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$$P'_k = \overbrace{(1-P)}^{\text{P}^n}$$

$$P_j(t + \Delta t) = \lambda_{j-1} \Delta t P_{j-1}(+) + (1 - \mu_j \Delta t - \lambda_j \Delta t) P_j(+) + \mu_{j+1} \Delta t P_{j+1}(+)$$

$$\lim_{\Delta t \rightarrow 0} \frac{P_j(t + \Delta t) - P_j(+)}{\Delta t} = \lambda_{j-1} P_{j-1}(+) - (\mu_j + \lambda_j) P_j(+) + \mu_{j+1} \lambda_{j+1}(+)$$

$$\frac{dP_j(+)}{dt} = \lambda_{j-1} P_{j-1}(+) -$$

$$P_{j+1} = \left\{ \frac{\lambda_j + \mu_j}{\mu_{j+1}} \right\} P_j - \frac{\lambda_{j-1}}{\mu_{j+1}} P_{j-1}$$

$$P_1 = \frac{\lambda_0}{\mu_1} P_0$$

On a network gateway measurements show that the packets arrive at a mean state

$$P \{ n(t + \Delta t) = j+1 \}$$

$$= \lambda_j \Delta t$$

$$P \{ n(t + \Delta t) = j-1 \} \\ = \mu_j \Delta t$$

$$P \{ n(t + \Delta t) = j \mid n(t) = j \}$$

$$1 - \lambda_j \Delta t - \mu_j \Delta t$$

$$P_0(t + \Delta t) = (1 - \lambda_0 \Delta t) P_0(t) +$$

$$\mu_1 \Delta t P_1(t)$$

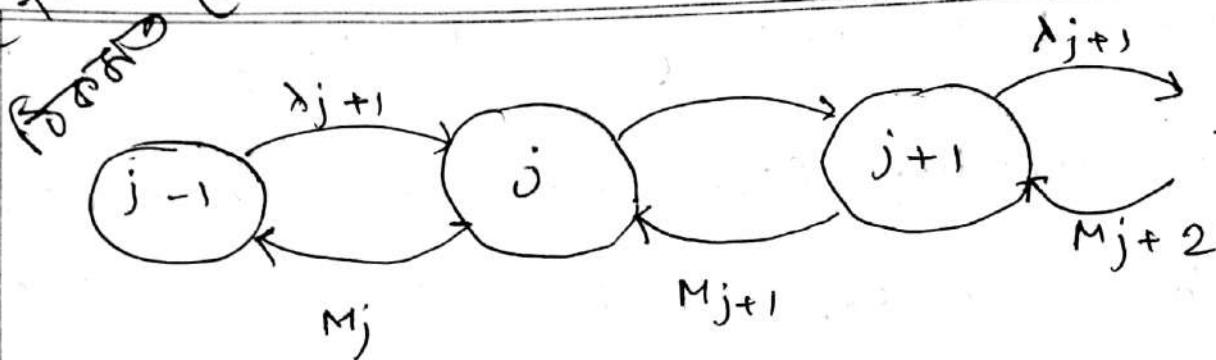
$$P_1(t + \Delta t) = \lambda_0 \Delta t P_0(t) +$$

13       $\begin{matrix} 25 \\ 25 \\ 10 \end{matrix}$       01       $12^{\text{00}}$        $\begin{matrix} 16 \\ 4 \end{matrix}$       100 mi      300 ACTIVE  
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time  $P_j$   
 $\Delta t \rightarrow 0$

\*\* Steady state probability \*\*

let future  $\left( \frac{\lambda^2}{\mu^2} \right)^2$



When the system is in state  $n$

it has  $n$  jobs in it

→ The new arrival take place at rate  $\lambda_n$

→ The service rate is  $M_n$

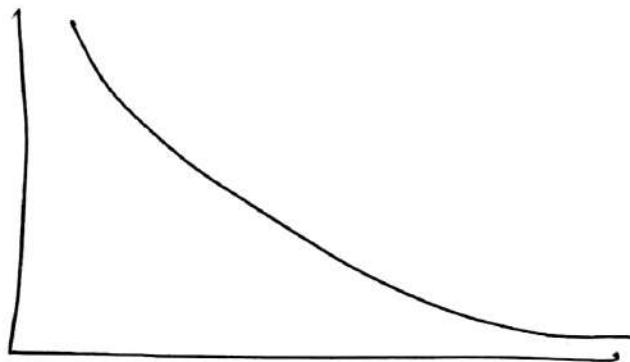
→ we assume that both the arrival times and service times are exponentially distributed suppose the system is in the state  $j$  at time  $t$ . There are  $j$  jobs in the system.

In the next time

system, both waiting and being serviced.

Time waiting ( $T_w$ ) the average time each customers waits in the queue -

$$T_a = T_w + S$$



$$\begin{array}{r} 185 \\ \times 2 \\ \hline 370 \end{array}$$

Service time ( $s$ ) → The average time required to service one customer.

Number waiting ( $w$ ) → The average no. of customer waiting.

Number in the system ( $Q$ ) — The average total number of customers in the system.

### Assumptions:

- Independent arrivals.
- Exponential distributions.
- customers do not leave or change queues
- large queues do not discourage customers.

Time in the system ( $T_2$ ) — the average time each customer is in the



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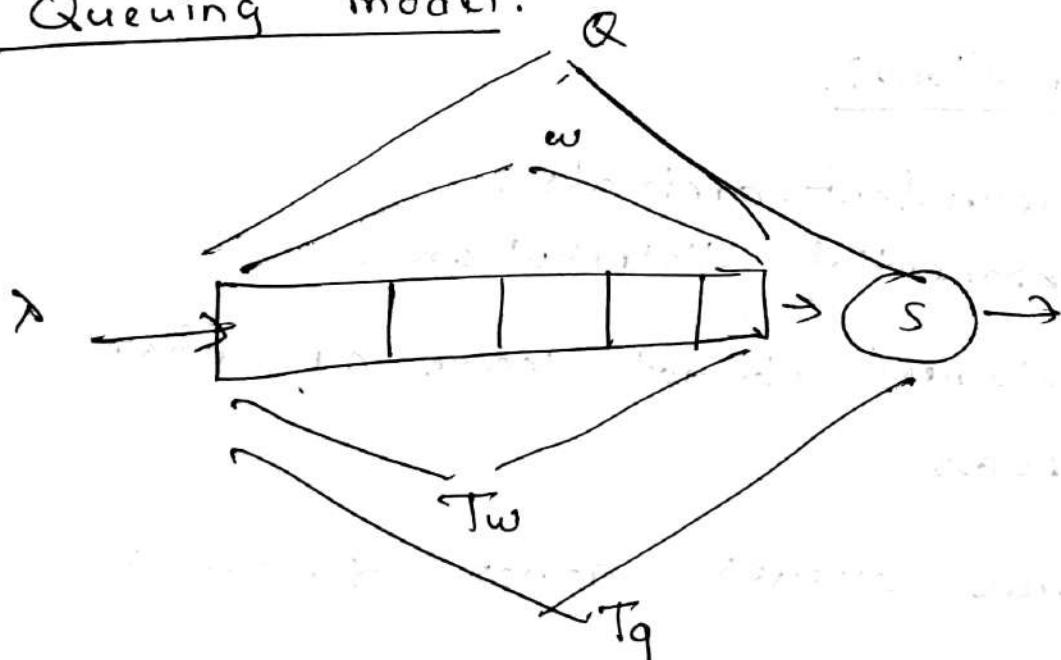
$$\Rightarrow N = \frac{\lambda}{\mu - \lambda}$$

Average waiting time,  $w = \frac{1}{\mu - \lambda} - \frac{1}{\mu}$   
 $= \frac{P}{\mu - \lambda}$

Average number of customer in queue.

$$N_Q = \lambda w = \frac{P}{1-P}$$

Queuing model:



Arrival rate ( $\lambda$ ) = The average rate at which customer arrive.

So, we have,

$$\frac{1}{1-p} P_0 = 1$$

$$\Rightarrow P_0 = 1 - p$$

that is,

$$P_k = p^k (1-p)$$

$p$  must be less than 1 or else the system is unstable.

Some formulas:

Average no. of customer

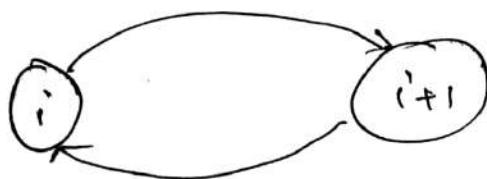
$$N = \sum_{k=0}^{\infty} k P_k = (1-p) \sum_{k=0}^{\infty} p^k$$

Average delay per customer (time in queue plus service time)

$$T = \frac{N}{\lambda}$$

$$\Rightarrow \frac{N}{\lambda} = \frac{1}{\mu - \lambda}$$

$$\Rightarrow N(\mu - \lambda) = \lambda$$



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In equilibrium, the rate of movement in both directions should be equal.

Let,

$P_i = P$  { system in state  $i$ }

\* we have,

$$\lambda P_i = \mu P_{i+1}$$

From state transition diagram,

$$\lambda P_0 = \mu P_1$$

$$\lambda P_1 = \mu P_2$$

$$\lambda P_2 = \mu P_3$$

$$P_1 = \frac{\lambda}{\mu} P_0 = \rho P_0 \quad \left[ \text{where } \rho = \frac{\lambda}{\mu} \right]$$

$$P_k = \rho^k P_0$$

Since,

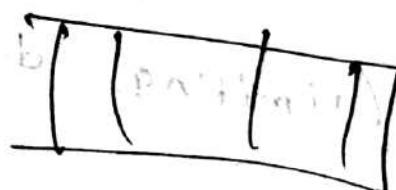
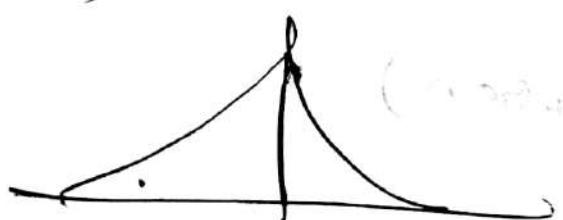
$$\sum_{k=0}^{\infty} P_k = \sum_{k=0}^{\infty} \rho^k P_0 = 1$$

concern of probabilistic analysis

can be summarised by the number of customer in the system at  $i$ .

(The past history (now we don't get them) - does not matter)

→ when a customer arrives or departs the system moves to an adjacent state. (either  $i+1$  or  $i-1$ )



$\text{iid} = \text{discrete distribution}$

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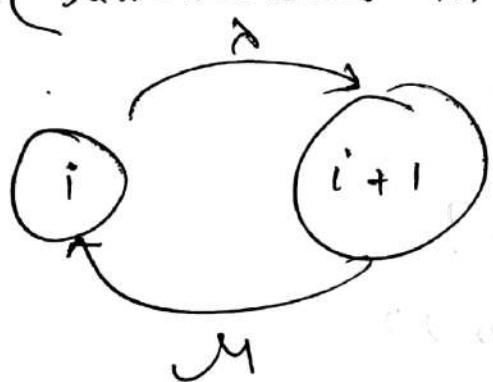
we have a single server, an infinite limiting line, the customer interarrival times are iid and exponentially distributed,  $\lambda$ , and the customer service times are also iid and we are mainly (in  $\downarrow$  for ~~for~~ start) in steady state solution. i.e when the system after a long running time tends to reach a static, where the distribution of customers in the system does not change (limiting distribution)

→ Due to memoryless property of the exponential distribution the entire state of a system as far as the

- The buffer is assumed to be infinite.
- The queuing discipline is **FCFS**.  
(First come first serve)

Memoryless property (Markov system)

\*\*(summarised in the current state)

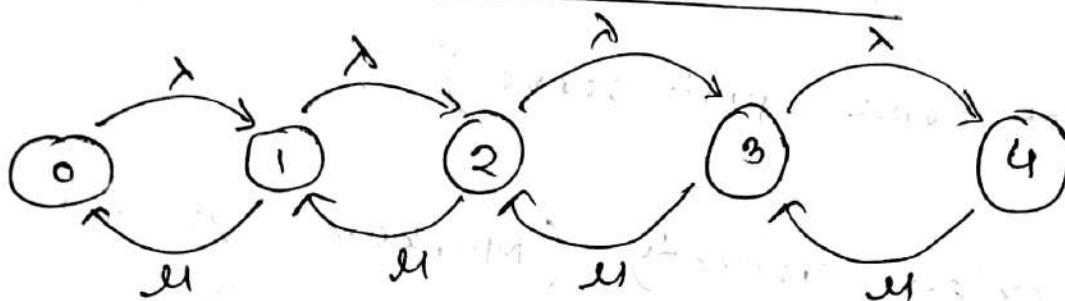


Taking the Markov system, the most simple queuing system, the **M/M/1** system (with **FIFO** service) can then be described as follows:-

In equilibrium, the rate of movement in both direction should be equal-

## Basic queuing theory:

State transition diagram:



What is state?

what is state function?

