing the gain or bandwidth. This is accomplished simply by changing R_2 to R_2' so that

$$R_2' = R_2\left(\frac{f_C}{f_C'}\right)^2 \tag{7-15}$$

(see Example 7-8).

EXAMPLE 7-8

a. Design the bandpass filter shown in Figure 7-13(a) so that f_C = 1 kHz, $Q = 3$, and $A_F = 10$.

b. Change the center frequency to 1.5 kHz, keeping A_F and the bandwidth constant.

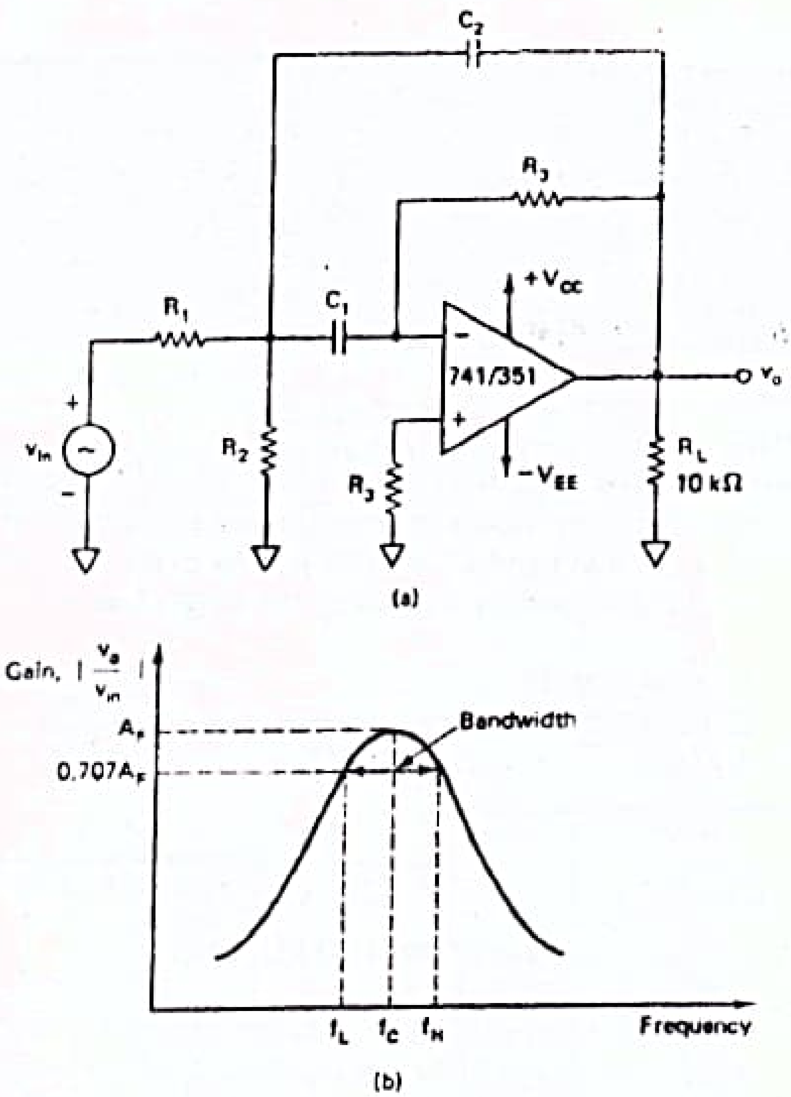


FIGURE 7–13 (a) Multiple-feedback narrow band-pass filter. (b) Its frequency response

SOLUTION

a. Choose the values of C_1 and C_2 first and then calculate the values of R_1, R_2, and R_3 from Equations (7–11) through (7–13). Let $C_1 = C_2 = C = 0.01\ \mu F$.

$$R_1 = \frac{3}{(2\pi)(10^3)(10^{-8})(10)} = 4.77\ k\Omega$$

$$R_2 = \frac{3}{(2\pi)(10^3)(10^{-8})[2(3)^2 - 10]} = 5.97\ k\Omega$$

$$R_3 = \frac{3}{(\pi)(10^3)(10^{-8})} = 95.5\ k\Omega$$

Use $R_1 = 4.7\ k\Omega$, $R_2 = 6.2\ k\Omega$, and $R_3 = 100\ k\Omega$.

b. Using Equation (7–15), the value of R_2' required to change the center frequency from 1 kHz to 1.5 kHz is

$$R_2' = (5.97\ \text{k}\Omega)\left(\frac{1(10^3)}{1.5(10^3)}\right)^2 = 2.65\ \text{k}\Omega$$

(Use $R_2' = 2.7\ \text{k}\Omega$.)

