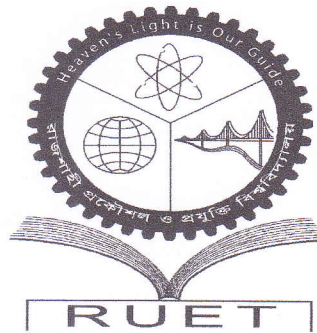


HEAVEN'S LIGHT IS OUR GUIDE

Rajshahi University Of Engineering And Technology

Rajshahi

Department of computer science and engineering



Class Assignment

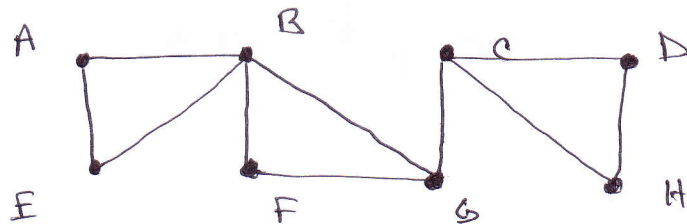
Subject: Discrete Mathematics

SUBMITTED BY,

NAME: Shaikh Md. Abu Hasan

Class: 2nd year, 3rd sem

Roll: 083043

Graph theory:Problem 2.33:

(a) The degree of each vertex:

$$A = 2$$

$$D = 3$$

$$G = 3$$

the sum of the degree = 21

$$B = 4$$

$$E = 2$$

$$H = 2$$

total edges = 8

$$C = 3$$

$$F = 2$$

(b) Simple path (A to G)

ABG

ABFG

AEBG

AEBFG

(c) All trails from B to C

BGC

BAEBGC

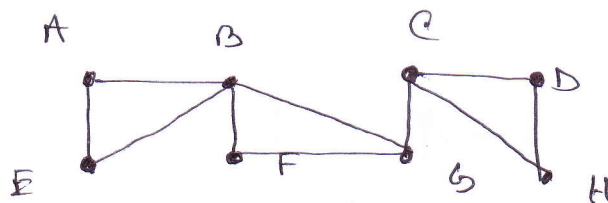
BFGC

BAEBFGC

(d) $d(A, C)$ the distance from A to C

$$\rightarrow A \rightarrow B \rightarrow G \rightarrow C = 3$$

$$(e) \text{diam}(G) = 4$$



(a) All cycles :

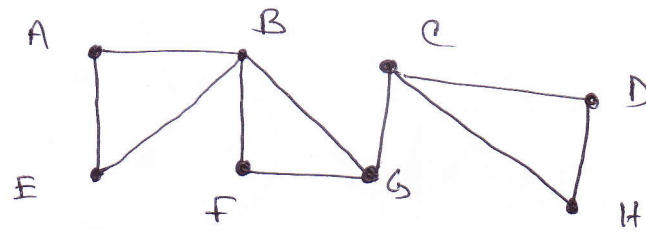
- i) ABEA
- ii) BFGB
- iii) CDHC

(b) All cut points:

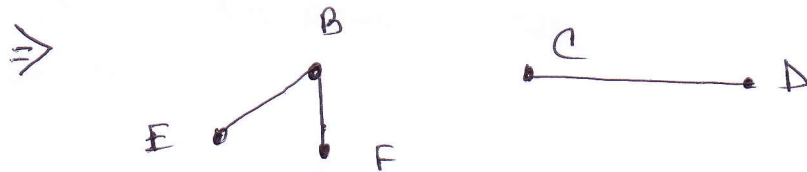
- i) B
- ii) G
- iii) C

(c) All bridges : $\{ G, C \}$

Q. 35



$$(a) V' = \{B, C, D, E, F\}$$



$$E' = \{BE, BF, CD\}$$

$$(b) V' = \{A, C, E, G, H\}$$



$$E' = \{AE, CG, CH\}$$

$$(c) V' = \{B, D, E, H\}$$



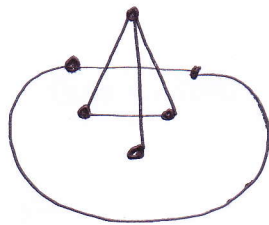
$$E' = \{BE, DH\}$$

$$(d) V' = \{C, F, G, H\}$$



$$E' = \{FG, GC, CH\}$$

em
8.36



(i)



(ii)



(iii)

(a) (iii) is connected. (ii) and (i) have 2 connected components.

