Image Basics

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Fundamental Steps in DIP

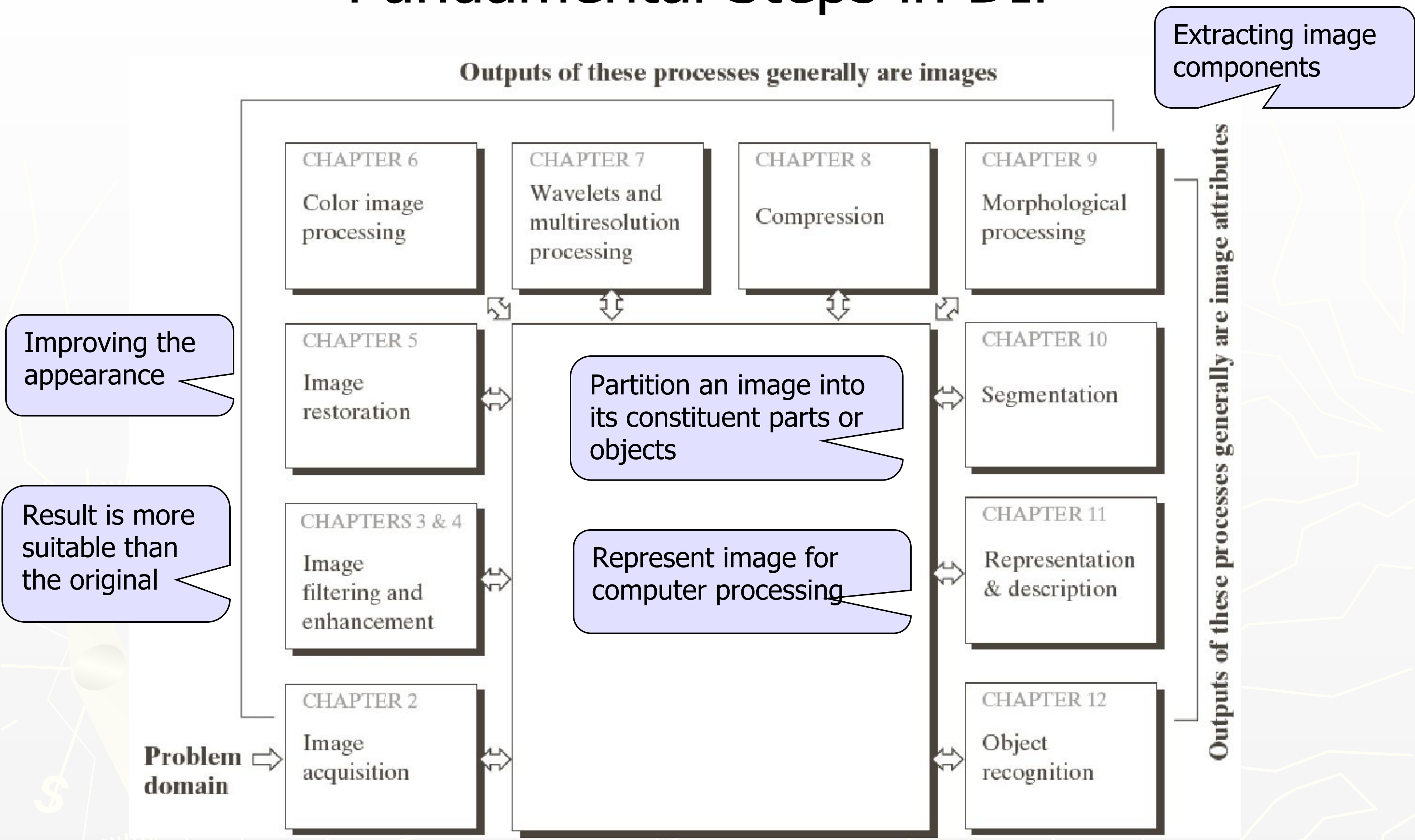


Image Interpolation

- **Interpolation** — Process of using known data to estimate unknown values

  e.g., zooming, shrinking, rotating, and geometric correction

- **Interpolation** (sometimes called ***resampling***) — an imaging method to increase (or decrease) the number of pixels in a digital image.

  Some digital cameras use interpolation to produce a larger image than the sensor captured or to create digital zoom

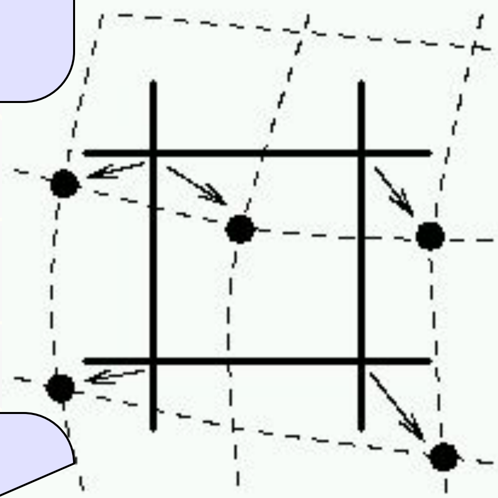
  http://www.dpreview.com/learn/?/key=interpolation

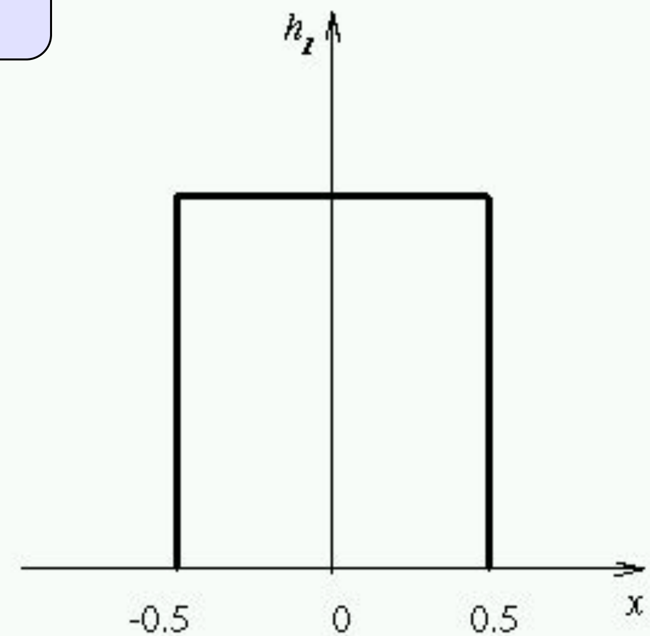
Image Interpolation:

Nearest Neighbor Interpolation

$f_1(x_2,y_2) =$
$f(\mathrm{round}(x_2), \mathrm{round}(y_2))$
$=f(x_1,y_1)$

$f(x_1,y_1)$

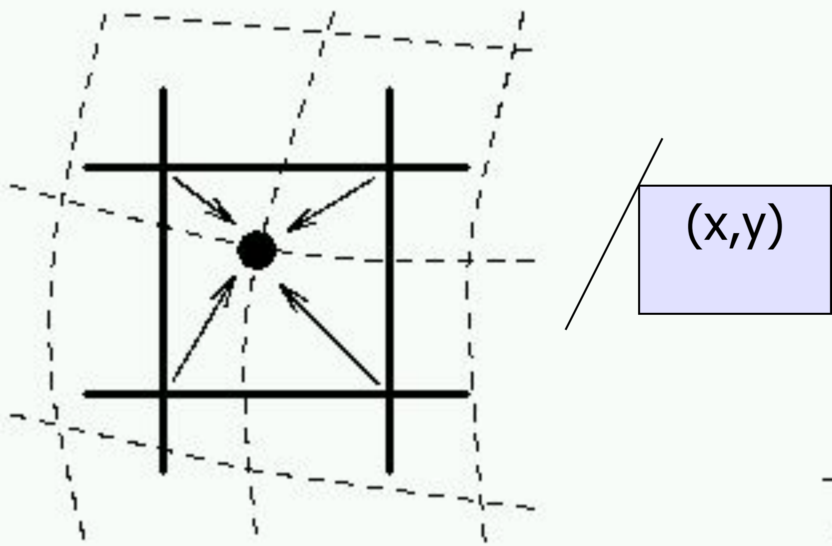


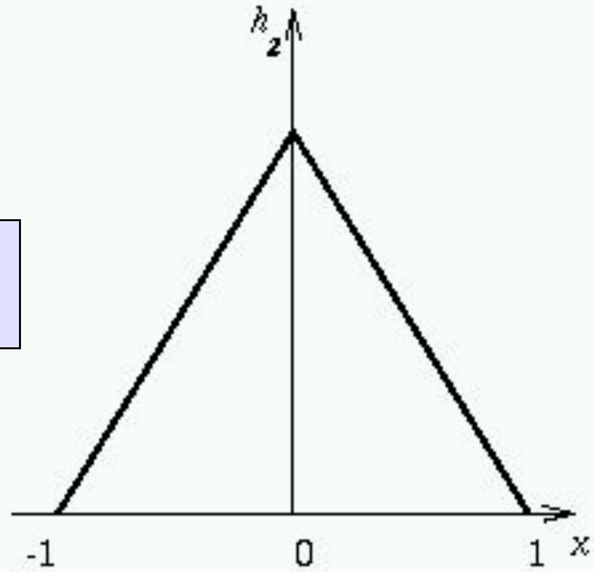


$f_1(x_3,y_3) =$
$f(\mathrm{round}(x_3), \mathrm{round}(y_3))$
$=f(x_1,y_1)$

Image Interpolation:

Bilinear Interpolation





$$f_2(x,y)$$
$$= (1-a)\cdot(1-b)\cdot f(l,k) + a\cdot(1-b)\cdot f(l+1,k)$$
$$+(1-a)\cdot b\cdot f(l,k+1) + a\cdot b\cdot f(l+1,k+1)$$
$$l = floor(x), k = floor(y), a = x-l, b = y-k.$$

Image Interpolation:
Bicubic Interpolation

- The intensity value assigned to point (x,y) is obtained by the following equation

$$f_3(x,y) = \sum_{i=0}^{3}\sum_{j=0}^{3} a_{ij} x^i y^j$$

- The sixteen coefficients are determined by using the sixteen nearest neighbors.