What is memoryless property?

what is Markov system??

M/M/1 = Queuing system:

- ① Interarrival times are exponentially distributed with average arrival rate  $\lambda$ .
- ② Service times are exponentially distributed with average service rate  $\mu$ .
- ③ There is only one server.

(ii) Number of servers

- Queuing disciplines (how customer are taken from queue)
- Number of buffers which customer use to wait for service.

Most common notation:

A/B/m, where m is the number of servers and A and B are chosen from

M :- Markov (exponential distribution)

D : Deterministic

G : General (Arbitrary distribution)



# Basic Queuing theory

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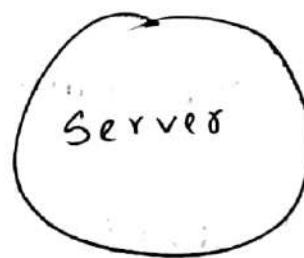
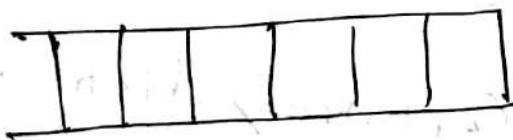
## Multi threading



Queuing theory is the mathematical basis for understanding and predicting any communication network.

Queuing system is a system consisting of a server or number of servers that perform a specific task and a queue where the processes waits to be performed.

Arrivals  
→



→ Departure

Major parameters:

- ① The inter arrival time distribution
- ② Service time distribution.



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Fig caption ফিল ২৮১

~~ক্ষেত্র~~ ক্ষেত্র সুষ্ঠু পর্যবেক্ষণ ক্ষেত্র পর্যবেক্ষণ ক্ষেত্র

Given below, ~~tab~~ Here is a ~~tab~~ table

ক্ষেত্র type terms are এই প্রয়োজন,

\* Table ৬ unit ফিল ২৮১,

\* Algorithm comparison করার প্রস্তাৱ আবশ্যিক  
environment পরিস্থিতি ক্ষেত্র পর্যবেক্ষণ ক্ষেত্র

in early part of the nineteenth century.

{ property, mean,  $\mu = \lambda$   
 variance,  $\sigma^2 = \lambda$

problems  
note

	Normal	Binomial	Poisson
Mean, $\mu$	$\mu$	$Np$	$\lambda$
Std deviation	$\sigma$	$\sqrt{Npq}$	$\sqrt{\lambda}$
Variance	$\sigma^2$	$Npq$	$\lambda$

two ordinates  $x=a$  and  $x=b$ , where  $a < b$ , represents the probability that  $x$  lies between  $a$  and  $b$ . This is denoted by  $\Pr \{ a < x < b \}$

$$\text{if } z = \frac{x - \mu}{\sigma}$$

Then the standard form of the normal distribution is —

$$Y = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2z^2}}$$

In each case we say that  $z$  is normally distributed with mean and variance 1.

The poisson distribution:

The discrete probability distribution

$$P(x) = \frac{\lambda^x e^{-\lambda}}{x!}, x = 0, 1, 2, \dots \text{ is called}$$

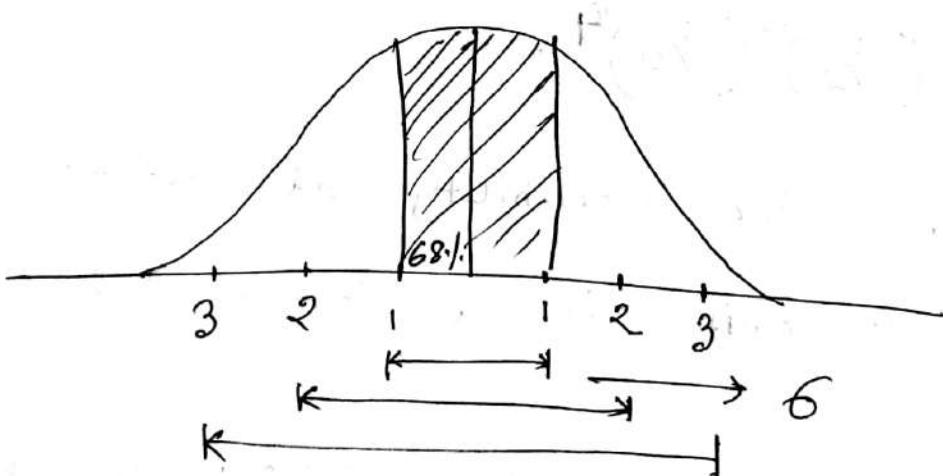
the poisson distribution after simeon-denis poisson who discovered it

One of the most important examples of continuous probability distribution is the normal distribution, normal curve or Gaussian distribution it is defined by the equation:

$$\Psi = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} (x-\mu)^2 / \sigma^2}$$

where,

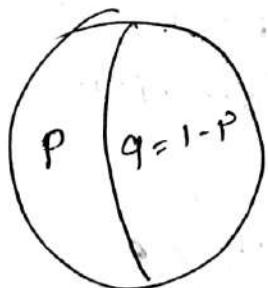
$\mu$  = mean,  $\sigma$  = standard deviation.



The total area bounded by the curve and the X axis is 1; hence the area under the curve between

occur) is given by,

$$P(x) = \binom{N}{x} p^x q^{N-x} = \frac{N!}{x!(N-x)!} p^x q^{N-x}$$



The probability of getting <sup>exactly</sup> two heads in six tosses of a fair coin.

$$\binom{6}{2} \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^4$$

# Find out the probability of getting at least 4 heads in 6 tosses of a fair coin?

$$\begin{aligned} & \binom{6}{4} \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^2 + \binom{6}{5} \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^1 \\ & + \binom{6}{6} \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^0 \end{aligned}$$

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Binomial distribution

$$P(x) = \binom{N}{x} p^x q^{N-x} = \frac{N!}{x!(N-x)!} p^x q^{N-x}$$

where  $x=0, 1, 2, \dots, N$ 

$$N! = N(N-1) \dots 1 \quad \text{and } 0! = 1$$

Binomial formula : —

$$(q+p)^N = q^N + \binom{N}{1} q^{N-1} p^1 + \binom{N}{2} q^{N-2} p^2 + \dots + p^N$$

If  $p$  is the probability that any an event will happen (probability of success) and  $q = 1-p$  is the probability that it will fail to happen any single time (probability of a failure), then the probability that the event will happen exactly  $x$  times in  $N$  trials (i.e  $x$  successes and  $N-x$  failures will

population parameter, the statistic is called an unbiased estimator of the parameter. Otherwise it is called a biased estimator.

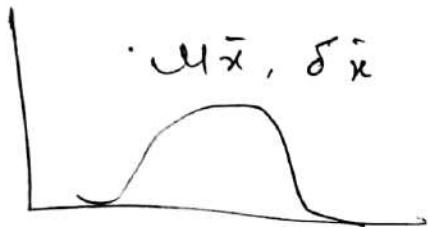
The corresponding values of such statistics are called unbiased or biased estimators respectively.

### Efficient estimates:

If the sampling distributions have the same mean or expectation, then the statistics with smaller variance is called an efficient estimator of the mean, while the other statistics is called an inefficient estimators. The corresponding values of or inefficient statistics are called efficient estimators respectively.

size

$$N_p > N$$



Sampling distribution কোর্কে definition

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### Statistical estimation theory

What is biased estimation?

" " unbiased estimator ?

" " efficient estimates ?

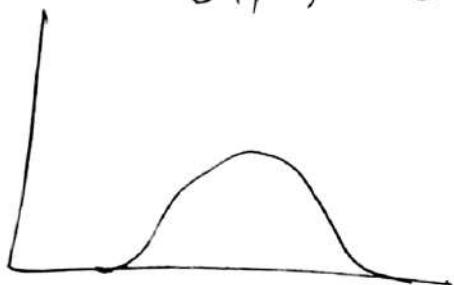
" " point estimates and interval  
estimates ?

#### Definition:

If the mean of the sampling distribution  
of a statistic equals the corresponding

Sampling distribution:

$$\mu_1, \mu_2, \mu_3, \dots, \mu_{1000}$$



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Sample 24-65

Sampling distribution of mean.

The statistics ~~like~~ sample mean, variance etc will vary sample to sample. The distribution obtained of the statistics is called its sampling distribution.

Sampling distribution of mean;

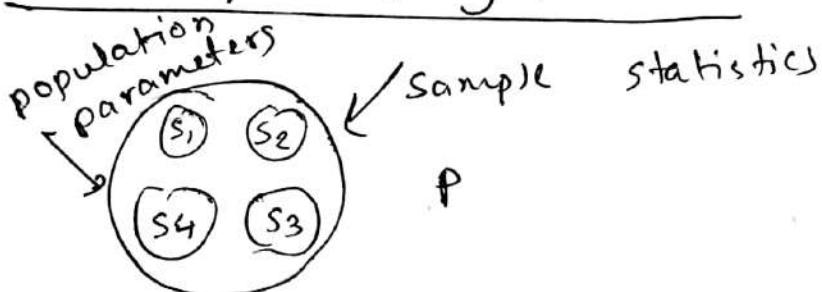
$$\mu_{\bar{x}} = \mu \quad \text{and} \quad \delta_{\bar{x}} = \frac{\sigma}{\sqrt{N}} \sqrt{\frac{N-p-N}{N-p-1}}$$

All possible sample size N are drawn without replacement from a finite population of

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## Elementary sampling theory



Sample is a subset of population, representative of whole population.

Sampling theory is a study of the relationship existing between population and sample drawn from the population.

## Sampling with and without replacement:

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$${}^5C_2 \cdot \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^3 \quad x \leq 1$$

$$= 10 \times \frac{1}{36} \times \frac{125}{216}$$

$$n = 2$$

=

$$1 - e^{-2x}$$

$$\underline{2e^{-2x}}$$

$$2 \int_2^\infty e^{-2x}$$

$$x \leq \int_{-3}^4 1$$

$$e^{-2}$$

$$\underline{\underline{2[-e^{-4}]}}$$

$$q = \frac{5}{6}$$

$${}^5C_1 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^3 \quad p = q^{n-r} \quad a = 0$$

T

$$=$$

If  $E_1$  and  $E_2$  are two events, the probability that  $E_2$  occurs given that  $E_1$  has occurred is denoted by  $\Pr\{E_2|E_1\}$ , or  $\Pr\{E_2 \text{ given } E_1\}$  and is called the conditional probability of  $E_2$  given that  $E_1$  has occurred.

