Optimal Monetary Policy with Rationally Inattentive Firms *

Ruoyun Mao †

Grinnell College

March 15, 2022

ABSTRACT

This paper derives optimal monetary policy under a framework where firms are rationally inattentive and monetary policy influences how firms process information. This effect is labeled the "informational effect" of monetary policy. The real effect of monetary policy is jointly determined by its informational effect and direct effect and depends on the marginal cost of attention. With inefficient flexible price allocation, when the marginal cost of attention is small, a monetary policy aimed at stabilizing output gap attracts attention from the private sector and generates inefficient price dispersion; increasing the marginal cost of attention can eliminate the trade-off between the central bank's dual mandates. Comparison between a rule-based policy and a discretionary policy shows welfare gain from commitment. Firms pay extra attention to the policy signal when it is discretionary, which generates more price dispersion and harms welfare.

Keywords: Optimal monetary policy; Incomplete information; Rational inattention; Shannon capacity; Nominal demand management

JEL Classification: E3; E5; D8

*I am grateful to Grey Gordon, Eric Leeper, Kwangyong Park, Daniela Puzzello, Todd Walker, and seminar participants at Midwest Theory Meeting, North American Summer Meeting of the Econometric Society, Washington University in St. Louis Graduate Student Conference, and Indiana University for helpful discussions and comments. All errors are mine.

†Email: maoruoyun@grinnell.edu

1 Introduction

The real effect of monetary policy is determined by the degree of information imperfection. Without nominal rigidity, perfect information leaves monetary policy neutral; on the other hand, imperfect information prevents full adjustment of price and creates a real role for monetary policy. When private agents can choose how to process information in response to monetary policy, the degree of information rigidity is not invariant to monetary policy, adding new implications for the optimal monetary policy.

This paper studies the optimal monetary policy under such an environment where firms only have limited capacity[1] to process information and need to optimally decide what information to observe and how much attention to dedicate to the information, following Sims (2003) and Maćkowiak and Wiederholt (2009).[2] In response to different monetary policies, firms process information differently and set different prices accordingly. This additional informational effect of monetary policy interacts with its well-studied direct effect to determine its effectiveness on the real economy. An aggressive response from the central bank might be as effective as a small response in influencing real output, since the more aggressive response could attract more attention from firms and come out less surprisingly. The real effect of monetary policy is therefore determined by the rate at which monetary policy attracts attention, and it depends on the marginal cost and marginal benefit of the attention. Meanwhile, the level of attention also affects individual firms' processing errors, which transmit into pricing decisions and create price dispersion. With these two effects combined, the marginal cost of paying attention shapes the trade-off between the central bank's dual mandates. When the marginal cost of paying attention is low, monetary policy stabilizing output gap easily attracts attention and creates inefficient price dispersion. When the marginal cost of paying attention is high, the trade-off between the central bank's dual mandates could disappear. Moreover, comparison between a rule-based policy and a discretionary policy confirms welfare gain from commitment. Under a discretionary policy, firms pay extra attention to the policy signal, generating more inefficient price dispersion compared with the case under a rule-based policy. There is a similar conclusion in the standard New Keynesian literature with nominal rigidity, whereas the channel here under rational inattention is very different.

Analysis in this paper is conducted under the framework in Adam (2007), with modifications on the information structure. In the economy, price is flexible, and there are three types of agents:

[1]Modeled by a finite Shannon (1948) capacity.

[2]See Mackowiak et al. (2018) for a detailed survey on rational inattention.

one representative household, a continuum of monopolistically competitive firms, and a monetary authority maximizing the welfare of the representative household. The model is static, and uncertainty comes from two types of fundamental shocks: a labor supply shock and a markup shock,[1] with different implications for welfare. The labor supply shock leads to fluctuations in both the equilibrium output and efficient level of output, while the markup shock only affects the equilibrium output. The presence of the markup shock leads to a trade-off between the central bank's dual mandates just as in the standard New Keynesian literature.[2] The central bank's policy instrument is nominal demand and responds to the above two fundamental shocks. Optimal monetary policy is derived by solving a Ramsey-type problem of the central bank. Both the household and the central bank are fully informed and can process information perfectly, while the firms only have limited information processing capacity and choose what information to process. When firms pay more attention to a certain signal, their pricing decisions respond more to it. Therefore, the real effect of monetary policy could be dampened if the central bank's response attracts a great deal of attention and most of the effect is absorbed by the firms' price adjustment. As for the implications on price dispersion, since firms only have limited information processing capacity, whenever they pay attention, they process information with individual errors that are reflected in their pricing decisions and create inefficient price dispersion.

