Rajshahi University of Engineering & Technology

B.Sc. Engineering 1st Year Even Semester Examination, 2014

Department of Computer Science & Engineering

Course No.: **Math 1213** Course Title: Co-Ordinate Geometry & Ordinary Differential Equation

Full marks: 72 Time: Three (03) hours

N.B. Answer any **six** questions, taking **three** from each section.
All questions are of equal value.
Use separate answer script for each section.

SECTION-A

Marks

Q1. (a) What is meant by transformation of coordinates? If by the rotation of the rectangular axes about the origin the expression $ax^2+2hxy+by^2$ changes to $a'x'^2+2h'x'y'+b'y'^2$, then prove that $a+b=a'+b'$ and $ab-h^2=a'b'-h'^2$. 06

(b) If the equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a pair of straight lines, prove that the square of the distance of their point of intersection from the origin is $\dfrac{\{c(a+b)-f^2-g^2\}}{(ab-h^2)}$. 06

Q2. (a) Find the condition that the general equation of the second degree $ax^2+2hxy+by^2+2gx+2fy+c=0$ may represent a pair of straight lines. 04

(b) Find the area of the triangle formed by the lines $ax^2+2hxy+by^2=0$ and $lx+my+n=0$. 04

(c) Find the angle between the lines represented by the equation $ax^2+2hxy+by^2=0$. 04

Q3. (a) What are the direction cosines and direction ratios of a straight line? Prove that $l^2+m^2+n^2=1$, where l, m, n represent the direction cosines. 04

(b) A variable plane is at a constant distance p from the origin and meets the axes at A, B, C. Show that the locus of the centroid of the tetrahedron OABC is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=16\frac{1}{p^2}$. 04

(c) Find the equation of the plane passing through the straight line $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z-3}{4}$ and perpendicular to the plane $x+2y+z=12$. 04

Q4. (a) Show that the equation to the plane containing the line $\frac{y}{b}+\frac{z}{c}=1,\ x=0$; and parallel to the line $\frac{x}{a}-\frac{z}{c}=1,\ y=0$ is $\frac{x}{a}-\frac{y}{b}-\frac{z}{c}+1=0$ and if 2d is the shortest distance between them, prove that $\frac{1}{d^2}=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}$. 06

(b) Find in symmetrical form of the line $x+y+z+1=0,\ 4x+y-2z+2=0$ and find also its direction cosines. 06

SECTION-B

14
15/30

Q5. (a) What is the ordinary differential equation? Give some examples. Find the differential equation of the family of parabolas with foci at the origin and axes along the x-axis. 04

(b) Solve 08

i) $y^2+x^2\frac{dy}{dx}=xy\frac{dy}{dx}$

ii) $\frac{dy}{dx}=(4x+y+1)^2$

Q6. (a) Solve 08

i) $\left(y+\sqrt{x^2+y^2}\right)dx-xdy=0,\quad y(1)=0.$

ii) $(2x\cos y+3x^2y)dx+(x^3-x^2\sin y-y)dy=0,\ y(0)=2.$

(b) What is the integrating factor? Solve $x\frac{dy}{dx}+(x+1)y=x^3$. 04

Rajshahi University of Engineering & Technology
B.Sc. Engineering 1st Year 2nd Semester Examination, 2013
Department of Computer Science & Engineering
Course no: Math 207 Course Title: Mathematics-II
Full marks: 70 Time: Three (03) hours.

N.B. Answer six questions, taking three from each section
The questions are of equal value
Use separate answer script for each section

SECTION-A

Q1. (a) If by the rotation of the rectangular co-ordinates axes about the origin the expression $ax^2+2hxy+by^2$ changes to $a'x'^2+2h'x'y'+b'y'^2$ then prove that $a+b=a'+b'$ and $ab-h^2=a'b'-h'^2$. 06

(b) Prove that the straight lines joining the origin to the points of intersection of the straight line $kx+hy=2hk$ with the curve $(x-h)^2+(y-k)^2=c^2$ are at right angles if $h^2+k^2=c^2$. $05\frac{2}{3}$

Q2 (a) A Conic given by the equation $(1+k^2)(x^2+y^2)-4kxy+2k(x+y)+2=0$, where k may take any value. Show that the conic is a parabola with the standard form $\sqrt{2}Y^2=X$ for $k=1$. 06

