(17)

Algebra/complete restoring

Let,

$x+3=5$

$\Rightarrow x+3-3=5-3$

$\therefore x=2$

$a=b$

$\Rightarrow a^2=ab$

$\Rightarrow a^2-b^2=ab-b^2$

$\Rightarrow (a-b)(a+b)=b(a-b)$ [$a=b$ তখন কাটাকাটি হবে না]

$\Rightarrow a+b=b$

$\Rightarrow a+a=a$

$\Rightarrow 2a=a$

$\therefore 2=1$

তখন, $-2=-2$

$\Rightarrow (1)^2-2\times1\times 3/2+(3/2)^2=$

$(2)^2-2\times2\times3/2+(3/2)^2$

$\Rightarrow (1-3/2)^2=(2-3/2)^2$

$\Rightarrow 1-3/2=2-3/2$ wrong

$\therefore 1=2$

combination 4 টা হবে

$\left.\begin{array}{l}++,\\ --,\end{array}\right\}$ combination নেয়া যাবে না

$$\sum_{J=0}^{n} ar^J = \begin{cases} \dfrac{ar^{n+1}-a}{r-1} & \text{if } r \neq \\ (n+1)a & \text{if } r = \end{cases}$$

$$S = \sum_{J=0}^{n} ar^J$$

$$\Rightarrow rS = r\sum_{J=0}^{n} ar^J$$

$$\Rightarrow rS = \sum_{J=0}^{n} ar^{J+1}$$

$$\Rightarrow rS = \sum_{J=0}^{n} ar^{J+1}$$

$$\Rightarrow rS = \sum_{K=1}^{n+1} ar^K$$

$$\Rightarrow rS = \sum_{K=0}^{n} ar^K + (ar^{n+1}-a)$$

$$\Rightarrow rS = S + (ar^{n+1}-a)$$

$$\Rightarrow S(r-1) = \frac{ar^{n+1}-a}{1}$$

$$\therefore S = \frac{ar^{n+1}-a}{r-1} \text{ A.}$$

$$\sum_{i=1}^{4}\sum_{J=1}^{3} iJ$$

$$\sum_{i=1}^{4} i + 2i + 3i$$

$$\sum_{i=1}^{4} 6i$$

$$= 6 + 12 + 18 + 24$$

$$= 60 \text{ A.}$$

$$\sum_{i=1}^{4}\sum_{J=1}^{3} iJ$$

$$= \sum_{i=1}^{4} i + 2i + 3i$$

$$= \sum_{i=1}^{4} 6i$$

$$= 6\times1 + 6\times2 + 6\times3 + 6$$

$$= 6 + 12 + 18 + 24$$

$$60 \text{ A.}$$

$$=$$

$$\sum_{i=1}^{4} i + 2i + 3i$$

$$= 6i$$

$$= 6\times1 + 6\times2 + 6\times3 + 6\times4$$

$$= 6 + 12 + 18 + 24$$

$$= 60 \text{ A.}$$

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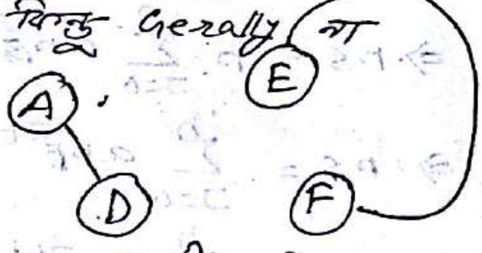
weigh টা দেয়া থাকলে matrix এ weight উল্লেখ করব

Isomorphism Graph:- The simple graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$

one to one & onto

$f(V_1) = V_2$

visualige করতে পারি কিন্তু Gerally না



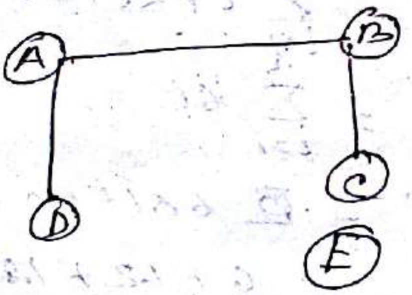
One to one বা onto না হলে তা isomorphism graph না

⊞ বলা অসম্ভব যে isomorphism না
⊞ " " না " "

} এই property টা হচ্ছে ivarient property.

দুইটি graph দেয়া থাকলে isomorphism কি না বলতে হবে

connectivity → path and circuit

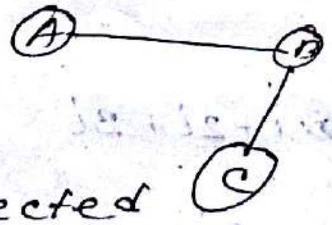


D থেকে C এর length = 3

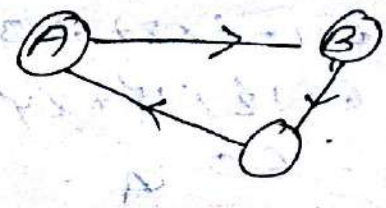
যেই node থেকে start করবে আবার ঐ node এ আসবে

$A \to B \to C \to D \to A = 4$

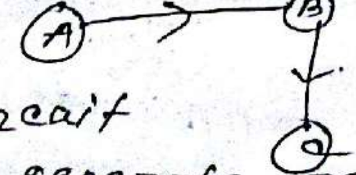
এটি connected না



connected



→ সবগুলো থেকে যখন excess করা যাবে তখন storngly graph



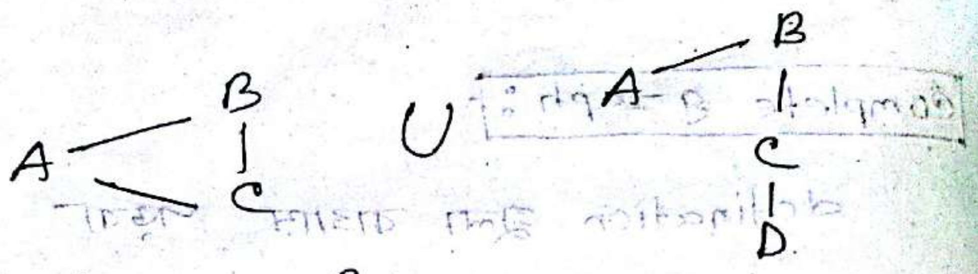
→ weak graph

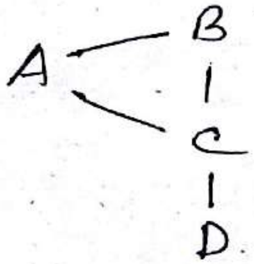
Oiler circuit

circuit generate করলে graph

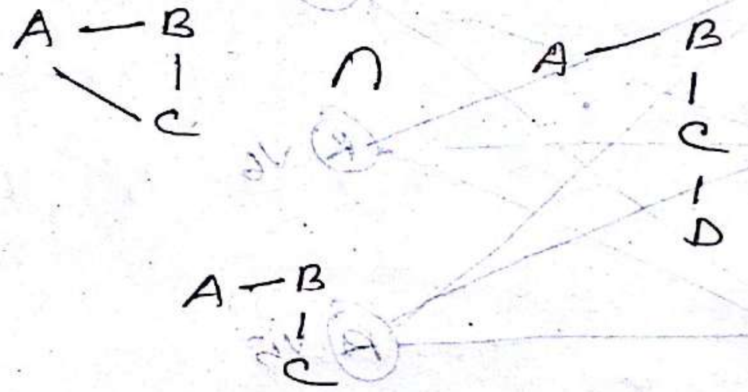
Hamilton থাকলে তার সব node থাকবে

দুইটো graph এর Union



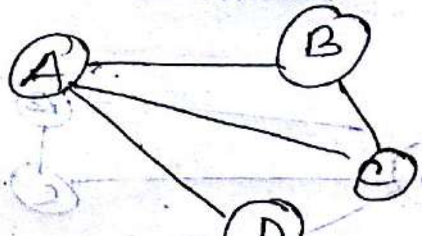


→ Resultant graph



=0=

Adjacent Matrix



A	B C D
B	A C
C	A, B
D	A

$$\begin{array}{c|cccc} & A & B & C & D \\ \hline A & 0 & 1 & 1 & 1 \\ B & & & & \\ C & & & & \\ D & & & & \end{array}$$

0 এর সংখ্যা বেশি হলে spar matri

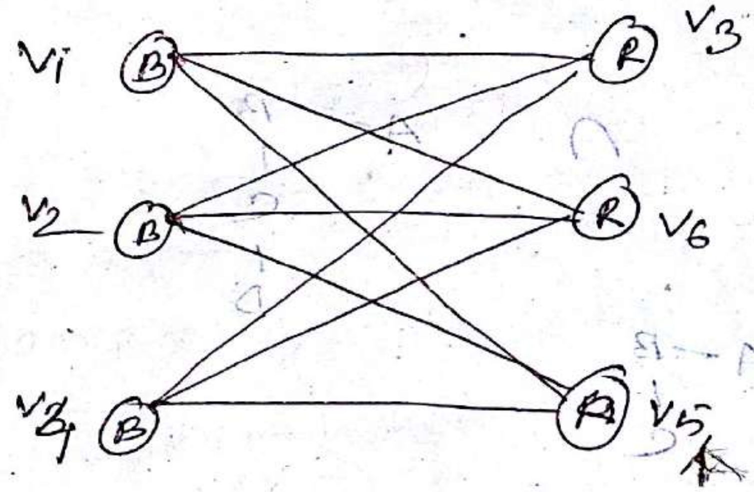
$\Rightarrow 4 = 4 = 4$

Complete graph :-

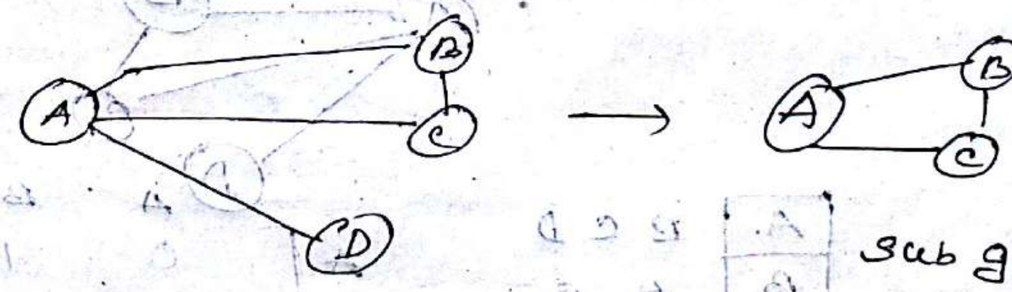
defination গুলা বাসায় পড়বা

Bipartite graph:

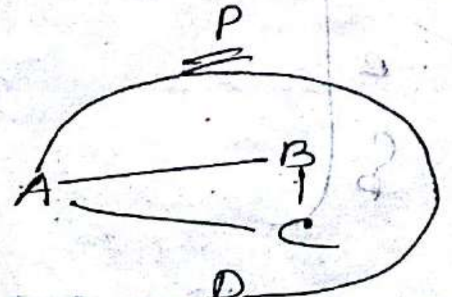
V, E



এখানে ২ টা coulour use করা হয়েছে তাই quantic number টি 2



sub graph of P



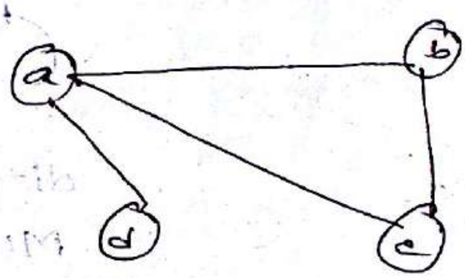
→ proper graph of p

Theorem:-

Handshaking theorem:-

Let $G = (V, E)$ be an undirected gr[aph] with e edges, then

$$2e = \sum_{v \in V} \deg(v)$$



$V = \{a, b, c, d$

$v = a$

$v \in V$

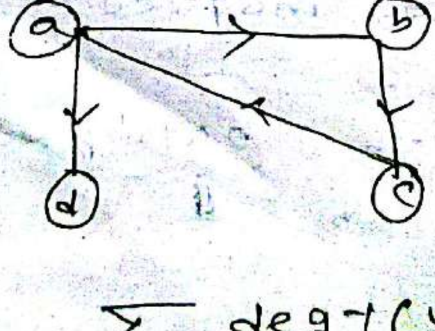
$$2e = \deg(a) + \deg(b) + \deg(c) +$$
$$= 3 + 2 + 2 + 1$$
$$= 8$$

$$2 \times e = 2 \times 4 = 8$$

(proved).

directed graph এর ক্ষেত্রে indegree, $\deg^{-}($

outdegree, $\deg^{+}($



$$\sum_{v \in V} \deg^{-}(v) = \sum_{v \in V} \deg^{+}(v) =$$

$$\Rightarrow 1+1+1+1 = 2+1+1+0 = 4$$

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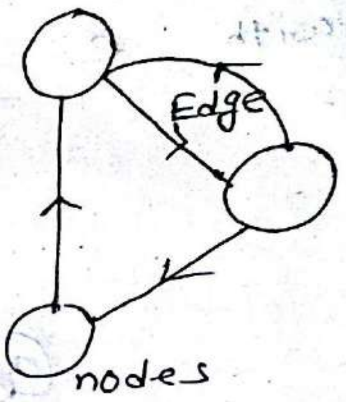
Graph colouring (Defination, ধারণা

=o=

Defination:-

$G(V, E)$

Vertex / Vertices / nodes — Edges



directed graph
Multiple graph

$V = \{A, B \dots\}$

$E = \{\langle A, B\rangle, \}$

Defination গুলো দেখে নিতে হবে,

Graph model:-

connection represent $\langle A, B\rangle$

$\langle u, v\rangle$

direction না থাকলে degree - 1

থাকলে In degree 1, 0 হয়

u এর indgree - 0

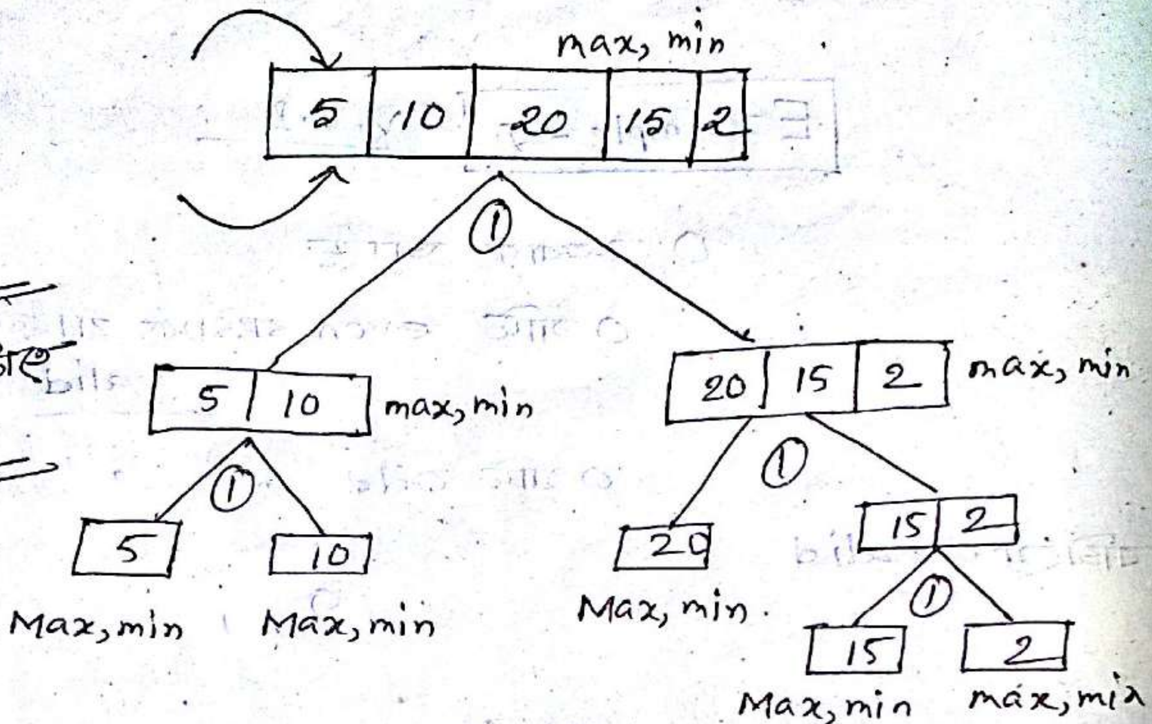
v " " - 1

u " outdegree - 1

v " " - 0

☐ Java তে multiple inheritance হয় কি না?