To understand how endogenous information affects optimal monetary policy, this paper considers three different information structures. First, in order to illustrate how the marginal cost of attention determines the real effect of monetary policy and shapes the trade-off between the central bank's dual mandates, we start from the case with least restrictions and let firms optimally choose the signal. Results show that when the marginal cost of paying attention is high, monetary policy that minimizes the output gap also eliminates the price dispersion. There is consistency between the central bank's two objectives because a low level of attention is paired with a large real effect as well as small price dispersion. In particular, if the marginal cost of paying attention is very high, not only is there no trade-off but the optimal monetary policy can achieve the first best result by not reacting to the markup shock at all because firms do not pay attention to it and do not respond to it anyway. Since both the central bank and firms do not respond to the markup shock, output does not respond to it either, and the response is efficient. The above case has been studied in Paciello and Wiederholt (2013),[3] and they conclude that complete price sta-

[1] Adam (2007) refers to the labor supply shock as the supply shock and the markup shock as the demand shock.
[2] See Galí (2015).
[3] See section 6.2 in Paciello and Wiederholt (2013).

bilization in response to the markup shock is also optimal, if information is endogenous and the cost function of attention satisfies certain restrictions, contradicting the case when information is exogenous. Here we revisit this case since the mechanism driving their conclusion is pervasive under endogenous information and has not been discussed much in the literature. Even when the cost function does not satisfy their restriction and their conclusion does not hold anymore, the above explained mechanism still exists and alters the optimal monetary policy under a traditional exogenous information environment, as shown in the following two cases.

The next two cases derive the optimal monetary policy when the marginal cost of paying attention is low and there is indeed trade-off between the central bank's two objectives. First, we consider a case where firms understand the policy rule and process independent signals. Although processing independent signals is not optimal for firms, this paper shows that when firms process independent signals, the welfare loss at the optimal monetary policy is smaller than the case where firms process the optimal signal. This is because only attention to the markup shock generates inefficient output gap fluctuation and therefore the trade-off, not the attention to the labor supply shock. When firms process independent signals, the level of attention paid to one shock can affect the marginal cost of paying attention to the other shock, and monetary policy can allocate attention between these two types of shocks by its responses. Combined with the above result that the existence of markup shock generates trade-off, under some cases it might be optimal for the central bank to adjust the level of attention to the labor supply shock so that the marginal cost of attention to the markup shock is high enough to eliminate the trade-off. However, this strategy does not exist when firms process the one-dimensional optimal signal since in that case there is no difference between the attention to the labor supply shock and the attention to the markup shock.

We then study the optimal monetary policy when firms do not understand the monetary policy rule and perceive the policy as an additional shock. Under current setup, we could interpret the comparison between this case and the previous case as a reinterpretation of rule versus discretion. If firms know how monetary policy depends on shocks, it is *as if* the central bank commits to its announced rule. On the contrary, when firms do not understand the rule and perceive the policy instrument as a separate shock, it is *as if* the central bank acts based on discretion every period, and the policy is drawn as a shock. The comparison shows that while under discretion the monetary policy strengthens its real effect in stabilizing output, its ability to decrease price dispersion is largely weakened. Because when firms believe the central bank acts upon discretion, they use their limited capacity to process one additional signal about the policy, which generates extra price dispersion and harms welfare.

In summary, this paper studies the optimal monetary policy and the nature of the trade-off faced by the central bank when the information structure is endogenous. The results show that different information settings result in different characterizations of optimal monetary policy but that they share a common mechanism to understand the trade-off between the central bank's dual mandates. Understanding this mechanism is the goal of this paper, while discussing which information structure is more realistic requires more careful empirical exploration and is beyond the discussion of this paper. In particular, this paper shows that it is vitally important to understand the marginal cost of paying attention in an economy since the level of marginal cost determines the real effect of monetary policy and the trade-off between the central bank's dual mandates.