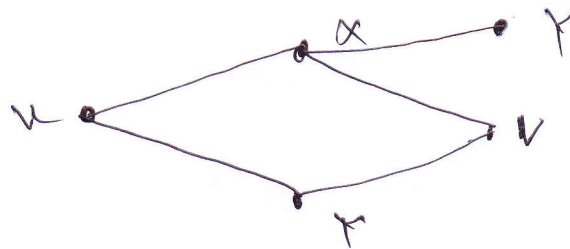
(b) none is cycle free.

(c) (i) and (iii) is loop free.

Problem 8.37:

Suppose a graph G contains two distinct paths from a vertex u to a vertex v . Show that G has a cycle.

If we consider a graph G :

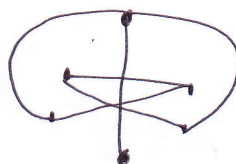
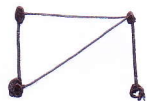


Here, two way to go from u to v . 1) $u \rightarrow x \rightarrow v$ 2) $u \rightarrow y \rightarrow v$

And there is a cycle $u \rightarrow y \rightarrow v \rightarrow x \rightarrow u$

Problem 8.39 :

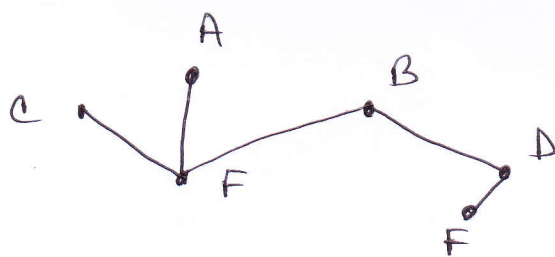
If we consider some connected graph then,



Here we can see that every graph having n number of vertices and all of them consist at least $n-1$ edges. (shown)

Problem 8.38 :

If we consider a cycle free graph G , then,



Here,

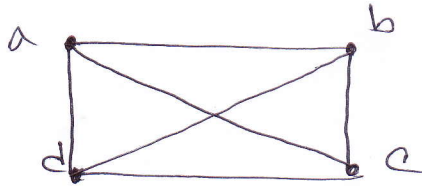
| | |
|---------------|---------------|
| $\deg(C) = 1$ | $\deg(B) = 2$ |
| $\deg(A) = 1$ | $\deg(D) = 1$ |
| $\deg(F) = 2$ | $\deg(E) = 1$ |

So, G , a finite cycle free graph with a least one edge has at least two vertices of degree 1

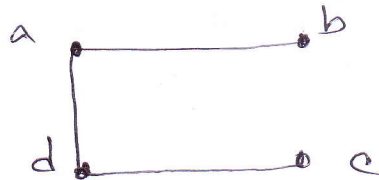
Problem 8.40

Find the number of connected graphs with four vertices.

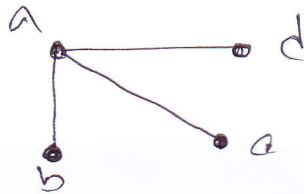
1)



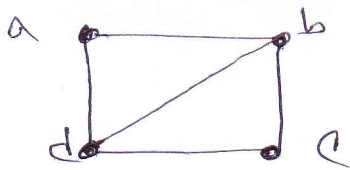
2)



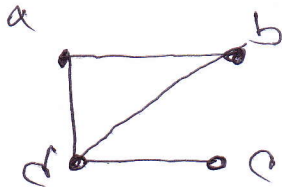
3)



4)