If the occurrence ~~or~~ or nonoccurrence of  $E_1$  does not affect the probability of occurrence of  $E_2$ , then  $\Pr\{E_2|E_1\} = \Pr\{E_2\}$  and we say that  $E_1$  and  $E_2$  are independent events;

$$\Pr\{E_1, E_2\} = \Pr\{E_1\} \Pr\{E_2|E_1\}$$

~~$$\Pr\{E_1, E_2\} = \Pr\{E_1\} \Pr\{E_2|E_1\}$$~~

$$= \frac{2}{5} \cdot \frac{1}{4} = \frac{1}{10}$$

If a variable  $X$  can assume a discrete set of values  $x_1, x_2, \dots, x_k$  with respective probabilities  $p_1, p_2, \dots, p_k$

where  $p_1 + p_2 + \dots + p_k = 1$

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$$= \frac{1}{b-a} \left[ \frac{b^v}{2} - \frac{a^v}{2} \right]$$

$$= \frac{1}{b-a} \frac{(b+a)(b-a)}{2}$$

$$= \frac{b+a}{2}$$

$\checkmark$  Suppose that,  $X$  is a discrete random variable which can take values  $-2, 0, 3$  and is with probabilities  $P[X = -2] = 0.1$ ,  $P[X = 0] = 0.3$ ,  $P[X = 3] = 0.4$  and  $P[X = 5] = 0.2$ . Find the mean of  $X$ .

$$\mu = E(X) = ?$$

$$= -2 \times 0.1 + 0 \times 0.3 + 4 \times 0.4 + 2 \times 0.2$$

$\checkmark^*$  Suppose, that, a continuous random variable follows density function as -

$$f(x) = \frac{1}{b-a}, \quad a \leq x \leq b$$

Find the mean of  $X$

$$\begin{aligned} E(X) &= \int_a^b x \cdot \frac{1}{b-a} dx \\ &= \frac{1}{b-a} \left[ \frac{x^2}{2} \right]_a^b \end{aligned}$$

$$\alpha f(x) = \frac{3}{4} (2x - x^2) dx$$

$$= \frac{3}{4} \cdot 2 \cdot \frac{x^2}{2} - \frac{3}{4} \cdot \frac{x^3}{3}$$

$$= \frac{3}{4} \int_{-0.5}^{1.5}$$

$$\frac{3}{4} \left\{ (1.5)^2 - (-0.5)^2 \right\} - \frac{3}{4} \left\{ (1.5)^3 - (-0.5)^3 \right\}$$

$$= \frac{3}{2} - \frac{13}{16}$$

$$= \frac{24 - 13}{16}$$

$$= \frac{11}{16} = \underline{\underline{0.6875}}$$

less than 1

$$\int_0^1$$

equal to 1

0

$$(iv) P[x \leq 0.2] = \int_{-\infty}^{0.2} f(x) dx$$
$$= \int_{-\infty}^{0.2} 0 = 0$$

$$(v) P[x > 3] = \int_3^{\infty} f(x) dx$$
$$= \int_3^{\infty} 0 \cdot du = 0$$

~~✓~~ Let,  $x$  be a continuous random variable

that is with PDF  
 $f(x) = \begin{cases} \frac{2}{3}x & \text{if } 1 < x < 2 \\ 0 & \text{otherwise} \end{cases}$

Find  $P[x \leq 1.2]$ ,  $P[x > 1.2]$ ,  $P[1.2 \leq x \leq 1.6]$

$P[x \leq 1.2]$  and  $P(x > 3)$

$$\rightarrow P[x \leq 1.2] = \int_1^{1.2} f(x) dx = \int_1^{1.2} \frac{2}{3}x dx = \left[ \frac{x^2}{3} \right]_1^{1.2}$$

$$= \frac{11}{75}$$

$$(ii) P[x > 1.2] = \int_{1.2}^2 \frac{2}{3}x dx = ? \quad \frac{64}{75}$$

$$= \frac{2}{3} \cdot \left[ \frac{x^2}{2} \right]_{1.2}^2$$

$$(iii) P[1.2 \leq x \leq 1.6] = \int_{1.2}^{1.6} \frac{2}{3}x dx$$

$$= \frac{1.6^2}{3} - \frac{1.2^2}{3}$$

$$= \frac{38}{75}$$

Sol<sup>n</sup>: find these probabilities values.

f(x) for  $x < -1 = P[x < -1] = 0$

f(x) for  $-1 \leq x < 2 = P[-1 \leq x < 2] = P[-1] = \frac{1}{4}$

f(x) for  $2 \leq x < 5 = P[2] \cancel{\text{prob}} = \frac{1}{2} *$

F(x) for  $x \leq 5 = \frac{1}{4} + \frac{1}{2} + \frac{1}{4} = 1$

$$F(x) = \begin{cases} 0 & \text{for } x < -1 \\ \frac{1}{4} & \text{for } -1 \leq x < 2 \\ \frac{3}{4} & \text{for } 2 \leq x < 5 \\ 1 & \text{for } x \leq 5 \end{cases}$$

Rules to find the probabilities for a continuous variable:

i)  $P[x = x] = 0$  for  $-\infty < x < \infty$

ii)  $P[x < x] = P[x \leq x]$

$$= \int_{-\infty}^x f(x) dx \text{ for } -\infty < x \leq x$$

iii)  $P[a \leq x \leq b] = P[a \leq x \leq b]$

$$= \left[ \int_a^b f(x) dx \right] \text{ for } a \leq b$$

$$(iii) P[x \leq 4] = P(-2) + P(0) + P(4)$$

$$= \frac{1}{10} + \frac{2}{10} + \frac{4}{10} = \frac{7}{10}$$

Another way:

$$1 - P(11)$$

$$= 1 - \frac{3}{10}$$

$$= \frac{7}{10}$$

Let,  $x$  be a discrete random variable whose only possible values are  $-1, 2$  and  $5$ . Suppose that the probability function of  $x$  is

$$f(x) = \begin{cases} \frac{1}{9}, & \text{for } x = -1 \\ \frac{1}{2}, & \text{for } x = 2 \\ \frac{1}{4}, & \text{for } x = 5 \end{cases}$$

Find the alternative distribution function of  $x$ .

~~Let,  $x$  be a random variable~~

with probability function defined by

$$f(-2) = \frac{1}{10}, f(0) = \frac{2}{10}, f(4) = \frac{4}{10} \text{ and}$$

$$f(11) = \frac{3}{10}. \text{ find (i) } P[-2 \leq x \leq 4]$$

$$(ii) P[x > 0]$$

$$(iii) P[x \leq 4]$$

Ans: Here random variable  $x$  takes two values  $-2, 0, 4$  and ~~11~~ with repulsive probabilities

values of $x$	-2	0	4	11
$f(x)$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{4}{10}$	$\frac{3}{10}$

$$(i) P[-2 \leq x \leq 4] = P(-2) + P(0)$$

$$= \frac{1}{10} + \frac{2}{10}$$

$$(ii) P[x > 0] = P(4) + P(11)$$

$$y = p(x)$$

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$$P_r\{E_1 + E_2\} = P_r\{E_1\} + P_r\{E_2\} - P_r\{E_1, E_2\}$$

If a variable  $X$  can assume a discrete set of values  $x_1, x_2, \dots, x_k$  with respective probabilities

$p_1, p_2, \dots, p_k$  where  $p_1 + p_2 + \dots + p_k = 1$

we say that a discrete probability function for  $X$  has been defined. The function  $p(x)$  which has the respective values  $p_1, p_2, \dots, p_k$  for  $X = x_1, x_2, \dots, x_k$  is called the probability

Continuous:

$$y = p(x)$$

The total area under this curve between lines  $x=a$  and  $x=b$  gives the probability that  $X$  lies between  $a$  and  $b$ , which can be denoted by  $P_r\{a < X < b\}$

Rough

Suppose that, an event  $E$  can happen in  $h$  ways out of a total of  $n$  possible equally likely ways. Then the probability of occurrence of the event is denoted by

bij

$$P = \Pr\{E\} = \frac{h}{n}$$

If  $E_1$  &  $E_2$  are two events, the probability that  $E_2$  occurs + given that  $E_1$  has occurred is denoted by  $\Pr\{E_2|E_1\}$ , or  $\Pr\{E_2 \text{ given } E_1\}$ , and is called the conditional probability of  $E_2$  given that  $E_1$  has occurred

$$\Pr\{E_1, E_2\} = \Pr\{E_1\} \Pr\{E_2|E_1\}$$

$$\Pr\{E_1, E_2\} = \Pr\{E_1\} \Pr\{E_2\}$$

$$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

first ball drawn is black  $\frac{2}{5}$   
 second " " " "  $\frac{1}{4}$

$$\frac{2}{5} \cdot \Pr\{E_1, E_2\} = \Pr\{E_1\} \Pr\{E_2\} = \frac{1}{4} \cdot \frac{2}{5} = \frac{1}{10}$$

5) a) The distribution function of the random variable  $x$  is  $f(x) = \begin{cases} 1 - e^{-x}; & x > 0 \\ 0 & x \leq 0 \end{cases}$

Find (i) The density function

(ii) The probability that  $n > 2$

(iii) The probability that  $-3 < x \leq 4$

6) Find the probability that 5 tosses of fair dice, a 3 appear

(i) at no time

$$\boxed{6} \rightarrow 5$$

(ii) once

$$\frac{5}{6}$$

(iii) twice



$$\frac{1}{6}$$

(iv) three times

(v) four times

$$1 - \frac{1}{6}$$

$$P(1) + P(2) = \frac{5}{6} \quad \frac{1}{6}$$

$\alpha \times d^n \quad \frac{\alpha}{2} (1.5)^n - 1 \quad 1 - \frac{1}{6}$

day in this region, the rainfall is

- (a) not more than 1 inch
- (b) greater than 1.5 inches
- (c) between .5 and 1.5 inches.
- (d) equal to 1 inches
- (e) less than 1 inches.

Q3 Let,  $x$  be a continuous random variable with PDF.

$$f(x) = \begin{cases} ax & \text{if } 0 \leq x < 1 \\ \frac{a}{2} & \text{if } 1 \leq x < 2 \\ -a + 3x & \text{if } 2 \leq x \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

(i) determine the constant

(ii) compute  $P[x \leq 1.5]$

✓ Find the expectation of continuous random variable  $x$

$$f(x) = \frac{1}{\pi(1+x^2)} \quad -\infty < x < \infty$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

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$$(iii) P[a \leq x \leq b] = P[a \leq x < b] = P[a < x \leq b]$$

$$= P[a < x \leq b] = \int_a^b f(x) dx \text{ for } a < b$$

~~Ex 1~~ A bag contains 4 red, 6 black and 7 white marbles. A marble is chosen at random from the bag. If the marble is not white, what is

the probability that it is red?

~~Q 2.~~ Suppose that in a certain region the daily rainfall (in inches) is a continuous random variable  $X$  with probability density function  $f(x)$  given

$$\text{by } f(x) = \begin{cases} \frac{3}{4}(2x - x^2); & \text{if } 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

Find the probability that on a given



i)  $\int_R f(x) dx = \int_{-\infty}^{\infty} f(x) dx = 1$

ii) The probability of any event A defined by  $x < x$

$$P[A] = P[x < x]$$

$$= \int_{-\infty}^x f(x) dx$$

Moreover, if B is an event defined by

$a \leq x \leq b$ , then,

$$P[B] = P[a \leq x \leq b] = \int_a^b f(x) dx$$

Rules for finding probabilities for a continuous random variable:

(i)  $P[x=x] = 0$  for  $-\infty < x < \infty$

(ii)  $P[x < x] = P[x \leq x] = \int_{-\infty}^x f(x) dx$   
for  $-\infty < x \leq x$

### Mathematical expectation:

The discrete random variable  $X$  can assume the values  $x_1, x_2, \dots, x_n$  with respective probabilities  $p_1, p_2, \dots, p_n$  where  $p_1 + p_2 + \dots + p_n = 1$  then the mathematical expectation of  $X$  denoted as  $E(X)$  is defined as

$$\text{E}(X) = p_1 x_1 + p_2 x_2 + \dots + p_k x_k =$$

$$\sum_{j=1}^k p_j x_j = \sum p x$$

A man's possibility to win \$10 is  $\frac{1}{2}$

$$\text{His expectation } E(X) = \frac{1}{2} \times \$10 \\ = \$5 = 5\$$$

x. This makes events  $\{X=x\}$  disjoint,

$$\sum_x p(x) = \sum_x p\{X=x\} = 1$$

$$\lim_{x \downarrow -\infty} F(x) = 0 \quad \text{and} \quad \lim_{x \uparrow +\infty} F(x) = 1$$

cdf  $F(x)$  is a non decreasing function.  
of  $x$ , always between 0 & 1.

The probability of an event can be calculated by adding the probabilities of all few outcome of it.

Hence, for any set, A,

$$P\{X \in A\} = \sum_{x \in A} p(x)$$

where A is an interval, its probability

can be computed directly from the cdf,

$$f(x)$$

$$P\{a < X \leq b\} = F(b) - F(a)$$

~~domain~~  $\mathbb{R}$

Domain of a variable is the sample space  $\Omega$ .

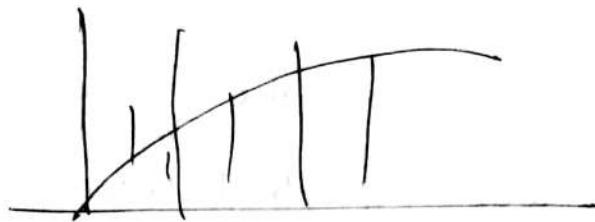
Its range can be the set of all real numbers  $\mathbb{R}$  or only the positive numbers  $(0, \infty)$  or the integers  $\mathbb{Z}$  or the interval  $(0, 1)$  etc.