(b) Find the area of the triangle formed by the lines $ax^2+2hxy+by^2=0$ and $lx+my+n=0$. $05\frac{2}{3}$

Q3 (a) Find the condition that the equation, $ax^2+2hxy+by^2+2gx+2fy+c=0$ may represent two parallel straight lines. 06

(b) A line makes angles $\alpha,\beta,\gamma,\delta$ with the four diagonals of a cube. Prove that $\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta=\frac{4}{3}$. $05\frac{2}{3}$

Q4 (a) Find the equation of the plane through the points (2,2,1) and (9,3,6) and perpendicular to the plane $2x+6y+6z=9$. $3\frac{2}{3}$

(b) Find the equation of the line of intersection of the planes $2x-y+2z+7=0$ and $x+2y-3z+6=0$. 04

(c) Find the length of the shortest distance between the lines 04

$$\frac{x-x_1}{l_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{n_1} \text{ and } \frac{x-x_2}{l_2}=\frac{y-y_2}{m_2}=\frac{z-z_2}{n_2}$$

SECTION-B

Q5. (a) Form a differential equation for $xy=ae^x+be^{-x}$. $05\frac{2}{3}$

(b) Solve the initial value problem $(3x^2y^2-y^3+2x)dx+(2x^3y-3xy^2+1)dy=0, y(-2)=1$. 06

Q6. (a) If $M(x,y)dx+N(x,y)dy=0$ is a homogeneous equation, then show that the change of variables $y=vx$ transforms the above homogeneous equation into a separable equation in the variables v and x. 04

(b) Solve i) $\sin^{-1}\left(\frac{dy}{dx}\right)=x+y$ $07\frac{2}{3}$

ii) $x\frac{dy}{dx}+y=(xy)^{\frac{3}{2}}$

Q7. Find the complete solution of

i) $(D^3-2D^2-5D+6)y=e^{3x}+e^{2x}$ 04

ii) $(D^2+2D+1)y=e^x\sin 2x$ $03\frac{2}{3}$

iii) $(D^2-D-2)y=e^x+\sin 2x$, where $D=\frac{d}{dx}$ 04

Q8. (a) Solve the nonlinear differential equation $2\frac{d^2y}{dx^2}-\left(\frac{dy}{dx}\right)^2+4=0$. 06

(b) Solve $\frac{d^2y}{dx^2}-3\frac{dy}{dx}-4y=16x-12e^{2x}$ by the method of undetermined coefficient. $05\frac{2}{3}$

THE END

"Heaven's light is our guide"

Rajshahi University of Engineering & Technology

B.Sc. Engineering 1st Year 2nd Semester Examination, 2010

Department of Computer Science & Engineering

Course no: MATH 207 Course Title: Mathematics II

Full marks: 70 Time: Three (03) hours

N.B. Answer six questions, taking three from each section
The questions are of equal value.
Use separate answer script for each section.

SECTION-A

Q1. (a) By transforming to parallel axes through a properly chosen point (h, k), prove that the equation $3x^2 - 5xy + y^2 + 7x + 5y - 23 = 0$ can be reduced to one containing only terms of the second degree. $5\frac{2}{3}$

(b) Show that the area of the triangle formed by the lines $lx + my = 1$, $ax^2 + 2hxy + by^2 = 0$ is $\dfrac{\sqrt{(h^2 - ab)}}{(am^2 - 2hlm + bl^2)}$ 6

Q2. (a) In the general equation of the 2nd degree $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines, then the equation $ax^2 + 2hxy + by^2 = 0$ represents a pair of straight lines parallel to them through the origin. 6

(b) A conic is given by the equation $(1 + \lambda^2)(x^2 + y^2) - 4\lambda xy + 2\lambda(x + y) + 2 = 0$, where λ may take any real value. Show that the conic is a parabola with the standard form $\sqrt{2}y^2 = x$ for $\lambda = 1$. $5\frac{2}{3}$

Q3 (a) Show that the equation to the plane containing the line $\frac{y}{b} + \frac{z}{c} = 1, x = 0$; and parallel to the line $\frac{x}{a} - \frac{z}{c} = 1, y = 0$ is $\frac{x}{a} - \frac{y}{b} - \frac{z}{c} + 1 = 0$, and if $2d$ is the shortest distance prove that $\frac{1}{d^2} = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$. 6

(b) Show that the lines $\frac{x+1}{2} = \frac{y-2}{2} = \frac{z}{1}$ and $\frac{x-1}{6} = \frac{y+1}{1} = \frac{z-3}{5}$ are coplanar. Also find their point of intersection.

