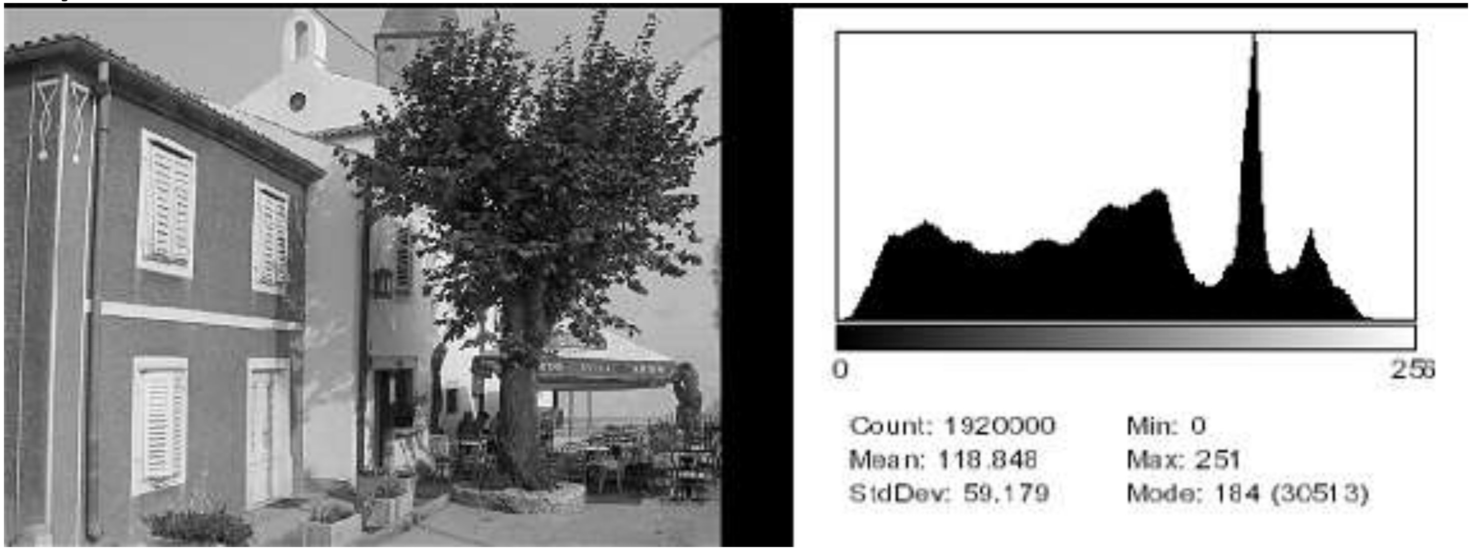
Digital Image Processing

Contents

- Histogram Processing
  - Histogram Equalization
  - Histogram Matching
  - Local Histogram processing
  - Using histogram statistics for image enhancement

Histograms

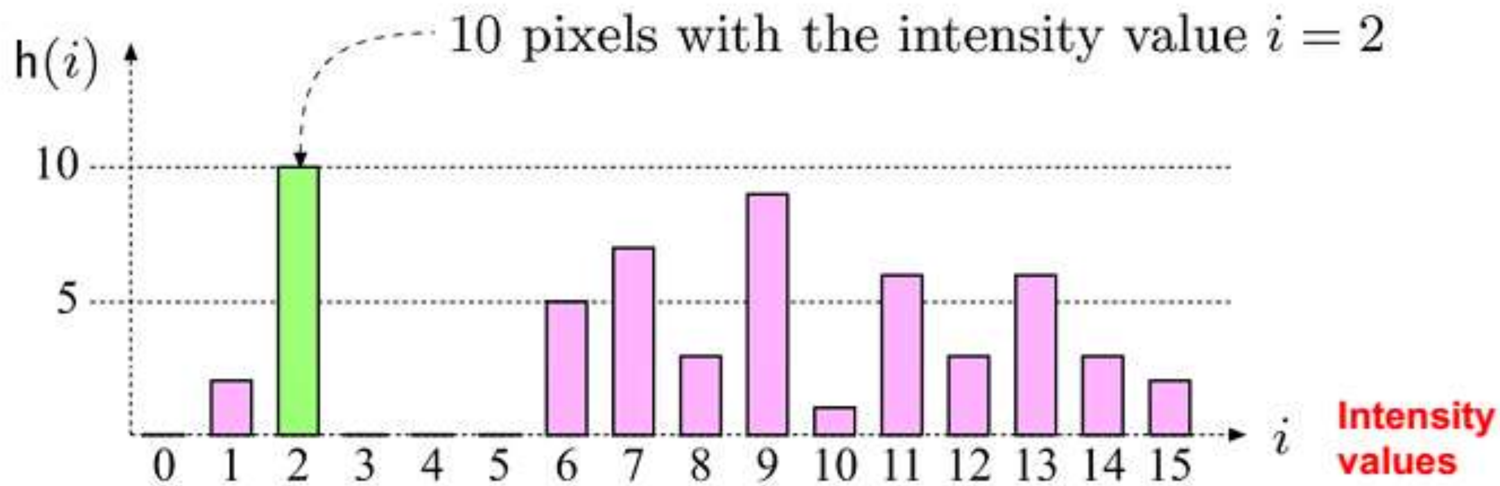
- **Histograms plots show many times (frequency) each intensity value in image occurs**
- Example:
  - Image (left) has 256 distinct gray-levels (8bits)
  - Histogram (right) shows frequency (how many times)each gray-level occurs



Histograms

- Many cameras display real time histograms of scene
- Helps avoid taking over-exposed pictures
- Also easier to detect types of processing previously applied to image

Histograms

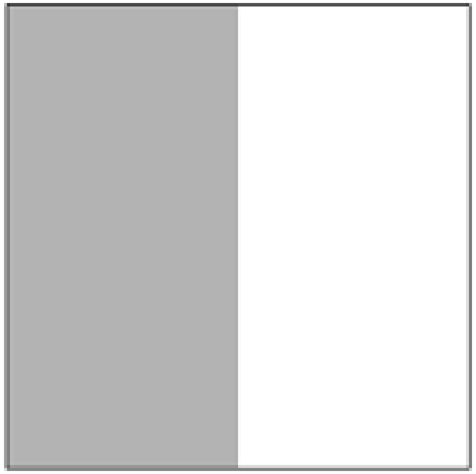


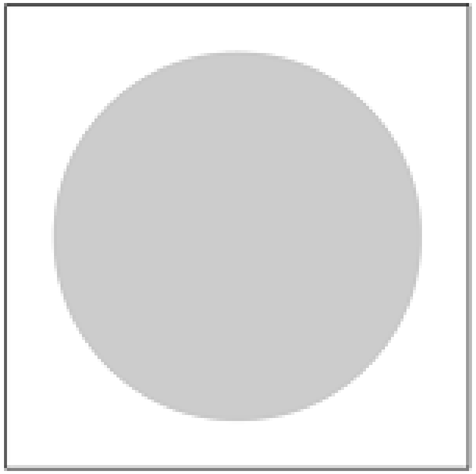
h(i)	0	2	10	0	0	0	5	7	3	9	1	6	3	6	3	2
i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

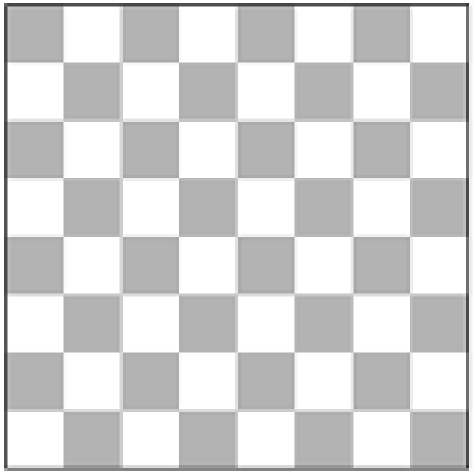
- **E.g. *L* = 16, 10 pixels have intensity value = 2**
- **Histograms: only statistical information**
- **No indication of location of pixels**

Histograms

- **Draw the histogram of following images. Consider each image of size 64x64 (2-levels).**

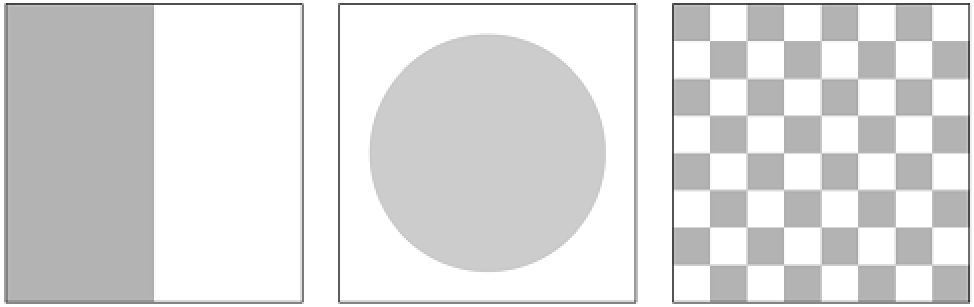






Histograms

- Different images can have same histogram
- 3 images below have same histogram



- Half of pixels are gray, half are white
  - Same histogram = same statistics
  - Distribution of intensities could be different
- Can we reconstruct image from histogram?

