



Controller Design in MATLAB®

Level 3 Control and Signal Processing

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Abstract

This report describes the design of two controllers for an industrial plant process in MATLAB®. Each controller was designed to meet a different performance specification.

Nomenclature

s	Laplace domain variable	$[\text{rad s}^{-1}]$
t	Time domain variable	$[\text{s}]$
ω_n	Natural frequency	
ζ	Damping factor	
G	Industrial plant process transfer function	
G_c	Controller transfer function	
\mathcal{L}	Laplace transform operator	
M_0	Peak overshoot	$[\%]$
SSE	Steady state error	
T_r	Eighty percent rise time	$[\text{s}]$
T_s	Two percent settling time	$[\text{s}]$

Percentage overshoot M_0 is the the peak response in proportion to the final output value and is solely dependent on the damping factor ζ for second order systems:

$$M_0 = \exp\left(-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right)^* \quad (1)$$

The settling time T_s of the controller is the time for the output oscillations to decay to within some margin of the final value: 2% is used in this report. Settling time is also dependent on the natural frequency ω_n :

$$T_s = -\frac{\ln(2\%)}{\zeta\omega_n} \approx \frac{4}{\zeta\omega_n}^* \quad (2)$$

The rise time of the controller is the time for the output to transition from 10% to 90% of the final value:

$$T_r = \frac{\ln(90\%/10\%)}{\zeta\omega_n} \approx \frac{2.2}{\zeta\omega_n}^* \quad (3)$$

1 Introduction

2 Background

Figure 1 illustrates a closed loop system. The controller is designed to drive the output $y(s)$ of the plant process to the desired output $r(s)$ by minimising the error $e(s)$.

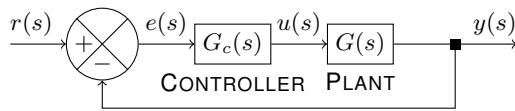


FIGURE 1: Block diagram of the closed loop system.

For low frequency applications the controller performance can be characterised by analysing the step response.

These three parameters are typically specified as upper bounds and constitute the dynamic specification.

In controller design there is a tradeoff between the dynamic specification and the steady state error. The steady state error is defined as follows:

$$SSE := \lim_{t \rightarrow \infty} e(t) \quad (4)$$

One of the most common types of controller in industry is the PID controller, which can be easily implemented using microcontrollers.

*Equality holds for second order systems only, but may be used to approximate higher order characteristics.

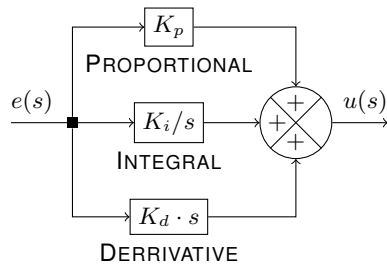


FIGURE 2: Block diagram of a PID controller

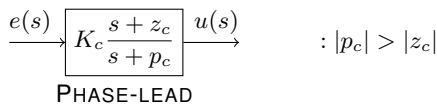


FIGURE 3: Block diagram of a phase-lead controller.

3 Specification

The transfer function for the industrial plant process $G(s)$ was given as follows:

$$G(s) = \frac{s + 1}{s(s^2 + 4s + 5)} \quad (5)$$

The transfer function $G_c(s)$ for the two controllers was chosen to meet the system performance specification detailed in Table 1.

TABLE 1: System performance specification.

SYSTEM	$M_0[\%]$	$T_r[s]$	$T_2\%$	$SSE[\%]$
1	5	1.0	4	—
2	20	0.5	—	5

4 Design

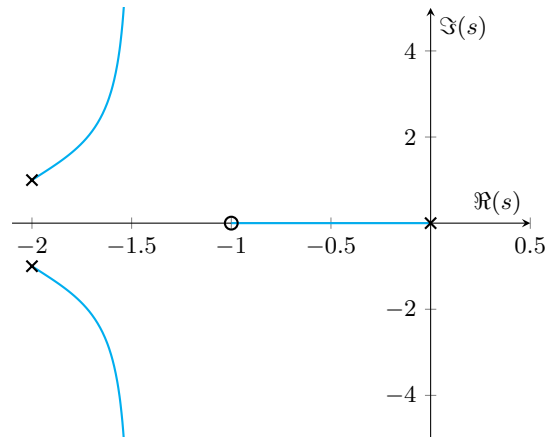


FIGURE 4: Root locus of system poles from closed loop poles to closed loop zeros as G_c varies from zero to infinity.

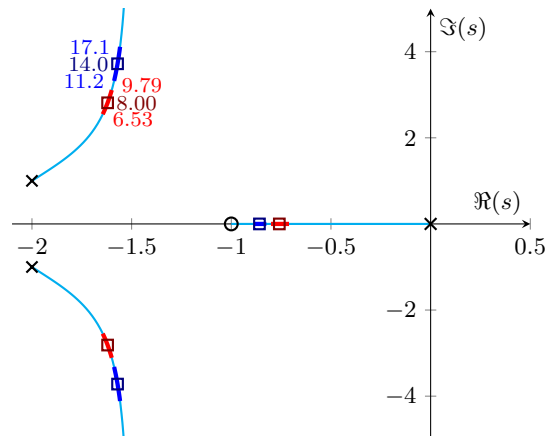


FIGURE 5: Chosen closed loop pole locations, gains and acceptable limits for the gain compensator G_c designed for **SYSTEM 1** and **SYSTEM 2**, marked on the root locus.