Q4. (a) A lines makes angles $\alpha, \beta, \gamma, \delta$ with four diagonals of a cube, prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$ $5\frac{2}{3}$

(b) Find the equation of the plane through the points (2, 2, 1) and (9, 3, 6) and perpendicular to the plane $2x + 6y + 6z = 9$. 6

SECTION-B

Q5. (a) From the differential equation corresponding to the family of curves $y = ae^{2x} + be^{-3x} + ce^{x}$ where a, b, & c are arbitrary constants. $6\frac{2}{3}$

(b) Solve the initial-value problem 5

$$(2x\cos y + 3x^2 y)dx + (x^3 - x^2\sin y - y)dy = 0, y(0) = 2.$$

Q6. (a) If $M(x, y)dx + N(x, y)dy = 0$ is a homogeneous equation then show that the change of variable y=vx transforms the above equation into a separable equation in the variables v & x. Solve the equation $(x^2 - 3y^2)dx + 2xydy = 0$ $6\frac{2}{3}$

(b) Solve the initial-value problem $\frac{dx}{dt} - x = \sin 2t, x(0) = 0$. 5

Q7. Solve the following differential equations

(a) $(D^3 + 2D^2 + D)y = e^{2x} + x^2 + x$ $3\frac{2}{3}$

(b) $(D^3 - 3D^2 + 3D - 1)y = xe^x + e^x$ 04

(c) $(D^3 - 2D + 4)y = e^x \cos x$ 04

Q8. (a) Solve $2\frac{d^2y}{dx^2} - (\frac{dy}{dx})^2 + 4 = 0$ $5\frac{2}{3}$

(b) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = e^{-2x}(\sec^2 x)(1 + 2\tan x)$$

Rajshahi University of Engineering & Technology
B.Sc Engineering [illegible] Year 2nd Semester Examination, [illegible]
Department of Computer Science and Engineering
Course no: Math 207 Course Title: Mathematics-II
Full marks: 70 Time: Three (03) hours

(C-6)

N.B. Answer six questions taking three from each section
The questions are of equal value
Use separate answer script for each section

SECTION-A

Q1. (a) What is the integrating factor? Find the integrating factor to solve $\frac{dy}{dx} + 2xy = [illegible]$ and hence find the solution of this equation. $6\frac{2}{3}$

(b) Solve $\frac{dy}{dx} = [illegible]$.

Q2. (a) Find the solution of the differential equation $\frac{d^2y}{dx^2} + 4y = [illegible]$ by the method of variation of parameters. $5\frac{2}{3}$

(b) Find the solution of the simple pendulum equation subject to the condition that $\frac{d\theta}{dt} = 0$ and $\theta = \theta_0$ when $t = 0$. 6

Q3. (a) Solve $\frac{dN}{dt} = \lambda N(K - \frac{N}{C})$, $N(0) = N_0$. Show that $N \to CK$ as $t \to \infty$. Plot the solution when (i) $N > CK$ and $N < CK$. 6

(b) Solve $m\frac{d^2h}{dt^2} + c\frac{dh}{dt} = g$. Find the velocity when $t \to \infty$. $5\frac{2}{3}$

Q4. Solve [illegible] $11\frac{2}{3}$

SECTION-B

Q5. (a) Transform the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ to $ax^2 + 2hxy + by^2 = 0$. Find the condition that this equation has a pair of straight lines perpendicular to one another. $6\frac{2}{3}$

(b) Transform the equation $\sqrt{x} + \sqrt{y} = \sqrt{a}$ to its standard form. 5

Q6. (a) Prove that the straight lines represented by the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ will be equidistant from the origin if $f^4 - g^4 = c(bf^2 - ag^2)$. 6

(b) Find the area of the triangle formed by the lines $ax^2 + 2hxy + by^2 = 0$ and $lx + my + n = 0$. $5\frac{2}{3}$

Q7. (a) Define direction ratio and direction cosine of a line. A line makes angles α, β, γ and δ with the four diagonals of a cube, prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$. 6

