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Assignment-2

1. Determine whether the following systems are linear or non-linear.

(a) $y(n) = nx(n^2)$

(b) $y(n) = x(n+2)$

(c) $y(n) = x^2(n+1)$

Answer:

(a) ~~For~~ Given that,

$$y(n) = nx(n^2)$$

$$\therefore y_1(n) = nx_1(n^2)$$

$$y_2(n) = nx_2(n^2)$$

$$y'(n) = a_1 n_1 x_1(n^2) + a_2 n_2 x_2(n^2)$$

$$y(n) = H(a_1 x_1(n^2) + a_2 x_2(n^2))$$

$$= n[a_1 x_1(n^2) + a_2 x_2(n^2)]$$

$$= a_1 n_1 x_1(n^2) + a_2 n_2 x_2(n^2)$$

$$\therefore y(n) = y'(n)$$

Hence, the system is linear.

(b) Given that,

$$y(n) = x(n+2)$$

$y_1(n) = x_1(n+2)$

$y_2(n) = x_2(n+2)$

$$y'(n) = a_1 y_1(n) + a_2 y_2(n)$$
$$= a_1 x_1(n+2) + a_2 x_2(n+2)$$

$$y(n) = H[a_1 x_1(n) + a_2 x_2(n)]$$
$$= a_1 x_1(n+2) + a_2 x_2(n+2)$$

$\therefore y(n) = y'(n)$

Hence, the system is linear.

(c) Given that,

$$y(n) = x^2(n+1)$$

$y_1(n) = x_1^2(n+1)$

$y_2(n) = x_2^2(n+1)$

$$y'(n) = a_1 y_1(n) + a_2 y_2(n)$$
$$= a_1 x_1^2(n+1) + a_2 x_2^2(n+1)$$

$$y(n) = H[a_1 x_1(n) + a_2 x_2(n)]$$
$$= [a_1 x_1(n+1) + a_2 x_2(n+1)]^2$$
$$= a_1^2 x_1^2(n+1) + 2a_1 a_2 x_1(n+1) x_2(n+1) + a_2^2 x_2^2(n+1)$$

Since, $y(n) \neq y'(n)$, the system is non-linear.

2. The impulse response of a linear time-invariant system is

$$h(n) = \{1, 2, \underset{\uparrow}{3}, -2, 3\}$$

Now, determine the response of the system to the input signal

$$x(n) = \{5, 4, 3, \underset{\uparrow}{2}, -1\}$$

Answer:

$$h(-k) = \{3, -2, \underset{\uparrow}{3}, 2, 1\}$$

$$y_0(n) = \sum x(n)\, h(-k)$$

$$= \sum \{0, 12, -6, 6, -2, 0\}$$

$$= 10$$

$$h(1-k) = \{3, \underset{\uparrow}{-2}, 3, 2, 1\}$$

$$y_1(n) = \sum x(n)\, h(1-k)$$

$$= \sum \{0, 0, 9, -4, -3, 0, 0\}$$

$$= 2$$

$$h(2-k) = \{\underset{\uparrow}{3}, -2, 3, 2, 1\}$$

$$y_2(n) = \sum x(n)\, h(2-k)$$

$= \sum\{0,0,0,6,2,0,0,0\}$

$= 8$

$h(3-k) = \{\underset{\uparrow}{0},3,-2,3,2,1\}$

$y_3(n) = \sum x(n)h(3-k)$

$= \sum\{0,0,0,0,-3,0,0,0,0\}$

$= -3$

$h(4-k) = \{\underset{\uparrow}{0},0,3,-2,3,2,1\}$

$y_4(n) = \sum x(n)h(4-k)$

$= \sum\{0,0,0,0,0,0,0,0,0,0,0\}$

$= 0$

$y_5(n) = \sum x(n)h(5-k)$

$= 0$

$h(-1-k) = \{3,-2,3,\underset{\uparrow}{2},1\}$

$y_{-1}(n) = \sum x(n)h(-1-k)$

$= \sum\{15,-8,9,4,-1\}$

$= 19$

$h(-2-k) = \{3,-2,3,2,\underset{\uparrow}{1}\}$

$$y_{-2}(n) = \sum x(n)\, h(-2-k)$$

$$= \sum \{0, -10, 12, 6, 2, 0\}$$

$$= 10$$

$$h(-3-k) = \{3, -2, 3, 2, 1, \underset{\uparrow}{0}\}$$

$$y_3(n) = \sum x(n)\, h(-3-k)$$

$$= \sum \{0, 0, 15, 8, 3, 0, 0\}$$

$$= 26$$

$$h(-4-k) = \{3, -2, 3, 2, 1, 0, \underset{\uparrow}{0}\}$$

$$y_{-4}(n) = \sum x(n)\, h(-4-k)$$

$$= \sum \{0, 0, 0, 10, 4, 0, 0, 0\}$$

$$= 14$$

$$h(-5-k) = \{3, -2, 3, 2, 1, 0, 0, \underset{\uparrow}{0}\}$$

$$y_{-5}(n) = \sum x(n)\, h(-5-k)$$

$$= \sum \{0, 0, 0, 0, 5, 0, 0, 0, 0\}$$

$$= 5$$

~~y_{-6}~~ $y_{-6}(n) = \sum x(n)\, h(-6-k)$

$$= 0$$

$y_{7}(n) = \sum x(n)\, h(-7-k)$

$= 0$

$\therefore y(n) = \{\ldots, 0, 0, 5, 14, 26, 10, 19, \underset{\uparrow}{10}, 2, 8, -3, 0, 0, \ldots\}$

Ans.

⑥

3. With proper example and mathematical equation briefly explain-

(a) Zero State response

(b) Zero Input response

Answer:

(a) If the system is relaxed initially at time $n=0$, then its memory should be zero. Hence, $y(-1)=0$. Thus, a recursive system is relaxed if it starts with zero initial conditions. Since, the memory of the systems describes its state, we say that the system is at zero state, and its corresponding output is called the zero state response. It is denoted by $y_{zs}(n)$. The zero state response is given by -

$$y_{zs}(n) = \sum_{k=0}^{n} a^k x(n-k) \quad ; n \geq 0$$

where, the input-output equation of the recursive system is,

$$y(n) = ay(n-1) + x(n)$$

(b) For a recursive system with an input output equation

$$y(n) = ay(n-1) + x(n)$$

If the system is initially non-relaxed, i.e. $y(-1) \neq 0$, and the input $x(n) = 0$ for all n, then the output of the system with zero input is called the zero input response or natural response. It is denoted by $y_{zi}(n)$. The zero input response is given by -

$$y_{zi}(n) = a^{n+1} y(-1), \quad n \geq 0$$

4. Given two signal,

$$x(n) = \{7, 2, \underset{\uparrow}{3}, -2, 3\} \text{ and,}$$

$$y(n) = \{5, 4, 3, \underset{\uparrow}{2}, -1\}$$

Find out and sketch the cross-correlation between the signals and also determine and sketch the auto-correlation of $x(n)$.

