Optimal Production Planning Using Linear Programming

An Application using LP to Maximize Profit Under Resource Constraints

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1. Overview

This exercise applies linear programming (LP) to determine the optimal production quantities of three products (A, B, and C) in a resource-constrained environment. Using Excel Solver, the problem was modeled with the objective of maximizing profit while considering restrictions on glass, labour, and machine time.

The results show that the company should produce 0 units of Product A, 60 units of Product B, and 45 units of Product C. This plan achieves a maximum profit of 1,545 while fully utilizing the available labour hours.

The analysis highlights the importance of mathematical optimization in decision-making and demonstrates how LP can improve profitability in real-world production scenarios.

2. Intended Audience

This report is intended for:

- **Business managers** seeking data-driven approaches to production planning
- **Operations analysts and industrial engineers** interested in resource optimization
- **Students and professionals** learning about applied operations research methods

3. Purpose of Exercise

The purpose of this exercise is to:

- Demonstrate the application of **linear programming** to optimize production planning.
- Illustrate how **Excel Solver**, a widely available and accessible tool, can be applied for solving real-world optimization problems.
- Provide insights into resource allocation, constraint management, and profit maximization.

- Demonstrate the use of the **sensitivity report** to show how changes in model parameters affect the optimal solution, enabling decision-makers to maximize profits (**Objective Function**).

4. Background & Context

A manufacturer produces three products (A, B, and C). Each product contributes differently to profit and requires varying amounts of glass, labour hours, and machine time. However, the company has limited resources and a pre-order requirement for Product C.

The problem is to find the best combination of A, B, and C to maximize profit while observing the constraints.

Key resource limits/constraints:

- Glass: **120 kg**
- Labour availability: **48 hours**
- Machine time availability: **130 hours**
- Pre-order: **To fulfill at least 45 units of Product C**

Resources	Resource Required to make a unit of Product A	Resource Required to make a unit of Product B	Resource Required to make a unit of Product C	Total Available
Glass (kg)	0.25	0.35	0.50	120
Labour (hrs)	0.25	0.35	0.60	48
Machine (hrs)	0.20	0.30	0.40	130

	Product A (/unit)	Product B (/unit)	Product C (/unit)	Total Available
Profit ($)	9	13	17	–

5. Problem Formulation

5.1 Decision Variables

- Let **A** be the number of units of Product A to produce
- Let **B** be the number of units of Product B to produce
- Let **C** be the number of units of Product C to produce

