Heaven's light is our guide

Rajshahi University of Engineering & Technology

B.Sc. Engineering 2nd Year Even Semester Examination, 2016

Department of Computer Science & Engineering

Course no: **CSE 2201** Course Title: **Computer Algorithms**

Full marks: 70 Time: Three (03) hours

N.B. Answer **six** questions, taking **three** from each section.
The questions are of equal value.
Use separate answer script for each section.

SECTION-A

Q1. (a) Let we have a sequence of n numbers $(a_1, a_2, \ldots\ldots a_n)$. Show that "the best case running time of insertion sort is a linear function of n and the worst case running time of insertion sort is a quadratic function of n". **04**

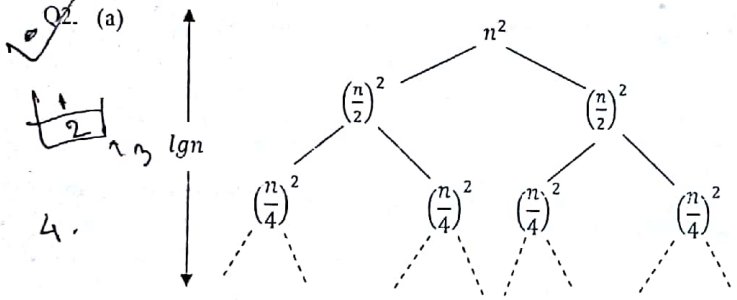
(b) Let we have $T(n)$ (The running time on a problem of size n). If the problem size is small enough, say $n \leq c$ for some constant c, the straightforward solution takes constant time, which we write as $\theta(1)$. Suppose we divide the problem into 'a' sub problems, each of which is $\frac{1}{b}$, the size of the original. If we take $D(n)$ time to divide the problem into sub problems and $C(n)$ time to combine the solutions to the sub problems into the solution to the original problem, your task is to find the recurrence of the running time of the original problem. **04**

(c) Use the mathematical induction to show that the solution of the recurrence **04**

$$T(n) = \begin{cases} 2 & if\ n = 2 \\ 2T\left(\frac{n}{2}\right) + n & if\ n = 2^k, k > 1 \end{cases}$$

is $T(n) = nlgn$.

Q2. (a)



04

The height of the tree is lgn. Find the recurrence of the tree and tight asymptotic bounds of that recurrence.

(b) Why do we not apply master method to find tight asymptotic bound for the following recurrence: $(n) = 2T\left(\frac{n}{2}\right) + nlgn$. **04**

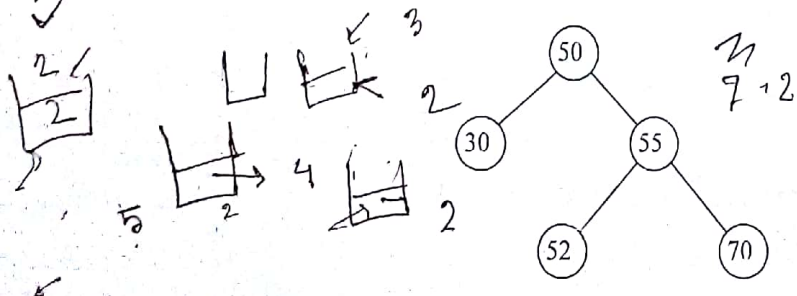
(c) Find out the upper bound of the following recurrence: $T(n) = 2T(\lfloor\sqrt{n}\rfloor) + lgn$ **04**

Q3 (a) Given following array of integers `a[6]={10,20,30,40,50,60};` If you apply quick sort without randomized partitioning what will happen? Explain briefly. **05**

(b) How does Quick sort and randomized Quick sort perform while sorting a random array? Explain in terms of θ. **03**

(c) What is amortized analysis? When is it better to consider amortized analysis for algorithm's complexity measurement? **04**

Q4. (a) Given following AVL tree what will happen if you insert a new node with value 85? **04**



(b) Prove the correctness of Dijkstra's shortest path algorithm. **06**

(c) Do you think "Asymptotic behavior keeps track of largest growing terms and drops all factor"? Why? **02**

SECTION-B

Q5. (a) Consider following set of numbers,

{0, 1, 4}, {2, 3, 5, 6}.

Above sets are stored in an array named 'data', so that two numbers u and v are in same set if data[u]=data[v]. The pseudocode used to perform union operation of a pair of numbers (u, v) is given bellow.

```
Union
 Input  : data[]    //Array containing set information
          N         //size of data
          u         // 1st number
          v         // 2nd number
 Output : data[]    //updated array
 Steps  : 1. Temp  := data[u];
          2. For i := 0 to N
               if(data[i] == Temp)
                  data[i] := data[v];
          3. Return data[];
```

(i) What will be the complexity (θ) of above code

(ii) Can this process be improved? If yes, write pseudocode and complexity analysis. Otherwise explain, why?

(b) How can Fibonacci heap improve the implementation of Prim's minimum spanning tree algorithm? Explain with necessary pseudocode and worst case complexity analysis or amortized analysis. **06**

Q6. (a) Consider the following instance of the Knapsack problem: total objects $n = 4$, Knapsack capacity $m = 6$, profit $(P_1, P_2, P_3, P_4) = (7,4,3,2)$, and weight $(w_1, w_2, w_3, w_4) = (5,2,1,3)$. Find the maximum possible profit using dynamic programming. **06**

i) Draw the matrix containing solutions.

ii) Write a pseudocode to find the selected objects from the matrix.

(b) Define: i) Intractable problems ii) NP hardness of a problem **03**

(c) Write the pseudocode to perform a non-deterministic search on a linear array of n elements.

Q7. (a) Consider the following matrix named LCS consisting of longest common subsequence value of sub problems for following strings. The strings are stored in two char type array names s1 and s2. S1: BACBAD S2: ABAZDC **06**

		B	A	C	B	A	D
	0	0	0	0	0	0	0
A	0	0	1	1	1	1	1
B	0	1	1	1	2	2	2
A	0	1	2	2	2	3	3
Z	0	1	2	2			
D	0	1	2	2			
C	0	1	2	3			

(i) Fill up the contents of LCS table.

(ii) Write the pseudocode to find the characters that form the longest common subsequence. You should track changes from the end of LCS matrix.

(b) Encode the message using variable number of bits, found from corresponding Huffman tree. How many bits are required to encode the total message in this way? **06**

Message: BACADAEAFABBAAAACAH

Q8. (a) Given a set s= {5, 10, 12, 13, 20}. Implement backtracking algorithm to draw the state space tree for finding all possible subsets of s for which the sum is 25. **06**

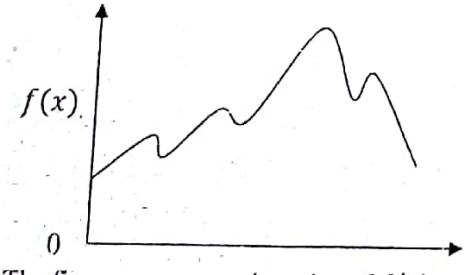
(i) How many nodes are skipped from the complete search space?

(ii) How much better the backtracking approach for this problem is, compared to brute force technique? Explain in term of worst case time complexity.

(b) Which properties of a problem lead the programmers towards dynamic programming for solution? **03**

(c) Why approximation algorithms are important for some problems? **01**

(c) Consider the following figure **02**



The figure represents the value of $f(x)$ as x changes to find the maximum value of $f(x)$, will you use greedy approach or dynamic approach? Why? [Assume that the values for x are given]

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Heaven's Light is our Guide

Rajshahi University of Engineering & Technology
B.Sc. Engineering 2nd Year Even Semester Examination, 2015
Department of Computer Science & Engineering
Course No. CSE 2201 Course Title. Computer Algorithms
Full Marks: 72 Time: THREE (03) hours

N.B: Answer SIX questions taking THREE from each section.
The questions are of equal value
Use separate answer script for each section.

SECTION A

Marks

Q.1(a) What do you mean time complexity and space complexity? Find the time complexity of the following codes: 06

(i)
```
for i:=1 to n/2 do
    for j:= 1 to i do
        //statements
    end
end
```

(ii)
```
for i:= 1 to n do
    for j:= 1 to n do
        //statements
    end
end
```

(b) Do you use master theorem to solve the recurrence $T(n) = 2T(n/2) + n\log n$? 03

(c) Prove that "if $f(n) = a_m n^m + \ldots\ldots + a_1 n + a_0$, then $f(n) = O(n^m)$". 03

Q.2(a) Explain-"the tree vertex splitting problem has a solution only if the maximum edge weight is <= tolerance limit δ." 04

(b) Given an array of jobs where every job has a deadline and associated profit if the job is finished before the deadline. It is also given that every job takes single unit of time. So the minimum possible deadline for any job is 1. How to maximize total profit if any one job can be scheduled at a time. Explain with a proper example. 04

(c) Explain-"Dijkstra's algorithm solves the single source shortest paths problem on a weighted directed graph G = (V, E) for the case in which all edge weights are nonnegative". 04

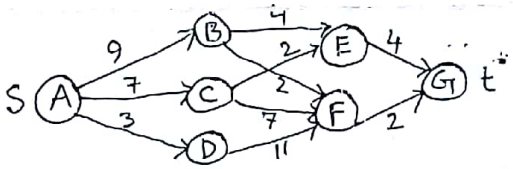
Q.3(a) Find the worst case complexity of merge sort and quick sort and compare them based on their performance. 06

(b) Let A and B be two 2 x 2 matrices. The product matrix, C = AB is also an 2 x 2 matrix, whose $C_{ij} = A_{ik}B_{kj}$. Strasen has discovered a way to compute the C_{ij}'s using only 7 multiplications and 18 additions or substraction. Now find out the complexity of this algorithm. 03

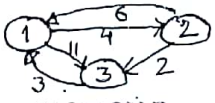
(c) Let $P_1, P_2, \ldots P_n$ be the vertices of the convex polygon Q in clockwise order. Let P be any other point. How do you determine whether P lies in the interior of Q or outside? Explain with a proper example. 03

Q.4(a) What do you mean by principle of optimality? What are the differences between greedy method and dynamic method? Explain briefly. 04

(b) Find the minimum cost path from s to t in the multistage graph of following figure using forward approach. 04



(c) Find all pairs shortest paths in the graph of the following figure using Floyd-Warshall algorithm. 04



SECTION B

Q.5(a) You are given as input an unsorted array of n distinct number, where $n = 2^k$ and k is interger. Give an algorithm to identify the second longest number in the array and that uses at most $n + \log_2 n - 2$ comparisons. 05