<u>Answer:</u>

Cross-correlation of $x(n)$:-

We know for cross-correlation -

$$r_{xy}(l) = \sum_{n=-\infty}^{\infty} x(n)\, y(n-l)$$

$$r_{xy}(0) = \sum x(n)\, y(n-0)$$

$$= \sum \{0, 4, 6, 6, 2, 0\}$$

$$= 18$$

$$r_{xy}(1) = \sum x(n)\, y(n-1)$$

$$= \sum \{5, 8, 9, -4, -3\}$$

$$= 15$$

$$r_{xy}(2) = \sum x(n)\, y(n-2)$$

$= \sum\{0, 10, 12, -6, 6, 0\}$

$= 22$

$r_{xy}(3) = \sum x(n)\, y(n-3)$

$= \sum\{0, 0, 15, -8, 9, 0, 0\}$

$= 16$

$r_{xy}(4) = \sum x(n)\, y(n-4)$

$= \sum\{0, 0, 0, -10, 12, 0, 0, 0\}$

$= 2$

$r_{xy}(5) = \sum x(n)\, y(n-5)$

$= \sum\{0, 0, 0, 0, 15, 0, 0, 0, 0\}$

$= 15$

$r_{xy}(6) = \sum x(n)\, y(n-6)$

$= 0$

$r_{xy}(-1) = \sum x(n)\, y(n+1)$

$= \sum\{0, 0, 3, 4, -3, 0, 0\}$

$= 4$

$$r_{xy}(-2) = \sum x(n)\, y(n+2)$$

$$= \sum \{0, 0, 0, 2, -2, 0, 0, 0\}$$

$$= 0$$

$$r_{xy}(-3) = \sum x(n)\, y(n+3)$$

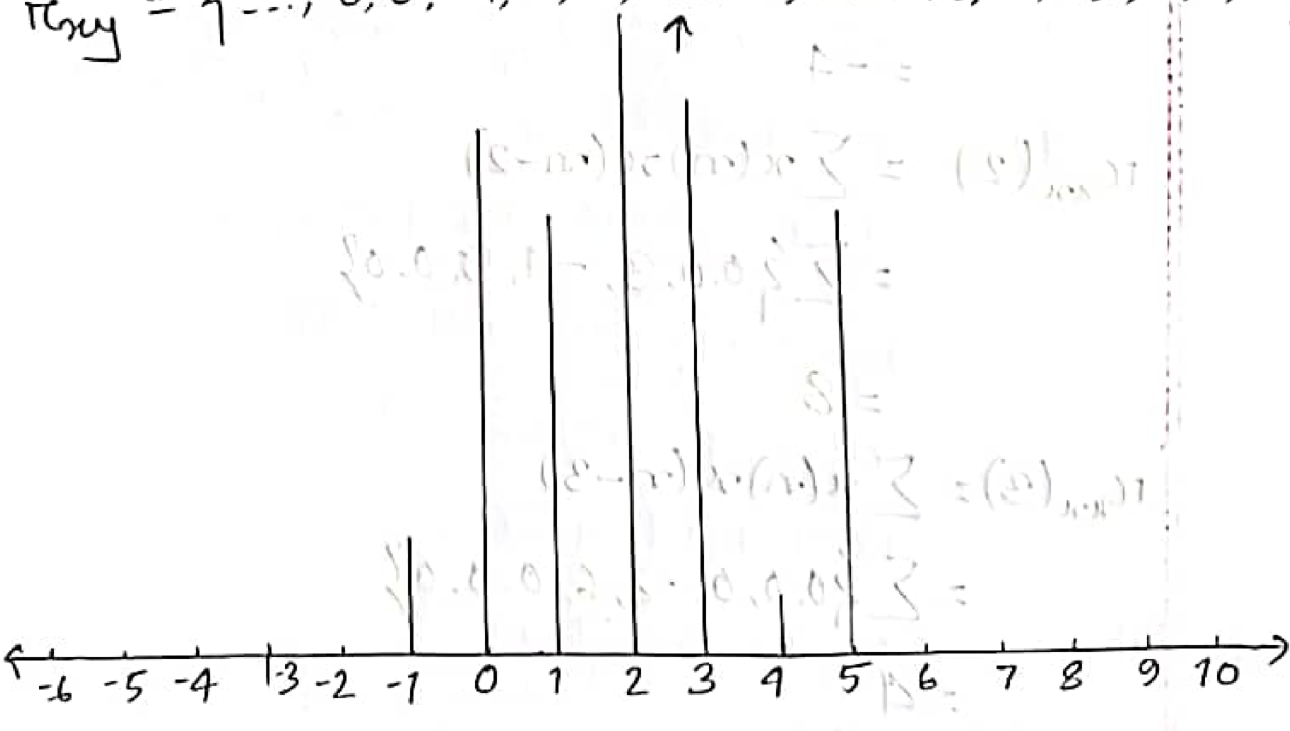
$$= \sum \{0, 0, 0, 0, -1, 0, 0, 0, 0\}$$