http://en.wikipedia.org/wiki/Bicubic_interpolation

Examples: Interpolation

Examples: Interpolation

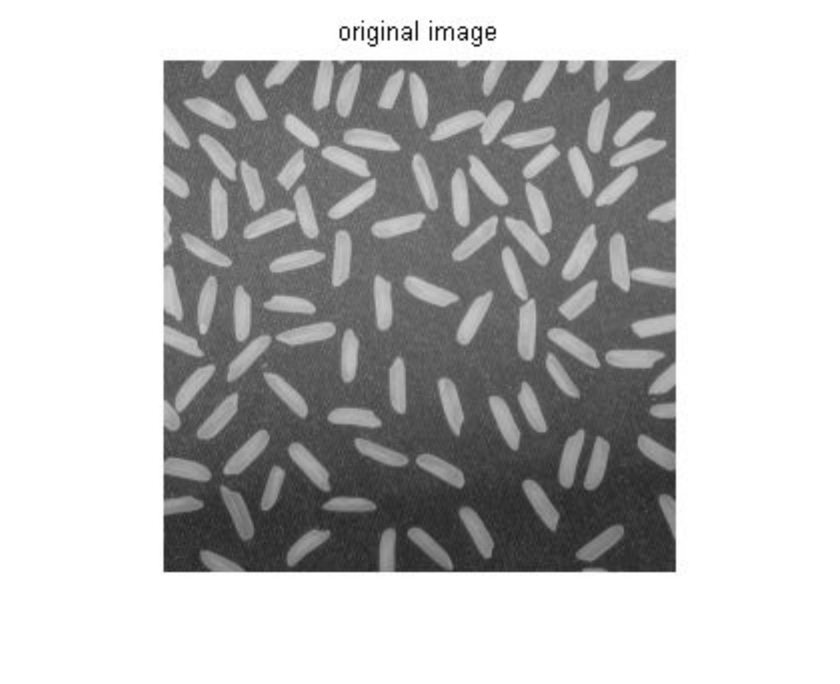
Nearest Neighbor Interpolation

Examples: Interpolation

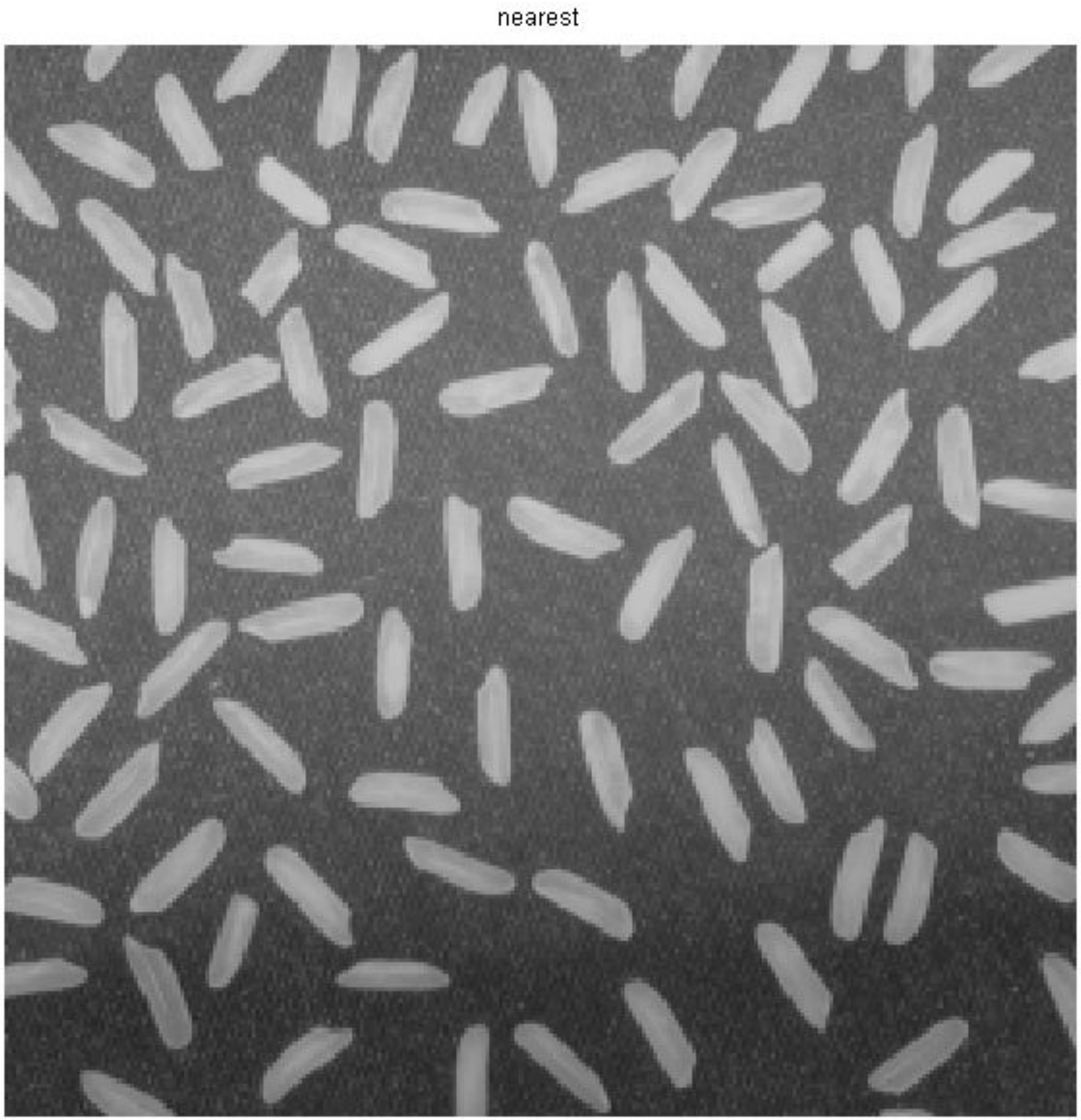
Examples: Interpolation

Bicubic Interpolation

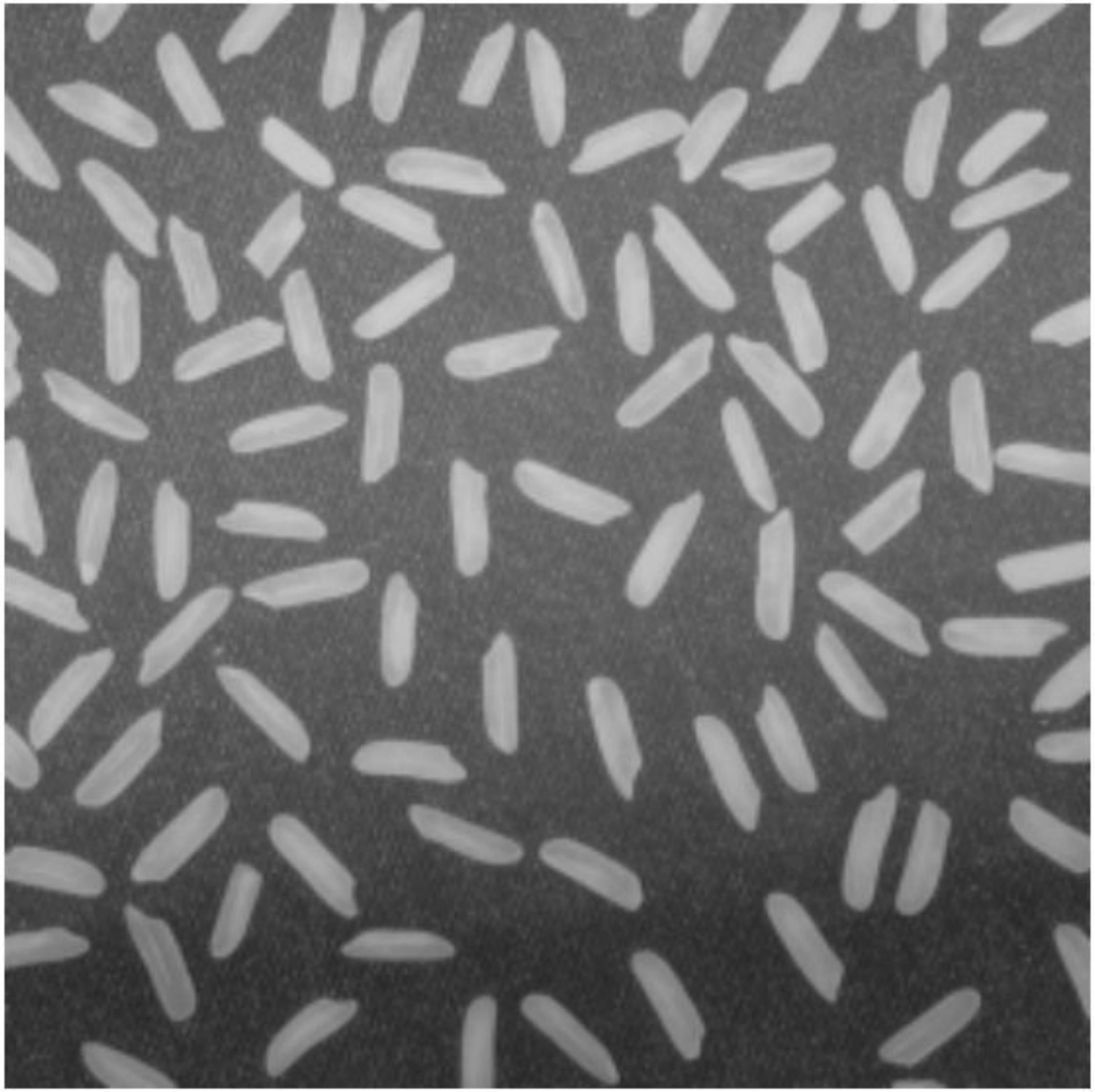
Examples: Interpolation



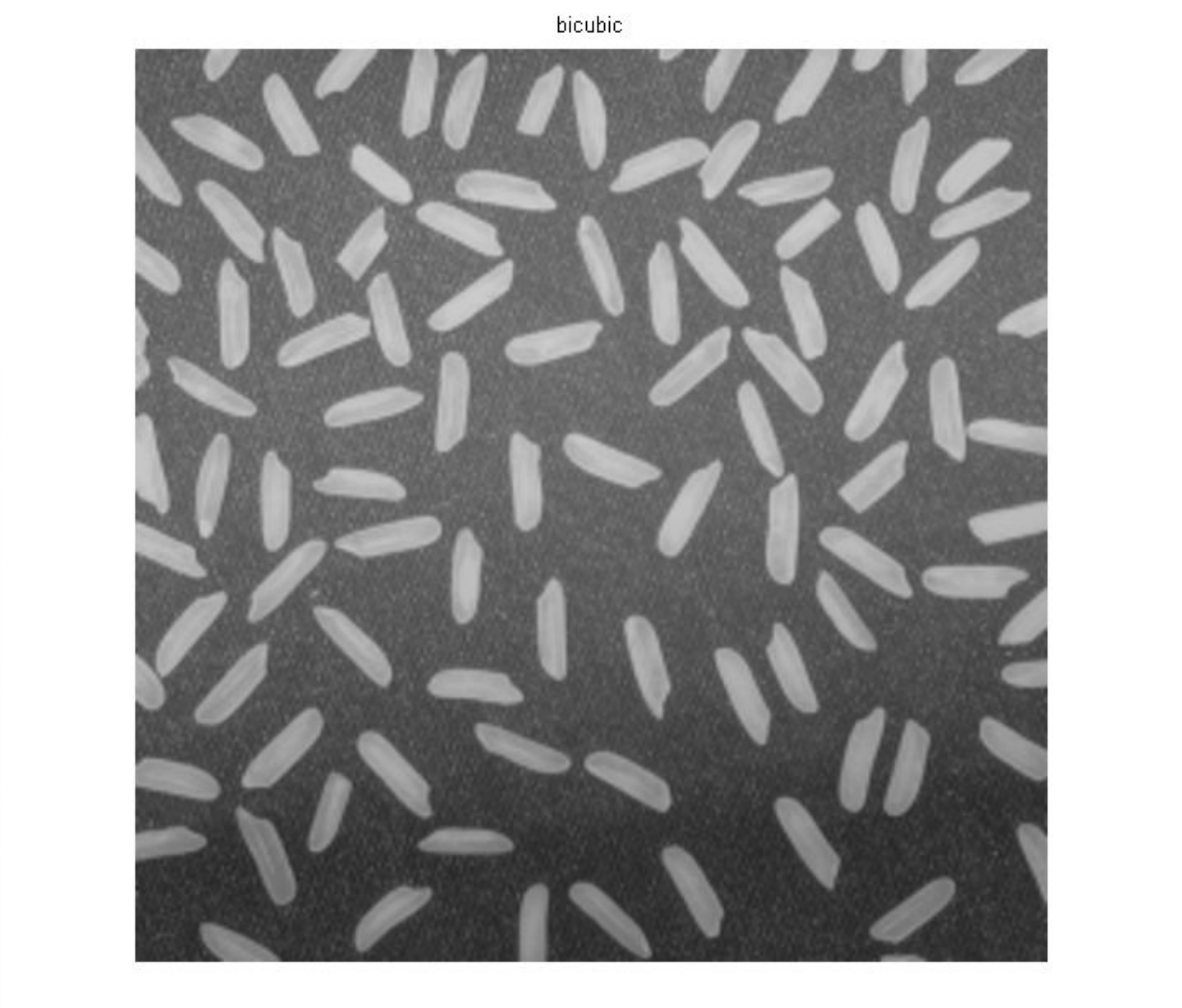
Examples: Interpolation



Examples: Interpolation



Examples: Interpolation



Basic Relationships Between Pixels

- Neighborhood
- Adjacency
- Connectivity
- Paths
- Regions and boundaries

Basic Relationships Between Pixels

- **Neighbors** of a pixel p at coordinates (x,y)
  - **4-neighbors of p**, denoted by $\mathbf{N_4(p)}$:
    (x-1, y), (x+1, y), (x,y-1), and (x, y+1).
  - **4 diagonal neighbors of p**, denoted by $\mathbf{N_D(p)}$:
    (x-1, y-1), (x+1, y+1), (x+1,y-1), and (x-1, y+1).
  - **8 neighbors of p**, denoted $\mathbf{N_8(p)}$
    $N_8(p) = N_4(p) \cup N_D(p)$

Basic Relationships Between Pixels

- **Adjacency**

  Let V be the set of intensity values

- **4-adjacency**: Two pixels p and q with values from V are 4-adjacent if q is in the set $N_4(p)$.

- **8-adjacency**: Two pixels p and q with values from V are 8-adjacent if q is in the set $N_8(p)$.

Basic Relationships Between Pixels

- **Adjacency**
  Let V be the set of intensity values

- **m-adjacency**: Two pixels p and q with values from V are m-adjacent if

  (i) q is in the set $N_4(p)$, or

  (ii) q is in the set $N_D(p)$ and the set $N_4(p) \cap N_4(q)$ has no pixels whose values are from V.

Basic Relationships Between Pixels

- **Path**
  - A (digital) path (or curve) from pixel p with coordinates (x_0, y_0) to pixel q with coordinates (x_n, y_n) is a sequence of distinct pixels with coordinates

$$(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$$

Where (x_i, y_i) and (x_{i-1}, y_{i-1}) are adjacent for $1 \le i \le n$.

  - Here n is the *length* of the path.
  - If $(x_0, y_0) = (x_n, y_n)$, the path is ***closed*** path.
  - We can define 4-, 8-, and m-paths based on the type of adjacency used.