(b) Suppose you are using ordered array for priority queue of the prim's minimum spanning tree algorithm. What will be the complexity (O) of the following Pseudo code if there are total V nodes and E edges? 05

```
for each v∈ adjacent [u] // for each edge adjacent to v
if (d[v] > d[u] + w[u,v])
    decrease key (v);  // updates the cost of node v in priority queue.
```

(c) What are the differences between AVL tree and Binary Search tree? 02

Q.6(a) Find the product of the polynomial $A(x) = 3x^2 + 4x + 1$ and $B(x) = x^2 + 2x + 5$ using (i) conventional multiplication process and (ii) evaluation and interpolation process. 06

(b) Define (i) answer state (ii) live node (iii) dead node and (iv) E-node. 02

(c) Define (i) Class P (ii) Class NP (iii) Class NP hard and (iv) Class NP complete. 04

Q.7(a) Define: (i) Criterion function (ii) explicit constraints and (iii) implicit constraints. 03

(b) Let w = {5,10,15,20,25} and m = 35. Find all possible subset of w that sum is equal to m. Draw the portion of state space tree that is generated. 05

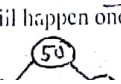
(c) Use back tracking algorithm and the dynamic partitioning scheme on the Knapsack problem for the following data; P = {11,21,31,33,43,53,55,65}, W = {1,11,21,23,33,43,45,55}, m = 110 and n = 8. 04

Q.8(a) Discuss the differences between greedy and dynamic algorithm. 02

(b) Write a Pseudo code of non deterministic algorithm to solve minimum vertex cover problem. 03

(c) Write a Pseudo code of non deterministic search algorithm to search from a linear array of n elements, and show that it is better (if works) than traditional linear search in terms of Big Oh (O) analysis. 04

(d) Given the following AVL tree, What will happen once you insert a new node with value 10? 03



Rajshahi University of Engineering & Technology
B.Sc. Engineering 2nd Year 4th Semester Examination, 2014
Department of Computer Science & Engineering
Course No. CSE 401 Course Title: Algorithms Design & Analysis
Full Marks: 70 Time: THREE (03) hours

N.B:
Answer SIX questions taking THREE from each section.
The questions are of equal value.
Use separate answer script for each section.

SECTION A

		Marks
Q.1(a)	What is an algorithm? Briefly describe about the characteristics of an algorithm.	04
(b)	Show that "the solution of $T(n) = T(\lceil n/2 \rceil) + 1$ is $O(\log n)$.	$03\frac{2}{3}$
(c)	Define Asymptotic tight bound. Find Asymptotic tight bound for the recurrance. $T(n) = 3n^2 - 2n$. Draw the behavior of the given function.	04
Q.2(a)	Let, a[1:n] is a global array. The task is to set max and min to the largest and smallest value in a[1:n] respectively. Design an algorithm to find the max and min recursively and analysis the complexity of your algorithm.	$05\frac{2}{3}$
(b)	Analyze the worst case complexity of Quick Sort and Merge Sort.	06
Q.3(a)	Suppose that you will drive your car for a long trip between Worcester, Massachusetts and San Francisco, California along Interstate-Highway. In preparation for your trip, you have downloaded a map that contains the distance in miles between all the gas stations in your route. Assume that your car's gas tank, when full, holds enough gas to travel n miles. Assume that the value n is given. The distance between Worcester and San Francisco is irrelevant for this problem. (i) Assume that you want to make the minimum number of stops possible along the way, without running out of gas at any point. Describe in detail an efficient method by which you can determine at which gas station you should stop.	04
(b)	Consider the following instance of the Knapsack problem: n = 3(total number of objects), m = 20(capacity of Knapsack), profit $(P_1,P_2,P_3) = (25,24,15)$ and weights$(W_1,W_2,W_3) = (18,15,10)$. Find out the four feasible solutions.	$03\frac{2}{3}$
(c)	Explain the statement with a proper example-"Tree vertex splitting problem (TVSP) has a solution only if the maximum edge weight is $<= \delta$, where δ indicates the tolerance limit.	04
Q.4(a)	A classic combinatorial problem is to place eight queens on an 8 x 8 chessboard so that no two "attack", that is, no two of them are on the same row, column or diagonal. Find out the explicit constraint and implicit constraint.	04
(b)	Explain Quick Hull algorithm to find convex hull from a set of points. Clarify each step with necessary diagram.	05
(c)	What are the drawbacks of using Divide & Conquer approach?	$02\frac{2}{3}$

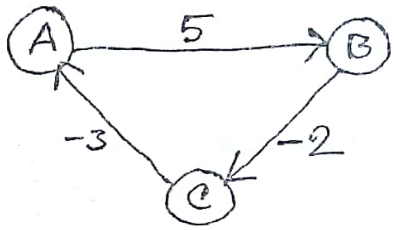
SECTION B

Q.5(a)	If A is an adjacent Matrix, $A^k(i,j)$ represents the length of the shortest path from i to j going through no vertex of index greater than k. Now prove the following equation for a graph with cycles of positive length, $A^k(i,j) = \min\{A^{k-1}(i,j),\ A^{k-1}(i,k) + A^{k-1}(k,j)\}$, where k>=1. What does the problem occur if a negative cycle exists. Explain it with an example.	04
(b)	Write Pseudo code of greedy knapsack problem where the first step is to sort the objects based on corresponding profit using Heap sort. Find the time complexity of your algorithm in terms of Big theta (θ).	04
(c)	How to determine whether a given algorithm is to be solved using greedy approach or not? Explain briefly.	$03\frac{2}{3}$
Q.6(a)	Suppose you are using an ordered (sorted) array to store the edges while implementing Prime's algorithm of spanning tree. You are also using an adjacency list representation of the given graph. What will be the complexity (Asymptotic upper bound) of the following Pseudo code if there are total V nodes and E edges? For each v e adjacent [u] /* for each v edges adjacent to u*/ If (d[v]>d[u] + w[u,v] Decrease key(v), /* update cost of node v & perform queue update*/	04

(b) You are given a choice to use an unordered array (no sorting) or Binary heap to represent a priority queue. Considering the Asymptotic upper bound, which will be better to perform the following operations? 04
(i) Delete min value (ii) decrease key value.

(c) What are the drawbacks of Dijkstra's algorithm? $03\frac{2}{3}$

Q.7(a) Given the following graph, show how Bellmanford algorithm will work to find the shortest path from the graph. What will be the final output for the following figure? 04



AC: A-A-E

(b) Using Dynamic programming strategy, find the longest common subsequence between the following strings: string1: ACABE string2: PAAE. 04

(c) What will be the time complexity Big Theta (θ) of solving the longest common subsequence problem using following choice and why? (i) Dynamic Programming technique and (ii) Brute Force technique. $03\frac{2}{3}$

Q.8(a) What are differences between NP-hard and NP-complete problem? Define deterministic algorithm. $03\frac{2}{3}$

(b) Write down the algorithm for Lagrange interpolation and analyze its complexity. 05

(c) Write the Pseudo code of a 2-apprximation algorithm to solve the vertex cover problem. With an example graph show that your Pseudo code is a 2-approximation approach. 03

Heaven's light is our guide

Rajshahi University of Engineering & Technology

B.Sc. Engineering 2nd Year 4th Semester Examination, 2013

Department of Computer Science & Engineering

Course no: **CSE 401** Course Title: **Algorithm Design & Analysis**

Full marks: 70 Time: Three (03) hours

N.B. Answer **six** questions, taking **three** from each section.
The questions are of equal value.
Use separate answer script for each section.

SECTION-A

Q1. (a) What is an algorithm? **01 2/3**

(b) Show that the following equalities are correct and describe them graphically: **06**

i) $5n^2 - 6n = \theta(n^2)$

ii) $n! = O(n^n)$

(c) Use the master method to give tight asymptotic bounds for the following recurrences: **04**

i) $T(n) = 9T(n/3) + n$

ii) $T(n) = T(2n/3) + 1$

Q2. (a) Find the recurrence relation for the recursive algorithm of finding maximum and minimum from a set of data. **02 2/3**

(b) Analysis the complexity of Quick-Sort and what are the differences between the complexity of quick-sort and merge-sort. **06**

(c) Prove that the overall computing time for strassen's matrix multiplication is $T(n) = O(n^{2.81})$. **03**

Q3. (a) The complexity of many divide and conquer algorithm is given by recurrences of the form: **04**

$$T(n) = T(1), \quad n = 1$$
$$= aT(n/b) + f(n), \quad n > 1$$

Where a and b are known constants. Assume $T(1)$=2 and n is a power of b (i.e., $n=b^k$). Determine the big O of $T(n)$.

(b) What is chromatic number of a graph? Define planar graph. **02**

(c) Explain how you will determine whether two consecutive line segment turn left to right. **03**

(d) Explain the traveling salesman problem. **02 2/3**

Q4. (a) Explain the difference between dynamic method and greedy method. **02 2/3**

(b) Let, $A^k(i,j)$ represents the length of a shortest path from i to j going through no vertex of index greater than k, Prove that **05**
$A^k(i,j) = \min\{ A^{k-1}(i,j), A^{k-1}(i,k) + A^{k-1}(k,j)\}, k \geq 1$
for graph with no cycles of negative length-If graph contains a negative length cycle, explain the problem occurred in above equation.

(c) Let, we have a set of data, D={5,3,8,2}, draw three different binary search tree, and let, the probability of success search={0.3,0.2,0.1,0.1} and the probability of un-success search={0.075,0.05,0.1,0.075}. Determine the optional binary search tree among them. **04**

SECTION-B

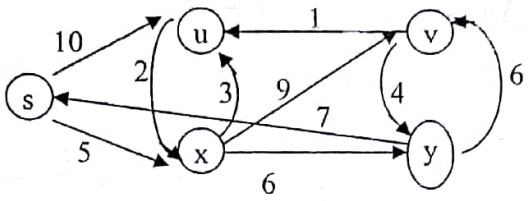
Q5. (a) A classic combinatorial problem is to place eight queen on an 8×8 chessboard so that no two "attack", that is, so that no two of them are on the same row, column or diagonal. Write down the explicit and implicit constraints of the above problem. **02 2/3**

(b) Define: i) State space tree, ii) E-node, iii) dead node, iv) live node. **04**

(c) Let w={5,7,10,12,15,18,20} and m=35. Find all possible subset of w that sum to m. Do this using recursive backtracking algorithm for sum of subsets problem. Draw the portion of the state space tree that is generated. **05**

Q6. (a) Distinguish between Breadth-First and Depth-First search. Is it always possible to find optimum solution using DFS? **04**

(b) Consider the following weighted directed graph. Use the Dijkstra's algorithm to obtain the shortest path that traverses all vertices **$05^{2/3}$**



(c) State the cook's algorithm. **02**

Q7 (a) Try out the backtracking algorithm and the dynamic partitioning scheme on the following 0/1 knapsack problem: **$05^{2/3}$**
Profit, P={11,21,31,33,43,53,55,65}, weight, w={1,11,21,23,33,43,45,55}, capacity, m=110 and number of object, n=8.