Maximum, Minimum:-



H.W.
10, 20টাকে করবা

09-08-16
12th (E) day

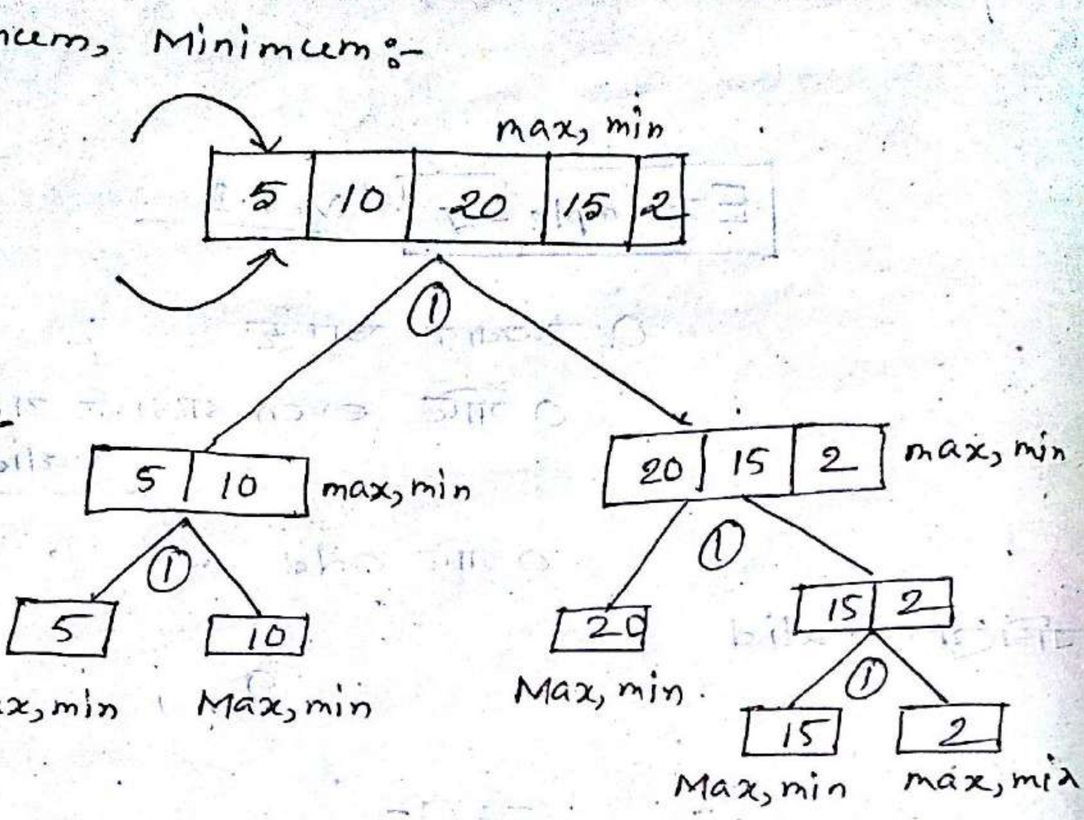
Graph

Graph model
" represent
Special simple graph.
** Bipartide Graph
Application of graph
** Isomorphism of graph
connectivity
** Boiler path / Hamilton path
*** Shortest path problem
** Dijkstra Algorith / Fluid
planner graph - Theorem (

planner gra
যদি একটা গ্রা
কোন crossing
যেইটা করে

Java তে multiple inheritance হয় কি না?

Maximum, Minimum:-



H.W.

10, 20 টাও করবা

09-08-16
12th (E) day

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planner graph - Theorem (

planner gra
যদি একটা গ্রা
কোন crossing
যেইটা করতে

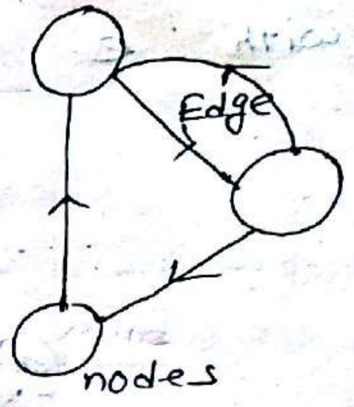
Graph colouring (Defination, প্রশ্ন)

=0=

Defination:-

G (V, E)

Vertex/Vertices/nodes — Edges



V = {A, B ---}

E = {⟨A, B⟩, }

directed graph
Multiple graph

Defination গুলা দেখে নিতে হবে,

Graph model:-

connection represent ⟨A, B⟩

⟨u, v⟩

direction না থাকলে degree - 1

থাকলে In degree 1, 0 হয়

u এর indgree - 0

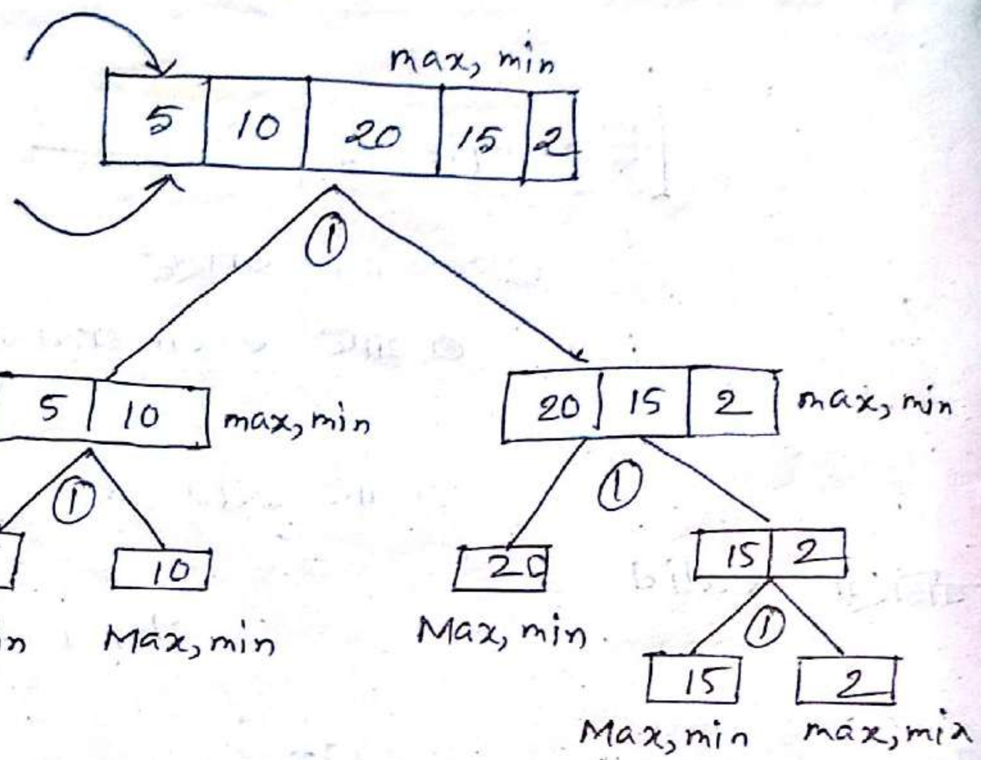
v " " - 1

u " outdegree - 1

u " " " - 0

Java তে multiple inheritance হয় কি না?

Maximum, Minimum:-



H.W.

10, 20টাও করবা

09-08-16

12th (E) day

Graph

Graph model

" represent

Special simple graph

** Bipartide Graph

Application of graph

** Isomorphism of graph

Connectivity

** Boiler path / Hamilton path

*** Shortest path problem

** Dijkstra Algorith / Fluid

planner graph - Theorem

planner graph

যদি একটা কাগজে

কোন crossing

যেইটা অঙ্কন

$a_1 = 9$

$\boxed{0} \rightarrow$ invalid

1
2
3
4
5
6
7
8
9

Example - 7

V.V.I

0 কতবার আছে

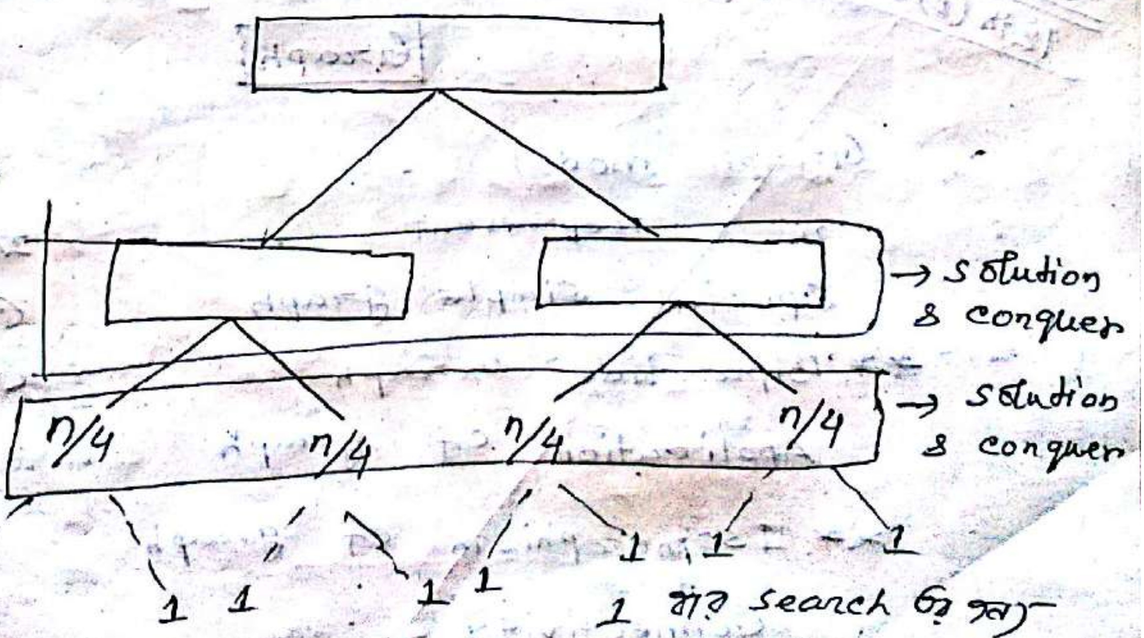
0 যদি even সংখ্যক থাকে তাহলে valid

0 যদি odd " " → invalid

0 বাদিয়ে valid $9a_{n-1}$

=0=

$\boxed{7.3}$



1 বার search এর জন্য

$f(n) = f(n/2) + \frac{1}{c}$

~~1 বাদ~~

2 বার search এর জন্য

$f(n) = f(n/c) + 2$

~~0 0 0 0~~
~~0 0 0 1~~
~~0 0 1 0~~
~~0 0 1 1~~
~~0 1 0 0~~
✓ 0 1 0 1
✓ 0 1 1 0 ✓
✓ 0 1 1 1
~~1 0 0 0~~
~~1 0 0 1~~
✓ 1 0 1 0 ✓
✓ 1 0 1 1
~~1 1 0 0~~
✓ 1 1 0 1
✓ 1 1 1 0 ✓
✓ 1 1 1 1

$a_4 = a_3 + a_2 + \cancel{a_1}$
$= 5 + 3 \quad 8$

$a_4 = \boxed{a_3 + a_2}$
$= 5 + 3$

$a_5 = a_4 + a_3$

$a_6 =$

Now In function,

$f(n) = f(n-1) + f(n-2)$

=0=

n length का string (0 - 9) digits

valid ⟶ 0 even | Recurance Relation
Invalid ⟶ 0 odd | Initial condition?

terminate condition

Problem: Recurrence relation টা বের কর?

5 bit এর str length:-
যেখানে প্রথম দুইটা 00 হবে না।

$\left.\begin{matrix}0\\1\end{matrix}\right\} 2$ $a_1 = 2$

$\left.\begin{matrix}\cancel{00}\\01\\10\\11\end{matrix}\right\}$ $a_2 = 3$

~~$a_3 = a_2$~~

$\left.\begin{matrix}\cancel{000}\\\cancel{001}\\010\\011\checkmark\\\cancel{100}\\101\checkmark\\110\\111\checkmark\end{matrix}\right\}$ $= a_3 = a_2 + a_1$

a_2 → 1 দিয়ে valid

a_1 → 0 দিয়ে valid

$a_4 = ?$

0000
0001
0010
0100
1000
1111
1110
1101
1001
0111

~~0100~~
0110
1010
~~1100~~
1110

3-08-16

12th (A) day

Discrete Mathematics

Recurrence Relations :- | computer Algorith

↓

complexity কমানোর জন্য.

$a_n = a_{n-1} + a_{n-2}$, $a_1 = 5$, $a_2 = 6$

find out, $a_{10} = ?$

Here $a_1 = 5$ & $a_2 = 6$ condition to be needed

$a_2 = a_1 + a_0 = 6 - 5$

$= 1$

$a_3 = a_2 + a_1 = 1 - 6$

$= -5$

$a_4 = a_3 - a_2 = -5 - 1$

$= -6$

$a_5 = a_5 - a_4 = -6 + 6$

$= 0$

$a_1 = a_9 + a_8$

$= a_8 + a_7 + a_6 + a_7$

$= a_7 + a_6 + a_6 + a_5 + a_4 + a_6 + a_5$

$= a_7 + 3a_6 + 2a_5 + a_4$

$=$

Fourth order R-K method:-

$x_0 = 0,\ y_0 = 2,\ h = 0.1$

$k_1 = hf(x_0, y_0) = 0.2$

$k_2 = hf(\underbrace{x_0 + \tfrac{1}{2}h}_{x}, \underbrace{y_0 + \tfrac{1}{2}k}_{y}) = 0.205$

$k_3 = 0.20525$

$k_4 = 0.21053$

$y(0.1) = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$= 2 + \frac{1}{6}(0.2 + 2 \times 0.205 + 2 \times 0.20525 + 0.21053)$

$= 2.2052$ (Ans.)

