$y(x) = Ax^2 + Bx + C$

Independent $\rightarrow x$

Dependent $\rightarrow y$

$\frac{dy}{dx}$

Ordinary derivative

$z = f(x, y) = x^2 + y^2$

more than one independent variable. for this

$$\frac{\partial z}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x, y+\Delta y) - f(x, y+\Delta y)}{\Delta x}$$

Partial derivative

$y = x$

$\Rightarrow \frac{dy}{dx} = 1$

$\Rightarrow \frac{dy}{dx} - 1 = 0$

Ordinary Differential Equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + y = 0$$

or,

$$\frac{d^3y}{dx^3} + 5\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$$

Order: The order of a differential equation is the order of highest ordered derivative involve in the differential equation.

Example:

$$\left(\frac{d^3y}{dx^3}\right)^2 + \frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$$

It is a third-order derivative or differential equation.

Degree: The degree of a differential equation is the power of the highest ordered derivative involve in the differential equation.

Example:

$$\left(\frac{d^3y}{dx^3}\right)^2 + \frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$$

It is a second-degree differential equation.

$$\frac{dy}{dx} = x^2+y \rightarrow \text{First order}$$

$$\frac{dy}{dx} = \frac{x^2+y}{x^3} \Rightarrow \frac{dy}{dx} = f(x,y).$$

$$\Rightarrow (x^2+y)\,dx - x^3\,dy = 0$$

$$M = x^2+y$$

$$N = -x^3$$

$$\Rightarrow M(x,y)\,dx + N(x,y)\,dy = 0$$

General form of first order first degree differential equation.

$$x\,dx + y\,dy = 0$$

$$\Rightarrow \frac{x^2}{2} + \frac{y^2}{2} = A \quad \text{[Integrating]}$$

Here, A is an integral constant.

21-08-2017 : 2E : Monday

Variable separable

$\rightarrow F(x)\,dx + G(y)\,dy = 0$ [Vari. Sepa.]

$\rightarrow F(y)\,dx + G(x)\,dy = 0$ [Vari. Sepa.]

$\Rightarrow F(y)\,dx = -G(x)\,dy$

$\Rightarrow \dfrac{dx}{G(x)} = -\dfrac{dy}{F(y)}$

$\Rightarrow \dfrac{dx}{G(x)} + \dfrac{dy}{F(y)} = 0$

$\rightarrow F(x) f(y)\,dx + G(x)\, g(y)\,dy = 0$ [Vari. Sepa.]

$\Rightarrow \dfrac{F(x)}{G(x)}\,dx + \dfrac{g(y)}{f(y)}\,dy = 0$

$[F(x) + f(y)]\,dx + dy = 0$ [Not Vari. Sepa.]

Exp - 2.8:

Solve the equation:

$(x-4)\,y^4\,dx - x^3(y^2-3)\,dy = 0$; $y(1) = \frac{\pi}{2}$

$\Rightarrow \left(\dfrac{x-4}{x^3}\right)dx - \left(\dfrac{y^2-3}{y^4}\right)dy = 0$

$\Rightarrow \left(\dfrac{1}{x^2} - \dfrac{4}{x^3}\right)dx - \left(\dfrac{1}{y^2} - \dfrac{3}{y^4}\right)dy = 0$

By integrating we get,

$$-\frac{1}{x}+\frac{2}{x^2}+\frac{1}{y}-\frac{1}{y^3}+c=0$$

Here, c is an integrating constant.

Ex-2.9:

Solve the equation

$$x\sin y\,dx+(x^2+1)\cos y\,dy=0$$

$$\Rightarrow \frac{x}{x^2+1}dx+\frac{\cos y}{\sin y}dy=0$$

$$\Rightarrow \frac{1}{2}\left(\frac{2x}{x^2+1}\right)dx+\frac{\cos y}{\sin y}dy=0$$

By integrating we get,

$$\frac{1}{2}\ln(x^2+1)+\ln(\sin y)=\ln c$$

$$\Rightarrow \ln\sqrt{x^2+1}+\ln(\sin y)=\ln c$$

$$\Rightarrow \ln\left(\sin y\sqrt{x^2+1}\right)=\ln c$$

$$\Rightarrow \sin y\sqrt{x^2+1}=c$$

~~⇒ sir~~ using initial condition

$$\sin\frac{\pi}{2}\sqrt{1^2+1}=c$$

$$\Rightarrow c=\sqrt{2}$$

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→ solve the equation:

$$(4x^3+4xy)\,dx + (2x^2+2y)\,dy = 0$$

$$\Rightarrow 4x^3dx + 4xy\,dx + 2x^2dy + 2y\,dy = 0$$

$$\Rightarrow 4x^3dx + d(2x^2y) + 2y\,dy = 0$$

$$\Rightarrow d(x^4) + d(2x^2y) + d(y^2) = 0$$

By integrating,

$$x^4 + 2x^2y + y^2 + c = 0$$

→ Exact differential equation: (First order differential equ^n)

$M\,dx + N\,dy = 0$ is said to be exact differential equation if we can find a function $\psi(x,y) = c$ such that $M\,dx + N\,dy = d\psi = 0$

$$\therefore d\psi = \frac{\partial \psi}{\partial x}dx + \frac{\partial \psi}{\partial y}dy$$

→ Total differentiation:

$$M\,dx + N\,dy = d\psi = \frac{\partial \psi}{\partial x}dx + \frac{\partial \psi}{\partial y}dy$$

If $M = \dfrac{\partial \psi}{\partial x}$	$N = \dfrac{\partial \psi}{\partial y}$
$\Rightarrow \dfrac{\partial M}{\partial y} = \dfrac{\partial^2 \psi}{\partial y \partial x}$	$\Rightarrow \dfrac{\partial N}{\partial x} = \dfrac{\partial^2 \psi}{\partial x \partial y}$

since, $\frac{\partial^2 \psi}{\partial y \partial x} = \frac{\partial^2 \psi}{\partial x \partial y}$

so, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ (condition for exactness)

$\rightarrow (2x^3+4xy).dx + (2x^2+2y)\,dy = 0$

$M = 2x^3 + 4xy$	$N = 2x^2 + 2y$
$\Rightarrow \frac{\partial M}{\partial y} = 0 + \cancel{4y}\, 4x$	$\frac{\partial N}{\partial x} = 4x$

28-08-2017: 3E: Monday:

solve the equation:

$$(3x^2+4xy)\,dx + (2x^2+2y)\,dy = 0$$

solution:

Given that,

$$(3x^2+4xy)\,dx + (2x^2+2y)\,dy = 0 \quad \text{------ (i)}$$

Let, $M = 3x^2 + 4xy$	$\frac{\partial M}{\partial y} = 4x$ ------ (ii
$N = 2x^2 + 2y$	$\frac{\partial N}{\partial x} = 4x$ ------ (

since $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$. So, the equation (i) is exact.

Therefore, we can find a function $\psi(x,y)=0$

such that $\frac{\partial \psi}{\partial x} = 3x^2+4xy$ ---------- (iv)

and $\frac{\partial \psi}{\partial y} = 2x^2+2y$ ---------- (v)

integrating (iv) w.r.t. x we get,

$$\psi = x^3+2x^2y+f(y) \quad \text{----------} \quad \text{(vi)}$$

Here, $f(y)$ is an integrating constant.

Differentiating (vi) w.r. to y partially,

$$\frac{\partial \psi}{\partial y} = 2x^2+f'(y)$$

$$\Rightarrow 2x^2+2y = 2x^2+f'(y)$$

$$\Rightarrow f'(y) = 2y$$

$$\Rightarrow f(y) = y^2+c \text{ [By integrating]}$$

putting the value of f(y) in ~~eq~~ equation (vi)

$$\psi = x^3+2x^2y+y^2+c.$$

$$\rightarrow z = (\overset{\nearrow f(x,y)}{u}, \overset{\nearrow f(x,y)}{v}) = f(x,y)$$

$$z = (u+v, u^2+2v)$$

$u = x+y+z$

$v = x^2+y^2+z^2$

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$\rightarrow z = f(u,v)$ $\quad u = F(x,y)$ & $v = G(x,y)$

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$$

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y} + \frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}$$

$u = F(x)$
$v = G(x)$

$$\frac{dz}{dx} = \frac{\partial z}{\partial u}\cdot\frac{du}{dx} + \frac{\partial z}{\partial v}\cdot\frac{dv}{dx}$$

$\rightarrow z = f(u)$

$u = F(x,y)$

$$\frac{\partial z}{\partial x} = \frac{dz}{du}\cdot\frac{\partial u}{\partial x}$$

$$\frac{\partial z}{\partial y} = \frac{dz}{du}\cdot\frac{\partial u}{\partial y}$$

Example: 2.3:

solve the equation:

$y^2dx + 2xy\,dy = 0$ & $y\,dx + 2x\,dy = 0$

sol: solution:

Given that,

$y^2dx + 2xy\,dy = 0$ ------- (i)

Let,

$M = y^2$ | $\frac{\partial M}{\partial y} = 2y$

$N = 2xy$ | $\frac{\partial N}{\partial x} = 2y$

Since $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ so ~~that~~ the equation (i) is exact.

Given that

$$y\,dx + 2x\,dy = 0 \quad \text{-------- (ii)}$$

Let,

$M = y$ | $\frac{\partial M}{\partial y} = 1$

$N = 2x$ | $\frac{\partial N}{\partial x} = 2$

Since, $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ so the equation (ii) is not exact.

Example → 2.4:

solve the equation:

$$(2x\sin y + y^3 e^x)\,dx + (x^2\cos y + 3y^2 e^x)\,dy = 0$$

solution:

Given that,

$$(2x\sin y + y^3 e^x)\,dx + (x^2\cos y + 3y^2 e^x)\,dy = 0 \quad \text{---- (i)}$$

Let,

$M = 2x\sin y + y^3 e^x$ | $\frac{\partial M}{\partial y} = 2x\cos y + 3y^2 e^x$

$N = x^2\cos y + 3y^2 e^x$ | $\frac{\partial N}{\partial x} = 2x\cos y + 3y^2 e^x$

Since, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$. So, the equation (i) is exact.

Example→2.5:

solve the equation:

$$(3x^2+4xy)\,dx + (2x^2+2y)\,dy = 0$$

Solution:

Given that,

$$(3x^2+4xy)\,dx + (2x^2+2y)\,dy = 0 \quad \text{-------- (i)}$$

Let, $M = 3x^2+4xy$ | $\frac{\partial M}{\partial y} = 4x$

$N = 2x^2+2y$ | $\frac{\partial N}{\partial x} = 4x$

since, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, so, the equation (i) is exact.

Therefore, we can find a function $\psi(x,y)=0$ such that,

$$\frac{\partial \psi}{\partial x} = 3x^2 + 4xy \quad \text{------ (ii)}$$

$$\frac{\partial \psi}{\partial y} = 2x^2 + 2y \quad \text{------ (iii)}$$

integrating (iii) w.r. to y we get,

$$\psi = y^2 + f(x) \quad \text{------------(iv)}$$

Here, $f(x)$ is an integrating constant.

Differentiating (iv) w.r. to x partially,

$$\frac{\partial \psi}{\partial x} = f'(x)$$

$\Rightarrow 3x^2 + 4xy = f'(x)$ - [from equation (ii)]

$\Rightarrow f(x) = x^3 + 2x^2y + c$ [By integrating]

putting the value of $f(x)$ into equation (iv),

$$\psi = y^2 + x^3 + 2x^2y + c.$$

Example → 2·6:

solve the initial value problem,

$$(2x\cos y + 3x^2y)\,dx + (x^3 - x^2\sin y - y)\,dy = 0$$

$$y(0) = 2$$

Solution: Given that,

$$(2x\cos y + 3x^2y)\,dx + (x^3 - x^2\sin y - y)\,dy = 0 \quad \text{----(i)}$$

Let, $M = 2x\cos y + 3x^2y$ | $\frac{\partial M}{\partial y} = 3x^2 - 2x\sin y$

$N = x^3 - x^2\sin y - y$ | $\frac{\partial N}{\partial x} = 3x^2 - 2x\sin y$

Since, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$. so the equation (i) is exact.

Therefore, we can find a function $\psi(x,y) = 0$

such that $\frac{\partial \psi}{\partial x} = 2x\cos y + 3x^2 y$ - - - - - - (ii)

$\frac{\partial \psi}{\partial y} = x^3 - x^2 \sin y - y$ - - - - - - (iii)

integrating (ii) w.r. to x we get,

$\psi = x^2\cos y + x^3 y + f(y)$ - - - - - - - (iv)

Here, $f(y)$ is an integrating constant.