Examples: Adjacency and Path

V = {1, 2}

0	1	1
0	2	0
0	0	1

0	1	1
0	2	0
0	0	1

0	1	1
0	2	0
0	0	1

Examples: Adjacency and Path

V = {1, 2}

0	1	1
0	2	0
0	0	1

0	1	1
0	2	0
0	0	1

0	1	1
0	2	0
0	0	1

8-adjacent

Examples: Adjacency and Path

V = {1, 2}

$$\begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix} \qquad \begin{matrix} 0 & 1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{matrix}$$

8-adjacent **m-adjacent**

Examples: Adjacency and Path

V = {1, 2}

$$
\begin{array}{ccc}
0_{1,1} & 1_{1,2} & 1_{1,3} \\
0_{2,1} & 2_{2,2} & 0_{2,3} \\
0_{3,1} & 0_{3,2} & 1_{3,3}
\end{array}
\qquad
\begin{array}{ccc}
0 & 1 & 1 \\
0 & 2 & 0 \\
0 & 0 & 1
\end{array}
\qquad
\begin{array}{ccc}
0 & 1 & 1 \\
0 & 2 & 0 \\
0 & 0 & 1
\end{array}
$$

8-adjacent **m-adjacent**

The 8-path from (1,3) to (3,3):

(i) (1,3), (1,2), (2,2), (3,3)

(ii) (1,3), (2,2), (3,3)

The m-path from (1,3) to (3,3):

(1,3), (1,2), (2,2), (3,3)

Basic Relationships Between Pixels

- **Connected in S**

  Let S represent a subset of pixels in an image. Two pixels p with coordinates (x_0, y_0) and q with coordinates (x_n, y_n) are said to be **connected in S** if there exists a path

  $$(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$$

  Where $\forall i, 0 \le i \le n, (x_i, y_i) \in S$

Basic Relationships Between Pixels

Let S represent a subset of pixels in an image

- For every pixel *p* in S, the set of pixels in S that are connected to *p* is called a ***connected component*** of S.

- If S has only one connected component, then S is called ***Connected Set.***

- We call R a **region** of the image if R is a connected set

- Two regions, R_i and R_j are said to be ***adjacent*** if their union forms a connected set.
- Regions that are not to be adjacent are said to be ***disjoint.***

Basic Relationships Between Pixels

- **Boundary (or border)**

  - The ***boundary*** of the region R is the set of pixels in the region that have one or more neighbors that are not in R.
  - If R happens to be an entire image, then its boundary is defined as the set of pixels in the first and last rows and columns of the image.

- **Foreground and background**

  - An image contains K disjoint regions, R_k, $k = 1, 2, \ldots, K$. Let R_u denote the union of all the K regions, and let $(R_u)^c$ denote its complement.

  All the points in R_u is called **foreground**;
  All the points in $(R_u)^c$ is called **background**.

Question 1

- **In the following arrangement of pixels, are the two regions (of 1s) adjacent? (if 8-adjacency is used)**

1	1	1
1	0	1
0	1	0
0	0	1
1	1	1
1	1	1

Region 1

Region 2

Question 2

- **In the following arrangement of pixels, are the two parts (of 1s) adjacent? (if 4-adjacency is used)**

1	1	1
1	0	1
0	1	0
0	0	1
1	1	1
1	1	1

Part 1

Part 2

- **In the following arrangement of pixels, the two regions (of 1s) are disjoint (if 4-adjacency is used)**

1	1	1
1	0	1
0	1	0
0	0	1
1	1	1
1	1	1

Region 1

Region 2

- **In the following arrangement of pixels, the two regions (of 1s) are disjoint (if 4-adjacency is used)**

1	1	1
1	0	1
0	1	0
0	0	1
1	1	1
1	1	1

foreground

background

Question 3

- **In the following arrangement of pixels, the circled point is part of the boundary of the 1-valued pixels if 8-adjacency is used, true or false?**

0	0	0	0	0
0	1	1	0	0
0	1	1	0	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

Question 4

- **In the following arrangement of pixels, the circled point is part of the boundary of the 1-valued pixels if 4-adjacency is used, true or false?**

0	0	0	0	0
0	1	1	0	0
0	1	1	0	0
0	1	1	1	0
0	1	1	1	0
0	0	0	0	0

Distance Measures

- Given pixels *p, q* and *z* with coordinates (x, y), (s, t), (u, v) respectively, the distance function D has following properties:

a. $D(p, q) \geq 0 \quad [D(p, q) = 0, \text{ iff } p = q]$

b. $D(p, q) = D(q, p)$

c. $D(p, z) \leq D(p, q) + D(q, z)$

Distance Measures

The following are the different Distance measures:

a. Euclidean Distance :

$D_e(p, q) = [(x-s)^2 + (y-t)^2]^{1/2}$

b. City Block Distance:

$D_4(p, q) = |x-s| + |y-t|$

		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

c. Chess Board Distance:

$D_8(p, q) = \max(|x-s|, |y-t|)$

2	2	2	2	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

Question 5

- **In the following arrangement of pixels, what's the value of the chessboard distance between the circled two points?**

0	0	0	0	0
0	0	1	1	0
0	1	1	0	0
0	1	0	0	0
0	0	0	0	0
0	0	0	0	0

Question 6

- **In the following arrangement of pixels, what's the value of the city-block distance between the circled two points?**

0	0	0	0	0
0	0	1	1	0
0	1	1	0	0
0	1	0	0	0
0	0	0	0	0
0	0	0	0	0

Question 7

- **In the following arrangement of pixels, what's the value of the length of the m-path between the circled two points?**

0	0	0	0	0
0	0	1	1	0
0	1	1	0	0
0	1	0	0	0
0	0	0	0	0
0	0	0	0	0

Question 8

- **In the following arrangement of pixels, what's the value of the length of the m-path between the circled two points?**

0	0	0	0	0
0	0	1	1	0
0	0	1	0	0
0	1	0	0	0
0	0	0	0	0
0	0	0	0	0

Introduction to Mathematical Operations in DIP

- **Array vs. Matrix Operation**

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

Array product operator

$$A \;.*\; B = \begin{bmatrix} a_{11}b_{11} & a_{12}b_{12} \\ a_{21}b_{21} & a_{22}b_{22} \end{bmatrix}$$

Array product

Matrix product operator

$$A * B = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

Matrix product

Introduction to Mathematical Operations in DIP

- **Linear vs. Nonlinear Operation**

$$H\left[f(x,y)\right]=g(x,y)$$

$$H\left[a_i f_i(x,y)+a_j f_j(x,y)\right]$$

$$=H\left[a_i f_i(x,y)\right]+H\left[a_j f_j(x,y)\right]$$

Additivity

$$=a_i H\left[f_i(x,y)\right]+a_j H\left[f_j(x,y)\right]$$

Homogeneity

$$=a_i g_i(x,y)+a_j g_j(x,y)$$

H is said to be a **linear operator;**

H is said to be a **nonlinear operator** if it does not meet the above qualification.

