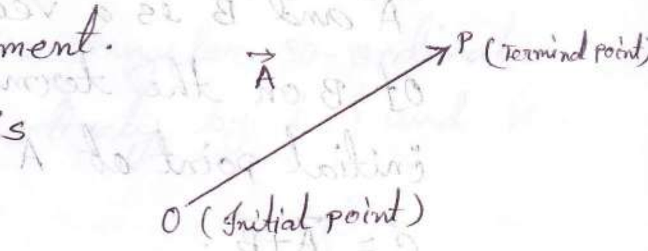
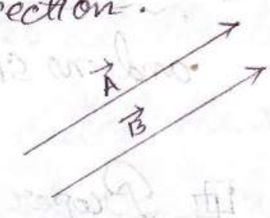
Vector Analysis

VECTOR :- A vector is a quantity having both magnitude and direction. Such as displacement.
The end of the arrow point O is called initial point & head P is called terminal point.



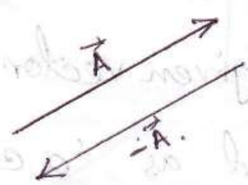
Scalar :- A scalar is a quantity having magnitude but no direction. Such as mass, length, time temperature and any real number.

Equal Vector :- Two vectors $\vec{A}$ and $\vec{B}$ are said to be equal of they have the same magnitude and direction.
Symbolically $\vec{A} = \vec{B}$

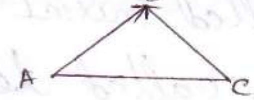


Negetive Vector :-

A vector having direction opposite to that of a vector $\vec{A}$ but having the same magnitude is denoted by $-\vec{A}$.



□ Sum of two vector:- The sum of resultant of vectors $\vec{A}$ and $\vec{B}$ is a vector formed by placing the initial point of B on the terminal point of A and then joining the initial point of $\vec{A}$ to the terminal point of B. Thus $\vec{C} = \vec{A} + \vec{B}$.



□ Difference of two vectors:- The difference of vectors $\vec{A}$ and $\vec{B}$ represented by $\vec{A} - \vec{B}$ is that a vector $\vec{c}$ which is added to $\vec{B}$ yield vector $\vec{A}$

$$\vec{C} = \vec{A} - \vec{B}$$

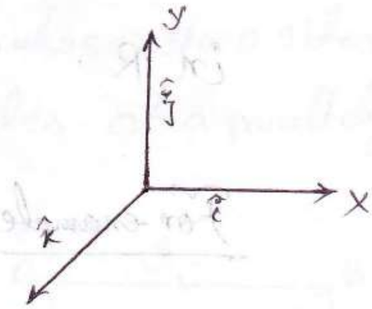
□ Null vector / zero vector:- A vector which has zero magnitude and no specific direction is called null or zero vector.

□ Proper vector:- A vector which is not null is a proper vector.

□ Unit vector:- A unit vector is a vector having unit magnitude. And having the same direction as that of a given vector $\vec{a}$ is usually denoted by the symbol $\hat{a}$ and read as 'a cap', Then we have $\hat{a} = \frac{a}{|a|}$.

(আয়ত-একক-ভেক্টর)

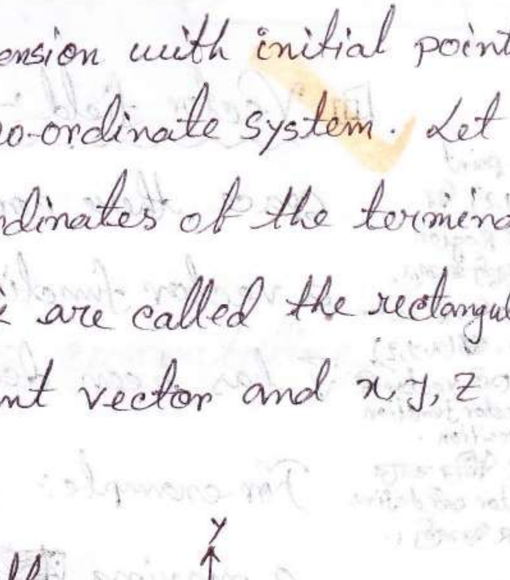
Rectangular Unit Vector:- An important set of unit vectors are those having the direction of the positive x, y and z axis of a three dimentional rectangular co-ordinate system and are dinoted respectively by $\hat{i}$, $\hat{j}$ and $\hat{k}$.



Component of a vector:-

Any vector of A & B dimension with initial point at the origin O of a rectangular co-ordinate system. Let (A_1, A_2, A_3) be the rectangular co-ordinates of the terminal point at O. The vector $A_1\hat{i}$, $A_2\hat{j}$, $A_3\hat{k}$ are called the rectangular component vector or simply component vector and x, y, z are the directions respectively.