$$= -1$$

$$r_{xy}(-4) = \sum x(n)\, y(n+4) = 0$$

$$r_{xy}(-5) = \sum x(n)\, y(n+5) = 0$$

$$\therefore r_{xy} = \{\ldots, 0, 0, -1, 0, 4, \underset{\uparrow}{18}, 15, 22, 16, 2, 15, 0, 0, \ldots\}$$



Auto-correlation of $x(n)$:-

We know for auto-correlation -

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)\,x(n-l)$$

$$r_{xx}(0) = \sum x(n)\,x(n-0)$$
$$= \sum \{1, 4, 9, 4, 9\}$$
$$= 27$$

$$r_{xx}(1) = \sum x(n)\,x(n-1)$$
$$= \sum \{0, 2, 6, -6, -6, 0\}$$
$$= -4$$

$$r_{xx}(2) = \sum x(n)\,x(n-2)$$
$$= \sum \{0, 0, 3, -4, 9, 0, 0\}$$
$$= 8$$

$$r_{xx}(3) = \sum x(n)\,x(n-3)$$
$$= \sum \{0, 0, 0, -2, 6, 0, 0, 0\}$$
$$= 4$$

$$r_{xx}(4) = \sum x(n)\,x(n-4)$$
$$= \sum \{0, 0, 0, 0, 3, 0, 0, 0, 0\}$$

$= 3$

$r_{xy}(5) = \sum x(n) y(n-5) = 0$

$r_{xy}(6) = \sum x(n) x(n-6) = 0$

$r_{xy}(-1) = \sum x(n) x(n+1)$

$= \sum \{0, 2, 6, -6, -6, 0\}$

$= -4$

$r_{xy}(-2) = \sum x(n) x(n+2)$

$= \sum \{0, 0, 3, -4, 9, 0, 0\}$

$= 8$

$r_{xy}(-3) = \sum x(n) x(n+3)$

$= \sum \{0, 0, 0, -2, 6, 0, 0, 0\}$

$= 4$

$r_{xy}(-4) = \sum x(n) x(n+4)$

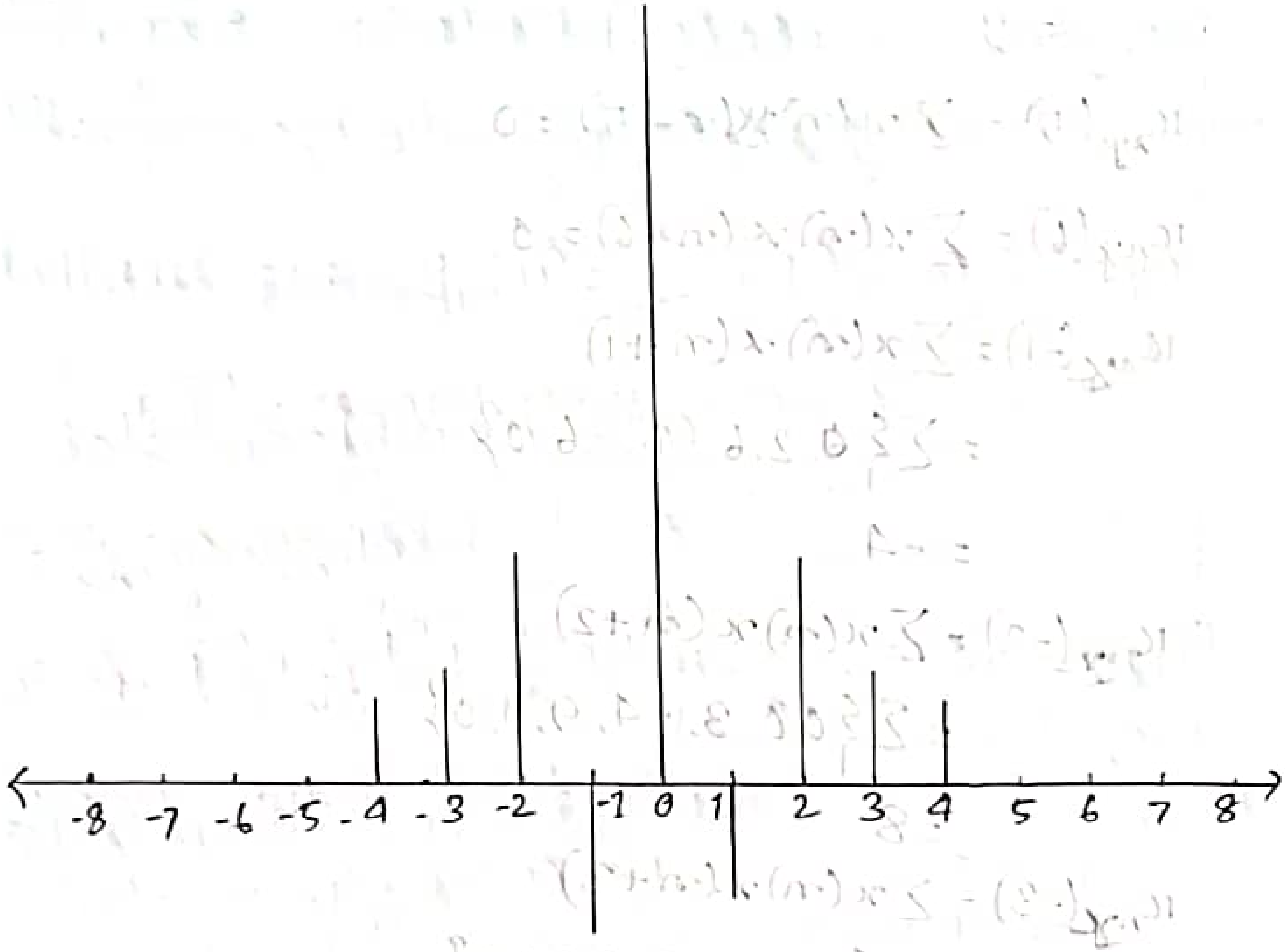
$= \sum \{0, 0, 0, 0, 3, 0, 0, 0, 0\}$

$= 3$

$r_{xx}(-5) = \sum x(n) x(n+5) = 0$

