Inverse function: $f(x) = 2x-3$.

Let, $y = f(x) = 2x-3$

$\therefore x = f^{-1}(y) = 2x-3$

$\therefore y = 2x-3$

$\Rightarrow 2x = y+3$

$\Rightarrow x = \frac{y+3}{2}$

$\therefore f^{-1}(y) = \frac{y+3}{2}$.

* Function composition:

$f(x) = 2x+1$.

$g(x) = x^2-2$

i) $g \circ f = g(f(x))$

$= g(2x+1)$

$= (2x+1)^2 - 2$

$= 4x^2+4x+1-2$

$= 4x^2+4x-1$

ii) $f \circ g = f(g(x))$

$= f(x^2-2)$

$= 2(x^2-2)+1$

$= 2x^2-3$

$\cancel{f(g)}\ (f \circ g)(2)$

$= 2(2)^2-3$

$= 2 \times 4 - 3$

$= 5$

* Let, m, a, b are integer. If $ac \equiv bc \mod m$ and $\gcd(c, m) = 1$, then $a \equiv b \mod m$

Since $ac \equiv bc \mod m$. So m must devide $(a-b) \cdot c$.

$\frac{ac-bc}{m} = k$

$\therefore ac - bc = mk$ — (1)

As, $\gcd(c,m) = 1$.

so, m devides $a-b$

$\boxed{\therefore a \equiv b \mod m}$ proved

* $7 \equiv 2 \pmod 5$ and $11 \equiv 1 \pmod 5$

$\frac{7-2}{5} = 1$ $\qquad$ $\frac{11-1}{5} = 2$

$\therefore 7 = 2 + 5 \cdot 1$ — (i) $\qquad$ $\therefore 11 = 1 + 5 \cdot 2$ — (ii)

(i) + (ii) 2(i).

$7 + 11 = 2 + 1 + 5 \cdot 1 + 5 \cdot 2$

$\Rightarrow 18 = 3 + 5(1+2)$

$\Rightarrow \frac{15}{5} = 3$

(i) $\times$ (ii) 2(i).

$7 \cdot 11 = (2 + 5 \cdot 1)(1 + 5 \cdot 2)$

$= 2 \cdot 1 + 2 \cdot 5 \cdot 2 + 5 \cdot 1 \cdot 1 + 5 \cdot 1 \cdot 5 \cdot 2$

$= 2 + 5(4 + 1 + 10)$

$77 = 2 + 5 \cdot 15$

$77 - 2$

Inverse function: $f(x) = 2x-3$.

Let, $y = f(x) = 2x-3$

$\therefore x = f^{-1}(y) = 2x-3$

$\therefore y = 2x-3$

$\Rightarrow 2x = y+3$

$\Rightarrow x = \frac{y+3}{2}$

$\therefore f^{-1}(y) = \frac{y+3}{2}$.

* Function composition:

$f(x) = 2x+1$.

$g(x) = x^2-2$

i) $gof = g(f(x))$

$= g(2x+1)$

$= (2x+1)^2-2$

$= 4x^2+4x+1-2$

$= 4x^2+4x-1$

ii) $fog = f(g(x))$

$= f(x^2-2)$

$= 2(x^2-2)+1$

$= 2x^2-3$

$\cancel{f(g)}(fog)(2)$

$= 2(2)^2-3$

$= 2\times4-3$

$= 5$

* Let,

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Since $ac \equiv bc$ mod m. So m must devide $(a-b)\cdot c$.

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$\therefore 7 = 2+5\cdot1$ — (i) $\qquad$ $\therefore 11 = 1+5\cdot2$ — (ii)

(i) + (ii) 2(i),

$7+11 = 2+1+5\cdot1+5\cdot2$

$\Rightarrow 18 = 3+5(1+2)$

$\Rightarrow \frac{15}{5} = 3$

(i) × (ii) 2(i),

$7\cdot11 = (2+5\cdot1)(1+5\cdot2)$

$= 2\cdot1+2\cdot5\cdot2+5\cdot1\cdot1+5\cdot1\cdot5\cdot2$

$= 2+5(4+1+10)$

$77 = 2+5\cdot15$

$77-2$

tractable: A problem that is solvable using an algorithm with polynomial worest case complexity is called tractable.

* intractable: The situation is much worse for problems that can not be solved by using an algorithm with worst-case polynomial time complexity. such problem is called intractable

* unsolvable problem: some problem even exist for which it can be shown that no algorithm exist for solving them is called

$= a^2 b a^3 b^2$, $v = bab^2$,

i) $uv = a^2 b a^3 b^3 ab$ (ii) $vu = bab^2 a^2 b a^3 b^2$

iii) $v^2 = v \times v = bab^3ab^2$

* linear congruence:-

(a) $x = 2 \pmod 3$

(b) $x = 3 \pmod 5$

(c) $x = 2 \pmod 7$.

(a) $x = 2 \pmod 3$ (b) $x = 3 \pmod 5$

Unique solution modulo

$m = 3 \times 5 = 15$. We obtain the following solution of (b) which are less than 15

3, 8 13

Testing each solution of (b) in equation (a) we find that 8 is the only solution of both equation

$x = 8 \pmod{15}$ $x = 2 \pmod 7$

$m = 15 \times 7 = 105$

8 23 38 53 68 83 98

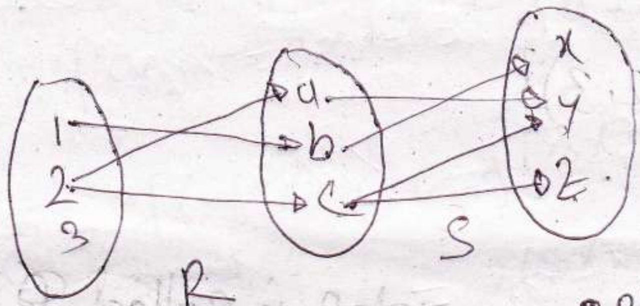
Relation

Let, $A=\{1,2,3\}$, $B=\{a,b,c\}$, $C=\{x,y,z\}$.

The relations R and S from A to B and from B to C.

$R=\{(1,b),(2,a),(2,c)\}$, $S=\{(a,y),(b,x),(c,y),(c,z)\}$

a) Draw the arrow diagram of the relation R & S.



(a) $1\,(RoS)\,x$ since $1Rb$ and bSx.

$2\,(RoS)\,y$ since $(2Ra)$ and (aSy)

since $(2Rc)$ and $(2Sy)$

$2\,(RoS)\,y$ since $(2Rc)$; (cSz)

$2\,(RoS)\,z$

$\therefore RoS = \{(1,x),(2,y),(2,z)\}$

(b) $M_R = \begin{array}{c|ccc} & a & b & c \\ \hline 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 1 \\ 3 & 0 & 0 & 0 \end{array}$, $M_S = \begin{array}{c|ccc} & x & y & z \\ \hline a & 0 & 1 & 0 \\ b & 1 & 0 & 0 \\ c & 0 & 1 & 1 \end{array}$

(c) $M_{RoS} = \begin{array}{c|ccc} & x & y & z \\ \hline 1 & & & \\ 2 & & & \\ 3 & & & \end{array}$

$$M_{R\circ S} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \begin{pmatrix} 1 & & \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{pmatrix} \end{array}$$

Q

* R on $A = \{1, 2, 3\}$.

a) R is both symmetric & antisymmetric.

b) R is neither symmetric nor antisymmetric

c) R is transitive but $R \cup R^{-1}$ is not transitive

a) $R = \{(1,1), (2,2)\}$.

b) $R = \{(1,2), (2,1), (2,3)\}$

c) $R = \{(1,2)\}$

Symmetric: A Relation R on a set A is called symmetric if $(b,a) \in R$ whenever $(a,b) \in R$ for all $(a,b) \in A$.

Antisymmetric: A Relation on a set A such that $(a,b) \in R$ and $(b,a) \in R$ only if $a=b$, for all $a, b \in A$ is called Antisymmetric.

Transitive: A Relation on a set A is called transitive if whenever $(a,b) \in R$ & $(b,c) \in R$ then $(a,c) \in R$ for all $a, b, c \in A$.

Set & function.

f be the function from $\{a, b, c\}$ to $\{1, 2, 3\}$
ch that $f(a) = 2$, $f(b) = 3$, $f(c) = 1$. is
nvertible and iff it is what is inverse.

Solution: The function is invertible.
ince it is one to one function.
So, the inverse will be the reverse of the
correspondence given by f.

$f^{-1}(2) = a$

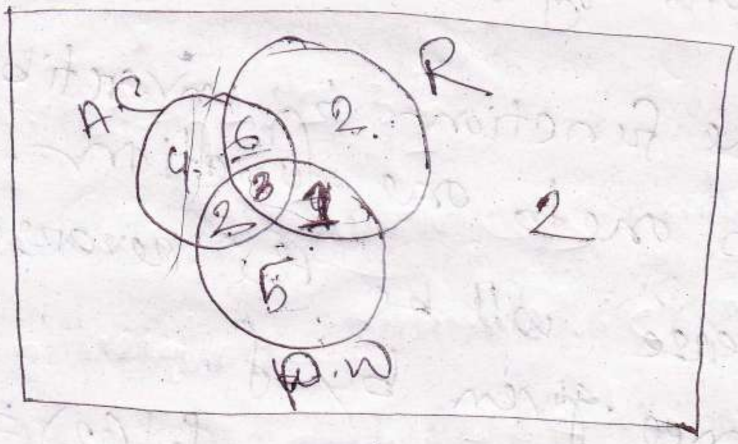
$f^{-1}(3) = b$,

$f^{-1}(1) = c$,

$f(x) = 2x + 3$, $g(x) = 3x + 2$.

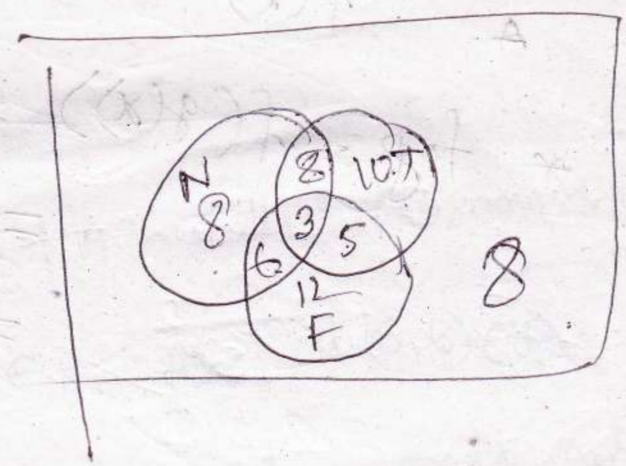
$$
\begin{aligned}
fog = f(g(x)) &= f(3x+2) \\
&= 2(3x+2) + 3 \\
&= 6x + 4 + 3 \\
&= 6x + 7
\end{aligned}
$$

$$
\begin{aligned}
gof = g(f(x)) &= g(2x+3) \\
&= 3(2x+3) + 2 \\
&= 6x + 9 + 2 \\
&= 6x + 11
\end{aligned}
$$



Total = 23

I) 5 (II) 4, (III) 2, (IV) 4 (V) 9 (VI) 3



~~A~~ Total = 52

(b)

^{644} mod 645. $x = 1$ power $= 2 \bmod 645 = 2$

$x = 1$ power $= 5 \bmod = 2 \ldots = 5$

$i = 0$ becoz $a_0 = 1$ wh $x = 1 \cdot 5 \bmod 13 = 5$ power $= 25 \; m \; 13 = 12$

$i = 1$ b $a_1 = 1$ $x = 5 \cdot 12 \bmod 13 = 8$, $P = 144 \; m \; 13 = 1$

$i = 2$ b $a_2 = 0$ $x = 8$, $p = 1 \times 1 \bmod 13 = 1$

$i = 3$ b $a_3 = 0$ $x = 8$ $p = 1 \times 1 \bmod 13 = 1$

$i = 4$ b $a_4 = 1$ $x = 8 \cdot 1 \bmod 13 = 8$, $P = 1 \times 1 \; m \; 13 = 1$

$i = 5$ b $a_5 = 0$ $x = 8$ $p = 1 \times 1 \; m \; 13 = 1$

$i = 6$ b $a_6 = 1$ $x = 8 \cdot 1 \; m \; 13 = 8$ $P = 1$

$i = 7$ b $a_7 = 1$ $x = 8$

$i = 8$ b $a_8 = 1$

$i = 9$ b $a_9 = 1$

$i = 10$ b $a_{10} = 1$, $x = 8 \bmod 13 = 8$, $P = 1$

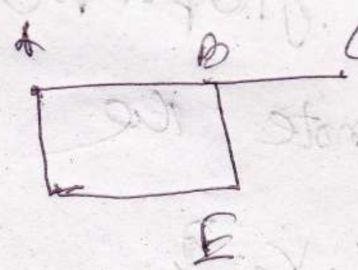
There are two standard ways to of maintaining a graph G in the memory of a computer. they are called

i) sequential representation

ii) linked representation. they are given below.