Differentiating (iv) w.r. to y partially

$$\frac{\partial \psi}{\partial y} = -x^2\sin y + x^3 + f'(y)$$

$\Rightarrow x^3 - x^2\sin y - y = -x^2\sin y + x^3 + f'(y)$

$\Rightarrow f'(y) = -y$

$\Rightarrow f(y) = -\frac{y^2}{2} + c$ [By integrating]

putting the value of $f(y)$ in equation (iv)

$\psi = x^2\cos y + x^3 y - \frac{y^2}{2} + c$ - - - - - - - (v)

$\therefore y(0) = 2$. Here, if $x = 0$ then $y = 2$ & $\psi(x,y)$

$$0 = 0 + 0 - \frac{4}{2} + c$$

$$\Rightarrow c = 2$$

putting the vale of c in equation (v)

$$\psi = x^2\cos y + x^3 y + \frac{y^2}{2} + 2$$

→ Linear differential equation: A first order differential equation of the form $\frac{dy}{dx} + p(x)\, y = Q(x)$ is called linear differential equation.

→ Non-linear differential equation: A differential equation is said to be non-linear equation if there exist product of dependent variable and their derivatives and transcendental function of dependent variable.

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + xy^2 = 0$$

Ⅱ → If there is no power on "y" then it is linear differential equation and if there is power on "y" then it is non-linear differential equation.

12-09-2017 : 4E : Tuesday :

→ Linear differential equation :

$$\frac{dy}{dx} + P(x)\, y = Q(x)$$

$$\frac{dy}{dx} = \ln x$$

Independent variable - এর transcendental function

Non-linear differential equation :

$$\frac{dy}{dx} + xy^2 = x^3$$

$$\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + xy = 0$$

$$\frac{dy}{dx} = \ln y,\ \cos y,\ \sin y,\ e^y$$

Dependent variable - এর transcendental function

$$\Rightarrow dy + P(x)\, y\, dx = Q(x)\, dx$$

$$\Rightarrow [P(x)\, y - Q(x)]\, dx + dy = 0$$

Let,

$$M = P(x)\, y - Q(x) \qquad N = 1$$

$$\frac{\partial M}{\partial y} = P(x) \qquad \frac{\partial N}{\partial x} = 0$$

since, $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

so, linear equation is not exact.

→ Integrating factor:

— If a first order differential equation $Mdx + Ndy$ is not exact and if multiplying by a factor μ of the equation becomes exact then the factor μ is called integrating factor of the equation.

→ Linear equation:

$$[P(x)y - Q(x)]\,dx + dy = 0 \quad \text{------------} (i)$$

Let, $\mu(x)$ is an integrating factor of equation (i),

$$\mu(x)[P(x)y - Q(x)]\,dx + \mu(x)\,dy = 0$$

Let,

$$M = \mu(x)P(x)y - \mu(x)Q(x) \qquad \frac{\partial M}{\partial y} = \mu(x)P(x)$$

$$N = \mu(x) \qquad \frac{\partial N}{\partial x} = \mu'(x)$$

since, $\mu(x)$ is an integrating factor,

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\Rightarrow \mu(x)P(x) = \mu'(x)$$

$$\Rightarrow \frac{\mu'(x)}{\mu(x)} = P(x)$$

By integrating we get,

$$\ln \mu(x) = \int p(x)\,dx$$

$$\Rightarrow \mu(x) = e^{\int p(x)\,dx}$$

multiplying (i) with $e^{\int p(x)\,dx}$ we get,

$$e^{\int p(x)dx}\left[p(x)y - Q(x)\right]dx + e^{\int p(x)dx}\,dy = 0$$

$$\Rightarrow e^{\int p(x)dx}\,p(x)y\,dx - e^{\int p(x)dx}\,Q(x)\,dx + e^{\int p(x)dx}\,dy =$$

$$\Rightarrow e^{\int p(x)dx}\,p(x)y\,dx + e^{\int p(x)dx}\,dy = e^{\int p(x)dx}\,Q(x)\,dx$$

$$\Rightarrow d\left(e^{\int p(x)dx}\,y\right) = e^{\int p(x)dx}\,Q(x)\,dx$$ [Grouping Method]

⇒ By integrating we get,

$$e^{\int p(x)dx}\,y = \int\left\{e^{\int p(x)dx}\,Q(x)\,dx\right\} + c$$

$$\boxed{\Rightarrow y = e^{-\int p(x)dx}\int\left\{e^{\int p(x)dx}\,Q(x)\,dx\right\} + c}$$

Here, c is an integrating constant.

This is the required solution of linear equation.

Example → 2.14:

Solve the equation

$$\frac{dy}{dx} + \left(\frac{2x+1}{x}\right) y = e^{-2x}$$

Solution: Given that

$$\frac{dy}{dx} + \left(\frac{2x+1}{x}\right) y = e^{-2x} \quad \text{------- (i)}$$

$$\mu(x) = e^{\int \left(\frac{2x+1}{x}\right) dx}$$

$$\Rightarrow \mu(x) = e^{\int \left(2 + \frac{1}{x}\right) dx}$$

$$= e^{2x + \ln x}$$

$$= e^{2x} \cdot e^{\ln x}$$

$$= xe^{2x}$$

multiplying (i) by xe^{2x} we get,

$$xe^{2x} dy + \frac{2x+1}{x} y \cdot xe^{2x} dx = e^{-2x} xe^{2x} dx$$

$$\Rightarrow d(xe^{2x} y) = x\,dx$$

By integrating we get,

$$xe^{2x} y = \frac{x^2}{2} + c$$

Here, c is an integrating constant.

Example → 2.15:

Solve the initial value problem that consist of the differential equation,

$$(x^2+1)\frac{dy}{dx} + 4xy = x$$

and the initial condition,

$$y(2) = 1$$

Solution:

Given that,

$$(x^2+1)\frac{dy}{dx} + 4xy = x$$

$$\Rightarrow \frac{dy}{dx} + \frac{4xy}{(x^2+1)} = \frac{x}{(x^2+1)}$$ [dividing both side by x^2+1] ------ (i)

from equation (i) we get,

$$\mu(x) = e^{\int\left(\frac{4x}{x^2+1}\right)dx}$$

$$= e^{\int\left(\frac{2x}{x^2+1} + \frac{2x}{x^2+1}\right)dx}$$

$$= e^{\int\left(\frac{2x}{x^2+1}\right)dx + \int\left(\frac{2x}{x^2+1}\right)dx}$$

$$= e^{\ln(x^2+1) + \ln(x^2+1)}$$

$$= e^{\ln(x^2+1)^2}$$

$$= (x^2+1)^2$$

multiplying (i) by $(x^2+1)^2$ we get,

$$(x^2+1)^2\, dy + (x^2+1)\, 4xy\, dx = (x^2+1)\, x\, dx$$

$$\Rightarrow d\{(x^2+1)^2 y\} = (x^3+x)\, dx$$

By integrating we get,

$$(x^2+1)^2 y = \frac{x^4}{4} + \frac{x^2}{2} + c \text{ ------- (ii)}$$

Here, c is an integrating constant,

from initial condition. we get,

if $x=2$ then $y=1$

putting value of x & y in equation (ii)

$$25 = 4+2+c$$

$$\Rightarrow c = 19$$

putting the value of c in equation (ii)

$$(x^2+1)^2 y = \frac{x^4}{4} + \frac{x^2}{2} + 19$$

Example → 2.16:

Consider the differential equation,

$$y^2 dx + (3xy - 1) dy = 0$$

Sol. Solution:

$$y^2 dx + (3xy - 1) dy = 0$$

$$\Rightarrow y^2 + (3xy - 1)\frac{dy}{dx} = 0$$ [dividing both sindes by dx

$$\Rightarrow \frac{y^2}{3xy-1} + \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{y^2}{1-3xy}$$

$$\Rightarrow \frac{dx}{dy} = \frac{1-3xy}{y^2}$$

$$\Rightarrow \frac{dx}{dy} = \frac{1}{y^2} - \frac{3}{y}x$$

$$\Rightarrow \frac{dx}{dy} + \frac{3}{y}x = \frac{1}{y^2} \quad \text{-----} (i)$$

from equation (i) we get,

$$\mu(y) = e^{\int \frac{3}{y} dy}$$

$$= e^{3\ln y}$$

$$= e^{\ln y^3}$$

$$= y^3$$

multiplying (i) by y^3 we gets

$$y^3 dx + 3y^2 x\, dy = y\, dy.$$

$$\Rightarrow d(y^3 x) = y\, dy$$

By integrating we get,

$$y^3 x = \frac{y^2}{2} + c$$ [Here, c is an integrating constant.]

$$\Rightarrow x = \frac{1}{2y} + \frac{c}{y^3}$$

→ Bernoulli Equation:

— A first order differential equation of the form $\frac{dy}{dx} + P(x) y = Q(x) y^n$, $n \neq (0, 1)$ is said to be Bernoulli Equation.

$$\frac{dy}{dx} + P(x) y = Q(x) y^n ; \; n \neq 0, 1$$

It is a Bernoulli Equation.

02-10-2017 : 6E : Monday :

$$\frac{dy}{dx} + p(x)\, y = Q(x)\, y^n$$

It is a linear equation

Integrating factor, $\mu = e^{\int p(x)\,dx}$

$$y^{-n}\frac{dy}{dx} + p(x)\, y^{-n+1} = Q(x)$$

Let, $y^{1-n} = v$

$$\Rightarrow (1-n)\, y^{-n}\frac{dy}{dx} = \frac{dv}{dx}$$

$$\Rightarrow y^{-n}\frac{dy}{dx} = \frac{1}{(1-n)}\frac{dv}{dx}$$

$$\therefore \frac{1}{(1-n)}\frac{dv}{dx} + p(x)\, v = Q(x)$$

$$\Rightarrow \frac{dv}{dx} + (1-n)\, p(x)\, v = (1-n)\, Q(x)$$ [linear equation of v]

$$\mu = e^{\int (1-n)\, p(x)\, dx}$$

Example $\to$ 2.17 :

$$\frac{dy}{dx} + y = xy^3$$

$$\Rightarrow y^{-3}\frac{dy}{dx} + y^{-2} = x \qquad \text{---------- (i)}$$

Let,
$$y^{-2} = v$$

$$\Rightarrow -2y^{-3}\frac{dy}{dx} = \frac{dv}{dx}$$

$$\Rightarrow y^{-3}\frac{dy}{dx} = -\frac{1}{2}\frac{dv}{dx}$$

From equation (i)

$$-\frac{1}{2}\frac{dv}{dx} + v = x$$

$$\Rightarrow \frac{dv}{dx} - 2v = -2x \quad \text{-------------- (ii)}$$

$$\mu = e^{\int -2\,dx} = e^{-2x}$$

multiplying equation (ii) by e^{-2x}

$$e^{-2x}\frac{dv}{dx} - 2ve^{-2x} = -2xe^{-2x}$$

$$\Rightarrow e^{-2x}dv - 2ve^{-2x}dx = -2xe^{-2x}dx$$

$$\Rightarrow d\left(e^{-2x}v\right) = -2xe^{-2x}dx$$

By integrating,

$$e^{-2x}v = \int -2xe^{-2x}dx$$

$$\Rightarrow e^{-2x}v = -2\int xe^{-2x}dx$$

$$\Rightarrow e^{-2x}v = -2\left[x\int e^{-2x}dx - \int \frac{e^{-2x}}{-2}dx\right]$$

$\Rightarrow e^{-2x} v = -2\left[-\frac{xe^{-2x}}{2} - \frac{e^{-2x}}{4}\right] + c$ [Here, c is an integrating constant]

$\Rightarrow e^{-2x} v = xe^{-2x} + \frac{e^{-2x}}{2} + c$

$\Rightarrow e^{-2x} y^{-2} = xe^{-2x} + \frac{e^{-2x}}{2} + c$

$\Rightarrow y^{-2} = x + \frac{1}{2} + ce^{2x}$. Answer.

→ Homogeneous Equation:

- A first order differential equation of the form $\frac{dy}{dx} = x^n \phi\left(\frac{y}{x}\right)$ or $\frac{dy}{dx} = y^n \psi\left(\frac{x}{y}\right)$ is called homogeneous differential equation.