5.2 Objective Function

- Maximize Profit = 9A + 13B + 17C

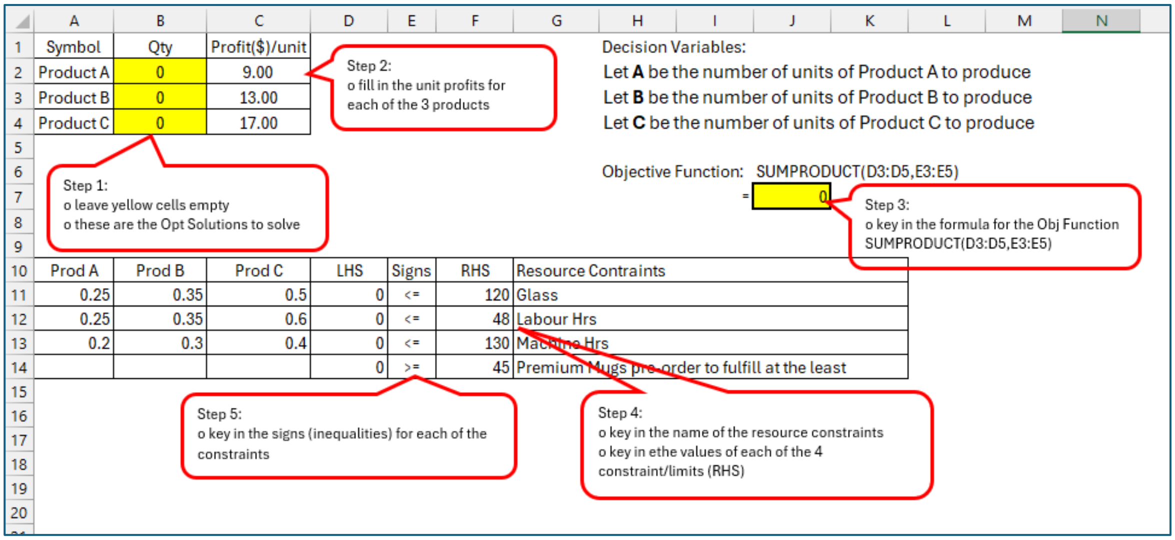
5.3 Constraints

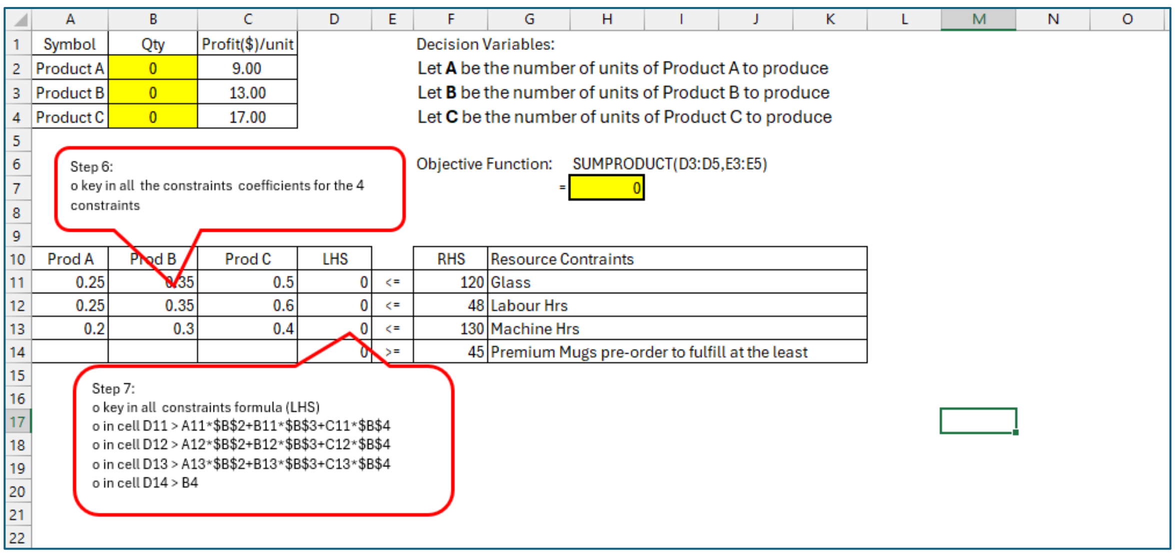
- **Glass**: 0.25A + 0.35B + 0.5C ≤ 120
- **Labour**: 0.25A + 0.35B + 0.6C ≤ 48
- **Machine**: 0.2A + 0.3B + 0.4C ≤ 130
- **Pre-order**: C ≥ 45
- Non-negativity: A, B, C ≥ 0

6. Solution Method

6.1 Solver Setup (& configuration)

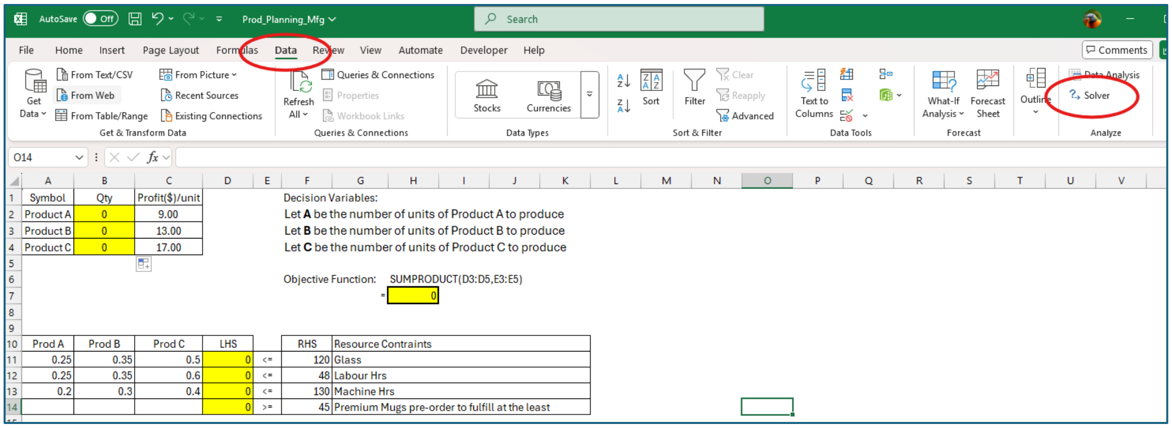
- Solver should be configured by following the sequence of steps outlined in the diagrams below