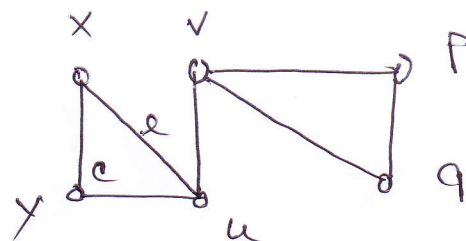
5)



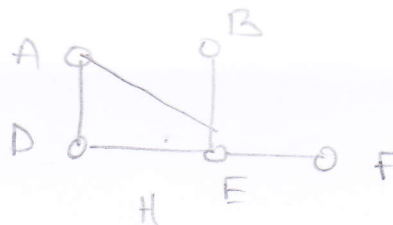
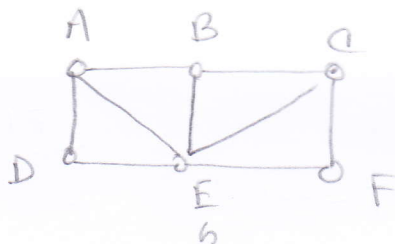
8.41

a. Consider a graph:

If we deleted e then, $G - e$ is still connected.



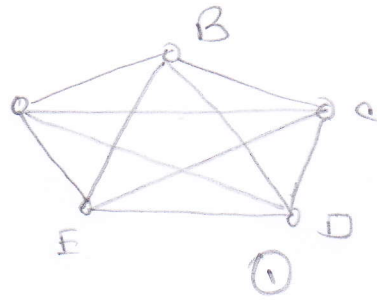
b. Let $e = \{u, v\}$. If we deleted the edge $\{u, v\}$ then the graph is disconnected. Then $x y u$ and $v p q$ are the components of $G - e$.

8.42

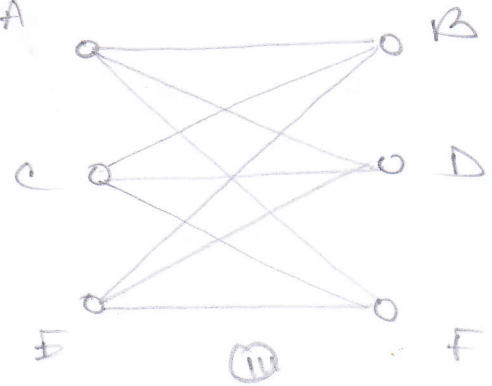
Let G is a connected graph. First we delete an edge AB . Then we delete C and all edges connected with C . Then we get a graph H , which is a subgraph of G .

Problem 8.43:

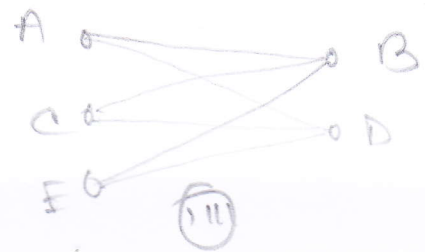
The graph is Eulerian because every vertex has even number of edges. The Eulerian path is $A B C D E A C E B D A$. A



The graph is not Eulerian because every vertex has odd number of edges.



The graph is not Eulerian. It has Eulerian path beginning in B and end in D or vice versa.



Problem 8.44: In the previous graph:

i) The Hamiltonian path or a Hamiltonian circuit will be $A B C D E A$.

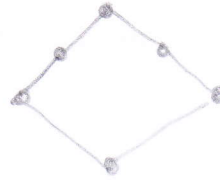
From graph, ii) The Hamiltonian path will be $A B C D E F A$.

From graph iii) It is not Hamiltonian because B or D must be visited twice in any closed path

including all vertices.

Problem 8.45 :

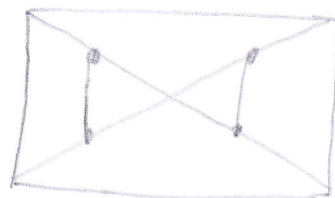
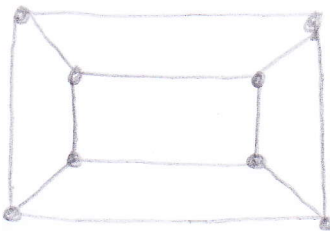
Ans: Eight.

