ok 05.02.10

Interpolation

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Since $y=0$ when $x=1$ (i.e. $x-1=0$)
it follows that $x-1$ is a factor.
Let, $y(x)=(x-1)R(x)$
Then, $R(x)=\dfrac{y(x)}{x-1}$.

Now, we can tabulate the values of x and $R(x)$

x	$R(x)$
0	12
3	6
4	8

From Lagrange's formula,

$$f(x)=\frac{f(x_1)(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)}+\frac{f(x_2)(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)}+\frac{f(x_3)(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)}$$

$$\therefore R(x)=\frac{12(x-3)(x-4)}{(0-3)(0-4)}+\frac{6(x-0)(x-4)}{(3-0)(3-4)}+\frac{8(x-0)(x-3)}{(4-0)(4-3)}$$

$$=(x-3)(x-4)-2x(x-4)+2x(x-3)$$

$$=x^2-7x+12-2x^2+8x+2x^2-6x.$$

$$=x^2-5x+12$$

So,

$$y(x)=(x-1)R(x)$$
$$=(x-1)(x^2-5x+12)$$

2. From the values, we can form a difference table:

x	y	Δ	Δ^2	Δ^3
0	1			
		-1		
1	0		2	
		1		6
2	1		8	
		9		
3	10			

Here, $h=1$

$x_0=0$

We have,

$x = x_0 + ph$

$= 0 + p.1$

$\therefore x = p$

$\therefore p = x$

We know, F.F

$$y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

$$= 1 + x(-1) + \frac{x(x-1)}{2} \times 2 + \frac{x(x-1)(x-2)}{6} \times 6$$

$$= 1 - x + x(x-1) + x(x-1)(x-2)$$

$$= 1 - x + x^2 - x + (x^2 - x)(x-2)$$

$$= 1 - 2x + x^2 + x^3 - x^2 - 2x^2 + 2x$$

$$= x^3 - 2x^2 + 1$$

3.2.13 B.9.2

$$y(x) - \phi_n(x) = \frac{\pi_{n+1}(x)}{(n+1)!} y^{n+1}(\xi), \quad x_0 < \xi < x_n \quad \cdots\cdots \text{(i)}$$

Equation (i) is the required expression for the error in polynomial interpolation. This equation can be used to estimate the error of the Lagrange interpolation formula for the class of functions which have continuous derivatives of order up to $(n+1)$ on $[a,b]$. We therefore have,

$$y(x) - L_n(x) = R_n(x)$$

$$= \frac{\pi_{n+1}(x)}{(n+1)!} y^{n+1}(\xi), \quad a < \xi < b$$

and the quality E_L, where,

$$E_L = \max_{[a,b]} |R_n(x)|$$

may be taken as an estimate of error. Further if we assume that

$$|y^{(n+1)}(\xi)| \leq M_{n+1}, \quad a \leq \xi \leq b$$

then

$$E_L \leq \frac{M_{n+1}}{(n+1)!} \max_{[a,b]} |\pi_{n+1}(x)|$$

From book p-75 What

4\. ~~Let~~, $y=f(x)$ denote a function ~~which~~ takes ~~the~~ values y_0, y_1, y_2 ..... y_n ~~for~~ the equidistant values $x_0, x_1, x_2, \cdots x_n$ the independent variable x ~~and~~ Let $y(x)$ denote a polynomial of the nth degree. This polynomial may be written in the form

$$y(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + a_3(x-x_0)(x-x_1)(x-x_2) + \cdots$$
$$\cdots + a_n(x-x_0)(x-x_1)\cdots\cdots(x-x_{n-1}) \quad \cdots \quad (1)$$

We shall now determine the coefficients $a_0, a_1, a_2 \cdots a_n$ so as to make $y(x_0)=y_0$, $y(x_1)=y_1$, $y(x_2)=y_2$, $y(x_3)=y_3$; $\cdots$ $y(x_n)=y_n$.

We know,

$$x_1 - x_0 = h,$$
$$x_2 - x_0 = 2h$$
$$x_3 - x_0 = 3h$$
$$\vdots$$
$$x_n - x_0 = nh$$

Substituting in (1) the successive values $x_0, x_1, x_2 \cdots x_n$ for x.