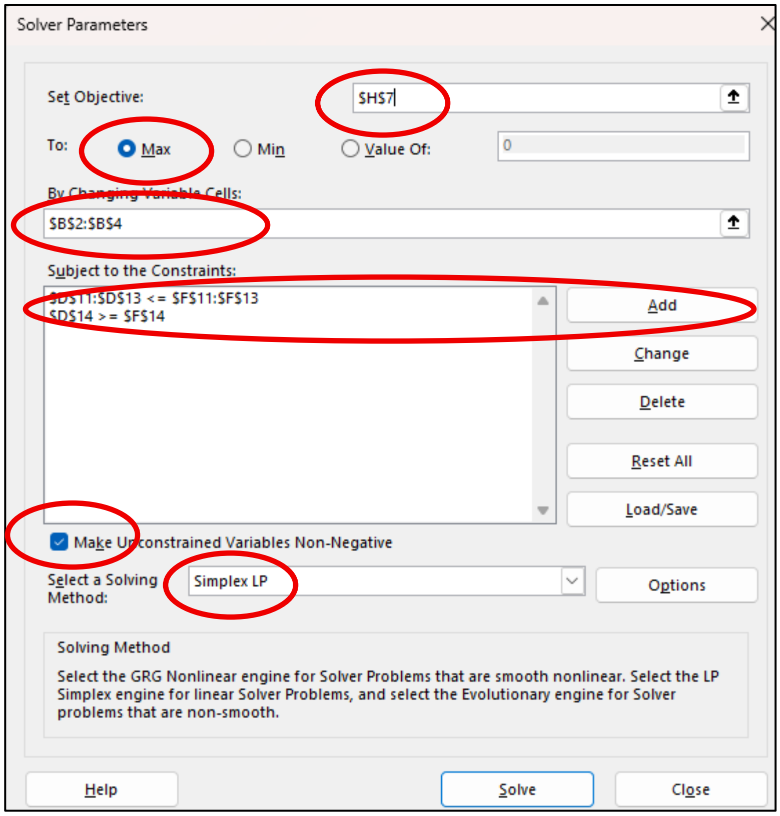


6.2 Executing the Solver Procedure

- On the menu bar, click *Data* at the top, then choose *Solver* on the far right (refer to the appendix for steps to enable Solver if it does not appear in the menu)



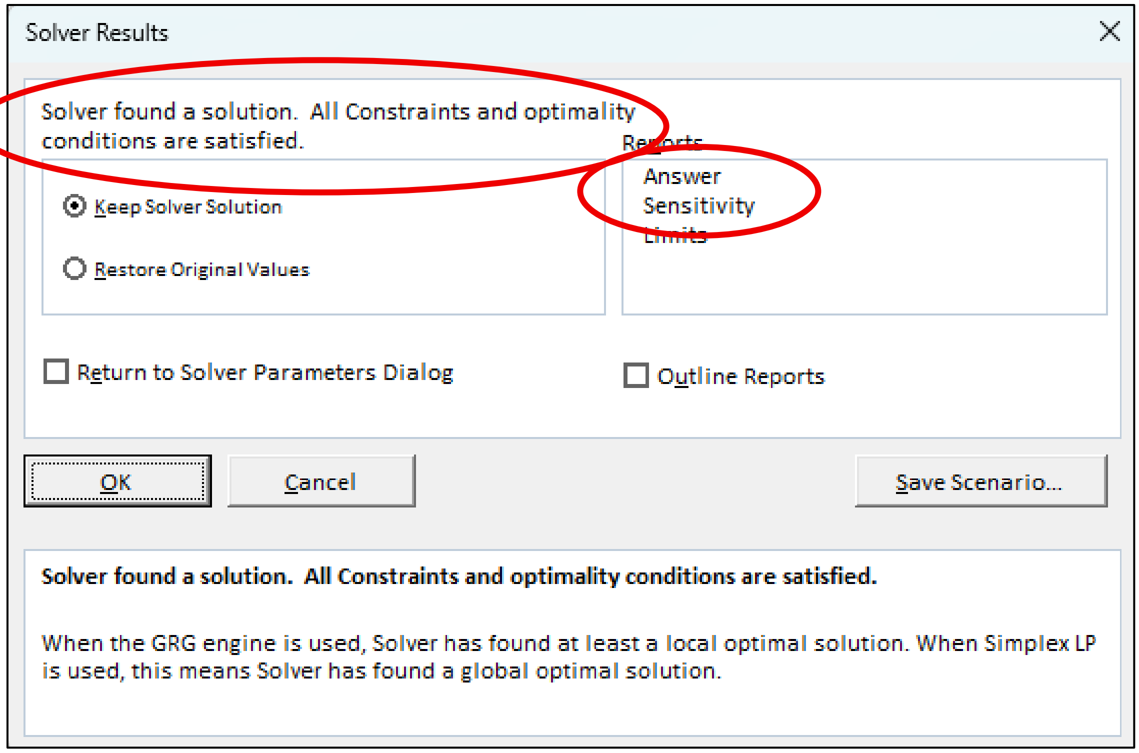
- In the Solver window (shown below), configure the following parameters:
  - **Set Objective**: Select the objective function representing total profit.
  - **To**: Choose Max to maximize profit.
  - **By Changing Variable Cells**: Specify the decision variables (optimal solutions to be determined by Solver).
  - **Subject to the Constraints**: Click Add to define all constraints.
  - **Make Unconstrained Variables Non-Negative**: Uncheck this option to ensure solutions are greater than or equal to 0.
  - **Select a Solving Method**: Choose Simplex LP.
- Run Solver to compute the optimal solution by clicking the Solve button.