Arithmetic Operations

- Arithmetic operations between images are array operations. The four arithmetic operations are denoted as

$$s(x,y) = f(x,y) + g(x,y)$$
$$d(x,y) = f(x,y) - g(x,y)$$
$$p(x,y) = f(x,y) \times g(x,y)$$
$$v(x,y) = f(x,y) \div g(x,y)$$

Example: Addition of Noisy Images for Noise Reduction

Noiseless image: $f(x,y)$

Noise: $n(x,y)$ (at every pair of coordinates (x,y), the noise is uncorrelated and has zero average value)

Corrupted image: $g(x,y)$

$$g(x,y) = f(x,y) + n(x,y)$$

Reducing the noise by adding a set of noisy images, $\{g_i(x,y)\}$

$$\overline{g}(x,y) = \frac{1}{K}\sum_{i=1}^{K} g_i(x,y)$$

Example: Addition of Noisy Images for Noise Reduction

$$\bar{g}(x,y) = \frac{1}{K}\sum_{i=1}^{K} g_i(x,y)$$

$$E\{\bar{g}(x,y)\} = E\left\{\frac{1}{K}\sum_{i=1}^{K} g_i(x,y)\right\}$$

$$= E\left\{\frac{1}{K}\sum_{i=1}^{K}\left[f(x,y)+n_i(x,y)\right]\right\}$$

$$= f(x,y) + E\left\{\frac{1}{K}\sum_{i=1}^{K} n_i(x,y)\right\}$$

$$= f(x,y)$$

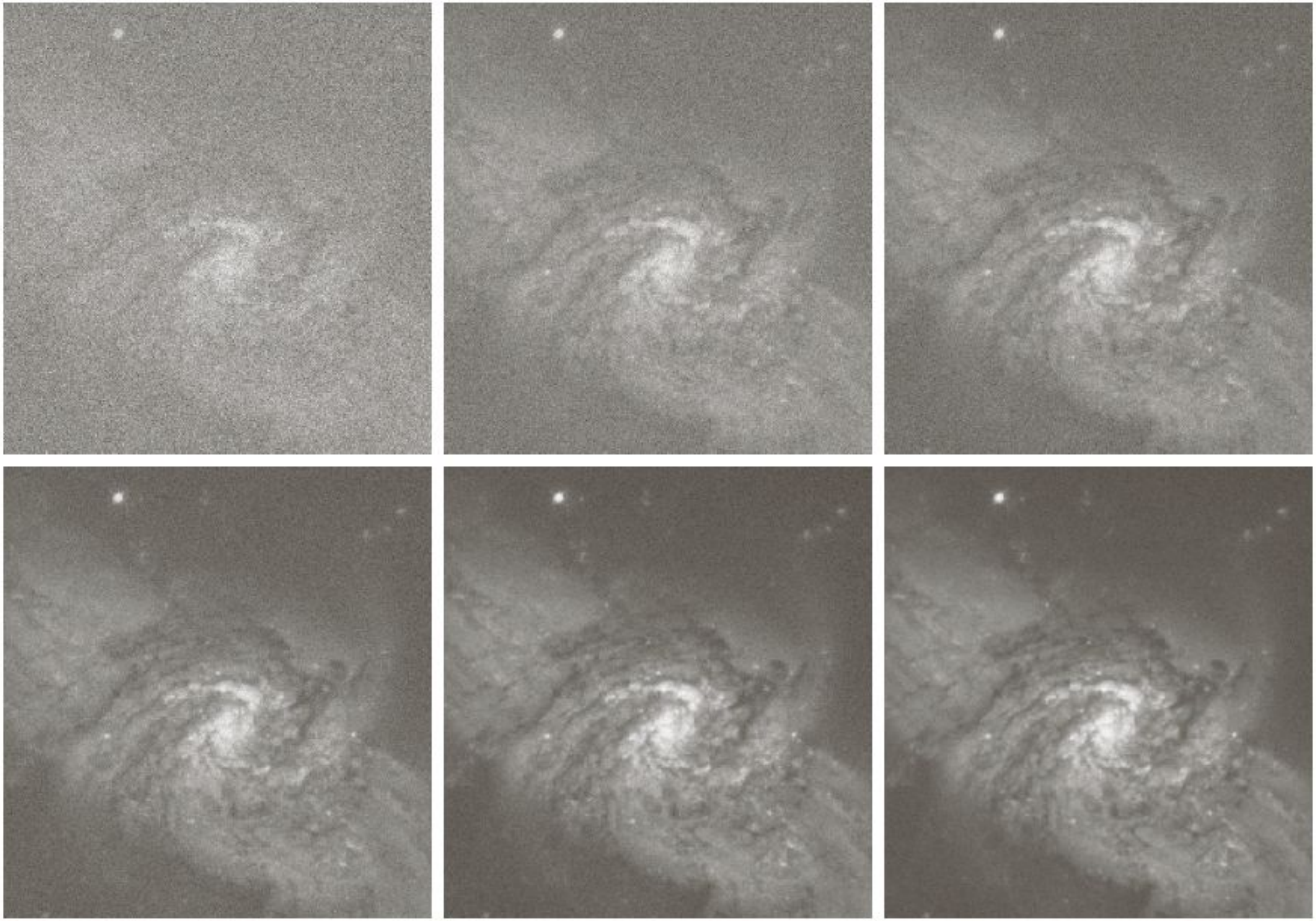
$$\sigma^2_{\bar{g}(x,y)} = \sigma^2_{\frac{1}{K}\sum_{i=1}^{K} g_i(x,y)}$$

$$= \sigma^2_{\frac{1}{K}\sum_{i=1}^{K} n_i(x,y)} = \frac{1}{K}\sigma^2_{n(x,y)}$$

Example: Addition of Noisy Images for Noise Reduction

- In astronomy, imaging under very low light levels frequently causes sensor noise to render single images virtually useless for analysis.

- In astronomical observations, similar sensors for noise reduction by observing the same scene over long periods of time. Image averaging is then used to reduce the noise.



a b c
d e f

FIGURE 2.26 (a) Image of Galaxy Pair NGC 3314 corrupted by additive Gaussian noise. (b)–(f) Results of averaging 5, 10, 20, 50, and 100 noisy images, respectively. (Original image courtesy of NASA.)

An Example of Image Subtraction: Mask Mode Radiography

Mask $h(x,y)$: an X-ray image of a region of a patient's body

Live images $f(x,y)$: X-ray images captured at TV rates after injection of the contrast medium

Enhanced detail $g(x,y)$

$$g(x,y) = f(x,y) - h(x,y)$$

The procedure gives a movie showing how the contrast medium propagates through the various arteries in the area being observed.

