1. Prove : $\sin^4\theta + \cos^4\theta = 1 - \frac{1}{2}\sin^2 2\theta$

Solution by Dhan Raut

$\text{LHS} = \sin^4\theta + \cos^4\theta$

$= (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta$

$= 1 - 2\sin^2\theta\cos^2\theta$

$= 1 - \frac{1}{2}(2\sin\theta\cos\theta)^2$

$= 1 - \frac{1}{2}\sin^2 2\theta$

Formulas used :

$\sin^2\theta + \cos^2\theta = 1$

$2\sin\theta\cos\theta = \sin 2\theta$

$a^4 + b^4 = (a^2 + b^2)^2 - 2a^2b^2$

2. Prove : $\dfrac{1 + \sin\theta - \cos\theta}{1 + \sin\theta + \cos\theta} = \tan\dfrac{\theta}{2}$

Solution by Dhan Raut

$1 - \cos\theta = 2\sin^2\dfrac{\theta}{2}$

$1 + \cos\theta = 2\cos^2\dfrac{\theta}{2}$

$\sin\theta = 2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}$

$\text{Numerator} = 2\sin^2\dfrac{\theta}{2} + 2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}$

$\text{Denominator} = 2\cos^2\dfrac{\theta}{2} + 2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}$

$= \dfrac{2\sin\frac{\theta}{2}\left(\sin\frac{\theta}{2} + \cos\frac{\theta}{2}\right)}{2\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right)}$

$= \dfrac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} = \tan\dfrac{\theta}{2}$

Formulas used :

$1 - \cos\theta = 2\sin^2\dfrac{\theta}{2}$

$1 + \cos\theta = 2\cos^2\dfrac{\theta}{2}$

$\sin\theta = 2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}$

CamScanner

3. Prove : $\cot A - \tan A = 2\cot 2A$

Solution by Dhan Raut

$$\cot A - \tan A = \frac{\cos A}{\sin A} - \frac{\sin A}{\cos A}$$

$$= \frac{\cos^2 A - \sin^2 A}{\sin A \cos A}$$

$$= \frac{\cos 2A}{\frac{1}{2}\sin 2A}$$

$$= 2 \cdot \frac{\cos 2A}{\sin 2A}$$

$$= 2\cot 2A$$

Formulas used :

$$\cot A = \frac{\cos A}{\sin A}, \quad \tan A = \frac{\sin A}{\cos A}$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\sin 2A = 2\sin A \cos A$$

4. Prove : $\cos^6\theta + \sin^6\theta = 1 - \frac{3}{4}\sin^2 2\theta$

Solution by Dhan Raut

$$\cos^6\theta + \sin^6\theta = (\cos^2\theta)^3 + (\sin^2\theta)^3$$

$$= (\cos^2\theta + \sin^2\theta)(\cos^4\theta - \cos^2\theta\sin^2\theta + \sin^4\theta)$$

$$= 1 \cdot \left[(\sin^4\theta + \cos^4\theta) - \sin^2\theta\cos^2\theta\right]$$

$$= \left[1 - 2\sin^2\theta\cos^2\theta\right] - \sin^2\theta\cos^2\theta$$

$$= 1 - 3\sin^2\theta\cos^2\theta$$

$$= 1 - 3 \cdot \frac{\sin^2 2\theta}{4}$$

$$= 1 - \frac{3}{4}\sin^2 2\theta$$

Formulas used :

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

CamScanner

5. Prove : $\tan\left(\frac{\pi}{4}+\theta\right)=\frac{1+\tan\theta}{1-\tan\theta}$

Solution by Dhan Raut

$$\tan\left(\frac{\pi}{4}+\theta\right)=\frac{\tan\frac{\pi}{4}+\tan\theta}{1-\tan\frac{\pi}{4}\tan\theta}$$

$$=\frac{1+\tan\theta}{1-1\cdot\tan\theta}$$

$$=\frac{1+\tan\theta}{1-\tan\theta}$$

Formulas used :

$$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$$

$$\tan\frac{\pi}{4}=1$$

6. Prove : $\sin^2 A-\sin^2 B=\sin(A+B)\sin(A-B)$

Solution by Dhan Raut

$$RHS=\sin(A+B)\sin(A-B)$$

$$=(\sin A\cos B+\cos A\sin B)(\sin A\cos B-\cos A\sin B)$$

$$=(\sin A\cos B)^2-(\cos A\sin B)^2$$

$$=\sin^2 A\cos^2 B-\cos^2 A\sin^2 B$$

$$=\sin^2 A(1-\sin^2 B)-(1-\sin^2 A)\sin^2 B$$

$$=\sin^2 A-\sin^2 A\sin^2 B-\sin^2 B+\sin^2 A\sin^2 B$$

$$=\sin^2 A-\sin^2 B=LHS$$

Formulas used :

$$\sin(A\pm B)=\sin A\cos B\pm\cos A\sin B$$

$$\cos^2 x=1-\sin^2 x$$

CamScanner

7. Prove : $\dfrac{\sin 3\theta}{\sin\theta} - \dfrac{\cos 3\theta}{\cos\theta} = 2$

Solution by Dhan Raut

$$\frac{\sin 3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta} = \frac{\sin 3\theta\cos\theta - \cos 3\theta\sin\theta}{\sin\theta\cos\theta}$$

$$= \frac{\sin(3\theta - \theta)}{\sin\theta\cos\theta}$$

$$= \frac{\sin 2\theta}{\sin\theta\cos\theta}$$

$$= \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$$= 2$$

Formulas used :

$\sin(A - B) = \sin A\cos B - \cos A\sin B$

$\sin 2\theta = 2\sin\theta\cos\theta$

8. Prove : $\dfrac{\cos A + \cos B}{\sin A - \sin B} = \cot\dfrac{A - B}{2}$

Solution by Dhan Raut

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2}$$

$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$\frac{\cos A + \cos B}{\sin A - \sin B} = \frac{2\cos\frac{A+B}{2}\cos\frac{A-B}{2}}{2\cos\frac{A+B}{2}\sin\frac{A-B}{2}}$$

$$= \frac{\cos\frac{A-B}{2}}{\sin\frac{A-B}{2}} = \cot\frac{A-B}{2}$$

Formulas used :

$$\cos P + \cos Q = 2\cos\frac{P+Q}{2}\cos\frac{P-Q}{2}$$

$$\sin P - \sin Q = 2\cos\frac{P+Q}{2}\sin\frac{P-Q}{2}$$

$$\cot x = \frac{\cos x}{\sin x}$$

CamScanner

9. Prove : $\sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta$

Solution by Dhan Raut

$$\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}}$$

$$= \sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}}$$

$$= \frac{1-\sin\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= \sec\theta - \tan\theta$$

Formulas used :

$$1-\sin^2\theta = \cos^2\theta$$

$$\sec\theta = \frac{1}{\cos\theta}, \quad \tan\theta = \frac{\sin\theta}{\cos\theta}$$