Collection of all the probabilities related to  $X$  is the distribution of  $X$ . Function

$P(x) = P\{X = x\}$  is the ~~prob~~ pmf (probability mass function), so the cdf (cumulative distribution function) is

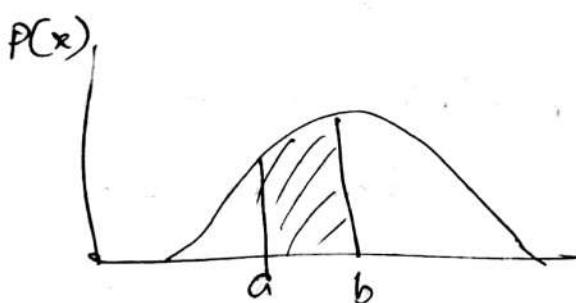
$$F(x) = P\{X \leq x\} = \sum_{y \leq x} p(y)$$

for every outcome  $w$ , the variable  $X$  takes  $\neq$  one and  $\wedge$  only one value



The area under the curve between line  $x=a$  and  $x=b$  gives the probability that  $x$  lies between  $a$  &  $b$ .

$$P_{\sigma} \{a < X < b\}$$



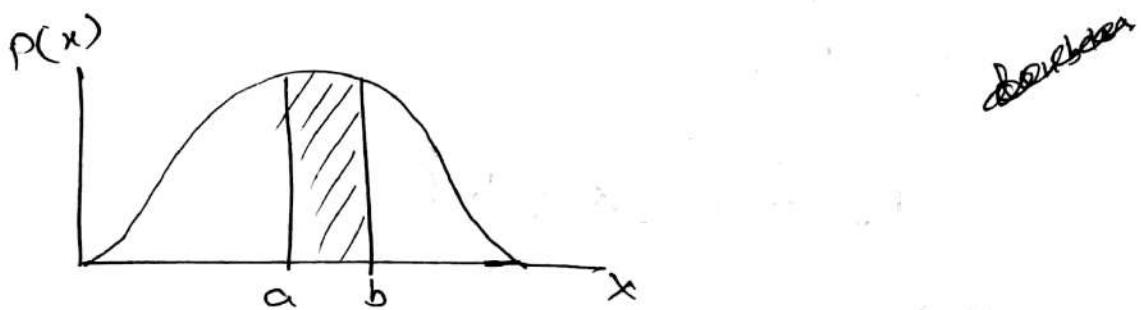
### Random variable:

A random variable is a function of an outcome. It is a quantity that depends on chance.  $X = f(\omega)$  with the

## Discrete Probability distribution :

Variable  $x$  can assume the values  $x_1, x_2, \dots, x_n$  with probabilities  $p_1, p_2, \dots, p_k$  where  $p_1 + p_2 + \dots + p_k = 1$

The probability function or discrete probability distribution for  $x$  is shown as  $p(x)$ .



The idea can be extended when the variable  $x$  may assume a continuous set of values. The relative frequency polygon of a sample becomes, in the theoretical or limiting case of a population, a continuous curve (given in figure) whose equation is  $y = p(x)$

$E_1 = \{\text{drawing an ace}\}$

$E_2 = \{\text{drawing an spade}\}$

If  $E_1$  &  $E_2$  are mutually exclusive

then  $P(E_1, E_2) = 0$

$$P\{E_1 + E_2\} = P\{E_1\} + P\{E_2\} - P\{E_1, E_2\}$$

If mutually exclusive  $P(E_1 + E_2) = P(E_1) + P(E_2)$

$E_1$  &  $E_2$  thus are not mutually exclusive

since the ace of spade can be drawn.

So the probability of drawing either  
an ace or a spade or both.

$$P\{E_1 + E_2\} = P\{E_1\} + P\{E_2\} - P\{E_1, E_2\}$$

$$= \frac{4}{52} + \frac{13}{53} - \frac{1}{13}$$

$$= \frac{16}{52} = \frac{4}{13}$$

$E_1$  = drawing an ace

$E_2$  = drawing a king

$$P\{E_1\} = \frac{4}{52} = \frac{1}{13}$$

$$P\{E_2\} = \frac{4}{52} = \frac{1}{13}$$

Since both ace and king can not be drawn in a single draw, thus mutually exclusive events,

$$P(E_1 + E_2) = P(E_1) + P(E_2)$$

$$= \frac{1}{13} + \frac{1}{13}$$

Probability of drawing an ace or a king in a single draw.

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The probability of drawing either an ace or a king in a single draw is

$$P\{E_1 + E_2\} = P\{E_1\} + P\{E_2\}$$

$$= \frac{1}{13} + \frac{1}{13}$$

$$= \frac{2}{13}$$

$$P\{E_1 \cap E_2\} = P\{E_1\} + P\{E_2\} - P\{E_1 \cup E_2\}$$

Proof by yourself



$$P\{E_1 \cup E_2\} = P\{E_1\} + P\{E_2\}$$

[for mutually exclusive events]

$$P\{E_1 \cup E_2 \cup \dots \cup E_n\} = P\{E_1\} + P\{E_2\} + \dots + P\{E_n\}$$

So here  $E_1 \cup E_2$  denotes either  $E_1$  or  $E_2$  or both occur.

" $E_1 \cup E_2$  denotes both  $E_1$  &  $E_2$  occur"

$E_1$  = drawing an ~~edge~~ ace from a deck of card.

$E_2$  = drawing an ~~key~~ king from a deck of card

$$\text{Probability of } E_1 = \frac{4}{52} = \frac{1}{13}$$

$$\text{and of } E_2 = \frac{4}{52} = \frac{1}{13}$$

The probability that the second ball drawn is black, given that the first ball drawn

comes black  $P\{E_2 | E_1\} = \frac{1}{4}$

$$P\{E_1, E_2\} = \left\{ \frac{1}{4} \times \frac{2}{5} \right\} = \frac{1}{10}$$

Independent event example:

$$E_1 = \{\text{head on 5th toss}\}$$

$$E_2 = \{\text{Head on 6th toss}\}$$

$$P\{E_1, E_2\} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Mutually exclusive event:

Two or more events are called mutually exclusive if the occurrence of one of them excludes the occurrences of others.

$$P\{E_1, E_2\} = 0$$

## Independent events:

Conditional probability

$$P\{E_1, E_2\} = P\{E_1\} \cdot P\{E_2 | E_1\}$$

↓

compound event

$$P\{E_2 | E_1\} = \frac{P\{E_1, E_2\}}{P\{E_1\}}$$

Independent events:

$$P\{E_1, E_2\} = ?$$

Let's have 3 red balls and 2 black balls in a box.

$E_1$  = "1<sup>st</sup> black ball is picked up"

$E_2$  = "2<sup>nd</sup>ly black ball is picked up"

$$P\{E_1\} = \frac{2}{5}$$

if  $\Pr\{E_1, E_2\}$   $E_1, E_2$  is conditional combined event,

$$\Pr\{E_1, E_2\} = \Pr\{E_1\} \Pr\{E_2 | E_1\}$$

$$\Pr(E_2 | E_1) = \frac{\Pr\{E_1, E_2\}}{\Pr\{E_1\}}$$

## Elementary Probability theory: (rps)

Probability is a measure of uncertainty.

Contd conditional probability:

Suppose that, an event  $E$  can occur in  $n$  ways out of a total of  $m$  possible likely ways. Then the probability of occurrence of the event (called its success) is denoted by

$$P = P_r(E) = \frac{h}{n}$$

If  $E_1$  and  $E_2$  are two events, the probability that  $E_2$  occurs ~~that~~ given that  $E_1$  has occurred is denoted by  $\Pr\{E_2|E_1\}$  or  $\{E_2 \text{ given } E_1\}$  and is called the conditional probability of  $E_2$  given that  $E_1$  has occurred.

### Prediction:

In general, prediction is the process of determining the magnitude of statistical variates at some future point of time.

The researcher divides the population into separate groups, called clusters. Then a single random sample of clusters is

### Difference between

5

In statistical modeling, regression analysis is a statistical process for estimating the relationships among variables. It includes many techniques for modeling and analyzing several variables,

(3v)

$$s = \sqrt{\frac{\sum_{j=1}^N (x_j - \bar{x})^2}{N}}$$

68. 27. ).

$$\bar{x} - s$$

$$s^2 = \frac{N_1 s_1^2 + N_2 s_2^2}{N_1 + N_2}$$

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{N}}$$

$$s^2 = \frac{\sum (x - \bar{x})^2}{N}$$

$$= \frac{\sum x^2 - 2\bar{x}x + \bar{x}^2}{N}$$

$$= \frac{\sum x^2}{N} - 2 \frac{\bar{x}x}{N} + \frac{N\bar{x}^2}{N}$$

$$= \frac{\sum x^2}{N} - 2\bar{x}^2 + \bar{x}^2$$

$$= \frac{\sum x^2}{N} - \bar{x}^2$$

$$= s = \sqrt{\frac{\sum x^2}{N} - \left(\frac{\sum x}{N}\right)^2} = \sqrt{\bar{x}^2 - \bar{x}^2}$$

$$s = \sqrt{\frac{\sum_{j=1}^N (x_j - \bar{x})^2}{N}} = \sqrt{\frac{\sum (x - \bar{x})^2}{N}} = \sqrt{\frac{\sum x^2}{N}}$$

$$\sqrt{(\bar{x} - \bar{x})^2}$$

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{N}}$$

$$s^2 = \frac{\sum (x - \bar{x})^2}{N}$$

$$= \frac{\sum (x^2 - 2\bar{x}x + \bar{x}^2)}{N}$$

$$= \frac{\sum x^2 - 2\bar{x} \sum x + N\bar{x}^2}{N}$$

$$= \frac{\sum x^2}{N} - 2\bar{x} \frac{\sum x}{N} + \bar{x}^2$$

$$= \frac{\sum x^2}{N} - 2\bar{x} \frac{\sum x}{N} + \bar{x}^2$$

$$= \frac{\sum x^2}{N} - 2\bar{x}^2 + \bar{x}^2$$

$$= \frac{\sum x^2}{N} - \bar{x}^2$$

$$= \bar{x}^2 = \bar{x}^2$$

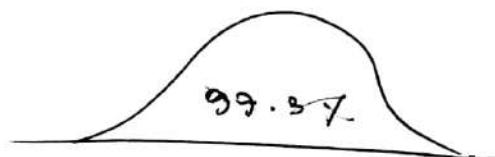
$$= \frac{\sum x^2}{N} - \left(\frac{\sum x}{N}\right)^2$$

$$\Rightarrow s = \sqrt{\frac{\sum x^2}{N} - \left(\frac{\sum x}{N}\right)^2}$$

~~This is~~

• 75	6.5	(• 78)
------	-----	--------

- (i) List the data height & weight  
~~(ii)~~ justify the normal behaviour



- (ii) Verify the symmetric or not  
symmetric properly .

Before we do anything with a data-

Look at it.

A quick look at a sample may clearly suggest -

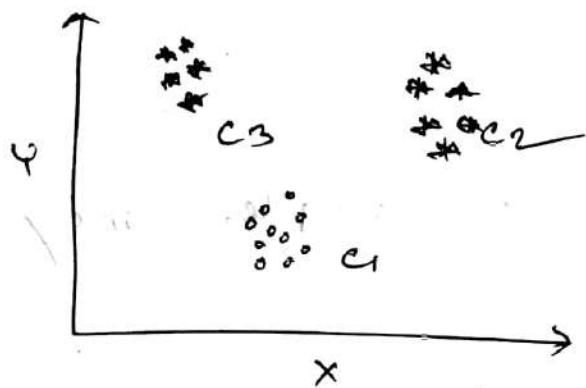
- A probability model i.e. a family of distribution to be used.
- Statistical methods suitable for the given data.
- Presence or absence of outliers.
- Existence of time trends and other pattern.
- Relation between 2 or several values.

Histogram / box plot

stem & leaf plots

Scatter plots

## Clusters & regression



$\sum$  sum square error  
min

clustering (process ~~in~~ random 25)

classifying

classification (process ~~in~~ supervised  
26)

## Regression:



$$Y' = Ax^2 + Bx + C$$

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Actually tells how two variables are related  
to each other.

Two variables are said to be correlated if there exists a non-zero linear relationship between them.