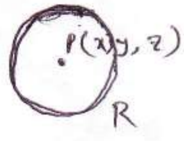
The sum of $A_1\hat{i}$, $A_2\hat{j}$, $A_3\hat{k}$ is the vector $\vec{A}$. So that we can write



$\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$.

The magnitude of A is $= |\vec{A}| = \sqrt{A_1^2 + A_2^2 + A_3^2}$.

Scalar field:- If to each point (x,y,z) of a region R in space there corresponds a number or scalar $\phi(x,y,z)$. Then ϕ is called a scalar function of position and also called that the scalar field ϕ has been defined in R.



For example:-

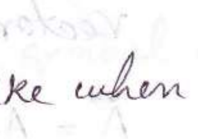
The temperature at any point on the earth surface at a certain time defining a scalar field.

Vector field:- If to each point (x,y,z) of a Region R in space there corresponds a vector $\vec{v}(x,y,z)$ then $\vec{v}$ is called a vector function of position and also called that a vector $\vec{v}$ has been defined in R.

কোন point (x,y,z) এর একটি Region এর মধ্যে থাকলে, যেখানে একটি vector থাকে। (x,y,z). তখন সেই vector টি a vector function of position. এবং তাকে বলবে vector যেটা define হয় R এর মধ্যে।

For example: If the velocity at any point (x,y,z) within a moving ~~[illegible]~~ fluid is known at a certain time, then a vector field is defined.

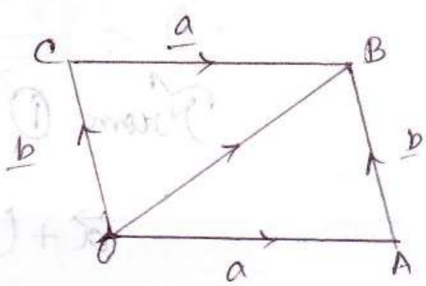
Like vector:- Vectors are said to be like when they have the same sense of direction.



(ii) Commutative law of addition:-

The commutative law of vector addition is normally represent that, $\vec{a}+\vec{b} = \vec{b}+\vec{a}$. where $\vec{a}$ and $\vec{b}$ are two vectors. If $\vec{OA} = \vec{a}$ and $\vec{AB} = \vec{b}$. Then $\vec{OB} = \vec{a}+\vec{b}$.

Now complete the parallelogram whose two sides are OA and AB. Since the opposite sides of a parallelogram are equal and parallel.



We say that,

$\underline{OA} = \underline{CB} = \underline{a}$

and $\underline{AB} = \underline{OC} = \underline{b}$.

$\underline{OA} + \underline{AB} = \underline{OB} = \underline{OC} + \underline{BC}$.

$\Rightarrow \underline{a} + \underline{b} = \underline{b} + \underline{a}$.

$\therefore$ The vector addition is commutative

Associative law of addition :-

Let $\vec{OA} = \vec{a}$, $\vec{AB} = \vec{b}$, $\vec{BC} = \vec{c}$.

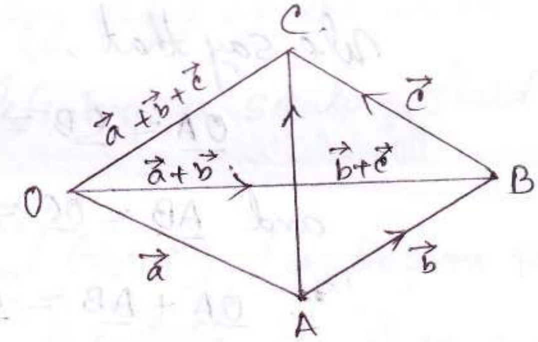
be any three vectors. Then using triangle law of addition of vectors, we have,

$$\vec{a} + (\vec{b} + \vec{c}) = \vec{OA} + (\vec{AB} + \vec{BC})$$
$$= \vec{OA} + \vec{AC}$$
$$= \vec{OC}$$

From (i) & (ii) we get,

$$\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}.$$

(Proved)



MORTUZA

Mathematics

"Vector"

Supplementary Problems:

57. a) $A = 3\hat{i} + 2\hat{j} - 6\hat{k}$ Find the angle between this two →

$B = 4\hat{i} - 3\hat{j} + \hat{k}$

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB} = \frac{(3\hat{i}+2\hat{j}-6\hat{k})\cdot(4\hat{i}-3\hat{j}+\hat{k})}{\sqrt{(3)^2+(2)^2+(-6)^2}\sqrt{(4)^2+(-3)^2+(1)^2}}$$

$$\cos\theta = \frac{0}{\sqrt{49}\cdot\sqrt{26}} = 0$$

$$\theta = 90^\circ$$

b) $C = 4\hat{i} - 2\hat{j} + 4\hat{k}$

$D = 3\hat{i} - 6\hat{j} + 2\hat{k}$

$$\vec{C}\cdot\vec{D} = CD\cos\theta$$

$$\cos\theta = \frac{\vec{C}\cdot\vec{D}}{CD} = \frac{(4\hat{i}-2\hat{j}+4\hat{k})\cdot(3\hat{i}-6\hat{j}-2\hat{k})}{\sqrt{(4)^2+(-2)^2+(4)^2}\sqrt{(3)^2+(6)^2+(-2)^2}}$$

$$\cos\theta = \frac{12+12-8}{6\cdot 7}$$

$$\cos\theta = \frac{\cancel{16}\,8}{\cancel{42}\,21} = \frac{8}{21}$$

$$\theta = \cos^{-1}\frac{8}{21}$$ Ans

(58) For what values

$A = a\hat{i} \cancel{+} 2\hat{j} + \hat{k}$ & $B = 2a\hat{i} + a\hat{j} - 4\hat{k}$ is Perpendicular

Perpendicular হওয়ার জন্য $A \cdot B = 0$ $\perp$

$$\vec{A} \cdot \vec{B} = 0$$

$$(a\hat{i} - 2\hat{j} + \hat{k}) \cdot (2a\hat{i} + a\hat{j} - 4\hat{k}) = 0$$

$$2a^2 - 2a - 4 = 0$$

$$a^2 - a - 2 = 0$$

$$a^2 - 2a + a - 2 = 0$$

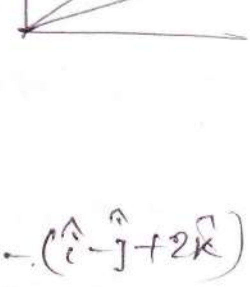
$$a(a-2) + 1(a-2) = 0$$

$$(a-2)(a+1) = 0$$

$\therefore a = 2, -1.$

Ans

(60) Find the direction cosine of the line joining the points $(3, 2, -4)$ & $(1, -1, 2)$.



Let,

$$P = 3\hat{i} + 2\hat{j} - 4\hat{k}$$

$$Q = \hat{i} - \hat{j} + 2\hat{k}$$

~~Direction co~~

joining the point $= (3\hat{i} + 2\hat{j} - 4\hat{k}) - (\hat{i} - \hat{j} + 2\hat{k})$

$= 2\hat{i} + 3\hat{j} - 6\hat{k}$.

Direction cosine of the line $\rightarrow \dfrac{2\hat{i}+3\hat{j}-6\hat{k}}{\sqrt{(2)^2+(3)^2+(-6)^2}}$

$$= \left(\frac{2}{7}\hat{i}+\frac{3}{7}\hat{j}-\frac{6}{7}\hat{k}\right)$$

$\therefore$ Points: $\frac{2}{7}, \frac{3}{7}, \frac{-6}{7}$. Ans.

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(63) Find the projection of vector $2\hat{i}-3\hat{j}+6\hat{k}$ on the vector $\hat{i}+2\hat{j}+2\hat{k}$.

Let, $\vec{A} = 2\hat{i}-3\hat{j}+6\hat{k}$
$\vec{B} = \hat{i}+2\hat{j}+2\hat{k}$.

$\therefore$ Projection of vector $= \dfrac{\vec{A}\cdot\vec{B}}{|\vec{B}|}$

$$= \frac{(2\hat{i}-3\hat{j}+6\hat{k})\cdot(\hat{i}+2\hat{j}+2\hat{k})}{\sqrt{1+4+4}}$$

$$= \frac{2-6+12}{3} = \frac{8}{3} \text{ Ans}$$

(64) Find the projection of a vector $4\hat{i}-3\hat{j}+\hat{k}$ on the line passing through the points $(2,3,-1)$ & $(-2,-4,3)$.