10. Prove : $\cos^2 A + \cos^2 B = 1 + \cos(A+B)\cos(A-B)$

Solution by Dhan Raut

$$\cos(A+B)\cos(A-B) = (\cos A\cos B - \sin A\sin B)(\cos A\cos B + \sin A$$

$$= \cos^2 A\cos^2 B - \sin^2 A\sin^2 B$$

$$= \cos^2 A\cos^2 B - (1-\cos^2 A)(1-\cos^2 B)$$

$$= \cos^2 A\cos^2 B - \left[1-\cos^2 A - \cos^2 B + \cos^2 A\cos^2 B\right]$$

$$= \cos^2 A\cos^2 B - 1 + \cos^2 A + \cos^2 B - \cos^2 A\cos^2 B$$

$$= \cos^2 A + \cos^2 B - 1$$

Hence, $\cos^2 A + \cos^2 B = 1 + \cos(A+B)\cos(A-B)$

Formulas used :

$$\cos(A\pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\sin^2 x = 1 - \cos^2 x$$

CS CamScanner

11. Prove : $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$

Solution by Dhan Raut

$$\tan 3\theta = \frac{\tan 2\theta + \tan\theta}{1 - \tan 2\theta \tan\theta}$$

$$= \frac{\frac{2\tan\theta}{1-\tan^2\theta} + \tan\theta}{1 - \frac{2\tan\theta}{1-\tan^2\theta} \cdot \tan\theta}$$

$$= \frac{\frac{2\tan\theta + \tan\theta(1-\tan^2\theta)}{1-\tan^2\theta}}{\frac{1-\tan^2\theta - 2\tan^2\theta}{1-\tan^2\theta}}$$

$$= \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$

Formulas used :

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

12. Prove : $\cos 20^\circ \cos 40^\circ \cos 80^\circ = \dfrac{1}{8}$

Solution by Dhan Raut

Let $A = \cos 20^\circ \cos 40^\circ \cos 80^\circ$

$$A = \frac{\sin 20^\circ \cos 20^\circ \cos 40^\circ \cos 80^\circ}{\sin 20^\circ}$$

$$= \frac{\frac{1}{2}\sin 40^\circ \cos 40^\circ \cos 80^\circ}{\sin 20^\circ}$$

$$= \frac{\frac{1}{4}\sin 80^\circ \cos 80^\circ}{\sin 20^\circ}$$

$$= \frac{\frac{1}{8}\sin 160^\circ}{\sin 20^\circ}$$

$$= \frac{\frac{1}{8}\sin(180^\circ - 20^\circ)}{\sin 20^\circ}$$

$$= \frac{\frac{1}{8}\sin 20^\circ}{\sin 20^\circ} = \frac{1}{8}$$

Formulas used :

$\sin 2A = 2\sin A \cos A$

$\sin(180^\circ - x) = \sin x$

CS CamScanner

13. Prove : $\dfrac{\sin 5\theta - \sin 3\theta}{\cos 5\theta + \cos 3\theta} = \tan\theta$

Solution by Dhan Raut

$\sin 5\theta - \sin 3\theta = 2\cos 4\theta \sin\theta$

$\cos 5\theta + \cos 3\theta = 2\cos 4\theta \cos\theta$

$$\frac{\sin 5\theta - \sin 3\theta}{\cos 5\theta + \cos 3\theta} = \frac{2\cos 4\theta \sin\theta}{2\cos 4\theta \cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta} = \tan\theta$$

Formulas used :

$$\sin P - \sin Q = 2\cos\frac{P+Q}{2}\sin\frac{P-Q}{2}$$

$$\cos P + \cos Q = 2\cos\frac{P+Q}{2}\cos\frac{P-Q}{2}$$

14. Prove : $(\sin\theta + \cos\theta)^2 = 1 + \sin 2\theta$

Solution by Dhan Raut

$(\sin\theta + \cos\theta)^2 = \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta$

$= 1 + 2\sin\theta\cos\theta$

$= 1 + \sin 2\theta$

Formulas used :

$\sin^2\theta + \cos^2\theta = 1$

$2\sin\theta\cos\theta = \sin 2\theta$

CS CamScanner

15. Prove : $\dfrac{1}{\sin 10^\circ} - \dfrac{\sqrt{3}}{\cos 10^\circ} = 4$

Solution by Dhan Raut

$$\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ} = \frac{\cos 10^\circ - \sqrt{3}\sin 10^\circ}{\sin 10^\circ \cos 10^\circ}$$

$$= \frac{2\left(\frac{1}{2}\cos 10^\circ - \frac{\sqrt{3}}{2}\sin 10^\circ\right)}{\frac{1}{2}\sin 20^\circ}$$

$$= \frac{2\left(\sin 30^\circ \cos 10^\circ - \cos 30^\circ \sin 10^\circ\right)}{\frac{1}{2}\sin 20^\circ}$$

$$= \frac{2\sin(30^\circ - 10^\circ)}{\frac{1}{2}\sin 20^\circ}$$

$$= \frac{2\sin 20^\circ}{\frac{1}{2}\sin 20^\circ} = 4$$

Formulas used :

$2\sin A\cos A = \sin 2A$

$\sin(A - B) = \sin A\cos B - \cos A\sin B$

$\sin 30^\circ = \dfrac{1}{2}, \ \cos 30^\circ = \dfrac{\sqrt{3}}{2}$

16. Prove : $\cos^2 15^\circ - \sin^2 15^\circ = \dfrac{\sqrt{3}}{2}$

Solution by Dhan Raut

$\cos^2 15^\circ - \sin^2 15^\circ = \cos 30^\circ$

$$= \frac{\sqrt{3}}{2}$$

Formulas used :

$\cos^2 A - \sin^2 A = \cos 2A$

$\cos 30^\circ = \dfrac{\sqrt{3}}{2}$

CS CamScanner

17. Prove : $\dfrac{\cos 7A + \cos 5A}{\sin 7A - \sin 5A} = \cot A$

Solution by Dhan Raut

$\cos 7A + \cos 5A = 2\cos 6A \cos A$

$\sin 7A - \sin 5A = 2\cos 6A \sin A$

$\dfrac{\cos 7A + \cos 5A}{\sin 7A - \sin 5A} = \dfrac{2\cos 6A \cos A}{2\cos 6A \sin A}$

$= \dfrac{\cos A}{\sin A} = \cot A$

Formulas used :

$\cos P + \cos Q = 2\cos\dfrac{P+Q}{2}\cos\dfrac{P-Q}{2}$

$\sin P - \sin Q = 2\cos\dfrac{P+Q}{2}\sin\dfrac{P-Q}{2}$

18. Prove : $\cos A\cos(60^\circ - A)\cos(60^\circ + A) = \dfrac{1}{4}\cos 3A$

Solution by Dhan Raut

$\text{LHS} = \cos A \cdot \dfrac{1}{2}\left[\cos(2A) + \cos 120^\circ\right]$

$= \cos A \cdot \dfrac{1}{2}\left[\cos 2A - \dfrac{1}{2}\right]$

$= \dfrac{1}{2}\cos A\cos 2A - \dfrac{1}{4}\cos A$

$= \dfrac{1}{4}\left[2\cos A\cos 2A - \cos A\right]$

$= \dfrac{1}{4}\left[\cos 3A + \cos A - \cos A\right]$

$= \dfrac{1}{4}\cos 3A$

Formulas used :