$y(0.2)$. এখানে,

$x_0 = 0.1$

$y_0 = 2.2052$

$k_2 = hf(x_0 + h, y_0 + hf_0)$

$= 0.1 \, f(.1, 2.2)$

$= 0.1 \times 2.1$

$= 0.21$

$y_0 + hf_0$
$= 2 + 0.1 * 2$
$= 2 + 0.2$
$= 2.2$

$\frac{dy}{dx} = y - x$
$= 2.2 - 0.1$
$= 2.1$

$y_1 = y_0 + \frac{1}{2}(k_1 + k_2)$

$= 2 + 1/2 (0.2 + 0.21)$

$= 2.2050$ (Ans.)

y_2 বের করতে হলে আগের y_0 এর জায়গায় হবে y_1 same for y_3, y_4 — — —

$y_2 = y(0.2)$

$x_0 = 0.1$

$y_0 = y_1 = 2.2050$

$k_1 = 0.1 (2.105) = 0.2105$

$k_2 = 0.1 (2.4155 - 0.2) = 0.22155$

$x_0 = x_1 = 0.1$

~~$y_2 = x_0 = 0.1$~~

$\frac{dy}{dx} = y - x$

$= y_1 - x$

$x =$

$y_2 = 2.2050 + 1/2 (0.2105 + 0.22105)$

$= 2.4210$

2-08-13

11th (E) ব্যাচ

আগের Lecture টা important

SZM

Numerical Method

$f_0 = f(x_0, y_0)$

Runge - Kutta method:-

$$y_1 = y_0 + h/2 \left[f(x_0, y_0) + f(x_1, y_1) \right]$$

$$y_1 = y_0 + hf(x_0, y_0)$$

$$y_1 = y_0 + h/2 \left[f(x_0, y_0) + f(x_1, y_0 + hf(x_0, y_0)) \right]$$

$$\cancel{= y_0 + h/2 \left[f_0 + f \right.}$$

$$= y_0 + h/2 \left[f_0 + f(x_0 + h, y_0 + hf_0) \right]$$

$$k_1 = hf_0, \quad k_2 = hf(x_0 + h, y_0 + hf_0)$$

$$y_1 = y_0 + \frac{1}{2}(k_1 + k_2) \rightarrow \text{2nd order R-K method}$$

Fourth order R-K method:-

$$k_1 = hf(x_0, y_0)$$

$$k_2 = hf(x_0 + \tfrac{1}{2}h, y_0 + \tfrac{1}{2}k_1)$$

$$k_3 = hf(x_0 + \tfrac{1}{2}h, y_0 + \tfrac{1}{2}k_2)$$

$$k_4 = hf(x_0 + h, y_0 + k_3)$$

$$y_1 = y_0 + \tfrac{1}{6}(k_1 + 2k_1 + 2k_3 + k_4)$$

$\boxed{\text{Example - 7.8}}$

$$\frac{dy}{dx} = y - x \quad y(0) = 2$$

Find $y(0.1)$, $y(0.2)$

y_1 এর জন্য

$x_0 = 0$

(i) 2nd order R-K : $h = 0.1$

$$k_1 = hf(x_0, y_0) = 0.1(2 - 0)$$

$$= 0.2$$

A function f from a set with $k+1$ or more elements to a set with k elements. f is not

one-to-one

বাকি সব → ঐ chapter থেকে দেখে নিবি

Theorem → 2

Theorem → 3 (উদাহরণ theory টা দেখে রাখবে)

Randomly 10 টা সংখ্যা। increasingly

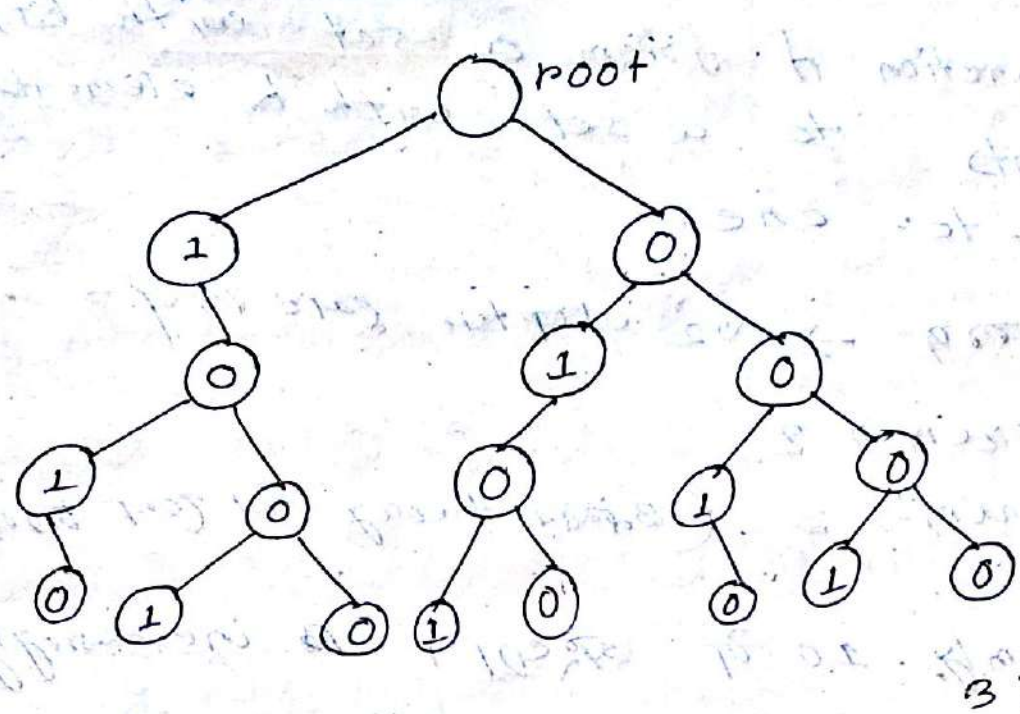
permutation & combination

Binomial Cofficients

Generaliged permutations & combination

** Distributing objects in to Boxes

$\boxed{\text{Generating permutations and combinations (এইটা lab এর কাজ)}}$



01010 010. → solution

$3 \times 2 - 1$
$= 6 - 1$

Tree complexity কমিয়ে দেয়, এটা log এ নিয়ে আসার চেষ্টা করে।

Game implement করা হয় Tree এর মাধ্যমে

আমার কাজ: 01 টা game design করতে হবে। যে team 3 টা match জিতবে সে জিতবে এবং programme হবে terminate (programme)

৩.২ The pigeonhole principale:-

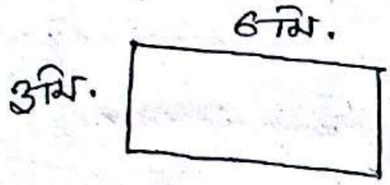
person → n+1
produce → n
তাহলে 1 টা product দুইজন ভাগ করে নিতে হবে

কবুতর → n+1 সংখ্যক
বাসা → n
At least 1 টা বাসায় দুইজন থাকতে লাগবে,

IPV6 → AAAA : AAAA ---------- : AAAA

8 | 128 | 16
 8
 48
 48
 0

APNIC → IP controll করে।



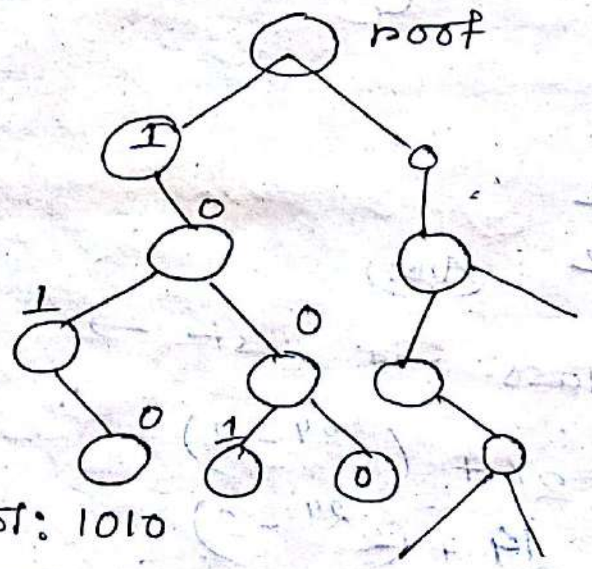
= 3 × 6 বর্গ মি.

= 18বর্গমি → কেন বর্গ লিখি ?

বর্গের আপেক্ষে পরিমাপ করি তাই বর্গ

Tree দিয়ে counting:-

4 length বিশিষ্ট string generate করবো



Sol: 1010

Internet addressing:-

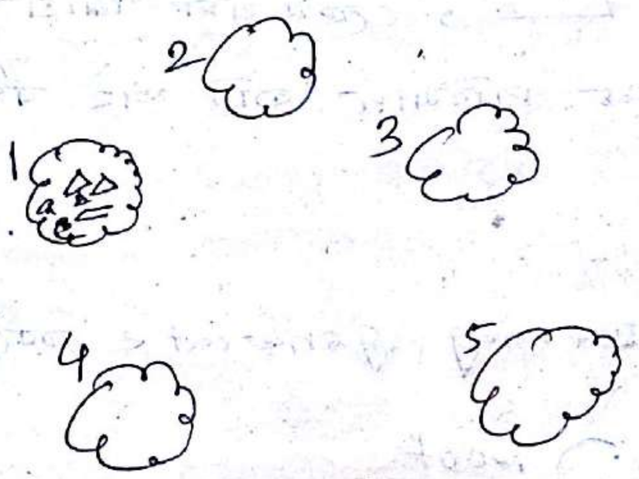
IPV4 $\longrightarrow$ continue + over

· IPV6

IPV4 — 32 bit
IPV6 — 64 bit } bit বাড়িয়ে দিয়ে

IP address $\overline{172}^{8\,bit} . \overline{16}^{8\,bit} . \overline{1}^{8\,bit} . \overline{1}^{8\,bit}$

mask $\leftarrow$ 255.255.0.0



1, 2, 3, 4, 5 $\rightarrow$ Network
a, b, c $\rightarrow$ Host id

$2^8 + 2^{24} - 2$

$= 2^8 + 2^{24} - 2$ (Ans:)

2 B class এর host $\rightarrow$

A class $\rightarrow 2^7 + (2^{24} - 2)$

B class $\rightarrow 2^{14} + (2^{24} - 2)$

host $\rightarrow$

C " $\longrightarrow 2^{21} + (2^{24} - 2)$

8 bit $\rightarrow$ host $(2^8 - 2)$

~~A et~~ Total $= \{2^7 + (2^{24} - 2)\} + \{2^{14} + (2^{24} - 2)\} + \{2^{21} + (2^{24} - 2)\}$

7-16

(A) day

More complex counting problems:

(digit)
number + letter

□ □ → string length = 2

letter upper/lower

$V_1 = 26$

$V_2 = \underbrace{26 * 36}_{\text{product rule}} - 5 = 931$

$V = V_1 + V_2$ (sum rules)

$= 26 + 931 = 957$

password generate:-

First we get six character long,

$P_6 = 36^6 - 26^6$

$= 1867866560$

$P_7 = 36^7 - 26^7$

$= 70332353920$

$P_8 = 36^8 - 26^8$

$=$

Ans: $P = P_6 + P_7 + P_8$

26-07-16
10th (E)

Counting:-

Rule → <u>product Rule</u>: Suppose that a procedure can be broken down into a sequence of two task. if there are n_1 ways to do the first task and for each of this ways of doing the first task, there are n_2 ways to do the second task, then there are $n_1 n_2$ ways to do the procedure.

কয়টা function generate করতে পারব a set with m element to a set with n element.

$$n \cdot n \cdot n \cdot \text{-------} = n^m$$

=0=

Every each word we found 100 key, so 26 key we found, $26 \times 100 = 2600$ key

<u>The sum Rule:</u> If a task can be done either in one of n_1 ways or in one of n_2 ways, where <u>none of the set of n_1 ways is the same as any of the set of n_2 ways</u>, then there are $n_1 + n_2$ ways to do the task.

List 1 — n

List 2 — m

List 3 — p

প্রত্যেকটা থেকে 1 টা নেব $n + m + p$, উপায়ে (এদের মধ্যে সম্পর্ক নাই তাই যোগ করছি)

Uniqe National ID কত হবে ?

Telephone Number ?

Home works:- Ex-3

Find an inverse of 3 modulo 7

The equation goes,

$$7 = 2 \cdot 3 + 1$$

So the the inverse = -2

Ex-4

$$3x \equiv 4 \pmod 7$$

find $x = ?$

we find, $7 = 2 \cdot 3 + 1$

inverse $= -2$

$$-2 \cdot 3x \equiv -2 \cdot 4 \pmod 7$$

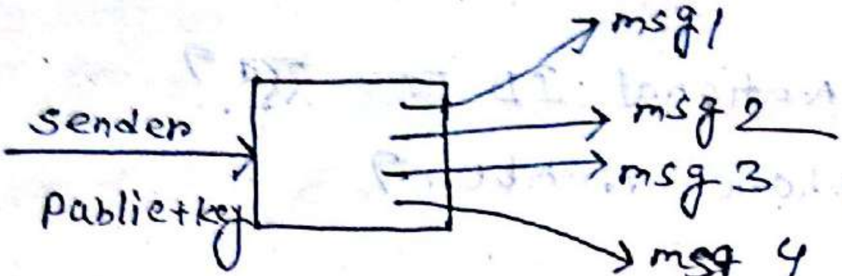
Now, $x \equiv -8 \equiv 6$

Now, applying $x = 6$ we find,

$$3x \equiv 3 \cdot 6 \equiv 18 \equiv 4 \pmod 7$$

Now the $x = 6, 13, 20, \ldots$

$x = -1, -8, -15, \ldots$



একটা public key থাকবে, এবং সবাই sms গুলা পাবে

Matrices

$3x + 2y = 5$
$5x + 3y = 6$

$$\begin{bmatrix} 3 & 2 \\ 5 & \end{bmatrix}$$

Counting

1 or 00 $\longrightarrow$ 8 bit (0 or 1)

1 - - - - - - -
2^7

- - - - - - 00
2^6

1 - - - - - 00
2^5

combination $= 2^7 + 2^6 - 2^5$

And হলে হবে
[illegible] + [illegible] + 2^5

$\Rightarrow$ National I D তৈরী হবে,
প্রথম চারটা digit, পরের 12 টা small letter
parmutation কত?

The chinese Remainder Theorem:-

$$x \equiv 2 \pmod 3$$
$$x \equiv 3 \pmod 5$$
$$x \equiv 2 \pmod 7$$
$$x = ?$$

প্রথমে m এর মান বের করতে হবে,

1. $m = \cancel{2 \times 3 \times 2 = 12} \quad 3 \times 5 \times 7 = 105$

2. $M_i = \frac{m}{m_i} =$

$$M_1 = \frac{3}{105} = \frac{1}{35}$$
$$M_2 = \frac{5}{105} = \frac{1}{21}$$
$$M_3 = \frac{7}{105} = \frac{1}{15}$$

$$\gcd(M_i, M_i) \equiv 1$$

Inverse $y_i = 1$

$$y_i M_i \equiv 1 \pmod{m_i}$$

$$\Rightarrow 1 \cdot M_i \equiv 1 \pmod m$$

$\Rightarrow$ $\boxed{\text{এই problem টার জন্য বই দেখ}}$

RSA → Lab Manual দেখো

Example:-

$$3x \equiv 1 \pmod{7}$$

3 ও 7 -এর gcd 1 তাই

$$3x \equiv 1 \pmod{7}$$

তাই solution পাওয়া যাবে,

$$3s + 7t = 1$$

$$\Rightarrow 3(-2) + 7 \times 1 = 1$$

$$\therefore s = -2$$

$$t = 1$$

$$\cancel{3s + 7t = 1}$$

$$\Rightarrow$$

সমাধান, $s = -2$

$\therefore$ solution হবে, $-2 + 7k,\ k \in \mathbb{Z}$

*** **বাড়ির কাজ :**

① $3x \equiv 4 \pmod{7}$

Linear combination কেন দরকার:-

ছোটকে ছোট করার জন্য।

$$5 \longrightarrow 1 \longrightarrow 5$$

Linear confluence :-

$$ax \equiv b \pmod m$$

x দরকার ?