7–9 BAND-REJECT FILTERS

The band-reject filter is also called a *band-stop* or *band-elimination* filter. In this filter, frequencies are attenuated in the stopband while they are passed outside this band, as shown in Figure 7–1(d). As with band-pass filters, the band-reject filters can also be classified as (1) wide band-reject or (2) narrow band-reject. The narrow band-reject filter is uncommonly called the *notch filter*. Because of its higher Q (>10), the bandwidth of the narrow band-reject filter is much smaller than that of the wide band-reject filter.

7–9–1 Wide Band-Reject Filter

Figure 7–14(a) shows a wide band-reject filter using a low-pass filter, a high-pass filter, and a summing amplifier. To realize a band-reject response, the low cutoff frequency f_L of the high-pass filter must be larger than the high cutoff frequency f_H of the low-pass filter. In addition, the passband gain of both the high-pass and low-pass sections must be equal (see Example 7–9). The frequency response of the wide band-reject filter is shown in Figure 7–14(b).

EXAMPLE 7–9

Design a wide band-reject filter having $f_H = 200$ Hz and $f_L = 1$ kHz.

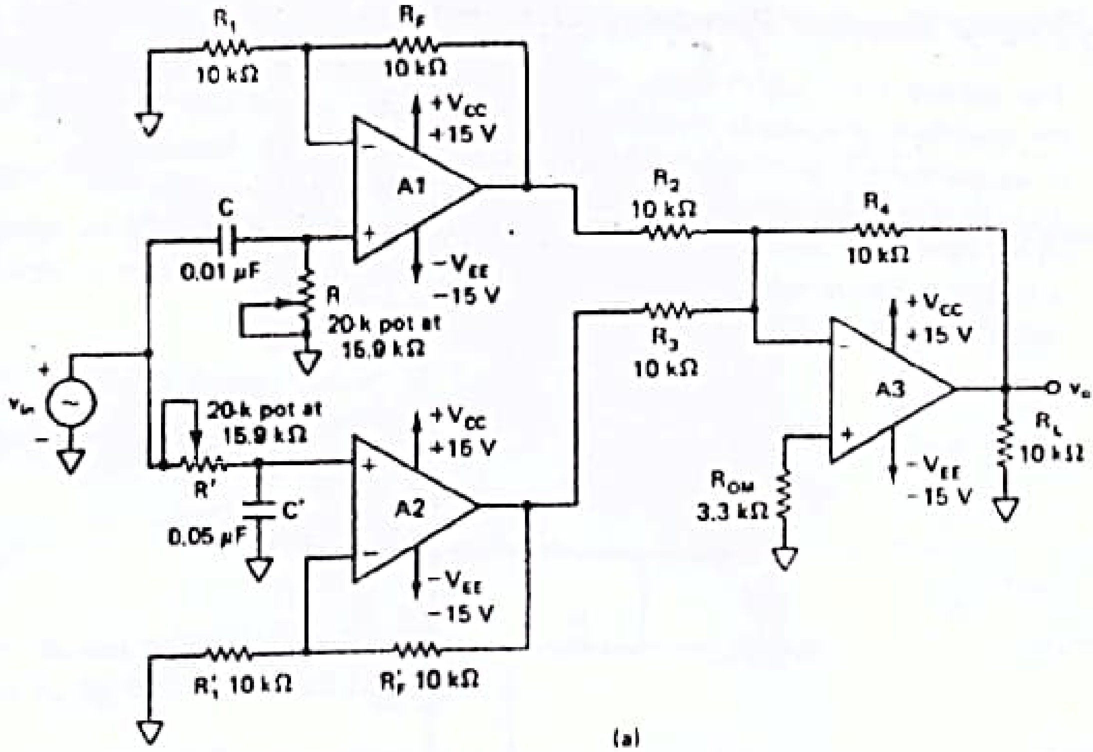
SOLUTION

In Example 7–7, a wide band-pass filter was designed with $f_L = 200$ Hz and $f_H = 1$ kHz. In this example these band frequencies are interchanged, that is, $f_L = 1$ kHz and $f_H = 200$ Hz. This means that we can use the same components as in Example 7–7, but interchanged between high-pass and low-pass sections. Therefore, for the low-pass section, $R' = 15.9\ \text{k}\Omega$ and $C' = 0.05\ \mu\text{F}$, while for the high-pass section