$2\cos x\cos y = \cos(x+y) + \cos(x-y)$

$\cos 120^\circ = -\dfrac{1}{2}$

$\cos 3A = 4\cos^3 A - 3\cos A$

CamScanner

19. Prove : $\dfrac{\sin 4\theta}{1+\cos 4\theta}\cdot\dfrac{\cos 2\theta}{1+\cos 2\theta}=\tan\theta$

Solution by Dhan Raut

$$\frac{\sin 4\theta}{1+\cos 4\theta}=\frac{2\sin 2\theta\cos 2\theta}{2\cos^2 2\theta}=\tan 2\theta$$

$$\frac{\cos 2\theta}{1+\cos 2\theta}=\frac{\cos 2\theta}{2\cos^2\theta}$$

$$\text{LHS}=\tan 2\theta\cdot\frac{\cos 2\theta}{2\cos^2\theta}$$

$$=\frac{\sin 2\theta}{\cos 2\theta}\cdot\frac{\cos 2\theta}{2\cos^2\theta}$$

$$=\frac{\sin 2\theta}{2\cos^2\theta}$$

$$=\frac{2\sin\theta\cos\theta}{2\cos^2\theta}$$

$$=\frac{\sin\theta}{\cos\theta}=\tan\theta$$

Formulas used :

$\sin 2\theta=2\sin\theta\cos\theta$

$1+\cos 2\theta=2\cos^2\theta$

$1+\cos 4\theta=2\cos^2 2\theta$

20. Prove : $\sqrt{2+\sqrt{2+\sqrt{2+2\cos 8\theta}}}=2\cos\theta$

Solution by Dhan Raut

$2+2\cos 8\theta=4\cos^2 4\theta$

$$\sqrt{2+\sqrt{2+2\cos 8\theta}}=\sqrt{2+\sqrt{4\cos^2 4\theta}}$$

$$=\sqrt{2+2\cos 4\theta}$$

$$=\sqrt{4\cos^2 2\theta}=2\cos 2\theta$$

$$\sqrt{2+2\cos 2\theta}=\sqrt{4\cos^2\theta}=2\cos\theta$$

Formulas used :

$1+\cos 2A=2\cos^2 A$

$\sqrt{\cos^2 A}=|\cos A|$ (taking positive root)

CS CamScanner

21. Prove : $\dfrac{\tan\theta}{1-\cot\theta}+\dfrac{\cot\theta}{1-\tan\theta}=1+\sec\theta\csc\theta$

Solution by Dhan Raut

$$\frac{\tan\theta}{1-\frac{1}{\tan\theta}}+\frac{\cot\theta}{1-\tan\theta}=\frac{\tan^2\theta}{\tan\theta-1}+\frac{\cot\theta}{1-\tan\theta}$$

$$=\frac{\tan^2\theta}{\tan\theta-1}-\frac{\cot\theta}{\tan\theta-1}$$

$$=\frac{\tan^2\theta-\cot\theta}{\tan\theta-1}$$

$$=\frac{\tan^2\theta-\frac{1}{\tan\theta}}{\tan\theta-1}$$

$$=\frac{\tan^3\theta-1}{\tan\theta(\tan\theta-1)}$$

$$=\frac{(\tan\theta-1)(\tan^2\theta+\tan\theta+1)}{\tan\theta(\tan\theta-1)}$$

$$=\frac{\tan^2\theta+\tan\theta+1}{\tan\theta}$$

$$=\tan\theta+1+\cot\theta$$

$$=1+\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}$$

$$=1+\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}$$

$$=1+\frac{1}{\sin\theta\cos\theta}$$

$$=1+\sec\theta\csc\theta$$

Formulas used :

$$\cot\theta=\frac{1}{\tan\theta}$$

$$a^3-b^3=(a-b)(a^2+ab+b^2)$$

$$\sec\theta=\frac{1}{\cos\theta},\quad \csc\theta=\frac{1}{\sin\theta}$$

22. Prove : $\sin^2 A\cos^2 B-\cos^2 A\sin^2 B=\sin^2 A-\sin^2 B$

Solution by Dhan Raut

$$\sin^2 A\cos^2 B-\cos^2 A\sin^2 B$$

$$=\sin^2 A(1-\sin^2 B)-(1-\sin^2 A)\sin^2 B$$

$$=\sin^2 A-\sin^2 A\sin^2 B-\sin^2 B+\sin^2 A\sin^2 B$$

$$=\sin^2 A-\sin^2 B$$

Formulas used :

$$\cos^2 x=1-\sin^2 x$$

CS CamScanner

23. Prove : $\tan 70^\circ = \tan 20^\circ + 2\tan 50^\circ$

Solution by Dhan Raut

$$\tan 70^\circ = \cot 20^\circ = \frac{1}{\tan 20^\circ}$$

Let $x = \tan 20^\circ$. Then $\tan 50^\circ = \tan(70^\circ - 20^\circ)$.

$$\tan 50^\circ = \frac{\tan 70^\circ - \tan 20^\circ}{1 + \tan 70^\circ \tan 20^\circ}$$

$$= \frac{\frac{1}{x} - x}{1 + \frac{1}{x}\cdot x} = \frac{\frac{1-x^2}{x}}{2} = \frac{1-x^2}{2x}$$

$$2\tan 50^\circ = \frac{1-x^2}{x} = \frac{1}{x} - x = \tan 70^\circ - \tan 20^\circ$$

Hence, $\tan 70^\circ = \tan 20^\circ + 2\tan 50^\circ$.

Formulas used :

$$\tan(90^\circ - A) = \cot A, \quad \tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

24. Prove : $\dfrac{\sin A + \sin 3A + \sin 5A}{\cos A + \cos 3A + \cos 5A} = \tan 3A$

Solution by Dhan Raut

$\sin A + \sin 5A = 2\sin 3A \cos 2A$

$\cos A + \cos 5A = 2\cos 3A \cos 2A$

Numerator $= 2\sin 3A \cos 2A + \sin 3A = \sin 3A(2\cos 2A + 1)$

Denominator $= 2\cos 3A \cos 2A + \cos 3A = \cos 3A(2\cos 2A + 1)$

$$\frac{\sin A + \sin 3A + \sin 5A}{\cos A + \cos 3A + \cos 5A} = \frac{\sin 3A(2\cos 2A + 1)}{\cos 3A(2\cos 2A + 1)}$$

$$= \frac{\sin 3A}{\cos 3A} = \tan 3A$$

Formulas used :

$$\sin P + \sin Q = 2\sin\frac{P+Q}{2}\cos\frac{P-Q}{2}$$

$$\cos P + \cos Q = 2\cos\frac{P+Q}{2}\cos\frac{P-Q}{2}$$

CamScanner

fb: Important Notes

follow for more...