সাধারণ নিয়মে,

$x \equiv \frac{b}{a}$ করা যাবে না

এটা Linear confluence

$$\bar{a}\, a \equiv 1$$

Theorem:-

If $\gcd(a, m) = 1$ (R.P), th

$\bar{a}a \equiv 1 \pmod m$ has a unique

Otherwise it has no solution.

proof:- $\gcd(a, m) = 1$

$$sa + tm = 1$$

$$(sa + tm) \bmod m = 1 \bmod m$$

$$sa \bmod m = 1 \bmod m$$

$$sa \equiv 1 \pmod m$$

$s = \bar{a}$ $\therefore \bar{a}a \equiv 1 \pmod m$

$\gcd(252, 198) = 18$

$252 = 198 \times 1 + 54$

$198 = 54 \times 3 + 36$

$54 = 36 \times 1 + 18$

$36 = 18 \times 2$

we can write,

$18 = 54 - 36 \times 1$

$36 = 198 - 3 \cdot 54$

$54 = 252 - 198 \times 1$

$54 = 252 - 198 \times 1$

এখন, $18 = 54 - 36(198 + 3 \times 54)$

$= \cancel{4(252 - 198 \times 1)} - 198$

$= 252 - 198 - 36(198 + 3 \times 54)$

$= 4 \times 54 - 198$

বাকিগুলা বাসায় বুঝতে হবে

20-07-16

10th (A) day

$$\gcd(a,b)$$

$r_0 = a;\ r_1 = b,\ a \geq b$

$r_0 = q_1 \times r_1 + r_2 \quad 0 < r_2 < r_1$

$r_1 = q_2 \times r_2 + r_3 \quad 0 < r_3 < r_2$

$\vdots$

$r_{n-1} = q_{n-1} \times r_n$

$\gcd(a,b) = r_n$

$$\gcd(15,3)$$

$r_0 = 15,\ r_1 = 3 \quad 15 > 3$

$\cancel{r_0} = \cancel{\times 3 +}$

$\cancel{15 = \times 3 + r_2}$

$15 = 2 \times 7 + 1$

$7 = 7 \times 1$

$\boxed{2.7} \longrightarrow$ Application of Number Theory:-

If a and b are positive integer Then their exist integer s and t such that $\gcd(a,b) = sa + tb$.

$3^{644} \bmod 645 = ?$

$(644)_{10} = (1010000100)_2$

ধাপ result, $x = 1$, power $= 3 \bmod 645$
$= 3$

$a_0 = 0 \longrightarrow x = 1$, power $= 3^2 \bmod 645$
$= 9$

$a_1 = 0 \longrightarrow x = 1$, power $= 9^2 \bmod 645$
$= 81$

$a_2 = 1 \longrightarrow x = 1 \times 81 \bmod 645$, power
$= 81^2 \bmod 625$

$x = 81$, power $81^2 \bmod$

$a_3 = 0 \longrightarrow x = 81$, power $81^3 \bmod 64$

$a_4 = 0 \longrightarrow x = 81$ power $81^4 \bmod 64$

$a_5 = 0 \longrightarrow x = 81$ power $81^5 \bmod 64$

$a_6 = 0 \longrightarrow x = 81$ power $81^6 \bmod 64$

$a_7 = 1 \longrightarrow x =$

$2^{10} \times 2^{10} =$

$(2^{120})^{?} \times 12$

$= (2^{120} \times 2^2) \% 7$

$2x$

$(2^{19} \times 20$

$/2$

$ab = \gcd(a,b) \times lcm(a,b)$

=0=

$n = a_k b^k + a_{k-1} b^{k-1} + \text{———} a_1 b + a_0$

$(10)_{10} = 1 \times 10^1 + 0 \times 10^0 = 1 \times 10^1 + 0$

$(87)_{10} = 8 \times 10^1 + 7$

$(FE)_{16} = F \times 16^1 + E$

যে কোন Number কে Represent করা যায়।

=0=

$3^{644} \bmod 645$

$2^{122} \bmod 7$

$= (2^{10} \bmod 7 \times 2^{5} \bmod 7 \times \cdots \times 2^2 \bmod 7) \bmod 7$

$= (2^{12} \times 4) \bmod 7$

$= (2^{10} \times 4 \times 4) \bmod 7$

$= (2^{10} \bmod 7 \times 4 \bmod 7 \times 4 \bmod 7) \bmod 7$

$= (2 \times 4 \times 4) \bmod 7$

$= (32) \bmod 7$

$= 4$

$2^{122} \bmod 7$

$= (2^{12} \times 4) \bmod 7$

$= (2^{10} \times 4 \times 4) \bmod 7$

$= (2 \times 4 \times 4) \bmod 7$

$= (32) \bmod 7 = 4$

$$\gcd(28 \times 4) = 7^{\min(a,b)} \times 2^{\min(a,b)}$$

$$= 7 \times 2$$

$$= 14$$

$$2 \overset{4}{8} | 2^5$$
$$16$$

56, 112

$56 = 2 \times 28$

$112 = 2 \times 56$

$= 2 \times 28 \times 2$

$= 2^2 \times 28$

$$\gcd(56, 112) = 2^1 \times 28$$

$$= 56$$

LCM → List common multiple

$$a = p_1^{a_1}, p_2^{a_2}, p_3^{a_3} \ldots p_n^{a_n}$$

$$b = p_1^{b_1}, p_2^{b_2}, p_3 \ldots p_n^{b_n}$$

$$\text{LCM}(a,b) = p_1^{\max(a_1,b_1)} p_2^{\max(a_2,b_2)} p_3^{\max(a_3,b_3)} \ldots$$

3

$$\begin{array}{r} 56 \\ \times 57 \\ \hline 392 \\ 2800 \\ \hline 3192 \end{array}$$

4

$56 = 2 \times 28$

$= 2 \times 2 \times 14$

$= \underline{2 \times 2 \times 2 \times 7} = 2^3 \times 7$

$114 = 2 \times 57$

$= 2 \times 3 \times 19$

$2^3 \times 7 \times 57$

$=$

Mersenne prime

$2^{P}-1 \rightarrow P$ prime

P যদি prime হয় তবে $2^{P}-1$ হবে prime

2006 সাল পর্যন্ত:-

$2^{30402457}-1 \rightarrow$ হল prime Number.

$\boxed{f(n) = n^2 - n + 41} \rightarrow$ prime
$n < 40$

Gold bach's conjecture,

Even Number $\longrightarrow$ Sum of two prime

Twin prime: দুইটা prime যাদের differen হবে 2

GCD :- Find x

$x|a$, $x|b$

or,

LCM: Find x

$a|x$, $b|x$

Relative prime: a ও b দুইটা number যাদের GCD = 1, তখন তাদের কে বলা হয় Relatively prime.

$28 = 7 \times 4$

$= 7 \times 2^2$

$14 = 7 \times 2$

Hash function:-

Lab implement

$\boxed{\dfrac{d \% 5 + 1}{4}}$ → Ruet gate → entry

cryptology:-

A → D

B → E

A, B, C

A B C

$(1+3) \% 26$

$4 \% 26$

4

RUET → 3 letter পর পর

↳ Encrypt

$\boxed{(P \% 26) + 3}$

$\boxed{(P+3) \% 26}$ → Encrypt

Home work

function টা কি one to one না onto। Inver

ফাংশন না থাকলে decrypt করা যাবে না।

Prime Number:-

$$\left.\begin{array}{l} P > 1, \quad q \nmid P \\ 1 < q < P \end{array}\right\} \text{prime}$$

$$\left.\begin{array}{l} P > 1, \quad q \mid P \\ 1 < q < P \end{array}\right\} \text{composite}$$

There are infinitely many primes.

যোগ করে,

$$a = b + k_1 m$$
$$c = d + k_2 m$$

$$(a+c) = (b+d) + m(k_1+k_2)$$

$$\Rightarrow (a+c) = (b+d) + mK$$

$$\Rightarrow (a+c) \equiv b+d \pmod{m}$$

গুণ করে,

$$a = b + k_1 m$$
$$c = d + k_2 m$$

$$ac = bd + bk_2 m + dk_1 m + k_1 k_2 m$$

$$\Rightarrow ac = bd + (bk_2 + dk_1 + k_1 k_2) m$$

$$\therefore ac \equiv bd \pmod{m}$$

Corrollary:-

$$(a+b) \bmod m = ((a \bmod m) + (b \bmod m)) \bmod m$$

and

$$ab \bmod m = ((a \bmod m)(b \bmod m)) \bmod m$$

$$114 \bmod 7$$
$$= (56+58) \bmod 7$$
$$= ((56 \bmod 7) + (58 \bmod 7)) \bmod 7$$
$$= (0+2) \bmod 7 = 2 \bmod 7$$
$$= 2$$

Theorem - 5

13-07-16
9th (A)

Modular Arithmetic :-

Theorem:- $a, b \in \mathbb{Z}$, $m \in \mathbb{Z}^+$, $a \equiv b \pmod m$ if

$(a \bmod m) = (b \bmod m)$

Theorem:- $m \in \mathbb{Z}^+$, $a, b \in \mathbb{Z}^+$ $\quad a \equiv b \bmod m \leftrightarrow$

$\exists k > 0 \; (a = b + km)$

From defination:

$a \equiv b \bmod m$

$\Rightarrow m \mid (a-b) \xrightarrow[\text{of division}]{\text{defination}} mk = (a-b)$

Again

$a = b + km$ [পক্ষান্তর করে]

k = positive integer.

Theorem:-

$m \in \mathbb{Z}^+$, $a, b \in \mathbb{Z}^+$

$a \equiv b \pmod m$ and $c \equiv d \pmod m \rightarrow$

$(a+c) \equiv b+d \pmod m$ and $ac \equiv bd \pmod m$

From

অনুরূপ,

$a \equiv b \pmod m \longrightarrow a = b + km$

$c \equiv d \pmod m \longrightarrow c = d + km$

Next Theorem:-

$$\frac{b}{a}$$

$$= \frac{\cancel{a} \times m}{\cancel{a}}$$

$$= m$$

কিন্তু $\frac{b}{a}$

$$= \frac{a \times m \times n}{a}$$

= অসম্ভব হবে না

$\boxed{\text{Defination : 2}}$

Non - gruand এর defination:-

m divides a - b

$a \equiv b \pmod{m}$

or, $a \bmod m = b \bmod m$

$\underline{17} \equiv \underline{5} \pmod{6}$

17 mod 6

= 5

5 mod 6

= 5

Next Theorem:-

$$\frac{b}{a}$$

$$= \frac{\cancel{a} \times m}{\cancel{a}}$$

$$= m$$

কিন্তু $= \frac{b}{a}$

$$= \frac{a \times m \times n}{a}$$

= সম্ভব হবে না

$\boxed{\text{Defination : 2}}$

Non - congruand এর defination:-

m divides $a - b$

$a \equiv b \pmod{m}$

or, $a \bmod m = b \bmod m$

$\underline{17} \equiv \underline{5} \pmod{6}$

17 mod 6

$= 5$

5 mod 6

$= 5$

Generated by CamScanner from intsig.com

⇒ n এর উপর power 1 থাকলে Linear complexity

⇒ n " " power 1 এর বেশি " polynomial "

$\boxed{\text{Integer and division:-}}$

$\boxed{\text{Integer and division:-}}$

$\exists_c (ac = b)$ if $a | b$

যদি $a \overline{) b (} c$ হয়
0

তাহলে $ac = b$ হবে।

$\boxed{\text{Theorem - 1}}$

$a, b, c \in \mathbb{Z}$

if $a|b$ and $a|c$ then $a|(b+c)$

$$b = na \qquad n, m \in \mathbb{Z}$$
$$c = ma$$
$$\overline{(b+c) = (n+m)a}$$

$$\Rightarrow (b+c) = Ma$$

defination অনুসারে,

$$a|(b+c)$$

$$\frac{n}{2^p} = 1$$

$$\Rightarrow 2^p = n$$

$$\Rightarrow \log_2 2^p = \log_2 n$$

$$\therefore p = \log_2 n$$

$$\therefore f(n) = \log_2 n$$

$$\therefore f(n) \text{ is } O(\log_2 n)$$

Table টা দেখাবা

দুইটা user input যেইটা বড় সেইটা displ

$a, b \longrightarrow 1$

if $(a>b) \longrightarrow 1$

printf (a)

else

printf (b)

$\}+1$

$f(n) = 3$

দুইটা matrix এর গুণ :-

int a[10][10], b[10][10];

for (i=0

$$C_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$$

$1 \longrightarrow n^2$

$n^2 \longrightarrow n \times n^2$

$= n^3$ (polynomial

12-07-16
8th (E) day

দুই ধরনের complexity

(i) Time complexity (worst case, average case, base case)

(ii) space "

Maximum Element :-

Example 3.1 :-

procedure max ($a_1, a_2, a_3, \ldots a_n$: integer)

max := a_1 ——— 1

for i = 2 to n ——— n-1

if max < a_i then max := a_i $\longrightarrow m \le n-1$

{max is the largest element}

$n-1+1+n-1$

$= 2n-1$

Here Best, average and worst case, $f(n) = 2n-1$

$f(n)$ is $O(n)$

এখানে max না নিলে data একটা loss হবে ফলে space complexity কমবে।

Next example :-

Linear search algorithm :-

*** প্রত্যেকটা উদাহরণ বানায়. practice :-

Binary search algorithm

2	6	9	12	15

1st $\longrightarrow$ $n/2$

2nd $\longrightarrow$ $n/2/2 = n/4$

3rd $\longrightarrow$ $n/2^3 = n/8$

p step = $n/2^p = 1$

-07-16
8th (E) day

দুই ধরনের complexity

(i) Time complexity (worst case, average case, base case)

(ii) space ,,

Maximum Element:-

Example 3.1:-

procedure max ($a_1, a_2, a_3, \ldots a_n$: integer)

max := a_1 —— 1

for i = 2 to n —— n-1

if max < a_i then max := a_i → $m \le n-1$

{max is the largest element}

$n-1+1+n-1$

$= 2n-1$

Here Best, average and worst case, $f(n) = 2n-1$

$f(n)$ is $O(n)$

এখানে max না নিলে data একটা loss হবে, ফলে space complexity কমবে।

Next example:-

Linear search algorithm:-

** প্রত্যেকটা উদাহরণ বানাও. practice:-

Binary search algorithm

2	6	9	12	15

1st → $n/2$

2nd → $\frac{n/2}{2} = n/4$ p step

3rd → $n/2^3 = n/8$

52

$n+2 > n$ $\quad c=1$

$f(n) \geq c\, g(n)$ $\quad g(n)=n$

$n > 0$ হলে হবে অত

$\therefore f(n)$ is $\Omega(n)$

অর্থাৎ upper limit n and lower

$\therefore$ $f(n)$ is $\Theta(n)$

=0=

- ~~3.3 chapter~~

Linear search এর জন্য
best case
worst "
Avg "
Binary search এর জন্য
Best case → 1
worst " → logn
Avg "