Let, $\vec{A} = 4\hat{i}-3\hat{j}+\hat{k}$

Passing through the points $= (-2-2)\hat{i}+(-4-3)\hat{j}+(3+1)\hat{k}$

$$= -4\hat{i}-7\hat{j}+4\hat{k}.$$

Let it $= \vec{B} = -4\hat{i}-7\hat{j}+4\hat{k}$

Projection of this goes to $= \dfrac{\vec{A}\cdot\vec{B}}{|\vec{B}|}$

$$= \frac{(4\hat{i}-3\hat{j}+\hat{k})\cdot(-4\hat{i}-7\hat{j}+4\hat{k})}{\sqrt{(-4)^2+(-7)^2+(4)^2}}$$

$$= \frac{-16+21+4}{\sqrt{16+49+16}}$$

$$= \frac{9}{\sqrt{81}} = \frac{9}{9} = 1 \text{ Ans.}$$

(65) If $A = 4\hat{i}-\hat{j}+3\hat{k}$ & $B = -2\hat{i}+\hat{j}-2\hat{k}$, find a unit vector perpendicular to both A & B.

Let, C is a unit vector perpendicular to both A & B.

$$A\times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 3 \\ -2 & 1 & -2 \end{vmatrix} = \hat{i}(2-3)+\hat{j}(-6+8)+\hat{k}(4-2)$$
$$= (-\hat{i}+2\hat{j}+2\hat{k})$$

$$|A\times B| = \sqrt{(-1)^2+(2)^2+(2)^2} = \sqrt{1+4+4} = \sqrt{9} = 3.$$

$\therefore$ Unit vector of perpendicular both A & B

$$= \pm\frac{-\hat{i}+2\hat{j}+2\hat{k}}{3}$$

Ans.

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69. Find the work done in moving an object along a straight line from $(3,2,-1)$ to $(2,-1,4)$ in a force field given by $\vec{F} = 4\hat{i} - 3\hat{j} + 2\hat{k}$.

Moving along a straight line the point $= (2-3)\hat{i} + (-1-2)\hat{j} + (4+1)\hat{k}$

$= -\hat{i} - 3\hat{j} + 5\hat{k}$.

Let the point ~~$\vec{r} = -\hat{i} + 3\hat{j} - 5\hat{k}$~~ $r = -\hat{i} - 3\hat{j} + 5\hat{k}$.

$\therefore$ Work done $= \vec{F} \cdot \vec{r}$

$= (4\hat{i} - 3\hat{j} + 2\hat{k}) \cdot (-\hat{i} - 3\hat{j} + 5\hat{k})$

$= \cancel{(4-9-10)}\ (-4-9+10)$

$= +15$ $\therefore$ Ans.

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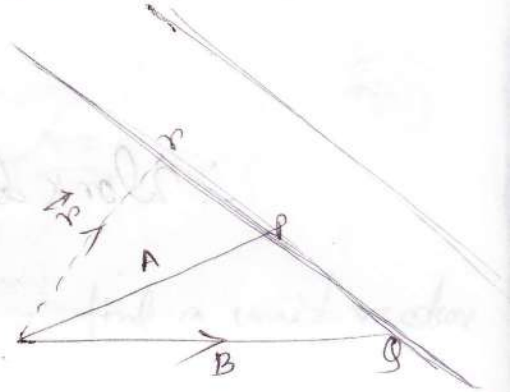
Basic:-

77. Given that, $A = 3\hat{i} + \hat{j} + 2\hat{k}$, $B = \hat{i} - 2\hat{j} - 4\hat{k}$ are the position vectors of points P & Q respectively.

a) Find the eq^n for the plane passing through Q and perpendicular to line PQ.

b) What is the distance from the point $(-1, 1, 1)$ to the plane



Ans: $\boxed{a}$

Let the position vector

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$\therefore$ Though the plane passing through $\underset{B}{Q}$ and perpendicular to line PQ so eq^n is $(\vec{r} - \vec{B}) \cdot (\vec{A} - \vec{B}) = 0$.

$$(\vec{r} - \vec{B}) = (x\hat{i} + y\hat{j} + z\hat{k}) - (\hat{i} - 2\hat{j} - 4\hat{k})$$

$$= (x-1)\hat{i} + (y+2)\hat{j} + (z+4)\hat{k}$$

$$(\vec{A} - \vec{B}) = (3-1)\hat{i} + (1+2)\hat{j} + (2+4)\hat{k}$$

$$= 2\hat{i} + 3\hat{j} + 6\hat{k}.$$

$$\therefore \text{eq}^n = (\vec{r} - \vec{B}) \cdot (\vec{A} - \vec{B}) = 0$$

$$= (x-1) \cdot 2 + (y+2) \cdot 3 + (z+4) \cdot 6 = 0$$

$$\dots 3y + 6 + 6z + 24 = 2x + 3y + 6z + 28 = 0$$

b) Distance from $(-1,1,1)$ to the plane is $\Rightarrow$

$2x+3y+6z+28=0 \longrightarrow$ eqn/plane.