Example: $(x^3 + xy^2)\,dx - (x^2y + xy^2)\,dxy = 0$

- when the power of every term is same or equal, then the equation is homogeneous

$$(x^3+y^3)\,dx - (x^2y+xy^2)\,dy = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^3+y^3}{x^2y+xy^2}$$

$$= \frac{1+\left(\frac{y}{x}\right)^3}{\frac{y}{x}+\left(\frac{y}{x}\right)^2}$$

$$= f\left(\frac{y}{x}\right)$$

if, $\frac{y}{x} = v$

$$\therefore f(v) = \frac{1+v^3}{v+v^2}$$

$$\frac{dy}{dx} = g\left(\frac{y}{x}\right)$$

$$\Rightarrow v + x\frac{dv}{dx} = g(v)$$

$$\Rightarrow x\frac{dv}{dx} = g(v) - v$$

$$\Rightarrow \int \frac{dv}{g(v)-v} = \int \frac{dx}{x}$$

As, $\frac{y}{x} = v$

$$\Rightarrow y = vx$$

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

Example → 2.10:

$$\frac{dy}{dx} = \frac{3y^2 - x^2}{2xy}$$

~~Let~~,

$$\Rightarrow \frac{dy}{dx} = \frac{\cancel{y}x^2\left(3\frac{y^2}{x^2} - 1\right)}{x^2 \cdot 2\frac{y}{x}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{3\left(\frac{y}{x}\right)^2 - 1}{2\left(\frac{y}{x}\right)}$$

Let,

$$\frac{y}{x} = v$$

$$\Rightarrow y = vx \quad \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\therefore \; v + x\frac{dv}{dx} = \frac{3v^2-1}{2v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{3v^2-1}{2v} - v$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v^2-1}{2v}$$

$$\Rightarrow \int \frac{2v\,dv}{v^2-1} = \int \frac{dx}{x}$$

$\Rightarrow \ln(v^2-1) = \ln x + \ln c$ [By integrating & c is an integrating constant

$$\Rightarrow v^2 - 1 = cx.$$

$\Rightarrow \dfrac{y^2}{x^2} - 1 = cx$ Answer.

⊕

Example → 2.11

$$\left(y + \sqrt{x^2+y^2}\right)dx - x\,dy = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{y + \sqrt{x^2+y^2}}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x\cdot\frac{y}{x} + x\sqrt{1+\frac{y^2}{x^2}}}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x} + \sqrt{1+\left(\frac{y}{x}\right)^2} \quad \text{- - - - (i)}$$

Let,

$$\frac{y}{x} = v$$

$$\Rightarrow y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

From equation (i)

$$\therefore v + x\frac{dv}{dx} = v + \sqrt{1-v^2}$$

$$\Rightarrow x\frac{dv}{dx} = \sqrt{1-v^2}$$

$$\Rightarrow \int \frac{dv}{\sqrt{1-v^2}} = \int \frac{dx}{x}$$

$$\Rightarrow \sin^{-1} v = \ln x + \ln c$$ [By integrainting. Here c is an integrating constant]

$$\Rightarrow v = \sin(\ln cx)$$

$$\Rightarrow \frac{y}{x} = \sin(\ln cx)$$

$$\Rightarrow y = x\sin(\ln cx)$$

Example → 2.128

$$(x^2 - 3y^2)\,dx + 2xy\,dy = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-x^2 + 3y^2}{2xy}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2\left(-1 + 3\frac{y^2}{x^2}\right)}{x^2\, 2\frac{y}{x}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-1 + 3\left(\frac{y}{x}\right)^2}{2\left(\frac{y}{x}\right)} \quad \text{------ (i)}$$

Let,

$$\frac{y}{x} = v$$

$$\Rightarrow y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

From equation (i)

$$v + x\frac{dv}{dx} = \frac{-1+3v^2}{2v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{-1+3v^2}{2v} - v$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v^2-1}{2v}$$

$$\Rightarrow \int \frac{2v\,dv}{v^2-1} = \int \frac{dx}{x}$$

$$\Rightarrow \ln(v^2-1) = \ln(x) + \ln(c)$$

[By integrating. Here, c is an integrating consta

$$\Rightarrow v^2 - 1 = cx$$

$$\Rightarrow \frac{y^2}{x^2} - 1 = cx$$

$$\Rightarrow y^2 - x^2 = cx^3$$ Answer.

Example → 2.13:

$$\left(y+\sqrt{x^2+y^2}\right)dx - x\,dy = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{y+\sqrt{x^2+y^2}}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x\cdot\frac{y}{x} + x\sqrt{1+\left(\frac{y}{x}\right)^2}}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x} + \sqrt{1+\left(\frac{y}{x}\right)^2} \quad \text{------------} \quad (i)$$

Given that, initial condition, $y(1)=0$

Let, $\frac{y}{x} = v$

$$\Rightarrow y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

From equation (i),

$$v + x\frac{dv}{dx} \cancel{+} = v + \sqrt{1+v^2}$$

$$\Rightarrow x\frac{dv}{dx} = \sqrt{1+v^2}$$

$$\Rightarrow \int \frac{dv}{\sqrt{1+v^2}} = \int \frac{dx}{x}$$

$$\Rightarrow \ln\left(v+\sqrt{1+v^2}\right) = \ln x + \ln c$$

[By integrating. Here c is an integrating constant]

$$\Rightarrow v + \sqrt{1+v^2} = cx$$

$$\Rightarrow \frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}} = cx$$

$$\Rightarrow y + \sqrt{y^2 + x^2} = cx^2$$

Now, if $x = 1$ then $y = 0$

$$0 + \sqrt{0 + 1} = c$$

$$\Rightarrow c = 1$$

$$\therefore y + \sqrt{y^2 + x^2} = x^2 .$$ Answer.

Ex-13:

$$\left(x^3 + y^2\sqrt{x^2+y^2}\right)dx - xy\sqrt{x^2+y^2}\cdot dy = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^3 + y^2\sqrt{x^2+y^2}}{xy\sqrt{x^2+y^2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{x^3}{\sqrt{x^2+y^2}} + y^2}{xy}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{x^2}{\sqrt{1+(\frac{y}{x})^2}} + y^2}{x^2\cdot\frac{y}{x}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2\left(\frac{1}{\sqrt{1+(\frac{y}{x})^2}} + \frac{y^2}{x^2}\right)}{x^2\cdot\frac{y}{x}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{1}{\sqrt{1+(\frac{y}{x})^2}} + (\frac{y}{x})^2}{\frac{y}{x}} \quad \text{-------- (i)}$$

Let, $\frac{y}{x} = v$

$$\Rightarrow y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

From equation (i)

$$v + x\frac{dv}{dx} = \frac{\frac{1}{\sqrt{1+v^2}} + v^2}{v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{\frac{1}{\sqrt{1+v^2}} + v^2}{v} - v$$

$$\Rightarrow x\frac{dv}{dx} = \frac{\frac{1}{\sqrt{1+v^2}} + v^2 - v^2}{v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1}{v\sqrt{1+v^2}}$$

$$\Rightarrow (v\sqrt{1+v^2})\,dv = \frac{dx}{x}$$

$$\cancel{\Rightarrow \int(\sqrt{v^2+v^4})\,dv = \int\frac{dx}{x}}$$

$$\Rightarrow \frac{1}{2}\int 2v\sqrt{1+v^2}\,dv = \frac{dx}{x} \quad \text{[By integrating]}$$

$$\Rightarrow \frac{1}{2}\times\frac{2}{3}\times(1+v^2)^{\frac{3}{2}} = \ln x + \ln c \quad \text{[Here, c is an integrating constant]}$$

$$\Rightarrow \frac{1}{3}\left(1+\frac{y^2}{x^2}\right)^{\frac{3}{2}} = \ln cx$$

$$\Rightarrow (x^2+y^2)^{\frac{3}{2}} = 3x^3 \ln cx.$$

~~Examp~~

09-10-2017 : 7E : Monday

Example → 2.18:

Consider the differential equation

$$(2x^2+y)\,dx + (x^2y - x)\,dy = 0 \quad ----- (i)$$

Let,

$M = (2x^2+y)$ | $N = (x^2y - x)$

$\frac{\partial M}{\partial y} = 1$ | $\frac{\partial N}{\partial x} = 2xy - 1$

$$\therefore \frac{1}{(x^2y-x)}\left[1-(2xy-1)\right] = \frac{2-2xy}{x^2y-x} = -\frac{2(1-xy)}{x(1-xy)}$$

$$= -\frac{2}{x}$$

This depends on x. So,

$$\therefore \exp\left(-\int \frac{2}{x}dx\right) = \exp(-2\ln x) = \exp\left(\ln\frac{1}{x^2}\right) = \frac{1}{x^2}$$

is an integrating factor of equation (i).
Multiplying ~~a~~ equation (i) by this integrating factor, we get,

$$\left(2+\frac{y}{x^2}\right)dx + \left(y-\frac{1}{x}\right)dy = 0$$

By integrating

$$2x - \frac{2y}{x^3} + \frac{y^2}{2} - \frac{y}{x} = c$$

The equation $\frac{dy}{dx} = A(x)y^2 + B(x)y + C(x)$ is called Riccate's equation.

a) Show that if $A(x) = 0$ for all x the equation is a linear equation where as if $C(x) = 0$ for all x the equation is a Bernoulli equation.

b) Show that if f is any solution then the transformation $y = f + \frac{1}{v}$ reduces the equation to linear equation of v.

Solution : b

Given, $$\frac{dy}{dx} = A(x)y^2 + B(x)y + C(x) \quad \text{------ (i)}$$

If f is any solution of (i) then,

$$\frac{df}{dx} = A(x)f^2 + B(x)f + C(x) \quad \text{------ (ii)}$$

Now,

$$y = f + \frac{1}{v}$$

$$\Rightarrow \frac{dy}{dx} = \frac{df}{dx} - \frac{1}{v^2}\frac{dv}{dx}$$

from equation (i),

$$\frac{df}{dx} - \frac{1}{v^2}\frac{dv}{dx} = A(x)\left(f + \frac{1}{v}\right)^2 + B(x)\left(f + \frac{1}{v}\right) + C(x)$$

~~te~~

$$\Rightarrow \frac{df}{dx} - \frac{1}{v^2}\frac{dv}{dx} = A(x) f^2 + 2A(x)\frac{f}{v} + A(x)\frac{1}{v^2} + B(x)\,f + B(x)\frac{1}{v} + C(x)$$

$$\Rightarrow \frac{df}{dx} - \frac{1}{v^2}\frac{dv}{dx} = A(x) f^2 + B(x) f + C(x) + 2A(x)\frac{f}{v} + \frac{A(x)}{v^2} + \frac{B(x)}{v}.$$

$$\Rightarrow \frac{df}{dx} - \frac{1}{v^2}\frac{dv}{dx} = \frac{df}{dx} + 2A(x)\frac{f}{v} + \frac{A(x)}{v^2} + \frac{B(x)}{v}$$

$$\Rightarrow -\frac{1}{v^2}\frac{dv}{dx} = 2A(x)\frac{f}{v} + \frac{A(x)}{v^2} + \frac{B(x)}{v}$$

$$\Rightarrow -\frac{dv}{dx} = 2A(x) f v + A(x) + B(x) v$$

$$\Rightarrow \frac{dv}{dx} + 2A(x) f v + B(x) v = -A(x)$$

$$\Rightarrow \frac{dv}{dx} + [2A(x) f + B(x)]\, v = -A(x)$$

it is a linear equation.

23-10-2017 : 8E : Monday :

Higher order ~~ordinary~~ differential equation

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$$

$$\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$$

$$\frac{d^3y}{dx^3} + y = 0$$

$$x\frac{d^2y}{dx^2} + y\frac{dy}{dx} + xy = 0$$

Solve:

$\rightarrow a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$

~~Lets~~ $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$ ----------------- (i)

Let, $y = e^{mx}$ be a trial solution of equation (i)

$$\frac{dy}{dx} = me^{mx}$$

$$\frac{d^2y}{dx^2} = m^2e^{mx}$$

From equation (i)

$$am^2e^{mx} + bme^{mx} + ce^{mx} = 0$$

$$\Rightarrow (am^2 + bm + c)e^{mx} = 0$$

$$\Rightarrow am^2 + bm + c = 0 \quad [\because e^{mx} \neq 0]$$

$$m = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$\therefore$ If $m \neq m_1 \neq m_2$ (real vale of m)

$\therefore y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$

If $m = m_1 = m_2$

$\therefore y = c_1 e^{mx} + c_2 e^{mx} x$

$= e^{mx}(c_1 + c_2 x)$

If, $m = m_1$ is conjucate of m_2 (imaginary vale of m)

$= a \pm ib$

$\therefore y = e^{ax}(c_1 \cos bx + c_2 \sin bx)$

→ Solve : $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$

Solution: $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$ ------- (i)

Let, $y = e^{mx}$ be a trial solution of equation (i)

$\therefore \frac{dy}{dx} = me^{mx}$

and $\frac{d^2y}{dx^2} = m^2e^{mx}$

From equation (i),

$m^2e^{mx} - 5me^{mx} + 6e^{mx} = 0$

$\Rightarrow m^2 - 5m + 6 = 0$ $[\because e^{mx} \neq 0]$

$\therefore m = 3, 2$

$\therefore y = c_1 e^{3x} + c_2 e^{2x}$

It is a general ~~equation~~. solution.