(b) Prove that the straight lines whose direction cosines are given by the relations $al + bm + cn = 0$ and $fmn + gnl + hlm = 0$ are perpendicular if $\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$ and parallel if $\sqrt{af} \pm \sqrt{bg} \pm \sqrt{ch} = 0$. $5\frac{2}{3}$ → 22 (ex)

Q8. (a) A variable plane is at a constant distance P from the origin and meet the axes in A, B, C. Show that the locus of the centroid of the tetrahedron OABC is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{16}{P^2}$. 6

(b) Find the shortest distance line of $\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$; $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$. $5\frac{2}{3}$

→ Pg → 52
Ex → 1[illegible]

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Rajshahi University of Engineering & Technology
B.Sc. Engineering 1st Year 2nd Semester Examination, 2007
Department of Computer Science & Engineering
Course no: Math 207 Course Title: Mathematics-II
Full marks: 70 Time: Three (03) hours

N.B. Answer SIX questions, taking THREE from each section.
The questions are of equal value
Use separate answer script for each section

SECTION-A

1. (a) Define ordinary and partial differential equation with example. Find the differential equation of all circles which have their centre of x-axis and have r given radius. 6½

(b) Solve $\frac{dy}{dx} = \frac{y-x}{y+x}$ by substituting $y = vx$, where v is a function of x only. 5

2. (a) What is exactness of a differential equation? Mention the integrating factor of the second order differential equation, $\frac{d^2y}{dx^2} + a^2y = 0$ and multiplying it on both side of this equation and integrate. Finally find the relation between y and x. 6½

(b) Solve $\frac{d^2y}{dx^2} + 2c\frac{dy}{dx} + b^2y = 0$ by reducing it to the form $\frac{d^2y}{dx^2} + a^2y = 0$. 5

3. (a) Solve (i) $\frac{d^2y}{dx^2} + y = \cos x$ and (ii) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{-x}$. 11½

4. (a) Solve the initial value problem 5½

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^x - 10\sin x;\quad y(0) = 2,\ y'(0) = 4$$

(b) Solve the differential equation 6

$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = x^2e^x$ by the method of undetermined coefficient.

SECTION-B

5. (a) Transform the equation $11x^2 + 24xy + 4y^2 - 20x - 40y - 5 = 0$ to rectangular axes through the point (2,-1) and inclined an angle $\tan^{-1}\left(\frac{4}{-3}\right)$. 5

(b) Derived the condition that the general equation of 2nd degree $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ will represents a pair of straight lines and hence show that $a:h = h:b = g:f$ if they are parallel. 6½

6. By suitable substitutions, change the equation $ax^2 + 2hxy + by^2 = 0$ to the form $a'x'^2 + 2h'x'y' + b'y'^2 = 0$. Also prove that $a + b = a' + b'$ and $ab - h^2 = a'b' - h'^2$. Utilize this rule to transfer the equation $xy = c^2$ to the form $x'^2 - y'^2 = a^2$. 11½

7. (a) Show that $\cos^2\alpha + \cos^2\beta + \cos^2\delta = 1$, where α, β and δ are the angles of a line with the coordinate axes. 5½

(b) Find the equations of the planes whose normal makes equal angles with the coordinate axes and having a distance 1 unit from the origin. 6

8. (a) Find the angle between the lines 5

$x + 2y - 2z = 0 = x - 2y + z - 7$ and $\frac{x-1}{1} = \frac{y+2}{-2} = \frac{z}{3}$.

(b) Find the distance of the point (-1,-5,-10) from the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y + z = 5$. 3

(c) Find the magnitude of the shortest distance between the lines 3½

$\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

..... The End .....

Rajshahi University of Engineering & Technology

B. Sc. Engineering 1st Year 2nd Semester Examination 2006

Course No. Math 207 — Course Title: Mathematics-II

Time: Three Hours — Full Marks: 70

Note:
- (i) Answer Six questions taking Three from each section
- (ii) Figures in the right margin indicate full marks
- (iii) Use separate answer script for each section

SECTION-A

Q1.(a) Define ordinary and partial differential equation with example. Find the differential equation of all circles passing through the origin and having their centers on the x-axis. $5\frac{2}{3}$

(b) Find the necessary and sufficient condition for a differential equation of first degree being exact. Also solve $(\cos x\tan y-\sin x\sec y)dx+(\sin x\sec^2 y+\cos x\tan y\sec y)dy=0$ 6

Q2.(a) Define integrating factor. Find the general and particular solution of the equation $[y-3(e^x+1)^2]dx+[e^x+1]dy=0,\ y(0)=4$. 6

(b) What do you mean by initial and boundary conditions of differential equation? $5\frac{2}{3}$

Solve the initial value problem: $(6x+4y+1)dx+(4x+2y+2)dy=0,\ y\left(\frac{1}{2}\right)=3$.