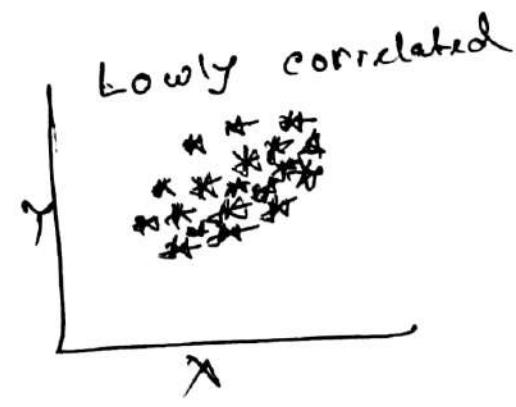
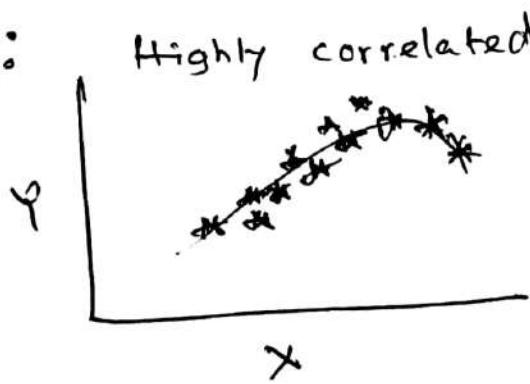
(iii) All the data points observe outside of this interval are assumed suspiciously far. They are the first candidate to be handled as outliers.

How to handle outlier:

Find out the history of outlier:

This rule comes from the assumption that the data are ~~normall~~ nearly normally distributed 99.7% population should appear within 1.3 $\sigma$  interquartile ranges from quartiles.

Correlation:



So we have to use those measures of variability to find defect outliers & which are not sensitive to outlier, i.e interquartile range.

### Outlier detection :

- (i) Find or measure the value  $1.5 \times IQR$  from the both side quartiles.
- (ii) Then find out the limits by measuring  $1.5 \times IQR$  from the both side quartiles.

$$L_1 = Q_1 - 1.5 \times IQR$$

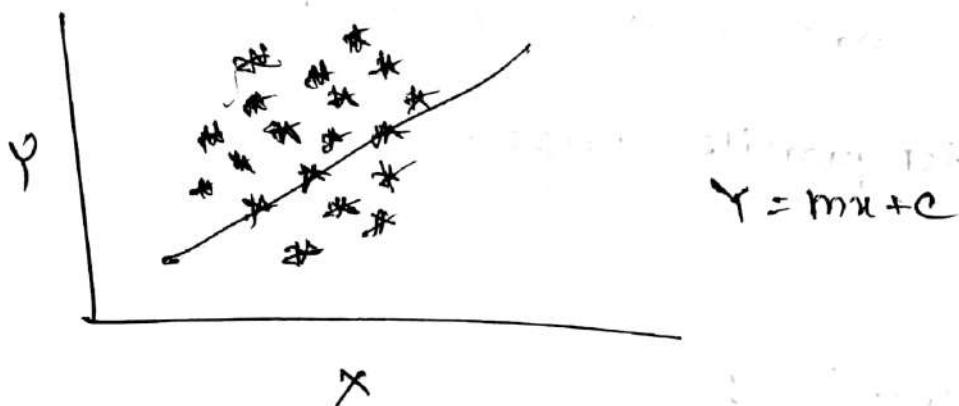
$$L_2 = Q_3 + 1.5 \times IQR$$

$$\text{Limit} = [L_1, L_2]$$

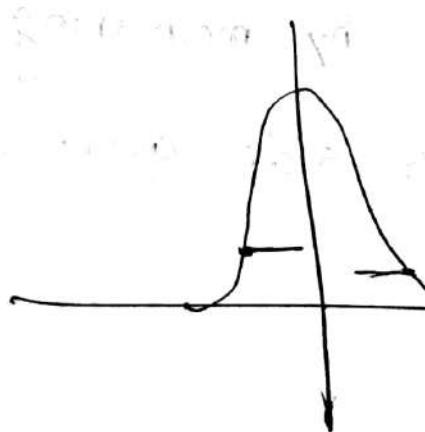


# Prediction / Forecasting and outliers

abrupt

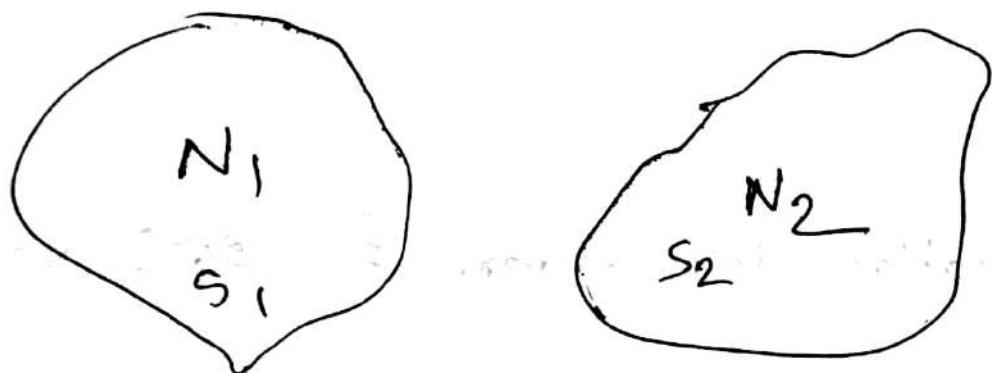
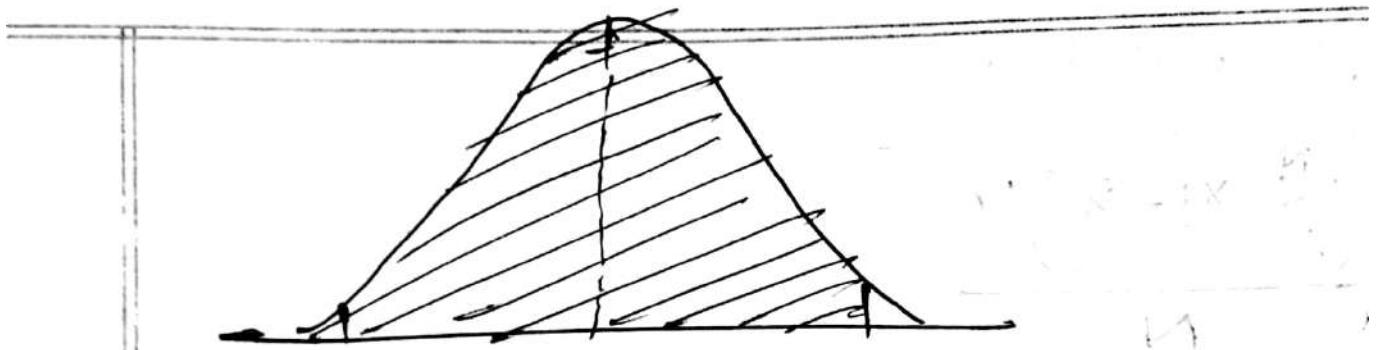


Forecast, extreme case 2nd 2m outliers.



Standard deviation, variance

Some measure of variability are sensitive to outlier.



Combined variance

$$S^2 = \frac{N_1 S_1^2 + N_2 S_2^2}{N_1 + N_2}$$

$$S = \sqrt{\frac{\sum x^2}{N} - \left(\frac{\sum x}{N}\right)^2}$$

$$= \sqrt{\bar{x}^2 - \bar{x}^2}$$

Standard deviation:

$$s = \sqrt{\frac{\sum_{j=1}^N (x_j - \bar{x})^2}{N}}$$

$$= \sqrt{(\Sigma x - \bar{x})^2}$$

Variance:

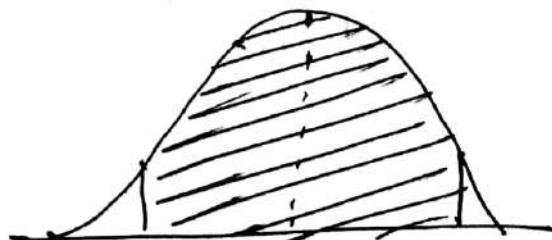
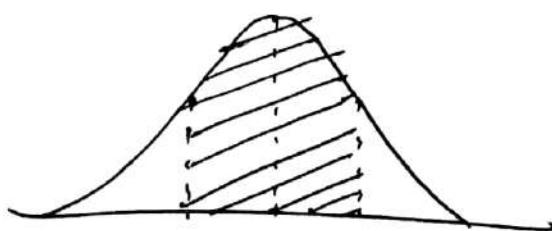
Standard deviation is square root of variance.

Properties of standard deviation:

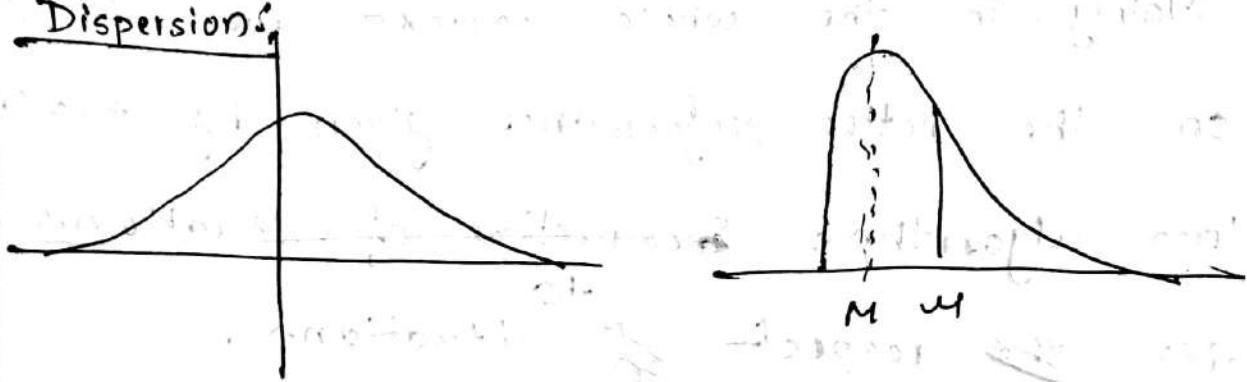
- i. The SD can be defined as

$$s = \sqrt{\frac{\sum_{j=1}^N (x_j - a)^2}{N}}$$

of all such standard deviation, the minimum is that for which  $a = \bar{x}$ .



### Dispersion:



### Range:

Mean deviation

$x_1, x_2 \dots x_N$

$$MD = \frac{\sum |x_i - \bar{x}|}{N}$$

### Interquartile range:

$$Q = \frac{Q_3 - Q_1}{2}$$

(0-90 percentile range)

$$\rightarrow P_{90} - P_{10}$$

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Mainly, in the whole context we will focus on the better performance given by these two algorithms ~~irrespective of situations~~ with ~~the~~ respect <sup>to</sup> situations.

value is called the dispersion or variation of the data.

Example of

The measures of dispersion

range, mean deviation, semi interquartile range, percentile, range, standard deviation.

Mostly used  $\rightarrow$  standard deviation.

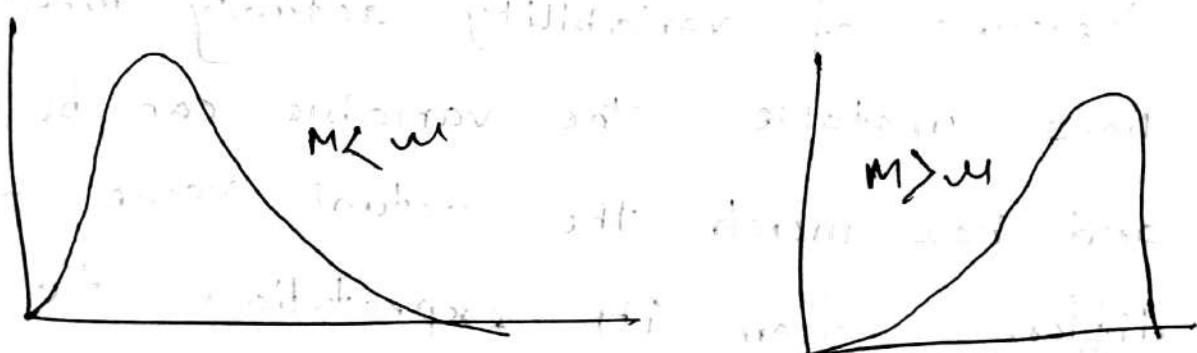
Measures of variability actually measures how unstable the variable can be and how much the actual value can differ from its expectation to

assessing reliability of our estimates and accuracy of our forecasts.

