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Assignment-1

1. What is sampling? Mathematically show aliasing occurs.

Answer:

Sampling is the conversion of a continuous time signal into a discrete time signal obtained by taking samples of the continuous time signal at discrete time instants. Thus, if $x_a(t)$ is the input to the sampler, the output is $x_a(nT) = x(n)$, where T is sampling interval.

If two signals after sampling becomes similar, then this event is called aliasing. and the signals are called alias of one another. This is not a good thing as it becomes impossible to distinguish the sampled signals.

Let's consider two signals,

$$x_1(t) = A\cos(2\pi F_0 t + \theta) \text{ and} \quad \text{---- (i)}$$

$$x_2(t) = \cancel{x_2} A\cos(2\pi F_k t + \theta) \quad \text{---- (ii)}$$

If the fundamental frequency is F_0, Sampling Frequency is F_s, and the frequency of the second signal is F_k, then for,

$$F_k = F_0 + kF_s \quad \text{----} \quad (iii)$$

the sampled signals are said to be alias of each other.

Now, for the signals (i) and (ii).

$$x_1(n) = A\cos\left(2\pi \frac{F_0}{F_s} n + \theta\right)$$

$$= A\cos(2\pi f_0 n + \theta)$$

and,

$$x_2(t) = A\cos[2\pi(F_0 + kF_k)t + \theta] \quad (\text{from } (iii))$$

$$\Rightarrow x_2(n) = A\cos\left[2\pi \frac{F_0 + kF_s}{F_s} n + \theta\right]$$

$$= A\cos(2\pi f_0 n + 2\pi kn + \theta)$$

$$= A\cos(2\pi f_0 n + \theta)$$

$$\therefore x_1(n) = x_2(n)$$

Hence, signals (i) and (ii) are alias of each other.

To avoid aliasing, we have to take the highest frequency of all the signals at hand and find the Nyquist rate from there.

2. Briefly explain multidimentional and multichannel signals with propere example

Answere:

A signal is called M-dimensional or multidimentional, if its value is a function of M independant variables. For example, a black and white television picture may be represented as $I(x, y, t)$, since each point is a function of three independant variables, where x and y describes the brightness of a point, and brightness is the function of time, t.

If $s_k(t)$, $k = 1, 2, 3$, denotes the electrical signal from the k-th sensore as a function of time, the set of $p=3$ signals can be represented by a vectore $S_3(t)$, where,

$$S_3(t) = \begin{bmatrix} s_1(t) \\ s_2(t) \\ s_3(t) \end{bmatrix}$$

Such a vectore of signals is referred as multichannel signal.

For example, in electrocardiography, 3-lead and 12-lead electrocardiograms (ECG) are often used in practice, which result in 3-channel and 12-channel signals.

An example of multichannel and multidimentional signal would be color tv picture. It can be described by the three intensity function of the form $I_r(x,y,t)$, $I_g(x,y,t)$ and $I_b(x,y,t)$ corresponding to the brightness of the three principle colors as functions of time. Hence, color tv picture is a three channel, three dimensional signal which can be represented by the vector

$$I(x,y,t) = \begin{bmatrix} I_r(x,y,t) \\ I_g(x,y,t) \\ I_b(x,y,t) \end{bmatrix}$$

3. Consider the signal-

$$x(t) = 4\cos 450\pi t + 7\cos 120\pi t + 6\cos 550\pi$$

(i) What is the Nyquist rate of this signal?

(ii) Evaluate the discrete time signal at a sample rate $F_s = 200$ samples/sec.

Answer:

(i) Given that,

$$x(t) = 4\cos 450\pi t + 7\cos 120\pi t + 6\cos 550\pi t$$

The frequencies present in the given signal are-

$F_1 = 225\ Hz$ $\quad F_2 = 60\ Hz$ $\quad F_3 = 275\ Hz$

Thus, $F_{max} = 275\ Hz$

Hence, the Nyquist rate of the signal-

$$F_N = 2F_{max}$$

$$= 2 \times 275\ Hz$$

$$= 550\ Hz$$

(ii) Given,

$$x(t) = 4\cos 450\pi t + 7\cos 120\pi t + 6\cos 550\pi t$$

and, $F_s = 200\,Hz$

$$\therefore x(n) = 4\cos \frac{450}{200}\pi t + 7\cos \frac{120}{200}\pi t + 6\cos \frac{550}{200}\pi t$$