$$R = 15.9\ \text{k}\Omega \text{ and } C = 0.01\ \mu\text{F}$$

Since there is no restriction on the passband gain, use a gain of 2 for each section. Hence let

$$R_1 = R_F = R_1' = R_F' = 10\ \text{k}\Omega$$



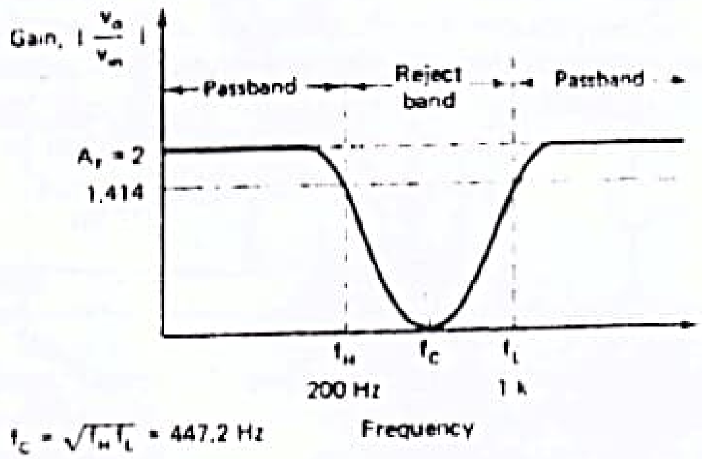


FIGURE 7–14 Wide band-reject filter. (a) Circuit. (b) Frequency response. For A_1, A_2, and A_3 use quad op-amp μAF774/MC34004.

Furthermore, the gain of the summing amplifier is set at 1; therefore,

$$R_2 = R_3 = R_4 = 10\text{ k}\Omega$$

Finally, the value of $R_{OM} = R_2 \| R_3 \| R_4 \cong 3.3\text{ k}\Omega$.

The complete circuit is shown in Figure 7–14(a), and its response is shown in Figure 7–14(b). The voltage gain changes at the rate of 20 dB/decade above f_H and below f_L, with a maximum attenuation occurring at f_C.

7-9-2 Narrow Band-Reject Filter

The narrow band-reject filter, often called the *notch filter*, is commonly used for the rejection of a single frequency such as 60-Hz power line frequency hum. The most commonly used notch filter is the *twin-T* network shown in Figure 7-15(a). This is a *passive filter* composed of two T-shaped networks. One T network is made up of two resistors and a capacitor, while the other uses two capacitors and a resistor. The *notch-out* frequency is the frequency at which maximum attenuation occurs; it is given by