B Long Questions:

1. A) $(\cos 2\alpha - \cos 2\beta)^2 + (\sin 2\alpha + \sin 2\beta)^2 = 4\sin^2(\alpha+\beta)$

LHS = $\underline{\cos^2 2\alpha} - 2\cos 2\alpha \cdot \cos 2\beta + \underline{\cos^2 2\beta} + \underline{\sin^2 2\alpha}$
$+ 2\sin 2\alpha \cdot \sin 2\beta + \underline{\sin^2 2\beta}$

$= 1 + 1 - 2\cos 2\alpha \cdot \cos 2\beta + 2\sin 2\alpha \cdot \sin 2\beta$
$= 2 - 2(\cos 2\alpha \cdot \cos 2\beta - \sin 2\alpha \cdot \sin 2\beta)$
$= 2 - 2\cos(2\alpha + 2\beta)$
$= 2 - 2\cos 2(\alpha+\beta)$
$= 2[1 - \cos 2(\alpha+\beta)]$
$= 2 \cdot 2\sin^2(\alpha+\beta)$
$= 4\sin^2(\alpha+\beta)$

B. $(\cos 2\alpha - \cos 2\beta)^2 + (\sin 2\alpha - \sin 2\beta)^2 = 4\sin^2(\alpha - \beta)$

LHS = $\cos^2 2\alpha - 2\cos 2\alpha \cdot \cos 2\beta + \cos^2 2\beta + \sin^2 2\alpha - 2\sin 2\alpha \cdot \sin 2\beta$
$+ \sin^2 2\beta$

$= 1 + 1 - 2\cos 2\alpha \cdot \cos 2\beta - 2\sin 2\alpha \cdot \sin 2\beta$
$= 2 - 2(\cos 2\alpha \cdot \cos 2\beta + \sin 2\alpha \cdot \sin 2\beta)$
$= 2 - 2\cos(2\alpha \mp 2\beta)$
$= \cancel{2 - 2\cos}\ 2[1 - \cos(2\alpha - 2\beta)]$
$= 2[1 - \cos 2 \cdot (\alpha - \beta)]$
$= 2 \cdot 2\sin^2(\alpha - \beta)$
$= 4\sin^2(\alpha - \beta)$

CamScanner

C. $(\cos 2A + \cos 2B)^2 + (\sin 2A + \sin 2B)^2 = 4\cos^2(A-B)$

LHS $= \cos^2 2A + 2\cos 2A \cos 2B + \cos^2 2B + \sin^2 2A + 2\sin 2A . \sin 2B + \sin^2 2B$

$= 1 + 1 + 2\cos 2A \cos 2B + 2\sin 2A . \sin 2B$

$= 2 + 2(\cos 2A . \cos 2B + \sin 2A . \sin 2B)$

$= 2 + 2\cos(2A - 2B)$

$= 2[1 + \cos(2A - 2B)]$

$= 2[1 + \cos 2(A-B)]$

$= 2 . 2\cos^2(A-B)$

$= 4\cos^2(A-B)$

D. $(\cos 2A + \cos 2B)^2 + (\sin 2A - \sin 2B)^2 = 4\cos^2(A+B)$

LHS $= \cos^2 2A + 2\cos 2A . \cos 2B + \cos^2 2B + \sin^2 2A - 2\sin 2A . \sin 2B + \sin^2 2B$

$= 1 + 1 + 2\cos 2A . \cos 2B - 2\sin 2A . \sin 2B$

$= 2 + 2(\cos 2A . \cos 2B - \sin 2A . \sin 2B)$

$= 2 + 2\cos(2A + 2B)$

$= 2[1 + \cos(2A + 2B)]$

$= 2[1 + \cos 2.(A+B)]$

$= 2 . 2\cos^2(A+B)$

$= 4\cos^2(A+B)$

Important Notes

CamScanner

2A. $(2\cos\theta+1)(2\cos\theta-1)(2\cos 2\theta-1) = 2\cos 4\theta+1$

$$\begin{aligned}
LHS &= [(2\cos\theta)^2-(1)^2](2\cos 2\theta-1) \\
&= (4\cos^2\theta-1)(2\cos 2\theta-1) \\
&= (2\cdot 2\cos^2\theta-1)(2\cos 2\theta-1) \\
&= [2(1+\cos 2\theta)-1](2\cos 2\theta-1) \\
&= (2+2\cos 2\theta-1)(2\cos 2\theta-1) \\
&= (2\cos 2\theta+1)(2\cos 2\theta-1) \\
&= (2\cos 2\theta)^2-(1)^2 \\
&= 4\cos^2 2\theta-1 \\
&= 2\cdot 2\cos^2 2\theta-1 \\
&= 2(1+\cos 4\theta)-1 \\
&= 2+2\cos 4\theta-1 \\
&= 2\cos 4\theta+1
\end{aligned}$$

B. $(2\cos\theta+1)(2\cos\theta-1)(2\cos 2\theta-1)(2\cos 4\theta-1) = 2\cos 8\theta+1$

$$\begin{aligned}
LHS &= (4\cos^2\theta-1)(2\cos 2\theta-1)(2\cos 4\theta-1) \\
&= (2\cdot 2\cos^2\theta-1)(2\cos 2\theta-1)(2\cos 4\theta-1) \\
&= [2(1+\cos 2\theta)-1](2\cos 2\theta-1)(2\cos 4\theta-1) \\
&= (2\cos 2\theta+1)(2\cos 2\theta-1)(2\cos 4\theta-1) \\
&= (4\cos^2 2\theta-1)(2\cos 4\theta-1) \\
&= (2\cdot 2\cos^2 2\theta-1)(2\cos 4\theta-1) \\
&= [2(1+\cos 4\theta)-1](2\cos 4\theta-1) \\
&= (2\cos 4\theta+1)(2\cos 4\theta-1) \\
&= (4\cos^2 4\theta-1) \\
&= (2\cdot 2\cos^2 4\theta-1) \\
&= [2(1+\cos 8\theta)-1] \\
&= 2\cos 8\theta+1
\end{aligned}$$

Important Notes

CamScanner

fb: Important Notes
ollow for more...

Page : Date :

3A. $\frac{4(\cos^4 A + \sin^4 A)}{\cos^4 A - \sin^4 A} = (3+\cos 4A)\sec 2A$

$$LHS = \frac{4[(\cos^2 A)^2 + (\sin^2 A)^2]}{(\cos^2 A)^2 - (\sin^2 A)^2}$$

$$= \frac{4[(\cos^2 A + \sin^2 A)^2 - 2\cos^2 A\cdot \sin^2 A]}{(\cos^2 A + \sin^2 A)(\cos^2 A - \sin^2 A)}$$

$$= \frac{4\left[(1)^2 - \frac{1}{2} \times 2\cdot 2\cos^2 A \sin^2 A\right]}{1\cdot \cos 2A}$$

$$= \frac{4\left[1 - \frac{1}{2}(2\cos A\cdot \sin A)^2\right]}{\cos 2A}$$

$$= 4\left(1 - \frac{1}{2}\sin^2 2A\right)\cdot \frac{1}{\cos 2A}$$

$$= \left(4 - \frac{4}{2}\sin^2 2A\right)\sec 2A$$

$$= (4 - 2\sin^2 2A)\sec 2A$$

$$= [4 - (1 - \cos 4A)]\sec 2A$$

$$= (4 - 1 + \cos 4A)\sec 2A$$

$$= (3 + \cos 4A)\sec 2A$$

$$= RHS$$

Important Notes
CamScanner

fb: Important Notes

follow for more...