$$= 4\cos \frac{9}{4}\pi t + 7\cos \frac{3}{5}\pi t + 6\cos \frac{11}{4}\pi t$$

$$= 4\cos\left(2\pi + \frac{1}{4}\pi\right)t + 7\cos \frac{3}{5}\pi t + 6\cos\left(2\pi + \frac{3}{4}\pi\right)t$$

$$= 4\cos \frac{\pi}{4} t + 7\cos \frac{3\pi}{5} t + 6\cos \frac{3\pi}{4} t$$

Ans.

4. Determine whether the following signals are periodic or not. If periodic, determine their fundamental period as well.

(a) $\cos 5\pi n$

(b) $\sin 3n$

(c) $x(n) = 3\cos(5t + \frac{\pi}{6})$

<u>Answer:</u>

We know,

$$A = \cos(2\pi f n + \theta) \quad , \text{ where } f = \frac{k}{N}$$

(a) $\cos 5\pi n$

$= \cos 2\pi \frac{5}{2} n$

$\therefore f = \frac{5}{2}$

$\therefore N = 2$, which is an integer number. Hence the signal is periodic and its period is 2, and fundamental period $T = \frac{2}{5}$.

(b) $\sin 3n$

$= \sin 2\pi \frac{3}{2\pi} n$

$\therefore f = \frac{3}{2\pi}$ and $N = 2\pi$, which is a real number. So the signal is not periodic.

(c) $x(n) = 3\cos\left(5t + \frac{\pi}{6}\right)$

$= 3\cos\left(5nT + \frac{\pi}{6}\right)$ [as $t = nT$]

$\therefore N = \frac{\pi}{6}$, which is not a real number.

Hence, the signal is not periodic.

(c) $x(t) = 3\cos\left(5t + \frac{\pi}{6}\right)$

We know that,

$x(t) = A\cos(\omega t + \theta)$

$\therefore \omega = 5$

$\Rightarrow 2\pi f = 5$

$\Rightarrow f = \frac{5}{2\pi}$; which is not a real number.

Hence, the signal ~~signal~~ is not periodic.

5. Consider the signal-

$$x(n) = \{\ldots, 0, 0, 2, 1, 3, -2, 1, \underset{\uparrow}{-4}, 1, 2, -3, -1, -2, 0, 0, \ldots\}$$

(a) Determine and sketch the even parts of $x(n)$.

(b) Determine and sketch the ~~even~~ odd parts of $x(n)$.

(c) Determine and graphically show the response of the system described by-

$$y(n) = -x(-2n+2)$$

(d) Determine and graphically show the response of the system described by -

$$y(n) = x\left(-\frac{n}{2} - 2\right)$$

Answer:

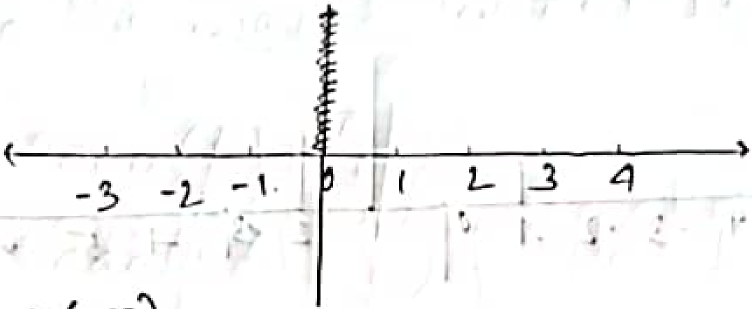
(a) Even parts of $x(n)$, $x_e(n)$

$$= \frac{x(n) + x(-n)}{2}$$

$$x(-n) = \{\ldots, 0, 0, -2, -1, -3, 2, 1, \underset{\uparrow}{-4}, -1, -2, 3, 1, 2, 0, 0, \ldots\}$$

$$\therefore x_e(n) = \{\ldots, 0, 0, 0, \underset{\uparrow}{-4}, 0, 0, 0, \ldots\}$$