$$f_N = \frac{1}{2\pi RC} \tag{7-16}$$

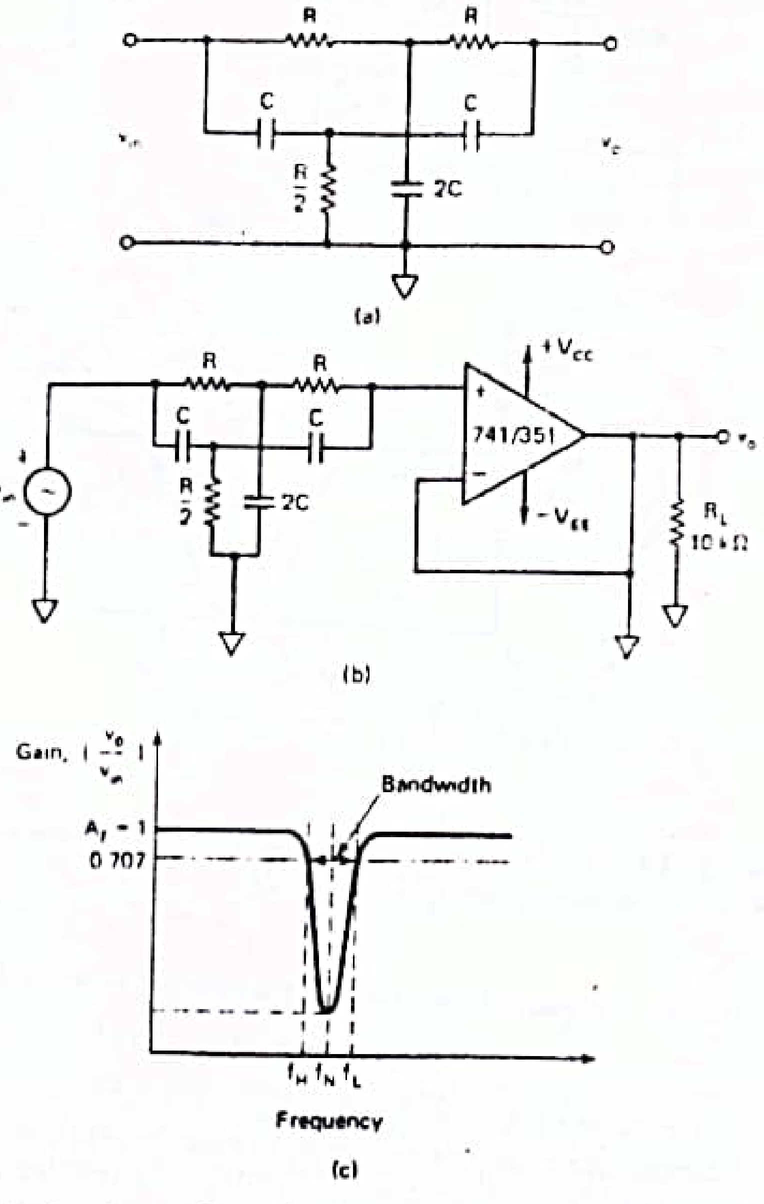


FIGURE 7-15 (a) Twin-T notch filter. (b) Active notch filter. (c) Frequency response of the active notch filter.

Unfortunately, the passive twin-T network has a relatively low figure of merit Q. The Q of the network can be increased significantly if it is used with the voltage follower as shown in Figure 7-15(b). The frequency response of the active notch filter of Figure 7-15(b) is shown in Figure 7-15(c). The most common use of notch filters is in communications and biomedical instruments for eliminating undesired frequencies. To design an active notch filter for a specific notch-out frequency f_N, choose the value of $C \leq 1\ \mu F$ and then calculate the required value of R from Equation (7-16). For the best response, the circuit components should be very close to their indicated values.

EXAMPLE 7-10

Design a 60-Hz active notch filter.

SOLUTION

Let $C = 0.068\ \mu F$. Then, from Equation (7-16), the value of R is

$$R = \frac{1}{2\pi f_N C} = \frac{1}{(2\pi)(60)(68)(10^{-9})} = 39.01\ k\Omega$$

(Use 39 kΩ.) For $R/2$, parallel two 39-kΩ resistors; for the $2C$ component, parallel two 0.068-μF capacitors.

7-10 ALL-PASS FILTER

As the name suggests, an all-pass filter passes all frequency components of the input signal without attenuation, while providing predictable phase shifts for different frequencies of the input signal. When signals are transmitted over transmission lines, such as telephone wires, they undergo change in phase. To compensate for these phase changes, all-pass filters are required. The all-pass filters are also called *delay equalizers* or *phase correctors*. Figure 7-16(a) shows an all-pass filter wherein $R_f = R_1$. The output voltage v_o of the filter can be obtained by using the superposition theorem:

$$v_o = -v_{in} + \frac{-jX_C}{R - jX_C} v_{in}(2) \tag{7-17}$$

But $-j = 1/j$ and $X_C = 1/2\pi fC$. Therefore, substituting for X_C and simplifying, we get