$y_0 = a_0 \Rightarrow a_0 = y_0$

$y_1 = a_0 + a_1(x_1 - x_0)$

$\Rightarrow y_1 = y_0 + a_1(x_1 - x_0)$

$\Rightarrow a_1 = \dfrac{y_1 - y_0}{x_1 - x_0}$

$\therefore a_1 = \dfrac{\Delta y_0}{h}$

$y_2 = a_0 + a_1(x_2 - x_0) + a_2(x_2 - x_0)(x_2 - x_1)$

$= y_0 + a_1 \cdot 2h + a_2\, 2h \cdot h$

$\Rightarrow y_2 = y_0 + 2\Delta y_0 + a_2 \cdot 2h^2$

$$\Rightarrow a_2 = \frac{y_2 - y_0 - 2\Delta y_0}{2h^2} = \frac{y_2 - y_0 - 2y_1 + 2y_0}{2h^2} = \frac{y_2 - 2y_1 + y_0}{2h^2}$$

$$\Rightarrow a_2 = \frac{(y_2 - y_1) - (y_1 - y_0)}{2h^2}$$

$$\therefore a_2 = \frac{\Delta^2 y_0}{2!h^2}$$

Again,

$$y_3 = a_0 + a_1(x_3 - x_0) + a_2(x_3 - x_0)(x_3 - x_1) + a_3(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)$$

$$= y_0 + \frac{\Delta y_0}{h}\cdot 3h + \frac{\Delta^2 y_0}{2h^2}\, 3h \cdot 2h + a_3 \cdot 3h \cdot 2h \cdot h$$

$$= y_0 + 3\Delta y_0 + 3\Delta^2 y_0 + a_3 \cdot 6h^3$$

$$\Rightarrow a_3 = \frac{y_3 - y_0 - 3\Delta y_0 - 3\Delta^2 y_0}{6h^3}$$

$$\cancel{= \frac{y_3 - y_0 - 3y_1 + 3y_0 - 3y_2 + 6y_1 - 3y_0}{6h^3}}$$

$$\therefore a_3 = \frac{\Delta^3 y_0}{3!h^3}$$

$$\therefore a_n = \frac{\Delta^n y_0}{n!h^n}$$

$\therefore$ (1) become,

$$y(x) = y_0 + \frac{\Delta y_0}{h}(x - x_0) + \frac{\Delta^2 y_0}{2h^2}(x - x_0)(x - x_1) + \frac{\Delta^3 y_0}{3!h^3}(x - x_0)(x - x_1)(x - x_2) + \cdots + \frac{\Delta^n y_0}{n!h^n}(x - x_0)(x - x_1)\cdots(x - x_{n-1})$$

Again, we know,

$$x = x_0 + ph$$

$$\therefore p = \frac{x - x_0}{h}$$

Then since $x_1 = x_0 + h$, $x_2 = x_0 + 2h$

we have,

$$\frac{x - x_1}{h} = \frac{x - x_0 - h}{h} = p - 1$$

$$\frac{x - x_2}{h} = \frac{x - x_0 - 2h}{h} = p - 2$$

$$\cdots \quad \cdots$$

$$\frac{x - x_{n-1}}{h} = \frac{x - x_0 - (n-1)h}{h} = p - n + 1$$

$\therefore$ (2) become.

$$y(x) = y(x_0 + ph) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots$$

$$\cdots + \frac{p(p-1)(p-2)\cdots(p-n+1)}{n!}\Delta^n y_0$$

This is the form in which Newton's forward difference formula of interpolation is usually written.

5. We have,

$$f(x) = x^2 + 5x - 7$$

Now, we can tabulate the values of x and $f(x)$ with the function :

x	$y=f(x)$	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$	$\nabla^6 y$
-1	-13						
		6					
0	-7		0				
		6		6			
1	-1		6		0		
		12		6		0	
2	11		12		0		0
		24		6		0	
3	35		18		0		
		42		6			
4	77		24				
		66					
5	143						

To find $f(6)$.

