$$\sqrt{1+\sqrt{2+\sqrt{3+\sqrt{4\ldots}}}}$$

$$1-1+1-1+1\ldots\ldots\ldots = ?$$

$$\sqrt{1+\sqrt{2+\sqrt{3+\sqrt{4\ldots}}}}$$

Discrete mathematics

Basic Structures: Sets, Functions, Sequences, Sums, and Matrices

$$\exists_{x \in \Re} \exists_{y \in \Re} (x = y)$$

$$\forall_x (\Re \mid x)$$

$$\sum_{x=1}^{\infty} \frac{1}{x} = ?$$

$$\sum_{x=1}^{\infty} x = ?$$

Chapter 2

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Sets

Section 2.1

Section Summary

- Definition of sets
- Describing Sets
  - Roster Method
  - Set-Builder Notation
- Some Important Sets in Mathematics
- Empty Set and Universal Set
- Subsets and Set Equality
- Cardinality of Sets
- Tuples
- Cartesian Product

Introduction

- Sets are one of the basic building blocks for the types of objects considered in discrete mathematics.
  - Important for counting.
  - Programming languages have set operations.
- Set theory is an important branch of mathematics.
  - Many different systems of axioms have been used to develop set theory.
  - Here we are not concerned with a formal set of axioms for set theory. Instead, we will use what is called naïve set theory.

Sets

- A *set* is an unordered collection of objects.
  – the students in this class
  – the chairs in this room
- The objects in a set are called the *elements*, or *members* of the set. A set is said to *contain* its elements.
- The notation $a \in A$ denotes that a is an element of the set A.
- If a is not a member of A, write $a \notin A$

Describing a Set: Roster Method

- $S = \{a, b, c, d\}$
- Order not important

  $S = \{a, b, c, d\} = \{b, c, a, d\}$
- Each distinct object is either a member or not; listing more than once does not change the set.

  $S = \{a, b, c, d\} = \{a, b, c, b, c, d\}$
- Elipses (…) may be used to describe a set without listing all of the members when the pattern is clear.

  $S = \{a, b, c, d, \ldots\ldots, z\}$

Roster Method

- Set of all vowels in the English alphabet:

  $V = \{a,e,i,o,u\}$

- Set of all odd positive integers less than 10:

  $O = \{1,3,5,7,9\}$

- Set of all positive integers less than 100:

  $S = \{1,2,3,\ldots\ldots..,99\}$

- Set of all integers less than 0:

  $S = \{\ldots., -3,-2,-1\}$

Some Important Sets

N = *natural numbers* = {0,1,2,3….}

Z = *integers* = {…,-3,-2,-1,0,1,2,3,…}

Z⁺ = *positive integers* = {1,2,3,…..}

R = set of *real numbers*

R⁺ = set of *positive real numbers*

C = set of *complex numbers*.

Q = set of rational numbers

Set-Builder Notation

- Specify the property or properties that all members must satisfy:

  $S = \{x \mid x \text{ is a positive integer less than } 100\}$

  $O = \{x \mid x \text{ is an odd positive integer less than } 10\}$

  $O = \{x \in \mathbf{Z}^{+} \mid x \text{ is odd and } x < 10\}$

- A predicate may be used:

  $$S = \{x \mid \mathrm{P}(x)\}$$

- Example: $S = \{x \mid \mathrm{Prime}(x)\}$
- Positive rational numbers:

  $$\mathbf{Q}^{+} = \{x \in \mathbf{R} \mid x = p/q, \text{ for some positive integers } p, q\}$$

Interval Notation

$$[a,b] = \{x \mid a \leq x \leq b\}$$
$$[a,b) = \{x \mid a \leq x < b\}$$
$$(a,b] = \{x \mid a < x \leq b\}$$
$$(a,b) = \{x \mid a < x < b\}$$

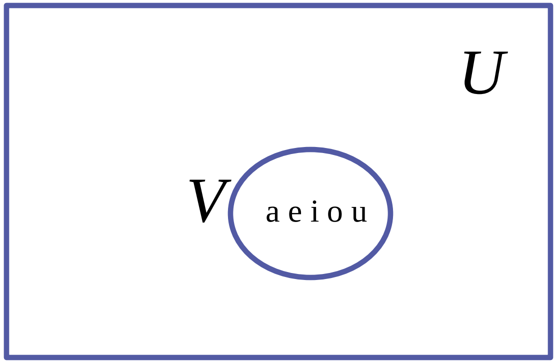
closed interval [a,b]

open interval (a,b)

Universal Set and Empty Set

- The *universal set* U is the set containing everything currently under consideration.
  - Sometimes implicit
  - Sometimes explicitly stated.
  - Contents depend on the context.
- The empty set is the set with no elements. Symbolized $\emptyset$, but $\{\}$ also used.