Population median  $M$  is a number that is exceeded with probability no greater than 0.5 and is preceeded with probability no greater than 0.5. That is,  $M$  is such that

$$P\{X > M\} \leq 0.5$$

$$P\{X < M\} \leq 0.5$$



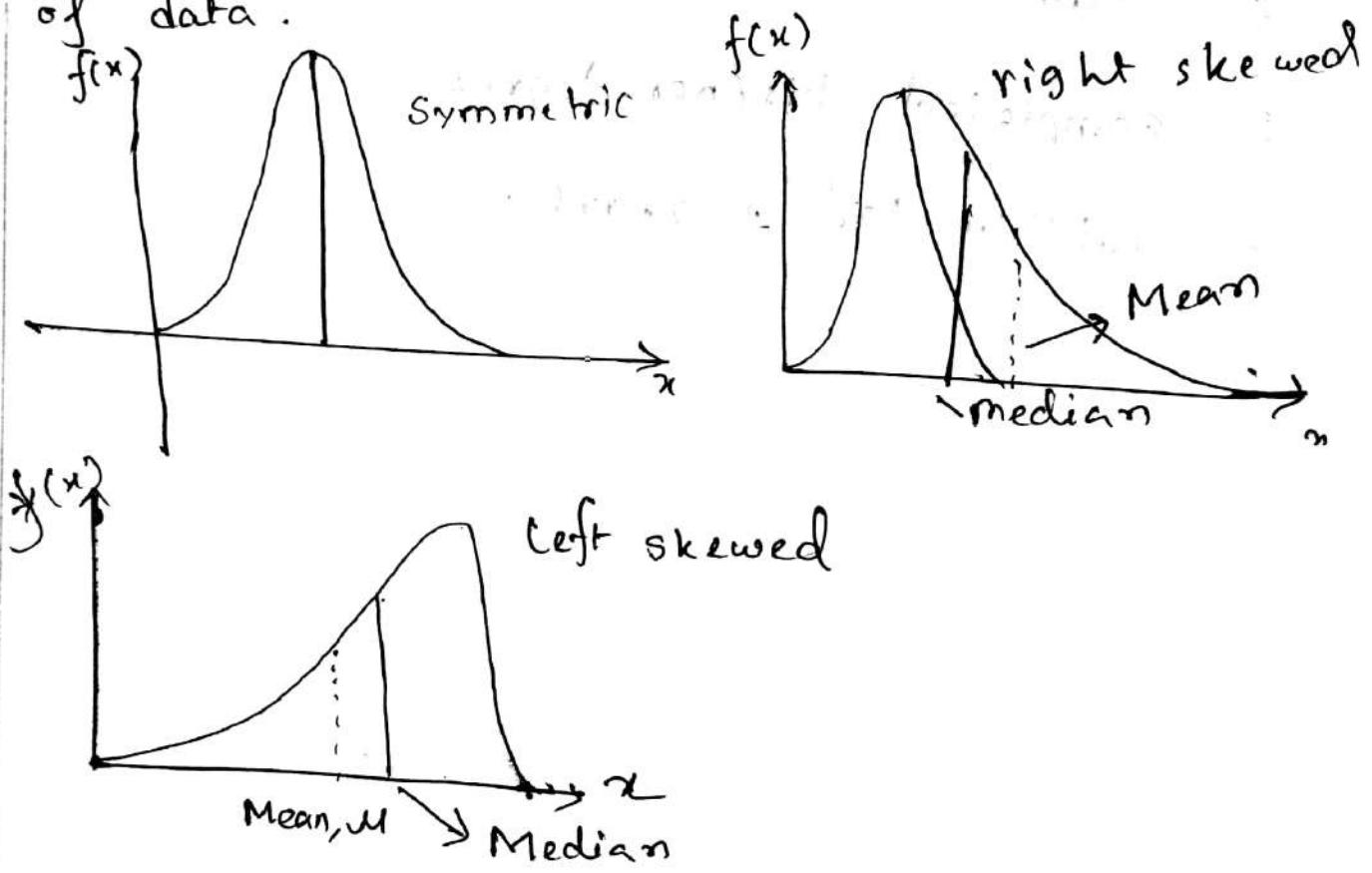
Measures of central tendency  
Measures of dispersion

### dispersion:

The degree to which numerical data tend to spread about an average

$$\frac{5+6}{2}$$

Variance, standard deviation or interquartile range measures the variability or spread of data.



Median means a "central" value.

Sample median is a number that is exceeded by at most a half of observations and is preceeded by at most a half of observations.

### Population

- = Bangladeshi student
- = admitted into European university
- = completed his/her degree.
- = within last 5 years

$$= \frac{\sum Af_j + \sum f_j dj}{N}$$

$$= \frac{A \sum f_j + \sum f_j dj}{N}$$

$$= \frac{AN + \sum f_j dj}{N}$$

$$= A + \frac{\sum f_j dj}{N}$$

$$= A + \frac{\sum fd}{N}$$

$$\bar{x} = \frac{\sum x}{N} = \frac{\sum fx}{N}$$

Some problems

from book HW

Dispersion -

where we have used  $\Sigma$  in place of  $\sum_{j=1}^N$  for briefly.

(b) In case  $x_1, x_2, \dots, x_k$  have respective frequencies  $f_1, f_2, \dots, f_k$  and  $d_1 = x_1 - A, d_2 = x_2 - A, d_k = x_k - A$

Show that the result in part A is replaced with,

$$\bar{x} = A + \frac{\sum_{j=1}^k f_j d_j}{\sum_{j=1}^k f_j}$$

$$= A + \frac{\sum f_j d_j}{N} \text{ where } \sum_{j=1}^k f_j = \sum f = N$$

SOLN:  $\bar{x} = \frac{\sum_{j=1}^k f_j x_j}{\sum_{j=1}^k f_j}$

$$= \frac{\sum f_j x_j}{N}$$

$$= \frac{\sum f_j (A + d_j)}{N}$$

(@) If  $N$  numbers  $x_1, x_2, \dots, x_n$  have deviation from any number  $A$  given by  $d_1 = x_1 - A, d_2 = x_2 - A, \dots, d_N = x_N - A$  respectively, then prove that,

$$\bar{x} = A + \frac{\sum_{j=1}^N d_j}{N}$$

$$= A + \frac{\sum d_j}{N}$$

Soln: Since  $d_j = x_j - A$  and  $x_j = A + d_j$  we

have,

$$\bar{x} = \frac{\sum x_j}{N}$$

$$= \frac{\sum (A+d_j)}{N}$$

$$= \frac{\sum A + \sum d_j}{N}$$

$$= \frac{NA + \sum d_j}{N}$$

$$= A + \frac{\sum d_j}{N}$$

Here,  $w=a$ ,

$$P = -2 \frac{\sum x}{N}$$

$$q = \frac{\sum x^w}{N}$$

$$a = -\frac{1}{2} p$$

$$= \frac{\sum x}{N}$$

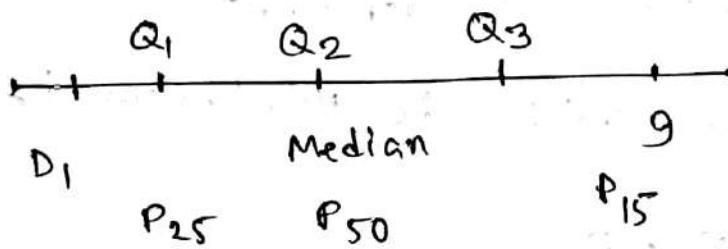
$\therefore \frac{\sum (x-a)^w}{N}$  it will be

minimum, if and only if

$$= \bar{x}$$

$$a = \bar{x}$$

Q = Quartiles



quartiles

Deciles = D

percentile = P,

Prove that sum of the squares of the deviation of a set of number  $x_j$  from any number  $a$  is minimum if and only if  $a = \bar{x}$ .

Let's take statement,

$$w^v + pw + q$$

$$\begin{aligned} &= w^v + 2w \cdot \frac{1}{2} p + \left(\frac{1}{2} p\right)^v + q - \frac{1}{4} p^v \\ &= \left(w + \frac{1}{2} p\right)^v + \frac{q - \frac{1}{4} p^v}{\text{constant}} \end{aligned}$$

The statement will be minimum if

$$w + \frac{1}{2} p = 0$$

$$\text{or, } w = -\frac{1}{2} p$$

$$\begin{aligned} \frac{\sum (x-a)^v}{N} &= \frac{\sum (x^v - 2ax + a^v)}{N} \\ &= \frac{\sum x^v - 2a \sum x + Na^v}{N} \\ &= a^v - 2a \sum \frac{x}{N} + \frac{\sum x^v}{N} \end{aligned}$$

Prove that algebraic sum of the derivations of a set of numbers from their arithmetic mean is zero.

Let,  $x_1, x_2, \dots, x_N$  is the set of numbers and mean is  $\bar{x}$ .

Then the derivations, deviations

$$d_1 = x_1 - \bar{x}$$

$$d_2 = x_2 - \bar{x}$$

$$d_N = x_N - \bar{x}$$

Sum of the derivations,  $\sum d = \sum_{j=1}^N (x_j - \bar{x})$

$$\sum d = \sum_j x_j - \sum_j \bar{x}$$

$$= \sum_j x_j - N\bar{x}$$

$$= \sum_j x_j - N \frac{\sum x_j}{N}$$

$$= \sum_j x_j - \sum_j x_j$$

$$= 0$$

if and only if  $a = \bar{x}$

→ If  $f_1$  numbers have mean  $m_1$ ,  $f_2$  numbers have mean  $m_2$ , the numbers have mean  $m_k$ , the mean of all numbers is —

$$\bar{x} = \frac{f_1 m_1 + f_2 m_2 + \dots + f_k m_k}{f_1 + f_2 + \dots + f_k} \text{ weighted AM}$$

of all the means.

10, 20, 30, 15, 25, 35

$$\text{AM, } \bar{x} = \frac{10+20+30+15, 25, 35}{6} = \frac{135}{6}$$

$$= 22.5$$

(i) -12.5

(ii) -2.5

(iii) 7.5      Arithmetic mean of them

(iv) -7.5      are 22.5

(v) 2.5

(vi) 12.5

Weight same at 27m<sup>2</sup>,

Weight arithmetic mean:

$$x_1 - w_1$$

$$x_2 - w_2$$

$$x_N - w_N$$

$$\bar{x} = \frac{x_1 w_1 + x_2 w_2 + \dots + x_N w_N}{w_1 + w_2 + \dots + w_N}$$

$$= \frac{\sum_{j=1}^k x_j w_j}{\sum_{j=1}^k w_j}$$

$$= \frac{\sum x_w}{\sum w}$$

Properties:

→ Algebraic sum of the derivation of a set of numbers from their arithmetic mean is zero.

→ The sum of the squares of the derivation of a set of numbers

$x_j$  from any number  $a$  is minimum

Bell shaped / symmetrical

- Equidistance from the max will have same frequency
- Normal distribution

Kurtosis

एक्टर distribution, spike रूप से आवेदन, उमर distribution flat रूप से।

Arithmetic mean (AM) :

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{N}$$

$$= \sum_{j=1}^k \frac{x_j}{N}$$

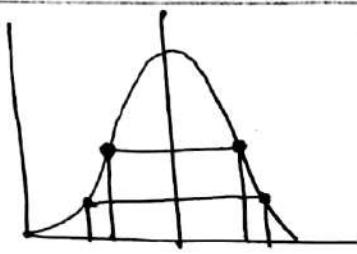
$$= \frac{\sum x}{N}$$

$$x_1 - f_1 = \frac{x_1 f_1 + x_2 f_2 + \dots + x_N f_N}{f_1 + f_2 + \dots + f_n}$$

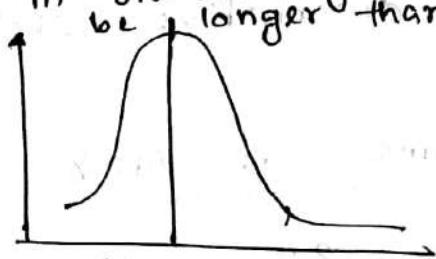
$$x_2 - f_2$$

$$= \sum_{j=1}^k \frac{f_j x_j}{f_j}$$

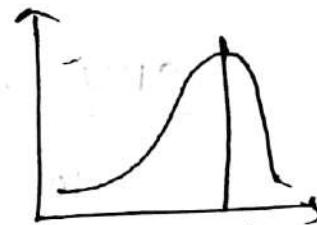
$$x_n - f_n = \frac{\sum f x}{N}$$



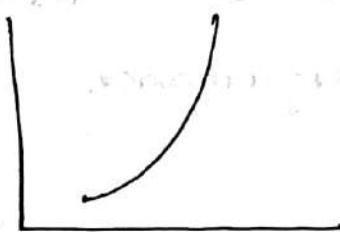
Bell shaped  
symmetrical



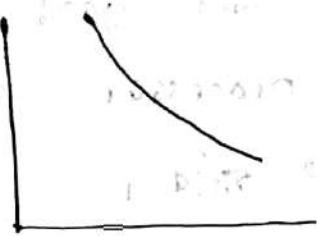
Right skewed  
(positive skewed)



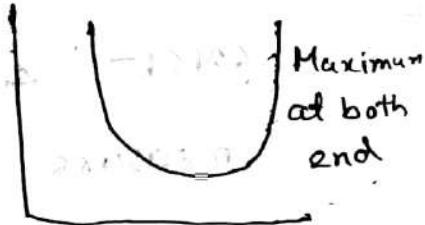
left skewed  
(negative skewed)



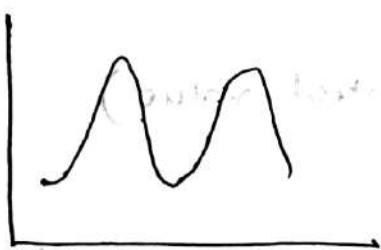
maximum at one end



reverse J shape



U shaped (minimum at both ends)  
count symmetric



bimodal



multi trimodal .