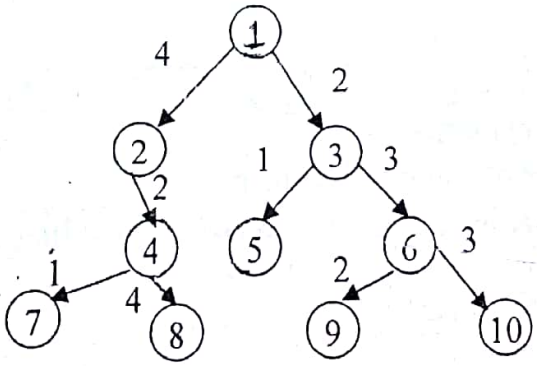
(b) Write down the pseudo code for Least cost search. **04**

(c) LC search uses a cost function $\hat{c} = f(h(x)) + \hat{g}(x)$ to select the next E-node would always choose for its next E-node a live node with least $\hat{c}(.)$. Show that BFS and D-search are special case of LC-search. **02**

Q8 (a) What is the difference between NP-hard and NP-complete problem? Define deterministic algorithm. **$03^{2/3}$**

(b) Create a convex hull for the set of points, **04**
P={(10,0),(15,10),(20,15),(12,12),(8,15),(5,15),(0,10)} by using Graham-Scan algorithm and draw the each step to construct the convex hull.

(c) For the following tree, solve the tree vertex splitting problem when a tolerance limit δ=5. **04**



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"Heaven's Light is our Guide"

Rajshahi University of Engineering and Technology

B.Sc. Engineering 2nd Year 4th Semester Examination, 2012

Department of Computer Science and Engineering

Course No. **CSE 401** Course Title: **Algorithm Design & Analysis**

Full Marks: **70** Time: **THREE** (03) hours

N.B.

Answer **SIX** questions taking **THREE** from each section.
The questions are of equal value.
Use separate answer script for each section.

SECTION A

		Marks
Q1(a)	Write down the general characteristic of an algorithm	03
(b)	What do you mean by average-case, worst-case and best-case of an algorithm?	03
(c)	Prove that $f(n) = O(n^m)$ where $f(n) = a_m n^m + \ldots\ldots + a_1 n + a_0$	02⅔
(d)	Show that the following equalities are correct:	03

i) $2n^2 2^n + n\log n = \Theta(n^2 2^n)$

ii) $6n^3 / (\log n + 1) = O(n^3)$

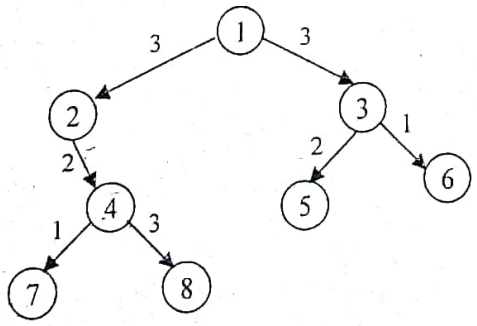
iii) $n^{2n} + 6 * 2^n = \Theta(n^{2^n})$

Q2(a)	Let us consider the following 15 entities of a data set: -10, -5, 1, 0, 7, 9, 15, 20, 50, 10, -2, 3, 82, 100, 25. Show the steps binary search to find an element n=25	04
(b)	If the time of the merging operation is proportional to n, then the computing time for merge sort is described by the recurrence relation,	04⅔

$$T(n) = \begin{cases} a & n = 1, a \text{ is a const} \\ 2T(n/2) + cn, & n > 1, c \text{ is a const} \end{cases}$$

When n is a power of 2 then find the *big*O of $T(n)$.

(c)	What is difference between big O, big omega and big theta?	03
Q3(a)	Apply the quick sort algorithm on the following dataset: 65, 45, 75, 80, 85, 60, 55, 50, 45	04⅔
(b)	Write down the pseudo code for divide and conquer algorithm.	03
(c)	For the following tree, solve the TVSP when δ = 4.	04



Q4(a)	Write down the pseudocode of divide-and-conquer algorithm to solve convex hull problem.	03
(b)	Explain the greedy approach to solve knapsack problem.	03⅔
(c)	Obtain a set of optimal Huffman codes for the messages (M_1, ……., M_7) with relative frequencies (q_1…….q_7) = (4, 5, 7, 8, 10, 12, 20). Draw the decode tree for this set of codes.	05

SECTION B

Q5(a) Use Strassen's algorithm to compute the matrix product **03⅔**

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

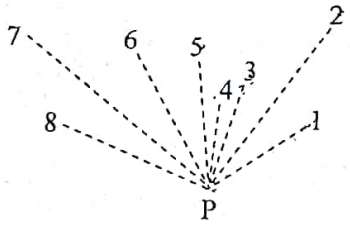
(b) What is the difference between NP-hard and NP-complete problem? Define deterministic algorithm. **04**

(c) Write down the N-Queens algorithm. **04**

Q6(a) What is dynamic programming? Write down two properties of dynamic programming. **03**

(b) Design a dynamic programming approach to solve the 8-queen problem. **04⅔**

(c) What is convex hull? Create a convex hull for the following set of points by using Graham-Scan algorithm and draw each step to construct the convex hull. **04**



Q7(a) Write the Dijkstra's algorithm to find the shortest path between a pair of verties. **03**

(b) Find out the optimal parenthesization of the following matrix chain, Also draw the 'm' and 's' tables. **05⅔**

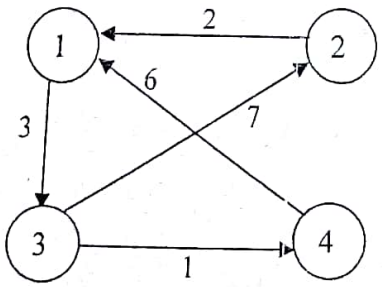
Matrix	A1	A2	A3	A4	A5	A6
Dimension	30×35	35×15	15×5	5×10	10×20	20×25
	0 1	2	3	4	5	6

(c) Explain a graph coloring algorithm. **03**

Q8(a) Prove that the fractional Knapsack problem has the greedy-choice property. **03**

(b) What is non-deterministic algorithm? **03**

(c) Obtain the shortest paths from every vertex to other vertices in the directed graph given below **05⅔**



1,5
2,6

********** END **********

Heaven's light is our guide

Rajshahi University of Engineering & Technology

B.Sc. Engineering 2nd Year 4th Semester Examination, 2011

Department of Computer Science & Engineering

Course no: **CSE 401** Course Title: **Algorithms Design & Analysis**

Full marks: 70 Time: Three (03) hours

N.B. Answer **six** questions, taking **three** from each section.
The questions are of equal value.
Use separate answer script for each section.

SECTION-A

Q1. (a) You win a shopping spree to walmart. The rules of spree are that you can only take 100 lbs total of items. You may have any weight of any item up to the weight of that item that is available. What is the maximal value of your shopping spree given that walmart has the following items with the following worth? 3

Item	Weight Available	Price	Value/lb
Watches	10	1200	$120
Cds	35	2100	$60
Cosmetics	30	2700	$90
Candy	80	2400	$5
Pots	30	2250	$75

(b) Trace multiplying the two decimal integers 87 and 39 wring a slight adaptation of the recursive multiplication algorithm shown in the text. Here is the algorithm from the text: 5

Let I an f J denote the n-digit binary numbers being multiplied. Let I_l denote the left-half digits of I and I_r denote the right-half digit of I. define J_l and J_r similarly. Then we have

$$I \times J = (I_l \times J_l)2^n + [(I_l - I_r) \times (J_l - J_r) + (I_l \times J_l) + (I_r \times J_r)]2^{n/2} + (I_r \times J_r)$$

(c) What is an algorithm? What is a program? $3\frac{2}{3}$

Q2. (a) What is dynamic programming? Write down two properties of dynamic programming. 3

(b) What is the complexity of an algorithm? What are the main characteristics to calculate the computational complexity of an algorithm. $3\frac{2}{3}$

(c) The complexity of some divided and conquer algorithms is given by recurrences of the form 5

$$T(n) = T(1) \text{ when } n = 1$$
$$= aT(n/b) + f(n), \text{ when } n > 1$$

Where a and b are known constants. Assume $T(1)=2$ and n is a power of b (i.e. $n = b^k$). Determine the big "O" of $T(n)$.

Q3 (a) Judge Jill has created a website that allows people to file complaints about one another. Each complaint contains exactly two name: that of the person who filed it and that of the person he/she is complaining about. 7

Jill has hoped to resolve each complaint personally but the site has received so many complaints that she has realized she wants an automated approach.

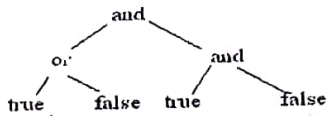
She decides to try to label each person as either good or evil. She only nude the labeling to be consistent, not necessarily correct. A labeling is consistent if every complaint labels one person as good and the other person as evil, and no person gets labeled both as good and evil in different complaints.

(i) Propose a way to model the consistent labeling problem as a graph problem.

(ii) Propose an efficient algorithm to consistently label all the names as good or evil, or to decide that no such classification exists. Use the graph model you proposed in the previous part of the problem. Analyze the running time of the algorithm.

(b) Write the general rules for in-order, pre-order and post-order tree traversal with example. $4\frac{2}{3}$

Q4. (a) At Aperture Bakeries every cake comes with a binary Boolean valued tree indicating whether or not it is available. Each leaf in the tree has either a true or false value. Each of the remaining nodes has exactly two children and is labeled either "and" or "or", the value is the result of recursively applying the operator to the values of the children. On example is the following tree: 7



If the root of a tree evaluates to false, like the one above the cake is a lie and you cannot have it. Any true cake is free for the taking. You may modify a tree to make it true. The only thing you can do to change a tree is to turn a false leaf into a true leaf or vice versa. This costs $1 for each leaf you change. You can't alter the operators or the structure of the tree.

Cake is good. Cheap cake is even better. Describe an efficient algorithm [consider BFS, dynamic programming, recursive cases] to determine the minimum cost of a cake whose tree has n nodes and analyze its running time.

(b) Write down Horner's rule of evaluating a polynomial. Consider 3 inputs (0,5), (1,10) and (2,21); $4\frac{2}{3}$

Here $G_1(x) = 5$ and $D_1(x) = (x - x_1) = x$ then find the polynomial by Langrage formula.