Venn Diagram



Russell's Paradox

- Let S be the set of all sets which are not members of themselves. A paradox results from trying to answer the question "Is S a member of itself?"
- Related Paradox:
  - Henry is a barber who shaves all people who do not shave themselves. A paradox results from trying to answer the question "Does Henry shave himself?"

Some things to remember

- Sets can be elements of sets.

  $\{\{1,2,3\}, a, \{b, c\}\}$

  $\{\mathbf{N}, \mathbf{Z}, \mathbf{Q}, \mathbf{R}\}$

- The empty set is different from a set containing the empty set.

  $\emptyset \neq \{\emptyset\}$

Set Equality

Definition: Two sets are *equal* if and only if they have the same elements.

- Therefore if A and B are sets, then A and B are equal if and only if $\forall x (x \in A \leftrightarrow x \in B)$.
- We write $A = B$ if A and B are equal sets.

$$\{1,3,5\} = \{3, 5, 1\}$$

$$\{1,5,5,5,3,3,1\} = \{1,3,5\}$$

Subsets

Definition: The set A is a *subset* of B, if and only if every element of A is also an element of B.

- The notation $A \subseteq B$ is used to indicate that A is a subset of the set B.
- $A \subseteq B$ holds if and only if The image cannot be displayed. Your computer may not have enough memory to open the image, or the image may have been corrupted. Restart your computer, and then open the file again. If the red x still appears, you may have to delete the image and then insert it again. is true.
  1. Because $a \in \emptyset$ is always false, $\emptyset \subseteq S$,for every set S.
  2. Because $a \in S \rightarrow a \in S$, $S \subseteq S$, for every set S.

Showing a Set is or is not a Subset of Another Set

- **Showing that A is a Subset of B**: To show that $A \subseteq B$, show that if x belongs to A, then x also belongs to B.
- **Showing that A is not a Subset of B**: To show that A is not a subset of B, $A \nsubseteq B$, find an element $x \in A$ with $x \notin B$. (Such an x is a counterexample to the claim that $x \in A$ implies $x \in B$.)

Examples:

1. The set of all computer science majors at your school is a subset of all students at your school.
2. The set of integers with squares less than 100 is not a subset of the set of nonnegative integers.

Another look at Equality of Sets

- Recall that two sets A and B are *equal*, denoted by $A = B$, iff $\forall x(x \in A \leftrightarrow x \in B)$

- Using logical equivalences we have that $A = B$ iff

$$\forall x[(x \in A \rightarrow x \in B) \wedge (x \in B \rightarrow x \in A)]$$

- This is equivalent to

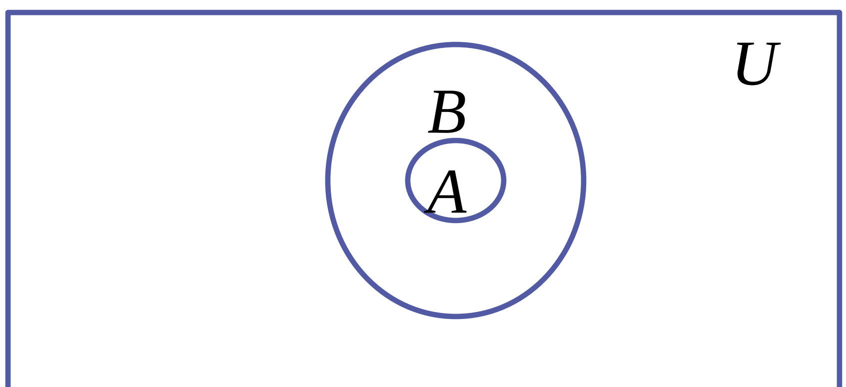
$$A \subseteq B \quad \text{and} \quad B \subseteq A$$

Proper Subsets

Definition: If $A \subseteq B$, but $A \neq B$, then we say A is a *proper subset* of B, denoted by $A \subset B$. If $A \subset B$, then

is true.