$\therefore$ Distance

$$\frac{2.-1+3.1+6.1+28}{\sqrt{(2)^2+(3)^2+(6)^2}} = \frac{-2+3+6+28}{\sqrt{49}}$$

$$= \frac{35}{7} = 5.$$

82. Find the area of a parallelogram having diagonals $\bar{A}=3\hat{i}+\hat{j}-2\hat{k}$ and $\vec{B}=\hat{i}-3\hat{j}+4\hat{k}$.

$\vec{A}=3\hat{i}+\hat{j}-2\hat{k}$.
$\vec{B}=\hat{i}-3\hat{j}+4\hat{k}$.

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix} = i(4-6)+j(2-12)+k(-9-1)$$

$$= -2\hat{i}-14\hat{j}-10k.$$

$|A\times B| = 10\sqrt{3}$.

Area of parallelogram $= \frac{1}{2}|A\times B| = \frac{1}{2}\times 10\sqrt{3}$.

$= 5\sqrt{3}$.

Ans

83. Find the area of a triangle with vertices at $(3,-1,2)$, $(1,-1,-3)$ & $(4,-3,1)$.

Let the points: $P(3,-1,2)$, $Q(1,-1,-3)$, $R(4,-3,1)$

$PQ = (1-3)\hat{i} + (-1+1)\hat{j} + (-3-2)\hat{k}$

$= -2\hat{i} + 0\hat{j} - 5\hat{k} = -2\hat{i} - 5\hat{k}$.

$PR = (4-3)\hat{i} + (-3+1)\hat{j} + (1-2)\hat{k}$.

$= \hat{i} - 2\hat{j} - \hat{k}$

$$PQ \times PR = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & -5 \\ 1 & -2 & -1 \end{vmatrix} = -10\hat{i} - 7\hat{j} + 4\hat{k}$$

$$|PQ \times PR| = \sqrt{(-10)^2 + (-7)^2 + (4)^2} = \sqrt{100+49+16} = \sqrt{165}$$

$\therefore$ Area of a triangle $= \frac{1}{2}|PQ \times PR|$

$= \frac{1}{2} \times \sqrt{165}$ Ans

(84) If $A = 2\hat{i} + \hat{j} - 3\hat{k}$ and $B = \hat{i} - 2\hat{j} + \hat{k}$. find a vector of magnitude 5 perpendicular to both A & B.

~~Ans - 5~~

$$A \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -3 \\ 1 & -2 & 1 \end{vmatrix} = \hat{i}(1-6) + \hat{j}(-3-2) + (-4-1)\hat{k}$$

$= -5\hat{i} - 5\hat{j} - 5\hat{k}$.

$\therefore |\vec{A} \times \vec{B}| = \sqrt{(-5)^2 + (-5)^2 + (-5)^2} = \sqrt{75} = 5\sqrt{3}$.

Let C is a vector perpendicular to both A & B.

$$C = \pm 5 \cdot \frac{A \times B}{|A \times B|} = \pm \cancel{5} \cdot \frac{\times 5(\hat{i}+\hat{j}+\hat{k})}{\cancel{5}\sqrt{3}} = \frac{5\sqrt{3}(\hat{i}+\hat{j}+\hat{k})}{\sqrt{3} \times \sqrt{3}}.$$

$$= \pm \frac{5\sqrt{3}(\hat{i}+\hat{j}+\hat{k})}{3}$$ Ans

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(86) $F = 3\hat{i} + 2\hat{j} - 4\hat{k}$ is applied at the point $(1,-1,2)$, Find the <u>moment</u> of F about the point $(2,-1,3)$

$F \times r$

Let, $P(1,-1,2)$ $\qquad \vec{F} = 3i + 2j - 4k$

$Q(2,-1,3)$. $\qquad \vec{r} = i + k$.

$\therefore \vec{r} = PQ = (2-1)\hat{i} + (-1+1)\hat{j} + (3-2)\hat{k} = \hat{i} + \hat{k}$

$$\text{Moment} = \vec{F} \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & -4 \\ 1 & 0 & 1 \end{vmatrix}$$

$$= i(2-0) + j(-4-3) + k(0-2)$$

$$= 2\hat{i} - 7\hat{j} - 2\hat{k}$$ Ans

(90) Find the volume of the parallelepiped whose edges are represented by $A = 2\hat{i} - 3\hat{j} + 4k$

$B = i + 2\hat{j} - k$

$C = 3i - j + 2k$.

Volume of the parallelepiped is $A \cdot (B \times C)$.

$$\therefore B \times C = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 3 & -1 & 2 \end{vmatrix}$$

$$= i(4-1) - j(2+3) + \hat{k}(-1-6)$$

$$= 3i - 5j + 7k$$

$$= 3i - 5j - 7k.$$

$\therefore$ Volume of parallelepiped is $= A \cdot (B \times C)$

$$(2i - 3j + 4k) \cdot (3\hat{i} - 5j - 7k)$$

$$= 6 + 15 - 28 = -7$$

$\therefore$ Ans: 7 unit.

Parallelopied → $A \cdot (B \times C)$

Co-planar → $A \cdot (B \times C) = 0$

(92) Find the constant ~~of~~ a such that the vectors $2i-j+k$, $\hat{c}+2j-3k$ and $3i+aj+5k$ are coplanar.

The formulla of co-planar is $A\cdot(B\times C)=0$

$\therefore$ Let, $A=2\hat{i}-j+k$, $B=i+2j-3k$, $C=3i+aj+5k$.

$$\therefore B\times C=\begin{vmatrix} i & j & k \\ 1 & 2 & -3 \\ 3 & a & +5 \end{vmatrix}$$

$$= i(10+3a)-j(5+9)+k(a-6)$$

$\therefore$ Co-planar $= A\cdot(B\times C)=0$.

$$\Rightarrow (2i-j+k)\{(10+3a)i-14j+(a-6)k\}=0$$

$$\Rightarrow 20+6a+14+a-6=0$$

$$\Rightarrow 28+7a=0$$

$$\Rightarrow a=-4.$$ Ans

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<u>Question → 2000</u>

Prove that, $A\cdot(B\times C) = B\cdot(C\times A) = C\cdot(A\times B)$

we know, $A\cdot(B\times C) = \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}$

By the theor on determinants which states that interchange of two rows of a determinant changes its sign we have,

প্রথমে রাখব $B\cdot(C\times A)$ তারপর B উপরে C পরে, পরে A

$$\begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix} = -\begin{vmatrix} B_1 & B_2 & B_3 \\ A_1 & A_2 & A_3 \\ C_1 & C_2 & C_3 \end{vmatrix} = \begin{vmatrix} B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \\ A_1 & A_2 & A_3 \end{vmatrix}$$

Then

$C(A\times B)$ C উপরে পরে A, পরে B

$$\begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix} = -\begin{vmatrix} C_1 & C_2 & C_3 \\ B_1 & B_2 & B_3 \\ A_1 & A_2 & A_3 \end{vmatrix} = \begin{vmatrix} C_1 & C_2 & C_3 \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix}$$

$$\therefore\ A\cdot(B\times C) = B\cdot(C\times A) = C\cdot(A\times B)$$

<u>Question: 2006</u> Prove that the area of parallelogram with sides A & B is $|A\times B|$.

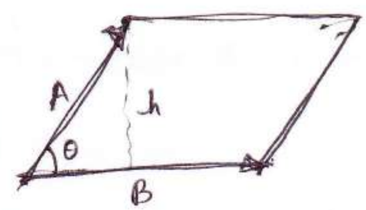
$\sin\theta = \frac{h}{A}$

$h = |A|\sin\theta$

Area of parallelogram $= h\,|B|$

$= |A|\sin\theta\,|B|$

$= |A\times B|$.



The area of the triangle with sides A & B

is $= \frac{1}{2}|A\times B|$

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BOARD SOLUTION

:VECTOR:-

MATH-08: Find a unit vector parallel to the resultant vectors $\vec{r}_1 = 2\vec{i}+4\vec{j}-5\vec{k}$, $\vec{r}_2 = \vec{i}+2\vec{j}+3\vec{k}$.

Resultant vector = $\vec{r} = \vec{r}_1 + \vec{r}_2$

$$\vec{r} = (2\vec{i}+4\vec{j}-5\vec{k}+\vec{i}+2\vec{j}+3\vec{k})$$

$$\vec{r} = 3\vec{i}+6\vec{j}-2\vec{k}.$$

$\frac{\vec{r}}{|\vec{r}|} = \frac{\vec{r}_1 + \vec{r}_2}{|\vec{r}|}$

∴ Unit vector parallel to the resultant vector

$$= \frac{\vec{r}}{|\vec{r}|} = \frac{3\vec{i}+6\vec{j}-2\vec{k}}{\sqrt{9+36+4}} = \frac{3}{7}i + \frac{6}{7}j - \frac{2}{7}k.$$

MATH-08 Find the constant 'a' such that the vector $2i-j+k$, $\vec{i}+2\vec{j}-3\vec{k}$ and $3\vec{i}+a\vec{j}+5\vec{k}$ are coplanar.

The basic of co-planar is = $A\cdot(B\times C)=0$

Let,

$A = 2i-j+k$

$B = i+2j-3k$

$C = 3i+aj+5k$

$$\therefore B\times C = \begin{vmatrix} i & j & k \\ 1 & 2 & -3 \\ 3 & a & 5 \end{vmatrix}$$

$$= [i(10+3a) - j(5+9) + k(a-6)]$$

$$= [i(10+3a) - 14j + k(a-6)]$$

Co-planar.

$\therefore A\cdot(B\times C) = 0.$

$= (2i-j+k)\cdot[i(10+3a)-14j+k(a-6)] = 0$

$= 20+6a+14+a-6 = 0 \Rightarrow a = -4.$ Ans.

MATH⇒08 Find the projection of the vector $\vec{A} = \hat{i} - 2\hat{j} + \hat{k}$ on the vector $\vec{B} = 4\hat{i} - 4\hat{j} + 7\hat{k}$

$$\therefore \text{Projection} = \frac{A \cdot B}{|B|}$$

$$= \frac{(\hat{i} - 2\hat{j} + \hat{k}) \cdot (4\hat{i} - 4\hat{j} + 7\hat{k})}{\sqrt{16 + 16 + 49}}$$

$$= \frac{4 + 8 + 7}{9} = \frac{19}{9} \quad \text{Ans}$$

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মোর্তুজা
23/04/05.

Question : 2006.

The DOT OR Scalar Product :-

The DOT or scaler product of two vector A & B denoted by A.B is defined as the product of the magnitudes of A & B and the cosine of the angle θ between them.

$$\therefore A \cdot B = AB\cos\theta$$

The CROSS OR Vector Product:-

The cross or vector product of two vector A & B is denoted by A×B. and it is defined as the product of magnitude of A & B of the sine of angle θ between them.

$$\therefore A\times B = AB\sin\theta$$

The direction of vector $C = A\times B$ is perpendicular to the plane of A & B. and such that A, B, C form a right handed system.

$$A\times B = AB\sin\theta\, u . \quad 0 < \theta < \pi$$

Question : 2000

$A\cdot(B\times C) = B\cdot(C\times A) = C\cdot(A\times B)$ = volume of a parallelopiped having A, B & C as edges, or the negative of this volume according as A, B & C do or do not form a right handed system. If

$A = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$ $\qquad B\times C = \begin{vmatrix} i & j & k \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}$

$B = B_1\hat{i} + B_2\hat{j} + B_3\hat{k}$ $\qquad$ then $A\cdot(B\times C) = (A_1 i + A_2 j + A_3 k)\begin{vmatrix} i & j & k \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}$

$C = C_1\hat{i} + C_2\hat{j} + C_3\hat{k}$

[illegible]

$$A\cdot(B\times C) = \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}$$ [proved]

Associated laws for cross product failed.

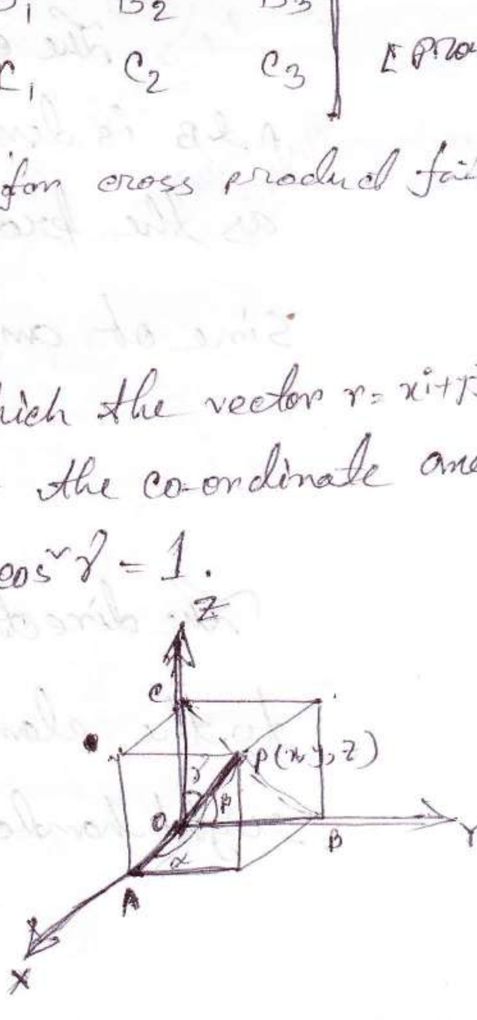
Question : 2004

Determine the angle α, β, γ which the vector $r = xi + yj + zk$ makes with positive directions of the co-ordinate axes and show that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.