* sequential / adjacency matrix:- let G be a graph with m vertices and the vertices have been ordered say, $v_1, v_2, v_3 \ldots v_n$. Then adjacency matrix $A = [a_{ij}]$ of the graph G is the $m \times n$ matrix defined by

$$a_{ij} = \begin{cases} 1 & \text{if } v_i \text{ is adjacent to } v_j \\ 0 & \text{otherwise.} \end{cases}$$



$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

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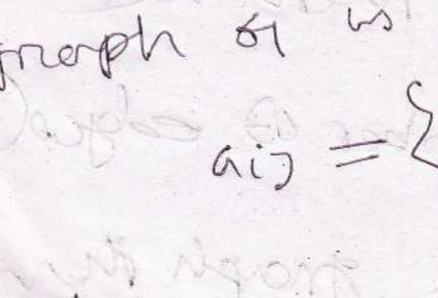

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

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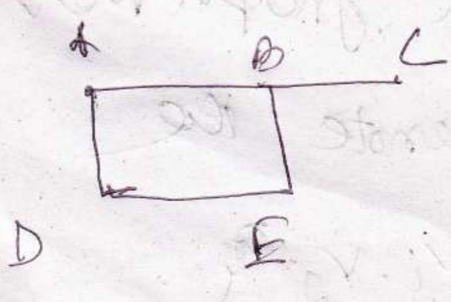


$$\begin{array}{c c} & \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

the matrix containing adjacency matrix graph G in Fig (a). Where the vert are ordered A B C D E F. observe that each edge $\{v_i, v_j\}$ of g represented twice by $a_{ij} = a_{ji} = 1$. tin adjacent matrix is Symmetric.

The adjacent matrix does not depend on the order in which the edges are input into the computer.

There are variations of above representation of G is a Multigraph then we usually let a_{ij} denote the number of edges (v_i, v_j)

If G is weighted graph then we may let ~~a_{ij}~~ a_{ij} denote $\boxed{\text{the}}$ weight of edge $\{v_i, v_j\}$

...oof: $\deg(fg) = \deg(f) + \deg(g)$.

Proof: proof follows directly from the definition of the product of polynomials. That is suppose

$f(t) = a_n t^n + \cdots + a_0$ and

$g(t) = b_m t^m + \cdots + b_0$ when $a_n \neq 0, b_m \neq 0$.

Then

$f(t)\,g(t) = a_n b_m t^{m+n} +$ terms of lower degree.

also since K is an integral domain with no zero divisors, $a_n b_m \neq 0$. Thus.

$\deg(fg) = \cancel{\deg} = m+n = \deg(f) + \deg(g)$.

* Let $S = N \times N$. Let $*$ be the operation on S defined by $(a,b)*(a',b') = (aa', bb')$.

a) show that $*$ is associative.

b) Define $f: (S,*) \longrightarrow (Q,\times)$ by $f(a,b) = a/b$.

show that f is a homomorphism.

c) Find the congruence relation $\sim$ in S determined by the homomorphism f. i.e. $x \sim y$ if

$f(x) = f(y)$.

d) Describe ~~$\circledast$~~ $S/\sim$. Does $S/\sim$ have an identity element? Does it have inverse?

solutions suppose $x = (a,b)$, $y = (c,d)$, $z = (e,f)$.

a) we have,

$$(xy)z = \cancel{(a,b)(c,d)} = (ac, bd) * (e,f)$$
$$= \cancel{(ace, bdf)}.$$
$$= [(ac)e, (bd)f]$$

$$x(yz) = (a,b) * (ce, df) = [a(ce), b(df)]$$

Since, (a,b,c,d,e,f) are (+ve) integers. $(ac)e = a(ce)$

and ~~ad~~ ~~b(df) =~~ b $(bd)f = b(df)$.

Thus $(xy)z = x(yz)$ and hence $*$ is associative

That is $(S,*)$ is a semigroup.

b) we have, $f(x*y) = f(ac, bd) = \cancel{ac}(bd)$

$$= (a/b)(c/d) = f(x) f(y)$$

Thus f is homomorphism.

ppose $f(x)=f(y)$. Then

~~Then~~ $\frac{a}{b}=\frac{c}{d}$ and hence, $ad=bc$.

f determines the congruence relation $\sim$ on S defined by $(a,b)\sim(c,d)$ if $ad=bc$.

ii) the image of f is Q^+, the set of p(+ve) real rational numbers. By theorem 12.3, $S/\sim$ is isomorphic to Q^+. Thus $S/\sim$ does have an identity element and every element has an inverse.

* Let:-

p = It is sunny this afternoon.

q = It is colder than yesterday.

r = we will go swimming.

s = we will take a ~~k~~ canoe trip.

t = we will be home by sunset.

The Hypothesis become:-

$\neg p \wedge q$, $r \to p$, $\neg r \to s$, $s \to t$.

* Let, p = You send me an e-mail Message.

q = I will finish the writing the program.

r = I will go to sleep early.

s = I will wake up feeling refreshed.

so, the Hypothesis will be.

$p \to q$, $\neg p \to r$, $r \to s$. $\neg q \to s$

$4!$ each partition of student can be arrange in $4! = 24$ ways

an ordered partition.

since there are $\frac{12!}{3!\,3!\,3!\,3!} = 369600$

such ordered partition there are

$369600/24 = 15400$

$$\sum_{n=50}^{200} k^2 \qquad \sum_{1}^{200} k^2 = \sum_{1}^{49} k^2 + \sum_{50}^{200} k^2$$

$$\sum_{n=1}^{n=49} - \Rightarrow \sum_{50}^{200} k^2 = \sum_{1}^{200} k^2 - \sum_{1}^{49} k^2$$

$$= \frac{200 \times 201 \times 401}{6} - \frac{49 \times 50 \times 101}{6}$$

$$= 2686700 - 41241.67$$

$$= 2645.43$$

$$\frac{n(n+1)(2n+1)}{6}$$

$$\sum_{99}^{200} K^3 = 60?$$

$$\sum_{99}^{200} K^3 \qquad \sum_{1}^{200} K^3 = \sum_{1}^{98} K^3 + \sum_{99}^{200} K^3$$

$$\Rightarrow \sum_{99}^{200} K^3 = \sum_{1}^{200} K^3 - \sum_{1}^{98} K^3$$

$$= \left(\frac{n(n+1)}{2}\right)^2 = \left(\frac{200 \times 201}{2}\right)^2 - \left(\frac{98 \times 99}{2}\right)^2$$

$$= (20100)^2 - (4851)^2$$

$$= 404010000 - 23532201$$

$$\boxed{= 380477799}$$

Let, $m = 3 \cdot 5 \cdot 7 = 105$

$M_1 = \frac{m}{3} = 35$

$m_2 = \frac{m}{5} = 21$

$m_3 = \frac{m}{7} = 15$

$a_1 = 2, a_2 = 3, a_3 = 2$

$y_1 = 35 \bmod 3 = 2$

$y_2 = 21 \bmod 5 = 1$

$y_3 = 15 \bmod 7 = 1$.

the solution of this system such that

$x = a_1 m_1 y_1 + a_2 m_2 y_2 + a_3 m_3 y_3$

$= 2 \times 35 \times 2 + 3 \times 21 \times 1 + 15 \times 2 \times 1$

$= 140 + 63 + 30 = 233$

$= 23 \bmod 105$

The smallest is 23

* pascal triangle: The number $\binom{n}{r}$ are called binomial coefficient. the coefficient of successive power of $(a+b)$ can be arrange in a triangle array this triangle is called pascal triangle.

1

1 1

1 2 1

1 3 3 1

1 4 4

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5

$3^{302} \bmod 5$

$= (3^{2})^{151} \bmod 5$

$(5)^{2003} \bmod 7.$

$= 5 \times (5^{2})^{1001} \bmod 7.$

$= 5 \times (4)^{1001} \bmod 7.$

$= 5 \times 4 \times (4)^{1000} \bmod 7.$

$= 5 \times 4 \times (4^{2})^{500} \bmod 7$

$= 5 \times 4 \times (2)^{500} \bmod 7.$

$= 5 \times 4 \times (2^{4})^{125} \bmod 7.$

$= 5 \times 4 \times (2)^{125} \bmod 7.$

$= 5 \times 4 \times (2^{5})^{25} \bmod 7.$

$= 5 \times 4 \times (4)^{25} \bmod 7.$

$= 5 \times 4 \times 4 \times (4)^{24} \bmod 7.$

$= 5 \times 4 \times 4 \times (4^{2})^{12} \bmod 7.$

$= 5 \times 4 \times 4 \times (2)^{12} \bmod 7.$

$= 5 \times 4 \times 4 \times (2^{4})^{3} \bmod 7.$

$= 5 \times 4 \times 4 \times 2^{3} \bmod 7.$

$= 5 \times 4 \times 4 \times 1$

$= 3.$

937

13

$p = 43, \; q = 59$

$\gcd(\phi(n), e)$

$\gcd(p-1, q-1, e)$

$\gcd(42, 58, 13)$

$= 1$

$n = pq = 2537$

$\gcd(n, e) = 1$

ϕ

$\gcd(235, e) = 1$

(42×58)

$= \boxed{2436}.$

~~$2436 = 0 \times 1 + 0$~~

~~$= 0$~~

$\{(513)^6 \times 1819\}$ m

$C = M^e \bmod(n)$.

$M = C^d \bmod(n)$.

$\phi(n) = (p-1)($

$$\begin{array}{r|l} 2 & 2436 \\ 3 & 609 \\ 7 & 203 \\ & 29 \end{array}$$

$2 \times 3 \times 7 \times 29$

0

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

2436,

$C = (1819)^{13} \bmod 2537$

$= \{(1819)^3 \times 2 \times 3 \times 4 \times 1819\} \bmod 2537$

$\Rightarrow \{(2068)^4 \times 1819\} \bmod 2537$

$\Rightarrow \{(1779)^2 \times 1819\} \bmod 2537$

$= (1202 \times 1819) \bmod 2537$

$= 2081$

$5^{2003} \bmod 7$

$\Rightarrow (5 \times 5^{2002}) \bmod 7$.

$\Rightarrow \{5 \times (5^2)^{1001}\} \bmod 7$

$\Rightarrow \{5 \times (4)^{1001}\} \bmod 7$

$\Rightarrow \{5 \times 4 \times (4^2)^{500}\} \bmod 7$

$\Rightarrow (5 \times 4 \times (2)^{500}) \bmod 7$

$\Rightarrow \cancel{5 \times 4 \times 2^{20} \times (2)^{498}} \bmod 7$

$\Rightarrow (5 \times 4 \times (2^{20}) \times (2^{30})^{16}\} \bmod 7$

$= (5 \times 4 \times 4 \times 1^{16}) \bmod 7$

$= 5 \times 4 \times (4)^{1000} \bmod 7$

$\Rightarrow 3$.

A_5, A_3, A_7, A_1, A_2, A_4, A_6, A_8.
G B W G W B W B

$A_5 = G = A_1$, $A_3 = A_4 = A_8 = B$

Total color is needed to paint 3.

Any planner graph is four colorable.