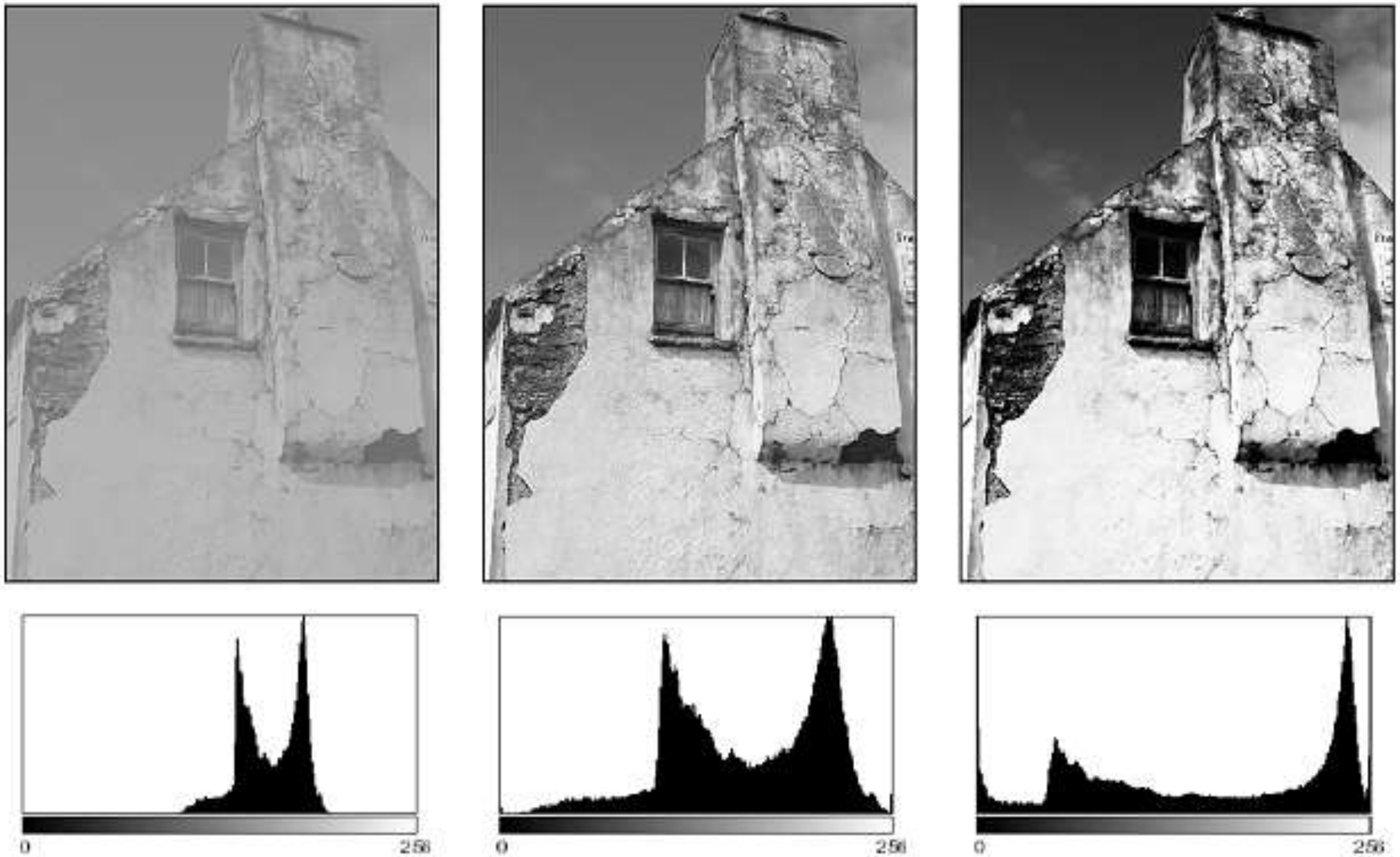
Histograms

- A histogram for a grayscale image with intensity values in range $I(x, y) \in [0, L—1]$ would contain exactly L entries
- E.g. 8-bit grayscale image, L = 2^8 = 256
- Each histogram entry is defined as:
- h(i) = number of pixels with intensity i for all 0 <= i< L.
- E.g: h(255) = number of pixels with intensity = 255

Image Contrast

- **The contrast of a grayscale image indicates how easily objects in the image can be distinguished**
- **High contrast image: many distinct intensity values**
- **Low contrast: image uses few intensity values**

Histogram and Image Contrast



Image

Histogram

(a) Low contrast

(b) Normal contrast

(c) High contrast

Histogram Processing

The *histogram* of a digital image, *f,* (with intensities $[0,L-1]$) is a discrete function

$$h(r_k) = n_k$$

Where r_k is the k^{th} intensity value and n_k is the number of pixels in f with intensity r_k

Normalizing the histogram is common practice

- Divide the components by the total number of pixels in the image
- Assuming an MxN image, this yields

$$p(r_k) = n_k/MN \text{ for } k=0,1,2,\ldots,L-1$$

- $p(r_k)$ is, basically, an estimate of the probability of occurrence of intensity level r_k in an image

$$\Sigma\, p(r_k) = 1$$

Uses for Histogram Processing

- **Image enhancements**
- **Image statistics**
- **Image compression**
- **Image segmentation**

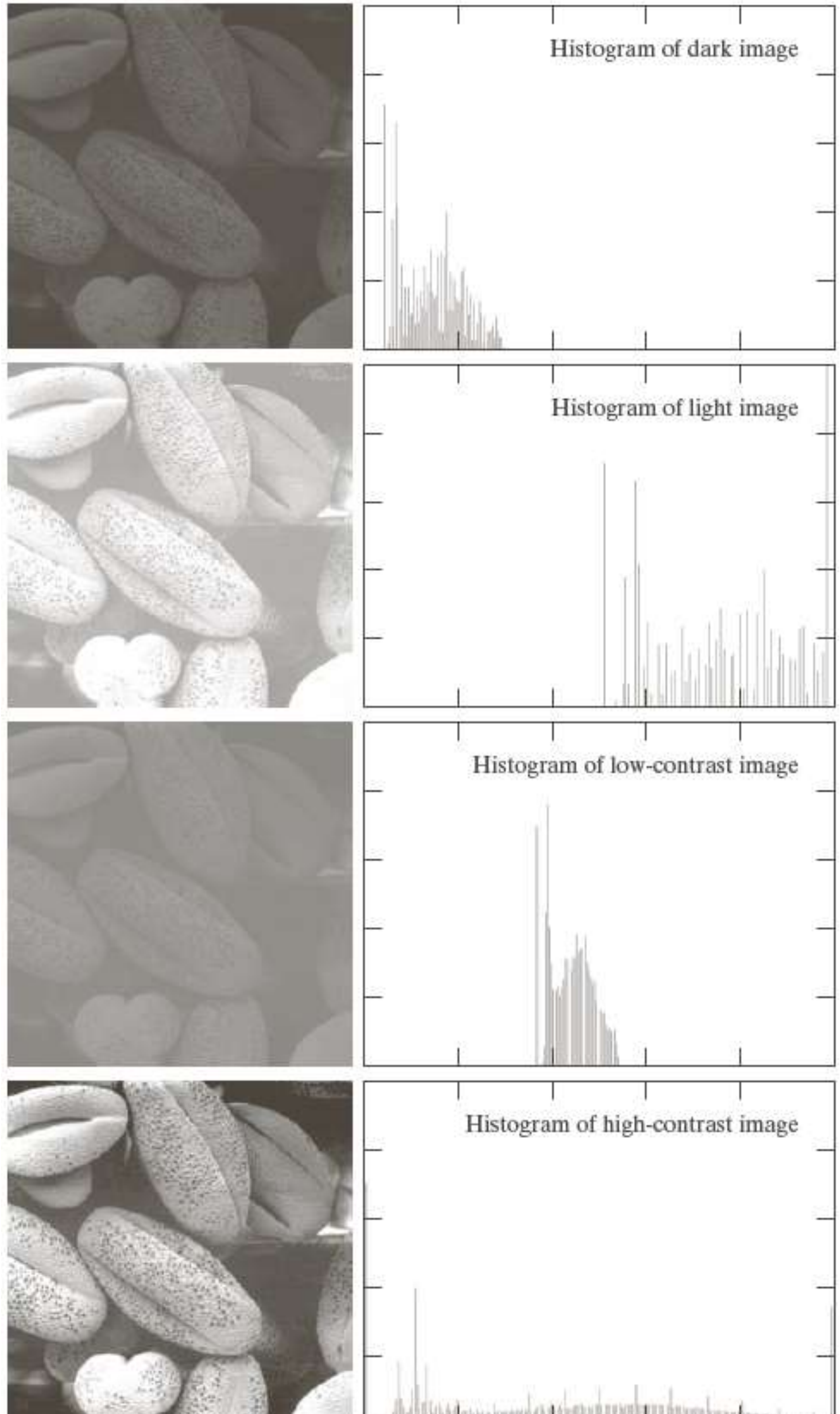
- Simple to calculate in software
- Economic hardware implementations
  - Popular tool in real-time image processing