SECTION-B

Q5. (a) Consider the following instance of the Knapsack problem 6 2/3

Item	a	b	c	d	e
benefit	15	12	9	16	17
weight	2	5	3	4	6

The maximum allowable total weight in the Knapsack is $W_{max} = 12$.

(i) Find an optimal solution for the fractional Knapsack problem given the weights and benefits above. Be sure to state both the value of the maximum benefit that you obtain as well as the item(s) that you need to obtain this benefit. Show some reasoning for your work.

(ii) Do the same as above but for the {0,1} Knapsack version of this problem (using the same weights and benefits). As above, show some reasoning for your work.

(b) What is the meaning of the following terms: 5

(i) $5n^2$ is $\Omega(n^2)$

(ii) $5n^2$ is $\Omega(n)$

(iii) $5n^2$ is $\Theta(n^2)$

Q6. (a) Suppose that S and T are sorted arrays of integers and let x be an integer. 6

There is an obvious $O(n^2)$ time algorithm to determine if there are integers a in S and b in T such that $x = a + b$ (Simply try all pairs of elements to see if they satisfy the condition).

Can you think of (and describe) a more sophisticated search procedure that has a running time that is quicker than the "try every pair" approach? What is the running time of your proposed procedure?

(b) What is the worst-case running time of the Quick Sort algorithm on a list of n-integer? What is the worst-case running time of the Merge Sort algorithm? 5 2/3

Q7. (a) Define greedy method. How this method is used as control abstraction for the subset paradigm? 2 2/3

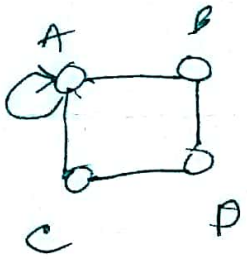
(b) What is Boruvka step? Where is it used and how? Explain with necessary diagram. 3

(c) How the Graham's scan algorithm is used to form a convex-hull from a set of points. Discuss briefly. 3

(d) Prove that the fractional Knapsack problem has the greedy-choice property. 3

Q8. (a) Draw a graph with the following adjacency matrix 3

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$



(b) Write the Welch-Power algorithm for coloring a graph G. 3

(c) Consider the definition 4

$h(0) = 1$

$h(n) = h(n-1) + h(floor(n/2)) + 1$

For example, $h(2) = h(1) + h(1) + 1 = 3, h(3) = h(2) + h(1) + 1 = 5$, etc. An obvious algorithm to complete h is a recursive one, based directly on the defining equations. Is that algorithm efficient? Why or why not? If not, how can the algorithm be improved? Justify your answer.

(d) What are the two representation of a univariate polynomial? 1 2/3

******

$h(4) = h(3) + h(2) + 1$

$= 5 + 3 + 1$

$h(1) = h(0) + h(0) + 1$

$= 1 + 1 + 1$

$= 3$

"Heaven's light is our guide"

Rajshahi University of Engineering & Technology

B Sc. Engineering 2nd year 4th Semester Examination, 2010

Department of Computer Science & Engineering

Course no: **CSE 401** Course Title: **Algorithms Design & Analysis**

Full marks: 70 Time: Three (03) hours

N.B. Answer **six** questions, taking **three** from each section.
The questions are of equal value.
Use separate answer script for each section.

SECTION-A

Q1. (a) Discuss how the performance of an algorithm can be analyzed. 3

(b) Find the asymptotic upper and lower bounds for $T(n)$ in each of the following recurrences. $4\frac{2}{3}$
Assume that $T(n)$ is constant for $n \le 1$.

i) $T(n) = \begin{cases} 1 & \text{if } n = 1 \\ 2T(n/2) + O(n) & \text{if } n > 1 \end{cases}$

(c) Draw the recursion tree for the Fibonacci numbers. 4

Q2. (a) Show by induction that a binary tree with n-nodes has height at least $\lfloor \log_2 n \rfloor$ $3\frac{2}{3}$

(b) Show that Quick sort's best-case running time is $\Omega(n \log_2 n)$ 4

(c) Consider an array of ten elements a[1:10] {'a', 'c', 'D', 'Q', 'P', 'a', 'Z', 'x', 'y', 'b'}. Find out the maximum and minimum using Devide and Conquer approach. Draw the recursion tree as well. 4

Q3 (a) What is tree diameter? Write an efficient algorithm to calculate the diameter of a tree. 4

(b) Find out the optimal parenthesization of the following matrix chain 5

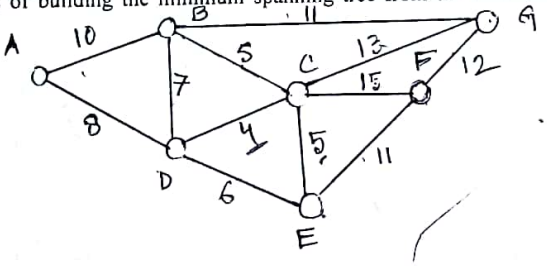
Matrix	Diameter
A_1	30 X 35
A_2	35 X 6
A_3	6 X 10
A_4	10 X 25
A_5	25 X 20

and draw the 'm' and 'S' tables.

(d) What are local and global optima? $2\frac{2}{3}$

Q4. (a) Distinguish between Breadth-First and Depth-First search. Is it always possible to find optimum solution using DFS? 4

(b) Show the various stages of building the minimum spanning tree from the following graph using Prim's algorithm. $4\frac{2}{3}$



(c) State the Cook's theorem. 3

SECTION-B

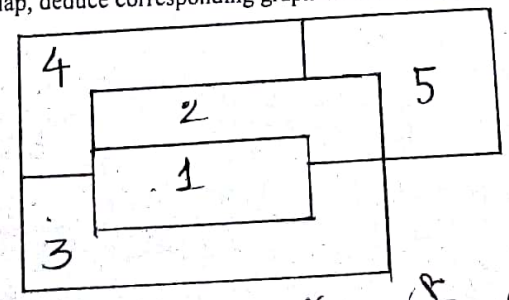
Q5. (a) Prove that $\left\lceil \log\left(\frac{m+n}{n}\right) \right\rceil \le MERGE(m,n) \le m+n-1$ 4

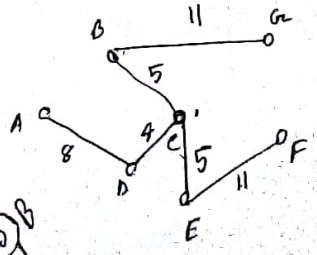
(b) "Any comparison-based algorithm that computes the largest and second largest of a set of n unordered elements requires $n-2+\lceil \log n \rceil$ comparison." Prove this. $4\frac{2}{3}$

(c) Describe the classes of NP-hard and NP-complete with examples. 3

Q6. (a) A sorting method is said to be stable if at the end of the method, identical elements occur in the same order as in the original unsorted set. Is merge sort a stable sorting method? 4

(b) From the map, deduce corresponding graph and then color it. $3\frac{2}{3}$





(c) Write down NQueens algorithm. 4

Q7. (a) Use Strasser's algorithm to compute the matrix product.

$$A = \begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix} \text{ and } B = \begin{bmatrix} 8 & 4 \\ 6 & 2 \end{bmatrix}.$$

Show your work.

(b) Show that the complexity of Strasser's matrix multiplication is $T(n) = O(n^{\log_2 7}) \approx O(n^{2.81})$. 5

(c) Define the 'P' class, 'NP'-class, 'NP-hard' and 'NP-complete' class of problems. 3

Q8. (a) Prove that the running time of an algorithm is $\theta(g^{(n)})$ if an only if its worst case running time is $O(g^{(n)})$ and the best case running time is $\Omega(g^{(n)})$. 3

(b) Write down the differences between greedy method and dynamic algorithm. 3

(c) Calculate the complexity of the Binary algorithm. $5\frac{2}{3}$

A_1 A_2 B_1 B_2

A_3 A_4 B_3 B_4

$P = (A_1 + A_4)(B_1 + B_4)$

$Q = (A_3 + A_4) B_1$

$R = A_1 (B_2 - B_4)$

$S = A_4 (B_3 - B_1)$

$T = (A_1 + A_2) B_4$

$U = (A_3 - A_1)(B_1 + B_2)$

$V = (A_2 - A_4)(B_3 + B_4)$

$2T(n/2) + O(n)$

2

$(A_1 + A_2)(B_1 + B_2)$

$(A_3 + A_4) B_1$

"Heaven's light is our guide"

Rajshahi University of Engineering & Technology

B.Sc Engineering 2nd Year 4th Semester Examination, 2009

Department of Computer Science and Engineering

Course no: CSE- 401 Course Title: Algorithm Design & Analysis

Full marks: 70 Time: Three (03) hours

N B. Answer **six** questions, taking **three** from each section.
The questions are of equal value.
Use separate answer script for each section

SECTION-A

			Marks
Q1.	(a)	Define time and Space complexity of an algorithm. "All algorithms must satisfy some criteria"- Discuss about those criteria.	04
	(b)	Show that the following equalities are incorrect: a) $n^2 \log n = \theta(n^2)$ b) $n^3 2^n + 6n^2 3^n = O(n^3 2^n)$	04
	(c)	Write GreedyKnapsack algorithm for greedy stategies.	$03\frac{2}{3}$
Q2.	(a)	Write down some advantages and disadvantages of randomized algorithms.	02
	(b)	Consider the array of ten elements $a[1:10] = (310,285,179,652,351,432,861,254,450,520)$. Sort the array using Merge Sort algorithm.	05
	(c)	In sum of subset problem m = 40 and $(w1, w2, w3, w4, w5) = (10,25,5,10,20)$, Draw the state space tree for backtracking algorithm to find out all possible solutions.	$04\frac{2}{3}$
Q3.	(a)	For the tree of the following figure, solve the TVSP when $\delta = 6$.	05
	(b)	Write down the differences between greedy method and dynamic algorithm.	02
	(c)	Show the various stages of building the minimum spanning tree from the graph below by Prim's algorithm.	$04\frac{2}{3}$
Q4.	(a)	What is convex hull? Why do we analyze the average case performance of a randomizes algorithm and not its worst case performance.	04
	(b)	Write an algorithm to delete an element x from a binary search tree t. What is the time complexity of your algorithm?	04
	(c)	Write down multistage graph algorithm corresponding to the forward approach.	$03\frac{2}{3}$

