

Sakir

Q11

Answer SIX questions taking THREE from each section.
 The questions are of equal value.
 Use separate answer script for each section.

SECTION A

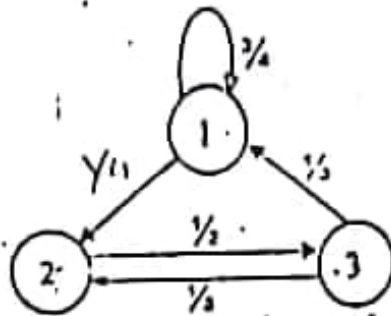
- Q1(a) What is the geometric mean? Give an example in which this mean is used. Marks
03
- (b) Let X_1, X_2, \dots, X_n denote the deviations of any class mark X_i in a frequency distribution from a given class mark A . Show that if all class intervals have equal size C , the arithmetic mean can be computed from $A + \frac{\sum f_i u_i}{N} \times C$, where $d_i = cu_i$ and $u_i = 0, \pm 1, \pm 2$. 04
- (c) Show that $\sum f_i (X_i - A)^2$ has a minimum value. Hence define the standard deviation (S.D). 04½
- Q2(a) The histogram derived from the pipe values of an in age shows that it is bimodal in nature. What can be inferred from the curve. 02½
- (b) Explain the correlation between voltage and current from the equation of a electrical network. 02
- (c) The distribution function of the random variable X is $F(x) = \begin{cases} e^{-2x}, & x > 0 \\ 0, & x < 0 \end{cases}$. 07
- Find (i) the density function (ii) the probability that $X > 2$ and (iii) the probability that $-3 < X < 4$.
- Q3(a) What is the Poisson's distribution? Show that its mean and variance have equal value. 05½
- (b) If 10% of the bolts produced by a machine are defective, determine the probability that out of 50 bolts chosen at random, (i) 1 and (ii) at most 1 bolts will be defective. Use both binomial and Poisson's distribution and compare the results. 06
- Q4(a) Find the Kurtosis of the distribution $\frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{x^2}{2\sigma^2}}$. 05½
- (b) The mean grade on a final examination was 72 and the S.D. was 9. The top 10% of the students are to receive A's. What is the minimum grade that a students must get in order to receive an A+? 03
- (c) Assume that the heights of 3000 male students at a university are normally distributed with mean 64 inches and S.D. 3 inches. If 80 samples consisting of 25 students each are obtained. What would be the expected mean and S.D. of the resulting sampling distribution of means? 03

SECTION B

- Q5(a) What are the confidence limits? A random sample of 50 mathematics grades out of 200 showed a mean 70 and a S.D. of 10. What are the 95% confidence limits for estimates of the mean of the 200 grades? 05
- (b) Derive the equation of the first and second moment about the origin of the binomial distribution. 04
- (c) Differentiate between point estimates and interval estimates. 02½
- Q6(a) To show that under the three conditions of a Poisson's process the number of arrivals in a fixed time follows the Poisson law i.e. if the probability of an arrival in time interval t and $t + \Delta t$ is $\lambda \Delta t + O(\Delta t)^2$, then $P_n(t) = \frac{e^{-\lambda t} (\lambda t)^n}{n!}$, $n = 0, 1, \dots, \infty$. 06
- (b) Why the exponential distribution lends itself well to model customer inter-arrival times or service times of a queuing system? 02½
- (c) Model the following process with respect to the basic queuing model: 03
- (i) the input process (ii) the output process (iii) Birth-death process
- Q7(a) On a network gateway, measurements show that the packets arrive at a mean rate of 125 packets per seconds (PPS) and the gateway takes about 2 milliseconds to forward them. Using an M/M/1 model, analyze the gateway, what is probability of buffer overflow if the gateway had only 13 buffers? How many buffers do we need to keep packet loss below one packet per million. 11½
- Q8(a) A pumping station has two identical pumps connected in parallel, each capable of pumping 3000 gallons/hr. If the failure rate and repair rate of each is 1.5/yr and 2.5/yr respectively. Calculate the average throughput of the pumping station. 07
- (b) Derive the probability distribution function of the normal distribution. 04½

What percentage of time the computer system will be in good condition?

Consider the 3 stage system shown below and the transition probabilities indicated. Calculate the steady-state probabilities.



Q7(a) Give a real life example of a queuing system and explain. Why do you need to study queuing model as a CSE graduate?

A potential customer enters the system with two servers as long as the first server is idle. The servers are in series and both servers need to be used. If the second server is busy then the customer needs to wait at first server until after getting service from the first server. If the arrival rate is λ and service rates are μ_1 and μ_2 . What proportion of the customers enters the system and the average time that a customer spends in the system? Draw the state space diagram when the customer does not wait at the first server after completion his service when the second server is not freed.