→ 5. Solve : $2y'' + y' - 6y = 0$

Solution : $2y'' + y' - 6y = 0$ ----------- (i)

Let, $y = e^{mx}$ be a trial solution of equation (i)

$\therefore$ The Auxiliary equation of equation (i) is

$2m^2 + m - 6 = 0$

$\Rightarrow (2m-3)(m+2) = 0$

$\therefore m = \frac{3}{2}, -2$

The required solution is

$y = c_1 e^{\frac{3x}{2}} + c_2 e^{-2x}$

28-10-2017 : 9C : Saturday

→ 19. Solve : $y'' - 4y' + 13y = 0$

Solution :

Given,

$$y'' - 4y' + 13y = 0 \text{ - - - - - - - } (i)$$

Let, $y = e^{mx}$ be a trial solution of equation (i)

The auxiliary equation of equation (i) is

$$m^2 - 4m + 13 = 0$$

$$\Rightarrow m = \frac{4 \pm \sqrt{16 - (4 \times 13)}}{2 \cdot 1} = \frac{4 \pm \sqrt{-36}}{2} = \frac{4 \pm 6i}{2}$$

$$= 2 \pm 3i$$

∴ The general solution is

$$y = e^{2x}(c_1 \cos 3x + c_2 \sin 3x)$$

→ 21. Solve : $y'' + 9y = 0$

Solution:

$$y'' + 9y = 0 \text{ - - - - - - - - } (i)$$

Let, $y = e^{mx}$ be a trial solution

The auxiliary ~~solution~~ equation of equation (i)

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$m^2+9=0$

$\Rightarrow m^2=-9$

$\Rightarrow m=\pm 3i$

The general solution is

$$y=c_1\cos 3x+c_2\sin 3x$$

11. Solve : $y'''-3y''-y'+3y=0$

Solution :

$$y'''-3y''-y'+3y=0 \quad \text{------} \quad (i)$$

Let, $y=e^{mx}$ be a trial solution of equation

The auxiliary equation of equation (i) is

$m^3-3m^2-m+3=0$

$\Rightarrow m^3-m^2-2m^2+2m-3m+3=0$

$\Rightarrow m^2(m-1)-2m(m-1)-3(m-1)=0$

$\Rightarrow (m-1)(m^2-2m-3)=0$

$\Rightarrow (m-1)(m-3)(m+1)=0$

$\therefore m=1, 3, -1$

The general solution

$$y=c_1e^{x}+c_2e^{3x}+c_3e^{-x}$$

→37. Solve: $y'' - y' - 12y = 0$, $y(0) = 3$, $y'(0) = 5$

Solution:

Given,

$$y'' - y' - 12y = 0 \text{ - - - - - - - - - (i)}$$

$$y(0) = 3, \; y'(0) = 5$$

Let, $y = e^{mx}$ be a trial solution of (i)

The auxiliary equation of equation (i) is

$$m^2 - m - 12 = 0$$

$$\Rightarrow (m-4)(m+3) = 0$$

$$\therefore m = 4, -3$$

The general solution of equation (i)

$$y = c_1 e^{4x} + c_2 e^{-3x}$$

If, $y(0) = 3$

$$3 = c_1 + c_2 \text{ - - - - - - - - - (ii)}$$

If $y'(0) = 5$ $\therefore y' = 4c_1 e^{4x} - 3c_2 e^{-3x}$

$$5 = 4c_1 - 3c_2 \text{ - - - - - - - - - (iii)}$$

solving (ii) & (iii)

$$c_1 = 2, \; c_2 = 1$$

$$\therefore y = 2e^{4x} + e^{-3x}$$

39. Solve: $y'' - 6y' + 8y = 0$, $y(0) = 1$, $y'(0) = 6$

Solution:

Given, $y'' - 6y' + 8y = 0$ - - - - - - - - - (i)

$y(0) = 1$, $y'(0) = 6$

Let, $y = e^{mx}$ be a trial solution of equation (i)

the auxiliary equation of equation (i)

$\cancel{m^2 - m - 12 = 0}$ $\quad m^2 - 6m + 8 = 0$

$\cancel{\Rightarrow m}$ $\quad \Rightarrow m^2 - 4m - 2m + 8 = 0$

$\Rightarrow (m-4)(m-2) = 0$

$\therefore m = 4, 2$

The general solution of equation (i)

$$y = c_1 e^{4x} + c_2 e^{2x}$$

If, $y(0) = 1$

$\therefore 1 = c_1 + c_2$ - - - - - - - (ii)

If, $y'(0) = 6$ $\quad \because y' = 4c_1 e^{4x} + 2c_2 e^{2x}$

$\therefore 6 = 4c_1 + 2c_2$ - - - - - - - - - (iii)

solving (ii) & (iii)

$c_1 = 2$, $c_2 = -1$

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39. Solve: $y'' - 6y' + 8y = 0$, $y(0) = 1$, $y'(0) = 6$

Solution:

Given, $y'' - 6y' + 8y = 0$ - - - - - - - - (i)

$y(0) = 1$, $y'(0) = 6$

Let, $y = e^{mx}$ be a trial solution of equation (i)

the auxiliary equation of equation (i)

$\cancel{m^2 - m - 12 = 0}$ $m^2 - 6m + 8 = 0$

$\cancel{\Rightarrow m}$ $\Rightarrow m^2 - 4m - 2m + 8 = 0$

$\Rightarrow (m-4)(m-2) = 0$

$\therefore m = 4, 2$

The general solution of equation (i)

$$y = c_1 e^{4x} + c_2 e^{2x}$$

If, $y(0) = 1$

$\therefore 1 = c_1 + c_2$ - - - - - - - (ii)

If, $y'(0) = 6$ $\because y' = 4c_1 e^{4x} + 2c_2 e^{2x}$

$\therefore 6 = 4c_1 + 2c_2$ - - - - - - - - - (iii)

solving (ii) & (iii)

$c_1 = 2$, $c_2 = -1$

$\therefore y = 2e^{4x} - e^{2x}$

→40. Solve: $3y'' + 4y' - 4y = 0$, $y(0) = 2$, $y'(0) = -4$

Solution:

Given,

$$3y'' + 4y' - 4y = 0$$

$$y(0) = 2, \quad y'(0) = -4$$

Let,

$y = e^{mx}$ be a trial solution of equation (i)

the auxiliary equation of equation (i) is

$$3m^2 + 4m - 4 = 0$$

$$\Rightarrow 3m^2 + 6m - 2m - 4 = 0$$

$$\Rightarrow 3m(m+2) - 2(m+2) = 0$$

$$\Rightarrow (m+2)(3m-2) = 0$$

$$\therefore m = -2, \frac{2}{3}$$

The general solution of equation (i)

$$y = c_1 e^{-2x} + c_2 e^{\frac{2x}{3}}$$

If, $y(0) = 2$

$\therefore 2 = c_1 + c_2$ ------------ (ii)

If, $y'(0) = -4$ $\quad \because y' = -2c_1 e^{-2x} + \frac{2}{3}c_2 e^{\frac{2x}{3}}$

$\therefore -4 = -2c_1 + \frac{2}{3}c_2$ ------------ (iii)

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~~(iii)~~ solving (ii) & (iii)

$c_1 = 2$, $c_2 = 0$

$y = 2e^{-2x}$

→ ~~44~~. 43. Solve: $y'' + 4y' + 4y = 0$, $y(0) = 3$, $y'(0) = 7$

Solution:

Given, $y'' + 4y' + 4y = 0$ ---------- (i)

$y(0) = 3$, $y'(0) = 7$

Let, $y = e^{mx}$ be a trial solution of equation (i)

the auxiliary equation of equation (i) is

$$m^2 + 4m + 4 = 0$$

$$\Rightarrow m^2 + 2m + 2m + 4 = 0$$

$$\Rightarrow (m+2)^2 = 0$$

$\therefore m = -2, -2$

$\therefore$ The general solution of equation (i)

$$y = (c_1 + c_2 x) e^{-2x}$$

If, $y(0) = 3$ $\therefore 3 = c_1 + \cancel{c_2}$ ---------- (ii)

If, $y'(0) = 7$ $\therefore y' = -2(c_1 + c_2) e^{-2x} + c_2 e^{-2x}$

$\therefore 7 = -2(c_1 + \cancel{c_2}) + c_2$ ---------- (iii)

~~(iii)~~ solving (ii) & (iii)

$c_1 = 2$, $c_2 = 0$

$y = 2e^{-2x}$

~~44~~ 43. Solve: $y'' + 4y' + 4y = 0$, $y(0) = 3$, $y'(0) = 7$

Solution: Given, $y'' + 4y' + 4y = 0$ ------------ (i)

$y(0) = 3$, $y'(0) = 7$

Let, $y = e^{mx}$ be a trial solution of equation (i)

the auxiliary equation of equation (i) is

$m^2 + 4m + 4 = 0$

$\Rightarrow m^2 + 2m + 2m + 4 = 0$

$\Rightarrow (m+2)^2 = 0$

$\therefore m = -2, -2$

$\therefore$ The general solution of equation (i)

$y = (c_1 + c_2 x)e^{-2x}$

If, $y(0) = 3$ $\therefore 3 = c_1$ ---------- (ii)

If, $y'(0) = 7$ $\therefore y' = -2(c_1 + c_2)e^{-2x} + c_2 e^{-2x}$

$\therefore 7 = -2c_1 + c_2$ ------- (iii) solving (ii) & (iii)

~~No solution~~ $c_1 = 3,\ c_2 = 13$

$\therefore$ ~~$y(0) \neq 4$ & or $y'(0) \neq 7$~~ $\therefore y = (3+13x)e^{-2x}$

→ 44. Solve: $9y'' - 6y' + y = 0$, $y(0) = 3$, $y'(0) = -1$

Solution:

Given,

$$9y'' - 6y' + y = 0 \quad \text{--------- (i)}$$

$$y(0) = 3,\ y'(0) = -1$$

Lets $y = e^{mx}$ be the trial solution of equation (i)

the auxiliary equation of equation (i) is

$$9m^2 - 6m + 1 = 0$$

$$\Rightarrow 9m^2 - 3m - 3m + 1 = 0$$

$$\Rightarrow (3m-1)(3m-1) = 0$$

$$\therefore m = \frac{1}{3}, \frac{1}{3}$$

$\therefore$ The general solution of equation (i)

$$y = (c_1 + c_2 x)e^{\frac{x}{3}}$$

If, $y(0) = 3 \therefore 3 = c_1$ --------- (ii)

If, $y'(0) = -1 \therefore y' = \frac{1}{3}(c_1 + c_2 x)e^{\frac{x}{3}} + c_2 e^{\frac{x}{3}}$

$\therefore -1 = \frac{1}{3}c_1 + c_2$ --------- (iii)

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solving (ii) & (iii)