$x = 6,\ h = 1,\quad p = \dfrac{x - x_n}{h} = \dfrac{6-5}{1} = 1$

$\cancel{y_5(6) = y_5 + p\nabla y_5 + p}$

$$y_5(6) = 143 + 1\times 66 + \frac{1(1+1)}{2}\times 24 + \frac{1(1+1)(1+2)}{6}\times 6 + 0 + 0$$

$$= 143 + 66 + 24 + 6.$$

$$= 239 \text{ (Ans.)}$$

To find $f(7)$, $x = 7,\ h = 1,\ p = \dfrac{7-5}{1} = 2$

$$y_5(7) = 143 + 2\times 66 + \frac{2(2+1)}{2}\times 24 + \frac{2(2+1)(2+2)}{6}\times 6 + 0 + 0$$

$$= 143 + 132 + 72 + 24$$

Q. Calculate the generalized formula of Lagrange interpolation

A: Let,
A polynomial of order 2 is

$$f(x) = c_1(x-x_2)(x-x_3) + c_2(x-x_1)(x-x_3) + c_3(x-x_1)(x-x_2)$$

By inspection of the above polynomial we have,

$$f(x_1) = c_1(x_1-x_2)(x_1-x_3)$$

$$\Rightarrow c_1 = \frac{f(x_1)}{(x_1-x_2)(x_1-x_3)} \quad \text{.......... (ii)}$$

$$f(x_2) = c_2(x_2-x_1)(x_2-x_3)$$

$$\Rightarrow c_2 = \frac{f(x_2)}{(x_2-x_1)(x_2-x_3)} \quad \text{.......... (iii)}$$

$$f(x_3) = c_3(x_3-x_1)(x_3-x_2)$$

$$\Rightarrow c_3 = \frac{f(x_3)}{(x_3-x_1)(x_3-x_2)} \quad \text{.......... (iv)}$$

It is seen that the coefficients of the polynomial $f(x)$ are directly obtained without solving simultaneous equations. From (ii), (iii) & (iv), the polynomial may be written as

$$f(x) = \frac{f(x_1)(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)} + \frac{f(x_2)(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)} + \frac{f(x_3)(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)}$$

and in a more concise notation as:

$$f(x) = \sum_{i=1}^{3} f(x_i) \prod_{\substack{j \neq i \\ j=1}}^{3} \frac{(x-x_j)}{(x_i-x_j)}$$

This polynomial is called the Lagrange polynomial and is simple to program on a computer.

The Lagrange polynomial may be generalized to the nth order as given below:

$$f(x) = \sum_{i=1}^{n+1} f(x_i) \prod_{\substack{j \neq i \\ j=1}}^{n+1} \frac{(x-x_j)}{(x_i-x_j)}$$

7.

$$R.H.S = u_x - n u_{x-1} + \frac{n(n-1)}{2} u_{x-2} + \cdots\cdots + (-1)^n u_{x-n}$$

$$= u_x - n E^{-1} u_x + \frac{n(n-1)}{2} E^{-2} u_x + \cdots + (-1)^n E^{-n} u_x \quad [\because u_{x-n} = E^{-n} u_x]$$

$$= [1 - n E^{-1} + \frac{n(n-1)}{2} E^{-2} + \cdots\cdots + (-1)^n E^{-n}] u_x$$

$$= (1-E^{-1})^n u_x$$

$$= (1-\frac{1}{E})^n u_x$$

$$= \left(\frac{E-1}{E}\right)^n u_x$$

$$= \frac{\Delta^n}{E^n} u_x \quad [\because E-1 = \Delta]$$

$$= \Delta^n E^{-n} u_x$$

$$= \Delta^n u_{x-n}$$

$$= L.H.S$$

$$\therefore \Delta^n u_{x-n} = u_x - n u_{x-1} + \frac{n(n-1)}{2} u_{x-2} + \cdots + (-1)^n u_{x-n}$$

(Proved)

8. Newton's forward difference formula is given below

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{6}\Delta^3 y_0 + \cdots$$

From this we obtain

$$p = \frac{1}{\Delta y_0}\left[y_p - y_0 - \frac{p(p-1)}{2}\Delta^2 y_0 - \frac{p(p-1)(p-2)}{6}\Delta^3 y_0 - \cdots\right]$$