$$v_o = v_{in}\left(-1 + \frac{2}{j2\pi fRC + 1}\right)$$

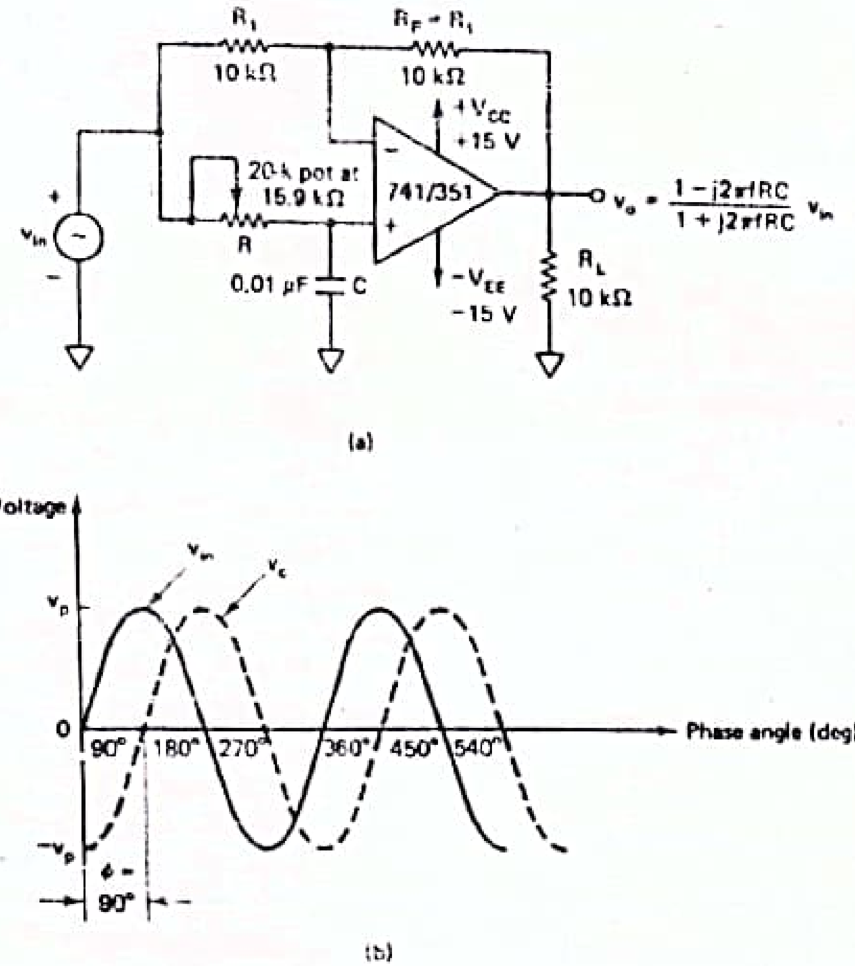


FIGURE 7–16 All-pass filter. (a) Circuit. (b) Phase shift between input and output voltages.

or

$$\frac{v_o}{v_{in}} = \frac{1 - j2\pi fRC}{1 + j2\pi fRC} \qquad (7\text{–}18)$$

where f is the frequency of the input signal in hertz.

Equation (7–18) indicates that the amplitude of v_o/v_{in} is unity; that is, $|v_o| = |v_{in}|$ throughout the useful frequency range, and the phase shift between v_o and v_{in} is a function of input frequency f. The phase angle ϕ is given by

$$\phi = -2 \tan^{-1}\left(\frac{2\pi fRC}{1}\right) \qquad (7\text{–}19)$$

where ϕ is in degrees, f in hertz, R in ohms, and C in farads. Equation (7–19) is used to find the phase angle ϕ if f, R, and C are known. Figure 7–16(b) shows a phase shift of 90° between the input v_{in} and output v_o. That is, v_o lags v_{in} by 90°. For fixed values of R and C, the phase angle ϕ changes from 0 to −180° as the frequency f is varied from 0 to ∞. In Figure 7–16(a), if the positions of R and C

are interchanged, the phase shift between input and output becomes positive. That is, output v_o leads input v_{in}.

EXAMPLE 7-11

For the all-pass filter of Figure 7-16(a), find the phase angle ϕ if the frequency of v_{in} is 1 kHz.

SOLUTION

From Equation (7-19),

$$\phi = -2\tan^{-1}\left[\frac{(2\pi)(10^3)(15.9)(10^3)(10^{-8})}{1}\right]$$
$$= -90^\circ$$

This means that the output voltage v_o has the same frequency and amplitude but lags v_{in} by 90°, as shown in Figure 7-16(b).

With the advance of integrated-circuit technology, a number of manufacturers now offer ready-to-use *universal filters* having simultaneous low-pass, high-pass, and band-pass output responses. Notch and all-pass functions are also available by combining these output responses in the uncommitted op-amp. Because of its versatility, this filter is called the *universal filter*. It provides the user with easy control of the gain and Q factor. The universal filter, sometimes called a *state-variable filter*, is presented in Chapter 9.

7-11 OSCILLATORS

Thus far we have examined op-amps wired as amplifiers or filters. This section will introduce the use of op-amps as oscillators capable of generating a variety of output waveforms. Basically, the function of an oscillator is to generate alternating current or voltage waveforms. More precisely, an oscillator is a circuit that generates a repetitive waveform of fixed amplitude and frequency without any external input signal. Oscillators are used in radio, television, computers, and communications. Although there are different types of oscillators, they all work on the same basic principle.

7-11-1 Oscillator Principles

An oscillator is a type of feedback amplifier in which part of the output is fed back to the input via a feedback circuit. If the signal fed back is of proper magnitude and phase, the circuit produces alternating currents or voltages. To visualize the requirements of an oscillator, consider the block diagram of Figure 7-17. This diagram looks identical to that of the feedback amplifiers of Chapter 3 (see Figures 3-3 and 3-9). However, here the input voltage is zero ($v_{in} = 0$). Also,