- A plot of this function for all values of *k* provides a global description of the appearance of the image (gives useful information for contrast enhancement)

Uses for Histogram Processing

- Four basic image types and their corresponding histograms
  - Dark
  - Light
  - Low contrast
  - High contrast
- Histograms commonly viewed in plots as

$$h(r_k) = n_k \text{ versus } r_k$$
$$p(r_k) = n_k / MN \text{ versus } r_k$$


Histogram of dark image
Histogram of light image
Histogram of low-contrast image
Histogram of high-contrast image

Histogram Equalization

Histogram equalization is a process for increasing the contrast in an image by spreading the histogram out to be approximately uniformly distributed

The gray levels of an image that has been subjected to histogram equalization are spread out and always reach white

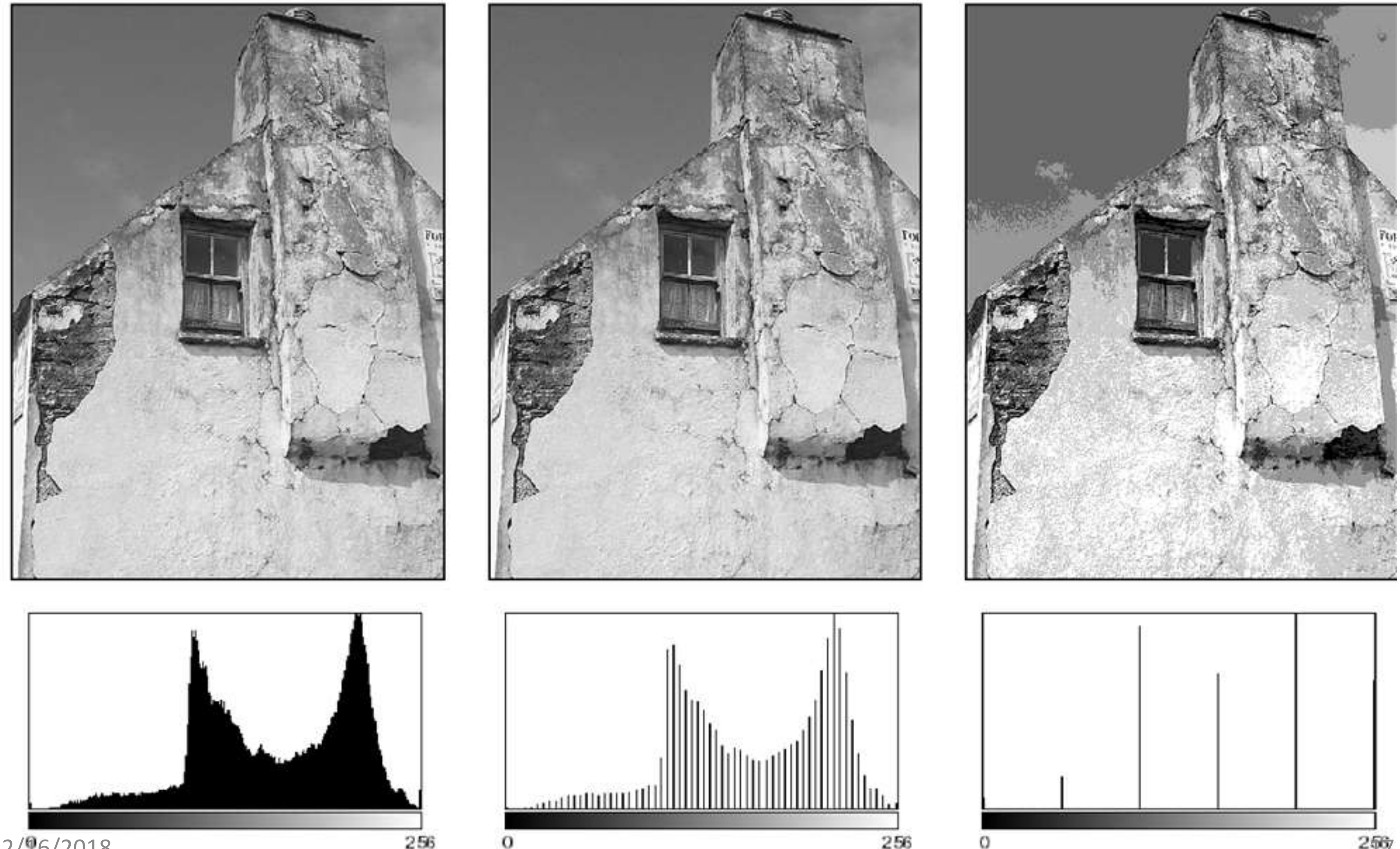
- The increase of dynamic range produces an increase in contrast

For images with low contrast, histogram equalization has the adverse effect of increasing visual graininess

Dynamic Range

- **Dynamic Range:** Number of distinct pixels in image
- **High dynamic range** means very bright and very dark parts in a single image (many distinct values)

Dynamic Range



Histogram Equalization

The intensity levels in an image may be viewed as random variables in the interval [0, L-1].

Let $p_r(r)$ and $p_s(s)$ denote the probability density function (PDF) of random variables r and s.