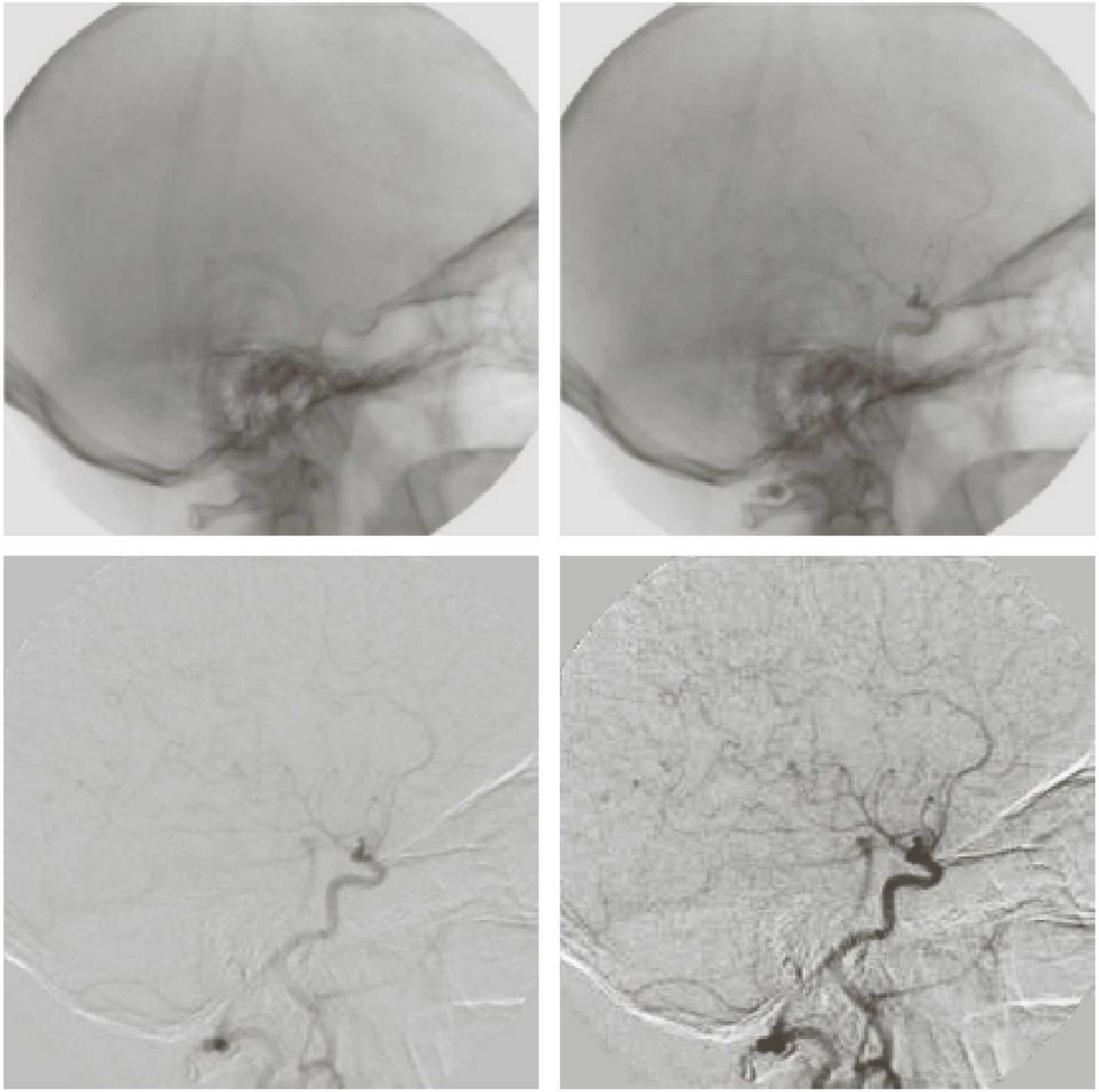


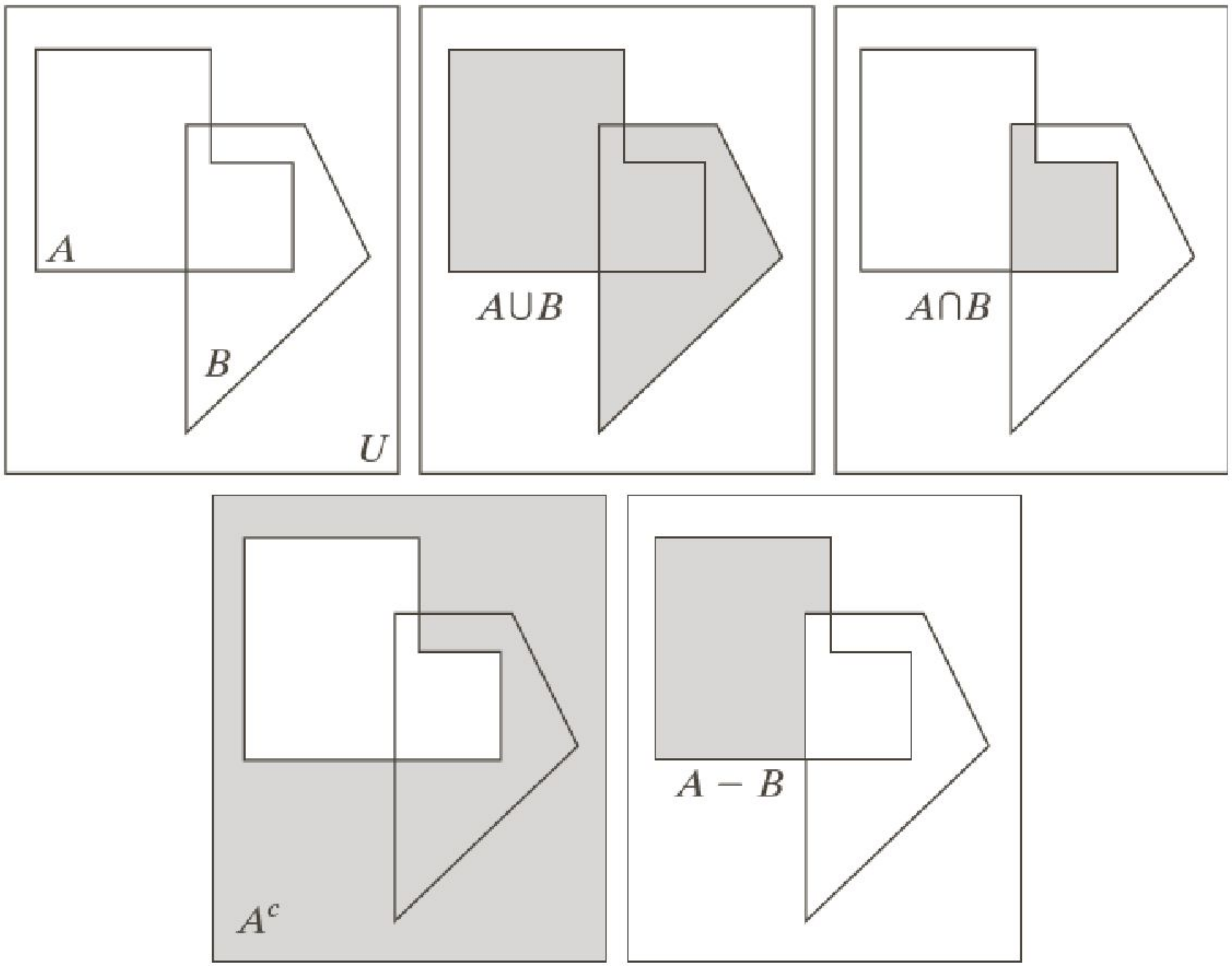
FIGURE 2.28 Digital subtraction angiography. (a) Mask image. (b) A live image. (c) Difference between (a) and (b). (d) Enhanced difference image. (Figures (a) and (b) courtesy of The Image Sciences Institute, University Medical Center, Utrecht, The Netherlands.)