B. $\cos^6\theta + \sin^6\theta = \frac{1}{4}(1+3\cos^2 2\theta)$

$LHS = (\cos^2\theta)^3 + (\sin^2\theta)^3$

$= (\cos^2\theta + \sin^2\theta)(\cos^4\theta - \cos^2\theta.\sin^2\theta + \sin^4\theta)$

$= 1. [(\cos^2\theta)^2 + (\sin^2\theta)^2 - \cos^2\theta.\sin^2\theta]$

$= [(\cos^2\theta - \sin^2\theta)^2 + 2\cos^2\theta.\sin^2\theta - \cos^2\theta.\sin^2\theta]$

$= [\cos^2 2\theta + \cos^2\theta.\sin^2\theta]$

$= \left[\cos^2 2\theta + \frac{1}{4}(2\cos\theta\sin\theta)^2\right]$

$= \left[\cos^2 2\theta + \frac{1}{4}\sin^2 2\theta\right]$

$= \left[\cos^2 2\theta + \frac{(1-\cos^2 2\theta)}{4}\right]$

$= \left[\frac{4\cos^2 2\theta + 1 - \cos^2 2\theta}{4}\right]$

$= \left[\frac{3\cos^2 2\theta + 1}{4}\right]$

$= \frac{1}{4}(1+3\cos^2 2\theta)$

C. $\cos^6 A - \sin^6 A = \frac{1}{4}(\cos^3 2A + 3\cos 2A)$

$LHS = (\cos^2 A)^3 - (\sin^2 A)^3$

$= (\cos^2 A - \sin^2 A)(\cos^4 A + \cos^2 A.\sin^2 A + \sin^4 A)$

$= \cos 2A[(\cos^2 A)^2 + (\sin^2 A)^2 + \cos^2 A.\sin^2 A]$

$= \cos 2A[(\cos^2 A + \sin^2 A)^2 - 2\cos^2 A.\sin^2 A + \cos^2 A.\sin^2 A]$

$= \cos 2A[1 - \cos^2 A.\sin^2 A]$

CamScanner

$= \cos 2A \left[1 - \frac{1}{4}(2\cos A \cdot \sin A)^2\right]$

$= \cos 2A \left[1 - \frac{1}{4} \sin^2 2A\right]$

$= \cos 2A \left[1 - \frac{1}{4}(1 - \cos^2 2A\right]$

$= \cos 2A \left[\frac{4 - 1 + \cos^2 2A}{4}\right]$

$= \cos 2A \left[\frac{3 + \cos^2 2A}{4}\right]$

$= \frac{3\cos 2A + \cos^3 2A}{4}$

$= \frac{1}{4}(\cos^3 2A + 3\cos 2A)$

$=$ RHS

D. $\cos^6 A - \sin^6 A = \cos 2A \left(1 - \frac{1}{4}\sin^2 2A\right)$

LHS $= (\cos^2 A)^3 - (\sin^2 A)^3$

$= (\cos^2 A - \sin^2 A)(\cos^4 A + \cos^2 A \cdot \sin^2 A + \sin^4 A)$

$= \cos 2A \left[(\cos^2 A)^2 + (\sin^2 A)^2 + \cos^2 A \cdot \sin^2 A\right]$

$= \cos 2A \left[(\cos^2 A + \sin^2 A)^2 - 2\cos^2 A \cdot \sin^2 A + \cos^2 A \cdot \sin^2 A\right]$

$= \cos 2A \left[1 - \cos^2 A \cdot \sin^2 A\right]$

$= \cos 2A \left[1 - \frac{1}{4}(2\cos A \cdot \sin A)^2\right]$

$= \cos 2A \left[1 - \frac{1}{4}\sin^2 2A\right]$

CamScanner

fb: Important Notes

follow for more...

$= \cos 2A \left[\frac{4-\sin^2 2A}{4}\right]$

$= \cos 2A \left[\frac{4-1+\cos^2 2A}{4}\right]$

$= \cos 2A$

E. $4(\cos^6\theta - \sin^6\theta) = \cos^3 2\theta + 3\cos 2\theta$

LHS $= 4\left[(\cos^2\theta)^3 - (\sin^2\theta)^3\right]$

$= 4\left[(\cos^2\theta - \sin^2\theta)(\cos^4\theta + \cos^2\theta\cdot\sin^2\theta + \sin^4\theta)\right]$

$= 4\cos 2\theta\left[(\cos^2\theta)^2 + (\sin^2\theta)^2 + \cos^2\theta\cdot\sin^2\theta\right]$

$= 4\cos 2\theta\left[(\cos^2\theta + \sin^2\theta)^2 - 2\cos^2\theta\cdot\sin^2\theta + \cos^2\theta\cdot\sin^2\theta\right]$

$= 4\cos 2\theta\left[1 - \cos^2\theta\cdot\sin^2\theta\right]$

$= 4\cos 2\theta\left[1 - \frac{1}{4}(2\cos\theta\cdot\sin\theta)^2\right]$

$= 4\cos 2\theta\left[1 - \frac{1}{4}\sin^2 2\theta\right]$

$= 4\cos 2\theta\left[\frac{4-\sin^2 2\theta}{4}\right]$

$= 4\cos 2\theta\left[\frac{4-1+\cos^2 2\theta}{4}\right]$

$= 4\cos 2\theta\left[\frac{3+\cos^2 2\theta}{4}\right]$

$= \cancel{4}\cos 2\theta\cdot\frac{1}{\cancel{4}}(3+\cos^2 2\theta)$

$= 3\cos 2\theta + \cos^3 2\theta$

$= \cos^3 2\theta + 3\cos 2\theta$

CamScanner

Q. $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x) = 13$

$$LHS = 3[(\sin x - \cos x)^2]^2 + 6(\sin^2 x + 2\sin x \cos x + \cos^2 x) + 4[(\sin^2 x)^3 + (\cos^2 x)^3]$$

$$= 3[\sin^2 x - 2\sin x \cos x + \cos^2 x]^2 + 6(1 + \sin 2x) + 4[(\sin^2 x + \cos^2 x)(\sin^4 x - \sin^2 x \cos^2 x + \cos^4 x)]$$

$$= 3(1 - \sin 2x)^2 + 6(1 + \sin 2x) + 4[(\sin^2 x)^2 + (\cos^2 x)^2 - \sin^2 x \cdot \cos^2 x]$$

$$= 3(1 - \sin 2x)^2 + 6(1 + \sin 2x) + 4[(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x - \sin^2 x \cos^2 x]$$

$$= 3(1 - \sin 2x)^2 + 6(1 + \sin 2x) + 4(1 - 3\sin^2 x \cos^2 x)$$

$$= 3(1 - 2\sin 2x + \sin^2 2x) + 6(1 + \sin 2x) + 4 - 12\sin^2 x \cos^2 x$$

$$= 3 - 6\sin 2x + 3\sin^2 2x + 6 + 6\sin 2x + 4 - 3 \cdot 4 \sin^2 x \cos^2 x$$

~~= 3 (1 −~~

$$= 13 + 3\sin^2 2x - 3 \cdot (2\sin x \cos x)^2$$

$$= 13 + 3\cancel{\sin^2 2x} - 3\cancel{\sin^2 2x}$$

$$= 13$$

fb: Important Notes

follow for more...