3.3 → complexity Analysis + table যে দেখ
অনির্ণেয় problem class NP prob

problem এর solution পাওয়া যায় না check করা যায়, একটা result ঠিক কি না।

NP complete problem → এই problem এর যদি solve করা যায়, তাহলে সব problem solve করা যাবে (~~~~ + NP problem

Growth function combination:-

if, $f_1(x)$ is $O(g_1(x))$

$f_2(x)$ is $O(g_2(x))$

$(f_1 + f_2)x$ is $O(?)$

From defination,

$$\cancel{f_1(x} \; f_1(x) \le c_1 \, g_1(x)$$

$$f_2(x) \le c_2 \, g_2(x)$$

$$(f_1 + f_2) \cdot x \le c_1 g_1(x) + c_2 g_2(x)$$

$$\le C g_1(x) + C g_2(x)$$

$$\le C (g_1(x) + g_2(x))$$

$$\le C (g_1(x) + g_1(x))$$

$$\le 2C g_1(x)$$

=0=

if $f_1(x)$ is $O(g_1(x))$

$f_2(x)$ is $O(g_2(x))$

$(f_1 \cdot f_2)x$ is $O(?)$

=0=

$f(n) = n+2$ is

$\Omega(?)$

$n > K$

$f(n) \ge C g(n)$

$$f(n) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0,$$

$$= x^n \left(a_n + \frac{a_{n-1}}{x} + \frac{a_{n-2}}{x^2} + \cdots\right)$$

$$\le x^n \left(a_n + a_{n-1} + a_{n-2} + \cdots\right)$$

$$f(n) \le C x^n, \quad x > 0$$

$f(n)$ is $O(x^n)$

= 0 =

তোমার একটা Algorithm এর complexity

$$f(x) = 2x^2 + 2x + 2$$

$$= a_n x^n + a_{n-1} + a_{n-2} x^{-2}$$

(n=

তাহলে অবস্থার complexity,

$f(n)$ is $O(x^2)$

$$f(n) = 1 + 2 + 3 + \cdots + n$$

$$= \frac{n}{2}(n+1)$$

$$= \frac{1}{2}n^2 + \frac{1}{2}n$$

$\therefore f(n)$ is $O(n^2)$

= 0 =

$$f(n) = n!$$

$$\cancel{= n(n-1)(n-2)\cdots 1}$$

$$= 1 \cdot 2 \cdot 3 \cdots n$$

$$\le n \cdot n \cdot n \cdots n$$

$$\le n^n$$

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$f(n) \in O(g(n))$ 	$\boxed{(c, k) \to \text{witness}}$

n = number of Data

Linear search Algorithm:-

$i = 1 \longrightarrow 1$

while (n)

$n \longrightarrow n$

$\text{if} \longrightarrow 1$

$f(n) \longrightarrow n+2 \leq n+n$ where $n \geq 2$

$\cancel{n+n}\, f(n) \leq 2n$ where $n \geq 2$

$n = g(n)$

$fn \leq 2g(n)$ where $n \geq 2$; n

From, defination,

$c = 2$

$k = 2$

$$\boxed{f(n) \text{ is } O(n) \quad (\text{Ans!})}$$ → v. v. I

$f(n) = n+2$

prove $f(n)$ is $O(n^4)$ → v. v. I.

$f(n) = n+2 \leq n^2 \quad n \geq 2$

$f(n) = n+2 \leq n^3 \quad n \geq 2$

$f(n) = n+2 \leq n^4 \quad n \geq 2$

prove, $f(n)$ is $O(1)$? → u no

Generated by CamScanner from intsig.com

Greedy Algorithm

Greedy Algorithm:-

=0=

~~কিছু কিছু~~ problem আছে ~~সাথে~~ যাদের কখনও solve হ
The Halith problem

Growth of function:-

~~Paul~~ paul Backman
Edmund Landau
Donald knuth
এরা আবিষ্কার কর

$O \longrightarrow$ Landau symbol

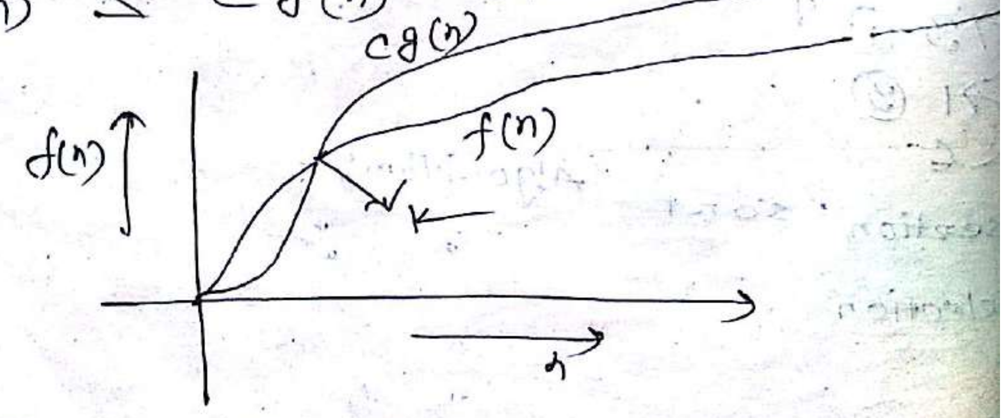
~~θ, δ~~

$\theta, \Omega \longrightarrow$

$O \rightarrow$ Big o notation

$f(n) = n^2 + 2n$

$f(n) \leq c\,g(n)$ where $n > k$



$f(n)$ is $O(g(n))$

$AD^2 = AC^2 + CD^2$

$AD = \sqrt{\frac{1}{4}AB^2 + CD^2}$

এভাবে AD এর মান বের করা যায়

Same ভাবে BD " " " "

⇒ π এর মান বের করার জন্য programme Lab-এ

⇒ Golden ratio " " "

=0=

Algorithm.

Number of list = $\{a, a_1, a_2, \ldots a_n\}$

Max বের করার Algorithm,

~~1. Take the input~~

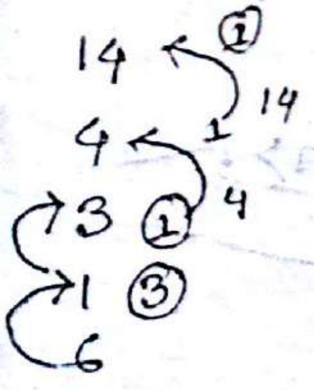
Linear search algorith:-

Binary search Algorithm:

এখানে ডাটাগুলো sorted থাকবে,

$a_1 > a_2 > a_3 > a_4 > \ldots > a_n$

Bubble sort Algorithm:-



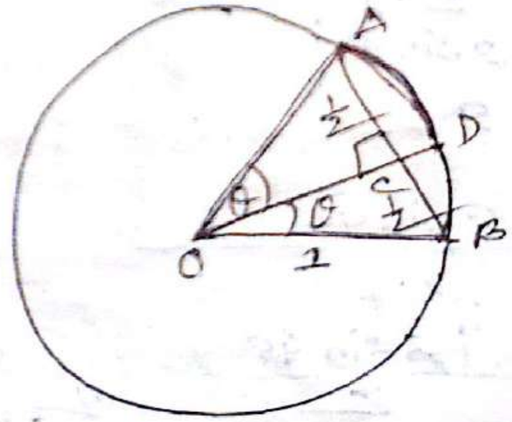
Insertion sort Algorithm:-

Selection " " :-

$$\pi = \frac{32 \sin 11 \cdot 12}{2 \cdot 1} =$$

$= 0 =$

বাইরের hexagon ও ভেতরের hexagon এর মধ্যে থাকে তারই π ।



$$\pi = \frac{n \sin 2\pi/n}{1/2 \cdot 2}$$

$$= n \sin \frac{2\pi}{n}$$

$AO^2 = OC^2 + AC^2$

$1^2 = OC^2 + AC^2$

$\Rightarrow OC = \sqrt{1 - 1/4 AB^2}$

$OD = OC + CD$

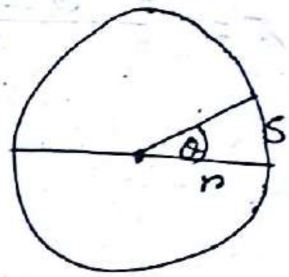
$CD = 1 - OC$

29-05-16

8th (A) day

(95)

$\pi \rightarrow$ calculation



$\pi = \frac{\text{পরিসীমা}}{\text{ব্যাস}}$

$= \frac{2\pi}{2r}$

$= \frac{\text{পরিসীমা}}{\text{ব্যাস}}$

$s = r\theta \quad [\sin\theta \approx \theta]$

$s = \theta_1 + \theta_2 + \theta_3 + \cdots + \theta_n$

[যখন, $r = 1$]

$= 2\pi$

$\theta = 90$

$s_1 = \sin\theta_1$

$= \sin 90$

$\pi = \frac{4\sin 90}{2 \cdot 1} \quad [r = 1]$

$= 2\sin 90$

$= 2$

$\pi = \frac{8\sin 45}{2 \cdot 1}$

$= \frac{\cancel{8}^{4}\sin 45}{\cancel{2}}$

$= 4\sin 45 = \cancel{4}^{2} \cdot \frac{1}{\cancel{2}}\sqrt{2}$

$= 2\sqrt{2}$

$\theta \longrightarrow 0$

$\pi = \frac{n\sin\theta}{2}$

$\lim_{\theta \to 0} \frac{n\sin\theta}{2} \qquad \frac{360}{16}$

$\pi = \frac{16\sin 22.25}{2 \cdot 1}$

$= \cancel{3.1416}$

(44)

~~Chan~~ Chadnowsky → brother

28-05-16
7th (E) day

Chapter - 3

The Fundamentals :-

Algorithm & it's complexity

Integer (Number theory)

Matrices

$\int \xrightarrow{\text{error}} \sum$

continuous → computerized করতে পারি না

discreate → computerized করতে পারি

Error কমাতে গেলে complexity বেড়ে যাচ্ছে

$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ → 3 dimension vector

3 dimension পর্যন্ত আমরা visualize করতে পারি

Function অর্থ হল maping.

** Algorithm:- থাকার বৈশিষ্ট্য গুলো / শর্তগুলো বই থেকে মুখস্থ করতে হবে।

CT2 → এর সিলেবাস
CT1 + study for CT2 ***

π এর মান

2013 → 12.1 trilion ~~12.1×10~~.

প্রথম π use হয়েছিল → $2\pi r$ → equation - এ

প্রথম 100 digit π এর calculate করা হয়েছিল :-

Arctan series :- (John Machin হলো আবিষ্কারক)

$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} + \cdots$

$\tan^{-1}(1) = 1 - 1/3 + 1/5 + \cdots$

$\pi/4 = \tan^{-1}(1)$

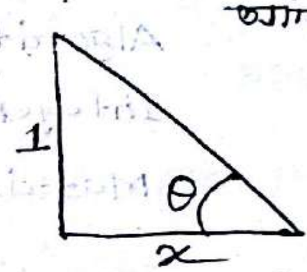
$= 4\tan^{-1}(1/5) - \tan^{-1}(1/239)$

$= 4 \times X - X$

5 টা term নিব

5 টা " "

এইসূত্র দিয়ে



$\tan\theta = \frac{1}{x}$

$\Rightarrow \tan\pi/4 = 1$

$\Rightarrow \pi/4 = \tan^{-1}(1)$

$\pi = 416x - 4y$

=0=

Libnin & Gregory :-

$$\int_0^1 \frac{dx}{1+x^2} = \tan^{-1}(1) = \pi/4$$

convergency Arctan series এর ভালো, কারণ অনেক digit পাওয়া যাবে decimal এর পর।

$$\int_0^1 \frac{dx}{(1+x^2)} = \int_0^1 1 + x^2 - x^4 + x^4 - x^5 + \cdots$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} = 1 - 1/3 + 1/5 - \cdots$$

42

Grandi's servies:-

$S = 1 - 1 + 1 - 1 + \cdots \infty$

$$S = \frac{1}{1-(-1)} = \frac{1}{2}$$

$= 0 =$

$S = 1 + 2 + 3 + \cdots \infty$

$S = 1 + 2 + 3 + \cdots \infty$

$0 = 1 + 1 + 1 + \cdots \infty$

$\boxed{1 + 1 + 1 + 1 + \cdots \infty = ?}$ H.W.

$$\sum_{k=50}^{100} k = ?$$

$$\frac{\cancel{1}50 \times \cancel{1}51}{2} = \frac{n(n+1)}{2}$$

$= \cancel{11,325}$ $\cancel{1275}$

$= 11,325$

finite : A set S is finite with cardinality $n \in \mathbb{N}$ if there is a bijection from the set $\underbrace{\{0, 1, \cdots, n-1\}}_{\text{domain}}$ to $\underbrace{S}_{\text{co.domain}}$

A set ~~A~~ infinite if it is not finite

$$S = \begin{cases} \dfrac{a(1-r^n)}{1-r} & r<1 \\ \dfrac{a(r^n-1)}{1-r} & r>1 \\ na & r=1 \end{cases}$$

$n \longrightarrow \infty$ -এ যায় এবং $r<1$ হয় তখন,

$$S = \frac{a}{1-r}$$

$$\underbrace{1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+ \text{------} \infty}$$

→ এই total portion টা অসীমে 1 হবে।

$$= \frac{1}{1-1/2}$$

$$= 2 \text{ (Ans)}$$

$1+2+3+ \text{------} \infty = -\frac{1}{12}$ Ramanujaan's নিয়ে।

~~কটু পরে~~ প্রমাণ করব,

$$S = 1+2+3+4+ \text{------} +\infty$$

$$4S = \quad 4+ \quad +8+ \text{------}$$

$$-3S = 1-2+3-4+ \text{------} \infty$$

$$\frac{1}{(1+x)^2} = (1-2x+3x^2-4x^3+ \text{------}) \quad [-1 \le x \le 1]$$

$$\Rightarrow \frac{1}{(1+1)^2} = (1-2+3-4+ \text{------} \infty)$$

$$\Rightarrow \frac{1}{4} = 1-2+3-4+ \text{------} \infty$$

$$-3S = \frac{1}{4}$$

$$\Rightarrow S = -\frac{1}{12}$$

$$1+2+3+ \text{------} \infty = -\frac{1}{12}$$

$1+2+ - \text{———} + 100$

$1+100 = 101$

$2+99 = 101$

$3+99 = 101$

$50 \times 101 = 5050 \longrightarrow$ complexity হল $O(1)$

$$= na + d \left\{ \frac{n(n-1)}{2} \right\}$$

$$= \frac{1}{2} n \left\{ 2a + d(n-1) \right\}$$

$$= \frac{1}{2} n \left\{ a + a + d(n-1) \right\}$$

$a + d(n-1) = l \longrightarrow n$ তম পদ

$$= \frac{1}{2} n \left\{ a + l \right\}$$

$= 0 =$

$$S = a + ar + ar^2 + ar^3 + - \text{———} + ar^{n-1}$$

$$= a \sum_{i=0}^{n-1} r^i$$

$$rS = ar + ar^2 + ar^3 + ar^4 + - \text{———} + ar^n$$

$$S - rS = a - ar^n$$

$$S(1-r) = a(1-r^n)$$

$$\therefore S = \frac{a(1-r^n)}{(1-r)} \quad [r<1]$$

Discrete math

11-05-16
7(A) day

Sequence & summessions:

S $\rightarrow$ 1 3 5 7 ----------

$f(n) = 2n+1$ দিয়ে represent হয়েছে $n \in \mathbb{N}$

Sequence: A sequence is a function from a subset of a set of integers to a set S

A geometric progression:-

$a, ar, ar^2, ar^3, \ldots$

An arithmetic progression:-

$a, a+d, a+2d, a+3d, \ldots$

Summession,

$a + ar + ar^2 + ar^3 + \ldots$

$a + (a+d) + (a+2d) + (a+3d) + \ldots$

$\Sigma_{i=1}^{n} i = \sum_{i=1}^{n} i \longrightarrow$ summession discreat এর জন্য

$\int_1^n \longrightarrow$ " continuous " "

$\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + 3^2 + 4^2 + \ldots$

$S = a + (a+d) + (a+2d) + (a+3d) + \ldots\ldots a+(n-1)d$

$= na + d\,(1+2+ \ldots\ldots (n-1))$

$= na + d \times \frac{(n-1)(n-1+1)}{2}$

$= \cancel{na} +$

প্রমাণ নিচে

(58)

Example→ 4.4

$$y = ae^{bx}$$

$X=$ x	y	$Y=$ $\ln y$	x^2	xY
2	4.077	1.405	4	2.810
4	11.084	2.405	16	9.620
6	30.128	3.405	36	20.430
8	81.897	4.405	64	35.240
10	222.62	5.405	100	54.050
30		17.025	220	122.150

$$y = ae^{bx}$$

$$\Rightarrow \ln y = \ln a + bx$$

$$Y = a_0 + a_1 x \quad (\ln y = Y,\ a_0 = \ln a,\ b = a_1$$

$$5a_0 + 30a_1 = 17.025$$

$$30a_0 + 220a_1 = 122.150$$

$$a_0 = 0.405,\quad a_1 = 0.5$$

$$a = e^{a_0} = e^{0.405} = 1.499$$

$$b = a_1 = 0.5$$

(Ans.)