Q3.(a) Solve $(1+x^2)\frac{dy}{dx}+2xy=4x^2$ and obtain the cubic curve satisfying the equation and passing through the origin. 5

(b) Solve the following differential equations: $6\frac{2}{3}$

(i) $\frac{d^2y}{dx^2}+\mu^2y=0$

(ii) $\frac{dy}{dx}+\frac{y}{x}\log y=\frac{y}{x^2}(\log y)^2$

Q4.(a) Discuss the method of variation of parameters, and hence solve the equation $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=\frac{1}{1+e^x}$ by this method. 6

(b) Solve $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+3y=9x^2+4$ by the method of undetermined coefficient subject to the conditions $y(0)=6$ and $y'(0)=8$. $5\frac{2}{3}$

SECTION-B

Q5.(a) If by the rotation of the rectangular co-ordinate axes about the origin the expression $ax^2+2hxy+by^2$ changes to $a'x'^2+2h'x'y'+b'y'^2$ then prove that $a+b=a'+b'$ and $ab-h^2=a'b'-h'^2$. 6

(b) Sow that the area of the triangle formed by the lines $ax^2+2hxy+by^2=0$ and $lx+my+n=0$ is $5\frac{2}{3}$

$$\frac{n^2\sqrt{(h^2-ab)}}{am^2-2hlm+bl^2}.$$

Q6.(a) Show that the straight lines joining the origin to the points of intersection of the straight line $kx+hy=2hk$ with the curve $(x-h)^2+(y-k)^2=c^2$ are at right angles if $h^2+k^2=c^2$. → Pg → 42; ex. 43 1st $5\frac{2}{3}$

(b) Reduce the equation $6x^2+5xy-6y^2-4x+7y+11=0$ to the standard form. Find also its lengths, equations and directions of the axes. Pg - 7.6, Ex - 6 2nd 6

Q7.(a) Define direction ratio and direction cosine of a line. Find the distance of (-2, 3, 4) from the line through the point (-1, 3, 2) whose direction cosines are proportional to 12, 3, -4. → Pg → 18; ex. 12(a) 6

(b) Prove that two lines whose direction cosines are connected by the relations $al+bm+cn=0$ and $ul^2+vm^2+wn^2=0$ are parallel if $\frac{a^2}{u}+\frac{b^2}{v}+\frac{c^2}{w}=0$. $5\frac{2}{3}$

Q8.(a) A plane meets the co-ordinate axes in A, B, C such that the centroid of the triangle ABC is the point (p, q, r). Show that equation of the plane is $\frac{x}{p}+\frac{y}{q}+\frac{z}{r}=3$. 6

(b) Find the magnitude and the equations of the line of shortest distance between the lines $\frac{x-8}{3}=\frac{y+9}{-16}=\frac{z-10}{7}$ and $\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$. → Pg → 231 (plane) Ex. 10 $5\frac{2}{3}$

"Heaven's light is our guide"

Rajshahi University of Engineering & Technology

B.Sc. Engineering 1st Year 2nd Semester Examination, 2005

Department of Computer Science and Engineering

Course no: Math 207 Course Title: Mathematics II

Full marks: 70 Time: Three (03) hours

N.B. Answer six questions, taking three from each section.
The questions are of equal value.
Use separate answer script for each section.

SECTION – A

2

1. (a) What is ordinary differential equation? What do you mean by the general solution of differential equation? Obtain the differential equation of the family of curves $y = e^x (A\cos x + B\sin x)$ 26.

(b) Find the condition that the D.E. $M(x,y)dx + N(x,y)dy = 0$ will be exact. Solve the D.E. $(y\sec^2 x + \sec x\tan x)dx + (\tan x + 2y)dy = 0$ 26, 36

2. (a) If $\frac{dy}{dx} + 2y\tan x = \sin x$ and if $y = 0$ when $x = \frac{\pi}{3}$ express y in terms of x.