~~No solution~~ $c_1 = 3$ & $c_2 = -2$

~~$\therefore y(0) \neq 2$~~ $\therefore y = (3 - 2x)e^{\frac{x}{3}}$

04-11-2017 : 10C : saturday

$$\frac{d^2y}{dx^2} + \alpha\frac{dy}{dx} + \beta y = e^{ax} / \cos ax / \sin ax / x^n / e^{ax}\cos bx / e^{ax}x^n / : x^n \sin bx$$

$$\frac{dy}{dx} \rightarrow Dy$$

$$\frac{d^2y}{dx^2} \rightarrow D^2y$$

$$\frac{d^3y}{dx^3} \rightarrow D^3y$$

$$(D^2 + \alpha D + \beta)y = f(x)$$

$$\Rightarrow f(D)\, y = f(x)$$

$$\therefore y = \frac{f(x)}{f(D)}$$

$f(D)y = e^{ax}$

particular solution

$$y_p = \frac{e^{ax}}{f(a)} \quad \text{if } f(a) \neq 0$$

$f(D)y = 0$

complementory solution

$$y_c = c_1 f(x) + c_2 f(x)$$

if $f(a) = 0$

$$y_p = x\frac{e^{ax}}{f'(a)} \quad \text{if } f'(a) \neq 0$$

if $f'(a) = 0$

$$y_p = x^2\frac{e^{ax}}{f''(a)} \quad \text{if } f''(a) \neq 0$$

→ Ex-2 :- Solve : $(D^2+4D+3)y = e^{-3x}$

Solution:

Given,

$$(D^2+4D+3)y = e^{-3x} \text{ ------- (i)}$$

The auxiliary equation of corresponding homogeneous equation of equation (i) is

$$D^2+4D+3=0$$

$$\Rightarrow (D+3)(D+1)=0$$

$$\therefore D=-3,-1$$

complementory solution:

$$y_c = c_1 e^{-3x} + c_2 e^{-x}$$

particular solution:

$$y_p = \frac{e^{-3x}}{D^2+4D+3}$$

$$= x\frac{e^{-3x}}{2D+4}$$

Here, from e^{ax} $a=-3$ and putting the value of 'a' in place of 'D' - $D^2+4D+3=0$ that's why "x" multiply and deno. differe

$$= -\frac{xe^{-3x}}{2}$$

$$\therefore y = y_c + y_p$$

$$= c_1 e^{-3x} + c_2 e^{-x} - \frac{xe^{-3x}}{2}$$

→ Ex : 5 :- Solve: $(D^3 - 2D^2 - 5D + 6)\, y = e^{3x}$ (Page-73; Ex-5)

Solution:

Given,

$$(D^3 - 2D^2 - 5D + 6)\, y = e^{3x} \quad \text{------- (i)}$$

The auxiliary equation of corresponding homogeneous equation of equation (i) is

$$D^3 - 2D^2 - 5D + 6 = 0$$

$$\Rightarrow (D-3)(D-1)(D+2) = 0$$

$$\therefore D = 3, 1, -2$$

complementorey solution:

$$y_c = c_1 e^{3x} + c_2 e^{x} + c_3 e^{-2x}$$

Particular solution:

$$y_p = \frac{e^{3x}}{D^3 - 2D^2 - 5D + 6}$$

$$= x \frac{e^{3x}}{3D^2-4D-5}$$

$$= \frac{xe^{3x}}{10}$$

$y = y_c + y_p$

$= c_1 e^{3x} + c_2 e^{x} + c_3 e^{-2x} + \frac{xe^{3x}}{10}$

11-11-2017 : 11C : Saturday :

$f(D)\, y = \cos ax / \sin ax$

$f(D^2)\, y = \cos ax / \sin ax$

$y_p = \frac{\cos ax / \sin ax}{f(-a^2)}$ if $f(-a^2) \neq 0$

if $f(-a^2) = 0$

$y_p = x \frac{\cos ax / \sin ax}{f'(-a^2)}$ if $f'(-a^2) \neq 0$

→ Solve: $(D^2+1)\, y = \cos x$

~~As for Example:~~

A.E. → Auxiliary Equation

A.E. : $D^2 + 1 = 0$

$\Rightarrow D^2 = -1$

$\therefore D = \pm i$

Complimentory solution:

$$y_c = c_1 \cos x + c_2 \sin x$$

Particular solution:

$$y_p = \frac{\cos ax}{D^2+1}$$

$$= x \frac{\cos x}{2D}$$

$$= \frac{x}{2} \frac{1}{D} (\cos x)$$

$$= \frac{x}{2} \sin x$$

Here, only $D^2 = -a^2 = -$ not D

$D \rightarrow$ differentiation

$\frac{1}{D} \rightarrow$ integration

$$y = y_c + y_p$$

$$= c_1 \cos x + c_2 \sin x + \frac{x}{2} \sin x$$

→ Solve: $(D^2 - 5D + 6) y = \cos 2x$

A.E.: $D^2 - 5D + 6 = 0$

$\Rightarrow (D-3)(D-2) = 0$

$\therefore D = 3, 2$

$$y_c = c_1 e^{3x} + c_2 e^{2x}$$

$$y_p = \frac{\cos 2x}{D^2 - 5D + 6} = \frac{\cos 2x}{-4 - 5D + 6} = \frac{\cos 2x}{2 - 5D}$$

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$$y_p = \frac{(2+5D)\cos 2x}{4-25D^2}$$

$$= \frac{(2+5D)\cdot\cos 2x}{4-25(-2^2)}$$

$$= \frac{2\cos 2x + 5D\cos 2x}{104}$$

$$= \frac{2\cos 2x - 10\sin 2x}{104} \quad \text{[Differentiation]}$$

$$= \frac{\cos 2x - 5\sin 2x}{52}$$

$$\therefore y = c_1 e^{3x} + c_2 e^{2x} + \frac{\cos 2x - 5\sin 2x}{52}$$

→ Solve : $(D^2+1)\,y = x^3$

A.E. : $D^2+1=0$

$\Rightarrow D^2 = -1$

$\Rightarrow D = \pm i$

$\therefore y_c = c_1\cos x + c_2 \sin x$

$$y_p = \frac{x^3}{D^2+1}$$

$$= (1+D^2)^{-1}\, x^3$$

$$= (1 - D^2 + D^4 - \cdots\cdots) x^3$$

$$= x^3 - 6x$$

$$y = c_1 \cos x + c_2 \sin x + x^3 - 6x$$

$$\frac{d^n}{dx^n}(x^n) = n!$$

$$\frac{d^{n+1}}{dx^{n+1}}(x^n) = 0$$

$$D \to \frac{d}{dx}$$

$$D^2 \to \frac{d^2}{dx^2}$$

$$1 \to \frac{d^0}{dx^0}$$

→ Solve: $(D^2 + 2D + 2)\, y = x^2$

A.E. : $D^2 + 2D + 2 = 0$

$$\Rightarrow D = \frac{-2 \pm \sqrt{4 - 8}}{2} = \frac{-2 \pm 2i}{2} = -1 \pm i$$

$$y_c = e^{-x}(c_1 \cos x + c_2 \sin x)$$

$$y_p = \frac{x^2}{D^2 + 2D + 2}$$

$$= \frac{x^2}{2\left(1 + D + \frac{D^2}{2}\right)}$$

$$= \frac{1}{2}\left(1 + D + \frac{D^2}{2}\right)^{-1} x^2$$

$$= \frac{1}{2}\left\{1 - D - \frac{D^2}{2} + \left(D + \frac{D^2}{2}\right)^2 - \cdots\right\} x^2$$

$$= \frac{1}{2}\left(1 - D - \frac{D^2}{2} + D^2 + D^3 + \cdots\right) x^2$$

$$= \frac{1}{2}\left(x^2 - 2x + 1 + 2 + 0 + \cdots\cdots\right)$$

$$= \frac{x^2 - 2x + 1}{2}$$

$$\therefore y = e^{-x}\left(c_1 \cos x + c_2 \sin x\right) + \frac{x^2 - 2x + 1}{2}$$

$\rightarrow$ Solve: $(D^2 - 5D + 6)y = xe^{2x} + e^{3x}\cos 4x$

A.E.: $D^2 - 5D + 6 = 0$

$\Rightarrow (D-3)(D-2) = 0$

$\therefore D = 3, 2$

$$y_c = c_1 e^{3x} + c_2 e^{2x}$$

$$y_p = \frac{xe^{2x}}{D^2 - 5D + 6} + \frac{e^{3x}\cos 4x}{D^2 - 5D + 6}$$

$$= e^{2x}\frac{1}{(D+2)^2 - 5(D+2) + 6}x + e^{3x}\frac{\cos 4x}{(D+3)^2 - 5(D+3) + 6}$$

[1st part: $a = 2$ from e^{2x} and $D \rightarrow D + a$
2nd part: $a = 3$ " e^{3x}]

$$= e^{2x}\frac{x}{D^2 - D} + e^{3x}\frac{\cos 4x}{-16 + D}$$

[~~D~~ 2nd part: $D^2 = -a^2 = -4^2 = -16$ ~~from~~
from $\cos 4x$]

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$$\Rightarrow e^{2x}\frac{x}{-D(1-D)}+e^{3x}\frac{(D+16)}{(D-16)(D+16)}\cos 4x$$

$$=-e^{2x}\frac{1}{D}(1-D)^{-1}x+e^{3x}\frac{D+16}{D^2-256}\cos 4x$$

$$=-e^{2x}\frac{1}{D}(1+D+\cdots)x+e^{3x}\frac{D(\cos 4x)+16\cos 4x}{-4^2-256}$$

$$=-e^{2x}\frac{1}{D}(x+1)+e^{3x}\frac{-4\sin 4x+16\cos 4x}{-272}$$

$$=-e^{2x}\left(\frac{x^2}{2}+x\right)-e^{3x}\frac{16\cos 4x-4\sin 4x}{272}$$

$$\therefore y=c_1e^{3x}+c_2e^{2x}-e^{2x}\left(\frac{x^2}{2}+x\right)-e^{3x}\frac{16\cos 4x-4\sin 4x}{272}$$

→ Page-73; Ex-6: Solve: $\frac{d^2y}{dx^2}+4\frac{dy}{dx}+4y=e^{2x}+e^{-2x}$

$$=2\sinh 2x$$

Solution:

A.E.: $D^2+4D+4=0$

$\Rightarrow (D+2)^2=0$

$\Rightarrow D=-2,-2$

$$\therefore y_c=(c_1+c_2x)e^{-2x}$$

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$$y_p = \frac{e^{2x}}{D^2+4D+4} + \frac{e^{-2x}}{D^2+4D+4}$$

$$= \frac{e^{2x}}{4+8+4} + \frac{x\,e^{-2x}}{2D+4}$$

$$= \frac{e^{2x}}{16} + \frac{x^2 e^{-2x}}{2}$$

$$\therefore y = (c_1 + c_2 x)\,e^{-2x} + \frac{e^{2x}}{16} + \frac{x^2 e^{-2x}}{2}$$

→ Page-75: Ex:-9: Solve: $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 2y = e^x + \cos x$

Solution:

A.E.: $D^3 - 3D^2 + 4D - 2 = 0$

$\Rightarrow (D-1)(D^2 - 2D + 2) = 0$

$\therefore D = 1,\ 1 \pm i$

$$\therefore y_c = c_1 e^x + e^x (c_2 \cos x + c_3 \sin x)$$

$$y_p = \frac{1}{D^3 - 3D^2 + 4D - 2} e^x + \frac{1}{D^3 - 3D^2 + 4D - 2} \cos x$$

$$= \frac{x e^x}{3D^2 - 6D + 4} + \frac{1}{(-1)D - 3(-1) \cancel{+4(-1)-2} + 4D - 2} \cos x$$

$$= \cancel{\frac{x e^x}{3(1)^2 - 6(1) + 4}} \cancel{+ \frac{\cos x}{}} = \cancel{\frac{x e^x}{3 - 6 + 4}} \cancel{+ \frac{\cos x}{3D+1}}$$

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$$\cancel{= \frac{xe^x}{3(1)^2-6(1)+4} + \frac{\cos x}{3D+1}} = \frac{xe^x}{3(1)^2-6(1)+4} + \frac{\cos x}{3D+1}$$

$$= xe^x + \frac{(3D-1)}{(9D^2-1)}\cos x$$

$$= xe^x + \frac{3D(\cos x) - \cos x}{9(-1)-1}$$

$$= xe^x + \frac{-(3\sin x + \cos x)}{-10} = xe^x + \frac{3\sin x + \cos x}{10}$$

$$\therefore y = c_1 e^x + e^x(c_2\cos x + c_3 \sin x) + xe^x + \frac{3\sin x + \cos x}{10}$$