Problem 8.46 :

Adding a vertex by dividing an edge does not change the degree of other original vertices and simply adds a vertex of degree 2. So G is Eulerian if and only if G^* is Eulerian. (shown)

Problem 8.47 :

The three-regular graphs are:



Problem 8.48:

It is not possible, because we know
 if we draw r -regular graph with 5 vertices then
 then $rs = \text{even number vertices}$.

Problem 8.51:

There are 8 such trees. They are:



Problem 8.52: 10

Problem 8.53: 15

TreesProblem: 8.51

There are eight such trees, as shown in Fig. The graph with one vertex and no edge is called trivial tree.

a

b

c

d

e

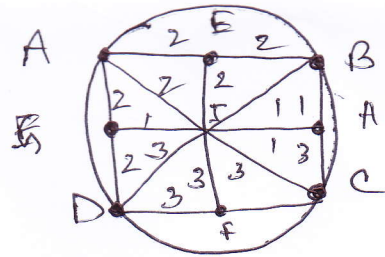
f

g

(h)

Problem 8.54

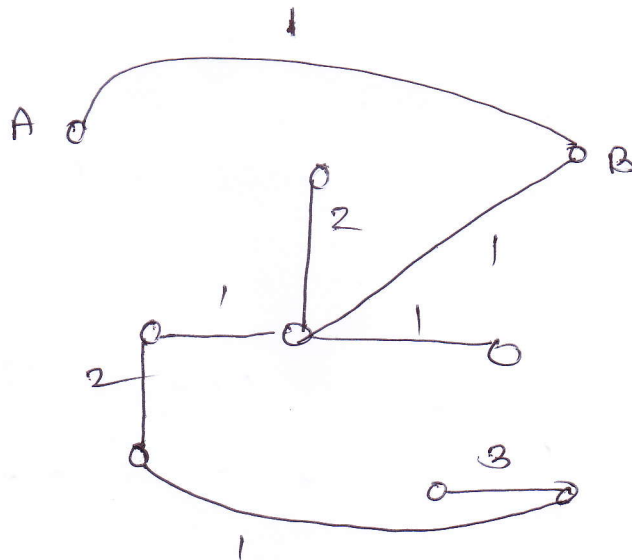
Given a weighted graph:



$$\frac{AB}{1} \quad \frac{BC}{2} \quad \frac{CD}{1} \quad \frac{AD}{3} \quad \frac{AE}{2} \quad \frac{BE}{2} \quad \frac{BH}{1}$$

$$\frac{CH}{3} \quad \frac{CF}{3} \quad \frac{BF}{3} \quad \frac{AG}{2} \quad \frac{DG}{2} \quad \frac{AI}{2} \quad \frac{DI}{3}$$

$$\frac{CI}{3} \quad \frac{BI}{1} \quad \frac{GI}{1} \quad \frac{EI}{2} \quad \frac{HI}{1} \quad \frac{FI}{3}$$

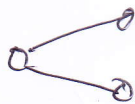


The weight of minimum spanning tree =

$$1+1+1+1+1+2+2+3=12$$

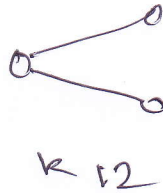
Problem 8.55:

Let a tree has three vertices and two edges.



We know, A graph G is said to be bipartite if its vertices V can be partitioned into two subsets M and N such that each edge of G connects a vertex of M to a vertex of N . Clearly the graph $K_{m,n}$ has mn edges.

If we consider one vertex is M and another two vertex N then graph has mn edges.

Problem 8.56 :Ans: $m=1$, $n=2$ Problem 8.58 :

The outside region has degree 8 and path
other two region have 5.

The map has 9 edges.

$$\therefore \text{Sum of the degrees} = 8 + 5 + 5 = 18$$

$$2E = 9 \times 2 = 18$$

\therefore Sum of the degree of the region is
equal to the twice of the number of edges.