SECTION-B

			Marks
Q5.	(a)	Distinguish between breadth first search and depth first search. Is it always possible to find optimum solution using DFS?	$03\frac{2}{3}$
	(b)	Draw a possible tree using back-tracking to solve 4-queen problem.	04
	(c)	Discuss about explicit and implicit constraints with suitable examples.	04
Q6.	(a)	Does the following graph contain Hamilton cycle?	02
	(b)	Prove that any comparison-based algorithm that computes the largest and second largest of a set of n unordered elements requires $n - 2 + \lceil \log n \rceil$ comparisons.	$05\frac{2}{3}$
	(c)	Define the following terms: i) NP-Hard problem. ii) NP-Complete problem.	04
Q7.	(a)	Determine the polynomial of the smallest degree that interpolates the points (0,9), (1,11) and (2,13).	04
	(b)	How does one use the comparison trees for deriving lower bounds and ordered searching?	04
	(c)	What is algebraic simplification? Determine sparse representation from dense representation.	$03\frac{2}{3}$
Q8.	(a)	Show that if G is a connected undirected graph, then no edge of G can be in two different bi-connected components.	04
	(b)	Explain the Chinese Postman Problem.	$03\frac{2}{3}$
		Define live node. Write the iterative procedure of Branch and Bound method.	04

"Heaven's Light is Our Guide"

Department of Computer Science& Engineering
Rajshahi University of Engineering & Technology
B.Sc. Engineering 2nd Year 4th Semester Examination, 2008
Course No: **CSE 401** Course Title: **Algorithm Design & Analysis**
Full Marks: **70** Time: Three **(03)** Hours

N.B. *(i) Answer **SIX** questions, taking **THREE** from each Section*
(ii) Figures in the margin indicate full marks
(iii) Use separated answer script for each section

SECTION- A

Marks

Q.1 (a) What is computational and space complexity of an algorithm? Discuss about the following asymptotic notation i) O(n) ii) Ω(n) iii)θ(n). 2

(b) Using substitution method, find out the complexity of the following recurrence relation. 4

$$T(n) = \begin{cases} T(1), & n = 1 \\ 2T(n/2) + n, & n > 1 \end{cases}$$

and $T(1) = 2$

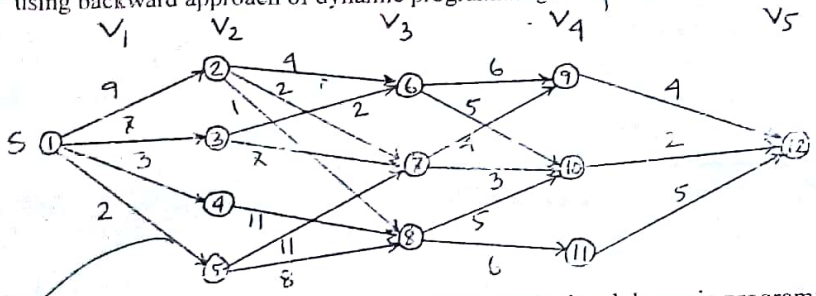
(c) Consider the array of ten elements a[1:10]= (310,285, 179, 652, 351, 432, 861, 254, 450, 520). Sort the array using Merge Sort Algorithm. 5½

Q.2 (a) What is convex hull? For the following 10 points (2,3), (-5,1), (3,6), (1,0), (-1,-4), (-3,-7), (7,3), (4,-5), (-3,-3, (1,1). Create a convex hull using Graham's Scan Algorithm. 5½

(b) Why do we analyze the average-case performance of a randomizes algorithm and not its worst-case performance? 2

(c) Let n = 4, $(P_1,P_2,P_3,P_4) = (100,10,15,27)$ and $(d_1,d_2,d_3,d_4) = (2,1,2,1)$ where P denotes profit, n denotes number of jobs and d their deadlines to perform. Show the feasible solutions and find out the optimal solution using greedy method. 4

Q.3 (a) Define multistage Graph. Find minimum-cost path from the following multistage Graph using backward approach of dynamic programming. 5½



(b) What is the basic difference between greedy method and dynamic programming? 2

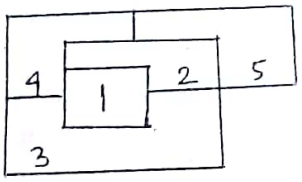
(c) Find an optimal paventhesization of a matrix-chain product whose sequence of dimensions is {5,10,3,12,5,50,6} 4

Q.4 (a) What is backtracking? Discuss about explicit and implicit constraints. 2

(b) Write down the algorithm of NQueens using backtracking. 4

(c) What is chromatic number? Define planar graph. 2

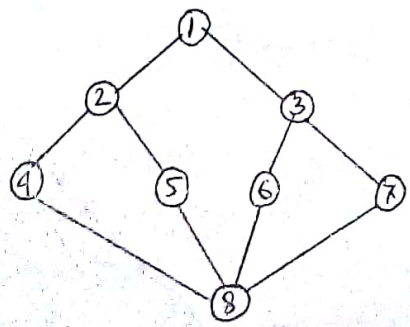
(d) Represent the following map into a planar graph. How many colors are required to color it? 3½



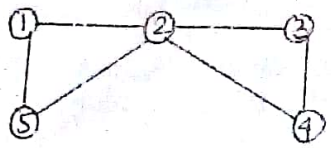
SECTION- B

Q.5 (a) Distinguish between breadth first search and depth first search. Is it always possible to find optimum solution using depth first search? Justify your answer. 3 4

(b) Show the adjacency list and traverse the following graph using BFS technique.



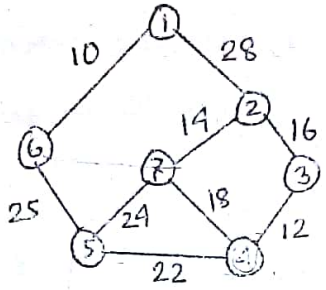
(c) Does the following graph contain Hamiltonian cycle?



→ not hamiltonian cycle

Q.6 (a) Write the Dijkstra's Algorithm to solve the single-source shortest path problem. 3½

Q.6 (a) Show the various stages of building the minimum spanning tree from the graph below by kruskal's algorithm. 4½



(b) In sum of subset problem m=31 and $(w_1,w_2,w_3,w_4) = (11,13,24,7)$. Draw the state space tree for backtracking algorithm to find out the all possible solutions. 4½

(c) Define the following terms: 2½
i) NP-Hard Problems
ii) NP- Complete Problems.

Q.7 (a) What is algebraic simplification? Differentiate Sparse representation from dense representation. 3

(b) Find out the polynomial by considering the following three inputs: (0,5), (1,10) and (2,21) using Newtonian interpolation. 4½

(c) Write down the Lagrange interpolation algorithm and deduce its multiplication complexity. 4

Q.8 (a) Write the Horner's rule algorithm for a Sparse representation of a polynomial, $A(x) = a_m x^{e_m} + \ldots\ldots + a_1 x^{e_1}$ where $a_i \neq 0$ and $e_m > e_{m-1} \ldots\ldots > e_1 \geq 0$. What is the required number of multiplication in this algorithm? 4½

(b) Prove that the fractional Knapsack problem has the greedy-choice property. 3½

(c) There are two sets by A[1:m] and B[1:n], where the items in A and the items in B are sorted. Find out the lower bound for algorithms that merge these two sets to give a single sorted array. 4½

***** The End *****

$\frac{5!}{5!\,3!}$

$\frac{5!}{3!\,(5-3)!}$

"Heaven's light is our guide"

Rajshahi University of Engineering & Technology

B.Sc. Engineering 2nd Year 4th Semester Examination, 2007

Department of Computer Science & Engineering

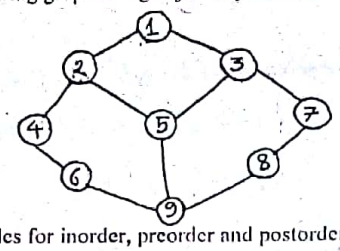
Course no: **CSE 401** Course Title: **Algorithm Design & Analysis**

Full marks: **70** Time: **Three (03) hours**

N.B. Answer **SIX** questions, taking **THREE** from each section.
The questions are of equal value.
Use separate answer script for each section.

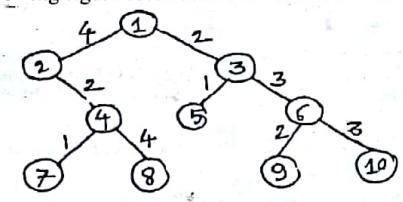
SECTION-A

Q1. (a) Represent the following graph using adjacency matrix and find out traversal path of the graph using BFS. 4



(b) Write the general rules for inorder, preorder and postorder tree traverse. $3\frac{2}{3}$

(c) For the tree of the following figure solve the TVSP when $\delta = 6$. 4



Q2. (a) Prove that the running time of an algorithm is $\theta(g(n))$ if an only if its worst-case running time is $O(g(n))$ and the best-case running time is $\Omega(g(n))$. 4

(b) if $a \geq 1$ and $b > 1$ are constants and $f(n)$ is a non-negative function defined on exact power of b. Define T(n) on exact powers of b by recurrence $4\frac{2}{3}$

$$T(n) = \begin{cases} \theta(1) & \text{if } n = 1 \\ aT(n/b) + f(n) & \text{if } n > 1 \text{ and } n = b^i \end{cases}$$

(c) Define the properties of an algorithm on which its performance can be measured. 3

Q3 (a) Using Strassen's Matrix multiplication calculate the product of the following two matrices. 4

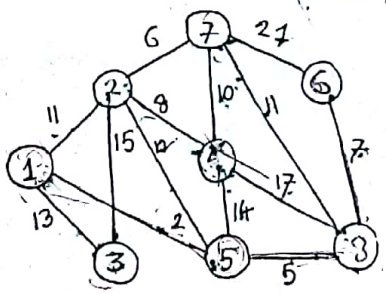
$$A = \begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix} \quad B = \begin{bmatrix} -3 & -2 \\ 5 & 7 \end{bmatrix}$$

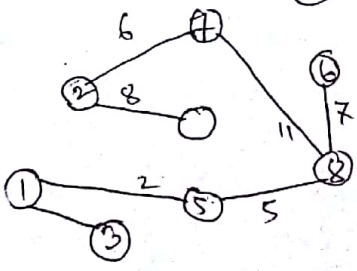
(b) What is convex hull? For the following 10 points. (2,3), (-5,1), (3,6), (1,0), (-1,-4), (-3,-7), (7,3), (4,-5), (-3,3), (1,1) create a convex hull using Graham's Scan algorithm. Show the details of the algorithm by writing initial contain of link list of all the points and when any point is delete from the list. $5\frac{2}{3}$

(c) Write down the differences between greedy method and dynamic algorithm. 2

Q4. (a) Briefly explain the ingredients of the problems that lend themselves to a greedy strategy. 4

(b) Compute a minimum cost spanning tree for the following graph using Prim's algorithm, $4\frac{2}{3}$





(c) Prove that the fractional Knapsack problem has the greedy-choice property. 3

SECTION-B

Q5. (a) What is Backtracking?