- If Solver identifies a valid solution, the Solver Results window will confirm with the message **'Solver found a solution. All constraints and optimality conditions are satisfied'**.

  Select the **Answer** and **Sensitivity** reports to view the results.

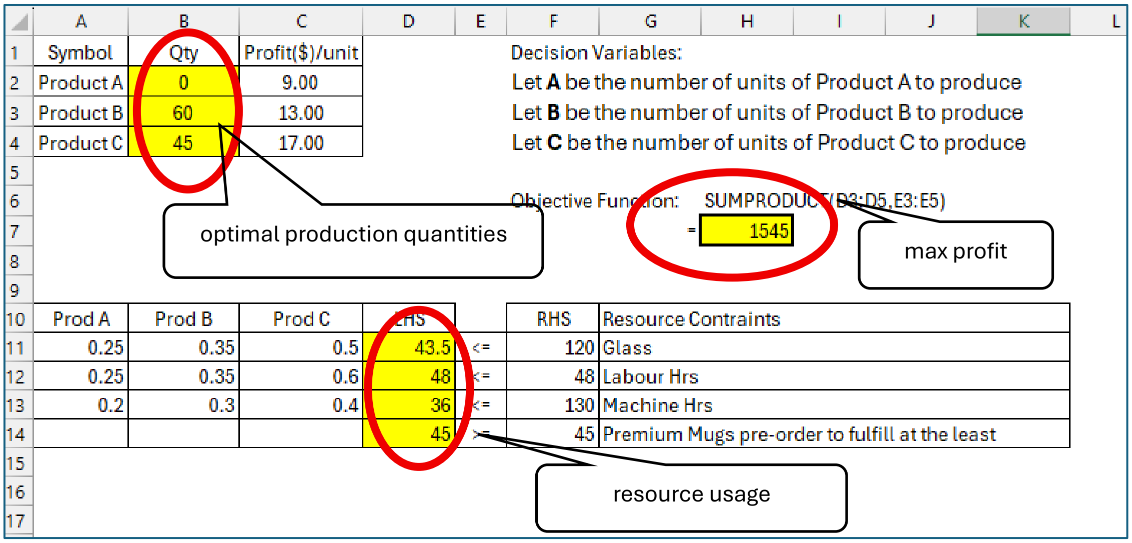
  Click the **OK** button.



6.3 Output

The screenshots below illustrate the following:

- **Solver output**:
  - Optimal Solution – to make 0, 60 and 45 units of Product A, Product B and Product C respectively
  - Objective Function – the maximum profit obtained from making the products is **$1545**
  - Resource Utilization -
    - Glass: **43.5 kg used** out of 120 kg available
    - Labour Hrs: **all 48hrs used**
    - Machine Hrs: **36hrs used** out of 130hrs available
    - Pre-Order: **All 45 units of Product C is fulfilled**