→ Page :- 75 - Ex: 10:- Solve: $(D^3-5D^2+7D-3)y = e^{2x}\cosh x$

Solution :

$$A.E.: D^3-5D^2+7D-3=0$$

$$\Rightarrow (D-1)^2(D-3)=0$$

$$\therefore D = 1, 1, 3$$

$$\therefore y_c = (c_1 + c_2 x)e^x + c_3 e^{3x}$$

$$\therefore y_p = \frac{e^{2x}\cosh x}{(D-1)^2(D-3)}$$

$$= \frac{e^{2x}\cdot\frac{1}{2}(e^x+e^{-x})}{(D-1)^2(D-3)}$$

$$= \frac{\frac{1}{2}(e^{3x}+e^x)}{D^3-5D^2+7D-3}$$

$$= \frac{\frac{1}{2} e^{3x}}{D^3 - 5D^2 + 7D - 3} + \frac{1}{2} \frac{e^x}{D^3 - 5D^2 + 7D - 3}$$

$$= \frac{x}{2} \frac{e^{3x}}{3D^2 - 10D + 7} + \frac{x}{2} \frac{e^x}{3D^2 - 10D + 7}$$

$$= \frac{x}{2} \frac{e^{3x}}{3 \times 3^2 - 10(3) + 7} + \frac{x^2}{2} \frac{e^x}{6D - 10}$$

$$= \frac{xe^{3x}}{8} - \frac{x^2 e^x}{8}$$

$$\therefore y = (c_1 + c_2 x) e^x + c_3 e^{3x} + \frac{xe^{3x}}{8} - \frac{x^2 e^x}{8}$$

→ solve: $(D^3 + 2D^2 + D) y = e^{2x} + x^2 + x$

Solution: A.E. is $D^3 + 2D^2 + D = 0$

$$\Rightarrow D(D+1)^2 = 0$$

$$\therefore D = 0, -1, -1$$

$$\therefore y_c = (c_1 + c_2 x) e^{-x} + c_3$$

$$y_p = \frac{e^{2x}}{D^3 + 2D^2 + D} + \frac{x^2 + x}{D^3 + 2D^2 + D}$$

$$= \frac{e^x}{8 + 8 + 2} + \frac{x^2 + x}{D(D+1)^2}$$

$$= \frac{e^x}{18} + \frac{1}{D}(1+D)^{-2}(x^2+x)$$

$$= \frac{e^x}{18} + \frac{1}{D}(1-2D+3D^2 \cdots)(x^2+x)$$

$$= \frac{e^x}{18} + \frac{1}{D}(x^2+x-4x-2+6)$$

$$= \frac{e^x}{18} + \frac{x^3}{3} + \frac{x^2}{2} - 2x^2 + 4x$$

$$= \frac{e^x}{18} + \frac{x^3}{3} - \frac{3x^2}{2} + 4x$$

$$\therefore y = (c_1 + c_2 x)e^{-x} + c_3 + \frac{e^x}{18} + \frac{x^3}{3} - \frac{3x^2}{3} + 4x$$

Page-77: Ex-1:- Solve: $\frac{d^2y}{dx^2} - 9y = 6e^{3x} + xe^{3x}$

Solution: A.E.: $D^2 - 9 = 0$

$$\therefore D = \pm 3$$

$$\therefore y_c = c_1 e^{3x} + c_2 e^{-3x}$$

$$y_p = \frac{1}{D^2-9} e^{3x}(6+x)$$

$$= \frac{1}{(D+3)^2+9} e^{3x}(6+x) \quad [D \to D+a]$$

$$= e^{3x}\frac{1}{D^2+6D}(6+x)$$

$$= e^{3x}\frac{1}{6D\left(1+\frac{D}{6}\right)}(6+x)$$

$$= \frac{e^{3x}}{6}\frac{1}{D}\left(1+\frac{D}{6}\right)^{-1}(6+x)$$

$$= \frac{e^{3x}}{6}\frac{1}{D}\left(1-\frac{D}{6}-\cdots\right)(6+x)$$

$$= \frac{e^{3x}}{6}\frac{1}{D}\left(6+x-0-\frac{1}{6}\right)$$

$$= \frac{e^{3x}}{6}\frac{1}{D}\left(\frac{35}{6}+x\right)$$

$$= \frac{e^{3x}}{6}\left(\frac{35x}{6}+\frac{x^2}{2}\right)$$

$$= \frac{e^{3x}(3x^2+35x)}{36}$$

$$\therefore y = c_1e^{3x}+c_2e^{-3x}+\frac{e^{3x}(3x^2+35x)}{36}$$

Page : 77 - Ex:2: Solve: $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - y = xe^x + e^x$

Solution:

$$A.E.: D^3 - 3D^2 + 3D - 1 = 0$$

$$\Rightarrow (D-1)^3 = 0$$

$$\therefore D = 1, 1, 1$$

$$\therefore y_c = (c_1 + c_2 x + c_3 x^2) e^x$$

$$y_p = \frac{1}{(D-1)^3} e^x (x+1)$$

$$= e^x \frac{x+1}{(D+1-1)^3} \quad [D \to D+a]$$

$$= e^x \frac{x+1}{D^3}$$

$$= e^x \frac{1}{D^3} (x+1)$$

$$= e^x \frac{1}{D^2} \frac{(x+1)^2}{2} \quad [\text{integrating}]$$

$$= e^x \frac{1}{D} \frac{(x+1)^3}{6} \quad [\text{integrating}]$$

$$= e^x \frac{(x+1)^4}{24}$$

$$\therefore y = (c_1 + c_2 x + c_3 x^2) e^x + \frac{e^x (x+1)^4}{24}$$

Page-78 : Ex:- 3 : Solve: $D^3 - 7D - 6 = e^{2x} . x^2$

Solution:

$$A.E. : D^3 - 7D - 6 = 0$$

$$\Rightarrow (D+1)(D-3)(D+2) = 0$$

$$\therefore D = -1, 3, -2$$

$$\therefore y_c = c_1 e^{-x} + c_2 e^{3x} + c_3 e^{-2x}$$

$$y_p = \frac{e^{2x} . x^2}{D^3 - 7D - 6}$$

$$= \frac{e^{2x} . x^2}{(D+2)^3 - 7(D+2) - 6} \quad [D \rightarrow D + a]$$

$$= e^{2x} \frac{1}{D^3 + 6D^2 + 5D - 12} x^2$$

$$= -\frac{e^{2x}}{12} \left(\frac{1}{1 - \frac{5}{12} D - \frac{1}{2} D^2 - \frac{1}{12} D^3} \right) x^2$$

$$= -\frac{e^{2x}}{12} \left(1 - \frac{5}{12} D - \frac{1}{2} D^2 - \frac{1}{12} D^3 \right)^{-1} x^2$$

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$$= -\frac{e^{2x}}{12}\left(1+\frac{5}{12}D+\frac{1}{2}D^2+\frac{5^2}{12^2}D^2\right)x^2$$

$$= -\frac{e^{2x}}{12}\left(x^2+\frac{5x}{6}+1+\frac{25}{144}\times 2\right)$$

$$= -\frac{e^{2x}}{12}\left(x^2+\frac{5x}{6}+\frac{97}{72}\right)$$

$$\therefore y = c_1 e^{-x}+c_2 e^{3x}+c_3 e^{-2x}-\frac{e^{2x}}{12}\left(x^2+\frac{5x}{6}+\frac{97}{72}\right)$$

Page: 78 - Ex:- 4: Solve: $\frac{d^3y}{dx^3}-2\frac{dy}{dx}+4y = e^x\cos x$

Solution:

A.E.: $D^3-2D+4=0$

$\Rightarrow (D+2)(D^2-2D+2)=0$

$\therefore D=-2,\ 1\pm i$

$$y_c = c_1 e^{-2x}+e^x(c_2\cos x+c_3\sin x)$$

$$y_p = \frac{1}{D^3-2D+4}e^x\cos x$$

$$= e^x\frac{1}{(D+1)^3-2(D+1)+4}\cos x \quad [D\to D+1]$$

$$= e^x\frac{1}{D^3+3D^2+D+3}\cos x$$

$$= -\frac{e^{2x}}{12}\left(1+\frac{5}{12}D+\frac{1}{2}D^2+\frac{5^2}{12^2}D^2\right)x^2$$

$$= -\frac{e^{2x}}{12}\left(x^2+\frac{5x}{6}+1+\frac{25}{144}\times 2\right)$$

$$= -\frac{e^{2x}}{12}\left(x^2+\frac{5x}{6}+\frac{97}{72}\right)$$

$$\therefore y = c_1 e^{-x}+c_2 e^{3x}+c_3 e^{-2x}-\frac{e^{2x}}{12}\left(x^2+\frac{5x}{6}+\frac{97}{72}\right)$$

Page : 78 - Ex :- 4 : Solve : $\frac{d^3y}{dx^3}-2\frac{dy}{dx}+4y=e^x\cos x$

Solution :

A.E. : $D^3-2D+4=0$

$$\Rightarrow (D+2)(D^2-2D+2)=0$$

$$\therefore D=-2,\ 1\pm i$$

$$y_c = c_1 e^{-2x}+e^x(c_2\cos x+c_3\sin x)$$

$$y_p = \frac{1}{D^3-2D+4}e^x\cos x$$

$$= e^x\frac{1}{(D+1)^3-2(D+1)+4}\cos x \quad [D\rightarrow D+1]$$

$$= e^x\frac{1}{D^3+3D^2+D+3}\cos x$$

$$= e^{x} \cdot x \frac{1}{3D^2+6D+1} \cos x$$

$$= xe^{x} \frac{1}{-3+6D+1} \cos x$$

$$= xe^{x} \frac{1}{6D-2} \cos x$$

$$= \frac{1}{2} xe^{x} \frac{3D+1}{9D^2-1} \cos x$$

$$= \frac{xe^{x}}{2} \frac{3D(\cos x)+\cos x}{-9-1}$$

$$= -\frac{xe^{x}}{20} \cancel{\cos}(-3\sin x+\cos x)$$

$$= \frac{xe^{x}}{20}(3\sin x-\cos x)$$

$$\therefore y = c_1 e^{-2x} + e^{x}(c_2\cos x + c_3 \sin x) + \frac{xe^{x}}{20}(3\sin x - \cos x)$$

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22-11-2017 : 12C : Wednesday :

First Order
- Seperable
- Homogeneous
- Linear
- Exact
- Bernoulli

Higher Order
- Homogeneous
- forced System

→ First order but not first degree $\boxed{p = \frac{dy}{dx}}$

Ex. 1 : Solve :- $p^4 - (x+2y+1)p^3 + (x+2y+2xy)p^2 - 2xyp = 0$

Solution :

$$p^4 - (x+2y+1)p^3 + (x+2y+2xy)p^2 - 2xyp = 0 \quad \text{------ (i)}$$

$$\Rightarrow p^4 - xp^3 - 2yp^3 - p^3 + xp^2 + 2yp^2 + 2xyp^2 - 2xyp = 0$$

$$\Rightarrow p(p-1)(p-x)(p-2y) = 0$$

$$\therefore p = 0 \;,\; p-1 = 0 \;,\; p - x = 0 \;,\; p - 2y = 0$$

$$\frac{dy}{dx} = 0 \;,\; \frac{dy}{dx} = 1 \;,\; \frac{dy}{dx} = x \;;\; \frac{dy}{dx} = 2y$$

By integrating

$$y = c \;,\; y = x + c \;,\; y = \frac{x^2}{2} + c \;,\; \frac{dy}{2y} = dx$$

$$\Rightarrow \frac{1}{2}\ln y = x + c$$

$$\Rightarrow \ln y = 2(x+c)$$

$$\Rightarrow y = e^{2x+c'}$$

$$\Rightarrow y = c'e^{2x}$$

$\therefore$ The general solution of equation (i)

$$(y-c)(y-x-c)(y-\frac{x^2}{2}-c)(y-c'e^{2x})=0$$

Page: 116 : B.D. Sharma.