Problem 8.59 :

a. 5, 8, 5

$$V - E + R = 5 - 8 + 5 = 2$$

b. 12, 17, 7

$$V - E + R = 12 - 17 + 7 = 2$$

c. 3, 6, 5

$$V - E + R = 3 - 6 + 5 = 2$$

d. 7, 12, 7

$$V - E + R = 7 - 12 + 7 = 2$$

Problem 8.60

a) 3

b) 3

c) 2

d) 3

Problem 8.62

a. $n = 3$

b. $n = 4$

Problem 8.63

a.

| | A | B | C | D |
|---|---|---|---|---|
| A | 0 | 1 | 0 | 1 |
| B | 1 | 0 | 1 | 1 |
| C | 0 | 1 | 0 | 1 |
| D | 1 | 1 | 1 | 0 |

b.

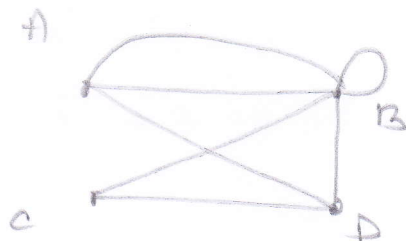
| | A | B | C | D |
|---|---|---|---|---|
| A | 0 | 1 | 2 | 0 |
| B | 1 | 0 | 1 | 1 |
| C | 2 | 1 | 0 | 0 |
| D | 0 | 1 | 0 | 0 |

c.

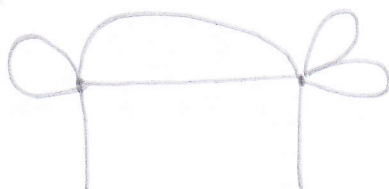
| A | A | B | C | D |
|---|---|---|---|---|
| A | 1 | 1 | 1 | 0 |
| B | 1 | 0 | 0 | 2 |
| C | 1 | 0 | 0 | 0 |
| D | 0 | 2 | 0 | 0 |

Problem 8.64:

a.



b.

Problem 8.66:

a. dist of the vertices which are appears in memory $B \rightarrow F \rightarrow A \rightarrow D \rightarrow E \rightarrow C$

$$b. G = [A: B; B: A, C, D, E; C: F; D: B, E; E: B; F: C]$$

Problem 8.67: Each vertex is adjacent to the other four vertices.

$$b. G = \left[\begin{array}{l} A: B, D, F; B: A, C, E; C: B, D, F; \\ D: A, C, E; E: B, D, F; F: A, C, E \end{array} \right]$$

$$c. G = \left[\begin{array}{l} A: B, D; B: A, C, E; C: B, D; \\ D: A, C, E; E: B, D \end{array} \right]$$

Problem 8.68:

Vertex File :Vertex $\rightarrow A \rightarrow B \rightarrow C \rightarrow D \rightarrow E \rightarrow F$ Ptr $1 \rightarrow 2 \rightarrow 9 \rightarrow 14 \rightarrow 8 \rightarrow 12$ Edge File :

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|--------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| number | 22 | 22 | 33 | 33 | 44 | 44 | 55 | 55 | 66 | 60 | 77 | 77 | 88 | 88 |
| adj | 2 | 1 | 6 | 5 | 4 | 2 | 5 | 2 | 6 | 3 | 6 | 2 | 4 | 1 |
| next | 13 | 9 | 0 | 0 | 7 | 0 | 11 | 3 | 0 | 4 | 0 | 10 | 0 | 6 |

Problem 8.69:

a. $G = \left[\begin{array}{l} A: B, E; \quad B: A, E, F, G; \quad C: D, G, H; \quad D: C, H, E; \\ A, B; \quad F: B, G; \quad G: B, C, F; \quad H: C, D \end{array} \right]$

Problem 8.70:

a. C, D, G, H, B, F, A, E
 B, A, E, F, G, C, D, H