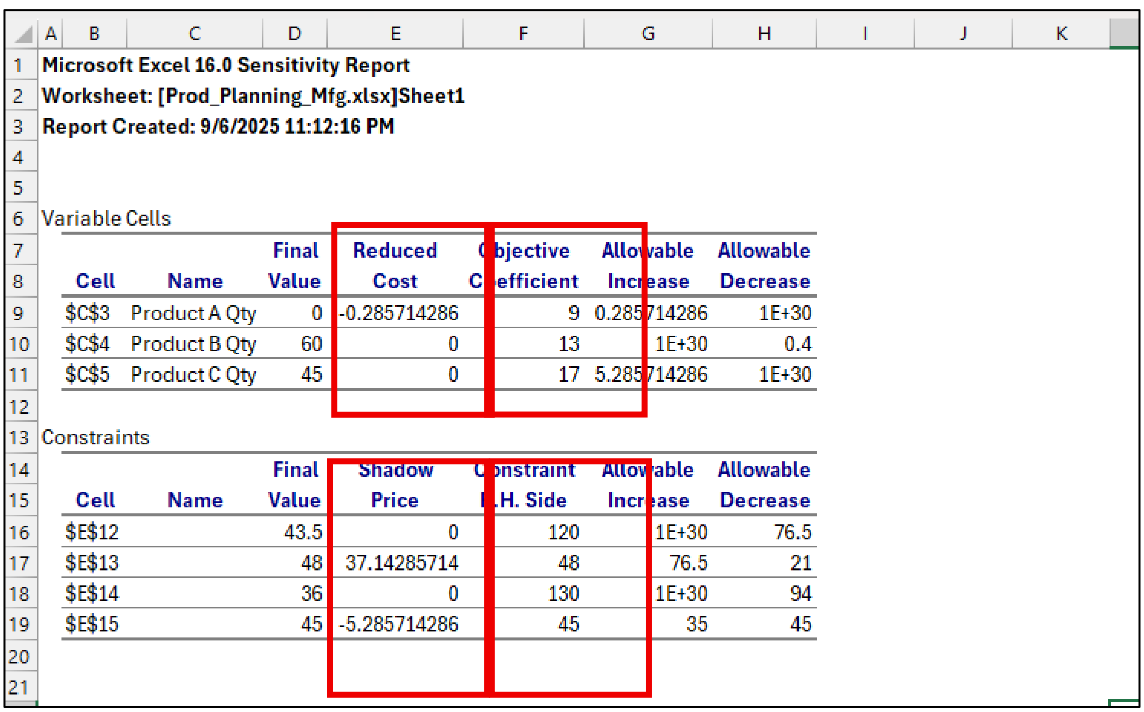
Symbol	Qty	Profit($)/unit
Product A	0	9.00
Product B	60	13.00
Product C	45	17.00

Decision Variables:
Let **A** be the number of units of Product A to produce
Let **B** be the number of units of Product B to produce
Let **C** be the number of units of Product C to produce

Objective Function: SUMPRODUCT(B3:D5,E3:E5)
= 1545

Prod A	Prod B	Prod C	LHS		RHS	Resource Contraints
0.25	0.35	0.5	43.5	<=	120	Glass
0.25	0.35	0.6	48	<=	48	Labour Hrs
0.2	0.3	0.4	36	<=	130	Machine Hrs
			45	>=	45	Premium Mugs pre-order to fulfill at the least

- **Sensitivity Report**: sensitivity analysis helps the decision-maker make informed decisions under uncertainty, anticipate the impact of changes, and identify critical constraints that could affect profits or resource utilization. It helps understand how robust the optimal solution is and how changes in the model's parameters affect outcomes.



Microsoft Excel 16.0 Sensitivity Report
Worksheet: [Prod_Planning_Mfg.xlsx]Sheet1
Report Created: 9/6/2025 11:12:16 PM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$C$3	Product A Qty	0	-0.285714286	9	0.285714286	1E+30
$C$4	Product B Qty	60	0	13	1E+30	0.4
$C$5	Product C Qty	45	0	17	5.285714286	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$E$12		43.5	0	120	1E+30	76.5
$E$13		48	37.14285714	48	76.5	21
$E$14		36	0	130	1E+30	94
$E$15		45	-5.285714286	45	35	45