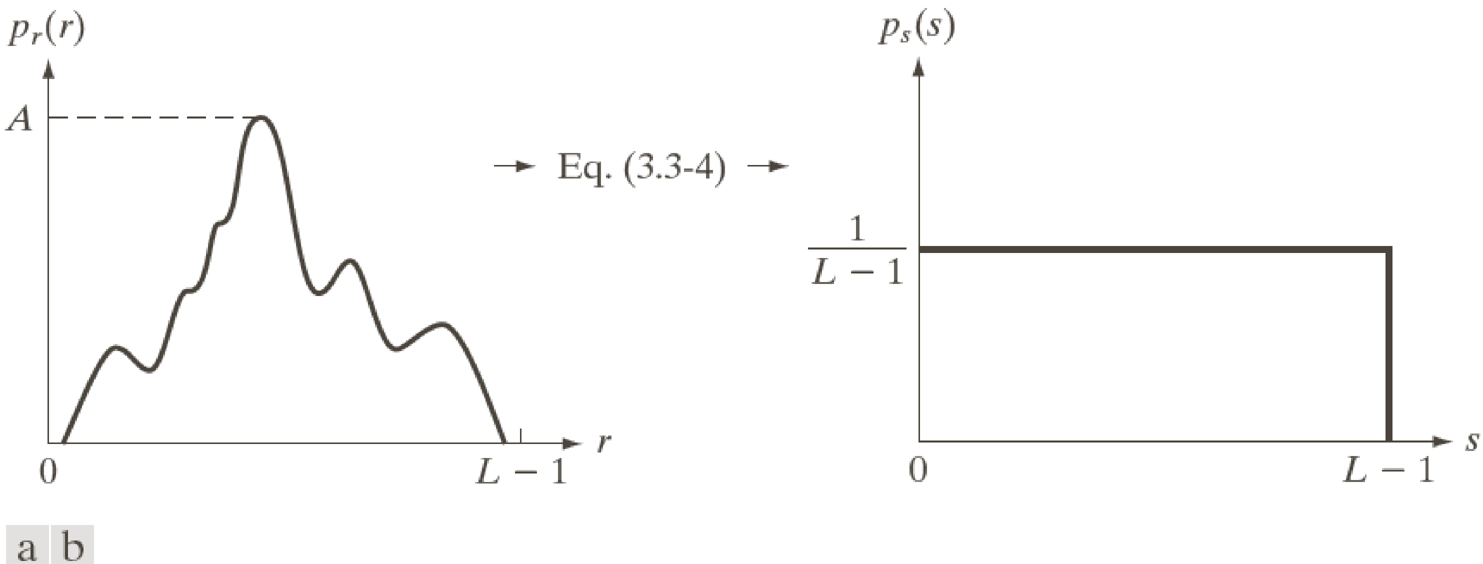


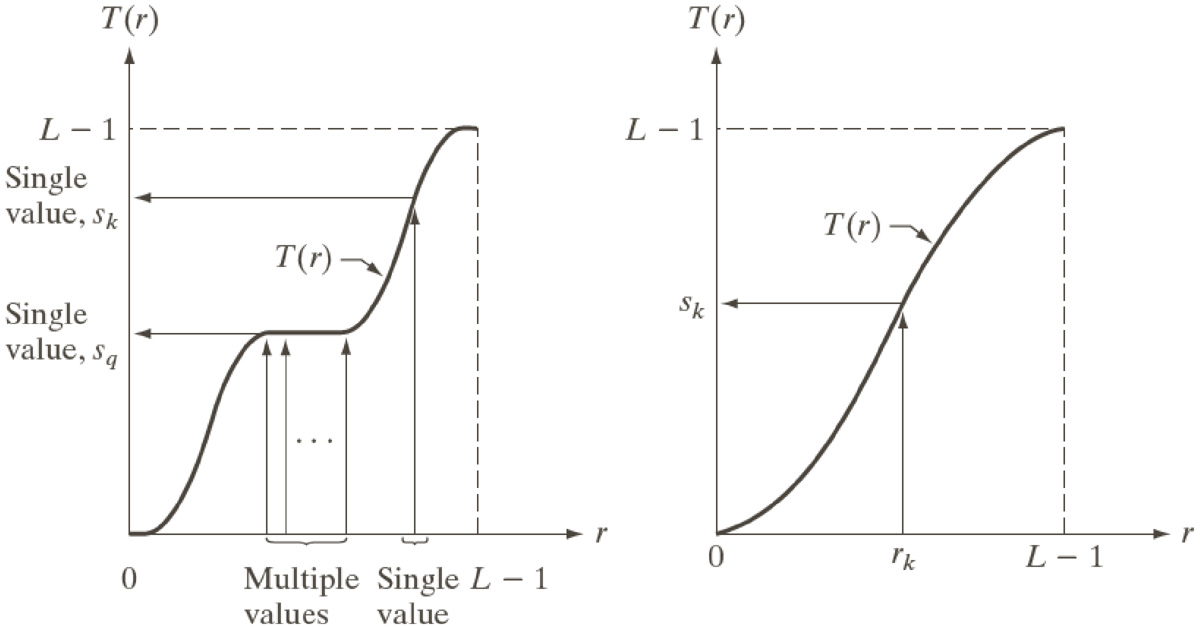
FIGURE 3.18 (a) An arbitrary PDF. (b) Result of applying the transformation in Eq. (3.3-4) to all intensity levels, r. The resulting intensities, s, have a uniform PDF, independently of the form of the PDF of the r's.

Histogram Equalization

$$s = T(r) \qquad 0 \le r \le L-1$$

$a.$ T(r) is a strictly monotonically increasing function in the interval $0 \le r \le L\text{-}1$;

$b.$ $0 \le T(r) \le L\text{-}1$ for $0 \le r \le L\text{-}1$.



a b

FIGURE 3.17 (a) Monotonically increasing function, showing how multiple values can map to a single value. (b) Strictly monotonically increasing function. This is a one-to-one mapping, both ways.

Histogram Equalization

$$s = T(r) \qquad 0 \le r \le L-1$$

$a.$ T(r) is a strictly monotonically increasing function in the interval $0 \le r \le L\text{-}1$;

$b.$ $0 \le T(r) \le L\text{-}1$ for $0 \le r \le L\text{-}1$.

$T(r)$ is continuous and differentiable.

$$p_s(s)ds = p_r(r)dr$$

Histogram Equalization

$$s = T(r) = (L-1)\int_0^r p_r(w)dw$$

$$\frac{ds}{dr} = \frac{dT(r)}{dr} = (L-1)\frac{d}{dr}\left[\int_0^r p_r(w)dw\right]$$

$$= (L-1)p_r(r)$$

$$p_s(s) = \frac{p_r(r)dr}{ds} = p_r(r) \Big/ \left(\frac{ds}{dr}\right) = p_r(r) \Big/ \left((L-1)p_r(r)\right) = \frac{1}{L-1}$$

Example

Suppose that the (continuous) intensity values in an image have the PDF

$$p_r(r) = \begin{cases} \dfrac{2r}{(L-1)^2}, & \text{for } 0 \le r \le L\text{-}1 \\ 0, & \text{otherwise} \end{cases}$$

Find the transformation function for equalizing the image histogram.

Example

$$s = T(r) = (L-1)\int_0^r p_r(w)dw$$

Histogram Equalization

Continuous case:

$$s = T(r) = (L-1)\int_0^r p_r(w)\,dw$$

Discrete values:

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j)$$

$$= (L-1)\sum_{j=0}^{k} \frac{n_j}{MN} = \frac{L-1}{MN}\sum_{j=0}^{k} n_j \qquad \text{k=0,1,..., L-1}$$

Example: Histogram Equalization

Suppose that a 3-bit image (L=8) of size 64 × 64 pixels (MN = 4096) has the intensity distribution shown in following table.

Get the histogram equalization transformation function and give the $p_s(s_k)$ for each s_k.

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

Example: Histogram Equalization

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

$$s_0 = T(r_0) = 7\sum_{j=0}^{0} p_r(r_j) = 7 \times 0.19 = 1.33 \qquad \rightarrow 1$$

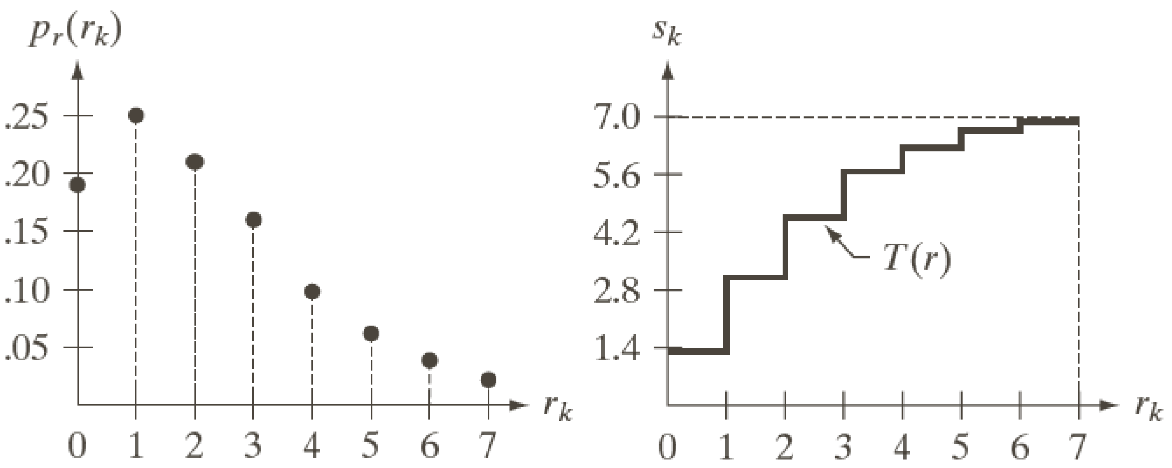
$$s_1 = T(r_1) = 7\sum_{j=0}^{1} p_r(r_j) = 7 \times (0.19 + 0.25) = 3.08 \quad \rightarrow 3$$