## Simple descriptive statistics

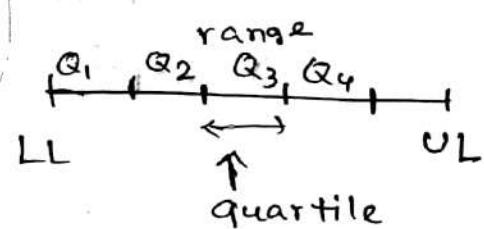
### CPU time of N=8 jobs

10      20      15      16      ↪ Good sample  
17      20      21      17

random sample এবং good sample কু তথ্য বা  
ক্ষণে - আমরা processor এর performance  
measure করতে পারি,

### Distribution

Knowledge of or information



→ Mean (Find out the average of a sample set)

→ Median (Measures the central value)



DATA

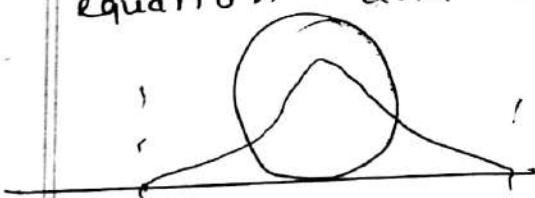
62	68	84	75	82	68	90	62	88	76	93
73	79	88	73	60	93	71	59	85	75	
61	65	75	87	74	62	95	78	63	72	
66	78	82	75	94	77	69				

→ find highest & lowest

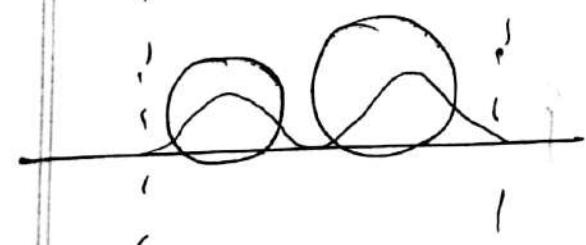
→ divide it into intervals

→ Make histogram

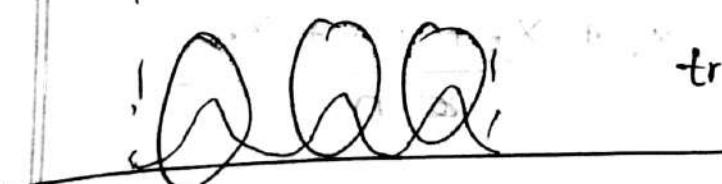
→ Again find mean, median, etc using equation and compare with histogram.



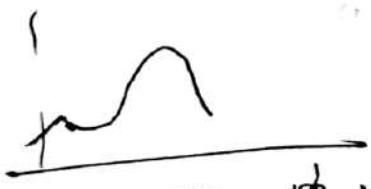
-single mode



bi mode



tri mode



if the graph is like that  
then the object is under  
background, or the object  
Data prominent

Mean: Measures the average value of a sample.

Median: Measures the central value.

Quartile / Quantile: actually shows where certain portion of a sample are located.

Variance, standard deviation, interquartile range: measures the variability or spread of data.

$X$  is a sample.

$$S = \{x_1, x_2, \dots, x_n\}$$

Mean:

$$\text{Mean, } M = E(X) = \frac{x_1 + x_2 + \dots + x_n}{n}$$

Sorted data:  $x_1, x_2, \dots, x_n$

$\rightarrow$  odd:  $x_i$

$$\rightarrow \text{even: } \frac{x_i + x_{i+1}}{2}$$

Simple descriptive Statistics:

CPU time of  $N = 8$  jobs

10    20    15    16

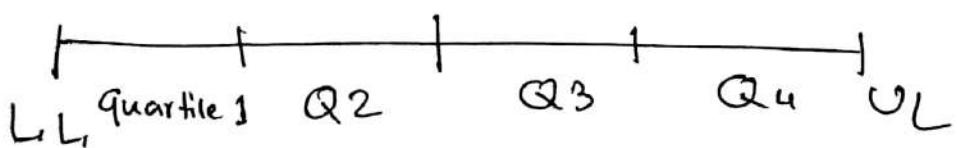
17    20    21    17

knowledge

or

information

- Mean
- Median
- Quantiles / Quartiles
- Variance, standard deviation
- Interquartile range



wrong

**ACTIVE**

Date : .....

Page :

choose first firm, Based 284r - 573 AT,

Saln 10
Admin 5
Marketing 30 HR

Note book

A

B

C

Collection of data for prediction का  
क्रम।

of a population is observed. For most of reasonable statistical procedures, sampling errors decrease as the sample size increases.

### Non sampling errors:

Non sampling errors are caused by inappropriate sampling skills, schemes or wrong statistical techniques.

### Literary digest magazine

1920

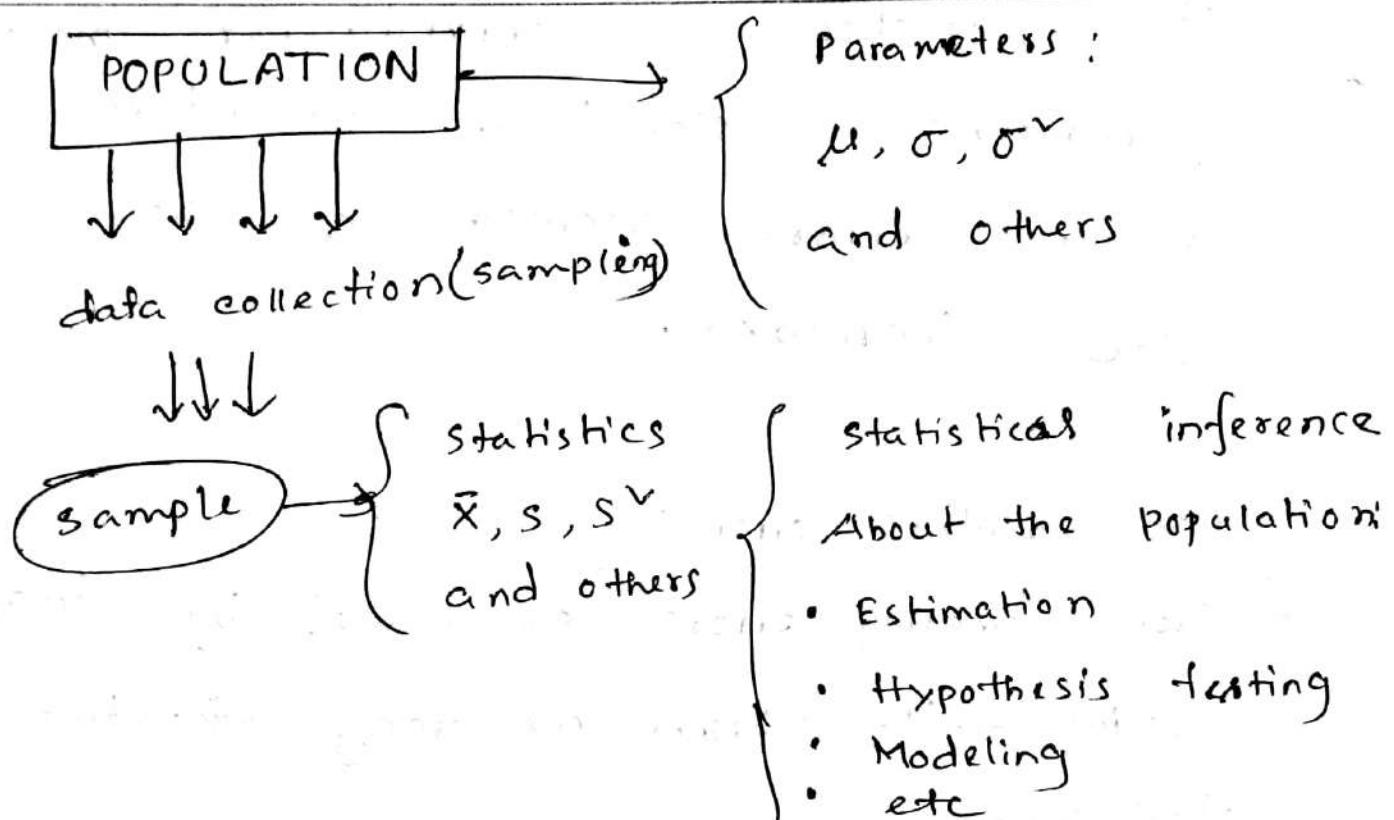
1924

1928

+ 1932 prediction  
presidential election  
USA prediction

1936 prediction fail ग़ले,

Because The Literary digest subscribers were mostly republicans. Wrong sample



Population → parameters

Sample → statistics

A properly collected sample of data can provide rather sufficient information about the parameters of the observe system.

### Sampling errors :

Sampling errors are cost by the mere fact that only a sample a proportion

### Algorithm

1. Input the ranges.
2. Select the intervals.
3. Check the frequency of each interval and find out the number
4. Repeat the process 3.

$H[1:n]$

for ( $i = 1:n$ )

if ( $m == i$ )

$H[i] += 1;$

### Central tendency

## Data representation

→ write down the algorithm to find out the histogram from array of table.

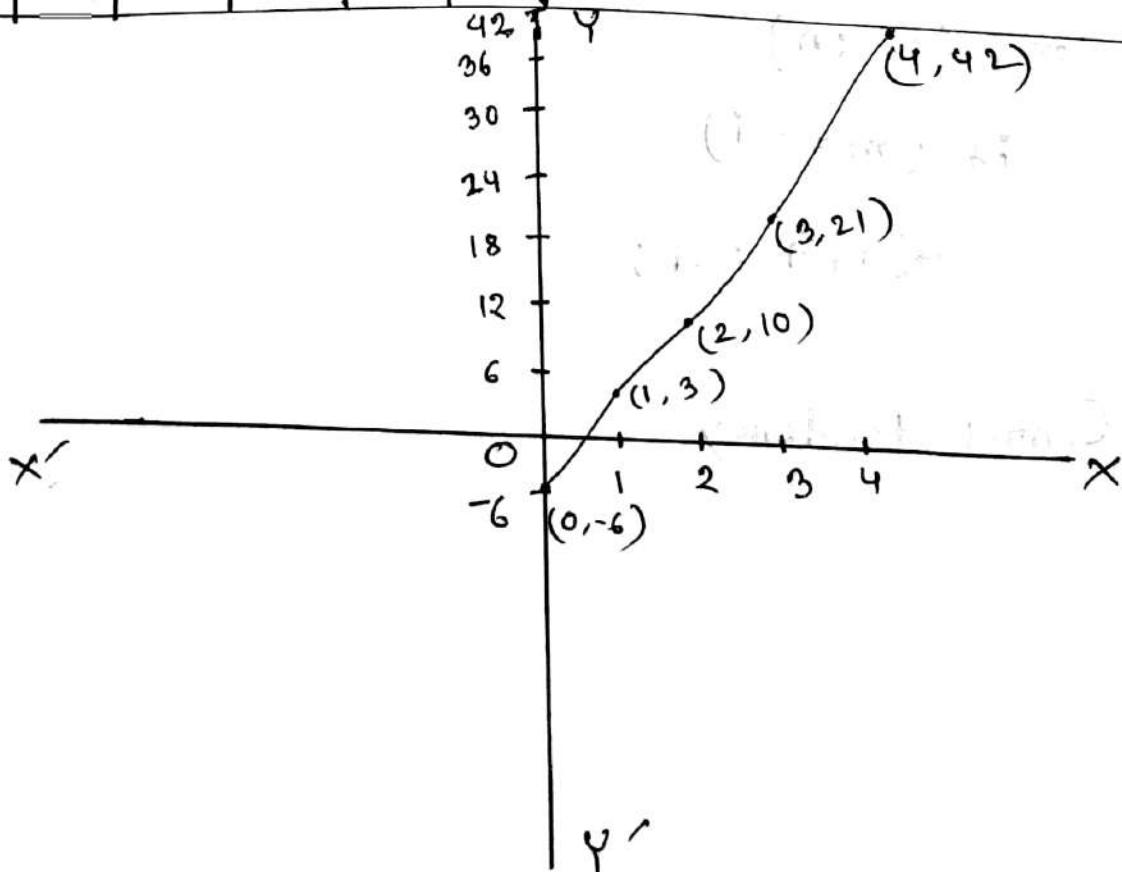
→ Graph the equation  $Y = X^3 - 4X^2 + 12X - 6$

$$\begin{array}{r} 27 - 4 \cdot 9 + 36 - 6 \\ \hline 8 - 4 \cdot 4 + 24 - 6 \end{array}$$