Venn Diagram



Set Cardinality

Definition: If there are exactly n distinct elements in S where n is a nonnegative integer, we say that S is *finite*. Otherwise it is *infinite*.

Definition: The *cardinality* of a finite set A, denoted by $|A|$, is the number of (distinct) elements of A.

Examples:

1. $|ø| = 0$
2. Let S be the letters of the English alphabet. Then $|S| = 26$
3. $|\{1,2,3\}| = 3$
4. $|\{ø\}| = 1$
5. The set of integers is infinite.

Power Sets

Definition: The set of all subsets of a set A, denoted $\mathcal{P}(A)$, is called the *power set* of A.

Example: If $A = \{a,b\}$ then

$$\mathcal{P}(A) = \{\emptyset, \{a\},\{b\},\{a,b\}\}$$

- If a set has n elements, then the cardinality of the power set is 2^n. (In Chapters 5 and 6, we will discuss different ways to show this.)

Tuples

- The *ordered n-tuple* $(a_1, a_2, \ldots, a_n)$ is the ordered collection that has a_1 as its first element and a_2 as its second element and so on until a_n as its last element.
- Two n-tuples are equal if and only if their corresponding elements are equal.
- 2-tuples are called *ordered pairs*.
- The ordered pairs (a,b) and (c,d) are equal if and only if $a = c$ and $b = d$.

Cartesian Product

Definition: The *Cartesian Product* of two sets A and B, denoted by $A \times B$ is the set of ordered pairs (a,b) where $a \in A$ and $b \in B$.

Example: $A \times B = \{(a, b) | a \in A \wedge b \in B\}$

$A = \{a, b\}$ $B = \{1,2,3\}$

$A \times B = \{(a,1),(a,2),(a,3), (b,1),(b,2),(b,3)\}$

- **Definition**: A subset R of the Cartesian product $A \times B$ is called a *relation* from the set A to the set B.

Cartesian Product

Definition: The cartesian products of the sets $A_1, A_2, \ldots\ldots, A_n$, denoted by $A_1 \times A_2 \times \ldots\ldots \times A_n$, is the set of ordered n-tuples $(a_1, a_2, \ldots\ldots, a_n)$ where a_i belongs to A_i for $i = 1, \ldots n$.

$$A_1 \times A_2 \times \cdots \times A_n = \{(a_1, a_2, \ldots, a_n) | a_i \in A_i \text{ for } i = 1, 2, \ldots n\}$$

Example: What is $A \times B \times \mathrm{C}$ where $A = \{0,1\}$, $B = \{1,2\}$ and $C = \{0,1,2\}$

Solution: $A \times B \times \mathrm{C} = \{(0,1,0), (0,1,1), (0,1,2), (0,2,0), (0,2,1), (0,2,2), (1,1,0), (1,1,1), (1,1,2), (1,2,0), (1,2,1), (1,2,2)\}$

Truth Sets of Quantifiers

- Given a predicate P and a domain D, we define the *truth set* of P to be the set of elements in D for which $P(x)$ is true. The truth set of $P(\mathrm{x})$ is denoted by

$$\{x \in D | P(x)\}$$

- **Example**: The truth set of $P(x)$ where the domain is the integers and $P(x)$ is "$|x| = 1$" is the set $\{-1,1\}$

Query???

$$\sqrt{1+\sqrt{2+\sqrt{3+\sqrt{4....}}}}$$

$$\exists_{x\in\Re}\exists_{y\in\Re}(x = y) = ?$$

$$\sum_{x=1}^{\infty} x = ?$$

$$\sum_{x=1}^{\infty}\frac{1}{x} = ?$$

$$\forall_{x}(\Re \mid x) = ?$$

$$\exists_{x\in\Re}\exists_{y\in\Re}(x = y) = ?$$

$$\sqrt{1+\sqrt{2+\sqrt{3+\sqrt{4....}}}} = ?$$

$$1-1+1-1+1......... = ?$$

$$\sum_{x=1}^{\infty}\frac{1}{x} = ?$$