6. $\operatorname{cosec} 20^\circ + \sqrt{3} \sec 20^\circ = 4 \cot 40^\circ$

$$LHS = \frac{1}{\sin 20^\circ} + \frac{\sqrt{3}}{\cos 20^\circ}$$

$$= \frac{\cos 20^\circ + \sqrt{3} \sin 20^\circ}{\sin 20^\circ \cdot \cos 20^\circ}$$

$$= \frac{\frac{1}{2} \cos 20^\circ + \frac{\sqrt{3}}{2} \sin 20^\circ}{\frac{\sin 20^\circ \cdot \cos 20^\circ}{2}}$$

$$= \frac{\cos 60^\circ \cos 20^\circ + \sin 60^\circ \sin 20^\circ}{\frac{\sin 20^\circ \cdot \cos 20^\circ}{2}}$$

$$= \frac{2 \cos (60^\circ - 20^\circ)}{\sin 20^\circ \cdot \cos 20^\circ} \times \frac{2}{2}$$

$$= \frac{4 \cos 40^\circ}{\sin 40^\circ}$$

$$= 4 \cot 40^\circ$$

CamScanner

fb: Important Notes
follow for more...

C. $\cos(56°+\theta).\cos(56°-\theta)+\cos(34°+\theta).\cos(34°-\theta)$
$= \cos 2\theta$

LHS $= \cos(56°+\theta).\cos(56°-\theta)+\sin\{90°-(34°+\theta)\}.$
$\sin\{90°-(34°-\theta)\}$

$= \cos(56°+\theta).\cos(56°-\theta)+\sin(90°-34°-\theta).$
$\sin(90°-34°+\theta)$

$= \cos(\underset{A}{56°+\theta}).\cos(\underset{B}{56°-\theta})+\sin(\underset{B}{54°-\theta}).\sin(\underset{A}{54°+\theta})$

$= \cos[56°+\theta-(56°-\theta)]$
$= \cos(\cancel{56°}+\theta-\cancel{56°}+\theta)$
$= \cos 2\theta$

D. $\cos(54°+\theta).\cos(54°-\theta)+\cos(36°+\theta).\cos(36°-\theta)$
$= \cos 2\theta$

LHS $= \cos(54°+\theta).\cos(54°-\theta)+\sin\{90°-(36°+\theta)\}$
$\sin\{90°-(36°-\theta)\}$

$= \cos(54°+\theta).\cos(54°-\theta)+\sin(90°-36°-\theta)$
$\sin(90°-36°+\theta)$

$= \cos(\underset{A}{54°+\theta}).\cos(\underset{B}{54°-\theta})+\sin(\underset{B}{54°-\theta}).\sin(\underset{A}{54°+\theta})$

$= \cos[54°+\theta-(54°-\theta)]$
$= \cos[\cancel{54°}+\theta-\cancel{54°}+\theta]$
$= \cos 2\theta$
$=$

CamScanner

- Prove that: $\sin^4 B + \cos^4 B = 1 - \frac{1}{2}\sin^2 2B$
- Here,

$$\sin^4 B + \cos^4 B = 1 - \frac{1}{2}\sin^2 2B$$

Solve by DhanRaut

Taking L.H.S,

$$= \sin^4 B + \cos^4 B$$
$$= \left(\sin^2 B\right)^2 + \left(\cos^2 B\right)^2$$
$$= \left(\sin^2 B + \cos^2 B\right)^2 - 2\sin^2 B \cdot \cos^2 B$$
$$= 1 - \tfrac{1}{2}(2\sin B \cdot \cos B)^2$$
$$= 1 - \tfrac{1}{2}(\sin 2B)^2$$
$$= 1 - \tfrac{1}{2}\sin^2 2B$$

CS CamScanner

- Prove that: $\frac{\sin\theta+\sin 2\theta}{1+\cos\theta+\cos 2\theta} = \tan\theta$
- Here,

$$\frac{\sin\theta + \sin 2\theta}{1 + \cos\theta + \cos 2\theta} = \tan\theta$$

Solve by DhanRaut

Taking L.H.S,

$$= \frac{\sin\theta+\sin 2\theta}{1+\cos\theta+\cos 2\theta}$$

$$= \frac{\sin\theta+2\sin\theta\cos\theta}{1+\cos\theta+2\cos^2\theta-1}$$

$$= \frac{\sin\theta(1+2\cos\theta)}{\cos\theta(1+2\cos\theta)}$$

$$\therefore \tan\theta = \text{ R.H.S}$$

CamScanner

- Prove that: $\operatorname{cosec} 2\theta - \cot 2\theta = \tan\theta$
- Here, $\operatorname{cosec} 2\theta - \cot 2\theta = \tan\theta$

Taking L.H.S,

$$= \operatorname{cosec} 2\theta - \cot 2\theta$$

$$= \frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta}$$

$$= \frac{1-\cos 2\theta}{\sin 2\theta}$$

$$= \frac{1-\left(2\cos^2\theta-1\right)}{\sin 2\theta}$$

$$= \frac{2-2\cos^2\theta}{\sin 2\theta}$$

$$= \frac{2(1-\cos^2\theta)}{2\sin\theta\cos\theta}$$

$$= \frac{\sin^2\theta}{\sin\theta\cos\theta}$$

$$\therefore \tan\theta = \text{ R.H.S}$$

Solve by DhanRaut

CS CamScanner

- Prove that: $\frac{\sin 2A}{1+\cos 2\text{ A}} = \tan \text{A}$
- Here,

$$\frac{\sin 2A}{1+\cos 2\text{ A}} = \tan \text{A}$$

Solve by DhanRaut

Taking L.H.S,

$= \frac{\sin 2A}{1+\cos 2\text{ A}}$

$= \frac{2\sin A\cos A}{1+2\cos^2 A-1}$

$\therefore \tan A = \text{ R.H.S}$

CamScanner

- $\frac{\cos 40^\circ - \sin 40^\circ}{\cos 40^\circ + \sin 40^\circ} = \tan 5^\circ$
- Here,

$$\frac{\cos 40^\circ - \sin 40^\circ}{\cos 40^\circ + \sin 40^\circ} = \tan 5^\circ$$