(b) Is the differential equation $(x^2 - 2xy)dx + 2xy\,dy = 0$ exact? Solve the differential equation.

(c) Show that $(4x + 3y + 1)dx + (3x + 2y + 1)dy = 0$ represents hyperbolas having as asymptotes $x + y = 0$, $2x + y + 1 = 0$.

3. (a) What is a complete solution and a singular solution?

(b). Solve the differential equations (any three)

(i) $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$, when $x = 0$, $y = 3$ and $y' = 0$.

(ii) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = x^2$.

(iii) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = x^2$, $y(0) = 1$, $y(1) = 1$.

(iv) $y'' + 9y = x\cos 2x$.

4. (a) Define clairaut form of D.E. Solve the D.E. $y = px + p(p-1)$, where $p = \frac{dy}{dx}$.

(b). Solve the differential equation $x^2\frac{d^2y}{dx^2} - 6y = \ldots$. Subject to the condition $y(1) = \frac{1}{6}$, $y'(1) = -\frac{1}{6}$.

SECTION – B

5\. (a) If by the rotation of the rectangular co-ordinate axes about the origin the expression $ax^2 + 2hxy + by^2$ changes to $a'x'^2 + 2h'x'y' + b'y'^2$, then $a + b = a' + b'$ and $ab - h^2 = a'b' - h'^2$.

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(b) If one of the straight lines given by the equation $ax^2 + 2hxy + by^2 = 0$ coincides with one of those given by $a_1x^2 + 2h_1xy + b_1y^2 = 0$ and the other lines represented by them be perpendicular, prove that $\frac{ha_1b_1}{b_1 - a_1} = \frac{h_1ab}{b - a} = \frac{1}{2}\sqrt{(-aa_1bb_1)}$.

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(c) Show that the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents two parallel lines, if $\frac{a}{h} = \frac{h}{b} = \frac{g}{f}$.

35

(d) Show that the area of the triangle formed by the lines $ax^2 + 2hxy + by^2 = 0$ and $lx + my + n = 0$ is $\frac{n^2\sqrt{(h^2 - ab)}}{am^2 - 2hlm + bl^2}$

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(e) A line makes angles α, β, γ and δ with the four diagonals of a cube. Prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta$ [illegible]

3d – 14

(f) Prove that the straight lines whose direction cosines are given by the relations $al + bm + cn = 0$ and $fmn + gnl + hlm = 0$ are perpendicular if $\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$ and parallel if $\sqrt{af} \pm \sqrt{bg} \pm \sqrt{ch} = 0$.

3d – 18

(g) [illegible] A, B, C such that the centroid of the triangle ABC is the point (p, q, r). Show that the equation of the plane is $\frac{x}{p} + \frac{y}{q} + \frac{z}{r} = 3$.

3d – 31

(h) Find the equation of the line of shortest distance and length of the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-4}{4}$, $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$.

– 3d – 60

Pg → 60 (End)

Ex → 46.

Rajshahi University of Engineering and Technology

B.Sc. Engineering 1st Year 2nd Semester Examination, 2003

Department of Computer Science and Engineering

Course no: Math 207 Course Title: Mathematics-II

Full marks: 70 Time: Three (03) hours

N.B. Answer six questions, taking three from each section.
The questions are of equal value.
Use separate answer script for each section.

SECTION A

Q1. (a) If by the rotation of the rectangular co-ordinate axes about the origin the expression $ax^2 + 2hxy + by^2$ changes to $a'x'^2 + 2h'x'y' + b'y'^2$, then $a + b = a' + b'$ and $ab - h^2 = a'b' - h'^2$.

(b) Reduce the equation $6x^2 + 5xy - 6y^2 - 4x + 7y + 1 = 0$ to the standard form. Find the lengths of the axes and equations of the axes.

Q2. (a) Find the condition that the general equation of second degree $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ may represents a pair of straight lines.

(b) Show that the area of the triangle formed by the lines $ax^2 + 2hxy + by^2 = 0$ and $lx + my + n = 0$ is $\left\{\frac{n^2\sqrt{h^2 - ab}}{am^2 - 2hlm + bl^2}\right\}$.

Q3. (a) A line makes angles α, β, γ, δ with the four diagonals of a cube, prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = 4/3$.