(b) Let W = { 15,17,10,12,15,18,2} and M = 55. Find all possible subsets of W that sum to M. Draw the portion of the state space tree that is generated.

(c) Write a backtracking algorithm for solving the knapsack optimization problem. 4

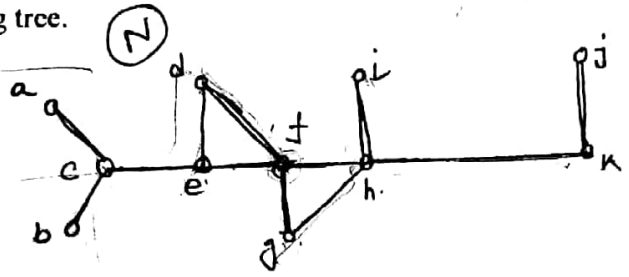
Q6. (a) Write an algorithm and draw a flow chart of a program that finds out the department from a roll number of your university. $4\frac{2}{3}$

(b) What are program code and pseudo code? Write the pseudo code for case statement used in C++. 4

(c) "Sometime space complexity must be considered", write your comments supporting this statement using proper example. 3

Q7. (a) What is NP-Hard and NP-Complete problems? How a NP-Hard problem is solved? $3\frac{2}{3}$

(b) Using breadth first search find out a minimum spanning tree of the graph. Consider f is the root of the spanning tree. 4

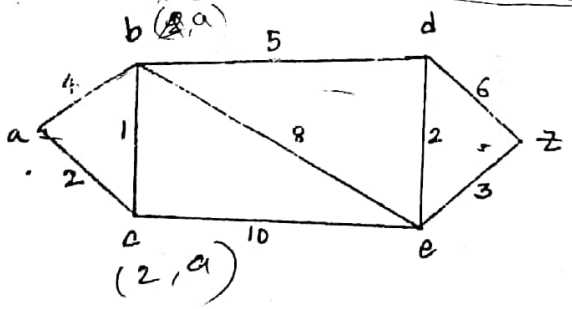


(c) How can you find the maximum and minimum number using recursive procedure? Find out the complexity of this algorithm. 4

Q8. (a) Show that any comparison-based algorithm that compute the largest and second largest of a set of n unordered elements requires $n-2+\lceil \log_{10} n \rceil$ comparison. $3\frac{2}{3}$

(b) Write an algorithm that finds out the response of a JK flip flop. 4

(c) Find out the shortest path using Dijkstra's algorithm from node a to z. 4



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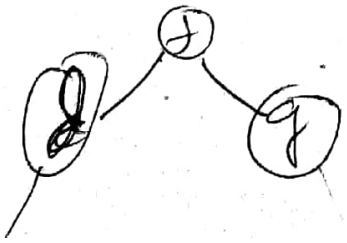


"Heaven's light is Our Guide"

Department of Computer Science & Engineering
Rajshahi University of Engineering & Technology

B. Sc. Engineering 2nd Year 4th Semester Examination 2006

Course No. CSE 401 Course Title: Algorithm Design & Analysis

Time: Three Hours Full Marks: 70

Note:
(i) Answer Six questions taking Three from each section
(ii) Figures in the right margin indicate full marks
(iii) Use separate answer script for each section

SECTION-A

Q1. (a) Define algorithm. Discuss the basic criteria of an algorithm.

(b) What do you mean by asymptotic notation? Discuss about the significance of (i) $O(n)$ (ii) $\Omega(n)$ (iii) $\Theta(n)$. 3

(c) Using substitution method, find out the complexity of the following recurrence relation. 3

$$T(n) = \begin{cases} T(1), n = 1 \\ 2T(n/2) + n, n > 1 \end{cases}$$

Let $T(1)=2$.

(d) Write insertion sort algorithm and find out its complexity. 3

Q2. (a) Define greedy method. How this method is used as control abstraction for the subset paradigm?

(b) What is Boruvka step? Where it is used and how? Explain with necessary diagram. 3

(c) How the Graham's scan algorithm is used to form a convex-hull from a set of points. Discuss briefly. 3

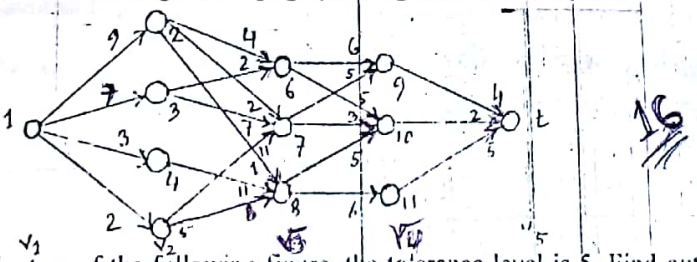
(d) Determine the space required by the following algorithms: 3

(i)
```
Sum (a, n) { s: = 0.0:
        for i: = 1 to n do
            s: = s + a[i];
        return s:}
```

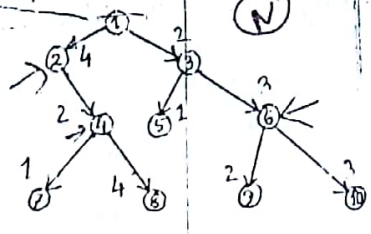
(ii)
```
Add (a, n) {
        if (n ≤ 0) then return 0.0;
            else return Add (a, n-1) + a[n];
        }
```

Q3. (a) Write down multistage graph algorithm corresponding to the forward approach. 3

(b) Solve the following five stage graph using the backward approach.

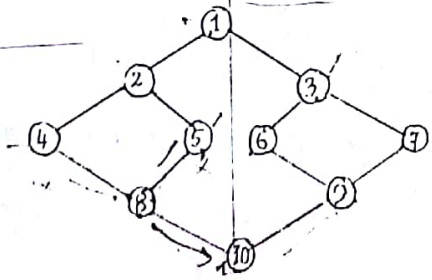


(c) In the tree of the following figure, the tolerance level is 5. Find out the nodes which are being splitted using the concept of TVSP. 4



Q4. (a) Distinguish between breadth first search and depth first search.

(b) Consider the following undirected graph G. Find out the adjacency list for G and traverse the graph using BFS. 3



(c) How does one find the maximum and minimum in recursive procedure? Find the complexity of this algorithm using recurrence relation. 4

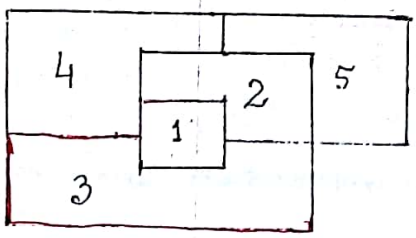
(d) What is backtracking? Discuss about explicit and implicit constraints? 3

SECTION-B

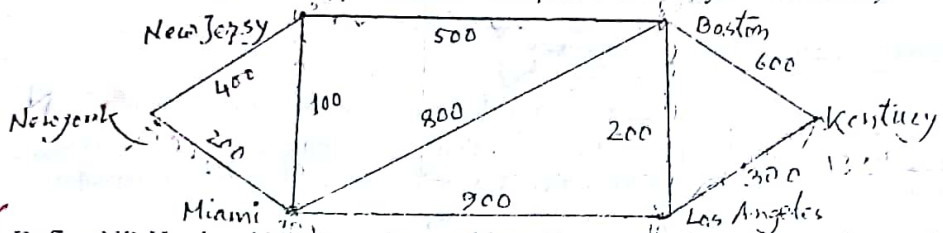
Q5. (a) Define n queens problem. How does one find a solution of the 8 queens problem? 4

(b) Describe briefly the problem of 15-puzzle arrangements. What is the necessary condition to reach the goal state? 3

(c) Represent the following map into a planar graph. How many colors are required to color the map? 2 3



Q6. (a) Use Dijkstra's algorithm to find the shortest path from Newyork to Kentucy. 2 3



(b) Define NP-Hard and NP-Complete problem. How can you test NP-Hardness of a problem? 3

(c) Discuss about the control abstraction of the least cost (LC) search. 4

Q7. (a) Write the Horner's rule algorithm for a sparse representation of a polynomial, $A(x) = a_m x^{e_m} + \ldots + a_1 x^{e_1}$, where $a_i \neq 0$ and $e_m > e_{m-1} > \ldots > e_1 \geq 0$. What is the required number of multiplication in this algorithm? 4

(b) How do you move up a binary tree to compute different functions at every node of a binary tree? Write the algorithm and find out it's complexity. 3

(c) How do you sort some numbers using convex hull algorithm? Explain with an example. 2 3

(d) What do you mean by "Locally Optimal"? 2

Q8. (a) Consider a set of tree items $A = \{1, 2, 3\}$. Draw a comparison tree for sorting the set and find out the complexity. 3

(b) Show that any comparison-based algorithm that computes the largest and second largest of a set of n unordered elements requires $n-2+\lceil \log n \rceil$ comparisons. 3

(c) Determine the polynomial of smallest degree that interpolates the points (0, 1), (1, 2) and (2, 3). 2 3

(d) Find out the complexity of Lagrange interpolation. 3

******* THE END *******

SHOVON

"Heaven's light is our guide"

Rajshahi University of Engineering & Technology

B.Sc. Engineering 2nd Year 4th Semester Examination, 2005

Department of Computer Science and Engineering

Course no: CSE 401C Course Title: Algorithm Design and Analysis

Full marks: 70 Time: Three (03) hours

N.B. Answer **six** questions, taking **three** from each section.
The questions are of equal value.
Use separate answer script for each section.

SECTION-A

Q1 (a) Define algorithm. Discuss its basic criteria.

(b) Discuss how the performance of an algorithm is analyzed?

(c) What is instance characteristics?

(d) Determine the space required by the following algorithms:

```
(i) Sum (a,n) { s:=0.0;
                for i:= 1 to n do
                    s:= s + a[i];
                return s; }
(ii) Add (a,n)
     { if (n ≤ 0) then return 0.0;
       else return Add(a, n-1) + a[n];
     }
```

Q2 (a) Illustrate the operation of Partition( ) in Quick sort on the Array, A = {13, 9, 5, 12, 8, 7, 4, 11, 2, 6, 21}. Consider A[p] and A[r] are the pivot elements, where p, r are used as the starting and last index of the array.

(b) Show that the running time of counting sort is O(n). Why counting sort is called stable and Non- Comparison sort?

(c) Determine the frequency counts for all statements in the following two algorithm segments.

```
(i) for i:= 1 to n do
        for j:= 1 to i do
            for k:= 1 to j do
                x:= x + 1;

(ii)  i:=1
      while (i ≤ n) do { x:= x + 1
                         i:= i + 1;}
```

Q3 (a) Write the control abstraction and explain the divide and Conquer algorithm.