Degree → 2 থাকলে, 4 টা নিব,
4 টা → x, x^2, x^{n+1}, x^{2n}
x^5 এর বের করতে বললে,
x, x^2, x^6, x^{10} বের করব,

=0=

Example → 4.3

x	y	x^2	x^3	x^4	xy	x^2y
0.0	1.0	0	0	0	0	0
1.0	6.0	1	1	1	6	6
2.0	17.0	4	8	16	34	68
Σ 3	24	5	9	17	40	74

Now,
$$3a_0 + 3a_1 + 5a_2 = 24$$
$$3a_0 + 5a_1 + 9a_2 = 40$$
$$5a_0 + 9a_1 + 17a_2 = 74$$
$$a_0 = 1, \; a_1 = 2, \; a_3 = 3$$

$$Y = 1 + 2x + 3x^2$$ (Ans)

Degree → 2 থাকলে, 4 টা নিব,

4 টা → x, x^2, x^{n+1}, x^{2n}

x^5 এর বের করতে বললে,

x, x^2, x^6, x^{10} বের করব,

=0=

Example → 4.3

	x	y	x^2	x^3	x^4	xy	x^2y
	0.0	1.0	0	0	0	0	0
	1.0	6.0	1	1	1	6	6
	2.0	17.0	4	8	16	34	68
Σ	3	24	5	9	17	40	74

Now,

$$3a_0 + 3a_1 + 5a_2 = 24$$
$$3a_0 + 5a_1 + 9a_2 = 40$$
$$5a_0 + 9a_1 + 17a_2 = 74$$
$$a_0 = 1, a_1 = 2, a_3 = 3$$

$$Y = 1 + 2x + 3x^2$$

(Ans.)

(38)

11-05-16
7 (A) day

SZM mam

Nonlinear curve fitting :-

(i) power function $\longrightarrow y = x^c$

(ii) polinomial of nth degree $\longrightarrow a_0 + a_1 x + a_2 x^2 + \cdots$

= 0 =

$y = ax^c$

$\Rightarrow \log y = \log a + c \log x$

$\Rightarrow Y = \log y$

$a_0 = \log a$

$a_1 = c, \; X = \log x$

$Y = a_0 + a_1 x$

$Y = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$

$S = [y_1 - (a_0 + a_1 x_1 + a_2 x_1^2 + \cdots + a_n x_1^n)]^2$
$+ [y_2 - (a_0 + a_1 x_2 + a_2 x_2^2 + \cdots + a_n x_2^n)]^2$

Generalized formula:-

$m a_0 + a_1 \sum_{i=0}^{m} x_i + a_2 \sum_{i=1}^{m} x_i^2 + \cdots + a_n \sum_{i=1}^{m} x_i^n = \sum_{i=1}^{m} y_i$

$a_0 \sum_{i=1}^{m} x_i + a_1 \sum_{i=1}^{m} x^2 + \cdots + a_n \sum_{i=1}^{m} x_i^{n+1} = \sum_{i=1}^{m} x_i y_i$

$a_0 \sum_{i=1}^{m} x_i^n + a_1 \sum_{i=1}^{m} x_i^{n+1} + \cdots + a_n \sum_{i=1}^{m} x_i^{2n} = \sum_{i=1}^{m} x_i^n y_i$

10-05-16
6(E) day

Sequence and summation:-

Sequence দিয়ে দিলে equation বের করতে হবে?

Arithmetic progression:-

$a, a+d, a+2d, a+3d$

Geometric progression:-

$a, ar, ar^2, \text{-----------}$

=0=

Odd number, $2n+1$ $n \in N$

$N = \{0, 1, 2, 3, \ldots\}$

Fibonacci sequence,

$1+1+2+3+5+ \text{-------}$

$5+6(n-1)$
$= 5+6n-6$
$= 6n-1$

Golden ratio,

$$\frac{F_{i+1}}{F_i}$$

Summession:

$1+2+3+ \text{-------} +n = ?$

$$\sum_{i=1}^{n} i = ?$$

$5+6(n-1)$
$= 5+6n-1$
$=$

** Prove that if x is a real number, then $\lfloor 2x \rfloor = \lfloor x \rfloor + \lfloor x + \frac{1}{2} \rfloor$

Let,

$x = n + \epsilon$ [where n is a positive integer and $0 \le \epsilon < 1$]

We first consider the case, $0 \le \epsilon < \frac{1}{2}$

In this case, $2x = 2n + 2\epsilon$ and

$\lfloor 2x \rfloor = 2n$ $[\because 0 \le 2\epsilon < 1]$

Again, $x + \frac{1}{2} = n + \frac{1}{2} + \epsilon$

$\Rightarrow \lfloor x + \frac{1}{2} \rfloor = n$ $[\because 0 < \frac{1}{2} + \epsilon < 1]$

$\therefore \lfloor 2x \rfloor = 2n$ and $\lfloor x + \frac{1}{2} \rfloor = n$

$\therefore \lfloor x \rfloor + \lfloor x + \frac{1}{2} \rfloor = n + n = 2n$

The second case, $\frac{1}{2} \le \epsilon < 1$,

In this case, $2x = 2n + 2\epsilon$

$= (2n+1) + (2\epsilon - 1)$

$[\because 0 \le 2\epsilon - 1 < 1]$

$\lfloor 2x \rfloor = 2n+1$

$\lfloor x + \frac{1}{2} \rfloor = \lfloor n + (\frac{1}{2} + \epsilon) \rfloor$

$= \lfloor (n+1) + (\epsilon - \frac{1}{2}) \rfloor$

$[\because 0 \le \epsilon - \frac{1}{2} < 1]$

$= n + 1$

$\lfloor 2x \rfloor = 2n+1$ and $\lfloor x \rfloor + \lfloor x + \frac{1}{2} \rfloor = n + (n+1)$
$= 2n+1$

Inverse function:-/ bijection

Defination :-

One to one আর যদি onto হয় তবে inverse function হয়

Onto → surjective

Bijection / one to one correspondence.

$f: \mathbb{R} \to \mathbb{R}$
$f(x) = x^2$
Find $f^{-1} = ?$

This is not one to one and it is not inverse function.

ceiling

$\lceil x \rceil$

$x < n$

↓

$\mathbb{Z}$

$\lceil 4.5 \rceil$

5

floor function

$\lfloor x \rfloor$

$\lfloor 4.5 \rfloor$

4

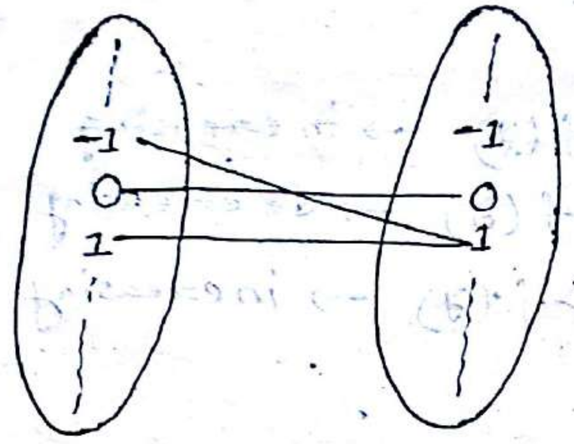
বাসার কাজঃ $\cancel{\lfloor 2x \rfloor}$. $\lfloor 2x \rfloor = \lfloor x \rfloor + \lfloor x + \frac{1}{2} \rfloor$

prove করে নিয়ে আসতে হবে

এই পর্যন্ত CT

$f: \mathbb{R} \longrightarrow \mathbb{R}$

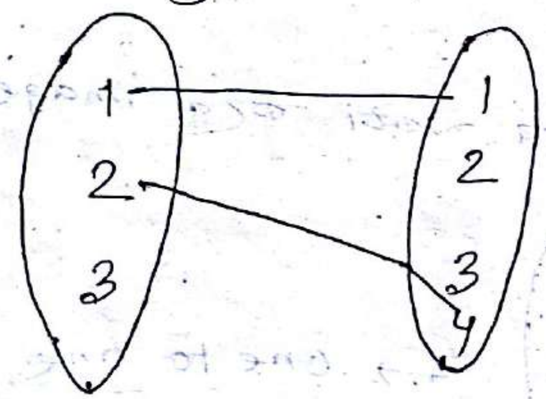
$f(x) = x^2$ onto



(Not onto,
Not one to one

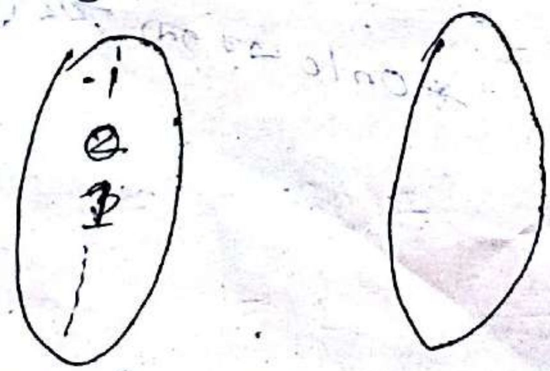
$f: \mathbb{Z}^+ \longrightarrow \mathbb{Z}^+$

$f(x) = x^2$



$f: \mathbb{R} \longrightarrow \mathbb{Z}^+$

$f(x) = x^2$



Not function

কিছু element এরকম co-domain নেই।

One to one / injective and onto function

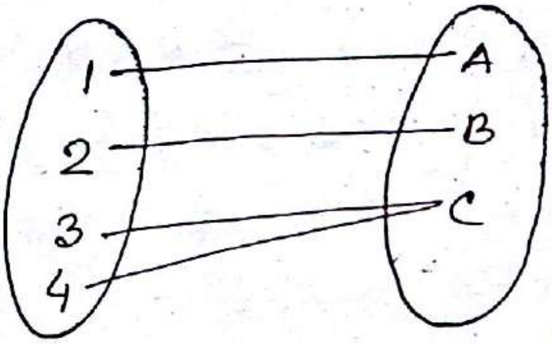
$f : A \to B,\ f(x)$

$\cancel{f(x)}$

$x < y \Rightarrow f(x) < f(y) \to$ ~~increasing~~

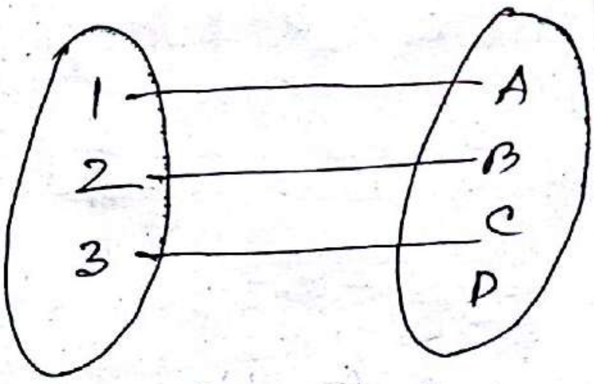
$x > y \Rightarrow f(x) < f(y) \to$ decreasing

$x < y \Rightarrow f(x) > f(y) \to$ increasing

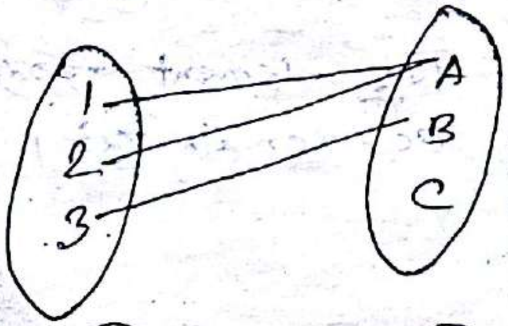


→ Not one to one

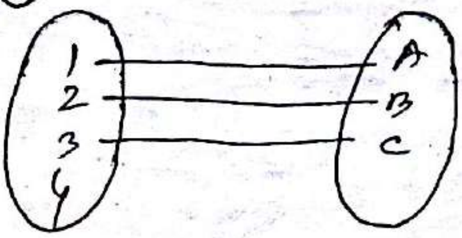
প্রত্যেকটা co-domain এর একটা করে image থাকবে



→ one to one



onto এর জন্য এইটে দেখা



→ function-ই নয়

$f: \mathbb{Z}^+ \longrightarrow \mathbb{Z}$

$f(x) = x^2 \longrightarrow$ one to one function

Onto function →

function defination 1 →

$f: A \rightarrow B$, a, $f(a)$

=0=

int main ( ) { → void

co-Domain

return $\boxed{0;}$ → Range

}

function नहीं Domain → void

co-Domain → int

Range → 0

Defination 3 → $f_1(x) : A \rightarrow B$, $f_2(x) : A \rightarrow B$

$f_1(x)$ $f_2(x)$

$f_1(x) + f_2(x) = (f_1 + f_2)(x)$

$(f_1 f_2)x = f_1(x) \times f_2(x)$

=0=

Defination → 4

$S \subset A$ → S subset A - का

$f(s) \subset$ Range ; $(s \in S)$

26-04-16
5A-day

$\boxed{\text{Functions}}$

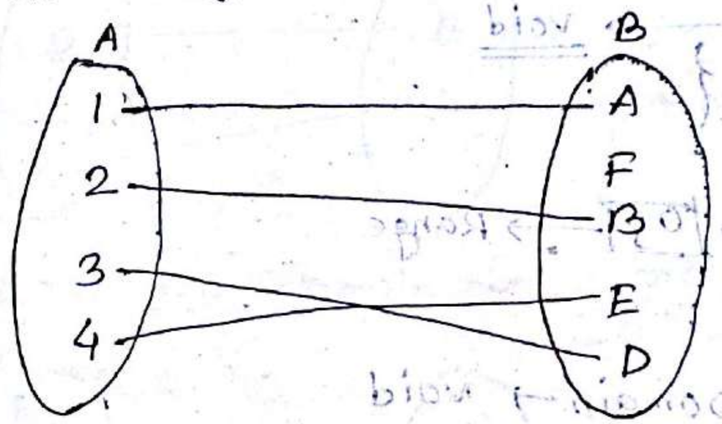
~~$f: A \to B$~~ $f: \underset{\text{Domain}}{\mathbb{R}} \to \underset{\text{co Domain}}{\mathbb{R}}$