(b) Find the angle between the lines whose direction cosines are given by the equations $l + m + n = 0$ and $l^2 + m^2 - n^2 = 0$.

(c) Show that the two lines whose direction cosines are connected by the relations $al + bm + cn = 0$ and $ul^2 + vm^2 + wn^2 = 0$ are perpendicular if $a^2(v + w) + b^2(w + u) + c^2(u + v) = 0$ and parallel if $a^2/u + b^2/v + c^2/w = 0$.

Q4. (a) Find the equation of the plane which is perpendicular to the plane $5x + 3y + 6z + 8 = 0$ and which contains the line of intersection of the planes $x + 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$.

(b) Find the shortest distance and direction cosine of the shortest line between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$, $\frac{x-2}{3} = \frac{y-z}{4} = \frac{z-4}{5}$, state whether the lines are coplanar or not.

SECTION B

Q5. (a) What is the degree and order of a differential equation? Solve $\frac{dy}{dx} + \frac{y^2 + y + 1}{x^2 + x + 1} = 0$.

(b) Find the condition that, the differential equation $M(x, y)dx + N(x, y)dy = 0$ is exact. Also solve $(2y\sin x\cos x + y^2\sin x)dx + (\sin^2 x - 2y\cos x)dy = 0$.

Q6. (a) Given that $y = x$ is a solution of $x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 4y = 0$. Find a linearly independent solution by reducing the order.

(b) Define integrating factor. Solve $(2x^2 + y)dx + (x^2y - x)dy = 0$.

Q7. Solve the followings:

(i) $x^2\frac{dy}{dx} + xy = \frac{y^3}{x}$, $y(1) = 1$.

(ii) $x\,dx + y\,dy + \frac{x\,dy - y\,dx}{x^2 + y^2} = 0$.

Q8.(a) Given that $y = x$ is a solution of $(x^2 - 1)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$. Find the general solution by reducing the order.

(b) Solve $y'' + 3y' + 2y = \frac{1}{1+e^x}$ by the method of variation of parameters.

"Heaven's Light is Our Guide"

Rajshahi University of Engineering and Technology

B.Sc. Engineering 1st Year 2nd Semester Examination, 2003

Department of Computer Science and Engineering

Course no: Math 207 Course Title: Mathematics-II

Full marks: 70 Time: Three (03) hours

N.B. Answer six questions, taking three from each section.
The questions are of equal value.
Use separate answer script for each section.

SECTION A

page 22 Ans 35 Q1. (a) If by the rotation of the rectangular co-ordinate axes about the origin the expression $ax^2 + 2hxy + by^2$ changes to $a'x'^2 + 2h'x'y' + b'y'^2$, then $a + b = a' + b'$ and $ab - h^2 = a'b' - h'^2$. LEC 06-12-2005 12th cycle (A-day), Physics wrong

page 76 Ex-6 (b) Reduce the equation $6x^2 + 5xy - 6y^2 - 4x + 7y + 1 = 0$ to the standard form. Find the lengths of the axes and equations of the axes.

page 30 Ans 40 Q2. (a) Find the condition that the general equation of second degree $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ may represents a pair of straight lines.

page 37 Ex-11 (b) Show that the area of the triangle formed by the lines $ax^2 + 2hxy + by^2 = 0$ and $lx + my + n = 0$ is $\left\{ \frac{n^2\sqrt{h^2 - ab}}{am^2 - 2hlm + bl^2} \right\}$.

page 14 Ex-6 Q3. (a) A line makes angles $\alpha, \beta, \gamma, \delta$ with the four diagonals of a cube, prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = 4/3$.

page 16 Ex-12 (b) Find the angle between the lines whose direction cosines are given by the equations $l + m + n = 0$ and $l^2 + m^2 - n^2 = 0$.

page 18 2274 (c) Show that the two lines whose direction cosines are connected by the relations $al + bm + cn = 0$ and $ul^2 + vm^2 + wn^2 = 0$ are perpendicular if $a^2(v + w) + b^2(w + u) + c^2(u + v) = 0$ and parallel if $a^2/u + b^2/v + c^2/w = 0$.

page 35 13.45 Q4. (a) Find the equation of the plane which is perpendicular to the plane $5x + 3y + 6z + 8 = 0$ and which contains the line of intersection of the planes $x + 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$.

page 60 45.2 (b) Find the shortest distance and direction cosine of the shortest line between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$, $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$, state whether the lines are coplanar or not.