Taking L.H.S,

Solve by DhanRaut

$$= \frac{\cos 40^\circ - \cos(90^\circ - 40^\circ)}{\cos 40^\circ + \cos(90^\circ - 40^\circ)}$$

$$= \frac{\cos 40^\circ - \cos 50^\circ}{\cos 40^\circ + \cos 50^\circ}$$

$$= \frac{2\sin\left(\frac{50^\circ - 40^\circ}{2}\right)\sin\left(\frac{50^\circ + 40^\circ}{2}\right)}{2\cos\left(\frac{40^\circ - 50^\circ}{2}\right)\cos\left(\frac{40^\circ - 50^\circ}{2}\right)}$$

$$= \frac{\sin 5^\circ \cdot \sin 45^\circ}{\cos 45^\circ \cdot \cos(-5)^\circ}$$

$$= \frac{\sin 5^\circ}{\cos 5^\circ}\left[\cos(-\theta) = \cos\theta, \sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}\right]$$

$$\therefore \tan 5^\circ = \text{R.H.S}$$

CS CamScanner

- $\frac{\sin A+\sin B}{\sin A-\sin B}=\tan\frac{A+B}{2}\cdot\cot\frac{A-B}{2}$
- Here,

$$\frac{\sin A+\sin B}{\sin A-\sin B}=\tan\frac{A+B}{2}\cdot\cot\frac{A-B}{2}$$

Solve by DhanRaut

Taking L.H.S,

$$=\frac{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{2\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)}$$

$$\therefore \tan\frac{A+B}{2}\cdot\cot\frac{A-B}{2} = \text{R.H.S}$$

CS CamScanner

- Without using the calculator or table, find the value of:

$\sin 100^\circ \cdot \sin 120^\circ \cdot \sin 140^\circ . \sin 160^\circ$

- Here,

$$
\begin{aligned}
&= \sin 100^\circ \cdot \sin 120^\circ \cdot \sin 140^\circ \cdot \sin 160^\circ \\
&= \sin(90^\circ + 10^\circ) \cdot \sin(90^\circ + 30^\circ) \cdot \sin(90^\circ + 50^\circ) \cdot \sin(90^\circ + 70^\circ) \\
&= \cos 10^\circ \cdot \cos 30^\circ \cdot \cos 50^\circ \cdot \cos 70^\circ \\
&= \cos 10^\circ \cdot \tfrac{\sqrt{3}}{2} \cdot \tfrac{2}{2} \cdot \cos 50^\circ \cdot \cos 70^\circ \\
&= \tfrac{\sqrt{3}}{4} \cdot \cos 10^\circ \cdot (2\cos 50^\circ \cdot \cos 70^\circ) \\
&= \tfrac{\sqrt{3}}{4} \cdot \cos 10^\circ \cdot \{\cos(50^\circ + 70^\circ) + \cos(50^\circ - 70^\circ)\} \\
&= \tfrac{\sqrt{3}}{4} \cdot \cos 10^\circ \{\cos 120^\circ + \cos(-20^\circ)\} \\
&= \tfrac{\sqrt{3}}{4} \cdot \tfrac{2}{2} \cdot (\cos 10^\circ \cdot \cos 120^\circ + \cos 10^\circ \cdot \cos 20^\circ)[\cos(-\theta) = \cos(\theta)] \\
&= \tfrac{\sqrt{3}}{8}(2\cos 10^\circ \cdot \cos 120^\circ + 2\cos 10^\circ \cdot \cos 20^\circ) \\
&= \tfrac{\sqrt{3}}{8}\left\{2\cos 10^\circ \cdot -\tfrac{1}{2} + \cos(10^\circ + 20^\circ) + \cos(10^\circ - 20^\circ)\right\} \\
&= \tfrac{\sqrt{3}}{8}(-\cos 10^\circ + \cos 30^\circ + \cos 10^\circ)[\cos(-\theta) = \cos(\theta)] \\
&= \tfrac{\sqrt{3}}{8} \times \tfrac{\sqrt{3}}{2} \\
&= \tfrac{3}{16}
\end{aligned}
$$

Solve by DhanRaut

CamScanner

- $\frac{\sec 4\theta - 1}{\sec 2\theta - 1} = \tan 4\theta \cdot \cot \theta$
- Here,

$$\frac{\sec 4\theta - 1}{\sec 2\theta - 1} = \tan 4\theta \cdot \cot \theta$$

Taking L.H.S,

Solve by DhanRaut

$$= \frac{\sec 4\theta - 1}{\sec 2\theta - 1}$$

$$= \frac{1 - \cos 4\theta}{\cos 4\theta} \times \frac{\cos 2\theta}{1 - \cos 2\theta}$$

$$= \frac{1 - \cos 2.2\theta}{\cos 4\theta} \times \frac{\cos 2\theta}{1 - (2\cos^2\theta - 1)}$$

$$= \frac{1 - (2\cos^2 2\theta - 1)}{\cos 4\theta} \times \frac{\cos 2\theta}{1 - 2\cos^2\theta + 1}$$

$$= \frac{2(1 - \cos^2 2\theta)}{\cos 4\theta} \times \frac{\cos 2\theta}{2(1 - \cos^2\theta)}$$

$$= \frac{\sin^2 2\theta}{\cos 4\theta} \times \frac{\cos 2\theta}{\sin 2\theta}$$

$$= \frac{\sin 2\theta \times 2\sin\theta\cos\theta}{\cos 4\theta} \times \frac{\cos 2\theta}{\sin 2\theta}$$

$$= \frac{2\sin 2\theta\cos 2\theta}{\cos 4\theta} \times \frac{\sin\theta\cos\theta}{\sin 2\theta}$$

$$= \frac{\sin 4\theta}{\cos 4\theta} \times \cot\theta$$

$$\therefore \tan 4\theta \cdot \cot\theta = \text{R.H.S}$$

CamScanner

- $8\left(\sin^6 p + \cos^6 p\right) = 5 + 3\cos 4p$
- Here,

$$8\left(\sin^6 p + \cos^6 p\right) = 5 + 3\cos 4p$$

Taking L.H.S,

Solve by DhanRaut

$$= 8\left\{\left(\sin^2 p\right)^3 + \left(\cos^2 p\right)^3\right\}$$
$$= 8\left(\sin^2 p + \cos^2 p\right)\left(\sin^4 p - \sin^2 p\cos^2 p + \cos^4 p\right)$$
$$= 8\left\{\left(\sin^2 p + \cos^2 p\right)^2 - 2\sin^2 p\cos^2 p - \sin^2 p\cos^2 p\right\}$$
$$= 8\left\{1 - 3\sin^2 p\cos^2 p\right\}$$
$$= 8\left\{1 - \tfrac{3}{4}(2\sin p\cos p)^2\right\}$$
$$= 8\left\{1 - \tfrac{3}{4}(\sin 2p)^2\right\}$$
$$= 8\left\{1 - \tfrac{3}{8}\left(2\sin^2 2p\right)\right\}$$
$$= \tfrac{8}{8}\{8 - 3(1 - \cos 2.2p)\}$$
$$= 8 - 3 + 3\cos 4p$$

$$\therefore 5 + 3\cos 4p = \text{ R.H.S}$$

CamScanner

- Prove that: $\frac{1-\cos\alpha+\sin\alpha}{1+\cos\alpha+\sin\alpha} = \tan\frac{\alpha}{2}$
- Here,

$$\frac{1-\cos\alpha+\sin\alpha}{1+\cos\alpha+\sin\alpha} = \tan\frac{\alpha}{2}$$