$f(x) = x^2+1$

যার Domain এর জন্য একটা করে image থাকবে এটাই function

~~Relation এর মধ্যে~~ sub

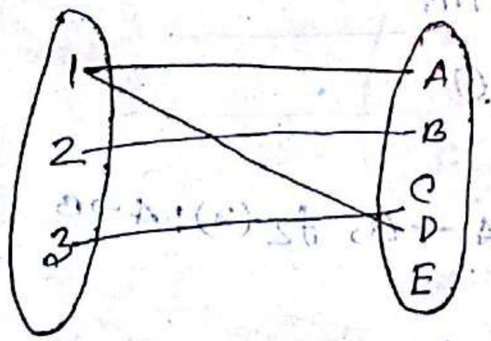
function তাহলে Relation এর sub set



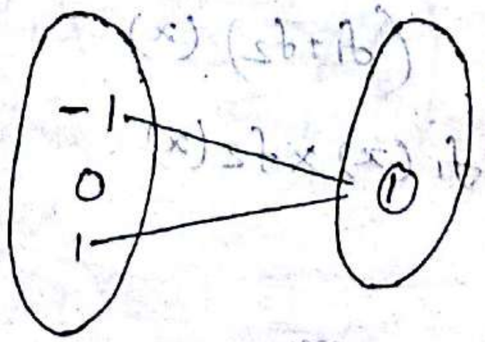
f ⊂ relation ⊂ A × B

→ ফাংশন

A, B, E, D → Range



→ ~~ফাংশন~~ না



$f(x) = x^2$

Range = {0, 1, 2, ~~...~~ }

= $\mathbb{N}$

= {0} ∪ $\mathbb{Z}^+$

Ex-5

$$x+y=2$$

$$\forall x \forall y \exists z \; Q(x,y,z) \rightarrow T$$

$$\exists z \forall x \forall y \; Q(x,y,z) \rightarrow F$$

Ex-6

$$\forall x \forall y \; ((x>0) \wedge (y>0) \longrightarrow (x+y)>0)$$

Or,

$$\forall x>0 \; \forall y>0 \; (x+y>0)$$

Or

$$\forall x \in \mathbb{Z}^+ \; \forall y \in \mathbb{Z}^+ \; (x+y>0)$$

Or,

$$\forall x \forall y \; (x+y>0) \text{ where}$$

$$x \in \mathbb{Z}^+, \; y \in \mathbb{Z}^+$$

Ex-8

$$\forall \epsilon>0 \; \exists \delta>0 \; \forall x \left(0<|x-a|<\delta \rightarrow |f(x)-L|\right)$$

Lab

$A \{1, 2, 3, \cdots\}$

y

$P(x)$ "$x+y=0$"

$x+2y+3$

$2, 0$

$= 0 =$

$x^2+y^2+z^2$

25

$(3, 4)$ or $(4, 3)$

$= 0 =$

$3n+1 \leftarrow$ কোন মানের জন্য even and odd

$= 0 =$

$\exists y \, \forall x \; P(x, y) \longrightarrow T$

$\forall x \, \exists y \; P(x, y) \longrightarrow T$

$= 0 =$

$\forall x \, \exists y \; P(x, y) \longrightarrow T$

$\exists y \, \forall x \; P(x, y) \longrightarrow T/F$

এটা true ওটা true

এইটা true. এটা False

Condition arise করে নিয়ে আসবে H.W.

(26)

$\exists x \exists y P(x, y)$

Algorithm All x All y () {

FOR x → All member of $\mathbb{R}$

FOR Y → All member of $\mathbb{R}$

IF P(x, y) is True

~~Return True~~ Return true

End

End ~~Return True~~.

End

Return False.

= 0 =

$\exists x \forall y P(x, y)$

End

End

End

Return true.

$\frac{3}{4}$ $\frac{-3}{-4}$ $(-1, -2, -3)$

$= 0 =$

$0 + 0 = 0$

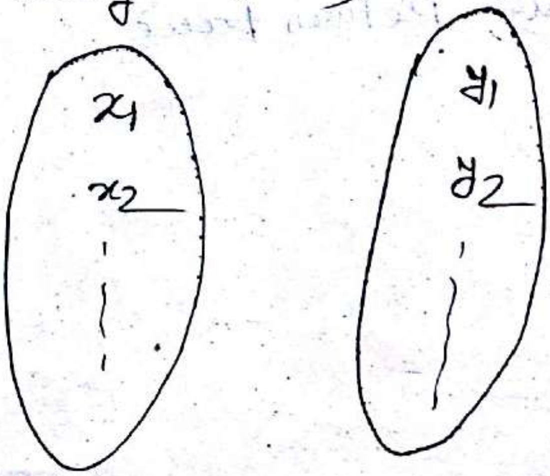
$0 + 1 = 1$

2 $\quad$ -1

3

4

$\forall x \, \exists y \; P(x, y)$



Algorithm All x ~~only~~ All y () {

FOR x → All member of $\mathbb{R}$

FOR y → All member of $\mathbb{R}$

IF $P(x, y)$ is True

Break

END

END

IF $P(x, y)$ is FALSE

Return FALSE

END

Return True.

25-04-16
4E-day

$$\forall x \; \exists y \; (x+y=0)$$

$P(x,y)$

Example-1

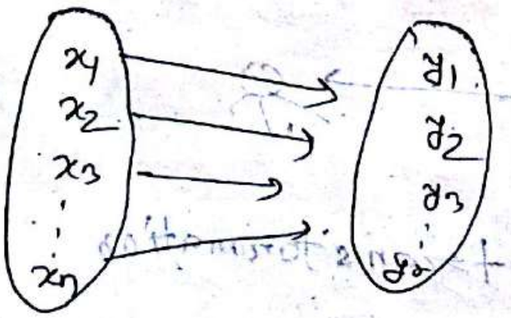
$$\forall x \; \forall y \; (x+y = y+x)$$

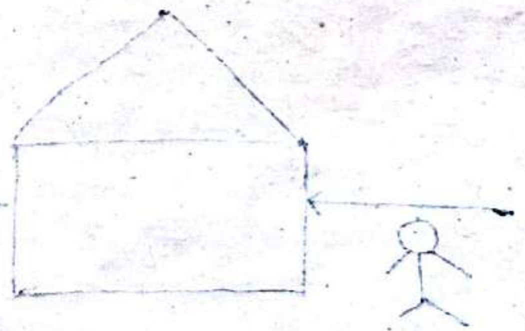
Example-2

$$\forall x \; \forall y \; ((x>0) \wedge (y<0) \rightarrow xy<0))$$

x_1
x_2
x_3
$\vdots$
x_n

$$\forall x \; \forall y \; P(x,y)$$





Algorithm: All x All y ( ) {
FOR $x \rightarrow$ All member of $\mathbb{R}$
FOR $y \longrightarrow$ all member of $\mathbb{R}$
IF $P(x,y)$ is FALSE
Return False.

Set কে computer - এ represent,

$A = \{1, 5, 7, 8\} = (0100010110000000)$

~~int A[]~~ এর limit $0 \le x_i \le 15$

x হল A এর element

$B = \{4, 3, 5\}$

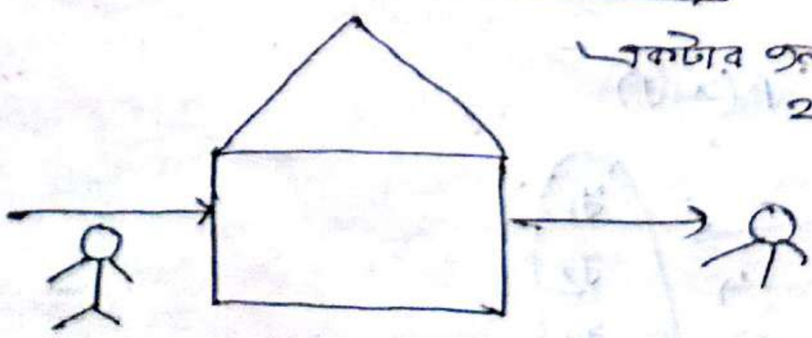
$A \wedge B$

$A \vee B$

A ও B এর মধ্যে common element না থাকলে তারা disjoint সেট

$\left\{ \text{Game devlope} \atop \text{সেটি গুলো bit wise দিয়ে করতে হবে} \right\}$

$\boxed{\text{Function}}$



একটার জন্য একটা imag থাকবে

Function ⟶ Map, transformation

$$\left.\begin{array}{l} f(x) = x^2 + 1 \\ f : A \rightarrow B \end{array}\right\} \text{function}$$

Generated by CamScanner from intsig.com

Set operation : কই ৫মেক

$A-B = \{x \mid x \in A \wedge x \notin B\}$

Identity law:-

$\boxed{\text{124 page}}$

De Morgans. → law → পড়ে নি।

$A \cup \emptyset = A$

$A \cup \emptyset = \{x \mid x \in A \wedge x \in \emptyset\}$

$= \{x \mid x \in A\}$

$= A$

$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup A_3 \cup \cdots \cup A_n$

$A_i = \{i+1, i+2\}$

$\bigcup_{i=1}^{3} A_i = \{\cancel{A_2 \cup A_3 \cup A_4 \cup A_5}\}$ 2, 3, 4, 5

$\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap A_3 \cdots \cap A_n$

$\bigcap_{i=1}^{3} A_i = \emptyset$

Page → 119

Truth set Quantifiers

$P(x) \rightarrow$ domain $\in$ set of integer

$P(n) \rightarrow$ " $|x| = 1$ "

result $\{1, -1\}$

=0=

$Q(x) \rightarrow$ domain $\in Z$

$Q(x) \rightarrow x^2 = 2$

result $= \phi$

=0=

$Q(x) \rightarrow$ doman $\in Q$

$Q(x) = x^2 = 2$

result $= \phi$

=0=

$Q(x) \rightarrow$ doman $\in Q'$

$Q(x) = x^2 = 2$

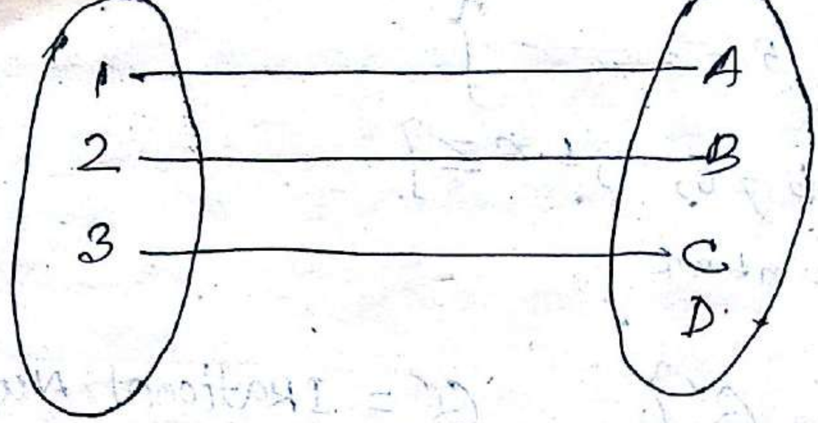
result $\{\pm\sqrt{2}\}$

=0=

$Q(x) \rightarrow$ domain $\in R$

$Q(x) = x^2 = 2$

result $\{\pm\sqrt{2}\}$



$A \subset B$

$x \mid (x \in A \rightarrow x \in B) \wedge (x \notin A \rightarrow x \in B)$

$\cancel{\emptyset \subset S}$ সব proper subset ই subset but সব subset ই proper sub set না

$\emptyset \subseteq S$

$= \{x \mid x \in \emptyset \rightarrow x \in S\} \longrightarrow T$

Vacuous proof

P	q	$P \rightarrow q$	
T	T	T	$\rightarrow$ D. proof
F	T	T	$\rightarrow$ vacuous proof.

$\forall x (x \in A \rightarrow x \in B) \wedge \exists x (x \notin A \wedge x \in B)$

Power set $A = \{1, 2, 3\}$

$P(A) \rightarrow$ সব সাব সেট হলো power set

$P(A) \rightarrow 2^3 = 8$

$Z^+ = \{1, 2, 3, 4, 5 \ldots\}$

$Q = \{x \mid a/b,\ b \neq 0,\ a, b \in Z\}$

$\hookrightarrow$ Rational Number

$R = \{N, Z, Q, Q'\}$ $\quad Q' =$ IRational Number

=0=

$S \subseteq S$

$\phi \subseteq S$

$\hookrightarrow$ subset

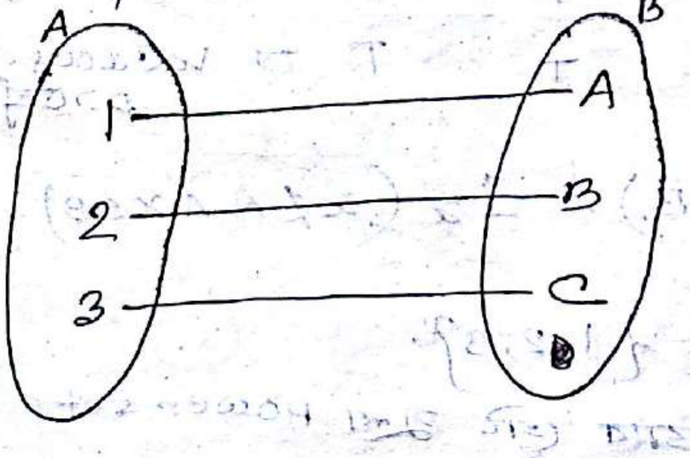
$\subset$

$\hookrightarrow$ proper subset

$A \subseteq B = \{x \mid x \in A \wedge y \in B\}$

$A \subset B = \{x \mid (x \in A \rightarrow y \in B) \wedge (x \notin A \rightarrow y \notin B)\}$

page → 114, 115



A সেটে যেগুলো আছে সেগুলো B সেটে আছে কিনা

$A \subseteq B$, $\{x \mid x \in A \rightarrow x \in B\}$

⑱

$n = 16$, if odd then $(3n+1)$, even $n/2$ করলে

las conclution হবে।

Basic structure, set, function sequences and sums

Set,

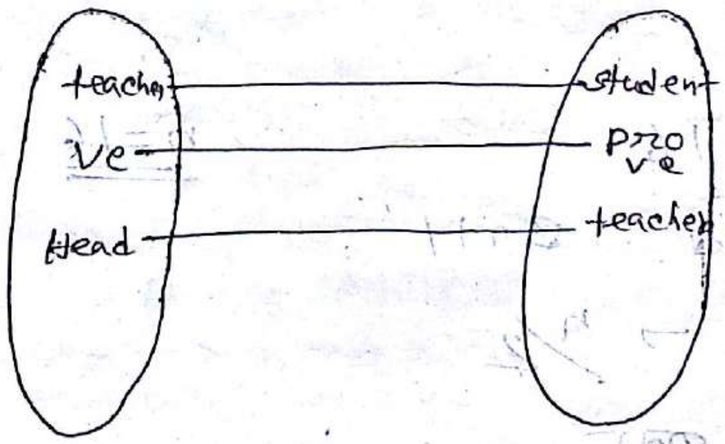
$A = \{1, 2, 3\}$

$B = \{B, C\}$

$A \times B = \{(1, B), (1, C), (2, B), (2, C), (3, B), (3, C)\}$

$R = \{(1, B), (2, B), (3, B)\}$

$R \subset (A \times B)$



combination ~~operation~~ এ উ আর main operation হবে

$N = \{0, 1, 2, \cdots\}$

$Z = \{\cdots -3, -2, -1, 0, 1, 2, 3, \cdots\}$

$N \subset Z$

$n = -b/a$

$a = 0$ হলে, $n = b/0$

then $an + b = 0$, False

conjectore: একটা proposition/সূত্র যেটা সত্য যেটা সত্য আমরা জানি, কিন্তু সেটা এখনও proof করা সম্ভব হয় নি।

$$x^n + y^n = z^n$$

$n = 2$, $x^2 + y^2 = z^2$

$n = 1$, $x + y = z$

$n = 3$, $x^3 + y^3 = z^3$

$n > 3$ এর জন্য কি হবে? এটা conjecture

$\cancel{30} = 0 =$

$3n+1$

Suppose, $n = 169$ $\underline{n = 16}$

if n odd $3n+1$

n even $\rightarrow n/2$

$7 \rightarrow$ এর জন্য

odd $\rightarrow 3 \cdot 7 + 1 = 22$

$22/2 = 11$

$11 \cdot 3 + 1 = 34$

$3 \times 17 + 1 = 51 + 1$

$= 52$

$26 \times 3 + 1 =$

19-04-16 (16)