SECTION B

#3

Q5. (a) What is the degree and order of a differential equation? Solve $\frac{dy}{dx} + \frac{y^2 + y + 1}{x^2 + x + 1} = 0$.

(b) Find the condition that, the differential equation $M(x, y)dx + N(x, y)dy = 0$ is exact. Also solve $(2y\sin x\cos x + y^2\sin x)dx + [illegible] x - 2y\cos x)dy = 0$.

Q6. (a) Given that [illegible] is a solution of [illegible] find a linearly independent [illegible] by reducing the order.

(b) Define [illegible]

[illegible]

Q7. Solve the followings:

(i) $x^2\dfrac{dy}{dx}+xy=\dfrac{y^3}{x}$, $y(1)=1$.

(ii) $xdx+ydy+\dfrac{xdy-ydx}{x^2+y^2}=0$.

Q8.(a) Given that $y=x$ is a solution of $\left(x^2-1\right)\dfrac{d^2y}{dx^2}-2x\dfrac{dy}{dx}+2y=0$. Find the general solution by reducing the order.

(b) Solve $y''+3y'+2y={}^{1}\!/_{1+e^{x}}$ by the method of variation of parameters.

Heaven's Light is Our Guide

Bangladesh Institute of Technology, Rajshahi

Computer Science and Engineering Department

B.Sc Engineering ... Year Second Semester Examination, 200?

Course no: Math 207, Course Title: Mathematics-II.

Full Marks: 70 Time: 3 hours

N.B. Answer six questions, taking three from each section.
The questions are of equal value.
Use separate answer script for each section.

Section A

Q.1(a) Turn the axes through an angle so that the second term of $ax^2+2hxy+by^2$ may vanish.

(b) Prove that the product of perpendiculars from the point (x_1,y_1) on the lines $ax^2+2hxy+by^2=0$ is $\frac{ax_1^2+2hx_1y_1+by_1^2}{\sqrt{(a-b)^2+4h^2}}$

Q.2(a) Prove that the pair of straight line joining the point of intersection of $lx+my+n=0$ and $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ will coincide if $a^2l^2+b^2m^2=n^2$.

(b) Test the nature of the conic $17x^2-12xy+8y^2+46x-28y+17=0$.

Q.3(a) Define direction ratio and direction cosine of a line and establish the relation $l^2+m^2+n^2=1$.

(b) Prove that two lines whose direction cosines are connected by the relations $al+bm+cn=0$ and $ul^2+vm^2+wn^2=0$ are perpendicular if $a^2(v+w)+b^2(w+u)+c^2(u+v)=0$.

Q.4(a) A variable plane is at a constant distance p from the origin and meets the axes in A,B,C. Through A,B,C planes are drawn parallel to the co-ordinate planes. Show that the locus of their point of intersection is $x^{-2}+y^{-2}+z^{-2}=p^{-2}$.

(b) Find the shortest distance between the lines $\frac{x-?}{?}=\frac{y-2}{?}=\frac{z-1}{-5}$ and $\frac{x+4}{4}=\frac{y}{3}=\frac{z-19}{-2}$.

Section B

Q.5(a) Define differential equation. Solve the differential equation,

$(2s^2+2st+t^2)ds+(s^2+2st-t^2)dt=0$.

(b) Prove that if $M(x,y)dx+N(x,y)dy=0$ is a homogeneous equation, then the change of variables $y=vx$ transform the equation into a separable equation in the variables v and x.

Q.6.(a) Define Bernoulli's differential equation. Suppose $n\neq 0,1$, then show that the transformation $v=y^{1-n}$ reduces the Bernoulli's equation to a linear equation in v.

(b) Solve the initial value problem:

$(6x+4y+1)dx+(4x+2y+2)dy=0,\ y(1/2)=3$

Q.7(a) What do you mean by complementary function? Solve the equation $y''+2y'+5y=3\sin 2x+5\cos 2x$ by the method of undetermined co-efficient.

(b) Solve $y''+3y'+2y=\frac{1}{1+e^x}$ by the method of variation of parameters.

Q.8. Given that $y=x$ is a solution of $(x^2-1)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0$, find a linearly independent solution by reducing the order.