From different values of  $X$  we find the corresponding values of  $Y$

$$\begin{array}{r} 64 - 64 + 96 - 6 \end{array}$$

$X$	0	1	2	3	4
$Y$	-6	3	10	21	42



Attendance - 10

Article - 30 [4-6]

page

How to write an article

325  
75  
1625  
750  
29325

to Algorithm

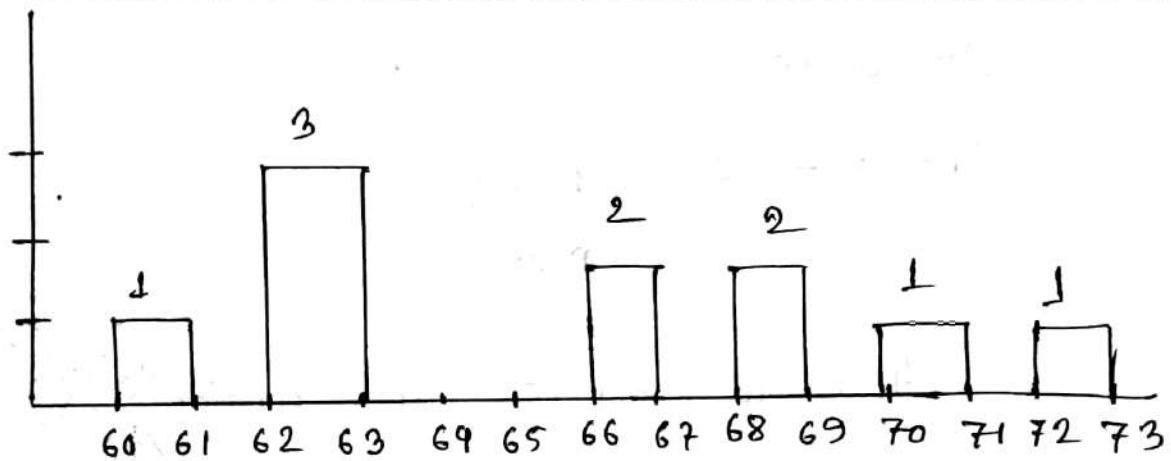
Next class → Title for article

Article 28,

Review report

(Opposition report) } (1-2) } page 20

Slide + Presentation (5+10)



Histogram representation

Q. What is relation between st

Q. Describe the role of statistics in computer science , find out a real life problem with algorithm to show the relation .

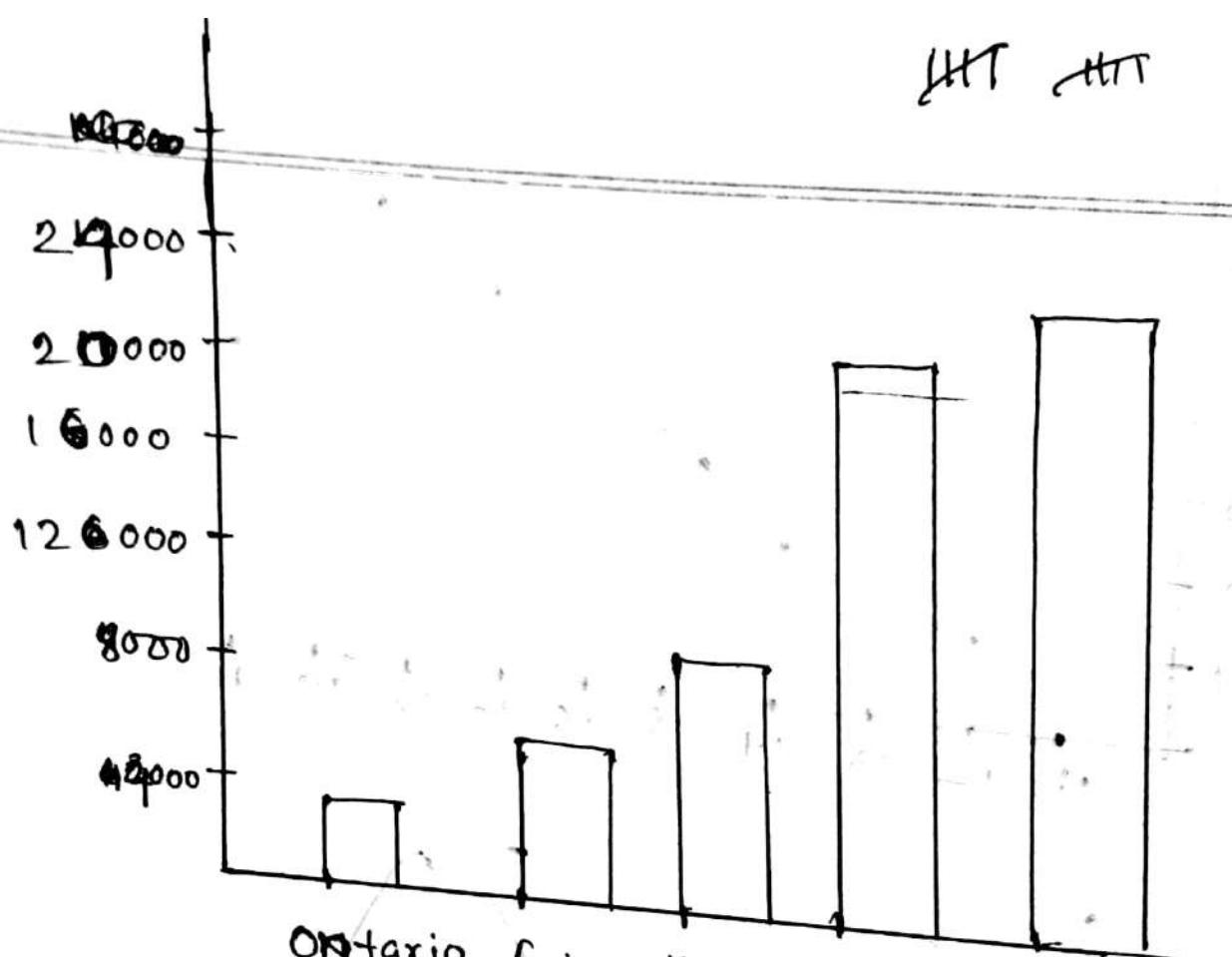
Assignment

JHT ATT

ACTIVE

Date : .....

Page :



Ontario Erie Huron Superior Michigan

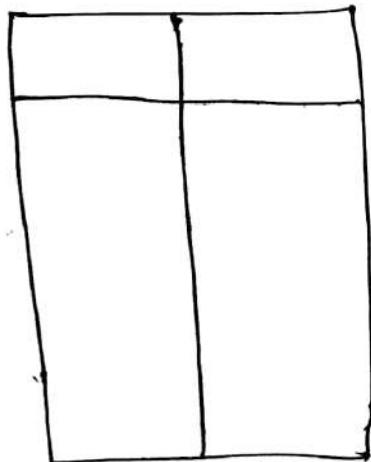
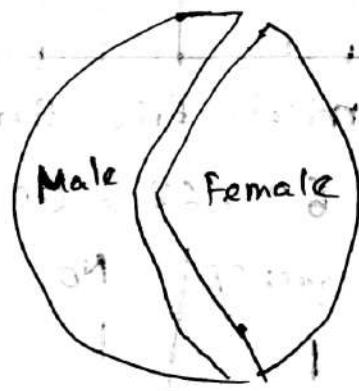
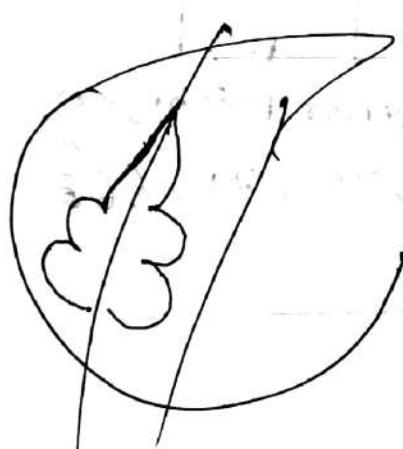
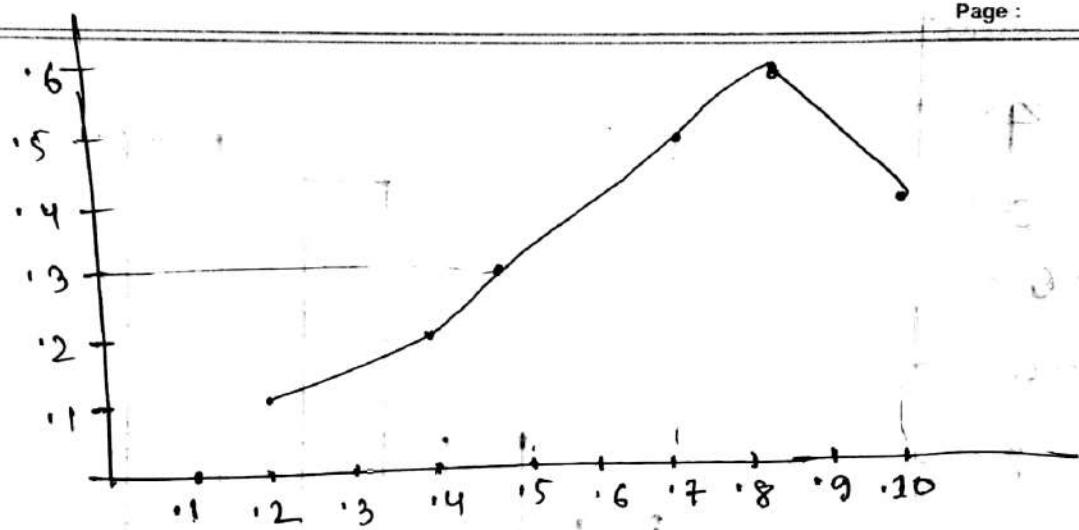
67, 63, 62, 62, 60, 68, 72, 71, 66<sup>4</sup>

Interval	frequency	No
60-64	1	1
62-63	1	3
64-65	.	.
65-67	1	2
68-69	1	2
70-71	1	1
72-73	1	1

**ACTIVE**

Date : .....

Page :



## → Descriptive statistics / Deductive statistics

→ Inductive statistics / inference

↳ deals with the condition

that ~~to~~ make a conclusion valid.

ଆଜିର ଏକାଟି ଫିଲ୍ସି ଅନୁମତି ଦିଲାଯାଇଛି । ଏହାରେ ଏକାଟି ଅନୁମତି ଦିଲାଯାଇଛି ।

Discrete variable: (can be represented by finite no)

Male	0.1	3	5	4	6	2
Female	2	5	7	10	8	4

- \* Statistics is gathering information out of data.
- \* Statistics is operation on data to infer something.

Statistics is the ~~summarize of~~ summarized representation of information from the data.

Sample: The part of population which is used for learning about the whole population.

Sample should be a ~~good~~ representative of the population.

A	B	C	D	E	F	G	H
1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8
1	2	3	4	5	6	7	8

## Statistics

Population & Sample

What is statistics?

What do you mean by population & samples?

- Data collection
- Data representation
- Data Analysis
- Inference (Draw some conclusions)

Definition:

Statistics is concerned with scientific methods for collecting, organising, summarising, presenting and analyzing data as well as drawing valid conclusions and making reasonable decisions on the basis of such analysis.