(b) For the following recurrence relation

$$T(n) = \begin{cases} b & \text{If } n < 3 \\ 3T(n/3) + bn & \text{If } n > 3 \end{cases}$$

Draw its recursion tree.

(c) Determine the big O of the following recursive equation

$$T(n) = 2T(n/2) + \log n$$
$$= b \quad \text{for } n < 2$$

Q4 (a) Compare the performance and efficiency of Recursive Max Min and Straight forward Max Min algorithm.

(b) What is chromatic number of a graph? Define planar graph.

(c) What is the benefit of incorporating insertion sort algorithm in side the merge sort?

(d) What is meant by 'Locally Optimal'?

SECTION-B

Q5 (a) How does one use the comparison trees for deriving lower bounds on ordered searching?

(b) What is meant by TVSP? On which condition a node will be splitted? Discuss briefly with an example

(c) Consider the algebraic expression, $E = (2x + y)(5a - b)^3$

(i) Draw the tree T which corresponds to the expression E.

(ii) Find the preorder of T.

Q6 (a) What is convex hull? Write the Graham's Scan algorithm.

(b) Explain the Greedy method by writing its Control abstraction.

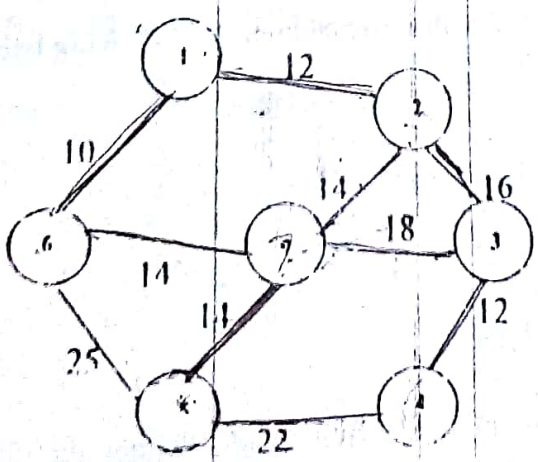
(c) Find an optimal solution to knapsack instance n = 7, m = 15, $(P_1, P_2, \ldots\ldots P_7) = (10,5,15,7,6,18,3)$, and $(W_1, W_2, \ldots\ldots W_7) = (2,3,5,7,1,4,1)$.

Q7 (a) What is algebraic simplification? Differentiate sparse representation from dense representation.

(b) Write the Horner's rule algorithm to evaluate the polynomial $A(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots\ldots + a_1 x + a_0$ at a point v. Is it advantageous over straight forward rule? Explain.

(c) Find out the polynomial by considering the following three inputs: (0,5), (1,10) and (2, 21), using Newtonian interpolation.

Q8 (a) Compute a minimum cost spanning tree for the following graph using Prim's algorithm.



(b) Define NP Hard and NP complete problem. How does one prove that whether a particular problem is NP Hard or not?

(c) When a vertex is said to be explored in the Breath first search method? What are the differences between Breath first search method and depth search method.

SHOVON

"Heaven's light is our Guide"

Computer Science and Engineering Department
Rajshahi University of Engineering & Technology
B. Sc. Engineering 2nd year 4th semester Examination, 2004
Course Title: Algorithm Design and Analysis, Course No. CSE 401C
Time :3 hours, Full marks:70

N.B: Answer six questions taking three from each section.
Use separate answer script for each section.
The Questions are of equal value

Section-A

Q 1 a) Write down some features to design an efficient algorithm.

b) What is computational and space complexity of an algorithm? What are the main considerations to calculate the computational complexity?

c) For the following algorithm determine the necessary stack space.

```
Rsum (a, n)
{if (n≤0) then return 0.0;
else return Rsum(a,n-1)+a[n];
}
```

d) Algorithm SeqSearch () finds out an item from a list. Construct a proper timing algorithm to measure its runtime complexity.

Q2 a) The complexity of many divide and conquer algorithms is given by recurences of the form

$$T(n) = \begin{cases} T(1) & n=1 \\ aT(n/b)+f(n) & n>1 \end{cases}$$

where a and b are known constants. Assume T(1)=2 and n is a power of b (i.e $n = b^k$) Determine the big O of T(n).

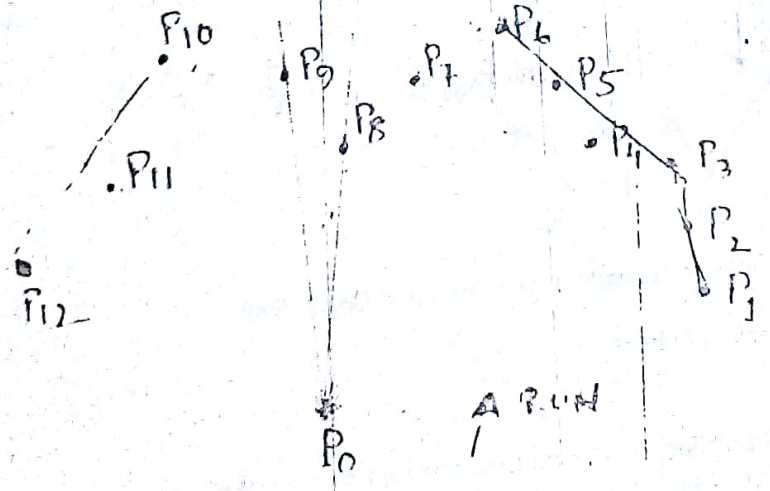
b) What is chromatic number of a graph? Define planar graph.

c) Explain how will you determine whether two consecutive line segment turn left or right.

d) Explain the travelling salesman problem.

Q3 a) Stating the control abstraction explain greedy method.

b) In case of execution GRAHAM SCAN algorithm for the following set of points. Draw each step to construct the convex hull.



c) Let A,B,C,D,E,F,G,H be eight letters with the following frequency in a text file:

letter	A	B	C	D	E	F	G	H
frequency	22	5	11	19	2	11	25	5

Constructing Huffman tree find out corresponding bit string for each of the letter and also determine the compression ratio in case of bit string representation of each letter in the file. Ignore the size of the header tree.

Q4 a) State and explain the principle of optimality for dynamic programming

b) Write the fractional Knapsack procedure and find out the optimal solution for n=7, m=15 (p1,p2,p3 ... p7) = (10,5,15,7,6,18,3) and (w1,w2,w3,w4,w7) = (2,3,5,7,1,4,1), where n is the number of weight, m is Knapsack capacity, p_i and w_i (1<i<7) are profit and corresponding weight respectively

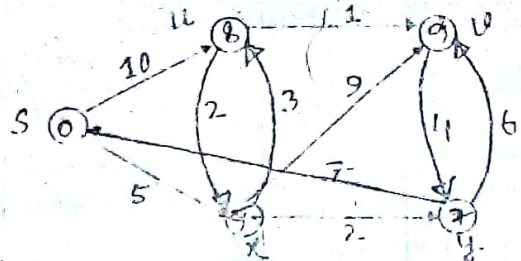
c) In Sum of Subset problem m=31 and (w1,w2,w3,w4) = (11,13,24,7). Draw the state space tree for backtracking algorithm to find out the all possible solution.

Section-B

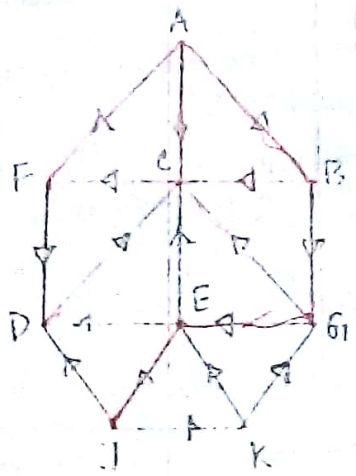
Q5 a) There may be some edges whose weights are negative in some instances of the single source shortest-path problem. Is there any possibility to form a negative weight cycle? Explain with proper example

b) Write the Dijkstra's algorithm to solve the single-source shortest path problem

c) Consider the following weighted directed graph. Use the Dijkstra's algorithm to obtain the shortest path that traverses all vertices.



Q6a) The following graph G represents the daily flights between cities of some airline, and suppose one wants to fly from city A to city J with the minimum number of stops. Using a Queue Q, write the algorithm and show the different state of Q to obtain the desired route

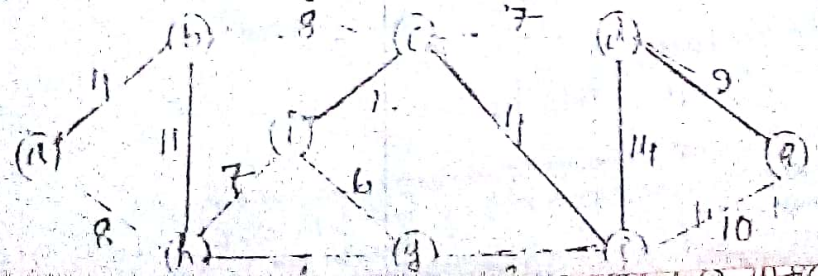


b) What are the differences between breadth first search and Depth first search?

c) Explain the max clique problem in brief

d) What is AND/OR graph decision problem?

Q7 a) Show the various stages of building the minimum spanning tree from the graph below by Kruskals algorithm



Q7 b) Consider a polynomial, $A(x)=a_n x^n + \ldots\ldots\ldots + a_0$ To evaluate it at any point, prove that Horner's rule is an improvement over the straight forward method by a factor of 2.

c) Determine the polynomial of the smallest degree that interpolates the points (0,9), (1,12) and (2,15).

Q8 a) How does one use the comparison trees for deriving lower bounds on sorting? Give an example.

b) There are two sets A[1:m] and B[1:n], where the items in A and the items in B are sorted. Find out the lower bounds for algorithms that merge these two sets to give a single sorted array.

c) How does the discrete Fourier transform correspond to the evaluation of a polynomial? Explain.

------THE END------

"Heaven's light is our guide"

Rajshahi University of Engineering & Technology

B.Sc. Engineering 2nd Year 4th Semester Examination, 2003

Department of Computer Science and Engineering

Course no: CSE 4012 Course Title: Algorithm Design and Analysis

Full marks: 70 Time: Three (03) hours

N.B. Answer six questions, taking three from each section.
The questions are of equal value.
Use separate answer script for each section.

SECTION-A

Q1 (a) Explain the methods of solving recurrences. Using substitution method prove that $T(n) = T(n/2) + 1$ is $O(\log n)$.

(b) Determine a good asymptotic upper bound on the recurrence $T(n) = 3T(n/2) + n$ by iteration. Show the recursion tree too.