Prove:- $V - E + R = 2$

• P

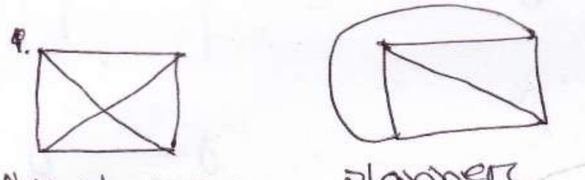
suppose the connected map M consists of a single vertex P. Then $V = 1$, $E = 0$ and $R = 1$.

Hence, $V - E + R = 1 - 0 + 1 = 2$.

$\boxed{\therefore V - E + R = 2}$ proved

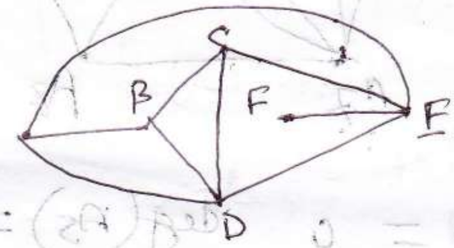
$\overline{(A \oplus B)} + A(B$

Plannere graph :- A graph is said to be plannere if one edge do not ~~eos~~ cross another edge.



Non-plannere   plannere

Map :- A particulare plannere ~~multigraph~~ representation of a finite plannere multigraph is called a map.

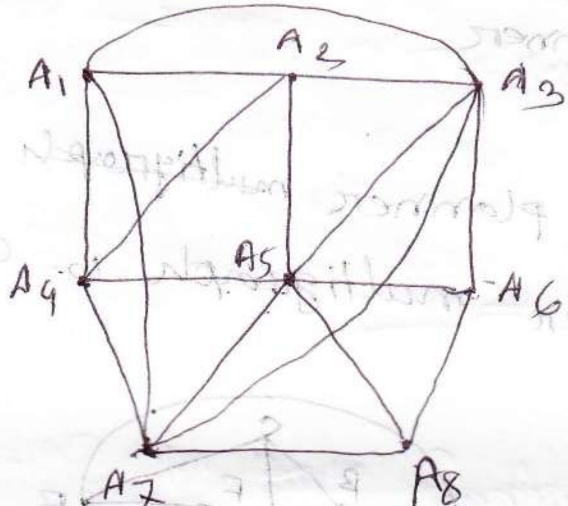


$$V - E + R = 2$$

$C'in(A \oplus B) + A(BC'n$

Graph coloring:-

Chromatic number :- The minimum number of color needed to paint G is called chromatic number.



$deg(A_1) = 4$

$deg(A_2) = 4$

$deg(A_3) = 5$

$deg(A_4) = 4$

$deg(A_5) = 6$

$deg(A_6) = 3$

$deg(A_7) = 5$

$deg(A_8) = 3$

$Adj(A_1) = A_2, A_3, A_4, A_7$

$Adj(A_2) = A_1, A_3, A_4, A_5$

$Adj(A_3) = A_1, A_2, A_5, A_6, A$

$Adj(A_4) = A_1, A_2, A_5, A_7.$

$Adj(A_5) = A_2, A_3, A_4, A_6, A_7, A_8.$

$Adj(A_6) = A_3, A_5, A_8.$

$Adj(A_7) = A_1, A_4, A_5, A_3, A_8.$

$Adj(A_8) = A_5, A_6, A_7.$

	B	Cin	S	Cout
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

2 Half adder with 1 OR gate

For exam

$A(\bar{B}\bar{C}in + BCin)$

$\bar{A}(\bar{B}Cin + B\bar{C}in)$

$\bar{A}(B \oplus Cin)$

$A + A + A = A$

$A \cdot A =$

$A + \bar{A} = 1$

Cin \ AB	00	01	11	10
0		1		1
1	1		1	

$S \Rightarrow \bar{A}\bar{B}Cin + \bar{A}B\bar{C}in + ABCin + A\bar{B}\bar{C}in$

$\Rightarrow \bar{A}\bar{B}Cin + ABCin + \bar{A}B\bar{C}in + A\bar{B}\bar{C}in + ABCin$

$\Rightarrow Cin(\bar{A}\bar{B} + AB) + B(\bar{A}\bar{C}in + A\bar{C}in)$

$Cin(A \odot B) + A(\bar{B}\bar{C}in + BCin)$

a Person is a student and has discrete math Course then he will be a CSE Student.

$x =$ is a student.

$S(x) = x$ is a student.

$D(x) = \cancel{ho}$ x has a discrete mathematics.

$CS(x) = x$ is a CSE student.

$$\exists x\left(\left(S(x) \land D(x)\right) \rightarrow CS(x)\right)$$

If a person is a student and has a DBcrete math course then he will be some ones friend in this class.

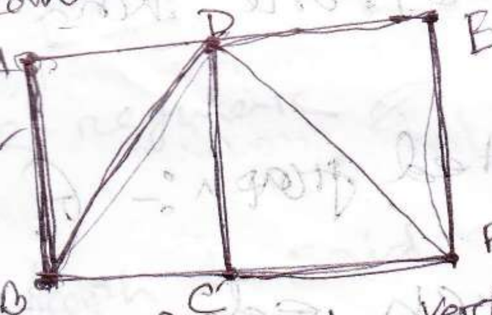
Let, $D(x) = x$ has discrete math.

$S(x) = x$ is a student.

$F(x,y) = x$ is a friend of y in this class.

$$\forall x\left(S(x) \land D(x) \rightarrow \cancel{\phi}\ \exists x(F(x,y))\right)$$

* Hamiltonian Graph :- Hamiltonian Graph is a closed path that visits every vertex in G exactly once, but may repeat edge is called a Hamiltonian graph.



A Graph will be Hamiltonian graph if it follows the condition.

$n \geq 3$ and $n \leq deg(v)$ for each vertex v in G.

* Eulerian graph :- Eulerian graph is a closed path that visits every edge exactly once but may be repeat vertices.

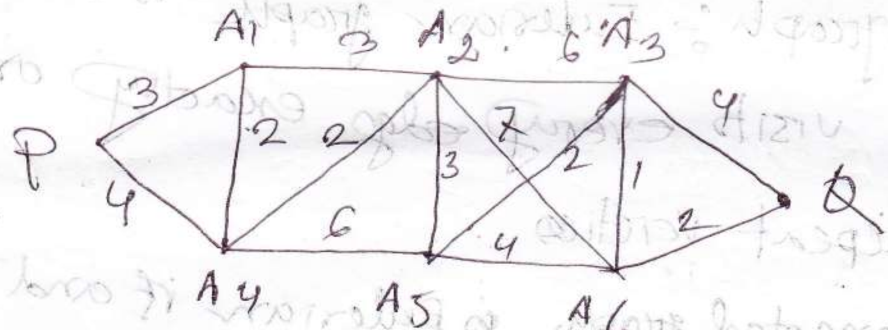
or. A finite connected graph is Eulerian if and only if each vertex has even degree.

Fig: Eulerian graph.

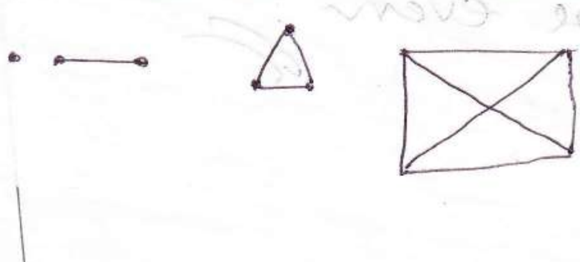
The Differences between H. & E graph is in H. graph number of degree for each vertices may be odd or even but in E. graph it will be even

Labeled graph:- A graph is called ~~to~~ a labe[led] graph if its edges and/or vertices are assigned data of one kind or another.

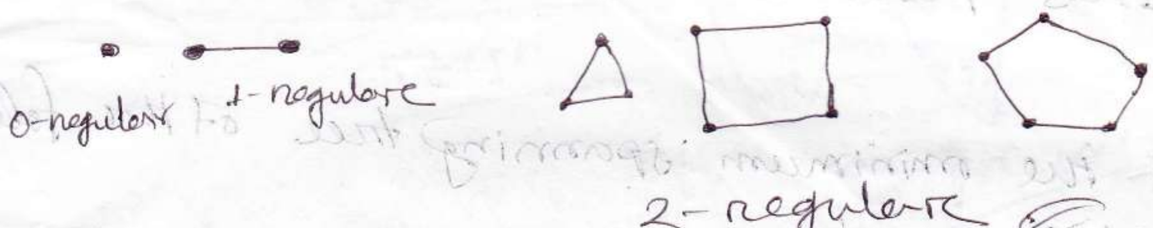
Weighted graph:- G is called a weighted graph if each edge e of G is assigned a non negative number w(e) called the weighted graph.



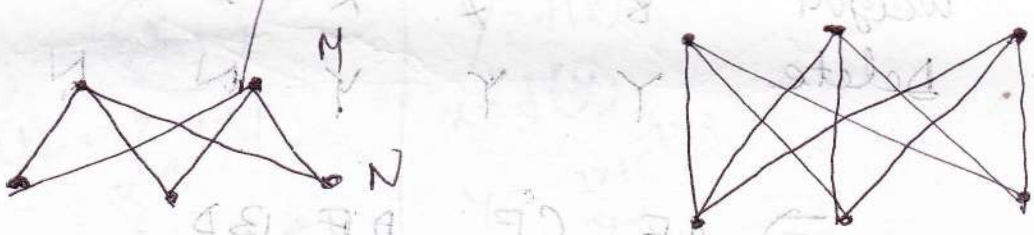
Complete graph:- A graph is said to be complete if every vertex ~~of~~ in G is connected to every other vertex in G. A complete graph with n vertices is denoted by Kn



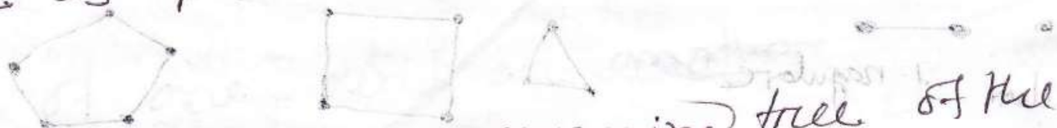
Regulare graph :- A graph is said to be regulare if every vertex has same degree.



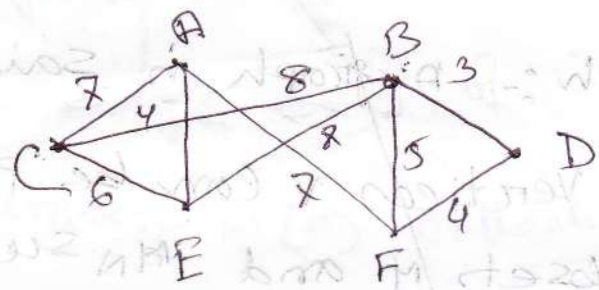
Bipartite Graph :- A graph is said to be bipartite if its vertices V can be partitioned into two subsets M and N such that ~~eve~~ each edge of G connects a vertex of M to a vertex of N.



* Minimum spanning tree :- A minimum spanning tree is spanning tree whose total weight is as small as possible.

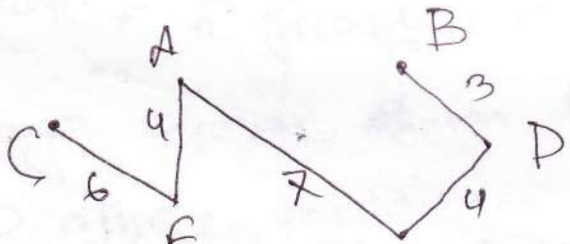


* Find the minimum spanning tree of the following weighted graph.

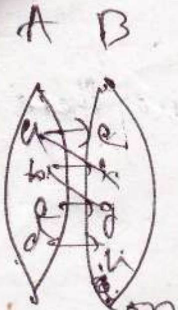


Edges	BC	AC	BE	AF	CE	BF	AE	DF	BD
Weight	8	7	7	7	6	5	4	4	3
Delete	Y	Y	Y	N	N	Y	N	N	N

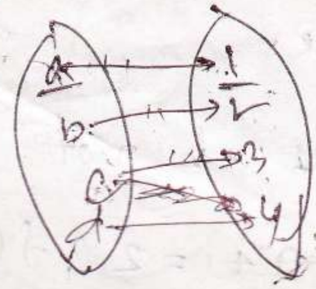
⇒ AF CE AE BD



Total weight is 24.



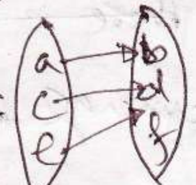
$\boxed{f(x) = x^2+1}$

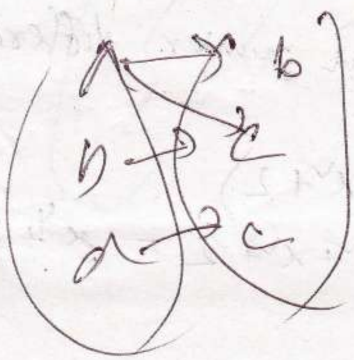


one to one

on to

on to





1) $f(x) = 2x+1$

$f(1) = 2\cdot 1+1 = 3$

$f(-1) = 2\cdot(-1)+1 = -1$

$f(2) = 2\cdot 2+1 = 5$

$f(-2) = 2\cdot(-2)+1 = -3$

then ....

$y = f(x) = 2x+1$

$x = f(y)$

$y = 2x+1$

$x = \frac{y-1}{2}$

$f(x) = \frac{x-1}{2}$

i) $f(x) = 2x+1$ is bijection because.

fore the value of (+ve) & (-ve) some value there is no

some image.

ii) $f(x) = x^2+1$ is not bijection because.

$f(-1) = (-1)^2+1 = 2$, $f(1) = 1^2+1 = 2$

So, image is same that is why it ~~d~~ is not bijection y

iii) $f(x) = x^3$, is bijection because ~~there is not~~

for different real number different image.

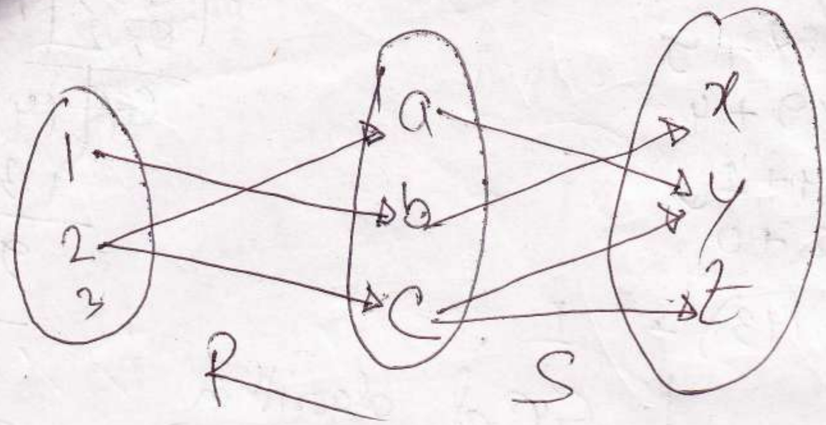
v) $f(x) = (x^2+1)(x^2+2)$

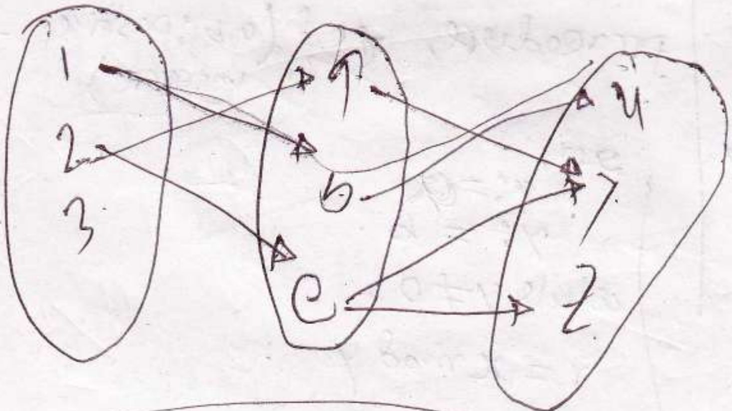
$= x^4+2x^2+x^2+2 = x^4+3x^2+2$

$f(1) = 6$

$f(-1) = 6$

so, it is not bijection.





$\boxed{1\,(R \circ S)\,u}$

$2\,(R \circ S)\,y$

$\cancel{2\,(R \circ S)\,y}$

$2\,(R \circ S)\,z$

$(R \circ S) = \{(1,u), (2,y)\ (2,z)\}$

Eucledun.

$639 = 4 \times 143 + 67$

$143 = 2 \times 67 + 9$

$67 = 7 \times 9 + 4$

$9 = 2 \times 4 + 1$ ✓

$4 = 2 \times 2 + 0$

$\therefore \gcd(639, 143) = 1.$

$$\begin{array}{r|l|l} 143 & 639 & 4 \\ & 572 & \\ \hline 67 & 143 & 2 \\ & 134 & \\ \hline 9 & 67 & 7 \\ & 63 & \\ \hline 4 & 9 & 2 \\ & 8 & \\ \hline 1 & 4 & 4 \\ & 4 & \\ \hline & 0 & \end{array}$$

begin:

~~r = x % y~~

~~x = y~~

~~y = r~~

GCd algorithm

procedure, gcd (a, b : positive integer)

set, $x := a$

$y := b$.

while $y \neq 0$,

$r = x \bmod y$

$x = y$

$y = r$.

end { gcd (a, b) is x }.

---

procedure, gcd (a, b are positive integers)

$x := a$;

$y := b$:

while $y \neq 0$

begin $r = x \bmod y$

$x = y$

$y = r$

end { gcd(a, b) is x }

$$\begin{array}{r|l} 3 & 639 \\ \hline 3 & 213 \\ \hline & 71 \end{array}$$

$\therefore$ prime factorization of

$639 = 3 \cdot 3 \cdot 71 = 3^2 \cdot 71$

$$\begin{array}{r|l} 13 & 143 \\ \hline & 11 \end{array}$$

$143 = 11 \cdot 13$.

* show that 101 is prime.

solution: The only primes not ~~exce~~ exceeding $\sqrt{101}$ are 2, 3, 5, 7. since 101 is not divisible by 2, 3, 5, 7 so, 101 is prime.

prime factorization of

$$7 \,\underline{|\, 7007}$$
$$7 \,\underline{|\, 1001}$$
$$11 \,\underline{|\, 143}$$
$$13$$

$7007 = 7 \cdot 7 \cdot 11 \cdot 13$
$\boxed{= 7^2 \cdot 11 \cdot 13}$

$$3 \,\underline{|\, 639}$$
$$3 \,\underline{|\, 213}$$
$$71$$

$639 = 3 \cdot 3 \cdot 71 = 3^2 \cdot 71$

$143 = 11 \times 13$

$\gcd(639, 143) = 1$

$50 = 2 \cdot 5 \cdot 5 = 2 \cdot 5^2$

$12 = 2 \cdot 2 \cdot 3 = 2^2 \cdot 3$

$\gcd(50, 12) = 2$

639, 143

$$\begin{array}{r|l} 3 & 639 \\ \hline 3 & 213 \\ \hline & 71 \end{array}$$

$639 = 3 \cdot 3 \cdot 71 = 3^2 \cdot 71$

$\begin{array}{|l} 143 \\ \hline \end{array}$

$143 = 11 \cdot 13$

$\therefore \gcd(639, 143) = 1$

$\boxed{n+1 > x}$?

$\longrightarrow$

$\boxed{x + \cancel{1} > x}$

* ~~pseudoprime~~: If n is a positive integer such that $b^{n-1} \equiv 1 \pmod{n}$, then n is called a pseudoprime to the base b.

$2^{340} \equiv 1 \bmod (11)$

$f(x) = x^2 = y \Rightarrow x = f(y)$

$\Rightarrow y = x^2 \therefore x = \pm\sqrt{y}$

$\Rightarrow f^{-1}(y) = \pm\sqrt{y}$

$\therefore f^{-1}(x) = \pm\sqrt{x}$

$\therefore f^{-1}(1) = \pm\sqrt{1} = \pm 1$

$f(x) = x^2+2x+1$.

Replace x by x^2.

$\therefore f'(x) = x^2+2x^2+1 = 3x^2+1$.

Now,

when, $x=1$ $f(x)=4$, $f'(x) = 4$

$x=2$ $f(x)=9$, $f'(x)=13$.

$x=3$, $f(x)=16$; $f'(x)=49$.

$\therefore$ ~~Big (O)~~ Big O is $O(x^2)$ Ans

$n = ab$ 71

$a \leq \sqrt{n},\ b \leq \sqrt{n}$

If n is a Composite integer, ~~then~~ it has a factor a. with $1 < a < n$. Hence $n = ab$ where $a, \& b$ are (+ve) integer greater than one. $a \leq \sqrt{n}, b \leq \sqrt{n}$ my

$\therefore ab = \sqrt{n} \cdot \sqrt{n} = n$. Hence n has a positive ~~integer~~ devisor, not exceding $\sqrt{n}$ //

$101 = 9 \cdot 11 + 2$

$\therefore$ quotient is 9 & reminder = 11 //

$36 = 1 \cdot 24 + 12$

$24 = 2 \cdot 12 + 0$

$\therefore \gcd = 12$.

~~$-11 = -4 \cdot 3 + 1$~~

$-11 = 3(-4) + 1$.

$\therefore\ q = -4, r = 1$.

$(7p+q)$,   Do not GO

28. a) $2^2 \cdot 3^3 \cdot 5^5 \cdot 2^5 \cdot 3^3 \cdot 5^2$

$2^{max(a,b)}\ 3^{(max\ a,b)}\ 5^{(max,\ a,b)}$

$= 2^5 \cdot 3^3 \cdot 5^5$

$1000 = 625 \cdot 1 + 375$
$625 = 375 \cdot 1 + 250$
$375 = 1 \cdot 250 + 125$
$250 = 2 \cdot 125 + 0$

$\therefore \gcd(1000, 625) = 125$

$$625\,|\,1000\,|\,1 \qquad 375\,|\,625\,|\,1 \qquad 250\,|\,375\,|\,1 \qquad 125\,|\,250\,|\,2$$

$$2\,|\,88,\ 2\,|\,44,\ 2\,|\,22,\ 11 \qquad 2\,|\,126,\ 3\,|\,63,\ 3\,|\,21,\ 7 \qquad 3\,|\,729,\ 3\,|\,243,\ 3\,|\,81,\ 3\,|\,27,\ 3\,|\,9,\ 3$$

$$7\,|\,1001,\ 11\,|\,143,\ 13 \qquad 11\,|\,1111,\ 101$$

a) $88 = 2 \cdot 2 \cdot 2 \cdot 11 = 2^3 \cdot 11$

b) $126 = 2 \cdot 3 \cdot 3 \cdot 7 = 2 \cdot 3^2 \cdot 7$

c) $729 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 3^6$

d) $1001 = 7 \cdot 11 \cdot 13$

e) $1111 = 11 \cdot 101$

— = 2081 / mouu25 BR

...T PASS Go.

$$...(p) = (3p + 7) \bmod 26$$

a	b	c	d	e	f	g
0	1	2	3	4	5	

g	h	i	j	k	l	m
6	7	8	9	10	11	12
n	o	p	q	r	s	t
13	14	15	16	17	18	19
u	v	w	x	y	z	
20	21	22	23	24	25	

D = 16
O = 23.
N = 20
T = 12

P = 0,
A = 7
S = 9
G = 25

16 mod 26

16 ÷ 26 = mod

So, Encryption = QX UXM AHJJ ZX

$3p + 7 - 26x = d$

$\therefore p = \frac{d + 26x - 7}{3}$

$\therefore p = \frac{d + 26x - 7}{3}$

Q: QX.

$$Q = \frac{16 + 26 \times 0 - 7}{3} = 3 = D$$

$$X = \frac{23 + 26 \times 1 - 7}{3} = 14 = O$$

$$U = \frac{20 + 26 - 7}{3} = \frac{46 - 7}{3} = 13 = N$$

$$M = \frac{12 + 26 \times 2 - 7}{3} = 19 = T$$

inverse of 35 modulu 3.

$$3\overline{)35}(11 \quad \frac{30}{5} \quad \frac{3}{2}$$

$$2\overline{)3}(1 \quad \frac{2}{1} \quad 1|2|1$$

since $\gcd(35,3)$ is $=1$. so.

inverse is exist

$35 = 11 \cdot 3 + 2$

$2 = -11 \cdot 3 + 35$

(-11) . so, -11 is the inverse of 35 modulu 3.

*. RSA Encryption: $p=43, q=59$.

so. $n = 43 \times 59 = 2537$, $e = 13$. STOP.

1819

$C = M^e \bmod n$

$= M^e \bmod 2537$

$= (1819)^{13} \bmod 2537$

$= ((1819^3)^4 \cdot 1819) \bmod 2537$

$= ((1819^3 \bmod 2537)^4 (1819 \bmod 2537)) \bmod 2337$

$= ((2068)^4 \cdot 1819) \bmod 2537$

$= ((2068)^3 \cdot 2068 \cdot 1819) \bmod 2537$

$= ((2068^3 \bmod 2537)(2068 \cdot 1819 \bmod 2537)) \bmod 2537$

$= (322 \cdot 1858 \bmod 2537) \bmod 2537$

$= 2081$

$OP = 1415$

$c = (1415)^{13} \bmod 2537$

$= (((1415)^3)^4 \cdot 1415) \bmod 2537$

$= (((1415)^3 \bmod 2537)^4 \cdot 1415 \bmod 2537) \bmod 2537$

$= ((1828)^4 \cdot 1415 \cancel{\bmod}) \bmod 2537$

$= ((1828)^3 \cdot 1828 \cdot 1415) \bmod 2537$

$= ((1828)^3 \bmod 2537 \cdot 1828 \cdot 1415 \bmod 2537) \bmod 2537$

$= (2005 \cdot 1417) \bmod 2537$

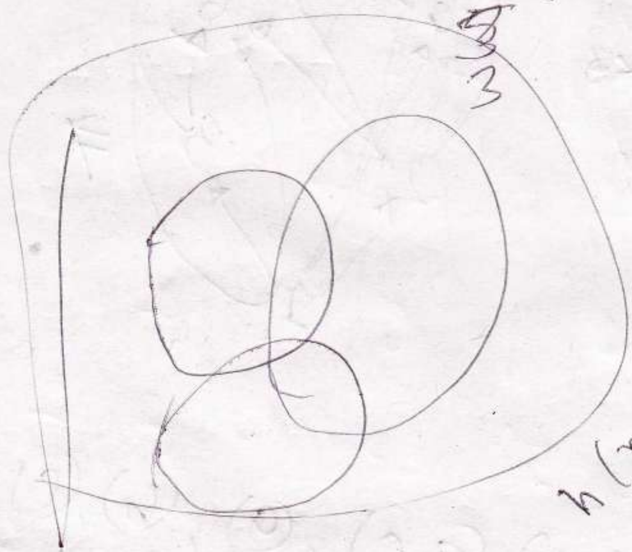
$= 2182 \bmod 2537$

$= 2182$

So, encryption ~~method~~ = 2081 , 2182

50

30 → 3
20
10 → 2 → 1
5
3



$n(M \cup E \cup B) =$

$n(S) = 50$

$n(M) = 30$

$n(E) = 20$

$n(B) = 10$

$n(M \cap E) = 10$

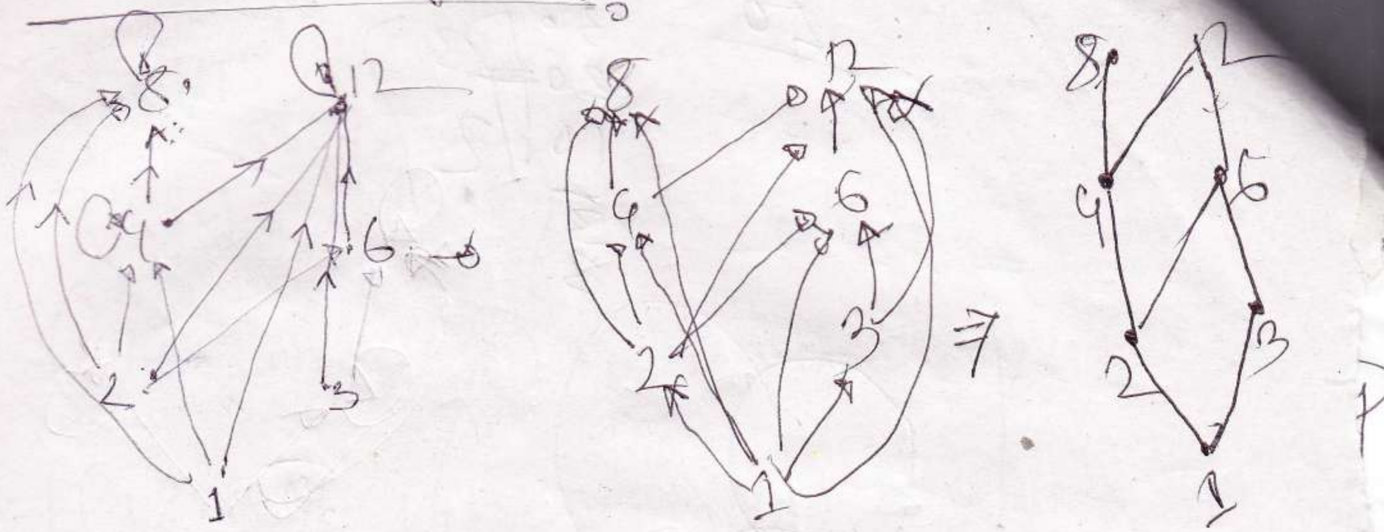
$n(B \cap M) = 5$

$n(E \cap B) = 8$

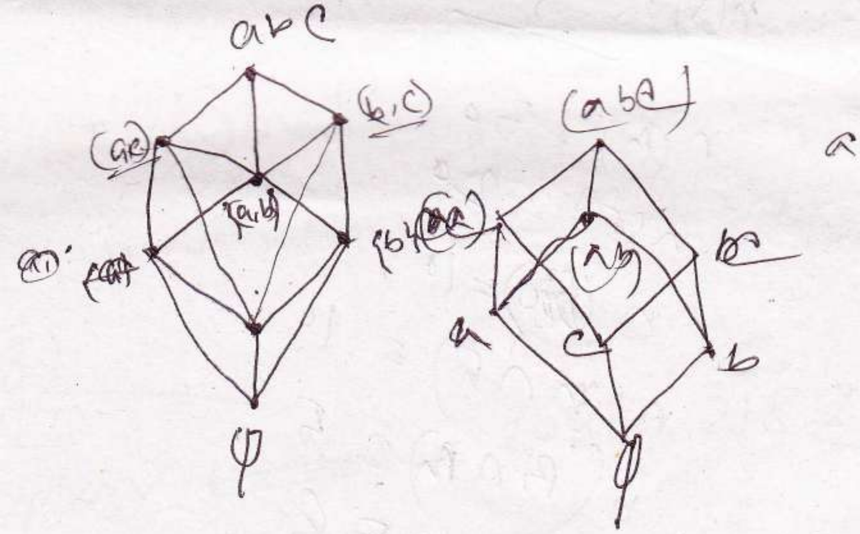
$n(B \cap M \cap E) = 3$

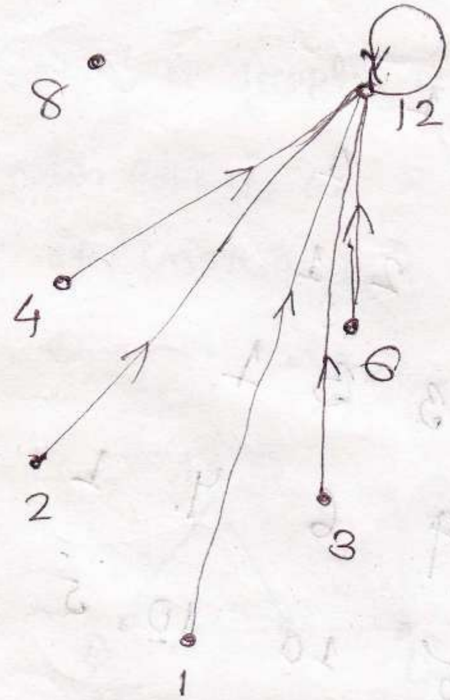
$$n(M \cup E \cup B) = n(M) + n(B) + n(E) - n(M \cap B) - n(B \cap E) - n(E \cap M) + n(M \cap B \cap E)$$

* Hasse diagram :—



(a,b,c), (a,b), (a,c), (a), (b), (c), ϕ, (b,c)





Hashee diagram :- 521 Page

are logically equivalent.

* Pascal triangle

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

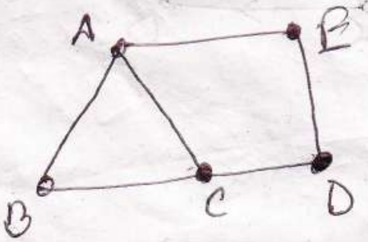
1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

Graph

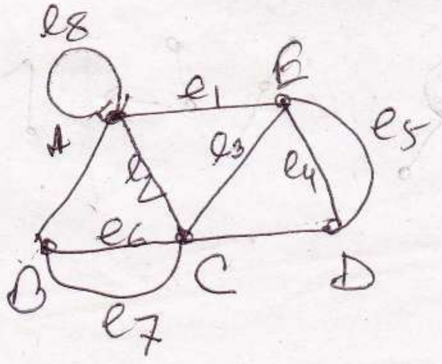
Simple graph: A graph is called simple graph when it ~~contain~~ consists of a non-empty set of vertices and a set of unordered pairs of distinct elements of vertices or ~~dex~~ edges.



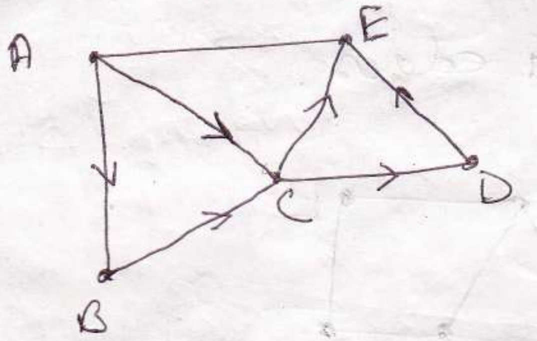
* Multigraph: A multigraph $G = (V,E)$ consists of a set ~~of~~ V of vertices, a set E of edges and a function f from E to $\{\{u,v\} \mid u, v \in V, u \neq v\}$.

The edges e_1 & e_2 are called multiple or parallel edges if $f(e_1) = f(e_2)$.

Since edges e_5, e_4 & e_6, e_7 has same endpoint and e_8 have same starting & endpoint. This digram is called multigraph.

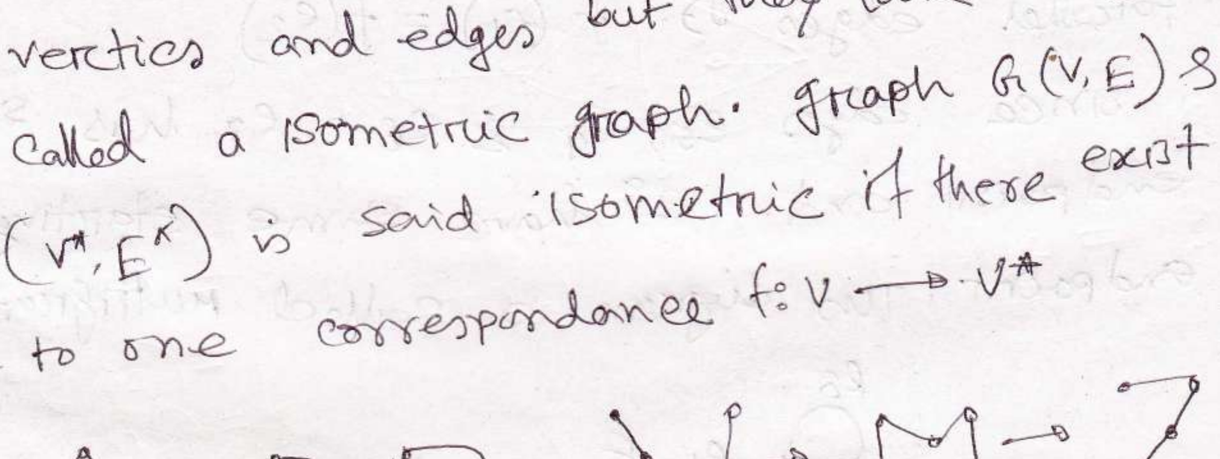


* Directed graph :- A directed graph (V, E) con[sists] of a set of verticies V and a set of edges E that are ordered pairs of elements of V.

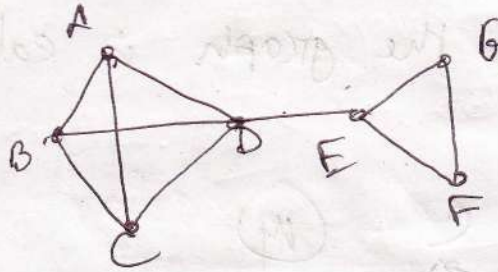


Trivial graph : A Finite graph with one vertex and no edge ie single point is called a trivial graph.

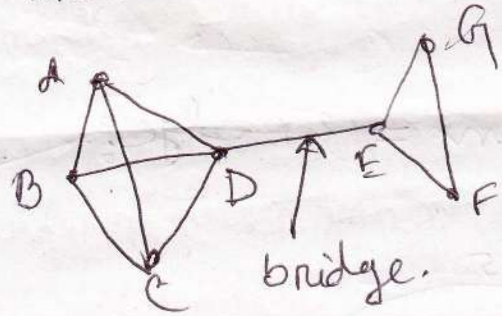
Isometric graph : If two graph has same number of vertics and edges but they looks different, is called a isometric graph. Graph G(V, E) & G*(V*, E*) is said isometric if there exist a one to one correspondance f: V ⟶ V*



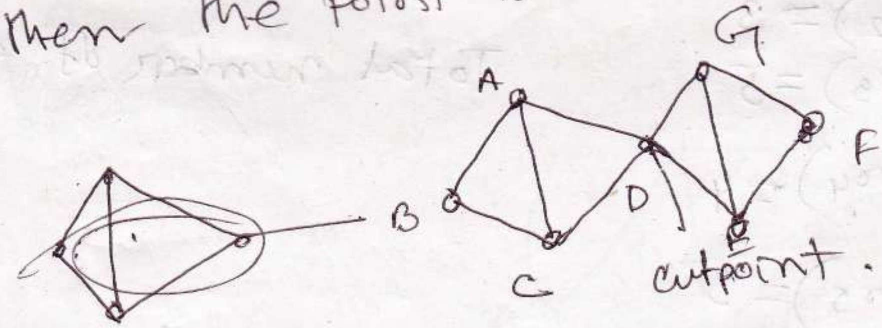
Connected graph: If in a graph all vertices are connected then the graph is called connected graph. If there is a path between two vertices.



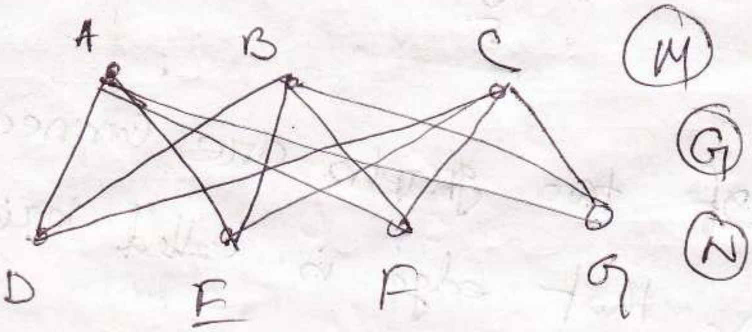
Bridge: If two graphs are connected by an edges then that edge is called bridge.



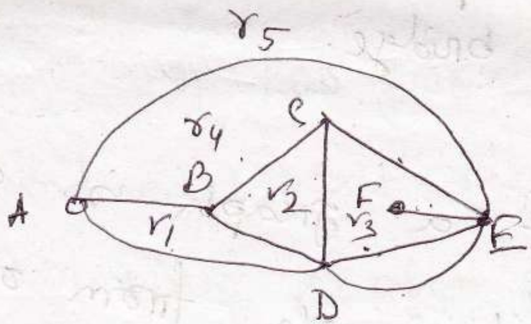
Cutpoint: If a graph is divided into two individual graph from one point or vertices then the point is called cutpoint.



* Bipartite graph: If there is a two region the vertice of one region is connected to the all vertices on the other region and vice-versa then the graph is called bipartite graph.



* The degree of a region = 2 × edge:-



$deg(r_1) = 3$

$deg(r_2) = 3$

$deg(r_3) = 5$

$deg(r_4) = 4$

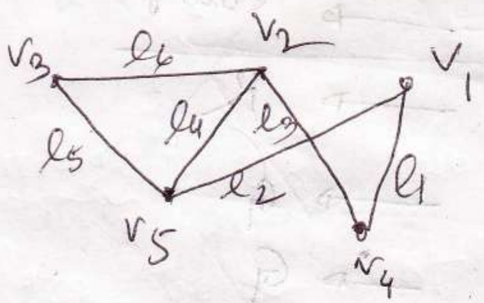
$deg(r_5) = 3$

Total number of degree = 18

Total number of edge = 9.

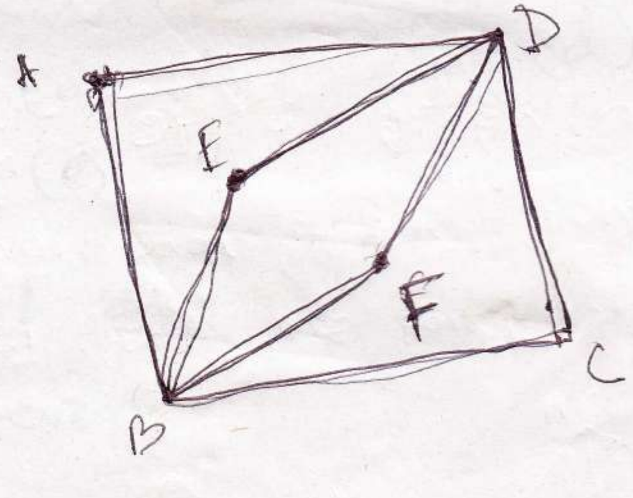
Spanning tree: If some edges are deleted from a graph, and the graph is then the grom is made a tree is called spanning tree.

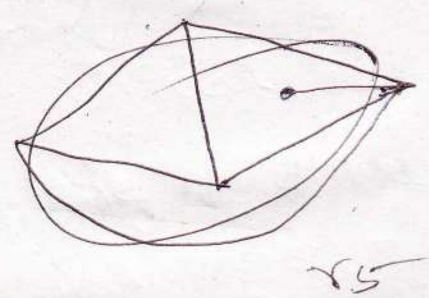
* Adjacent Matrix:-

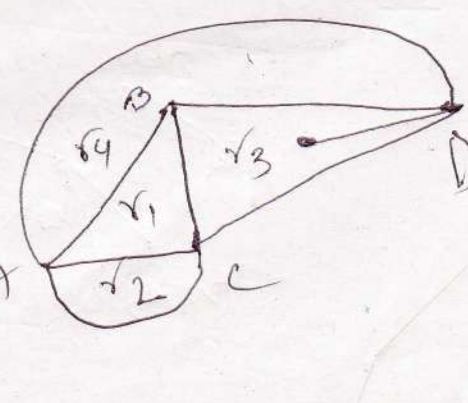


	e_1	e_2	e_3	e_4	e_5	e_6
v_1	1	1	0	0	0	0
v_2	0	0	1	1	0	1
v_3	0	0	0	0	1	1
v_4	1	0	1	0	0	0
v_5	0	1	0	1	1	0

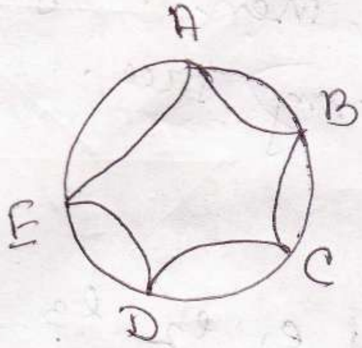
Cycle: If the starting & ending point are same.







Graph coloring:

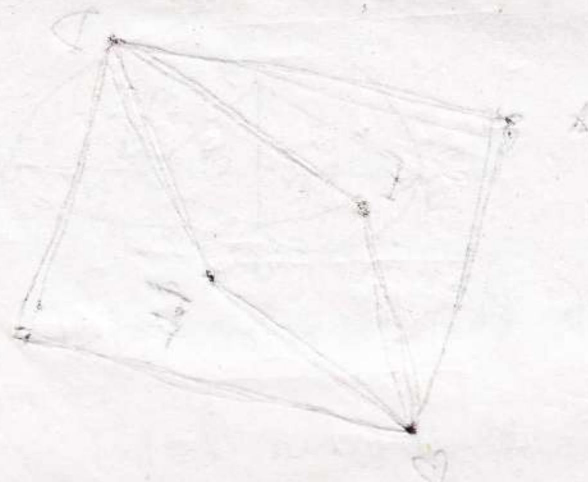


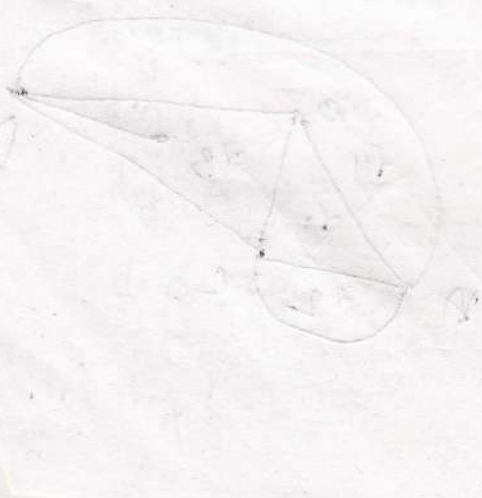
VER

Vertex	Adj
A	B, E
B	A, C
C	B, D
D	C, E
E	D, A

Vertex	→ color
A	→ R
B	→ G
E	→ G
C	→ B
D	→ R

Minimum Colore = 3.

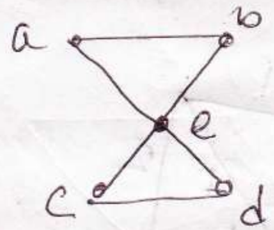




Prove that a finite Connected graph G is Eulerian if and only if each vertex has even degree.

Solution:-

Consider the following graph.



This graph is connected and has finite number of vertex.

The Path is $a \to e \to c \to d \to e \to b \to a$.

that contain every edges of graph G.

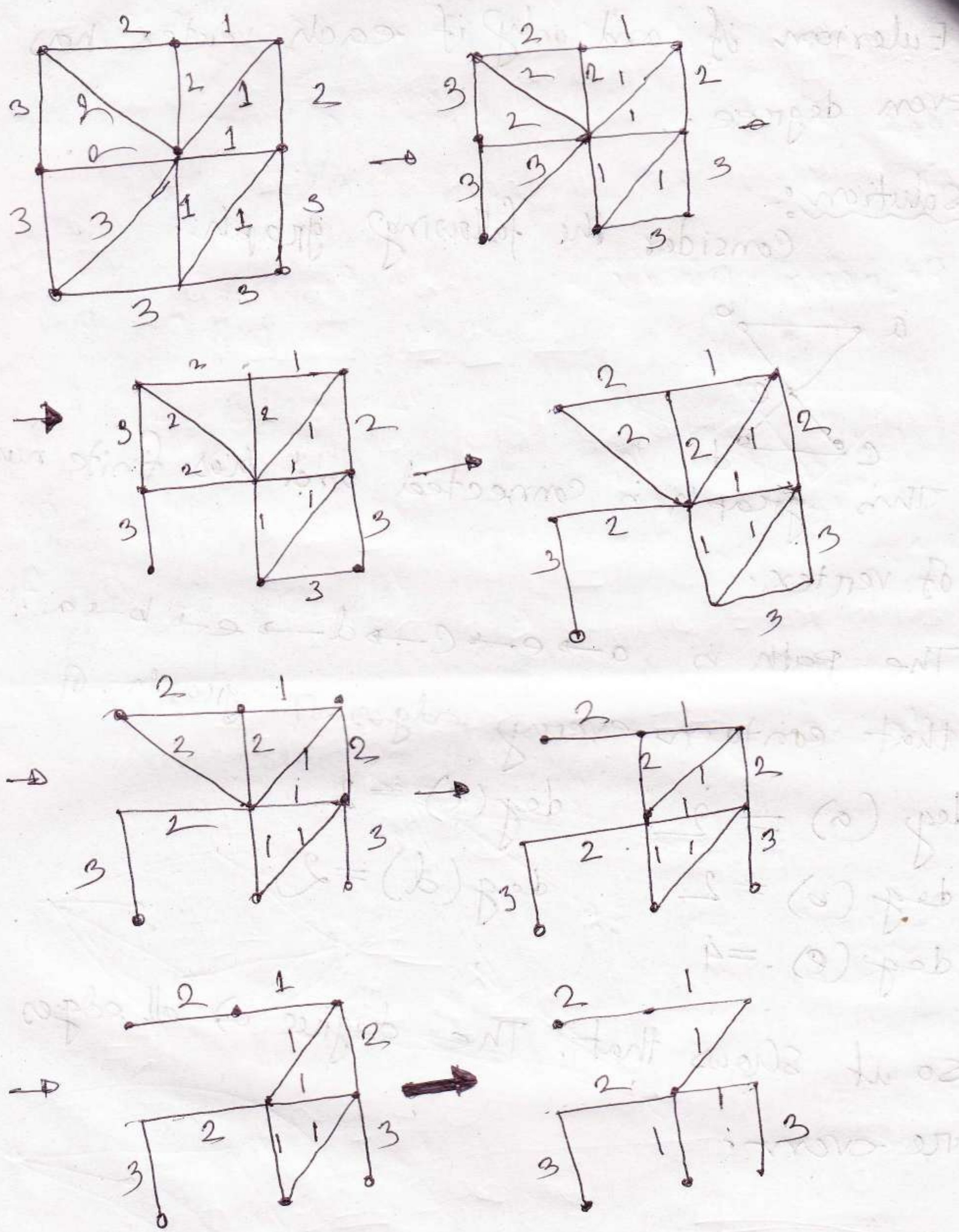
deg $(a) = 2$ deg $(c) = 2$

deg $(b) = 2$ deg $(d) = 2$

deg $(e) = 4$

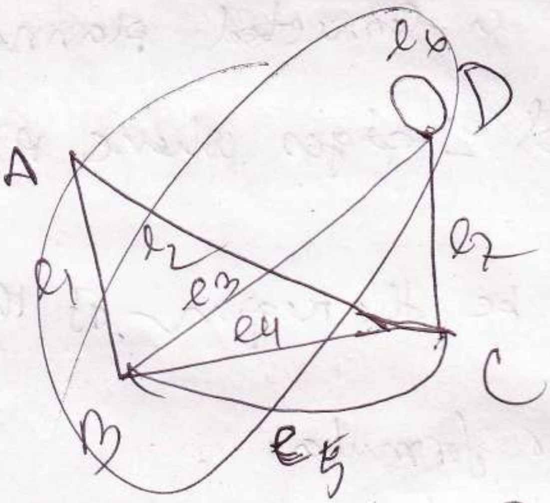
so it shows that. The degree of all edges are even.

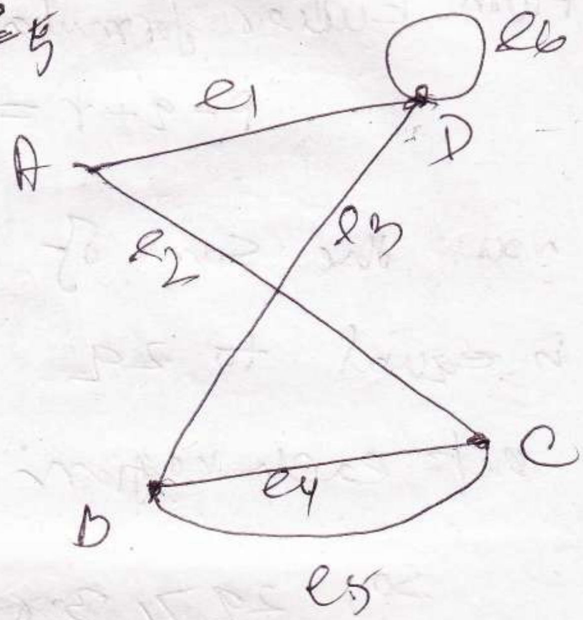
b) find the minimum spanning tree.



Total cost = 2+1+1+2+3+1+1+3

= 14





1 are logically equivalent.

* Let G be a connected planner graph with p vertex and q edges where $p \geq 3$ then $q \geq 3p-6$.

~~proof~~: Let r be ~~the~~ region of the planner graph

From Eulier formula,

$$p-q+r=2 \quad \text{--- (1)}$$

Now, the sum of total ~~edge~~ degree of the region is equal to $2q$.

But each region has degree 3 or more.

So, $2q \geq 3r \Rightarrow r = \frac{2q}{3}$.

$\therefore$ put it in (1).

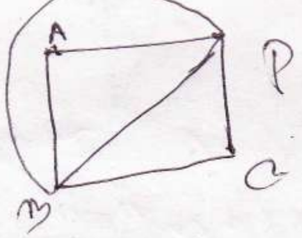
$$p-q+\frac{2q}{3} \leq 2$$

$$\Rightarrow 3p-3q+2q \leq 6$$

$$\Rightarrow 3p-q \leq 6$$

$$\Rightarrow \boxed{q \leq 3p-6} \text{ Proved}$$

~~nner~~ graph: A graph is said to be planner if no edges do not cross to another edges.



→ planners graph.

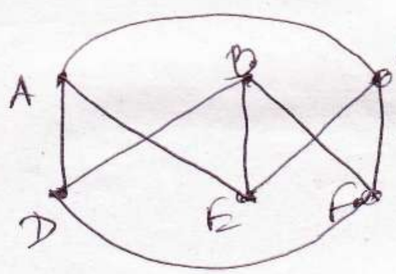
A graph will be planner if

$q \leq 3p-6$.

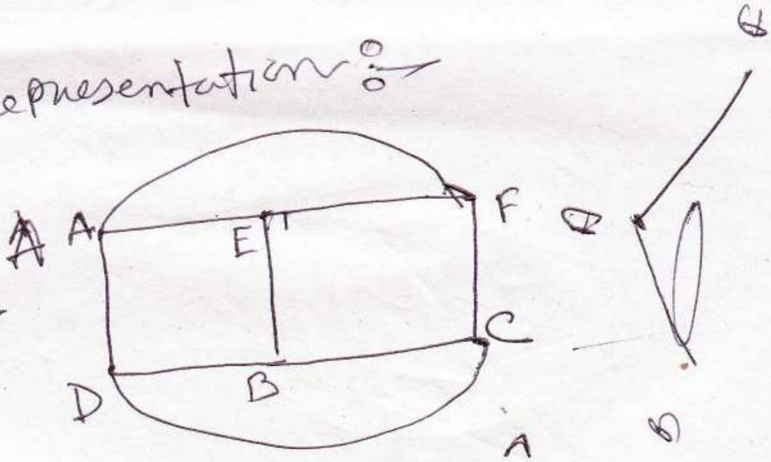
$p=6,\ q=9,\quad 9 \leq 18-6 \leq 12$

* Draw a planner representation :-

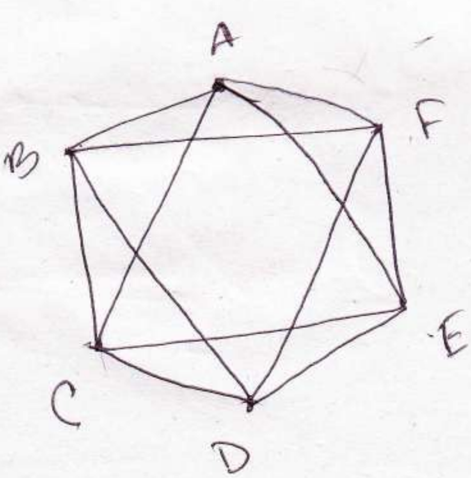
(a)



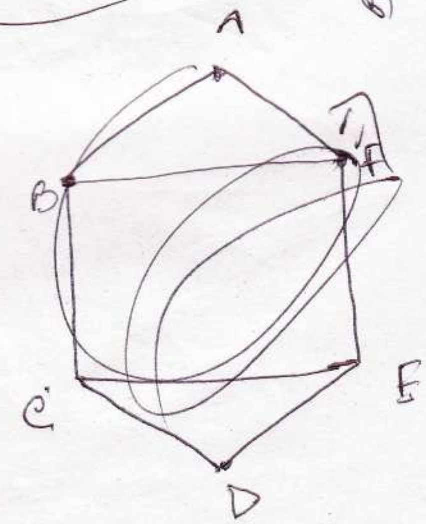
⇒

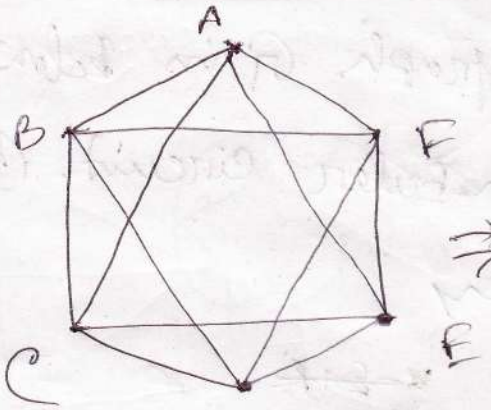


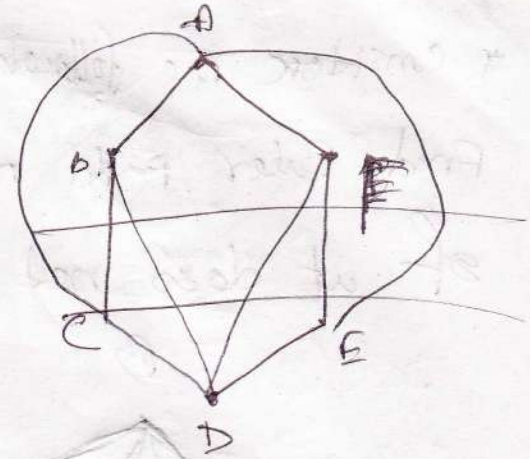
(b)

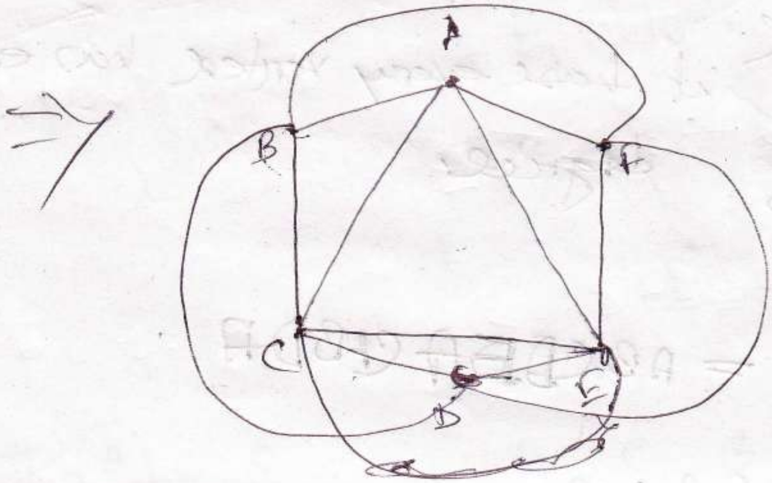


⇒



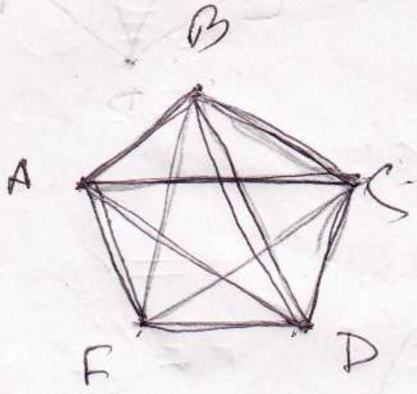






are logically equivalent,

* Consider the following graph G in below.
Find Euler path or an Euler circuit. If it every of it does not any

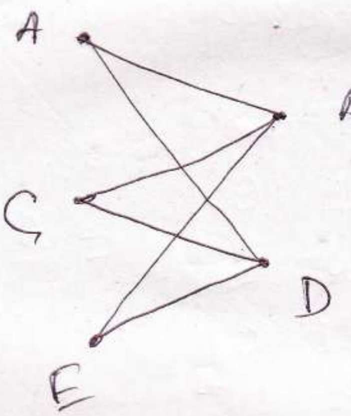
a) 

~~V-E+R~~.

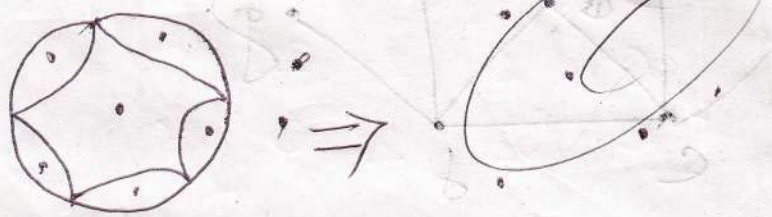
It is Eulerian circuit because it ~~have~~ every vertex has even degree.

The Euler Path = ABC~~DEA~~ CEBDA

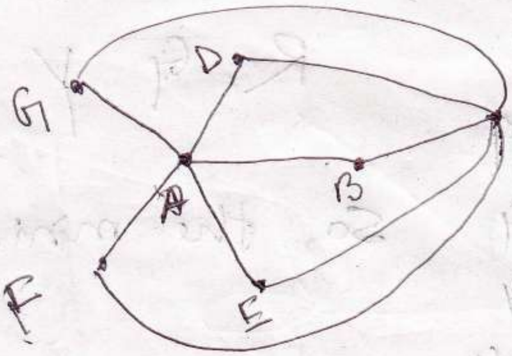
ABCDEACEBDA

b) 

It is Eulerian circuit.



=)



A	C	B	D	E	F	G
R	R	B	B	B	B	B

Chromatic number = 2

Deg(A) = 5

Deg(B) = 2

deg(C) = 4

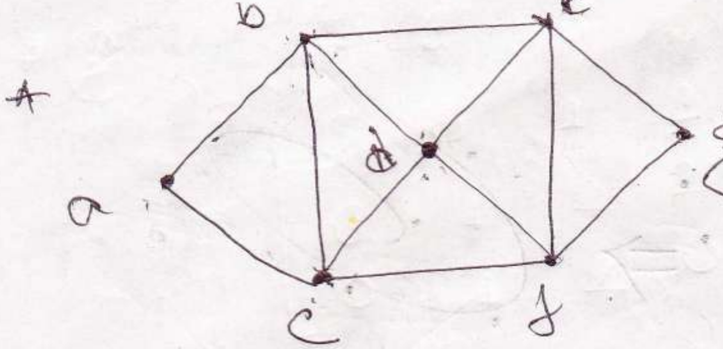
deg(D) = 2

deg(E) = 2

deg(F) = 2

deg(G) = 2

*



b c d e f a g.

R G Y G B R R

Ded (a) = 2

deg(b) = 4

deg(c) = 4

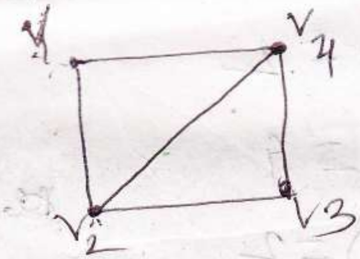
deg(d) = 4

deg(e) = 4

deg (f) = 4

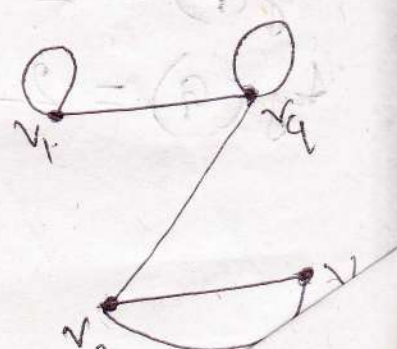
deg(g) = 2

So, the minimum colour = 4.



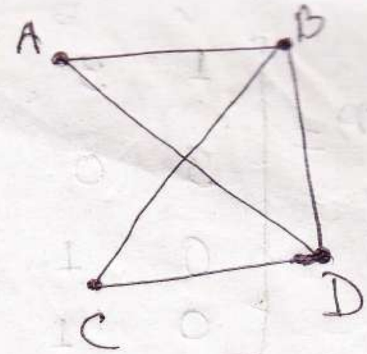
* Find the adsacent matrix:-

$$a)\ A = \begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

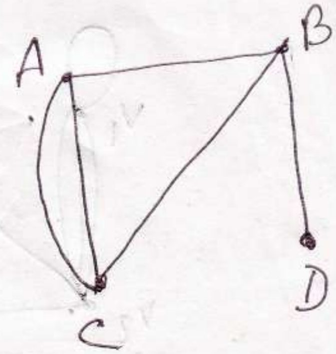


$$ii)\ A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 2 & 0 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

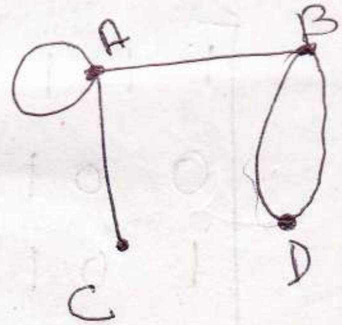
$$A = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$



B) $= A = \begin{bmatrix} 0 & 1 & 2 & 0 \\ 1 & 0 & 1 & 1 \\ 2 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$



c) $A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 2 \\ 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \end{bmatrix}$

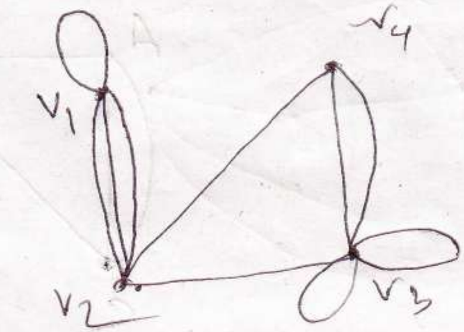


b

(b)

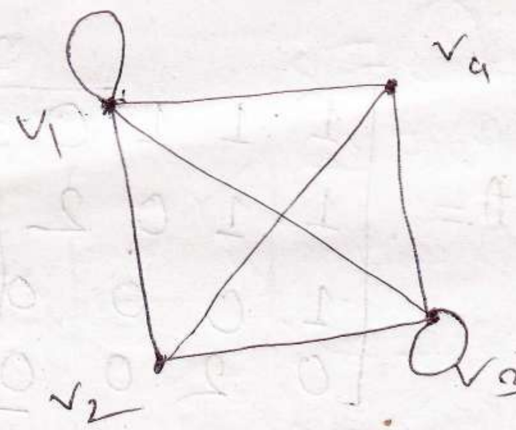
$$A = \begin{array}{c} \\ \\ \end{array}\begin{array}{cccc} a & b & c & d \end{array}$$

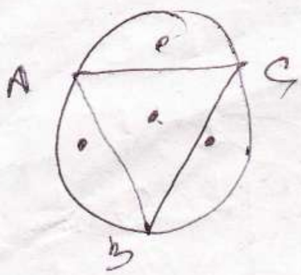
$$A = \begin{array}{c|c} & \begin{array}{cccc} a & b & c & d \end{array} \\ \hline \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix} \end{array}$$



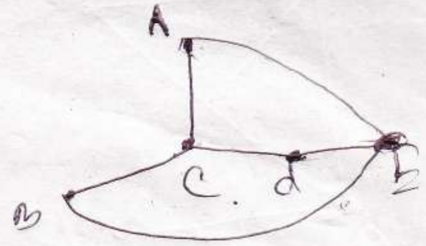
* (c)

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} \Rightarrow$$





$\Rightarrow$



~~E E C A B D~~

~~R R~~

C A B D

E C B B B

R R B

$deg(A) = 2$

$deg(B) = 2$

$deg(C) = 3$

$deg(d) = 2$

$deg(E) = 3$

Tautology: A compound proposition that ~~has~~ is always true, no matter what the truth values of the propositions that occur in it is called a tautology.

Contradiction: A compound proposition that is always false is called contradiction.

$p \vee \neg p$ is always true and $p \wedge \neg p$ is always false.

p	q	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
F		T	F

* Logical Equivalent: A compound proposition that have the same truth values in all possible cases are called logically equivalent.

Another: A proposition p and q are logically equivalent if $p \leftrightarrow q$ is a tautology.

The notation $p \equiv q$ denotes that p and q are logically equivalent.

show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent.

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$	$\neg p \wedge \neg q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

Table says that $\neg(p \vee q)$ and $(\neg p \wedge \neg q)$ is a tautology and that is these propositions are logically equivalent.

* show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent.

$$\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg(\neg p \wedge q) \quad \text{[De morgan's law]}$$
$$\equiv \neg p \wedge (p \vee \neg q)$$
$$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q) \quad \text{[distributive law]}$$
$$= F \vee (\neg p \wedge \neg q)$$
$$= \neg p \wedge \neg q \quad \text{[Identity law]}$$

~ranslating English Sentences into logical.

a) You can access the Internet from campus only if you are a computer science major or you are not freshman

solution: Let,

You can access Internet from campus = a

You are a computer ~~ma~~science major = c.

You are a freshman = f.

So, logical expression is

$$a \rightarrow (c \vee \neg f).$$

(b) ~~You~~ let,

You can ride roller coaster = q

" are under 4 feet tall = ~~s~~ r.

" " older than 16 years old = s.

$$(r \wedge \neg s) \rightarrow \neg q$$

Ex.

7. (a) p = It is below freezing ~~and snowing~~.

q = It is snowing.

a) logical Expression = $p \wedge q$

(b) " " = $p \wedge \neg q$.

(c) " " = $\neg p \wedge \neg q$.

(d) " " = $p \vee q$

" " = $p \rightarrow q$.

Implication: Let p and q be proposition. The c[…] $p \to q$ is the proposition that false when p is true [and] q is false, and true otherwise. In this case p is called the hypothesis and q is called the conclusion.

The truth table for the implication $p \to q$.

যদি তবে If, Then.

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional: Let p and q be propositions.

* The biconditional $p \leftrightarrow$ is the proposition that is true when p and q have the same truth values an[d] is false otherwise. [p if and only if q]

Truth table for biconditional. $p \leftrightarrow q$.

p	q	$p \Leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

[p if and only if q
p is necessary & sufficient for q]

that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

$$
\begin{aligned}
(p \wedge q) \rightarrow (p \vee q) &\equiv \neg(p \wedge q) \vee (p \vee q) \\
&\equiv (\neg p \vee \neg q) \vee (p \vee q) \\
&= (\neg p \vee p) \vee (\neg q \vee q) \\
&= T \vee T \\
&= T \quad \text{[By domination law]}
\end{aligned}
$$

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \rightarrow (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

This is tautology.

* Show that $(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$ is a tautology.

p	q	r	$\neg p$	$p \vee q$	$\neg p \vee r$	$q \vee r$	$(p \vee q) \wedge (\neg p \vee r)$	$(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$
T	T	T	F	T	T	T	T	T
T	T	F	F	T	T	T	T	T
T	F	T	F	T	T	T	T	T
T	F	F	F	T	F	F	F	T
F	T	T	T	T	T	T	T	T
F	F	T	T	F	T	T	F	T
F	T	F	T	T	T	T	T	T
F	F	F	T	F	T	F	F	T

This is a tautology