$$s_2 = 4.55 \quad \rightarrow 5 \qquad\qquad s_3 = 5.67 \quad \rightarrow 6$$

$$s_4 = 6.23 \quad \rightarrow 6 \qquad\qquad s_5 = 6.65 \quad \rightarrow 7$$

$$s_6 = 6.86 \quad \rightarrow 7 \qquad\qquad s_7 = 7.00 \quad \rightarrow 7$$

Example: Histogram Equalization



a b c

FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

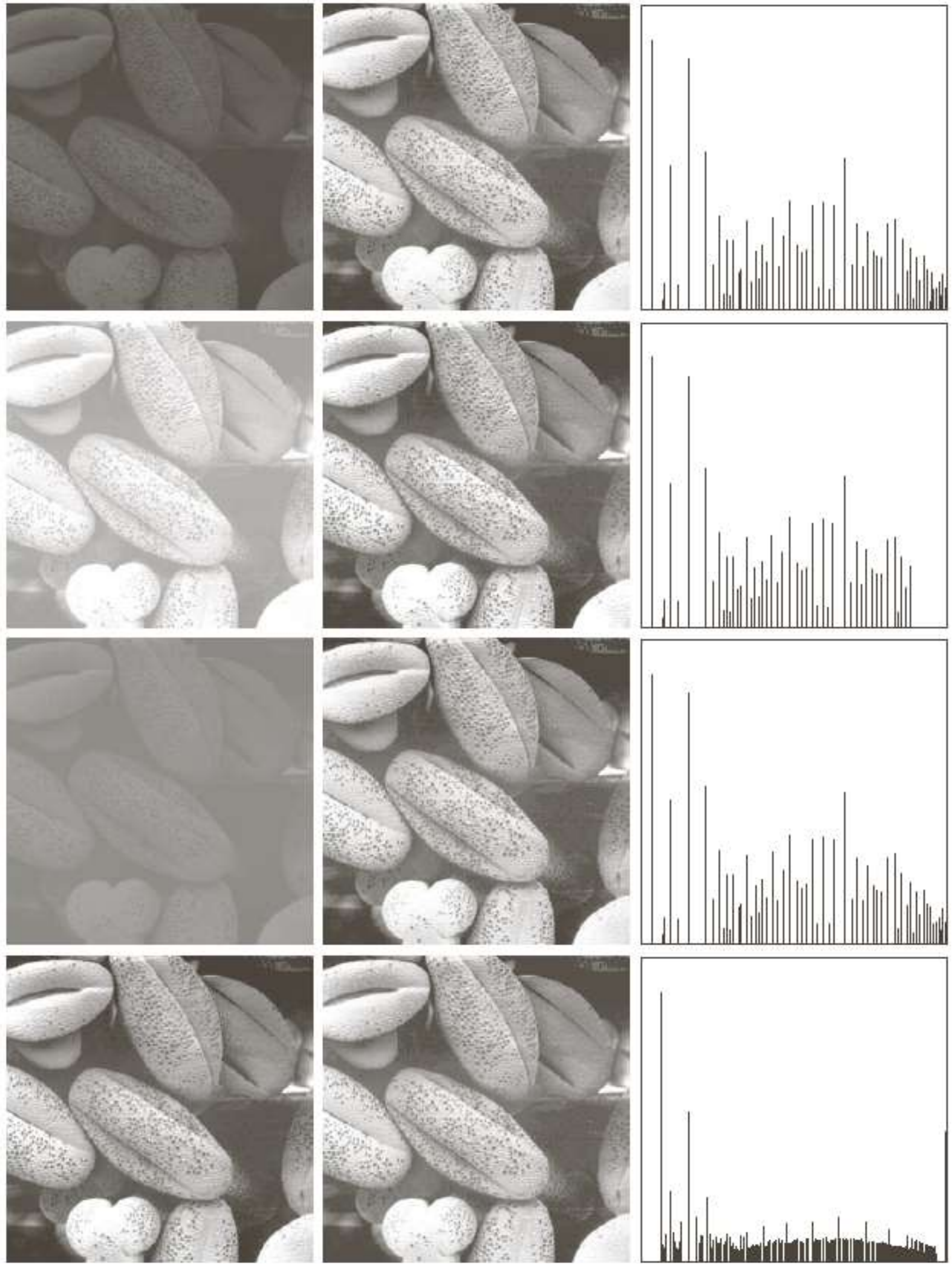


FIGURE 3.20 Left column: images from Fig. 3.16. Center column: corresponding histogram-equalized images. Right column: histograms of the images in the center column.

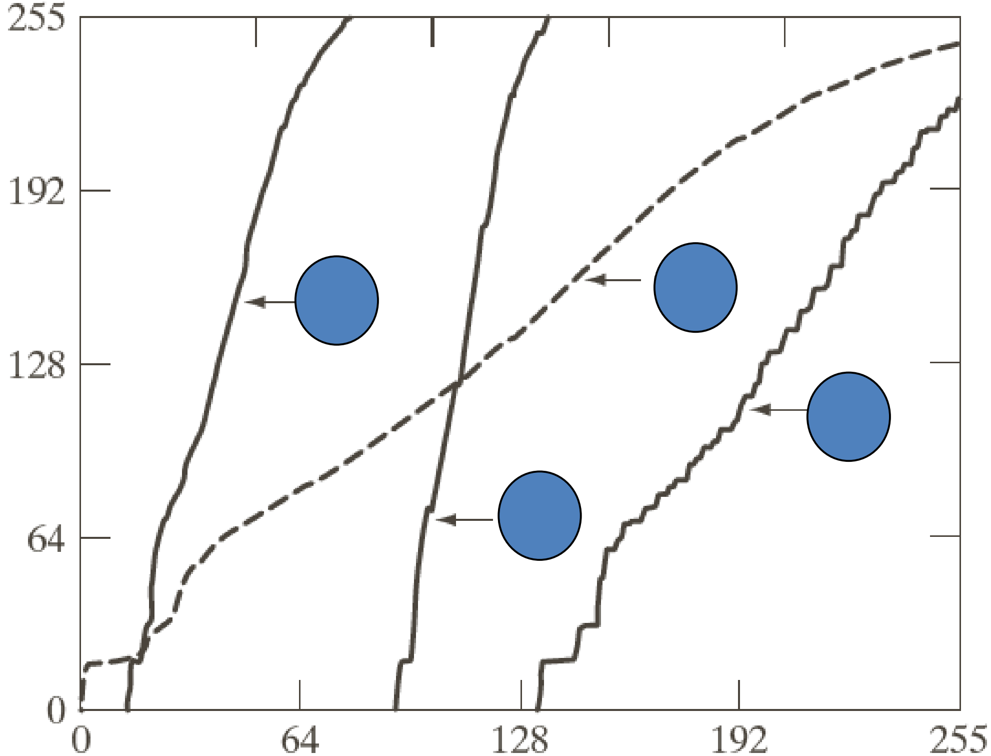


FIGURE 3.21 Transformation functions for histogram equalization. Transformations (1) through (4) were obtained from the histograms of the images (from top to bottom) in the left column of Fig. 3.20 using Eq. (3.3-8).

Question

Is histogram equalization always good?

Histogram Matching

Histogram matching (histogram specification)

— generate a processed image that has a specified histogram

Let $p_r(r)$ and $p_z(z)$ denote the continous probability density functions of the variables r and z. $p_z(z)$ is the specified probability density function.

Let s be the random variable with the probability

$$s = T(r) = (L-1)\int_0^r p_r(w)dw$$

Define a random variable z with the probability

$$G(z) = (L-1)\int_0^z p_z(t)dt = s$$

Histogram Matching

$$s = T(r) = (L-1)\int_0^r p_r(w)\,dw$$

$$G(z) = (L-1)\int_0^z p_z(t)\,dt = s$$

$$z = G^{-1}(s) = G^{-1}\left[T(r)\right]$$

Histogram Matching: Procedure

- Obtain $p_r(r)$ from the input image and then obtain the values of s

$$s = (L-1)\int_0^r p_r(w)dw$$

- Use the specified PDF and obtain the transformation function G(z)

$$G(z) = (L-1)\int_0^z p_z(t)dt = s$$

- Mapping from s to z

$$z = G^{-1}(s)$$

Histogram Matching: Example

Assuming continuous intensity values, suppose that an image has the intensity PDF

$$p_r(r) = \begin{cases} \dfrac{2r}{(L-1)^2}, & \text{for } 0 \le r \le L\text{-}1 \\ 0\ , & \text{otherwise} \end{cases}$$

Find the transformation function that will produce an image whose intensity PDF is

$$p_z(z) = \begin{cases} \dfrac{3z^2}{(L-1)^3}, & \text{for } 0 \le z \le (L\text{-}1) \\ 0, & \text{otherwise} \end{cases}$$

Histogram Matching: Example

Find the histogram equalization transformation for the input image

$$s = T(r) = (L-1)\int_0^r p_r(w)dw = (L-1)\int_0^r \frac{2w}{(L-1)^2}dw \ = \frac{r^2}{L-1}$$

Find the histogram equalization transformation for the specified histogram

$$G(z) = (L-1)\int_0^z p_z(t)dt = (L-1)\int_0^z \frac{3t^2}{(L-1)^3}dt = \frac{z^3}{(L-1)^2} = s$$

The transformation function

$$z = \left[(L-1)^2 s\right]^{1/3} = \left[(L-1)^2 \frac{r^2}{L-1}\right]^{1/3} = \left[(L-1)r^2\right]^{1/3}$$

Histogram Matching: Discrete Cases

- Obtain $p_r(r_j)$ from the input image and then obtain the values of s_k, round the value to the integer range [0, L-1].