(c) Mentioning the constraints write down the rules to solve recurrences using "Master" method

Q2 (a) With the help of figures illustrate the operation of Heap Extract Max on the heap A = {15, 13, 9, 5, 12, 8, 7, 4, 0, 6, 2, 1}

(b) Illustrate the operation of Partition( ) in Quick sort on the Array, A = {13, 19, 9, 5, 12, 8, 7, 4, 11, 2, 6, 21}

(i) Where A[p] is the pivot element

(ii) Where A[r] is the pivot element.

p, r are used as the starting and last index of the array

(c) Show that the running time of counting sort is O(n). Why counting sort is called stable and How Comparison sort?

Q3 (a) Write down the steps for developing an algorithm in dynamic programming approach

(b) What is an optimal Huffman code for the following frequencies:

a : 3; b : 5; c : 8; d : 13; e : 21; f : 34; g : 55; h : 89;

(c) What are the differences between Greedy and dynamic programming?

Q4 (a) Write down the place(i ,i) and N Queen (k,n) function for the N Queen problem

(b) Graphically show if N Queen (1,4) is called in N Queen problem where the dimension of the chess board is 4×4

(c) Illustrate the following terms with graphical representation

(i) o (ii) Ω (iii) w

SECTION-B

Q5 (a) How does one use the comparison trees for deriving lower bounds on ordered searching?

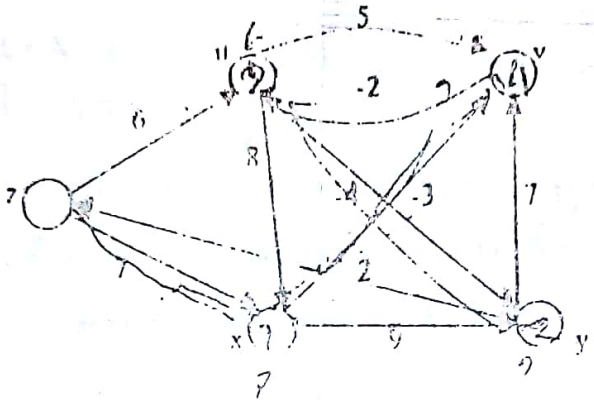
(b) There are two sets A[1..n] and B[1..n], where the items in A and the items in B are sorted. Find out the lower bounds for algorithms that merge these two sets to give a single sorted array.

(c) What is LVSP? On which condition a node will be spitted? Discuss briefly with an example

Q6.(a) There may be edges whose weights are negative in some instances of the single-source shortest path problem. Is there any possibility to form a negative weight cycle? Explain.

(b) Write the Bellman Ford algorithm to solve the single source shortest path problem.

(c) Consider the following weighted directed graph. Use Bellman-Ford algorithm to obtain the shortest path that traverses all vertices.

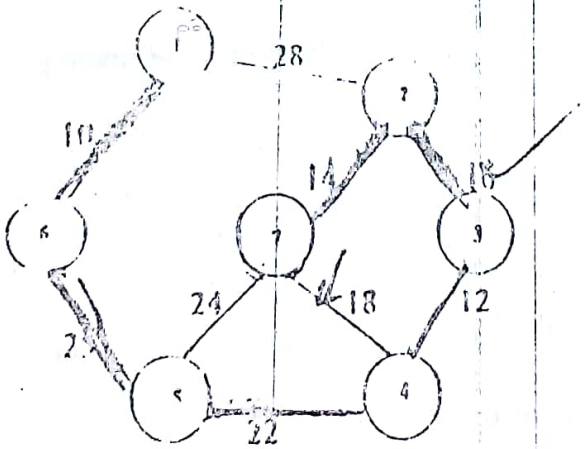


Q7.(a) What is algebraic simplification? Differentiate sparse representation from dense representation

(b) Write the Horner's rule algorithm to evaluate the polynomial

$A(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots\ldots\ldots + a_1 x + a_0$ at a point v. Is this advantageous over straight forward rule? Explain.

(c) Compute a minimum cost spanning tree for the following graph using Kruskal's algorithm



OR (a) Explain the Chinese Postman problem.

(b) When a vertex is said to be explored in the Breath first search method? Write the pseudo code for Breath first search method.

(c) Define live node. Write the iterative procedure of Branch and Bound method.

"Heaven lights our guide"

Bangladesh Institute of Technology (BIT), Rajshahi

B.Sc. Engineering 2nd Year 4th Semester Examination, 2002

Department of Computer Science and Engineering

Course no: CSE 4012 Course Title: Algorithm Design and Analysis.

Full marks: 70 Time: Three (03) hours.

N.B. Answer six questions, taking three from each section.

The questions are of equal value.

Use separate answer script for each section.

SECTION - A

Q.1 (a) Briefly explain the following criteria of algorithms:

(i) Definiteness (ii) Effectiveness (iii) Finiteness;

(b) What is meant by 'O' notation? Finding the value of C_1, C_2 and n_0 prove that

$\frac{1}{2}n^2 - 3n = \Theta(n)$

(c) Briefly explain the components of dynamic programming.

(d) Logically prove that Strassen matrix Multiplication takes less amount of time than Conventional matrix Multiplication.

Q.2 (a) What is the key difference between Greedy method and dynamic programming?

(b) Write an optimal Huffman code for the following characters a, b, c, d, e, f, g, h if they are having frequencies based on first eight numbers of fibonacci series.

(c) Let C be an alphabet in which each character $c \in C$ has frequency $f[c]$. Let x and y be the two characters in C having the lowest frequencies. Then prove that, there exists an optimal prefix code in C in which the code words for x and y have the same length and differ only in the last bit.

Q.3 (a) Write down the equation to show that in N queen problem two queens are placed in the same diagonal if their positions are at (i, j) and (k, l).

(b) Write down the control abstractions for LC search.

(c) Prove that worst case partitioning of Quicksort is $O(n^2)$.

Q.4 (a) Write Pseudo code to place a queen in n no. of chess board.

(b) Prove that BFS and DFS are the special cases of LC search.

(c) Find out the explicit and implicit constraints for 8 queen problem.

SECTION - B

Q.5 (a) Define the term Algebraic simplification.

(b) Suppose the polynomial $A(x) = a_m x^{e_m} + \dots + a_1 x^{e_1}$, where $a_i \neq 0$ and $e_m > e_{m-1} > \dots > e_1 \geq 0$ is represented using the sparse form. Write a function PAdd(r, s, t) that computes the sum of two such polynomials r and s and stores the result in t.

(c) Determine the polynomials of smallest degree that interpolates the points (0, 5), (1, 10) and (2, 21) using lagrange interpolation formula.

Q.6 (a) Consider the following univariate polynomials: $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$

Represent the above polynomial in Dense and sparse form.

"Heaven's light is our guide"

Rangpur Institute of Technology (RIT), Rajshahi

B.Sc. Engineering 2nd Year 4th Semester Examination, 2002

Department of Computer Science and Engineering

Course no: CSE 4101 Course Title: Algorithm Design and Analysis.

Full marks: 70 Time: Three (03) hours.

N.B. Answer six questions, taking three from each section.

The questions are of equal value.

Use separate answer script for each section.

SECTION - A

1. (a) Briefly explain the following criteria of algorithms:
   (i) Definiteness (ii) Effectiveness (iii) Finiteness;
   (b) What is meant by 'O' notation? Finding the value of C_1, C_2 and n_0 prove that $\frac{1}{2}n^2 - 3n = \Theta(n^2)$
   (c) Briefly explain the components of dynamic programming.
   (d) Logically prove that Strassen matrix Multiplication takes less amount of time than Conventional matrix Multiplication

2. (a) What is the key difference between Greedy method and dynamic programming?
   (b) Write an optimal Huffman code for the following characters a, b, c, d, e, f, g, h if they are having frequencies based on first eight numbers of fibonacci series.
   (c) Let C be an alphabet in which each character $c \in C$ has frequency $f[c]$. Let x and y be the two characters in C having the lowest frequencies then prove that, there exists an optimal code in C in which the code words for x and y have the same length and differ only in their last bit.

3. (a) Write down the equation to show that in N queen problem two queens are placed in the same diagonal if their positions are at (i, j) and (k, l)
   (b) Write down the control abstractions for LC search.
   (c) Prove that worst case partitioning of Quicksort is $\Theta(n^2)$

4. (a) Write Pseudo code to place a queen in n-n chess board
   (b) Prove that BFS and DFS are the special cases of LC search
   (c) Find out the explicit and implicit constraints for 8 queen problem.

SECTION - B

5. (a) Define the term Algebraic simplification
   (b) Suppose the polynomial $A(x) = a_m x^{e_m} + \ldots + a_1 x^{e_1}$, where $a_i \neq 0$ and $e_m, e_{m-1}, \ldots, e_1 \geq 0$ is represented using the sparse form. Write a function PAdd(a, b, t) that computes the sum of two such polynomials a and b and stores the result in t.
   (c) Determine the polynomials of smallest degree that interpolates the points (0, 5), (1, 10) and (3, 21) using lagrange interpolation formula.

6. (a) Consider the following univariate polynomials: $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$
   Represent the above polynomial in Dense and sparse form.

Q.6(a) Prove that an undirected graph has an even no. of vertices of odd degree.

(b) Define planar graph.

(c) Prove that if a connected planar simple graph has e edges and v vertices with $v \geq 3$ and no circuits of length 3, then $e \leq 2v - 4$.

(d) Suppose that a connected simple planar graph has 10 vertices, each of degree 5. Into how many regions does a representation of this planar graph split the plane?

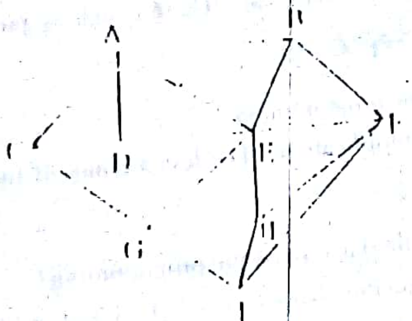
(e) What is the basic difference between the incidence matrix and adjacency matrix in respect of graph representation?

Q.7(a) Define Euler circuit and Hamilton circuit.

(b) Write an algorithm to construct Euler circuit.

(c) Discuss the four colour theorem.

(d) Consider the following graph. Use welch powell algorithm to paint the graph and find the chromatic number of the graph.



Q.8(a) Let A[1..n], n ≥ 1, contain n distinct elements, ordered so that A[1] < ... < A[n]. Let find(n) be the minimum number of comparisons needed, in the worst case, by any comparison based algorithm to recognize whether x ∈ A[1..n]. Then prove that find(n) ≥ ⌈log(n+1)⌉.

[illegible] puppet theater has [illegible] 600 seats. The admission fee to the theater is $3.25 per adult and $1.00 per child. Suppose $1172.25 was collected. Find the number of adults and children attending the performance.

(c) Use pruning algorithm to find the shortest path from u to w in the following figure.