4 (A) day

$\exists_y \forall x \; Q(x, y)$

$y = \{ a, b, c, d \}$

$x = \{ 1, 2, 3, 4 \}$

$x \in \mathbb{R}$

$\exists_y \{ \forall x \; Q(x, y) \}$

$1 + y = 0 \; , \; +2 + y = 0$

$\boxed{\text{Existence proofs}}$

constructive Exist:

Non constructive

$\times$	B	B	B	A
A	A	B	B	A
A	A	B	B	A

$\Rightarrow$ ওই game টা programme

$B \rightarrow$ winner

$A \rightarrow$ loser

uniqueness proofs:-

1. case property hold
2. " " NOT "

if a and b are real numbers and $a \neq 0$, then there is a unique real number r such that $ar + b = 0$

19-04-16 (16)
4 (A) दफ़ा

$\exists_y \forall x \, Q(x, y)$

$y = \{ a, b, c, d \}$

$x = \{ 1, 2, 3, 4 \}$

$x \in \mathbb{R}$

$\exists_y \{ \forall x \; Q(x, y) \}$

$1 + y = 0, \; +2 + y = 0$

$\boxed{\text{Existence proofs}}$

constructive Exist:

Non constructive

$\times$	B	B	B	A
A	A	B	B	A
A	A	B	B	A

$\Rightarrow$ गेमे game के programme

B $\rightarrow$ winner

A $\rightarrow$ loser

Uniqueness proofs:-

1. case property hold
2. " " NOT "

if a and b are real numbers and $a \neq 0$, then there is a unique real number r such that $ar + b = 0$

Proof method and strategy:-

Exhaustive proof and by case

Exhaustive proof

$(n+1)^2 \geq 3^n$ if n is a positive integer with $n \leq 4$

if n is an integer then $n^2 \geq n$

Existence proof:

একটা case এর জন্য আছে

There is a positive integer that can be written as the sum of cubes of positive integer in two different way.

$1729 = 10^3 + 9^3 = 12^3 + 1^3$

= 0 =

$x, y \longrightarrow$ irrational (অমূলদ)

$x^y =$ rational (মূলদ)

$x = \sqrt{2}$

$y = \sqrt{2}$

$x^y = (\sqrt{2})^{\sqrt{2}}$ irr / rational

যদি rational হলে তাহলে proved.

আর যদি irrational হয় তবে,

$x = \sqrt{2}^{\sqrt{2}}$

$y = \sqrt{2}$

$x^y = ((\sqrt{2})^{\sqrt{2}})^{\sqrt{2}}$

$= (\sqrt{2})^2 = 2$

Now Rational.

Indirect proof:-

Proof by contraposition

contraposition এর সূত্র $p \rightarrow q \equiv \neg q \rightarrow \neg p$

$\neg q$ যদি true করতে পারি, $\neg q \rightarrow \neg p$ করতে পারি, তবে তা হবে proof by contraposition

If n is an integeger and 3n+2 is odd, then n is odd.

$$3n+2 = 2k+1$$
$$\Rightarrow n = \frac{2k-1}{3}$$

এভাবে proof করতে পারছি না তাই Indirect proof করব,

Imagine,
n is not odd,
then $n = 2k$
then, $3n+2 = 6k+2$
$= \cancel{\text{even}}$
$= 2(3k+1)$

Contradiction:
যে কোন combination এর জন্য proposition টা false হবে

proof করতে বলল, p is true ?
তাহলে ধরি, $\neg p$ is false

$\neg p$	q	$\neg p \rightarrow q$
F	T	T

$$\forall x \forall y \forall z \, (x + (y+z) = (x+y) + z)$$

Introduction to proofs:-

Theorem:- is a true statement that can be shown to be true.

Propositions / facts / results → Less important theorem

Proof: A valid argument that establishes the truth of a theorm.

axioms / postulates: the statements used in a proof, we assume to be true.

Lemma: A less important theorem that is helpful in the proof of other results.

corollary: A theory that can be established directly from a theorem. that has been proved.

odd ← $a = 2k+1$
even ← $a = 2k$

Direct proof:

$p \to q \xrightarrow{\ T\ }$ Direct proof

"If n is an odd integer, then n^2 is odd"
proof it?

Direct proof:

$$n = 2k+1$$

$$n^2 = (2k+1)^2 = 4k^2 + 4k + 1$$
$$= 4(k^2 + k) + 1$$
$$= \text{even} + 1$$
$$= \text{Odd}$$

এটাই direct proof.

1. p, q, হচ্ছে দুইটা মান input নিতে / user input show করতে হবে

2. কয়টা input নিব সেটা user define করে দিবে
And / OR / XOR

1 1 1 → And

1 1 0 → And

1 0 1 ②

p	q	p ∧ q
1	0	~~0~~
0	1	0
0	0	0
1	1	1

1 0 0

0 1 1 if (

0 1 0

~~And~~ And (1, 1, 2, 2)

0 0 1 c = 1
if (2 == 2){
0 0 0 printf ("%d\n", c)

1

3. $(p \wedge q) \vee r \rightarrow r$ and (1, 1,
$(p \wedge q \vee r) \Rightarrow r$.

(10)

$((p \to q) \wedge q) \to p$

p	q	$p \to q$
T	F	F
F	T	T
F	F	T
T	T	T

$p \to q$	q	$(p \to q) \wedge q$
F	F	F
T	T	T
T	F	F
T	T	T

$(p \to q) \wedge q$	p	$((p \to q) \wedge q) \to p$
F	T	T
T	F	F
F	F	T
T	T	T

$12 + 3 - 2 \times 2$

12

+

3

p

∪

q

$\neg p \vee$

F

F

T

F

9

Rules of inference:-

fallacies premises

"if you have a current password, then you can log onto the network."

you have a current password.

Therefore

" you can log into the network ".

conclusion true হলে আর information true হলে
" false " " " false "

$p \rightarrow q$

If p

Ans q

$(p \wedge (p \rightarrow q)) \rightarrow q \rightarrow$ tattology

p	q	p → q
T	F	F
T	T	T
F	F	T
F	T	T

F	F	T
F	T	T
T	F	F
F	T	T

p	a	p →	q	r
T		T	F	F
F		F	T	F
T		F	T	T
F		–	T	F

Suma

Nested quantifiers

$\forall x \, \forall y \, (x+y = y+x)$ for all real Number

Example - 4

$Q(x,y) \longrightarrow x+y=0$

$\boxed{\exists y \, \forall x \, Q(x,y)} \longrightarrow$ সত্য

$\forall x \rightarrow$ সব real Number

$\exists y \rightarrow$ fixed একটা

$\exists y \, \exists x \, Q(x,y) \longrightarrow$ মিথ্যা

$\exists x \rightarrow$ সব real

$\exists y \rightarrow$ fixed

$\boxed{\forall y \, \forall x} \, Q(x,y)$

$\forall x \rightarrow$ All

$\forall y \rightarrow$ All

$P(x) \equiv "x+1 > x"$

$\forall x\, Px = ?$

for all real numbers (Domain)

সত্য হবে তাই এক্ষেত্রে $\forall x Px$ = True (1)

for all complex number এর ক্ষেত্রে,

$P(x) = x^2+1 > y^2$

$x \rightarrow$ ~~real~~ all

$y \rightarrow$ real

$\forall x\, P(x) = 0$

domain এর একটা false দেখাতে পারলে সেটা false

= 0 =

$P(x) = x > 3$

$\exists x\, P(x) = ?$ $\forall x\, P(x) = ?$

for all real Number

$\exists x\, P(x)$ = True (1) $\forall x\, P(x)$ = False (0)

= 0 =

$P(x) \equiv " x > 3 "$ Binding variables

$\rightarrow$ dependent

$P(x) \equiv " x + y > 3 "$

$\rightarrow$ free variables

$\rightarrow$ Independent

12-04-16

3 (A) day

⑥

Discrete mathmatics

$$\neg (p \vee (\neg p \wedge q))$$
$$= \neg p \cancel{\wedge} \neg (\neg p \wedge q)$$
$$= \neg p \wedge \neg(\neg p) \vee \neg q$$
$$= \neg p \wedge (p \vee \neg q)$$
$$= (\neg p \wedge p) \vee (\neg p \wedge \neg q)$$
$$= F \vee (\neg p \wedge \neg q)$$
$$= (\neg p \wedge \neg q) \vee F$$
$$= \neg p \wedge \neg q \quad [\text{identity law}]$$

Predicate:

x is greater than three.

variable — predicate

এটাকে লেখবো তখন $P(x)$

Page → 31 এর example → 1 দেখ

Quantifiers:-

Quantifiers
- universal quantifiers [$P(x)$ for all values of x in the domain]
  - domain এর একটা false দেখাতে পারলে false
  - '∀' is called universal quantifier
- Existenential quantifiers
  - [There is exists an element x in the domain such that $P(x)$]
  - '∃' → existential quantifier.

Proposition equivalence :-

Defination

(i) Tautology ✓

(ii) contradiction

(iii) contingency ✓

T
T
T
T

সব সত্য হলে → (i)

সব মিথ্যা " → (ii)

True, false mix → (iii)

p, q তখন (i) হবে যখন $p \leftrightarrow q$ সত্য হবে।

$\neg p \vee q$ $\qquad$ $p \rightarrow q$ $\qquad$ T

p	$\neg p$	q	$\neg p \vee q$
T	F	T	T
F	T	F	T
F	T	T	T
T	F	F	F

p	q	$p \rightarrow q$
T	F	T
F	T	T
F	T	T
F	F	T

$p \wedge T = p$

$p \vee T = T$

$p \wedge F = F$

$p \vee F = p$

11-04-16
2(E) day ①

$\boxed{\text{Discrete Matematics}}$

```
    1 1 0 = 6
XOR 0 1 1 = 3
    -----
    1 0 1
    Sum
```

```
   1 1 0
 + 0 1 1   And
 -------
  0 0 1 0
  carry
```

Left shift

~~0 1 0~~

```
0 1 0 1
0 1 0 0   and
-------
0 1 0 0
carry
```

```
0 1 0 1
0 1 0 0   XOR
-------
0 0 0 1
0 1 0 0
```

↓

0 0 0

```
0 0 0 0 1
0 1 0 0 0   XOR
---------
0 1 0 0 1 = 9
```

$\boxed{\text{এই process এ যোগ করে।}}$

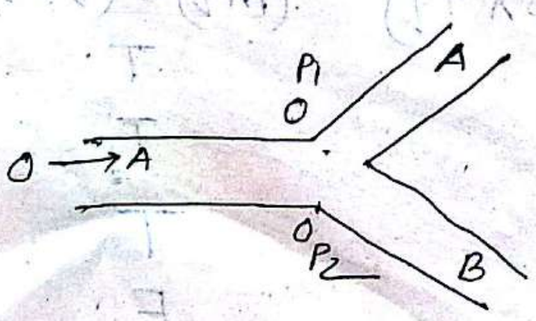
আবার,

```
0 0 0 0 1
0 1 0 0 0   And
---------
0 0 0 0 0
carry
```

1 1 1 = 7

Shift করলে 1 1 1 0 = 14

0 1 1 = 7/2 = 3



$P_1 \wedge P_2 = 0$

T F = 0

F T = 0

p	q	$p \wedge q$
T	F	F
F	T	F
F	F	F
T	T	T

q	p	$\neg p$	$q \vee \neg p$
F	F	T	T
T	F	T	T
F	F	T	T
T	T	F	~~F~~ T

p	$\neg p$	q	$\neg q$	$\neg p \wedge \neg q$
T	F	F	T	F
F	T	T	F	F
F	T	F	T	T
T	F	T	F	F

$(q \vee \neg p)$	$(\neg p \wedge \neg q)$	$(q \vee \neg p) \wedge (\neg p \wedge \neg q)$
T	F	F
T	F	F
T	T	T
T	F	F

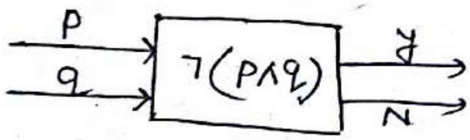
$(p \wedge q)$	$(q \vee \neg p) \wedge (\neg p \wedge \neg q)$	$(p \wedge q) \rightarrow (q \vee \neg p) \wedge (\neg p \wedge \neg q)$
F	F	T
F	F	T
F	T	T
T	F	F

$\neg p$ = Not p

$\sim p$ = Not p

$!p$ = Not p

$\neg(p \wedge q) \rightarrow$ আমি ম্যাম : ও CSE class নিচ্ছি না



Conditional statement :

$p \rightarrow q$ — If p, then q

p	q	r
T	T	T
T	F	F
F	F	T
F	T	T

$p \leftrightarrow q$ — If p and q

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

$(\leftrightarrow) \rightarrow$ Biconditional statement

$(p \wedge q) \rightarrow (q \vee \neg p) \wedge (\neg p \wedge \neg q)$

04-04-16
1(E)

proposition chapter টা দেখে আসবে?

111
+101 } computer কিভাবে এই কাজটা করে?

05-04-16
2(A)

ফ্লোচার্ট এর সাথে discrete math এর relation?

Discrete math কি?

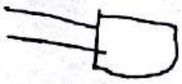
Linear programming কি?

Logic & proofs:-

proposition এর Opposite Fuzzy

proposition → True or False আছে

Fuzzy → True ~~or~~ and False both আছে

 → AND

 → OR

$P \oplus q$ → XOR

XOR → P ও Q বিপরীতধর্মী হলে True

p	q	r
1	0	1
0	1	1
1	1	0
0	0	0