Taking L.H.S,

Solve by DhanRaut

$$= \frac{1-\cos\alpha+\sin\alpha}{1+\cos\alpha+\sin\alpha}$$
$$= \frac{1-\cos 2\frac{\alpha}{2}+\sin\alpha}{1+\cos 2\cdot\frac{\alpha}{2}+\sin\alpha}$$
$$= \frac{2\sin^2\frac{\alpha}{2}+\sin 2\cdot\frac{\alpha}{2}}{2\cos^2\frac{\alpha}{2}+\sin 2\cdot\frac{\alpha}{2}}$$
$$= \frac{2\sin^2\frac{\alpha}{2}+2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{2\cos^2\frac{\alpha}{2}+2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}$$
$$= \frac{2\sin\frac{\alpha}{2}\left(\sin\frac{\alpha}{2}+\cos\frac{\alpha}{2}\right)}{2\cos\frac{\alpha}{2}\left(\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}\right)}$$
$$\therefore \tan\frac{\alpha}{2} = \text{R.H.S}$$

CamScanner

- $\tan\theta + 2\tan 2\theta + 4\cot 4\theta = \cot\theta$
- Here,

$\tan\theta + 2\tan 2\theta + 4\cot 4\theta = \cot\theta$

Taking L.H.S,

$$= \tan\theta + 2\tan 2\theta + 4\cot 4\theta$$
$$= \tan\theta + 2\tan 2\theta + \frac{4\cos 4\theta}{\sin 4\theta}$$
$$= \tan\theta + 2\tan 2\theta + \frac{4\cos 4\theta}{2\sin 2\theta\cdot\cos 2\theta}$$
$$= \tan\theta + 2\tan 2\theta + \frac{2\cos 4\theta}{2\sin\theta\cdot\cos\theta\cdot\cos 2\theta}$$
$$= \tan\theta + \frac{2\sin 2\theta}{\cos 2\theta} + \frac{\cos 4\theta}{\sin\theta\cdot\cos\theta\cdot\cos 2\theta}$$
$$= \tan\theta + \frac{2\sin 2\theta\cdot\sin\theta\cdot\cos\theta + \cos 4\theta}{\sin\theta\cdot\cos\theta\cdot\cos 2\theta}$$
$$= \tan\theta + \frac{\sin 2\theta\cdot\sin 2\theta + \cos 2\cdot 2\theta}{\sin\theta\cdot\cos\theta\cdot\cos 2\theta}$$
$$= \tan\theta + \frac{\sin^2 2\theta + 1 - 2\sin^2 2\theta}{\sin\theta\cdot\cos\theta\cdot\cos 2\theta}$$
$$= \tan\theta + \frac{1-\sin^2 2\theta}{\sin\theta\cdot\cos\theta\cdot\cos 2\theta}$$

Solve by DhanRaut

$$= \tan\theta + \frac{\cos^2 2\theta}{\sin\theta\cdot\cos\theta\cdot\cos 2\theta}$$
$$= \tan\theta + \frac{\cos 2\theta}{\sin\theta\cdot\cos\theta} \times \frac{2}{2}$$
$$= \tan\theta + \frac{2\cos 2\theta}{\sin 2\theta}$$
$$= \tan\theta + 2\cot 2\theta$$
$$= \frac{\sin\theta}{\cos\theta} + \frac{2\cos 2\theta}{2\sin\theta\cdot\cos\theta}$$
$$= \frac{\sin^2\theta + \cos 2\theta}{\sin\theta\cos\theta}$$
$$= \frac{\sin^2\theta + 1 - 2\sin^2\theta}{\sin\theta\cos\theta}$$
$$= \frac{1-\sin^2\theta}{\sin\theta\cos\theta}$$
$$= \frac{\cos^2\theta}{\sin\theta\cos\theta}$$
$$= \cot\theta$$

CamScanner

2nd Method)

- If $\frac{1}{\sin A}+\frac{1}{\cos A}=\frac{1}{\sin B}+\frac{1}{\cos B}$, prove that: $\cot\left(\frac{\mathrm{A+B}}{2}\right)=\tan \mathrm{A}\cdot\tan \mathrm{B}$
- Given: $\frac{1}{\sin A}+\frac{1}{\cos A}=\frac{1}{\sin B}+\frac{1}{\cos B}$

To prove: $\cot\left(\frac{\mathrm{A+B}}{2}\right)=\tan\mathrm{A}.\tan\mathrm{B}$

Now,

$$\frac{1}{\sin A}-\frac{1}{\cos B}=\frac{1}{\sin B}-\frac{1}{\cos A}$$

Solve by DhanRaut

or, $\frac{\cos\mathrm{B}-\sin\mathrm{A}}{\sin A\cdot\cos B}=\frac{\cos\mathrm{A}-\sin\mathrm{B}}{\sin B\cdot\cos A}$

or,

$\cos A\cdot\cos B\cdot\sin B-\sin A\cdot\sin B\cdot\cos A=\sin A\cdot\cos A\cdot\cos B-\sin A\cdot\sin B\cdot\cos B$

Dividing both sides by cosA.cosB.cosC, we get,

or, $\frac{\sin B}{\cos C}-\frac{\tan A\cdot\tan B\cdot\cos A}{\cos C}=\frac{\tan A\cdot\cos A}{\cos C}-\frac{\tan A\cdot\tan B\cdot\cos B}{\cos C}$

or, $\sin B-\tan A\cdot\tan B\cdot\cos A=\tan A\cdot\cos A-\tan A\cdot\tan B\cdot\cos B$

or, $\sin B-\tan A\cdot\cos A=\tan A\cdot\tan B(\cos A-\cos B)$

or, $\sin\mathrm{B}-\frac{\sin A\cdot\cos A}{\cos A}=\tan\mathrm{A}\cdot\tan\mathrm{B}(\cos\mathrm{A}-\cos\mathrm{B})$

or, $\frac{\sin\mathrm{B}-\sin\mathrm{A}}{\cos\mathrm{A}-\cos\mathrm{B}}=\tan\mathrm{A}\cdot\tan\mathrm{B}$

or, $\frac{2\sin\left(\frac{\mathrm{B-A}}{2}\right)\cos\left(\frac{\mathrm{B-A}}{2}\right)}{2\sin\left(\frac{\mathrm{B-A}}{2}\right)\sin\left(\frac{\mathrm{B-A}}{2}\right)}=\tan\mathrm{A}\cdot\tan\mathrm{B}$

or, $\cot\left(\frac{B+A}{2}\right)=\tan A\cdot\tan B$

$\therefore\cot\left(\frac{\mathrm{A+B}}{2}\right)=\tan A\cdot\tan\mathrm{B}$

CamScanner