Parameters to analyse

- Range of Optimality - range of the profit coefficients to maintain optimal solution
  - **Prod A <= 9 + 0.286**
  - **Prod B >= 13 – 0.4**
  - **Prod C <= 17 + 5.286**
- Range of Feasibility – range of resource availability can vary without changing the optimal solution. This would help in planning for resource fluctuations and identifying which constraints are “tight” or critical.
  - **Glass >= 120 – 76.5**
  - **48 - 21 <= Labour Hrs <= 48 + 79.5**
  - **Machine Hrs >= 130 - 94**
- Reduced Costs - the amount by which a decision variable’s contribution would need to improve before it enters the optimal solution. This helps which products are worth considering
  - Product A - **-$ 0.2857**
  - Product B - **$ 0**
  - Product C - **$ 0**
- Shadow Prices - the value of having one additional unit of a constrained resource. It guides resource allocation decisions and highlights bottlenecks
  - Labour Hrs - **$ 37.143**

- **Answer Report**: provides the maximum profit, the optimal solution, actual resource utilization, and identifies which resources are binding.

Microsoft Excel 16.0 Answer Report
Worksheet: [Prod_Planning_Mfg.xlsx]Sheet1
Report Created: 9/6/2025 11:12:16 PM
Result: Solver found a solution. All Constraints and optimality conditions are satisfied.
Solver Engine
Engine: Simplex LP
Solution Time: 0.016 Seconds.
Iterations: 4 Subproblems: 0
Solver Options
Max Time Unlimited, Iterations Unlimited, Precision 0.000001, Use Automatic Scaling
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume NonNegative

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$I$8	= SUMPRODUCT(D3:D5,E3:E5)	1545	1545

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$C$3	Product A Qty	0	0	Contin
$C$4	Product B Qty	60	60	Contin
$C$5	Product C Qty	45	45	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$E$12		43.5	$E$12<=$G$12	Not Binding	76.5
$E$13		48	$E$13<=$G$13	Binding	0
$E$14		36	$E$14<=$G$14	Not Binding	94
$E$15		45	$E$15>=$G$15	Binding	0

7. Results

The optimal solution is:

- Product A = **0 units**
- Product B = **60 units**
- Product C = **45 units**

Profit

- Maximum Profit = $ 1,545

Resource Utilization

- Glass used = 43.5 kg (≤ 120)
- Labour used = 48 hrs (**binding constraint**)
- Machine used = 36 hrs (≤ 130)

8. Discussion

- Labour hours were fully utilized, making them the limiting factor in production.

  Question: If labour hours were increased, could profits be further improved?

  Answer: Labour availability/resource is binding and has a shadow price of $37.143. As long as the increase (or decrease) is within the range of feasibility, profit will increase (or decrease) with every unit of labour hours made available (or removed).

  Example – if labour hour is **increased** to 50 hrs, profit will increase by

  profit = $37.143 * 2 = $ 74.286

  In short, if labour capacity were expanded, production could increase, and profits could rise further.

  Question: If additional labour costs $40 per hour and if manpower is available, should the manufacturer consider overtime production?

  Answer: Since additional labor capacity is valued at $37.143 per hour, running overtime at $40 per hour is not cost-effective.

- Product A was not produced because it yields less profit per labour hour compared to B.

  Question: What is the impact of force producing Product A?

Answer: Product A has a reduced cost of -$ 0.2857. Producing a unit of Product A will reduce profit by this amount.

Profit = 1545 – 0.2587 * 1 = $ 1544.714

To make it attractive to produce Product A, the manufacturer can either:

- increase the profit coefficient of Product A by $9 + $0.2587 = 9.2587 or
- reducing one of the resources required to make Product A

Example let's consider reducing the amount of labour hour, L.

0 = 9 – (0.25 * 0) – (L * 37.143) - (0.2 * 0)

L = 9 / 37.143 = 0.242 hours

In short, lowering the labor requirement for Product A from 0.25 to 0.242 hours increases its attractiveness for production.

9. Conclusion

This exercise demonstrates the value of **linear programming** in solving production planning problems. With Excel Solver, managers can optimize resource allocation and maximize profitability under real-world constraints.

The linear programming analysis produced a clear and actionable solution. To maximize profit under the given resource constraints, the manufacturer should produce:

- 0 units of Product A
- 60 units of Product B
- 45 units of Product C

This production plan achieves the **maximum possible profit of $1,545**, while fully utilizing the available **labour hours**, which proved to be the **binding constraint**. Glass and machine resources remain underutilized. Labour availability is the key limiting factor in production.

10. Appendix

10.1 Steps to enable Solver if it does not appear in the menu

- Go to *File* > *Options*> *Add-ins*
- Under the *Manage* field, select *Excel Add-ins* and click *Go*
- Then check *Solver Add-in* and click *OK*