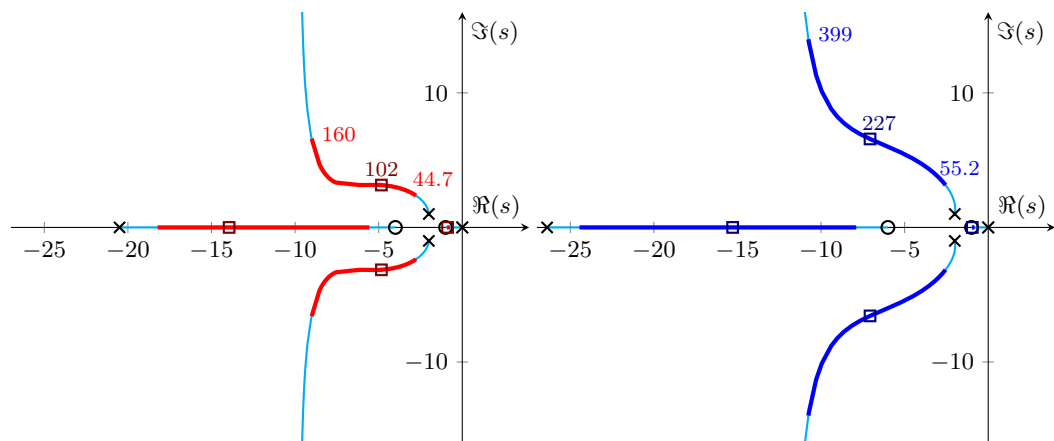


FIGURE 6: Chosen closed loop pole locations, gains and acceptable limits for the phase-lead compensator G_c designed for **SYSTEM 1** and **SYSTEM 2**, marked on the compensated root loci.

A Controller Derivations

A.1 Phase-lead Compensator

Figure 7 illustrates how the added poles and zeroes were chosen when designing the phase-lead controllers.

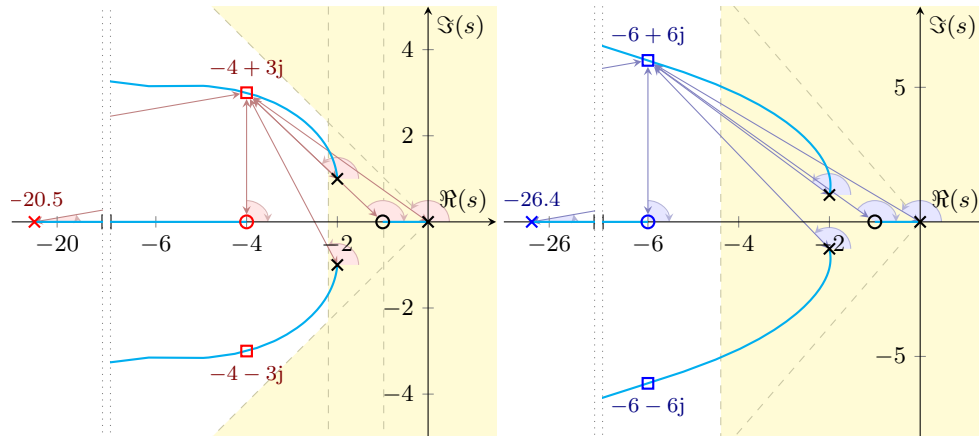


FIGURE 7: Ideal phase-lead compensated root locus for **SYSTEM 1** and **SYSTEM 2**, designed using the angle and magnitude criteria.

The yellow region marks the second order closed loop pole locations which did not satisfy the performance specification and the desired closed loop poles were placed outside this region. The added zero was placed on the real axis in line with desired closed loop poles. The angle criterion was used to uniquely identify the added real pole which ensured the root locus passed through the desired pole locations. This was done programmatically in MATLAB®:

```
1 function [zc, pc, Kc] = find_lead_compensator(G, s)
2 % Place real zero 'zc' below desired pole 's'
3 zc = real(s);
4 % Find real pole 'pc' using angle criterion
5 th = sum(angle(s-pole(G))) - sum(angle(s-zero(G))) - angle(s-zc);
6 th = mod(th-pi, 2*pi);
7 pc = zc + imag(s)/tan(th);
8 % Find gain 'Kc' using magnitude criterion
9 K = prod(abs(s-pole(G)))/prod(abs(s-zero(G))) * abs(s-pc)/abs(s-zc);
10 Kc = K / dcgain(G);
11 end % function
```

The magnitude criterion was used to find the gain corresponding to this desired pole, which can be seen on lines 9 and 10, but this was not the gain used in the final controller. The gain was varied in MATLAB® `sisotool()` to find the limiting gains which met the performance specification for each system. The median gains were chosen for the optimum controllers.