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} p_r(r_j) = \frac{(L-1)}{MN}\sum_{j=0}^{k} n_j$$

- Use the specified PDF and obtain the transformation function $G(z_q)$, round the value to the integer range [0, L-1].

$$G(z_q) = (L-1)\sum_{i=0}^{q} p_z(z_i) = s_k$$

- Mapping from s_k to z_q

$$z_q = G^{-1}(s_k)$$

Example: Histogram Matching

Suppose that a 3-bit image (L=8) of size 64 × 64 pixels (MN = 4096) has the intensity distribution shown in the following table (on the left). Get the histogram transformation function and make the output image with the specified histogram, listed in the table on the right.

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

z_q	Specified $p_z(z_q)$
$z_0 = 0$	0.00
$z_1 = 1$	0.00
$z_2 = 2$	0.00
$z_3 = 3$	0.15
$z_4 = 4$	0.20
$z_5 = 5$	0.30
$z_6 = 6$	0.20
$z_7 = 7$	0.15

Example: Histogram Matching

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

Obtain the scaled histogram-equalized values,

$$s_0 = 1, s_1 = 3, s_2 = 5, s_3 = 6, s_4 = 7,$$
$$s_5 = 7, s_6 = 7, s_7 = 7.$$

Compute all the values of the transformation function G,

$$G(z_0) = 7\sum_{j=0}^{0} p_z(z_j) = 0.00 \rightarrow 0$$

$$G(z_1) = 0.00 \rightarrow 0 \qquad G(z_2) = 0.00 \rightarrow 0$$
$$G(z_3) = 1.05 \rightarrow 1 \qquad G(z_4) = 2.45 \rightarrow 2$$
$$G(z_5) = 4.55 \rightarrow 5 \qquad G(z_6) = 5.95 \rightarrow 6$$
$$G(z_7) = 7.00 \rightarrow 7$$

z_q	Specified $p_z(z_q)$	Actual $p_z(z_k)$
$z_0 = 0$	0.00	0.00
$z_1 = 1$	0.00	0.00
$z_2 = 2$	0.00	0.00
$z_3 = 3$	0.15	0.19
$z_4 = 4$	0.20	0.25
$z_5 = 5$	0.30	0.21
$z_6 = 6$	0.20	0.24
$z_7 = 7$	0.15	0.11

Example: Histogram Matching

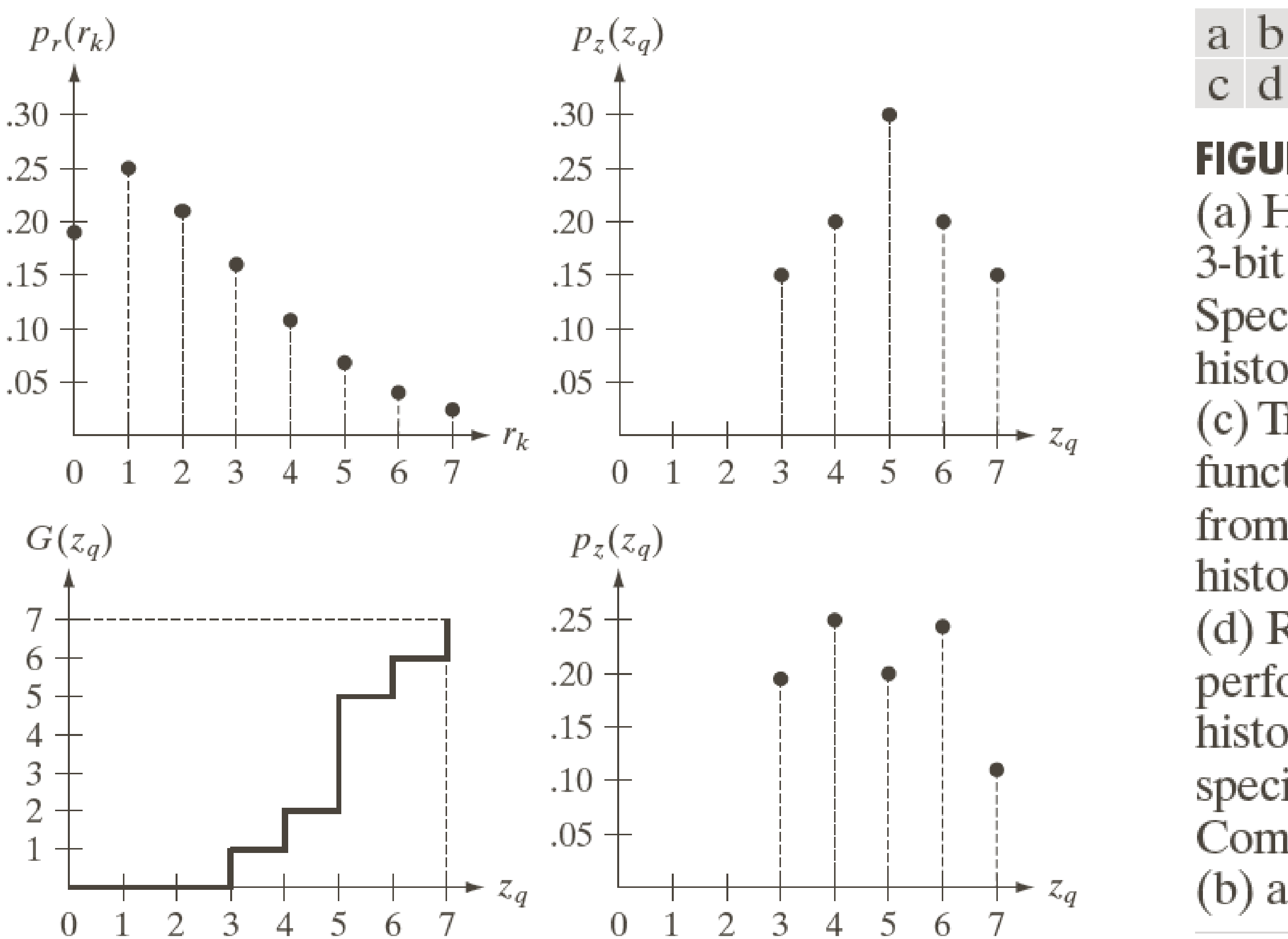


FIGURE 3.22 (a) Histogram of a 3-bit image. (b) Specified histogram. (c) Transformation function obtained from the specified histogram. (d) Result of performing histogram specification. Compare (b) and (d).

Example: Histogram Matching

Obtain the scaled histogram-equalized values,

$$s_0 = 1, s_1 = 3, s_2 = 5, s_3 = 6, s_4 = 7,$$
$$s_5 = 7, s_6 = 7, s_7 = 7.$$

Compute all the values of the transformation function G,

$$G(z_0) = 7\sum_{j=0}^{0} p_z(z_j) = 0.00 \rightarrow 0$$

$G(z_1) = 0.00 \rightarrow 0$ $\quad$ $G(z_2) = 0.00 \rightarrow 0$

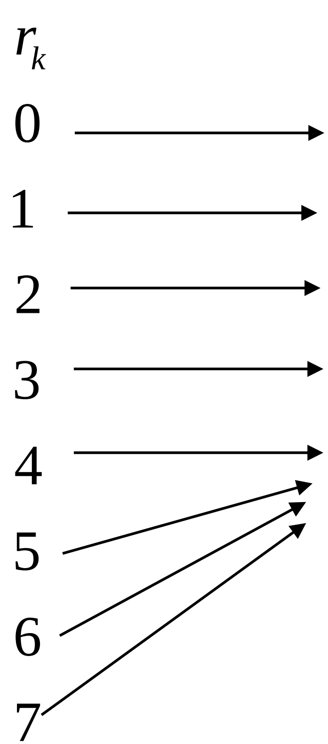
$G(z_3) = 1.05 \rightarrow 1$ $\quad$ $\mathbf{s_0}$ $\quad$ $G(z_4) = 2.45 \rightarrow 2$ $\quad$ $\mathbf{s_1}$