Neglecting the second and higher differences, we obtain the first approximation ~~to or and this we write as follows~~

$$p_1 = \frac{1}{\Delta y_0}(y_p - y_0)$$

Next, we obtain the second approximation to p by including the term containing the second differences. Thus,

$$p_2 = \frac{1}{\Delta y_0}\left[y_p - y_0 - \frac{p_1(p_1-1)}{2}\Delta^2 y_0\right]$$

where we have used the value of p_1 for p in the coefficient of $\Delta^2 y_0$. Similarly we obtain,

$$p_3 = \frac{1}{\Delta y_0}\left[y_p - y_0 - \frac{p_2(p_2-1)}{2}\Delta^2 y_0 - \frac{p_2(p_2-1)(p_2-2)}{6}\Delta^3 y_0\right]$$

and so on. This process should be continued till two successive approximations to p agree with each other to the required accuracy.

9. We can tabulate ~~y=x~~ $y=x^3$ for $x=2,3,4,5$.

x	$y=x^3$	Δ	Δ^2	Δ^3
2	8			
		19		
3	27		18	
		37		6
4	64		24	
		61		
5	125			

Here, $y_p = 10$, $y_0 = 8$, $\Delta y_0 = 19$, $\Delta^2 y_0 = 18$ & $\Delta^3 y_0 = 6$.
The succesive approximations to p are therefore

$$p_1 = \frac{1}{19}(10-8) = 0.105$$

$$p_2 = \frac{1}{19}\left[10-8-\frac{0.105(0.105-1)}{2}\times 18\right]$$

$$= 0.150$$

$$p_3 = \frac{1}{19}\left[10-8-\frac{0.15(0.15-1)}{2}\times 18 - \frac{0.15(0.15-1)(0.15-2)}{6}\times 6\right]$$

$$= \frac{1}{19}[2+1.1475-0.235875]$$

$$= 0.1532$$

$$p_4 = \frac{1}{19}\left[2-\frac{0.1532(0.1532-1)}{2}\times 18 - \frac{0.1532(0.1532-1)(0.1532-2)}{6}\right.$$

$$= .1541$$

$$p_5 = \frac{1}{19}\left[2-\frac{0.1541(0.1541-1)}{2}\times 18 - \frac{0.1541(0.1541-1)(0.1541-2)}{6}\right.$$

$$= 0.1542$$

We therefore take $p = 0.154$ correct to three decimal places

Hence the value of x (which corresponds to $y = 10$):

$x_0 + ph = 2 + 0.154 \times 1 = 2.154$ (Ans.)

10.

Using Lagrange Polynomial

$$\cancel{f(x)}\ y = \frac{y_1 (x-152)(x-154)(x-156)}{(150-152)(150-154)(150-156)} + \frac{y_2 (x-150)(x-154)(x-156)}{(152-150)(152-154)(152-156)}$$

$$+ \frac{y_3 (x-150)(x-152)(x-156)}{(154-150)(154-152)(154-156)} + \frac{y_4 (x-150)(x-152)(x-154)}{(156-150)(156-152)(156-154)}$$

$$= \frac{12.247(155-152)(155-154)(155-156)}{-48} + \frac{12.329(155-150)(155-154)(155-156)}{16}$$

$$+ \frac{12.40(155-150)(155-152)(155-156)}{-16} + \frac{12.490(155-150)(155-152)(155-154)}{48}$$

$$= 0.7654375 - 3.8528125 + 11.625 + 3.903125$$

$$= 12.44075$$

11.