Ex-2 :- Solve: $p^3+2xp^2-y^2p^2-2xy^2p=0$

Solution:

$$p^3+2xp^2-y^2p^2-2xy^2p=0 \text{ ---------- (i)}$$

$$\Rightarrow p(p^2+2xp-y^2p-2xy^2)=0$$

$$\Rightarrow p(p+2x)(p-y^2)=0$$

$$\therefore p=0,\ p+2x=0,\ p-y^2=0$$

$$\frac{dy}{dx}=0,\quad \frac{dy}{dx}=-2x,\quad \frac{dy}{dx}=y^2$$

By integrating,

$$y=c,\ y=-x^2+c,\quad \frac{dy}{y^2}=dx$$

$$\Rightarrow y-c=0,\ y+x^2-c=0$$

$$\Rightarrow -\frac{1}{y}=x+c$$

$$\Rightarrow -1=xy+cy$$

$$\Rightarrow xy+cy+1=0$$

The general solution of equation (i)

$$(y-c)(y+x^2-c)(xy+cy+1)=0$$

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Ex-3 :- Solve : $xy(p^2+1) = (x^2+y^2)p$

Solution :

$$xy(p^2+1) = (x^2+y^2)p \quad \text{------------ (i)}$$

$$\Rightarrow xyp^2 - xy - px^2 - py^2 = 0$$

$$\Rightarrow (yp-x)(xp-y) = 0$$

$$\therefore yp - x = 0 \;,\; xp - y = 0$$

$$\therefore \frac{dy}{dx} = \frac{x}{y} \;,\; \frac{dy}{dx} = \frac{y}{x}$$

By integrating

$$y\,dy = x\,dx \;, \qquad \frac{dy}{y} = \frac{dx}{x}$$

$$\Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + c \qquad \Rightarrow \ln y = \ln x + \ln c$$

$$\Rightarrow y = cx$$

$$\Rightarrow y^2 - x^2 - c' = 0 \qquad \Rightarrow y - cx = 0$$

$\therefore$ The general ~~equation~~ solution of equation (i)

$$(y^2 - x^2 - c')(y - cx) = 0$$

$\rightarrow$ Clairaut's Equation : $y = px + f(p)$ ---------- (i)

The differential equation of the form (i) is known a

Clairaut's equation.

To solve, $y = px + f(p)$

Differentiating it w.r.t. to x we gets

$$p = p + [x + f'(p)]\frac{dp}{dx} \text{ ------ (ii)}$$

If equation (ii) is true,

$$[x + f'(p)]\frac{dp}{dx} = 0$$

$$\Rightarrow \frac{dp}{dx} = 0 \quad [\because x + f'(p) \neq 0]$$

$$\Rightarrow p = c \text{ [By integrating]}$$

putting $p=c$ in equation (i) we get,

$$y = cx + f(c)$$

This is the required solution.

Ex-1:- Solve: $px - y + p^3 = \frac{m^3}{p^3}$

Solution:

$$px - y + p^3 = \frac{m^3}{p^3}$$

$$\Rightarrow y = px + p^3 - \frac{m^3}{p^3} \text{ ---------- (i)}$$

This equation is called Clairaut's equation. So the solution of the equation is

$$y = cx + c^3 - \frac{m^3}{c^3}$$

Ex-2 :- Solve : $y = px + p - p^2$

Solution :

$$y = px + p - p^2$$

This equation is called Clairaut's equation. So the solution of the equation is

$$y = cx + c - c^2$$

Dependent Variable Absent

Ex-1 :- Solve : $2\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 + 4 = 0$

Solution :

$$2\frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 + 4 = 0$$

Let, $p = \frac{dy}{dx}$

$\Rightarrow \frac{dp}{dx} = \frac{d^2y}{dx^2}$

$$\Rightarrow 2\frac{dp}{dx} - p^2 + 4 = 0$$

$$\Rightarrow 2\frac{dp}{dx} = p^2 - 4$$

$$\Rightarrow \frac{2dp}{p^2-4} = dx$$

$$\Rightarrow \frac{2}{2\cdot 2} \ln\frac{p-2}{p+2} = x + \ln c$$

$$\Rightarrow \ln\frac{p-2}{p+2} = 2x + 2\ln c$$

$$\Rightarrow \ln\frac{p-2}{p+2} = \ln(c^2e^{2x})$$

$$\Rightarrow \cancel{\ln}\ \frac{p-2}{p+2} = c^2e^{2x}$$

$$\Rightarrow p-2 = pc^2e^{2x}+2c^2e^{2x}$$

$$\Rightarrow p(1-c^2e^{2x}) = 2(1+c^2e^{2x})$$

$$\Rightarrow p = \frac{2(1+c^2e^{2x})}{1-c^2e^{2x}}$$

$$\Rightarrow \frac{dy}{dx} = \cancel{2x}\ 2\left(1+\frac{2c^2e^{2x}}{1-c^2e^{2x}}\right)$$

By integrating

$$y = 2\left[x - \ln(1-c^2e^{2x})\right] + c_1$$

Ex:2:- solve: $\frac{d^2y}{dx^2} + \frac{dy}{dx} + \left(\frac{dy}{dx}\right)^3 = 0$

Solution:

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + \left(\frac{dy}{dx}\right)^3 = 0$$

Let, $p = \frac{dy}{dx}$

$\Rightarrow \frac{dp}{dx} = \frac{d^2y}{dx^2}$

$$\Rightarrow \frac{dp}{dx} + p + p^3 = 0$$

$$\Rightarrow \frac{dp}{dx} = -p(p^2+1)$$

$$\Rightarrow \frac{dp}{p(p^2+1)} = -dx$$

$$\Rightarrow \left(\frac{1}{p} - \frac{p}{p^2+1}\right)dp = -dx$$

$$\Rightarrow \frac{1}{p}dp - \frac{1}{2}\left(\frac{2p}{p^2+1}\right)dp = -dx$$

$$\Rightarrow \ln p - \frac{1}{2}\ln(p^2+1) = -x + \ln c$$

$$\Rightarrow \ln\frac{p}{\sqrt{p^2+1}} = \ln(ce^{-x})$$

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$$\Rightarrow \frac{p^2}{p^2+1} = c^2e^{-2x}$$

$$\Rightarrow \frac{p^2+1}{p^2} = \frac{1}{c^2e^{-2x}}$$

$$\Rightarrow \frac{1}{p^2} = 1 - \frac{1}{c^2e^{-2x}}$$

$$\Rightarrow p^2 = \frac{c^2e^{-2x}}{1-c^2e^{-2x}}$$

$$\Rightarrow p = \frac{c^2e^{-x}}{\sqrt{1-c^2e^{-2x}}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{c^2e^{-x}}{\sqrt{1-c^2e^{-2x}}}$$

By integrating,

$$y = -\sin^{-1}(c^2e^{-x}) + c_1$$

Ex-3:- solve: $(1+x^2)\frac{d^2y}{dx^2} + 1 + \left(\frac{dy}{dx}\right)^2 = 0$

$$(1+x^2)\frac{d^2y}{dx^2} + 1 + \left(\frac{dy}{dx}\right)^2 = 0$$

Let, $p = \frac{dy}{dx}$

$\Rightarrow \frac{dp}{dx} = \frac{d^2y}{dx^2}$

$$\Rightarrow (1+x^2)\frac{dp}{dx} + 1 + p^2 = 0$$

$$\Rightarrow (1+x^2)\frac{dp}{dx} = -(1+p^2)$$

$$\Rightarrow \frac{dp}{1+p^2} + \frac{1}{1+x^2} = 0$$

$\cancel{\Rightarrow \tan^{-1}}$

By integrating,

$$\tan^{-1} p + \tan^{-1} x = \tan^{-1} c_1$$

$$\Rightarrow \tan^{-1} \frac{p+x}{1-px} = \tan^{-1} c_1$$

$$\Rightarrow \frac{p+x}{1-px} = c_1$$

$$\Rightarrow p+x = c_1(1-px)$$

$$\Rightarrow p(1+c_1x) = c_1 - x$$

$$\Rightarrow \frac{dy}{dx} = \frac{c_1-x}{1+c_1x} = \frac{1}{c_1}\left[\frac{c_1^2-c_1x}{1+c_1x}\right] = \frac{1}{c_1}\left[\frac{1+c_1^2}{1+c_1x} - 1\right]$$

$$= \frac{1+c_1^2}{c_1}\left(\frac{1}{1+c_1x}\right) - \frac{1}{c_1}$$

By integrating

$$y = \frac{1+c_1^2}{c_1^2}\log(1+c_1x) - \frac{x}{c_1} + c_2$$

Ex-4:- Solve: $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$

Solution:

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$$

Let, $p = \frac{dy}{dx}$

$\Rightarrow \frac{dp}{dx} = \frac{d^2y}{dx^2}$

$$\Rightarrow (1-x^2)\frac{dp}{dx} - xp = 2$$

$$\Rightarrow \frac{dp}{dx} - \frac{x}{1-x^2}p = \frac{2}{1-x^2}$$

~~This is a linear equation in p and x~~

~~Integrating factor~~ $\cancel{= e^{\int -\frac{x}{1-x^2}dx}}$

$$\cancel{= e^{\frac{1}{2}\int \frac{2x}{1-x^2}dx}}$$

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$$\cancel{= e^{\frac{1}{2}\log(1-x^2)}}$$

$$\cancel{= \sqrt{1-x^2}}$$

$$\cancel{\Rightarrow \frac{dp}{dx}}$$

This is a linear equation in p and x

$$\text{Integrating factor} = e^{\int -\frac{x}{1-x^2}dx}$$

$$= e^{-\frac{1}{2}\int \frac{2x}{1-x^2}dx}$$

$$= e^{\frac{1}{2}\ln(1-x^2)}$$

$$= \sqrt{1-x^2}$$

$$\therefore \sqrt{1-x^2}\,dp - \frac{x}{\sqrt{1-x^2}}\,p\,dx = \frac{2}{\sqrt{1-x^2}}\,dx$$

$$\Rightarrow d\left(p\sqrt{1-x^2}\right) = \frac{2}{\sqrt{1-x^2}}\,dx$$

By integrating,

$$p\sqrt{1-x^2} = 2\sin^{-1}x + c$$

$$\Rightarrow p = \frac{2\sin^{-1}x}{\sqrt{1-x^2}} + \frac{c}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2\sin^{-1}x}{\sqrt{1-x^2}} + \frac{c}{\sqrt{1-x^2}}$$

$\Rightarrow$ By integrating,

$$y = (\sin^{-1}x)^2 + c\sin^{-1}x + c_1$$

26-11-2017 : 12E : Sunday

Forcce free umdamped motion

$\rightarrow mx'' + kx = 0$ $\qquad x(0) = x_0$

$\Rightarrow x'' + \frac{k}{m}x = 0$ $\qquad x'(0) = V_0$

$\Rightarrow x'' + \omega^2 x = 0$

A.E.: $m^2 + \omega^2 = 0$

$\Rightarrow m = \pm \omega i$

$\therefore x(t) = c_1 \cos \omega t + c_2 \sin \omega t$ $\quad | \quad x(0) = c_1$

& $x'(t) = -c_1 \omega \sin \omega t + c_2 \omega \cos \omega t.$ $\quad | \quad x'(0) = c_2 \omega$

$\therefore x_0 = c_1$ & $V_0 = c_2 \omega$

$\Rightarrow c_2 = \frac{V_0}{\omega}$

$x(t) = x_0 \cos \omega t + \frac{V_0}{\omega} \sin \omega t$

$= c\left(\frac{x_0}{c} \cos \omega t + \frac{V_0/c}{\omega} \sin \omega t\right)$

where, $c = \sqrt{\left(\frac{V_0}{\omega}\right)^2 + x_0^2}$

Let, $\frac{V_0}{\omega} = -c \sin \phi$ $\quad \left[\because -1 < \frac{V_0}{c\omega} < 1\right]$