$G(z_5) = 4.55 \rightarrow 5$ $\quad$ $\mathbf{s_2}$ $\quad$ $G(z_6) = 5.95 \rightarrow 6$ $\quad$ $\mathbf{s_3}$

$G(z_7) = 7.00 \rightarrow 7$ $\quad$ $\mathbf{s_4}$ $\quad$ $\mathbf{s_5}$ $\quad$ $\mathbf{s_6}$ $\quad$ $\mathbf{s_7}$

Example: Histogram Matching

$$s_0 = 1, s_1 = 3, s_2 = 5, s_3 = 6, s_4 = 7,$$
$$s_5 = 7, s_6 = 7, s_7 = 7.$$



s_k	$\rightarrow$	z_q
1	$\rightarrow$	3
3	$\rightarrow$	4
5	$\rightarrow$	5
6	$\rightarrow$	6
7	$\rightarrow$	7

Example: Histogram Matching

$$r_k \rightarrow z_q$$

$$0 \rightarrow 3$$

$$1 \rightarrow 4$$

$$2 \rightarrow 5$$

$$3 \rightarrow 6$$

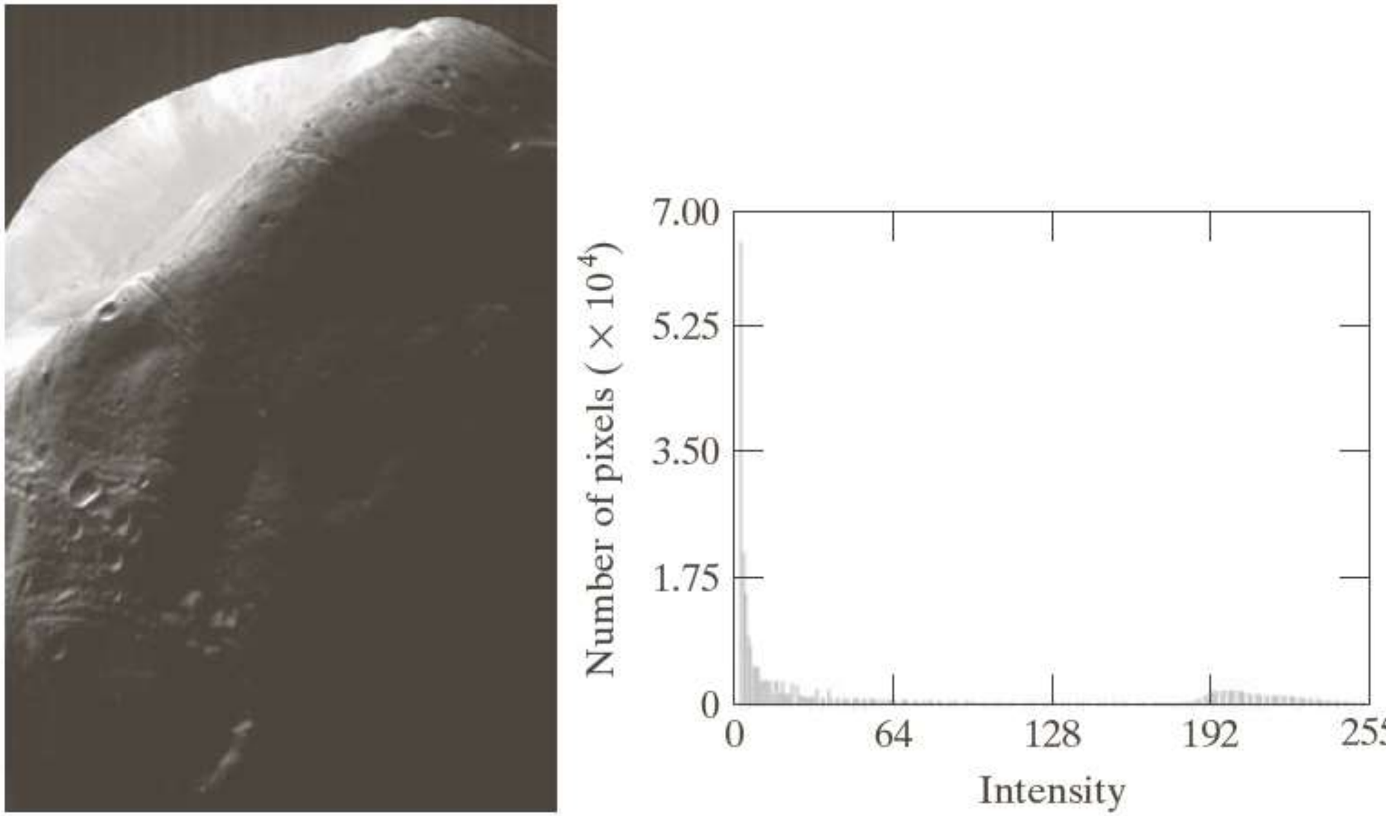
$$4 \rightarrow 7$$

$$5 \rightarrow 7$$

$$6 \rightarrow 7$$

$$7 \rightarrow 7$$

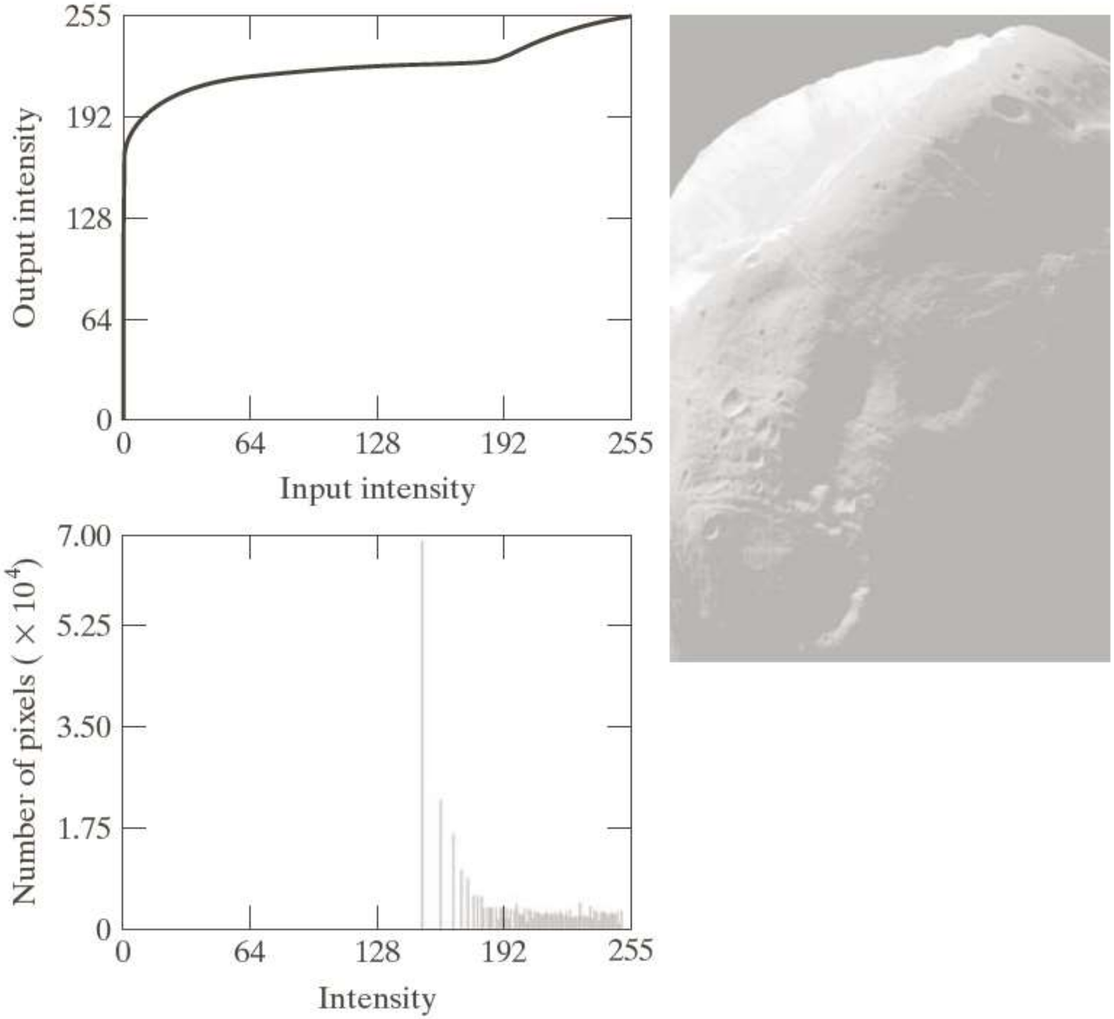
Example: Histogram Matching



a b

FIGURE 3.23 (a) Image of the Mars moon Phobos taken by NASA's *Mars Global Surveyor.* (b) Histogram. (Original image courtesy of NASA.)