Relation between Bessel's and Everett's formula:

The Bessel's and Everett's formula are closely related and it is posible to deduce one from the other by suitable arrangement. To see this we start with Bessel's formula,

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\frac{\Delta^2 y_{-1} + \Delta^2 y_0}{2} + \frac{p(p-1)(p-\frac{1}{2})}{3!}\Delta^3 y_{-1} + \frac{p(p+1)(p^2-1)(p-2)}{4!}\frac{\Delta^4 y_{-2} + \Delta^4 y_{-1}}{2} + \cdots$$

$$= y_0 + p(y_1 - y_0) + \frac{p(p-1)}{2!}\frac{\Delta^2 y_{-1} + \Delta^2 y_0}{2} + \frac{p(p-1)(p-\frac{1}{2})}{3!}(\Delta^2 y_0 - \Delta^2 y_{-1}) + \frac{(p+1)p(p-1)(p-2)}{4!}\frac{\Delta^4 y_{-2} + \Delta^4 y_{-1}}{2} + \cdots$$

$$= (1-p)y_0 + \left[\frac{p(p-1)}{4} - \frac{p(p-1)(p-\frac{1}{2})}{6}\right]\Delta^2 y_{-1} + \cdots + py_1 + \left[\frac{p(p-1)}{4} + \frac{p(p-1)(p-\frac{1}{2})}{6}\right]\Delta^2 y_0 + \cdots$$

$$= qy_0 + \frac{q(q^2-1^2)}{3!}\Delta^2 y_{-1} + \cdots + py_1 + \frac{p(p^2-1^2)}{3!}\Delta^2 y_0 + \cdots$$

which is Everett's formula truncated after second differences. Hence we have a result of practical importance that Everett's formula truncated after second differences is equivalent to Bessel's formula truncated after third differences. In a similar way, Bessel's formula may be deduced from Everett...

12.

We have,

$$1.17 = 1.15 + p(0.05)$$

$$\therefore p = \frac{1.17 - 1.15}{0.05} = 0.4$$

The diffence table is given below:

x	e^x	Δ	Δ^2	Δ^3	Δ^4
1.00	2.7183				
		0.1394			
1.05	2.8577		0.0071		
		0.1465		0.0004	
1.10	3.0042		0.0075		0
		0.1540		0.0004	
1.15	3.1582		0.0079		0
		0.1619		0.0004	
1.20	3.3201		0.0083		0.0001
		0.1702		0.0005	
1.25	3.4903		0.0088		
		0.1790			
1.30	3.6693				

Now,

$$e^{1.17} = y_0 + p\Delta y_0 + \frac{p(p-1)}{2}\Delta^2 y_{-1} + \frac{(p+1)p(p-1)}{6}\Delta^3 y_{-1}$$

$$= 3.1582 + 0.4 \times 0.1619 + \frac{0.4(0.4-1)}{2} \times 0.0079 + \frac{(0.4+1)0.4(0.4-1)}{6}$$

$$= 3.1582 + 0.06476 - 0.000948 - 0.0000224$$

$$= 3.2219896$$

14.

We first interpolate with respect to x keeping y constant. For $x = 2.5$, we obtain the following table using linear interpolation:

y	z
0	6.5
1	8.5
2	12.5
3	18.5
4	24.5

Now, we interpolate with respect to y using linear interpolation once again. For $y = 1.5$, we obtain

$$z = \frac{8.5 + 12.5}{2}$$

$$= 10.5$$

so that $z(2.5, 1.5) = 10.5$. Actually, the tabulated function is $z = x^2 + y^2 + y$ and hence $z(2.5, 1.5) = 10.0$, so that the computed value has an error of 5%.

15.

x	$y = e^x$	Δ	Δ^2	Δ^3	Δ^4	Δ^5
1.7	5.4739	0.5757	0.0606	0.0063 ~~0.0137~~	0.0007	0.0001 ~~0.0019~~
1.8	6.0496	0.6363	0.0669	0.0070	0.0008	
1.9	6.6859	0.7032	0.0739	0.0078		
2.0	7.3891	0.7771	0.0817			
2.1	8.1662	0.8588				
2.2	9.0250					

here,

$x_0 = 1.7$

$h = 1.8 - 1.7 = 0.1$

$x = \cancel{1.19}\ 1.91$

$\therefore\ p = \dfrac{1.91 - 1.70}{0.1} = 2.1$

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{6}\Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{24}\Delta^4 y_0$$