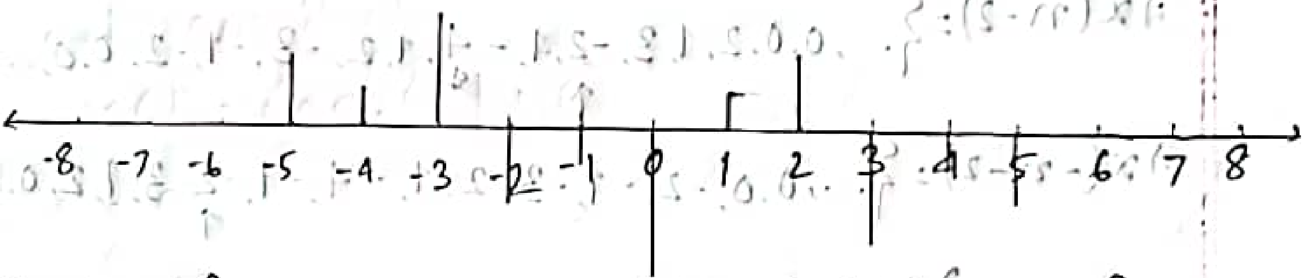
~~(b) odd parts of x(n)~~



(b) $x(n) - x(-n)$

$= \{\ldots, 0, 0, 4, 2, 6, -4, -2, \underset{\uparrow}{-8}, -2, 4, -6, -2, -4, 0, 0, \ldots\}$

$\therefore x_o(n) = \{\ldots, 0, 0, 2, 1, 3, -2, -1, \underset{\uparrow}{-4}, 1, 2, -3, -1, -2, 0, 0, \ldots\}$



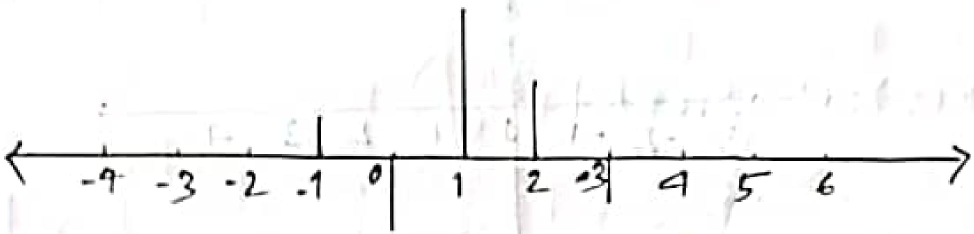
(c) $x(n) = \{\ldots, 0, 0, 2, 1, 3, -2, -1, \underset{\uparrow}{-4}, 1, 2, -3, -1, -2, 0, 0, \ldots\}$

$\Rightarrow x(n+2) = \{\ldots, 0, 0, 2, 1, 3, -2, -1, -4, 1, \underset{\uparrow}{2}, -3, -1, -2, 0, 0, \ldots\}$

$\Rightarrow x(-n+2) = \{\ldots, 0, 0, -2, -1, -3, \underset{\uparrow}{2}, 1, -4, -1, -2, 3, 1, 2, 0, 0, \ldots\}$

$\Rightarrow x(-2n+2) = \{\ldots, 0, -1, \underset{\uparrow}{2}, -4, -2, 1, 0, \ldots\}$

$$\Rightarrow -x(-2n+2) = \{\ldots, 0, 1, \underset{\uparrow}{-2}, 4, 2, -1, 0, \ldots\}$$



$$(d)\ x(n) = \{\ldots, 0, 0, 2, 1, 3, -2, 1, \underset{\uparrow}{-4}, 1, 2, -3, -1, 2, 0, 0, \ldots\}$$

$$\Rightarrow x(n-2) = \{\ldots, 0, 0, 2, 1, 3, \underset{\uparrow}{-2}, 1, -4, 1, 2, -3, -1, -2, 0, 0, \ldots\}$$

$$\Rightarrow x(-n-2) = \{\ldots, 0, 0, -2, -1, -3, 2, 1, -4, -1, \underset{\uparrow}{-2}, 3, 1, 2, 0, 0, \ldots\}$$

$$\Rightarrow x\left(-\frac{n}{2}-2\right) = \{\ldots, 0, 0, 0, 0, -2, 0, -1, 0, -3, 0, 2, 0, 1, 0, -4, 0, -1, 0, \underset{\uparrow}{-2}, 0, 3, 0, 1, 0, 2, 0, 0, 0, \ldots\}$$