$\Rightarrow \frac{V_0}{c\omega} = -\sin \phi$

$c = \sqrt{c^2 \sin^2 \phi + x_0^2}$

$\Rightarrow c^2 = c^2 \sin^2 \phi + x_0^2$

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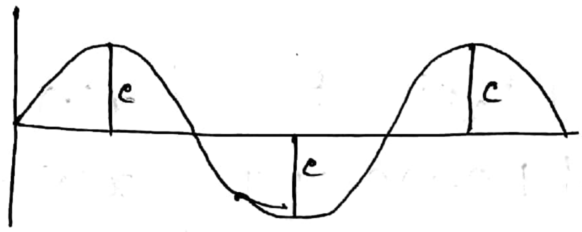
$$\Rightarrow c^2(1-\sin^2\phi) = x_0^2$$

$$\Rightarrow c^2\cos^2\phi = x_0^2$$

$$\Rightarrow \frac{x_0}{c} = \cos\phi$$

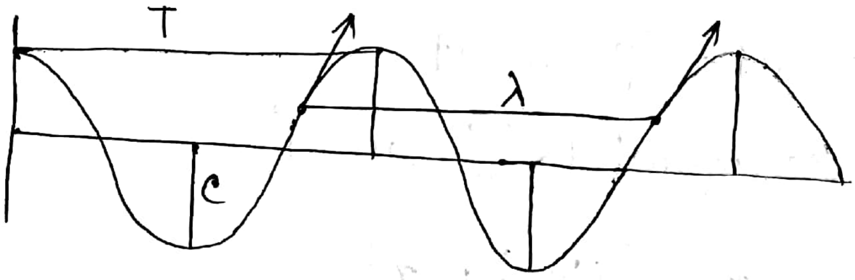
$$\therefore\ c(\cos\phi\cos\omega t - \sin\phi\sin\omega t)$$

$$= c\cos(\omega t+\phi)$$



$$x(t) = c\cos(\omega t+\phi)$$

$$= c\sin\left(\frac{\pi}{2}-\omega t-\phi\right)$$



$$\cos(\omega t+\phi) = \cos 2n\pi$$

$$\Rightarrow \omega t+\phi = 2n\pi$$

$$\Rightarrow t = \frac{2n\pi-\phi}{\omega}$$

$$\therefore\ t = \sqrt{\frac{m}{k}}\,(2n\pi-\phi)$$

$n=0,\quad t_0 = -\phi\sqrt{\frac{m}{k}}$

$n=1,\quad t_1 = (2\pi-\phi)\sqrt{\frac{m}{k}}$

$$T = (t_1 - t_0)$$
$$= 2\pi\sqrt{\frac{m}{k}}$$

$n=2, \quad t_2 = (4\pi - \phi)\sqrt{\frac{m}{k}}$

$$T = t_2 - t_1 = 2\pi\sqrt{\frac{m}{k}}$$

$$\therefore T = 2\pi\sqrt{\frac{m}{k}}$$

S.L. Ross: Page-194

Example - 5.1: An 8-lb weight is placed upon the lower end of a coil spring suspended from the ceiling. The weight comes to rest in its equilibrium position, thereby stretching the spring 6 inches. The weight is then pulled down 3 inches below its equilibrium position and released at $t=0$ with an initial velocity of 1 ft/sec, directed down-ward. Neglecting the resistance of the medium and assuming that no external forces are present, determine the amplitude, period and frequency of the resulting motion.

Solution:

This is clearly an example of free, undamped motion, hence

$$m\frac{d^2x}{dt^2} + kx = 0 \quad \text{--------- (i)}$$

Since, 8lb weight stretches the spring 6 inches $= \frac{1}{2}$ ft,

$\therefore$ According to Hooke's law,

also, $m = \frac{w}{g} = \frac{8}{32}$ (slugs)

$F = ks$

$\Rightarrow 8 = k \times \frac{1}{2}$

$\therefore k = 16$ lb/ft

putting the value of k and m in equation (i) we get

$$\frac{8}{32}\frac{d^2x}{dt^2} + 16x = 0$$

$$\Rightarrow \frac{d^2x}{dt^2} + 64x = 0 \quad \text{------} \quad (ii)$$

Since the weight was released with a downward initial velocity of 1 ft/sec from a point 3 inches ($= \frac{1}{4}$ ft) below its equilibrium position, we also have the initial conditions,

$$x(0) = \frac{1}{4}, \quad x'(0) = 1$$

A.E. of equation (ii)

$r^2 + 64 = 0$

$\therefore r = \pm 8i$

$\therefore$ The general solution of equation (ii)

$$x = c_1 \sin 8t + c_2 \cos 8t$$

$x'(0) = 8c_1 \cos 8t - 8c_2 \sin 8t$

Applying the initial condition

$c_2 = \frac{1}{4}$ & $1 = 8c_1 \Rightarrow c_1 = \frac{1}{8}$

$\therefore$ The solution of equation (ii) is

$$x = \frac{1}{8}\sin 8t + \frac{1}{4}\cos 8t \quad \text{------- (iii)}$$

putting the equation (iii) in form of

$$x = c\left(\frac{1}{8c}\sin 8t + \frac{1}{4c}\cos 8t\right)$$

where, $c = \sqrt{\left(\frac{1}{8}\right)^2 + \left(\frac{1}{4}\right)^2} = \frac{\sqrt{5}}{8}$

$$\therefore x = \frac{\sqrt{5}}{8}\left(\frac{\sqrt{5}}{5}\sin 8t + \frac{2\sqrt{5}}{5}\cos 8t\right)$$

Let, $\frac{2\sqrt{5}}{5} = \cos\phi \quad \therefore \phi \approx -0.46$ radia

$\Rightarrow \sin\phi = -\frac{\sqrt{5}}{5}$

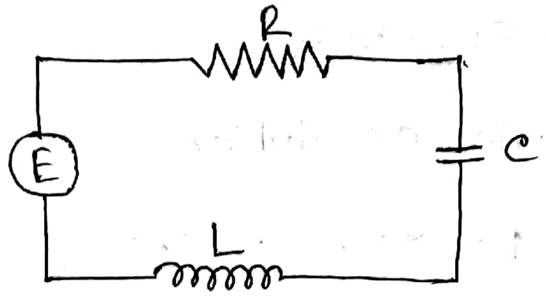
$$\therefore x = \frac{\sqrt{5}}{8}(\cos\phi \cos 8t - \sin\phi \sin 8t)$$

$$= \frac{\sqrt{5}}{8}\cos(8t + \phi)$$

$$= 0.280 \cos(8t \div 0.46)$$

From the formula $T = \frac{2\pi}{\omega}$ we get

Amplitude, $A = 0.28 ft$, $T = \frac{2\pi}{8} = \frac{\pi}{4}$ and frequency, $f = \frac{1}{T} = \frac{4}{\pi}$



$\mathcal{E} = L\frac{di}{dt}$

$i = \frac{dq}{dt}$

$\int i\,dt = q$

$Li' + \frac{q}{c} + iR = E$

$\Rightarrow L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{c} = E$ [differential equation]

or, $Li' + \frac{1}{c}\int i\,dt + iR = E$ [integral equation]

29-11-2017: 13C: Wednesday:

Example - 5.7: A circuit has in series an electromotive force given by $E = 100 \sin 40t$ V, a resistor of $10\,\Omega$ and an inductor of 0.5 H. If the initial current is 0. Find the current at time $t > 0$.

Solution: Here, the current equation is,

$$L\frac{di}{dt} + iR = E$$

$$\Rightarrow \frac{1}{2}\frac{di}{dt} + 10i = 100 \sin 40t$$

$$\Rightarrow \frac{di}{dt} + 20i = 200 \sin 40t \quad \text{------} \quad (1)$$

Equation (1) is first order linear equation

$$\therefore \mu = e^{\int 20\,dt}$$
$$= e^{20t}$$

multiplying equation (1) by e^{20t} we got,

$$e^{20t}\,di + 20 i e^{20t}\,dt = 200\, e^{20t} \sin 40t\, dt$$

$$\Rightarrow d\left(e^{20t} i\right) = 200\, e^{20t} \sin 40t\, dt$$

By integrating,

$$\cancel{e^{20t} i = 200\, e^{20t}\left(\frac{\sin 40t}{20} - \frac{\cos 40t}{40}\right)}$$

$$\Rightarrow e^{20t} i = 200 \frac{e^{20t}(20 \sin 40t - 40 \cos 40t)}{20^2 + 40^2} + c$$

$$\Rightarrow i = 2(\sin 40t - 2 \cos 40t) + c e^{-20t}$$

$i = 0$ when $t = 0$

$$0 = 2(\sin 0^\circ - 2 \cos 0^\circ) + c e^{-20\times 0}$$

$$\Rightarrow 0 = -4 + c$$
$$\Rightarrow c = 4$$

$$\therefore i(t) = 2(\sin 40t - 2 \cos 40t) + 4 e^{-20t}$$

$$i(t) = 2\sqrt{5}\left(\frac{1}{\sqrt{5}} \sin 40t - \frac{2}{\sqrt{5}} \cos 40t\right) + 4 e^{-20t}$$

Let,

$$\sin\phi = \frac{1}{\sqrt{5}}$$

$$\cos\phi = \frac{2}{\sqrt{5}}$$

$$= 2\sqrt{5}\,(\sin\phi \sin 40t - \cos\phi \cdot \cos 40t) + 4e^{-20t}$$

$$= 4.47 \cos(40t + \phi) + 4e^{-20t}$$

$$\Rightarrow i(t) = 4.47 \cos(40t + \phi) + 4e^{-20t}$$

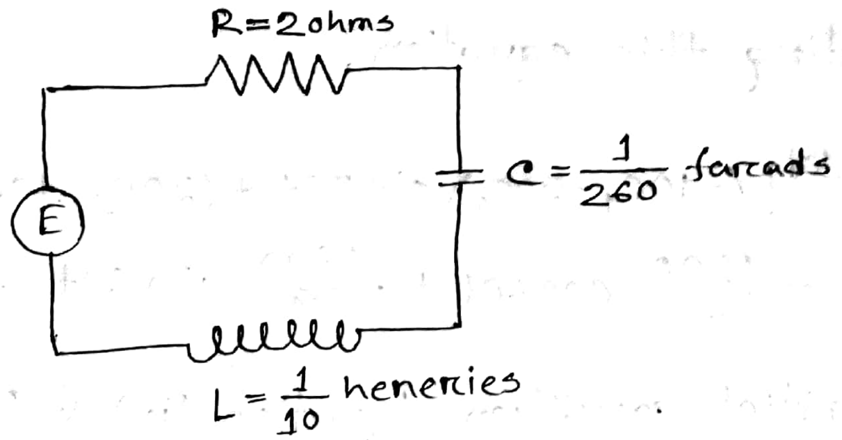
Example - 5.8: A circuit has in series an electromotive force given by $E = 100 \sin 60t$ V, a resistor of 2Ω, an inductor of 0.1 H and a capacitor of $\frac{1}{260}$ farads. If the initial current and the initial charge on the capacitor are both 0, find the charge on the capacitor at any time $t > 0$.

Solution: Here, the current equation is,

$$L\frac{di}{dt} + \frac{q}{c} + iR = E$$

$$\Rightarrow \frac{1}{10}\frac{d^2q}{dt^2} + 260q + 2\frac{dq}{dt} = 100 \sin 60t$$

$$\Rightarrow \frac{d^2q}{dt} + 20\frac{dq}{dt} + 2600q = 1000 \sin 60t \quad \text{----(i)}$$



Initial conditions. $q(0)=0$, $q'(0)=0$

A.E. of equation (i)

$$r^2+20r+2600=0$$

$$\Rightarrow r=-10\pm 50i$$

complimentary solution of equation (i)

$$q_c = e^{-10t}(c_1 \sin 50t + c_2 \cos 50t) \quad \text{------ (ii)}$$

particular solution of equation (i)

$$q_p = A \sin 60t + B \cos 60t \quad \text{---------- (iii)}$$

Applying initial condition on equation (iii) & its differentiating equation, we get,

$$A = -\frac{25}{61} \text{ and } B = -\frac{30}{61}$$

$\therefore$ The general solution of equation (i)

$$q = e^{-10t}(c_1 \sin 50t + c_2 \cos 50t) - \frac{25}{61}\sin 60t - \frac{30}{61}\cos 60t$$

(iv)---

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Differentiating this equation,

$$\frac{dq}{dt} = e^{-10t}\left[(-10c_1 - 50c_2)\sin 50t + (50c_1 - 10c_2)\cos 50t\right] - \frac{1500}{61}\cos 60t + \frac{1800}{61}\sin 60t \quad \text{- - - - (v)}$$

Applying initial condition équation (iv) & equation (v)

$$c_2 - \frac{30}{61} = 0 \quad \& \quad 50c_1 - 10c_2 - \frac{1500}{61} = 0$$

From these equations, we find that,

$$c_1 = \frac{36}{61} \quad \& \quad c_2 = \frac{30}{61}$$

∴ The solution of equation (i)

$$q = \frac{6e^{-10t}}{61}(6\sin 50t + 5\cos 50t) - \frac{5}{61}(5\sin 60t + 6\cos 60t)$$

$$\Rightarrow q = \frac{6\sqrt{61}}{61} e^{-10t}\cos(50t - \phi) - \frac{5\sqrt{61}}{61}\cos(60t - \theta)$$

where, $\cos\phi = \frac{5}{\sqrt{61}}$ & $\cos\theta = \frac{6}{\sqrt{61}}$

$\sin\phi = \frac{6}{\sqrt{61}}$ $\quad$ $\sin\theta = \frac{5}{\sqrt{61}}$

$\phi = 0.88$ rads $\quad$ $\theta = 0.69$ rads

$$\therefore q = 0.77\, e^{-10t}\cos(50t - 0.88) - 0.64\cos(60t - 0.69)$$