An Example of Image Multiplication

a b c

FIGURE 2.29 Shading correction. (a) Shaded SEM image of a tungsten filament and support, magnified approximately 130 times. (b) The shading pattern. (c) Product of (a) by the reciprocal of (b). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

Set and Logical Operations



a b c
d e

FIGURE 2.31
(a) Two sets of coordinates, A and B, in 2-D space. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B. In (b)–(e) the shaded areas represent the member of the set operation indicated.

Set and Logical Operations

- Let A be the elements of a gray-scale image

  The elements of A are triplets of the form (x, y, z), where x and y are spatial coordinates and z denotes the intensity at the point (x, y).

$$A = \{(x, y, z) \mid z = f(x, y)\}$$

- The complement of A is denoted A^c

$$A^c = \{(x, y, K - z) \mid (x, y, z) \in A\}$$

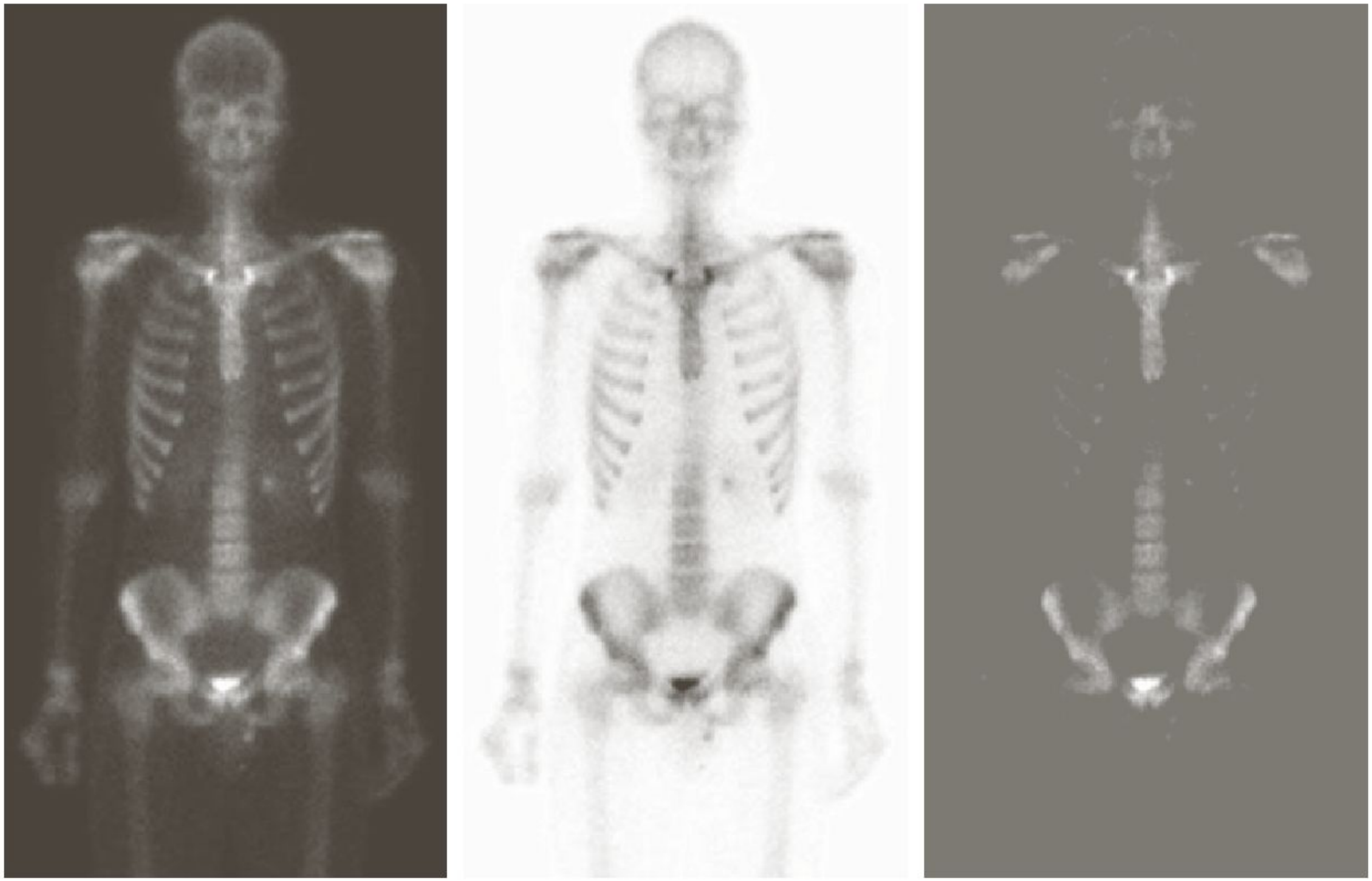
$$K = 2^k - 1; k \text{ is the number of intensity bits used to represent } z$$

Set and Logical Operations

- The union of two gray-scale images (sets) A and B is defined as the set

$$A \cup B = \{\max_z(a,b) \mid a \in A, b \in B\}$$

Set and Logical Operations



a b c

FIGURE 2.32 Set operations involving gray-scale images. (a) Original image. (b) Image negative obtained using set complementation. (c) The union of (a) and a constant image. (Original image courtesy of G.E. Medical Systems.)