Q8(a) Average repair time (exponential) for a computer is 20 minutes. The arrival rate of computers in the service station is 12 for an 8 hour day. How many computers are ahead of the average set just brought in? What percentage of time will the operator be idle?

(b) It is necessary to determine how much storage space to allocate to a particular work centre in a new factory. Jobs would arrive at this work centre according to a poisson process with a mean rate of three per hour and the time required to perform the necessary work has an exponential distribution with a mean rate of 0.3 hours. If each job would require 2 square feet of floor space while in the process storage at the work centre, how much space must be provided to accommodate all waiting jobs 50% of the time?

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SECTION A

- Q1(a) How statistics is related to probability? Mention the necessity of studying applied statistics and queuing theory as a computer engineer. 03
- (b) Suppose X is a continuous random variable with probability density function 06

$$f(x) = \begin{cases} K(x-1), & 1 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

 Calculate i) K ii) $E(X)$ iii) $V(X)$ and iv) $F(x)$
- (c) Describe the independent and stationary properties of Poisson distribution by real world example. 02%
- Q2(a) Define "Mutually exclusive" events and "Independent" events. Are mutually exclusive events independent? Explain. 03
- (b) It is conjectured that an impurity exists in 30% of all drinking wells in a certain rural community. In order to gain some insight on this problem, it is determined that some tests should be made. It is too expensive to test all of the many wells in the area, so 10 were randomly selected for testing. 06
 i) What is the probability that exactly three wells have the impurity assuming that the conjecture is correct?
 ii) What is the probability that more than three wells are impure?
 Why we dividing by $(n-1)$ instead of n when we are calculating the sample standard deviation? 02%
- Q3(a) Explain the memory less property and its effect on the exponential distribution. 02%
- (b) An Engineer commutes daily from his home to his office. The average time for a one way trip is 24 minutes, with a standard deviation of 3.8 minutes. Assume the distribution of the trip times to be normally distributed. 09
 i) What is the probability that a trip will take at least 1/2 hour?
 ii) If the office opens at 9:00AM and he leaves his house at 8:40AM, what percentage of time is he late for work?
 iii) Find the length of time above which we find the slowest 15% of the trips.
 iv) Find the probability that 7 out of 10 trips will take at least 1/2 hour.
- Q4(a) What is standard normal? Write some properties of normal distribution. 03
- (b) The probability that a patient recovers from a rare blood disease is 0.4. If 100 people are known to have contracted this disease, what is the probability that less than 30 survive? 02%
- (c) The average zinc concentration recovered from a sample of zinc measurement in 36 different locations is found to be 2.6 grams per milliliter. Find the 95% and 99% confidence intervals for the mean zinc concentration in the river. Assume that the population standard deviation is 0.3. 06

SECTION B

- Q5(a) What is the difference between correlation and covariance analysis? 02%
- (b) The following table shows the heights to the nearest inch and the weight to the nearest pound of a sample of 10 male students drawn at random from the third year student at RUET. 09
- | Height | X (in) | 62 | 65 | 67 | 70 | 68 | 61 | 64 | 69 | 64 | 65 |
|--------|--------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Weight | Y (lb) | 130 | 168 | 156 | 168 | 129 | 132 | 152 | 135 | 153 | 139 |
- Find the equation of linear regression line to weight from height.
 Estimate the weight when height is 66 inch.
 Estimate the correlation coefficient. 02
- Q6(a) What is meant by Hypothesis testing? Why do you need to do this? 03
- (b) Explain the Birth-Death process of queuing system. 06%
- (c) A random sample of 64 RAM capacity, on average, 5.23 GB with a standard deviation of 0.24 GB. Test the hypothesis that $\mu = 5.5$ GB against the alternative hypothesis, $\mu < 5.5$ GB at the 0.05 level of confidence. 03
- Q7(a) Write the properties of stochastic process. Explain the application of stochastic process in engineering. 04%
- (b) Derive the equation of steady state probability for M/M/1 queuing system. 04
- (c) A secretary makes 2 errors per page, on average. What is the probability that on the next page he or she will make 4 or more errors. 02%
- Q8(a) What is Markov process? Explain with example. 09
- (b) An airline has 15 flights leaving a base per day, each with a hostess. The airlines keep three hostesses so that they may be called in case the scheduled hostesses for a flight

- (b) Suppose that in a certain region the daily rainfall (in inches) is a continuous random variable X with probability density function $f(x)$ given by 04

$$f(x) = \begin{cases} \frac{1}{2}(3x - 2x^2); & \text{if } 0 < x < 3 \\ 0 & ; \text{ otherwise} \end{cases}$$