$r_{xx}(-6) = \sum x(n) x(n+6) = 0$

$\therefore r_{xx} = \{\ldots 0, 0, 3, 4, 8, -4, \underset{\uparrow}{27}, -4, 8, 4, 3, 0, 0, \ldots\}$


-8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8

5. Determine the response of $y(n), n \geq 0$, of the system described by the second order difference equation

$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$$

where, the input sequence is, $x(n) = 4^n u(n)$.

<u>Answer:</u>

For a homogeneous solution, $x(n) = 0$

$y_h(n) = \lambda^n$

$$\therefore y(n) - 3y(n-1) - 4y(n-2) = 0 \quad \text{——— (1)}$$

$$\Rightarrow \lambda^n - 3\lambda^{n-1} - 4\lambda^{n-2} = 0$$

$$\Rightarrow \lambda^{n-2}(\lambda^2 - 3\lambda - 4) = 0$$

$$\Rightarrow \lambda^{n-2}(\lambda^2 - 4\lambda + \lambda - 4) = 0$$

$$\Rightarrow \lambda^{n-2}[\lambda(\lambda - 4) + (\lambda - 4)] = 0$$

$$\Rightarrow \lambda^{n-2}(\lambda - 4)(\lambda + 4) = 0$$

Therefore, the roots are $\lambda = -1, 4$, and the general form of the ~~homogeneous~~ solution to the homogeneous solution is -

$$\cancel{c_1(-1)^n + c_2 4^n}$$

$$y_h(n) = c_1 \lambda_1^n + c_2 \lambda_2^n$$
$$= c_1(-1)^n + c_2 4^n \quad \text{——— (10)}$$

The particulare solution to the given equation would be of the form-

$$y_p(n) = K(4)^n \cdot u(n)$$

However, $y_p(n)$ is alread contained in the homogeneous solution. So, this particulare solution is redundant. Thus, Let's assume that,

$$y_p(n) = Kn4^n u(n) \quad \text{——— (11)}$$

Substituting eqn. (11) in the given equation-

$$Kn4^n u(n) - 3K(n-1)4^{n-1}u(n-1) - 4K(n-2)4^{n-2}u(n-2)$$
$$= 4^n u(n) + 2 \cdot 4^{n-1} u(n-1)$$

$$\Rightarrow K \cdot 2 \cdot 4^2 - 3K(2-1)4^{2-1} - 4K(2-2)4^{2-2} = 4^2 + 2 \cdot 4$$

[evaluating with $n \geq 2$ so that none of the unit steps vanish]

$$\Rightarrow 32K - 12K = 16 + 8$$

$$\Rightarrow 20K = 24$$

$\Rightarrow K = \frac{24}{20}$

$\Rightarrow K = \frac{6}{5}$

$\therefore$ Therefore, $y_p(n) = \frac{6}{5} n 4^n u(n)$ —— (iv)

$\therefore$ Total solution, $y(n)$

$= y_h(n) + y_p(n)$

$= c_1(-1)^n + c_2 4^n + \frac{6}{5} n 4^n u(n) \; ; n \geq 0$ —— (v)

From eqn. (1), we have,

$y(0) = 3y(-1) + 4y(-2) + 1$

$y(1) = 3y(0) + 4y(-1) + 6$

$= 3[3y(-1) + 4y(-2) + 1] + 4y(-1) + 6$

$= 9y(-1) + 12y(-2) + 4y(-1) + 3 + 6$

$\cancel{= 7y(-2) + 12y(-2)}$

$= 13y(-1) + 12y(-2) + 9$

From eqn. (v), we have

$y(0) = c_1 + c_2$

$y(1) = -c_1 + 4c_2 + \frac{24}{5}$

$$\therefore c_1 + c_2 = 3y(-1) + 4y(-2) + 1 \quad \text{——— (VI)}$$

$$-c_1 + 4c_2 + \frac{24}{5} = 13y(-1) + 12y(-2) + 9 \quad \text{——— (VII)}$$

From eqn. (VI) + (VII),

$$5c_2 + \frac{24}{5} = \cancel{3}\,16y(-1) + 16y(-2) + \cancel{9}\,10$$

$$\Rightarrow 5c_2 = 16y(-1) + 16y(-2) + \frac{26}{5}$$

$$\Rightarrow c_2 = \cancel{16}\,\frac{16}{5}y(-1) + \frac{16}{5}y(-2) + \frac{26}{25} \quad \text{——— (VIII)}$$

~~8~~ substituting the value of c_2 in eqn. (VI),

$$c_1 + \frac{16}{5}y(-1) + \frac{16}{5}y(-2) + \frac{26}{25} = 3y(-1) + 4y(-2) + 1$$

$$\Rightarrow c_1 = -\frac{1}{5}y(-1) + \frac{4}{5}y(-2) \cancel{-\frac{4}{25}} - \frac{1}{25} \quad \text{——— (IX)}$$

Setting $y(-1) = y(-2) = 0$

$$c_1 = -\frac{1}{25}$$

$$c_2 = \frac{26}{25}$$

$$\therefore y(n) = -\frac{1}{25}(-1)^n + \frac{26}{25}4^n + \frac{6}{5}n\,4^n\,u(n)$$

Ans.