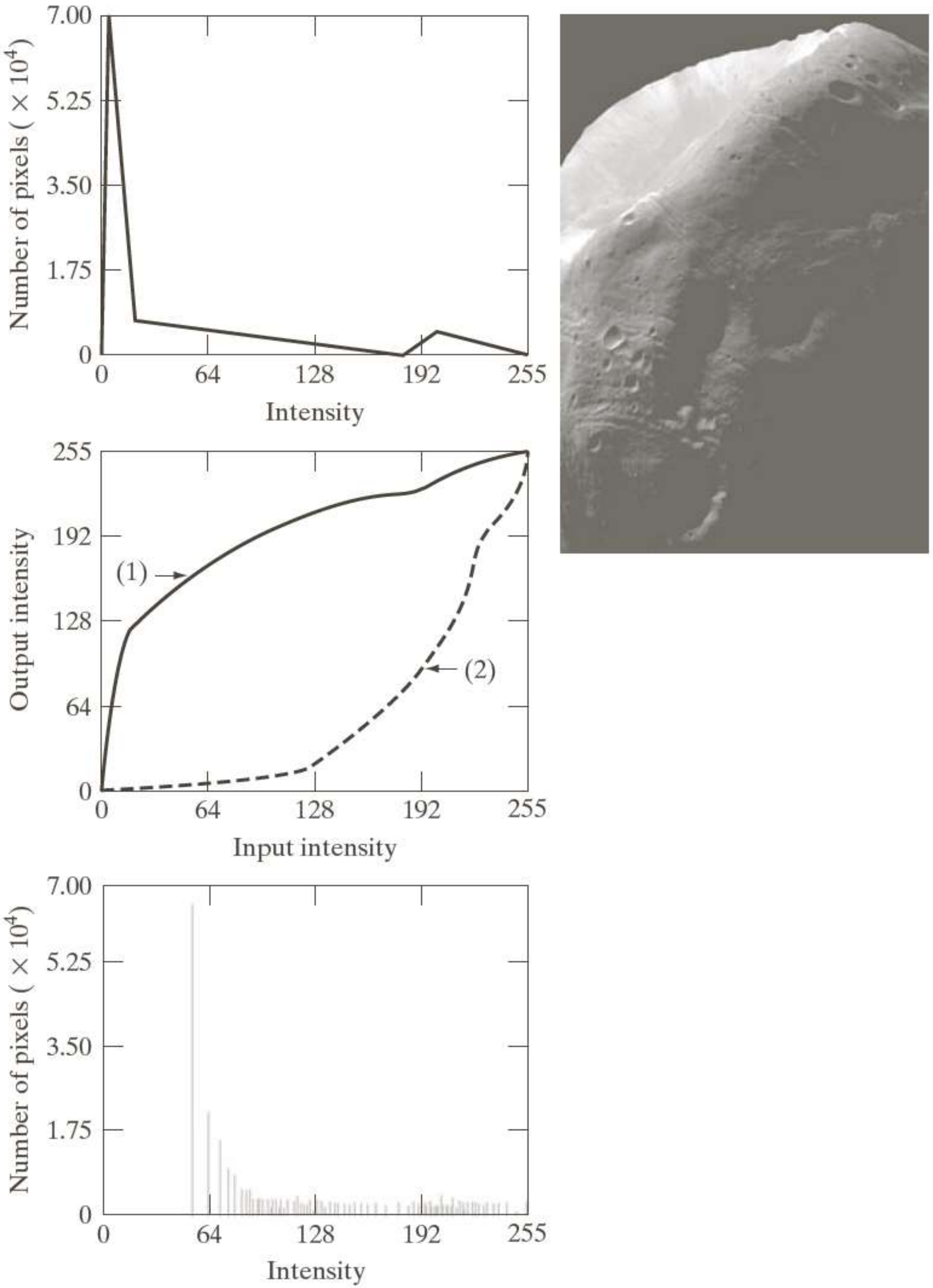
Example: Histogram Matching



a b
c

FIGURE 3.24
(a) Transformation function for histogram equalization.
(b) Histogram-equalized image (note the washed-out appearance).
(c) Histogram of (b).

Ex



a c
b
d

FIGURE 3.25
(a) Specified histogram.
(b) Transformations.
(c) Enhanced image using mappings from curve (2).
(d) Histogram of (c).

Local Histogram Processing

Define a neighborhood and move its center from pixel to pixel

At each location, the histogram of the points in the neighborhood is computed. Either histogram equalization or histogram specification transformation function is obtained

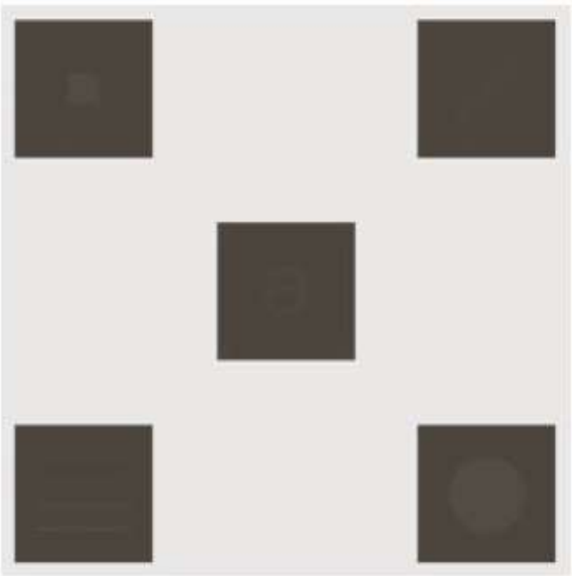
Map the intensity of the pixel centered in the neighborhood

Move to the next location and repeat the procedure

Local Histogram Processing

- The two histogram processing methods discussed:
  - Global
    - pixels are modified by a transformation function based on the gray-level content of an entire image.
- Global approach is suitable for overall enhancement
  - there are cases in which it is necessary to enhance details over small areas in an image

Local Histogram Processing: Example



a b c

FIGURE 3.26 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization applied to (a), using a neighborhood of size 3×3.

Using Histogram Statistics for Image Enhancement

Average Intensity

$$m = \sum_{i=0}^{L-1} r_i \, p(r_i) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y)$$

$$u_n(r) = \sum_{i=0}^{L-1} (r_i - m)^n p(r_i)$$

Variance

$$\sigma^2 = u_2(r) = \sum_{i=0}^{L-1} (r_i - m)^2 p(r_i) = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [f(x,y) - m]^2$$

Using Histogram Statistics for Image Enhancement

Local average intensity

$$m_{s_{xy}} = \sum_{i=0}^{L-1} r_i \, p_{s_{xy}}(r_i)$$

s_{xy} denotes a neighborhood

Local variance

$$\sigma_{s_{xy}}^2 = \sum_{i=0}^{L-1} (r_i - m_{s_{xy}})^2 \, p_{s_{xy}}(r_i)$$

Using Histogram Statistics for Image Enhancement: Example

$$g(x,y)=\begin{cases} E\,g f(x,y), & \text{if } m_{s_{xy}} \le k_0 m_G \text{ and } k_1\sigma_G \le \sigma_{s_{xy}} \le k_2\sigma_G \\ f(x,y), & \text{otherwise} \end{cases}$$

m_G : global mean; σ_G : global standard deviation

$k_0 = 0.4;\ \ k_1 = 0.02;\ k_2 = 0.4;\ E = 4$

a b c

FIGURE 3.27 (a) SEM image of a tungsten filament magnified approximately 130×. (b) Result of global histogram equalization. (c) Image enhanced using local histogram statistics. (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

Enhancement Using Arithmetic/Logic Operations

- Arithmetic/logic operations involving images are performed on a pixel-by-pixel basis between two or more images (this excludes the logic operation NOT, which is performed on a single image).
- The difference between two images f(x, y) and h(x, y), expressed as
- $G(x,y)=f(x,y)-h(x,y)$

Basics of Spatial Filtering

- The subimage is called a filter, mask,
- kernel, template, or window.
- The values in a filter subimage are referred to as coefficients , rather than pixels.
- Smoothing spatial filters