Find the probability that on a given day in this region the rainfall is,

- (i) not more than 2 inches.
 - (ii) greater than 1 inch.
 - (iii) between 1.5 and 2.0 inches
 - (iv) equal to 1 inch, and
 - (v) less than 2 inches.
- (c) A bag contains 4 red, 6 black and 7 white marbles. A marbles is chosen at random from the bag. If the marble is not white what is the probability that it is red? 04

- Q.6(a) Write down the properties of standard deviation. 03
- (b) Explain the differences: 03
- (i) point estimate Vs interval estimate.
 - (ii) Sample Vs population.
 - (iii) Bi-modal Vs multi-modal.
- (c) Show that Geometric mean \leq Arithmetic mean. 03
- (d) Find the standard deviation of 1, 2, 3,----- n 03

- Q.7(a) For an M/M/1 queuing system with the average inter arrival time of 5 minutes and the average service time of 3 munities, compute, 05
- (i) . The expected response time.
 - (ii) The fraction of time when there are fewer than 2 jobs in the system.
 - (iii) The fraction of customers S who have to wait before their service starts.
- (b) What is memory less property? 2.5
- (c) Explain Birth-death process. 2.5
- (d) Explain the relation between the binomial distribution and the normal distribution. 02

- Q.8(a) On a network gateway, measurements show that the packets arrive at a mean rate of 125 packets per seconds (PPS) and the gateway takes about 2 milliseconds to forward them. Using M/M/1 model, analyze the gateway, what is probability of buffer overflow if the gateway had only 13 buffers? How many buffers do we need to keep packet loss below one packet per million? 08
- (b) Find the probability that five tosses of a fair die a 3 appears 04
- (i) at no time,
 - (ii) once,
 - (iii) twice,
 - (iv) 3 times.

N.B:

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SECTION A

- Q.1(a)** Define statistics from the perspective of an investigation. What do you mean by anecdotal evidence and its generalization, discuss with a suitable example. **Marks 04**
- (b)** Discuss about different types of variables. For the following data matrix, determine the type of each variable: **04**

Country	Content-removal request	Content-removal comply	User-data request	hemisphere	HDI
Australia	21	100	134	Southern	High
USA	92	63	5950	Northern	Very high

(Google's transparency report)

- (c)** Write down the differences between observational study and experimental study. **04**
- Q.2(a)** Explain the factors that need to be considered for evaluating the relationship between 2 variables. **04**
- (b)** What is modality of a histogram? Given 3 histograms of images that are right skewed, left skewed and uniform. What can you guess about the images contents-Justify your answer. **04**
- (c)** Consider a statistic: $\frac{\text{mean}}{\text{median}}$. Given 3 distributions having this statistic >1 , <1 and $=1$. **04**
 Discuss about the shapes of the 3 distributions.

- Q.3(a)** "Sample is used to visualize, understand the patterns and make quick statement about system's behavior", explain the statement. **03**
- (b)** Describe non sampling error by examples. **03**
- (c)** The following data set represents the number of new computer accounts registered during ten consecutive days. **06**
 43, 37, 50, 51, 58, 105, 52, 45, 45, 10
 (i) Compute the mean, median, quartiles and standard deviation.
 (ii) Check for outliers using the 1.5 (IQR) rule.
 (iii) Delete the detected outliers and compute the mean, median, quartiles and standard deviation.
 (iv) Make a conclusion about the effect of outliers on basic descriptive statistics.

- Q.4(a)** Consider the following data sets: **08**
 (i) 56, 52, 13, 34, 33, 18, 44, 41, 48, 75, 24, 19, 35, 27, 46, 62, 71, 24, 66, 94, 40, 18, 15, 39, 33, 23, 41, 78, 15, 35
 (ii) 19, 24, 12, 19, 18, 24, 8, 5, 9, 20, 13, 11, 1, 12, 11, 10, 22, 21, 7, 16, 15, 15, 26, 16, 1, 13, 21, 21, 20, 19

For each data set, draw a histogram and determine whether the distribution is right skewed, left-skewed, or symmetric. Compute sample means and sample medians. Do they support your findings about skewness and symmetry? How?

- (b)** Prove that sum of the deviation of a group of numbers from their mean is equal to zero. **04**

SECTION B

- Q.5(a)** The distribution function of the random variable X is : **04**

$$F(x) = \begin{cases} 1 - e^{-x} & ; x > 0 \\ 0 & ; x < 0 \end{cases}$$

Find,

- (i) the density function
 (ii) The probability that $X > 2$, and
 (iii) The probability that $-3 < X \leq 4$

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