$\Delta OAP \rightarrow \cos\alpha = \frac{x}{|r|}$

$\Delta OBP \rightarrow \cos\beta = \frac{y}{|r|}$

$\Delta OCP \rightarrow \cos\gamma = \frac{z}{|r|}$



Also $|\vec{r}| = r = \sqrt{x^2+y^2+z^2}$.

Then $\cos\alpha = x/r$, $\cos\beta = y/r$, $\cos\gamma = z/r$ from which α, β, γ can be obtained. From these it follows that

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{x^2+y^2+z^2}{r^2} = 1.$$

The number of $\cos\alpha, \cos\beta, \cos\gamma$ are called direction Cosine of the vector OP.

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100% Imp

(68) show that $A = \frac{(2i-2j+K)}{3}$, $B = \frac{(i+2j+2K)}{3}$ and $C = \frac{2i+j-2K}{3}$ are mutually orthogonal unit vectors.

They are mutually orthogonal unit vector.

$A \perp B$, $B \perp C$, $C \perp A$

$$A \cdot B = \left(\frac{2i-2j+K}{3}\right) \cdot \left(\frac{i+2j+2K}{3}\right)$$

$$= \frac{1}{9} \cdot (2-4+2) = 0.$$

$$B \cdot C = \left(\frac{i+2j+2K}{3}\right) \cdot \left(\frac{2i+j-2K}{3}\right)$$

$$= \frac{1}{9}(2+2-4) = 0.$$

Hence the three vectors are orthogonal unit vector

proved.

Question :- 2004

* Show that $i+j+k$, $i-k$ and $i-2j+k$ are mutually orthogonal. Find x, y and z if $(i+j+2k)$; $(-i+zk)$ and $(2i+xj+yk)$ are mutually orthogonal.

$$A \perp B, \quad B \perp C$$

$$A \cdot B = (i+j+k)\cdot(i-k)$$
$$= 1+0-1 = 0$$

$$B \cdot C = (i-k)\cdot(i-2j+k)$$
$$= 1+0-1 = 0.$$

$\therefore$ Both are mutually orthogonal.

$A \perp B$; $B \perp C$; $A \perp C$.

Then,
$$A = i+j+2k$$
$$B = -i+zk.$$
$$C = 2i+xj+yk$$

$A \perp B$, $B \perp C$, $A \perp C$ Mutually orthogonal

$$A \cdot B = (i+j+2k)\cdot(-i+zk)$$
$$= -1+0+2z$$
$$= 2z-1.$$

$$2z-1=0$$
$$z = 1/2$$

$$B \cdot C = (-i+zk)\cdot(2i+xj+yk)$$
$$= -2+yz.$$

$$yz-2=0$$
$$y \times 1/2 - 2 = 0$$
$$\frac{y}{2} - 2 = 0$$
$$y-4=0$$
$$y=4$$

$$A \cdot C = (i+j+2k)\cdot(2i+xj+yk)$$
$$\Rightarrow 2+x+2y=0.$$
$$\Rightarrow 2+x+2\times 4=0 \Rightarrow x=-10.$$

$\therefore x, y, z = (-10, 4, 1/2)$. Ans

$$A\cdot(B\times C) = B\cdot(C\times A) = C\cdot(A\times B)$$

$$A\cdot(B\times C) = \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}$$

$$\begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix} = -\begin{vmatrix} B_1 & B_2 & B_3 \\ A_1 & A_2 & A_3 \\ C_1 & C_2 & C_3 \end{vmatrix} = \begin{vmatrix} B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \\ A_1 & A_2 & A_3 \end{vmatrix}$$

$$\begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix} = -\begin{vmatrix} C_1 & C_2 & C_3 \\ B_1 & B_2 & B_3 \\ A_1 & A_2 & A_3 \end{vmatrix} = \begin{vmatrix} C_1 & C_2 & C_3 \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix}$$

$A+B$ $(\hat{i}+\hat{j}+k)\cdot(i-2j) = 0$

$B+C$ $(i+j+2k)\cdot(-i+2k)$

$(-1+2z)=0$

$z = 1/2$

$(-i+2k)\cdot(2i+xj+yk)$

$\Rightarrow -2+yz=0$

$\Rightarrow -2+y\times\frac{1}{2}=0$

$= -4+y=0$

$y=4.$

$$A\cdot C = (i+j+2k)\cdot(2i+xj+yk)$$
$$= (2+x+2y)=0$$
$$\Rightarrow 2+x+8=0$$
$$x = -10$$

$x=-10,\ y=4,\ z=1/2$ Ans