→ Linear Property

$$F[c_1 x_1(t) + c_2 x_2(t)] = c_1 X_1(f) + c_2 X_2(f)$$

$$F[c_1 x_1(t) + c_2 x_2(t)] = \int_{-\alpha}^{\alpha} \{c_1 x_1(t) + c_2 x_2(t)\} e^{-i2\pi ft} dt$$

$$= \int_{-\alpha}^{\alpha} c_1 x_1(t) e^{-i2\pi ft} dt + \int_{-\alpha}^{\alpha} c_2 x_2(t) e^{-i2\pi ft} dt$$

$$= c_1 X_1(f) + c_2 X_2(f)$$

→ ~~Scale~~ Shifting Property

$$F\{g(t-a)\} = e^{-i2\pi fa} G(f)$$

$$F\{g(t-a)\} = \int_{-\alpha}^{\alpha} g(t-a) e^{-i2\pi ft} dt$$

$$= \int_{-\alpha}^{\alpha} g(u) e^{-i2\pi f(u+a)} du$$

Let
$t - a = u$
$dt = du$

$$= \int_{-\alpha}^{\alpha} g(u) e^{-i2\pi fu} e^{-i2\pi fa} du$$

$$= e^{-i2\pi fa} G(f)$$

→ Scale Property

$$F\{g(ct)\} = \frac{G\left(\frac{f}{c}\right)}{|c|}$$

$$F\{g(ct)\} = \int_{-\alpha}^{\alpha} g(ct)\, e^{-i2\pi f t}\, dt$$

$$= \frac{1}{c}\int_{-\alpha}^{\alpha} g(u)\, e^{-i2\pi f \frac{u}{c}}\, du$$

let
$ct = u$
$dt = \frac{1}{c} du$

if $c > 0$

$$F\{g(ct)\} = \frac{1}{c}\int_{-\alpha}^{\alpha} g(u)\, e^{-i2\pi \frac{f}{c} u}\, du$$

$$= \frac{1}{c}\, G\left(\frac{f}{c}\right) = \frac{G\left(\frac{f}{c}\right)}{|c|}$$

if $c < 0$

$$F\{g(ct)\} = \frac{1}{c}\int_{+\alpha}^{-\alpha} g(u)\, e^{-i2\pi f/c\, u}\, du$$

$$= -\frac{1}{c}\int_{-\alpha}^{\alpha} g(u)\, e^{-i2\pi f/c\, u}\, du$$

$$= -\frac{1}{c}\, G\left(\frac{f}{c}\right)$$

$$\therefore \cancel{F\{g(ct)\}(f)} = \frac{G\left(\frac{f}{c}\right)}{|c|}$$