$$+ \frac{p(p-1)(p-2)(p-3)(p-4)}{120}\Delta^5 y_0$$

$$= \cancel{0}\ 5.4739 + 2.1 \times 0.5757 + \frac{2.1(2.1-1)}{2} \times 0.0606 + \frac{2.1(2.1-1)(2.1-2)}{6} \times 0.006$$

$$+ \frac{2.1(2.1-1)(2.1-2)(2.1-3)}{24} \times 0.0007 + \frac{2.1(2.1-1)(2.1-2)(2.1-3)(2.1-4)}{120} \times 0.0$$

$$= 5.4739 + 1.20897 + 0.069993 + 0.00024255 - 0.00000606375$$

$$+ 3.29175 \times 10^{-7}$$

$$= 6.753099815$$

17.

Using Lagrange formula,

$$y(x) = \frac{19.97(x-15)(x-20)(x-25)}{(10-15)(10-20)(10-25)} + \frac{21.51(x-10)(x-20)(x-25)}{(15-10)(15-20)(15-25)}$$

$$+ \frac{22.47(x-10)(x-15)(x-25)}{(20-10)(20-15)(20-25)} + \frac{23.52(x-10)(x-15)(x-20)}{(25-10)(25-15)(25-20)}$$

$$= \frac{19.97(x-15)(x-20)(x-25)}{-750} + \frac{21.51(x-10)(x-20)(x-25)}{250}$$

$$+ \frac{22.47(x-10)(x-15)(x-25)}{-250} + \frac{23.52(x-10)(x-15)(x-20)}{750}$$

18.

The difference table is given below:

Year	Population (in thousands)	∇	∇^2	∇^3	∇^4
1921	46				
		20			
1931	66		-5		
		15		2	
1941	81		-3		-3
		12		-1	
1951	93		-4		
		8			
1961	101				

here,

$x_n = 1961$, $y_n = \cancel{46}\,101$

$h = 10$

$p = \cancel{1961}\ \dfrac{1955-1961}{10} = -0.6$

$$y_p = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \frac{p(p+1)(p+2)(p+3)}{4!}\nabla^4 y_n$$

$$= 101 + (-0.6)\times 8 + \frac{-0.6(-0.6+1)}{2}(-4) + \frac{-0.6(-0.6+1)(-0.6+2)}{6}(-1)$$

$$+ \frac{-0.6(-0.6+1)(-0.6+2)(-0.6+3)}{24}(-3)$$

$= 101 - 4.8 + 0.48 + 0.056 + 0.1008$

$= 96.8368$

19. [In this answer, first derive the ~~gp~~ generalized Lagrange polynomial of n^{th} order (in Q6), then write...]

From the generalized formula, we can see the number of addition is $(n+1)$ & the number of multiplicatio[n] is $n(n+1)$ needed to implement Lagrange interpolation of nth degree.

$2(n+1)$ – M

$(2n+1)$ – A

16. Interpolation & Extrapolation: The statement $y=f(x)$ $x_0 \le x \le x_n$ Given the set of ~~tabulata~~ tabular values $(x_0, y_0), (x_1, y_1), (x_2, y_2)$ ... (x_n, y_n) satisfying the relation $y=f(x)$ when the explicit natu[re] of $f(x)$ is not known, it is required to find a simpler functi[on] say $\phi(x)$, such that $f(x)$ and $\phi(x)$ agree at the set of tabulated point. Such a process is called interpolation.

The statement $y=f(x)$, ~~x_0~~ $x<x_0$ and $x_n<x$. given the set of tabular values $(x_0, y_0), (x_1, y_1)$ ...... (x_n, y_n) satisfying the relation $y=f(x)$ when the explicit nature is known, it is required to find a simpler function, say $\phi(x)$, such that $f(x)$ and $\phi(x)$ agree at the set of tabulated point. Such a process is called extrapolation.

In another word,

If the value of $f(x)$ is to be found at some point y in the interval $[x_1, x_n]$ and y is not one of the tabulated points then the value is estimated by using the known values of $f(x)$ at the ~~so~~ surrounding points. This is called interpolation.

If the point y is outside the interval $[x_1, x_n]$ then the estimation of $f(y)$ is called extrapolation.

How it is usefull?: