2(c)-Day Date: 9/5/2017

Mean → Arithmetic Mean
→ Geometrical Mean
→ Harmonic Mean

Median

Mode

~~6, 25, 29, 55, 71, 82,~~

10, 15, 18, 11, 12, 19, 17, 14, 11, 12,
16, 13, 20, 14, 18, 11, 15, 17, 16,
11, 17, 18, 14, 17, 15

10, 11, 11, 11, 11, 12, 12, 13, 14, 14, 14, 15,
15, 15, 16, 16, 17, 17, 17, 17, 18, 18, 18,
19, 20

~~The~~ Median → 15

(25) Odd → 13rd

(24) even → $\frac{12nd + 13rd}{2}$

*Mode

The mode is the value of the observation that appears most frequently.

Geometric Mean

$$GM = \sqrt[n]{(x_1)(x_2)(x_3)\cdots(x_n)}$$

application:

Interest rate, growth rate

2(D)-Day

Date: 13/5/2017

Graphical representation and tabular representation of data:

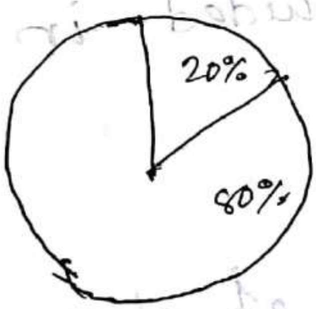
→ Pie chart

→ Bar graph

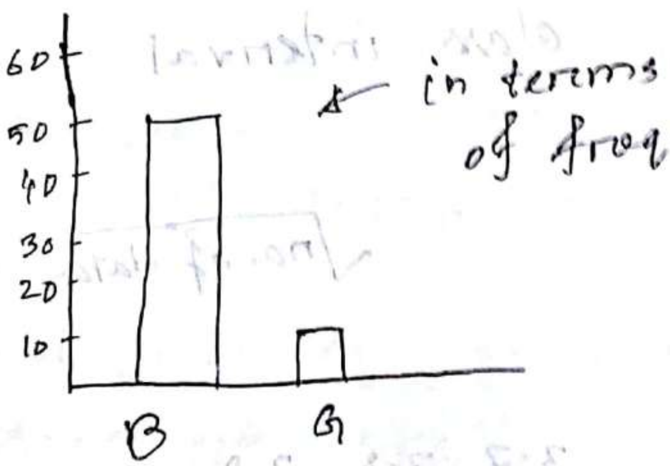
Parcentage → Relative frequency

No. of boys → Frequency

$$\text{Relative Frequency} = \frac{\text{Frequency}}{\text{Total no. of } \cancel{\text{students}} \text{ observation}} \times 100$$



Pie chart



Bar chart

Bar chart is more easier than pie chart

Quantitative Variable:

Frequency Distribution:

A list consisting pair of values.

Value + frequence → Frequency Distribution

Frequency Distribution:

→ Ungrouped → each value + freq.

→ Grouped → constructing set of class/group of values

* Right end level is not included in class interval

$\sqrt{\text{no. of data}}$ → no. of class [not fixed rule]

3.7 3.8 3.9 4.0 4.1 4.2 4.3 4.4 4.5 4.6 4.7

MATLAB → মাতলব

mint014 B/A

2(E)-Day

Date: 14/5/2017

* Stem & Leaf Display (Tabular)
  → Used when number of data points are limited

* Histogram

~~Bar গুলো~~
→ ~~No~~ বারগুলোর মাঝে ফাঁকা থাকে না।

Measures of Dispersion:

48 49 50 51 52   AM = 50
Median = 50

1 2 50 99 100   AM = 50
Median = 50

→ Measure of central tendency cannot characterize the data set alone.

Measure of dispersion: (common)

→ Range   Range = Highest − Lowest
$= 52 - 48$
$= 4$

→ Variance

→ Sample variance

$$s^2 = \frac{1}{n-1} \sum (x - \bar{x})^2$$

$$s^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}$$

→ standard deviation

$$s = \sqrt{s^2}$$

→ Population SD

3(c) - Day Date: 17/5/2017

→ Quartiles

→ Midquartiles

5- Number Summary

Interquartile range

↳ measure of despersion

8 9 10 2 5 3 7 12 15

2 3 5 7 8 9 10 12 15

Q_1 Q_2 Q_3

→ Parcentiles

$$I = n\frac{p}{100}$$

$n \rightarrow$ sample size

<u>3(D)-Day</u> **Date:** 20/5/2017

~~SD~~ Z-score ± 3 এবং বাইরে হলে ধরে নেয়া হয় যে সেখানে অনেক অনেক বিরোধী Data থাকতে ~~হ~~ পারে.

Standardized Variable

Mean $\rightarrow$ 0

SD $\rightarrow$ 1

<u>Next slide</u>

$\downarrow$

<u>Describing Distributions</u>

$$\text{Skewness} = \frac{\sqrt{n} \sum_{i=1}^{n} (x_i - \bar{x})^3}{s^3 - n}$$

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Z-score

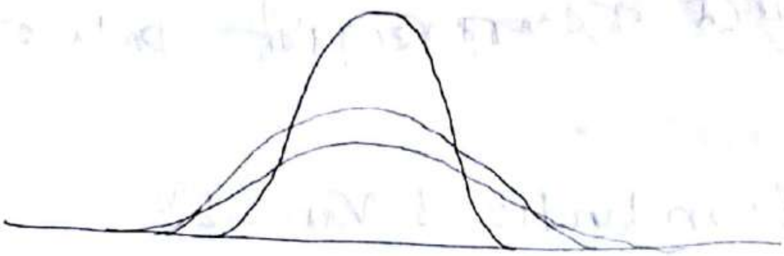
$$Z = \frac{\text{value} - \text{mean}}{\text{st. dev.}} = \frac{x - \bar{x}}{s}$$

Z-score का normal value -3 to $+3$

Z - score
- population Z-score $\rightarrow \mu$
- sample mean $\rightarrow \bar{x}$

$$Z = \frac{x - \mu}{\sigma} \quad \text{and} \quad Z = \frac{x - \bar{x}}{\sigma}$$

Kurtosis



~~Pickness~~ Peakness ধরনের জন্য

Skewness

Skewness ধরনের জন্য

Extreme value → outlier

3(E)-Day

Date: 21/5/2017

Outlier Determination:

→ Determine Q_1 & Q_3

⇒ $IQR = Q_3 - Q_1$

⇒ $Q_1 - 1.5 \times IQR$ & $Q_3 + 1.5 \times IQR$

⇒ $x < (Q_1 - 1.5 \times IQR)$ → x is an outlier
$x > (Q_3 + 1.5 \times IQR)$ →

Coefficient of variation,

$CV = \left(\frac{s}{\bar{x}}\right) 100\%$ → unit আলাদা থাকলেও comparison করা হয়।

Mutually Exclusive Events

$A_1, A_2, \ldots, A_K$ are mutually exclusive events if

$\rightarrow A_i \cap A_j = \phi, \; i \neq j$

$A_1, A_2, \ldots A_K$ are Exhaustive events if

$\rightarrow A_1 \cup A_2 \cup \cdots A_K = S$

মানুষের Experience থেকে আসে Probability
→ Subjective probability

* Exam script-এ measure এর মাধ্যমে probability নির্ণয়ে হবে শূন্য।

Probability is a real valued set-function P that assigns to each events of A, is called the ~~the~~ probability of the event A.

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4(C) - Day   Date: 4/7/2017

1 — Head
2 → Head
3 → Head
4 → Tale
5 → Head
6 → Head
7 → Tale
8 → Head
9 → Head
10 → Head

HH
HT
TH
TT

S set

$S = \{HH, HT, TH, TT\}$

$P(S) = \{\{HH\}, \{HT\}, \{TH\}, \{TT\}, \{HH, HT\}, \{HH, TH\}$
$\{HH, TT\}, \{HT, TH\}, \{HT, TT\},$
$\{TH, TT\}, \{HH, HT, TH\}, \{HH, HT, TT\}$
$\{HH, TH, TT\}, \{HT, TH, TT\},$
$\{HH, HT, TH, TT\}, \phi\}$

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P, A

$P(A) \rightarrow$ measure

i) $P(A) \geq 0$

ii) $P(S) = 1$

iii) If $A_1, A_2, \ldots$ are events and $A_i \cap A_j = \phi$ where $i \neq j$

then $P(A_1 \cup A_2 \ldots \cup A_k)$

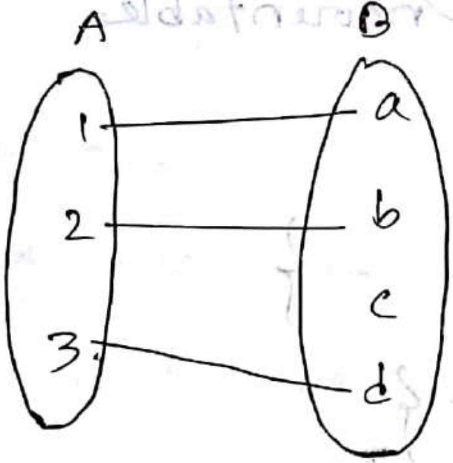
$= P(A_1) + P(A_2) + \cdots + P(A_k)$

for ~~th~~ each positive integer k and $P(A_1 \cup A_2 \cup \ldots) = P(A_1) + P(A_2) + \cdots$

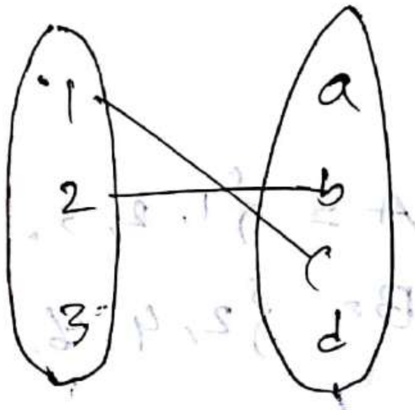
for an infinite but countable number of events.

$A = \{1, 2, 3\}$

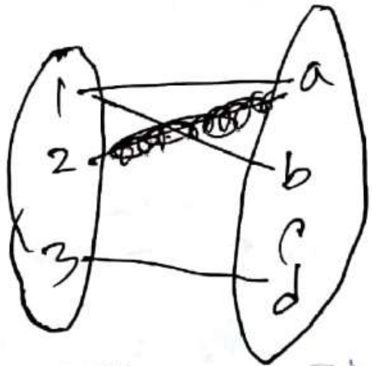
$B = \{a, b, c, d\}$



Function



Not function



Not function

set function → Domain এর element গুলো হবে set

Infinite

Countable Uncountable

$A = \{1, 2, 3, 4, 5, \ldots\}$

$B = \{2, 4, 6, 8, \ldots\}$

Theorem → 1.2-1

1.2-2

1.2-3

1.2-4

1.2-5

1.2-6

Methods of enumeration → দেখে জানতে হবে

Sampling with replacement

" without "

4(E) - Day

Date: 8/7/2017

1. For each event A

$$P(A) = 1 - P(A')$$

2. $P(\phi) = 0$

3. If two events A and B are such that $A \subset B$, then $P(A) \leq P(B)$

4. For each event A, $P(A) \leq 1$

5. If A and B are any two events, then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

① Let, A be the set and the universal set is S.

$\therefore A' = S - A$

Probability of A is $P(A)$

" " A' " $P(A')$

According to the definition of probability

$P(A) + P(A') = 1$

$\therefore P(A) = 1 - P(A')$ [Proved]

1

$S = A \cup A'$

$P(S) = \cancel{P(A) + P(A')}\ P(A \cup A')$

$\Rightarrow 1 = P(A) + P(A')$

$\therefore P(A) = 1 - P(A')$

$\because P(S) = 1$

$A \cap A' = \emptyset$

2

$\cancel{S = \phi \cup \phi'}$

$\cancel{P(S) = P(\phi \cup \phi')}$

$\because P(S) = 1$

$\phi \cap \phi' = \emptyset$

$A = \phi,\ S = A'$

$P(A) = 1 - P(A')$

$\Rightarrow P(\phi) = 1 - P(S)$

$= 1 - 1$

$= 0$

$P(B) = P(A) \cup (B \cap A'))$

$= P(A) + P(B \cap A') \geq P(A)$

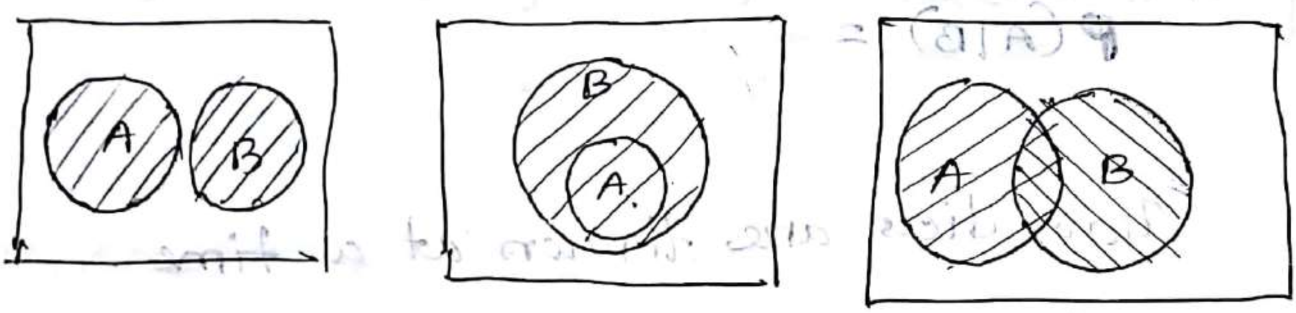
$P(B) \geq P(A)$

$\therefore P(A) \leq P(B)$

5

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$A \cup B = A \cup (A' \cap B)$



~~$A \cup (A' \cap B)$~~

$P(A \cup B) = P(A \cup (A' \cap B))$

$= P(A) + P(A' \cap B)$ — (i)

$B = (A \cap B) \cup (A' \cap B)$

$\therefore P(B) = P(A \cap B) + P(A' \cap B)$

$\Rightarrow P(A' \cap B) = P(B) - P(A \cap B)$

$\therefore$ from (i) $\Rightarrow$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

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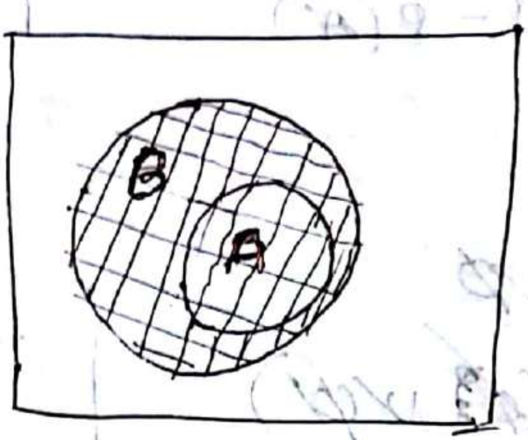
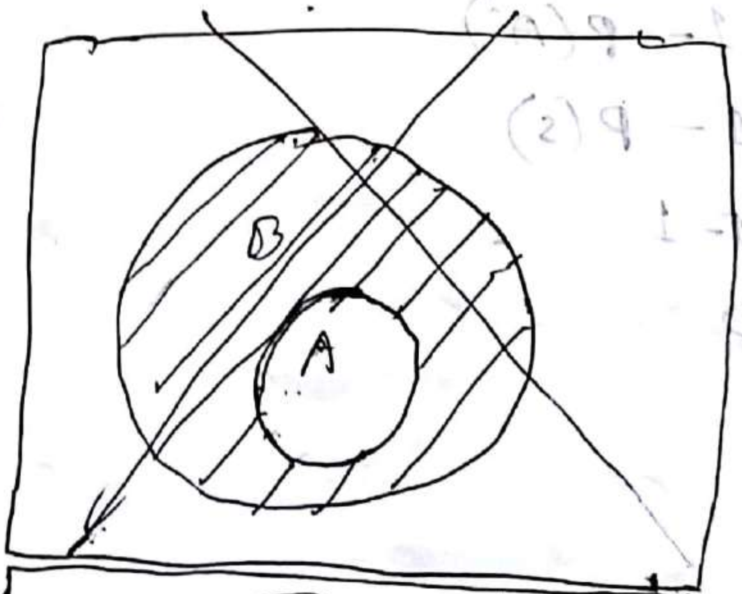
3

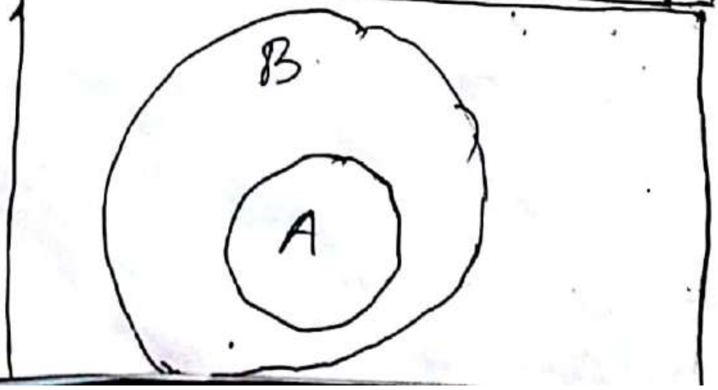
$B = A \cup (B \cap A')$

$A \cap (B \cap A') = \phi$



$A \subset B$





5(C) - Day Date: 11/7/2017

A = { Both are tails }

S = { HH, HT, TH, TT }

$\therefore P(A) = \frac{1}{4}$

B = { at least one coin shows head }

$P(B) = \frac{3}{4}$

A, B

$P(A|B) =$

Two dices are thrown at a time.

S = { (1,1), (1,2), (1,3), (1,4), (1,5), (1,6),
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6) }

$A = \{$ ~~same~~ sum of the values ~~are~~ of two dices will be greater than 12$\}$

$P(A) = 0$

$A' = \{- \quad - \longrightarrow 6\}$

$\therefore P(A) = \frac{21}{36}$

$B = \{$The value of the first dice will be 5$\}$

$P(B) = \frac{1}{6}$

$P(A \cap B) = \frac{21}{36}$

$B = \{$ ———— greater than 3$\}$

$P(B) = \frac{18}{36} = \frac{1}{2}$

$P(A \cap B) = \frac{15}{36} = \frac{5}{12}$

6(D)-Day

Date: 19/7/2017

Random variable

Variable →

Catagorical value

$S = \{HH, HT, TH, TT\}$

s

Numeric value

$X(s) = \{2, 1, 0\}$

→ discrete random variable

→ get some states

→ no states between any two states

→ Continuous random variable

$P(\{HH\}) = \frac{1}{4}$

$P(X=2) = \frac{1}{4}$

$S = \{HH, HT, TH, TT\}$

$A = \{$ heads on the first flip $\}$

$\therefore P(A) = \frac{2}{4} = \frac{1}{2}$

$B = \{$ tails on the second flip $\}$

$\therefore P(B) = \frac{1}{2}$

$C = \{$ tails on both flips $\}$

$P(C) = \frac{1}{4}$

$$P(B|C) = \frac{P(B \cap C)}{P(C)} = \frac{1/4}{1/4} = 1$$

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{4} \times \frac{2}{1} = \frac{1}{2}$$

$$P(B \cap A) = P(B)\,P(A) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$P(B \cap A) = \frac{1}{4}$

$\therefore$ A and B are independent events.

$P(A \cap B \cap C) = P(A)\,P(B)\,P(C)$

A, B and C are independent

probability of A ~~event B~~ depend B

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\frac{5}{12}}{\frac{1}{2}} = \frac{10}{12} = \frac{5}{6}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$= \frac{5/12}{21/36} = \frac{5}{12} \times \frac{36}{21} = \frac{5}{7}$$

$P(A \cap B) = P(B)\,P(A|B)$ → multiplication rule

$= P(A)\,P(B|A)$

Independent Events

Tow events A and B are independent if and only if

$$P(A \cap B) = P(A)\,P(B)$$

~~Ba~~ ~~is~~ a

p.m.f

$f(x)$

characteristics →

① $f(x) \geq 0, \ x \in S$

② $\sum f(x) = 1, \ x \in S$

③

$$P(X \in A) = \sum_{x \in A} f(x) \text{ where } A \subset S$$

1 dice is thrown →

$X(S) = \{1, 2, 3, 4, 5, 6\}$

$f(x) = \frac{1}{6}$

$f(x) = \frac{1}{m}$ ← Uniform probability distribution

Consider a pond. There are 50 types of fish in which 10 are tagged. If a fishermen catches 7 fish. What is the probability of catching two tagged fish.

$f(x=2) = ?$

~~$S = \{ (a_1, a_2, a_3, a$~~

$a_{50} = \{a_1, a_2, \ldots, a_{50}\}$

$a_{5T}, a_{10T}, a_{15T}, a_{20T}, a_{25T}, a_{30T}, a_{35T}, a_{40T}, a_{45T}, a_{50T}$

$S = \{ (a_1, a_2, a_3, a_4, a_{5T}, a_6, a_7),$
$(a_4, a_{5T}, a_{10T}, a_{17}, a_{36}, a_{49}, a_{50T})$
$(a_1, a_2, a_3, a_4, a_6, a_7, a_8)$
$(a_4, a_6, a_{11}, a_{13}, a_{33}, a_{43}, a_{44})$
$(a_1, a_2, a_3, a_7, a_8, a_9, a_{10T}),$
$(a_{30T}, a_{31}, a_{35T}, a_{39}, a_{41}, a_{46}, a_{49})$
$(a_2, a_4, a_9, a_{12}, a_{13}, a_{14}, a_{15T}),$
$(a_{5T}, a_{10T}, a_{15T}, a_{20T}, a_{30T}, a_{45T}, a_{50T})$
$(a_3, a_6, a_7, a_9, a_{11}, a_{16}, a_{17}),$
$(a_7, a_{10T}, a_{15T}, a_{17}, a_{18}, a_{22}, a_{24}).$

$^{50}C_7$

4 sided dice is throwon two times $\rightarrow$

$$S = \{ (1,1),\ (1,2),\ (1,3),\ (1,4),$$
$$(2,1),\ (2,2),\ (2,3),\ (2,4),$$
$$(3,1),\ (3,2),\ (3,3),\ (3,4),$$
$$(4,1),\ (4,2),\ (4,3),\ (4,4) \}$$

Let, x

$$X(s) = \{ 1,\ 2,\ 3,\ 4,$$
$$2,\ 2,\ 3,\ 4,$$
$$3,\ 3,\ 3,\ 4,$$
$$4,\ 4,\ 4,\ 4 \}$$

$$f(x=1) = \frac{1}{16} \qquad f(x=3) = \frac{5}{16}$$

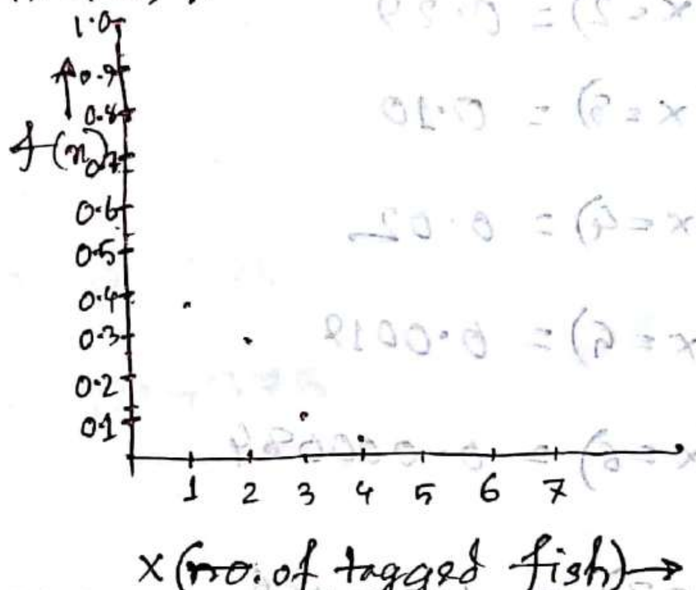
$$f(x=2) = \frac{3}{16} \qquad f(x=4) = \frac{7}{16}$$

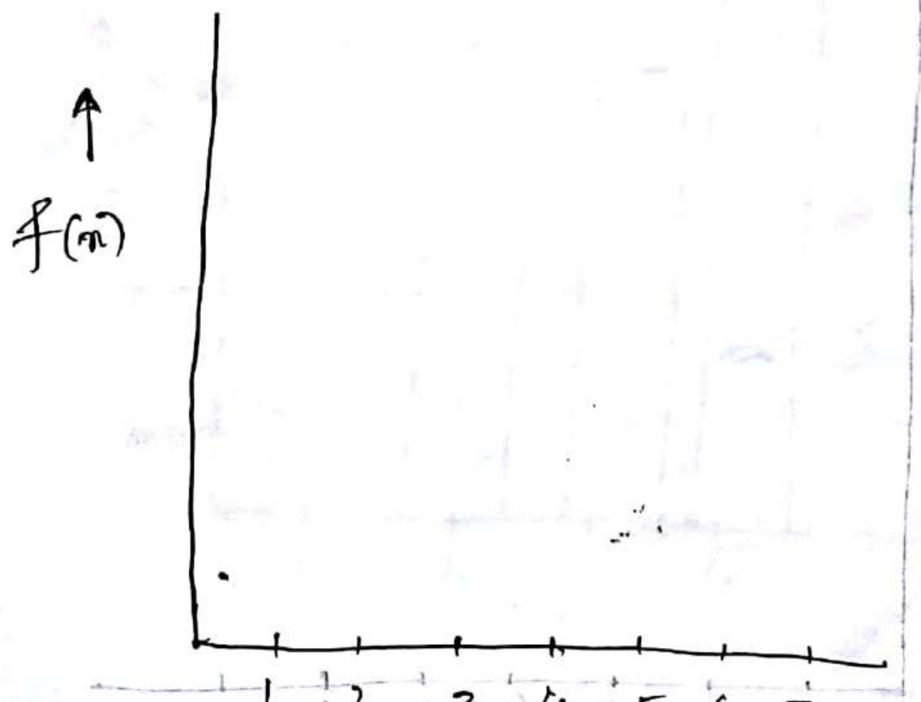
$$\therefore f(x) = \frac{2x-1}{16}$$

6(E)-Day

Date: 29/7/2017

X = 1, 2, 3, 4, 5, 6, 7





$$X(s) = \{1, 3, 0, 0, 1, 2, 1, 7, 0, 2\}$$

$$X = \{0, 1, 2, 3, 4, 5, 6, 7\}$$

$$f(x=2) = \frac{\binom{10}{2}\binom{40}{5}}{{}^{50}C_7}$$

$$\therefore f(x) = P(X=x) = \frac{\binom{N_1}{x}\binom{N_2}{n-x}}{\binom{N}{n}} \qquad N = N_1 + N_2$$

Hyper geometric distribution

$f(x=0) = 0.187$

$f(x=1) = 0.38$

$f(x=2) = 0.29$

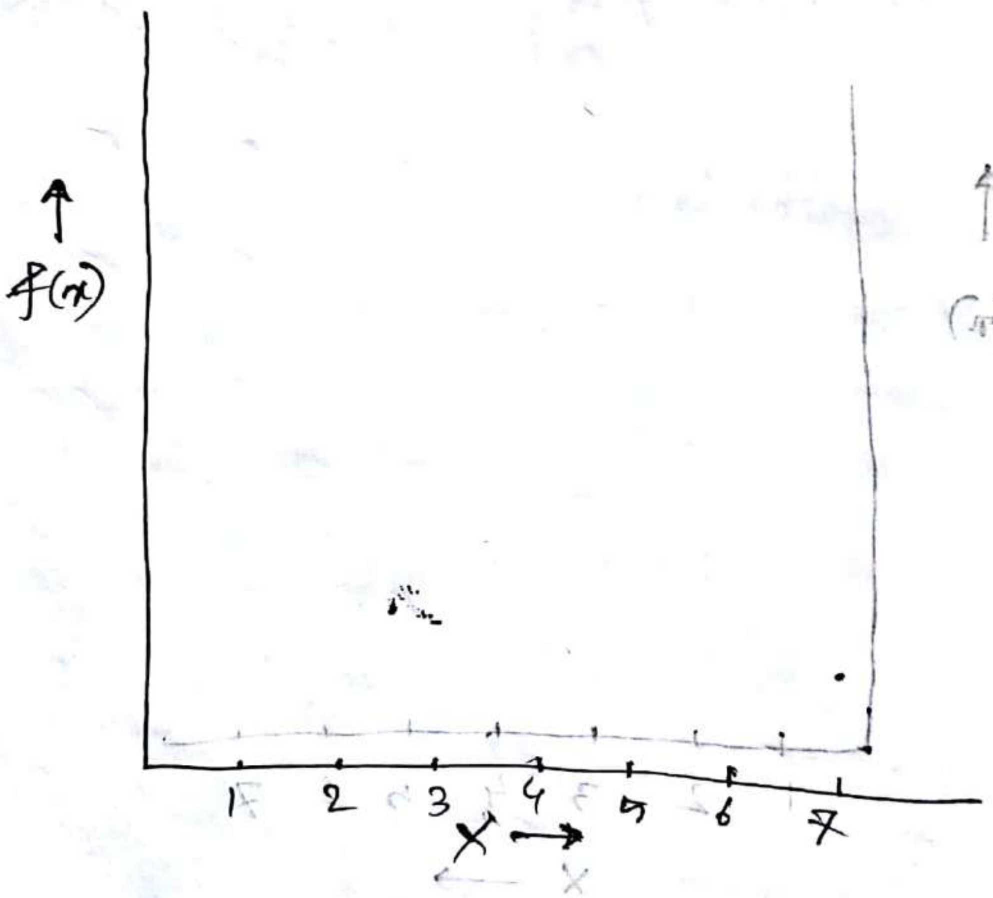
$f(x=3) = 0.10$

$f(x=4) = 0.02$

$f(x=5) = 0.0019$

$f(x=6) = 0.000084$

$f(x=7) = 1.2 \times 10^{-6} = 0.0000012$



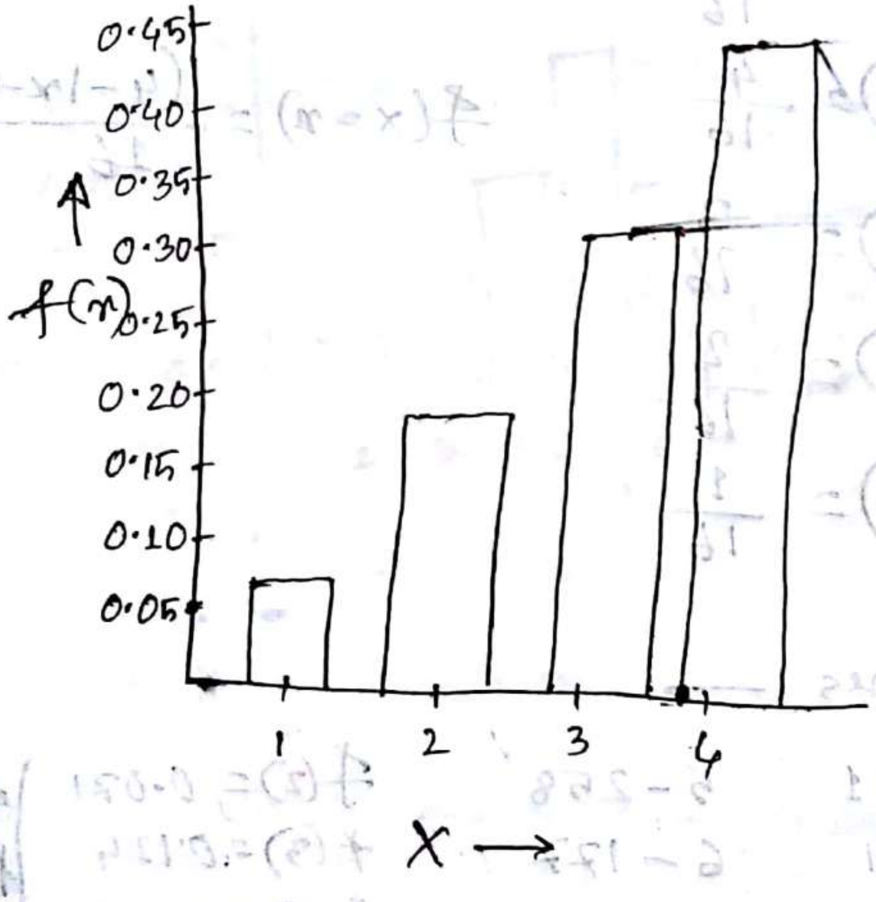
$$f(x) = \frac{2x-1}{16} \qquad x = 1, 2, 3, 4$$

$$f(1) = \frac{1}{16} = 0.0625$$

$$f(2) = \frac{3}{16} = 0.1875$$

$$f(3) = \frac{5}{16} = 0.3125$$

$$f(4) = \frac{7}{16} = 0.4375$$



* Four sided dices are thrown two times. x = sum of the value of two dices.

$$S = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4)\}$$

$f(x=2) = \frac{1}{16}$ ← True value

$f(x=3) = \frac{2}{16}$

$f(x=4) = \frac{3}{16}$

$f(x=5) = \frac{4}{16}$ $\qquad f(x=n) = \frac{(4-|x-5|)}{16}$

$f(x=6) = \frac{3}{16}$

$f(x=7) = \frac{2}{16}$

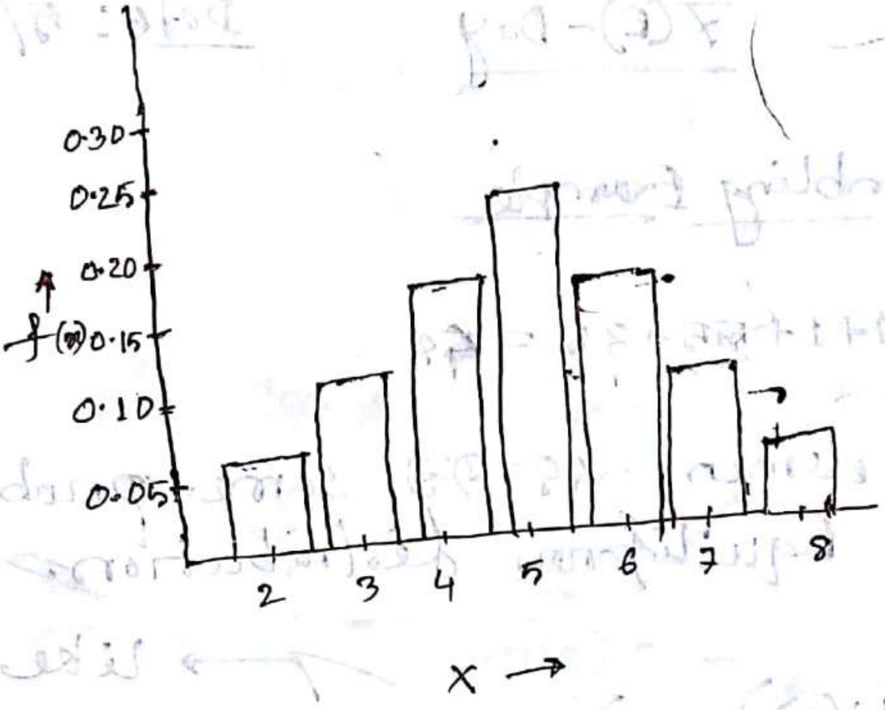
$f(x=8) = \frac{1}{16}$

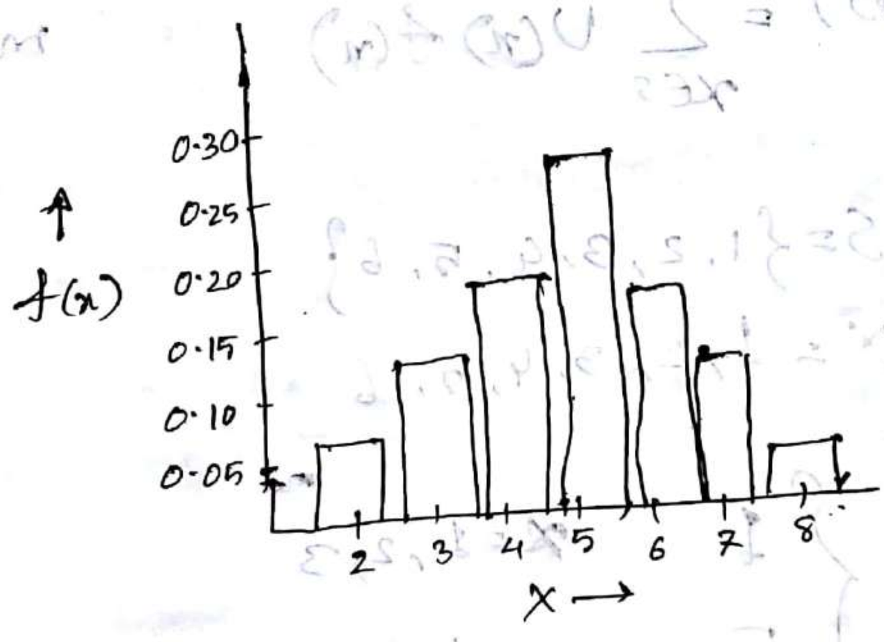
own 1000 times →

2 — 71	5 — 258	$f(2) = 0.071$
3 — 124	6 — 177	$f(3) = 0.124$
4 — 194	7 — 122	$f(4) = 0.194$
	8 — 54	$f(5) = 0.258$
		$f(6) = 0.177$

$f(7) = 0.122$

$f(8) = 0.054$





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* <u>4 sided dices</u>

$$S = \{ \underset{2}{(1,1)}, \underset{3}{(1,2)}, \underset{4}{(1,3)}, \underset{5}{(1,4)},$$
$$\underset{3}{(2,1)}, \underset{4}{(2,2)}, \underset{5}{(2,3)}, \underset{6}{(2,4)},$$
$$\underset{4}{(3,1)}, \underset{5}{(3,2)}, \underset{6}{(3,3)}, \underset{7}{(3,4)},$$
$$\underset{5}{(4,1)}, \underset{6}{(4,2)}, \underset{7}{(4,3)}, \underset{8}{(4,4)} \}$$

$X = 2, 3, 4, 5, 6, 7, 8$ ~~Mean = 2+3+4~~

$$\text{Mean} = \frac{2+3+4+5+3+4+5+6+4+5+6+7+5+6+7+8}{16} = 5$$

$f(2) = \frac{1}{16}$

$f(3) = \frac{2}{16}$

$f(4) = \frac{3}{16}$

$f(5) = \frac{4}{16}$

$f(6) = \frac{3}{16}$

$f(7) = \frac{2}{16}$

$f(8) = \frac{1}{16}$

$$f(x) = \frac{(4-|x-5|)}{16}$$

$$\sum v(x) f(x)$$

$$= 2\times\frac{1}{16} + 3\times\frac{2}{16} + 4\times\frac{3}{16} + 5\times\frac{4}{16} + 6\times\frac{3}{16} + 7\times\frac{2}{16} + 8$$

$$= 5$$

$$S = \{HH, HT, TH, TT\}$$

$$\begin{array}{cccc} HH & HT & TH & TT \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 2 & 1 & 1 & 0 \end{array}$$

$X = 0, 1, 2$

$f(0) = \frac{1}{4}$

$f(1) = \frac{1}{2} = \frac{2}{4}$ $\quad f(x) = \frac{\quad}{4}$

$f(2) = \frac{1}{4}$

$0 \cdot \frac{1}{4} + 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{4}$

$= \frac{0+2+2}{4}$

$= 1$

7(E)- Day

Date: 5/8/2017

Dice Gambling Example

$$1+1+1+5+5+35 = 48$$

প্রত্যেকটির element এর জন্য same probability হলে তা Equiliform distribution.

$$E(U(x)) = \sum_{x \in S} U(x) f(x)$$

→ Like weighted mean

$$S = \{1, 2, 3, 4, 5, 6\}$$

$$X = 1, 2, 3, 4, 5, 6$$

$$U(x) = \begin{cases} 1 & x = 1, 2, 3 \\ 5 & x = 4, 5 \\ 35 & x = 6 \end{cases}$$

$$\sum_{x=1}^{6} U(x) f(x)$$

$$= 1 \times \frac{1}{6} + 2 \cdot \frac{1}{6} + 3 \frac{1}{6} + \cdots + 6 \cdot \frac{1}{6}$$

$$= \frac{1+2+3+4+5+6}{6}$$

① Prove that $E(c) = c$

$$E(c) = \sum_{x \in S} c f(x) \qquad | \, U(x) = c$$

$$= c \sum_{x \in S} f(x)$$

$$= c \cdot 1$$

$$= c$$

② Prove that $E[cU(x)] = cE[U(x)]$

$$E[cU(x)] = \sum_{x \in S} cU(x) f(x)$$

$$= c \sum_{x \in S} U(x) f(x)$$

$$= c\, E[U(x)]$$

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$f(x) = \frac{1}{3} \quad x \in S$

$S = \{-1, 0, 1\}$

> x এর value specific হলে ব্যবহারে x lower case

$U(\cancel{X}) = x$

$U(\cancel{X}) = x^2$

$\mu = \frac{-1+0+1}{3} = 0$

$U(\cancel{X}) = x$

$\sum U(\cancel{x}) f(x) = -1 \times \frac{1}{3} + 0 \times \frac{1}{3} + 1 \times \frac{1}{3}$

$= 0$

$U(x) = x^2$

$\sum U(x) f(x) = 1 \times \frac{1}{3} + 0 \times \frac{1}{3} + 1 \times \frac{1}{3}$

$= \frac{2}{3}$

<u>Theorem</u>

If c is a constant, $\xrightarrow{}$ Expectation of

(i) $E(c) = c$

(ii) c is a constant, $U(x)$ is a function, $E[cU(x)] = cE[U(x)]$

(iii) c_1, c_2 constant.

$E[c_1 U_1(x) + c_2 U_2(x)] = c_1 E[U_1(x)] + c_2 E[U_2(x)]$

Variance

standard variance

$$N^2 = E[(x-\mu)^2]$$

$$= E[x^2 - 2\mu x + \mu^2]$$

$$= E[x^2] - 2\mu E[x] + \mu^2$$

$$= E[x^2] - 2\mu^2 + \mu^2 \quad [E[x] = \mu]$$

$$= E[x^2] - \mu^2$$

standard deviation

$$N = \sqrt{N^2}$$

$f(x) = \frac{x}{6}$, $x = 1, 2, 3$

$f(1) = \frac{1}{6}$

$f(2) = \frac{2}{6} = \frac{1}{3}$

$f(3) = \frac{3}{6} = \frac{1}{2}$

$$\sum_{x=1}^{3} v(x) f(x)$$

$$= 1 \times \frac{1}{6} + 2 \times \frac{1}{3} + 3 \times \frac{1}{2}$$

$$= \frac{1+4+9}{6} = \frac{14}{6}$$

$$= \frac{7}{3}$$

(3) Prove that $E[c_1 u_1(x) + c_2 u_2(x)]$.

$$= c_1 E[u_1(x)] + c_2 E[u_2(x)]$$

$$E[c_1 u_1(x) + c_2 u_2(x)]$$

$$= \sum_{x \in S} [c_1 u_1(x) + c_2 u_2(x)] f(x)$$

$$= \sum_{x \in S} c_1 u_1(x) f(x) + \sum_{x \in S} c_2 u_2(x) f(x)$$

$$= c_1 \sum_{x \in S} u_1(x) f(x) + c_2 \sum_{x \in S} u_2(x) f(x)$$

$$= c_1 E[u_1(x)] + c_2 E[u_2(x)]$$

$$\sum_{x=1}^{10} (a+b) x.$$

$$= \sum ax + \sum bx$$

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আবার Example

$N^2 = E[x^2] - \mu^2$

$= \frac{2}{3} - 0$

$= \frac{2}{3}$

$\therefore N = \sqrt{\frac{2}{3}}$

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$$\boxed{\sigma^2 = E[x^2] - \mu^2 = 6 - \cancel{\tfrac{49}{9}} = \frac{5}{9}}$$

$$E(x) = \sum_{x \in S} U(x) f(x)$$

$$= 1 \times \frac{1}{6} + 2 \times \frac{2}{6} + 3 \times \frac{3}{6}$$

$$= \frac{14}{6}$$

$$= \frac{7}{3}$$

$$\therefore E(x^2) = \sum_{x \in S} U(x) f(x)$$

$$\cancel{= 1 \times \frac{1}{36} + 2 \cdot \frac{1}{9} + 3 \cdot \frac{1}{4}}$$

$$\cancel{= \frac{1 + 8 + 27}{36}}$$

$$= 1 \times \frac{1}{6} + 4 \times \frac{1}{3} + 9 \times \frac{1}{2}$$

$$= \frac{1 + 8 + 27}{6}$$

$$= \frac{36}{6} = 6$$

Binomial Distribution

n times Bernoulli experiment

$p \to$ fix

$0101 \rightarrow 2$

$0000 \rightarrow 0$

X is $b(n, p)$

$p \to$ probability

$n \to$ কতবার draw করা হয়েছে

$x \to$ random variable

Lottery

20% winning tickets

$n = 8$

$0\ 0\ 0\ 0\ 1\ 1\ 0\ 0$

$.8\ \ .8\ \ .8\ \ .8\ \ .2\ \ .2\ \ .8\ \ .8 \rightarrow (.8)^6 \times (0.2)^2$

একটি Pattern এর Probability

8(c) - Day Date: 8/8/2017

Bernoulli Trials

Success $\longrightarrow 1 \longrightarrow p$

Failure $\longrightarrow 0 \longrightarrow 1-p$

$x = 0, 1$ $f(x) = p^x (1-p)^{1-x}$

$f(0) = 1-p$

$f(1) = p$

$$E(X) = \sum_{x \in S} U(x) f(x)$$

$$= (1-p) \times 0 + p \times 1$$

$$= p$$

Or,

$$E[(x-\mu)^2]$$

$$= \int_0^1 (x-\mu)^2 f(x)$$

$$E[X^2] = \sum_{x \in S} U(x) f(x)$$

$$= \cancel{(1-p) \times 0 + p \times 1^2} \quad (1-p) \times 0 + p \times 1^2$$

$$= p$$

$$\therefore \sigma^2 = E[X^2] - \mu^2$$

$$= p - p^2$$

$$\therefore \sigma = \sqrt{p - p^2}$$

Proof

$$P[A|B] = \frac{P[A \cap B]}{P[B]}$$

~~P(AC~~

$$P((A \cup B) \cap C)$$
$$= P(AC \cup BC)$$
$$= P(AC) + P(BC)$$

$$P\left(\bigcup_{i=1}^{n} A_i | B\right) = \frac{P\left(\bigcup_{i=1}^{n} A_i B\right)}{P(B)}$$

$$= \frac{P(A_1 B \cup A_2 B \cup \dots \cup A_n B)}{P(B)}$$

$$= \frac{P(A_1 B) + P(A_2 B) + \dots P(A_n B)}{P(B)}$$

$$= \frac{\sum_{i=1}^{n} P[A_i B]}{P(B)}$$

$$= \sum_{i=1}^{n} P(A_i|B)$$

Date: 9/8/2017

8(D)-Day

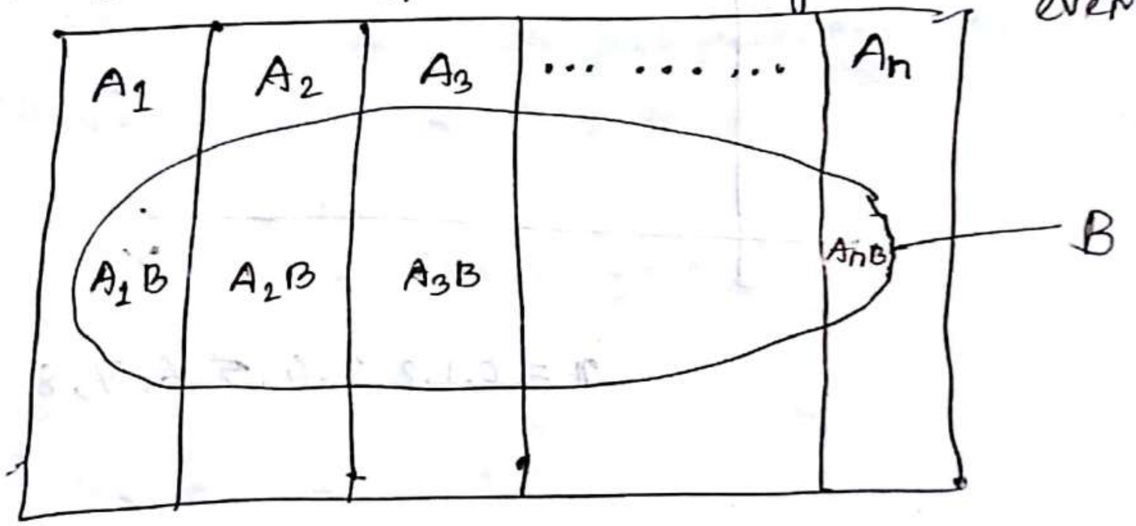
Conditional Probability
Independence

Conditional Probability

$$P[A|B] = \frac{P[AB]}{P[B]} \quad [P[B] \neq 0]$$

$$P[B|A] = \frac{P[AB]}{P[A]}$$

$A_1, A_2, \ldots A_n \rightarrow$ mutually exclusive event

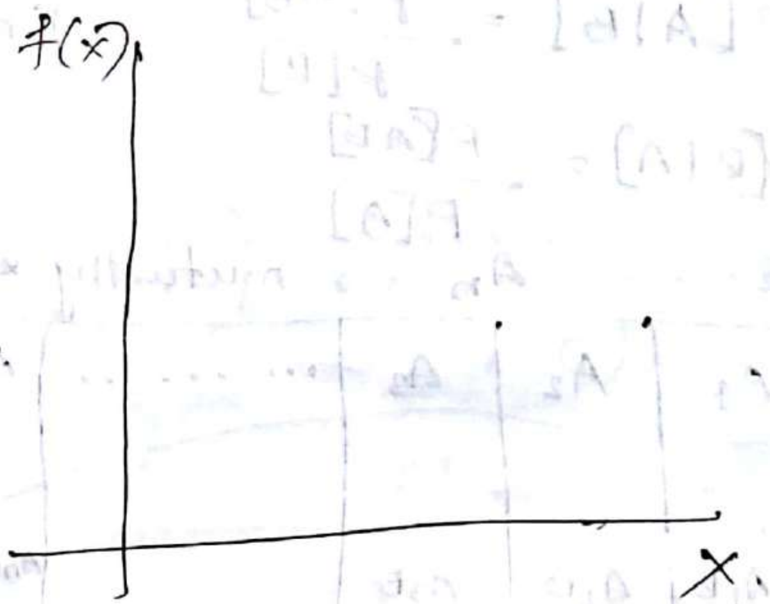


$$P\left[\bigcup_{i=1}^{n} A_n | B\right] = \sum_{i=1}^{n} P[A_i | B]$$

$$f(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$f(2) = \binom{8}{2} p^2 (1-p)^{8-2}$$

$b(8, 0.2)$



$x = 0, 1, 2, 3, 4, 5, 6, 7, 8$

~~Basic~~

Total Probability

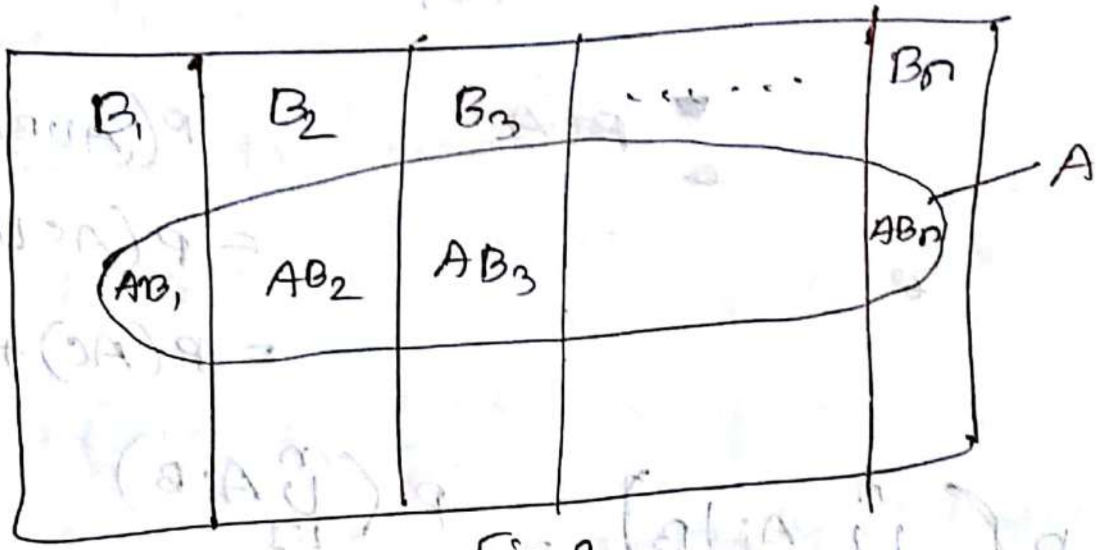


Fig. 2

Total sample space

$$S \text{ or } \Omega = \bigcup_{i=1}^{n} B_i \qquad P(B_i) > 0$$

Theorem of total probabilities →

$$P(A) = \sum_{i=1}^{n} P(B_i) P(A \mid B_i)$$

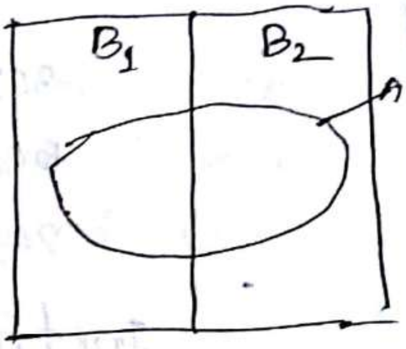
Proof → next class.

Slide → work

Sol^n

B_1 item produced by mA

B_2 " " " mB



A = item-defective

$\cancel{P(B_i|B)}$

$$P(B_1|A) = \frac{P(B_1)\,P(A|B_1)}{P(B_1)\,P(A|B_1) + P(B_2)\,P(A|B_2)}$$

$=$

$P(B_1) = 0.45$

$P(B_2) = 0.55$

$P(A|B_1) = \frac{9}{1000}$

$P(A|B_2) = \frac{2}{500} = \frac{1}{250}$

* ~~Mp. Ka~~i

Bayes ~~Base~~ Theorem

Fig. 2 ← Prev. fig.

$$\Omega = \bigcup_{i=1}^{n} B_i \qquad P(B_i) > 0$$
$$P(A) > 0$$

Then,

$$P(B_k \mid A) = \frac{P(B_k)\, P(A \mid B_k)}{\sum_{i=1}^{n} P(B_i)\, P(A \mid B_i)}$$

Example

At factory

	Produce
Machine A →	45%
Machine B →	the rest (55%)

9 items in 1000 are defective by A.

2 " " 500 " " by B.

20000 items a day. One item is picked and found defective. Probability produced by A.

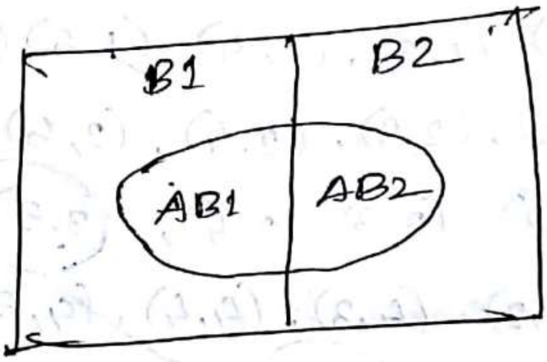
Theorem

5.3.3, 5.3.4, 5.3.5, ~~5.~~ 5.3.6, 5.3.7 — 5.4.3

→ বাদে

Example

5.5.1



Bayes theorem

$$P(B_K|A) = \frac{P(B_K)\,P(A|B_K)}{P(A)}$$

$$= \frac{P(B_K)\,P(A|B_K)}{\sum_{i=1}^{n} P(B_i)P(A|B_i)}$$ ← total prob. theorem থেকে

$$P(B_K A) = P(B_K)\,P(A|B_K)$$

→ theorem of multiplication rule.

9(c) - Day   Date: 19/8/2017

* In rolling two balanced dices, (6 sided) if the sum of the two value is 8. What is the probability that one ~~value~~ of the values is 3.

$$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),$$
$$(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),$$
$$(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),$$
$$(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),$$
$$(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),$$
$$(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$$

probability $= \frac{2}{5}$

B = the sum of the two values is 8

A = One of the values is 3.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{2}{36}}{\frac{5}{36}} = \frac{2}{5}$$

30% from country ~~A~~

50% " " B

20% " " C

Inefficient 0.02 % → from ~~A~~

0.01 % → B

0.03 % → C

Recently 500 Engineers are appointed. Found Mr. Jack inefficient. Probability from country C.

<u>Soln</u>

B_1

If independent →

$$P(A \cap B) = P(A)\, P(B)$$

If dependent →

$$P(A \cap B) = P(B)\, P(A|B)$$

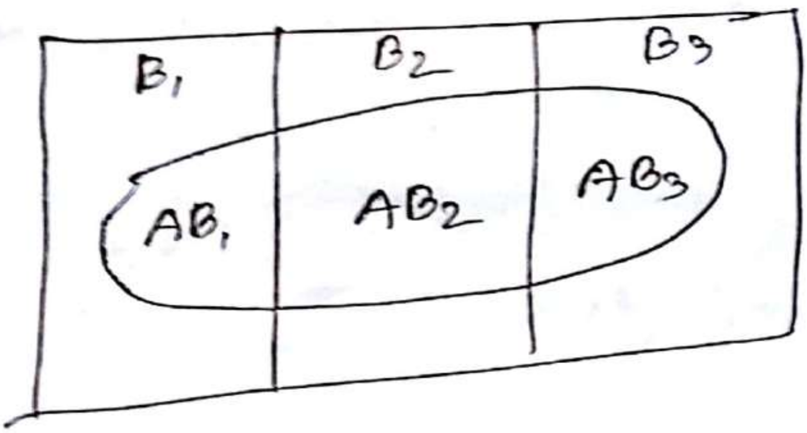
$$= P(A)\, P(B|A)$$

$\xrightarrow{\text{independent}}$ $P(A|B) = P(A)$

Example → 5.5.7

B1 → 30% by Mr. Ahmed
B_2 → 25% by Mrs. Ali
B_3 → 45% by Miss. Karim

A is a statement with error



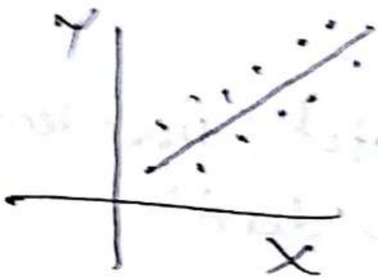
$$P(B_3 | A) = \frac{P(B_3)\, P(A | B_3)}{P(B_1) P(A|B_1) + P(B_2) P(A|B_2) + P(B_3) P(A|B_3)}$$

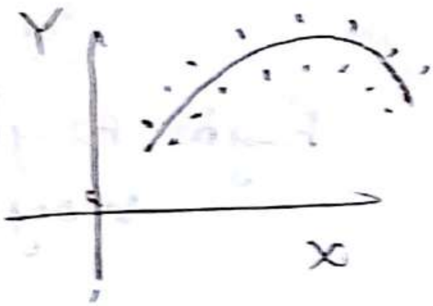
$=$

$P(B_3) = 45$
$P(A|B_3) = 0.00001$
$P(B_1) = 30$
~~$P(B_3) =$~~
$P(B_2) = 25$
$P(A|B_1) =$
$P(A|B_2) =$

Correlation

Linear — Non Linear





Methods of Studying →

→ Scatter Diagram

→ Karl Pearson's

9(D)-Day

Date: 20/8/2017

Double variable

x y

Correlation

~~high~~ height → predicted (dependent)
weight (independent)

* Every Causation is not correlation.

Correlation

positive

x ~~β~~ + ৭৫ দিলে
y + " "

$\boxed{\text{Height, weight}}$

Negative

x + ২০ দিলে
y − " "

On the basis of number of var.

Correlation

Simple

Multiple

Partial

Total

$$r_{x,y} = \frac{S_{x,y}}{S_x \cdot S_y}$$

$$S_{x,y} = \frac{\sum (y_i - \bar{y})(x_i - \bar{x})}{n-1}$$

$$= -322.5$$

$$S_x = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}}$$

$$= \sqrt{\frac{(0-30)^2 + (15-30)^2 + (30-30)^2 + (45-30)^2 + 30^2}{4}}$$

$$= \sqrt{\frac{2250}{4}}$$

$$= \sqrt{562.5} = 23.72$$

$$S_y = \sqrt{\frac{\sum (y - \bar{y})^2}{n-1}}$$

$$= \sqrt{\frac{(90-72)^2 + (84-72)^2 + (66-72)^2 + 10^2 + 14^2}{4}}$$

$$= \sqrt{\frac{800}{4}}$$

$$= \sqrt{200} = 10\sqrt{2}$$

Cigs(X)	Lung Cap(Y)	X × 3	Y × 2
0	45	0	90
5	42	15	84
10	33	30	66
15	31	45	62
20	29	60	58

$\bar{X} = 30$ $\quad \bar{Y} = 72$

$$S_{x,y} = \frac{\Sigma (Y_i - \bar{Y})(X_i - \bar{X})}{n-1}$$

$$= \frac{(90-72)(0-30) + (84-72)(15-30) + (66-72)(30-30) + (62-72)(45-30) + (58-72)(60-30)}{4}$$

$= -322.5$

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10(D)-Day

Date: 27/8/2017

Binomial distributio → for random discrete variable

1. Non overlapping interval এর number of change independent.

2. probab. $\lambda h \rightarrow h$ এর মধ্যে ঘটবে (1 টা change)
   short interval

3. 0 → " " " " (2 or more change)

Probability merge function

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!} \quad \Bigg| \begin{array}{l} x = 0, 1, 2, \ldots \\ \lambda > 0 \end{array}$$

$$E(e^{tx}) = \sum_{x \in S} e^{tx} f(x)$$

$$M(t) = E(tx) = \sum_{x=0}^{\infty} e^{tx} \frac{\lambda^x e^{-\lambda}}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{(e^t \lambda)^x}{x!}$$

$$= e^{-\lambda} e^{\lambda e^t} = e^{\lambda(e^t - 1)}$$

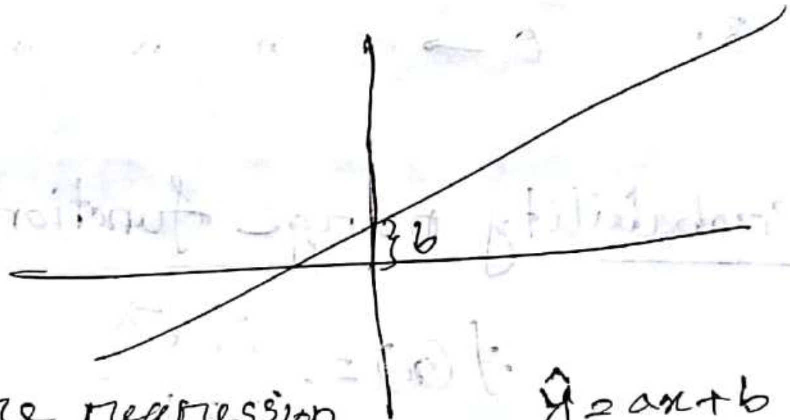
→ এখান থেকে

$$\therefore r_{x,y} = \frac{-322.5}{\cancel{562.5 \times 200}\ 23.72 \times 10\sqrt{2}}$$

$$= \cancel{-2.87 \times 10^{-3}}\ -0.96$$

→ Trained data x ও y দুইটাই আছে

→ Test " x আছে, y নেই



Least square regression $\hat{y} = ax + b$

Error $= y - \hat{y}$

$a = ?$ $b = ?$ where $\sum (y - \hat{y})^2$ min

$\sum (y - ax - b)^2$ ~~dem~~ differentiate

করলে min পাওয়া যাবে।

$$a = \frac{r \cdot S_y}{S_x}$$

| r = corr-coeff.
| S_y = standard dev. y
| S_x = " " x

Oct(29-30) 0 1 1 1 0 1 2 1 4 1 2 3

0 3 0 1 0 1 1 2 3 0 2 2

$$\text{Mean, } \bar{x} = \frac{32}{24} = \frac{16}{12} = \frac{4}{3} =$$

$$\text{Variance} = \frac{\Sigma (x-\bar{x})^2}{n-1}$$

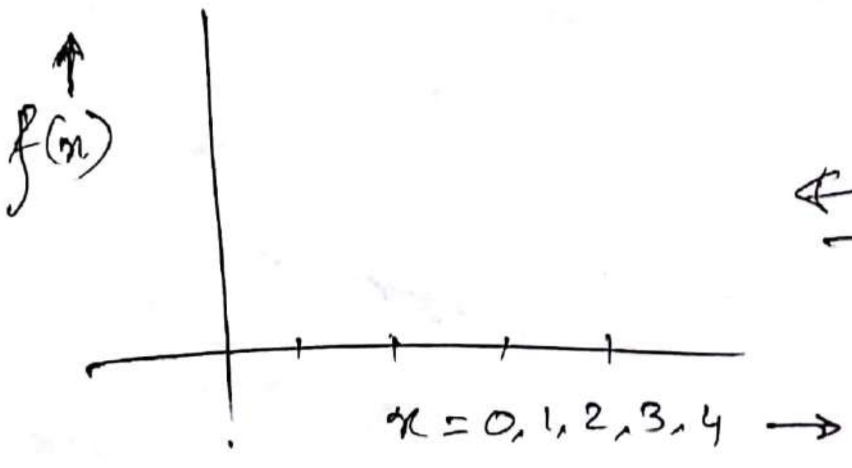
$$\frac{4(3-\frac{4}{3})^2 + 6(-\frac{4}{3})^2 + 9(1-\frac{4}{3})^2 + 5(2-\frac{4}{3})^2}{23}$$

=

Determining poison's distribution →

$\lambda = 1.33$

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!}$$



← Histogram

এর sample data এর Histogram তৈরি করে দেখাতে হবে।

$\cancel{\lambda e^{-\lambda} e}$

$$M'(t) = \frac{dM}{dt} = e^{-\lambda} e^{\lambda e^t} \cdot \lambda e^t = \lambda e^{(t-\lambda)} \cdot e^{\lambda e^t}$$

$$= \lambda e^{(t-\lambda)+\lambda e^t}$$

$\cancel{M''(t) = e^{-\lambda} e^{\lambda e^t}}$. when $t=0$.

$$M'(0) = \lambda$$

$$M''(t) = \lambda e^{(t-\lambda)+\lambda e^t} \cdot (1+\lambda e^t)$$

$$M''(0) = \lambda$$

$$E(X^2) - (E(X))^2 = \lambda - \lambda^2$$

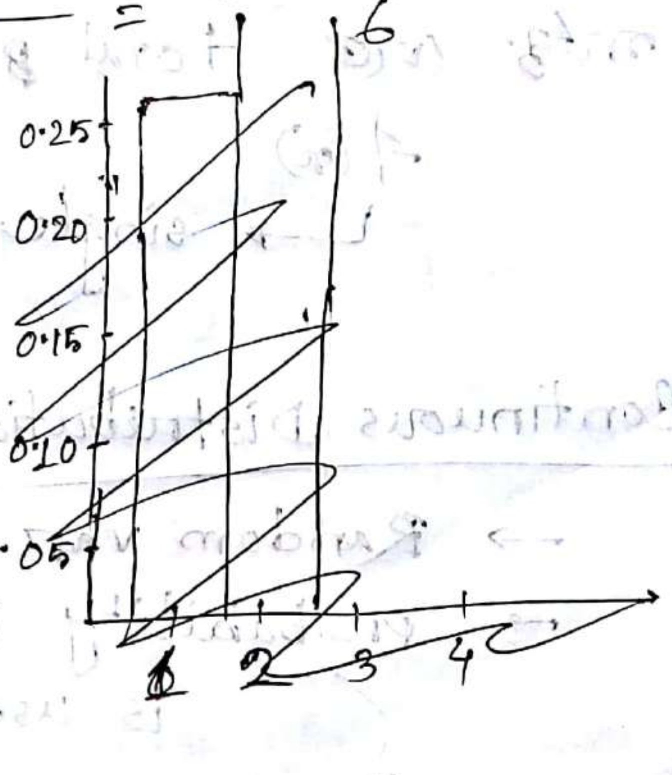
$$f(0) = \frac{\lambda^0 e^{-\lambda}}{0!} = \frac{(1.33)^0 e^{-1.33}}{1} = e^{-1.33} = 0.26$$

$$f(1) = \frac{(1.33)^1 e^{-1.33}}{1!} = 1.33 \times e^{-1.33} = 0.35$$

$$f(2) = \frac{(1.33)^2 \cdot e^{-1.33}}{2!} = \frac{(1.33)^2 \cdot e^{-1.33}}{4} = 0.11$$

$$f(3) = \frac{(1.33)^3 \cdot e^{-1.33}}{3!} = \frac{(1.33)^3 \cdot e^{-1.33}}{6} = 0.10$$

$$f(4) = 0.034$$



$0 \longrightarrow 6 \; 4$

$\therefore$ Prob $\rightarrow \frac{6}{24} = 0.25$

$$f(x \le 3) = \sum_{x=0}^{3} f(x)$$

$$f(x) = \frac{1}{20} e^{-x/20} \quad ; \quad S = 0 \le x < \alpha$$

(b)

$$\int f(x)\,dx = \int_0^{\alpha} \frac{1}{20} e^{-x/20}\,dx$$

$$= \frac{1}{20} \left[\frac{e^{-x/20}}{-\frac{1}{20}} \right]_0^{\alpha}$$

$$= \left[-e^{-x/20} \right]_0^{\alpha}$$

$$= \left[-0 + 1 \right]$$

$$= 1$$

$$\lim_{b \to \alpha} \left[-e^{-x/20} \right]_0^{b}$$

$$= 1 - \lim_{b \to \alpha} e^{-b/20}$$

$$= 1 - 0$$

$$= 1$$

Cumulative distribution function:

capital X

$$F(x) = P(X \le x) \qquad x = 1, 2, \dots, 10$$

$$P(X \le 5) = \sum_{x=1}^{5}$$

সবচেয়ে ছোট value থেকে ঐ value পর্যন্ত যোগ total sum of prob.

$f(x)$ → single value এর জন্য probability

Continuous Distribution

→ Random var. is continuous

→ probability density function is used

→ pdf $f(x)$

(a) $f(x) > 0, \; x \in S$

(b) $\int_S f(x)\, dx = 1$

A → event

(c) $x \in A, \; P(X \in A) = \int_A f(x)\, dx$

$$f(x) = \begin{cases} \frac{1}{20} e^{-x/20}, & 0 \leq x < \infty \\ 0; & \text{otherwise} \end{cases}$$

Continuous এর ক্ষেত্রে কোন specific point এর probability 0.

→ like wight কখনো সঠিক মিলবে না.

→ prob বের করা হচ্ছে একটি range এর মধ্যে।

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)\, dt$$

$$f(x) = \begin{cases} \frac{1}{20} e^{-x/20}, & 0 \le x < \infty \\ 0, & \text{otherwise} \end{cases}$$

Continuous এর ক্ষেত্রে কোন specific point এর probability 0.

↳ like wight করলে মান মিলবে না.

→ prob বের করা হচ্ছে একটি range এর মধ্যে।

$$f(x) = \frac{1}{20} e^{-x/20} \quad ; \quad S = 0 \le x < \infty$$

(b)

$$\int f(x)\,dx = \int_0^{\infty} \frac{1}{20} e^{-x/20}\,dx$$

$$= \frac{1}{20}\left[\frac{e^{-x/20}}{-\frac{1}{20}}\right]_0^{\infty}$$

$$= \left[-e^{-x/20}\right]_0^{\infty}$$

$$= \cancel{0}\,\left[-0+1\right]$$

$$= 1$$

$$\lim_{b \to \infty} \left[-e^{-x/20}\right]_0^{b}$$

$$= 1 - \lim_{b \to \infty} e^{-b/20}$$

$$= 1 - 0$$

$$= 1$$

4.1-3 (2)

for ~~Find~~ each of the following functions,

(i)

(a) $f(x) = \frac{x^3}{4}$, $0 < x < c$

(b) $f(x) = (3/16)x^2$, $-c < x < c$

(c) $f(x) = c/\sqrt{x}$, $0 < x < 1$

Find the mean and variance of each of the above distribution functions.

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10(A) - Day Date - 28/8/2017

4.1-2 (1)

For each of the following functions.

(i) find the constant c so that $f(x)$ is a p.d.f of a random variable x,

(ii) find the distribution function $F(x)$

$$F(x) = P(X \leq x)$$

(a) $f(x) = 4x^c \; ; \; 0 \leq x \leq 1$

(b) $f(x) = c\sqrt{x} \; , \; 0 \leq x \leq 4$

(c) $f(x) = c/x^{3/4} \; , \; 0 \leq x < 1$

~~mean, vari~~

Find the mean and variance of each of the above distribution functions.

mean, $\mu = E(X) = \int_{-\alpha}^{\alpha} x f(x)\, dx$

variance, $\sigma^2 = VAR(X) = E\left[(x-\mu)^2\right] = \int_{-\alpha}^{\alpha} (x-\mu)^2 f(x)\, dx$

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(4)

$$\boxed{F(x) = \int_{-\infty}^{x} f(t)\, dt}$$

~~$f(x) = x e^{x}$ $0 < x < 1$~~

(i)

$$f(x) = 2x \quad , \quad 0 < x \leq 1$$

$$F(x) = \begin{cases} 0 & , x < 0 \\ \int_0^x 2t\,dt = x^2 , & 0 \leq x \leq 1 \\ 1 & , 1 \leq x \end{cases}$$

$$\boxed{\mu = \int_0^1 x \cdot 2x\, dx}$$

$x\, f(x)\, dx$

$$\int_{-\infty}^{x \geq 1} f(t)\, dt$$

$$= \int_{-\infty}^{0} f(t)\,dt + \int_0^1 f(t)\, dt + \int_1^{\infty} f(t)\, dt$$

$$= 0 + 1 + 0 = 1$$

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4·1-4 (3)

Let the random variable X have the p.d.f $f(x) = \begin{cases} 2(1-x), & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$

(a) Find the mean and variance of the above function.

(b) Find (i) $P(0 \le X \le \frac{1}{2})$ (ii) $P(X = \frac{3}{4})$

(iii) $P(X \ge \frac{3}{4})$

4·1-5 (4)

Let $f(x) = xe^{x}$, $0 < x < 1$ be the p.d.f of X.

(a) Show that $f(x)$ is a p.d.f

(b) Find the values of μ and σ^2.

Finding C

(1) (a) $f(x) = 4x^c$ ~~be~~ a p, $0 \le x \le 1$

If $f(x)$ be a p.d.f then,

$$\int_{S} f(x)\, dx = 1$$

$$\Rightarrow \int_0^1 4x^c\, dx = 1$$

$$\Rightarrow 4\left[\frac{x^{c+1}}{c+1}\right]_0^1 = 1$$

$$\Rightarrow 4\left[\frac{1-0}{c+1}\right] = 1$$

$$\Rightarrow 4 = c+1$$

$$\therefore c = 3$$

(1)

(a) $f(x) = 4x^c \quad 0 \le x \le 1$

If $f(x)$ be p.d.f then

$$\int_0^1 4x^c \, dx = 1$$

$$\Rightarrow 4\left[\frac{x^{c+1}}{c+1}\right]_0^1 = 1.$$

$$\Rightarrow \frac{4}{c+1} = 1$$

$\therefore c = 3$

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$F(x) = P(X \le x)$

$$P(X \le 1) = \sum_{x=0}^{1} f(x)$$

$$= \sum_{x=0}^{1} 4x^3 \quad \Big| \; c = 3$$

$= 0 + 4$

$= 4$

$$F(x) = 4\left[\frac{x^{c+1}}{c+1}\right] \quad \Big| \; \text{Putting } c = 3$$

$$= \frac{4}{4}x^4$$

$= x^4$

[Ans].

(8)

$$f(x) = c\sqrt{x} \quad , \quad 0 \le x \le 4$$

$$\int_0^4 c\sqrt{x} = 1$$

$$\Rightarrow c\left[\frac{x^{3/2}}{3/2}\right]_0^4 = 1$$

$$\Rightarrow \frac{c}{3/2}\left[4^{3/2} - 0\right] = 1$$

$$\Rightarrow 8c = \frac{3}{2}$$

$$\therefore c = \frac{3}{16}$$

~~Now,~~ $\cancel{F(x) = P(X \le x)}$

$\cancel{\int_0^x 4\sqrt{t}\, dt = 4\left[\frac{t^{3/2}}{3/2}\right]_0^x = \frac{4x^{3/2}}{3/2} = \frac{8x^{3/2}}{3}}$

Now, $$\int_0^x \frac{3}{16}\sqrt{t}\, dt = \frac{3}{16}\left[\frac{t^{3/2}}{3/2}\right]_0^x$$

$$= \frac{x^{3/2}}{8}$$

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(t)\,dt$$

$$\cancel{= \int_0^1 4x^3\,dx}$$

$$\cancel{= 4\left[\frac{x^4}{4}\right]_0^1}$$

$$\cancel{= x^4}$$

$c = 3$

$$\int_0^x 4t^3\,dt$$

$$= 4\left[\frac{t^4}{4}\right]_0^x$$

$$= x^4$$

(b)

$\cancel{f(x) = \dots}$ $\quad$ $\cancel{0 \le x \le 1}$

$$\therefore F(x) = \begin{cases} 0 & , x < 0 \\ x^4 & , 0 \le x \le 1 \\ 1 & , x > 1 \end{cases}$$

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Moment generating function,

$$M(t) = \exp\left(\mu t + \frac{\sigma^2 t^2}{2}\right)$$

$$M'(t) = \exp\left(\mu t + \frac{\sigma^2 t^2}{2}\right) \cdot \left(\mu + \frac{2\sigma^2 t}{2}\right)$$

$$= \exp\left(\mu t + \frac{\sigma^2 t^2}{2}\right) \cdot \left(\mu + \sigma^2 t\right)$$

$$M'(0) = \mu$$

$$M''(t) = \exp\left(\mu t + \frac{\sigma^2 t^2}{2}\right) \cdot \mu \cdot (\mu + \sigma^2 t)$$

$$+ \sigma^2 \exp\left(\mu t + \frac{\sigma^2 t^2}{2}\right)$$

$$+ \sigma^2 t \exp\left(\mu t + \frac{\sigma^2 t^2}{2}\right) \cdot (\mu + \sigma^2 t)$$

$$M''(0) = \mu^2 + \sigma^2$$

Var(x)

$$= M''(0) - [M'(0)]^2$$

$$= \mu^2 + \sigma^2 - \mu^2$$

$$= \sigma^2$$

12(D)-Day Date: 17/9/2017

Chapter-3

Continuous Distribution

Normal Distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right], \quad -\infty < x < \infty$$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right] dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{z^2}{2}} \cdot \sigma\, dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{z^2}{2}}\, dz$$

Here

$z = \frac{x-\mu}{\sigma}$

$\therefore dz = \frac{1}{\sigma} dx$

$\therefore dx = \sigma\, dz$

Let,

$$I = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$$

H.W

If $I^2 = 1$ then $I = 1$

$$\therefore F(x) = \begin{cases} 0 & , x < 0 \\ \dfrac{x^{3/2}}{8} & , 0 \le x \le 4 \\ 1 & , x > 4 \end{cases}$$

(c) $f(x) = \dfrac{c}{x^{3/4}}$ $\quad 0 < x < 1$

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$$M(t) = \exp(5t + 12t^2)$$

$$\mu = 5 \quad \Big| \quad \sigma^2 = 24$$

$$N(5, 24)$$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$$

$$= \frac{1}{\sqrt{48\pi}} \exp\left[-\frac{(x-5)^2}{48}\right]$$

$N(\mu, \sigma^2)$

$$f(x) = \frac{1}{\sqrt{32\pi}} \exp\left(-\frac{(x+7)^2}{32}\right)$$

$\mu = ?$, $\sigma = ?$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

$\sigma = 4$ | $\mu = -7$

$\therefore \sigma^2 = 16$

X is $N(-7, 16) \rightarrow N(\mu, \sigma^2)$ so, what will be the PDA $\rightarrow$

$$f(x) = \frac{1}{4\sqrt{2\pi}} \exp\left(-\frac{(x+7)^2}{32}\right)$$

Moment generating function,

$$M(t) = \exp\left(-7t + \frac{16t^2}{2}\right)$$

$$= \exp\left(\frac{-14t + 16t^2}{2}\right)$$

$$= \exp(8t^2 - 7t)$$

13(B)-Day. Date: 23/9/2017

Stochastic Process

→ Set of random variables.

→ Discrete-time process $T=\{0,1,2,3,\ldots\}$

→ Continuous-time process $T=[0,\infty)$

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t-step/n-step transition probabilities-156

$$P(X_t=j \mid X_0=i)=P(X_{n+t}=j \mid X_n=i)=(X^t)_{ij}$$

Probability is a real valued set-function P that assigns, to each event of A in the sample space S, a number P(A), called the probability of the event A, such that the following properties are satisfied -

i) $P(A) \geq 0$

ii) $P(S) = 1$

iii) if $A_1, A_2, A_3, \ldots$ are events and $A_i \cap A_j = \phi$, $i \neq j$, then

$$P(A_1 \cup A_2 \cup \ldots \cup A_K) = P(A_1) + P(A_2) + \cdots + P(A_K)$$

for each positive integer K. and

$$P(A_1 \cup A_2 \ldots) = P(A_1) + P(A_2) + \ldots$$

for an infinite but countable number of events.

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Statistics

#CT 3

Probability

Properties of Probability

Probability

Probability is a real valued set-function P that assigns, to each event of A in the sample space S, a number P(A), called the probability of the event A.

Special terminology associated with events that is often used by statisticians includes the followings -

1. $A_1, A_2, \ldots, A_K$ are mutually exclusive events means that $A_i \cap A_j = \emptyset$, $i \neq j$; that is $A_1, A_2, \ldots, A_K$ are disjoint sets.

2. $A_1, A_2, \ldots, A_K$ are exclusive events means that $A_1 \cup A_2 \cup \ldots \cup A_k = S$.

So, if 1 & 2 then

$A_i \cap A_j = \emptyset$, $i \neq j$ and $A_1 \cup A_2 \cup \ldots \cup A_K = S$

13(C)-Day Date: 24/9/2017

Queuing Theory

Markovian → Poison/exponential

Service discipline/Queue discipline

T/X/C/∞/∞/FIFO

Interarrival time দেয়া থাকলে λ বের করতে হবে।

System ← service + queue

$L = L_s$

$\frac{\lambda}{\mu} < 1$

λ → arrival rate

μ → service rate

Example 3.3

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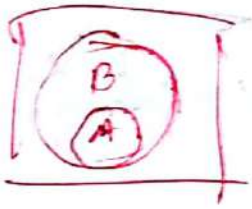
Theorem 3

If events A and B are such that $A \subset B$, then $P(A) \leq P(B)$



Proof

We have

$$B = A \cup (B \cap A') \quad \text{and} \quad A \cap (B \cap A') = \emptyset$$

Hence, from the third property of probability we obtain

$$P(B) = P(A) + P(B \cap A') \geq P(A)$$

because from 1st property.

$$P(B \cap A') \geq 0 .$$

[Proved]

Theorem 4

For each event A, $P(A) \leq 1$

Since, $A \subset S$, by theorem 3

$$P(A) \leq P(S)$$

$$\therefore P(A) \leq 1 .$$

[Proved]

$$0 \leq P(A) \leq 1$$

* ~~For a~~ Theorem 1

For each event A, $P(A) = 1 - P(A')$

Proof

We have,

$$S = A \cup A' \quad \text{and} \quad A \cap A' = \emptyset$$

$\therefore \quad P(S) = P(A \cup A')$

$\Rightarrow P(S) = P(A) + P(A')$

$\Rightarrow \quad P(A) = P(S) - P(A')$

$\therefore \quad P(A) = 1 - P(A')$

[Proved]

$P(S) = 1$

$\because P(A_1 \cup A_2 \cup \cdots \cup A_n) = P(A_1) + P(A_2) + \cdots + P(A_n)$

* Theorem 2

$$P(\emptyset) = 0$$

Proof

$$A = \emptyset \;,\; S = A'$$

We know from the properties of probability

$$P(A) = 1 - P(A')$$

$$= 1 - P(S)$$

$$= 1 - 1$$

$\therefore P(\emptyset) = 0$

[Proved]

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(B \cap C) - P((A \cap B) \cup (A \cap C))$ [From distributive Law]

$= P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap B) - P(A \cap C) + P((A \cap B) \cap (A \cap C))$

$\therefore P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

[Proved]

Theorem 5

If A and B are any two events Then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Proof

$$A \cup B = A \cup (A' \cap B)$$

$$\therefore P(A \cup B) = P(A) + P(A' \cap B) \quad \text{[from property (3)]} \quad \text{--- (i)}$$

Now,

$$B = (A \cap B) \cup (A' \cap B)$$

$$\therefore P(B) = P(A \cap B) + P(A' \cap B)$$

$$= P(A \cap B) + P(A \cup B) - P(A) \quad \text{[from (i)]}$$

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

[Proved]

Theorem 6

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

Proof

$$A \cup B \cup C = A \cup (B \cup C)$$

$$P(A \cup B \cup C) = P(A) + P(B \cup C) - P(A \cap (B \cup C))$$

$$= P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap (B \cup C))$$

Chapter-2

Random Variables of the Discrete Types

Random Variable

Given a random experiment with an outcome space S. a function X that assigns one and only one real number $X(s) = x$ to each element s in S is called a random variable.

pmf $\longrightarrow$ Probability Mass Function

Properties $\rightarrow$

a) $f(x) > 0$ when $x \in S$

if $x \notin S$ then $f(x) = 0$

b) $\sum_{x \in S} f(x) = 1$

c) $P(X \in A) = \sum_{x \in A} f(x)$, where $A \subset S$

Uniform distribution

$$f(x) = \frac{1}{m} \quad x = 1, 2, 3, - - m$$

Hyper Geometric Distribution

$$f(x) = P(X = x) = \frac{\binom{N_1}{x}\binom{N_2}{n-x}}{\binom{N}{n}} \quad N = N_1 + N_2$$

Example 2.1-5

$$P(X=2) = \frac{\binom{10}{2}\binom{40}{5}}{\binom{50}{7}}$$