This paper is mostly related to Adam (2007) and Paciello and Wiederholt (2013). Adam (2007) also studies the optimal monetary policy when firms have limited attention, but in his paper, information is exogenous, and the degree of information rigidity does not respond to monetary policy. This paper is mainly developed from Adam (2007); however the information structure here is endogenous. Paciello and Wiederholt (2013) also extend Adam (2007) and study the optimal monetary policy when the information structure is endogenous. They show that under endogenous information setup, complete price stabilization is also optimal in response to shocks that cause inefficient fluctuations, which is different from the result under an exogenous information structure where complete price stabilization is optimal only in response to shock that cause efficient fluctuations. This paper differs from Paciello and Wiederholt (2013) in that they focus on the conclusion that when the marginal cost of attention is high, the trade-off between reducing price dispersion and output gap fluctuation disappears, making complete pricing stabilization optimal. This paper concentrates more on how the marginal cost of attention and the informational effect of monetary policy influence the nature of the trade-off faced by the central bank, as well as characterize the optimal monetary policy when the marginal cost of paying attention is low and there is indeed a trade-off. In this regard, this paper complements Paciello and Wiederholt (2013)'s result. Moreover, this paper further assesses the value of commitment under rational inattention, adding one more argument for rule-based policy.

This paper is also related to the broader literature studying optimal monetary policy with information frictions. Most of them are under an exogenous information structure, although the sources of incomplete information are different from each other. The first category studies optimal monetary policy when the central bank's information is incomplete such as Aoki (2003), Orphanides (2003), and Svensson and Woodford (2003). The second category studies optimal monetary policy when private agents have incomplete information, and this paper is also in this category. For example, Ball et al. (2005) study optimal monetary policy when information is sticky and firms can only update their information infrequently with an exogenous probability. Angeletos and La'O (2011) study the optimal monetary policy under an environment where the information friction can lead to real rigidity. Berkelmans (2011) studies optimal monetary policy when there are multiple shocks and private agents cannot tell them apart. His paper concludes that the optimal policy's response to one shock depends on the existence of other shocks, contrary to a typical linear-quadratic framework. Our paper also finds such results, but the channel is different. In this paper, the central bank's response to one shock affects its response to others via changing the marginal cost of paying attention. Baeriswyl and Cornand (2010) combine the economy in Adam (2007) with sticky information à la Mankiw and Reis (2002) to study optimal monetary policy when the central bank explicitly takes the policy's signaling effects into consideration. They show that the central bank needs to distort its policy response to optimally control the information it conveys. The central bank has no incentive to provide more information about the markup shocks because those shocks cannot by neutralized. This last result is similar to the result in this paper despite different information environments: the central bank has an incentive to distract firms from paying attention to the markup shock.

Additionally, this paper also falls into the fast-growing strand of literature on rational inattention. The entire body of literature is large, and here we only focus on the discussions about monetary policy and rational inattention.[1] Maćkowiak and Wiederholt (2009) study how rational inattention affects firms' pricing setting with strategic complementarity and argue that the small and slow response of prices to aggregate conditions can be rationalized by firms paying more attention to the more volatile idiosyncratic shocks. With similar mechanisms, Paciello (2012) considers a monetary policy rule with feedback on inflation and output and shows how the feedback component of monetary policy affects the attention allocation decision by firms. Inflation adjusts faster to aggregate technology shocks than to monetary policy shocks since firms pay more attention to the more volatile aggregate technology shock. Afrouzi (2017) also studies firms' price setting with rational inattention but under the assumption of oligopolistic competition instead of

[1] For early applications of rational inattention to consumption decision, see Luo (2008) and Tutino (2013). For applications of rational inattention to portfolio choice, see Mondria (2010).

the widely used assumption of monopolistic competition. The paper finds that limited competition exaggerates firms' response of output and lowers firms' response of inflation. This is because with limited competition, firms tend to track their direct competitors' beliefs instead of monetary policy, amplifying the real effect of monetary policy. Sims (2010)'s handbook chapter also discusses the applications of rational inattention and implications for monetary policy and provides an overview. All these papers shed light on how the private sector's information processing influences the effect of monetary policy, while this paper focuses on the normative analysis.

2 The Model Setup

Following Adam (2007), the economy is static and price is flexible. There exists a representative household, a continuum of monopolistically competitive firms, and a central bank. The central bank is benevolent and maximizes the utility of the representative agent via controlling the nominal demand. Diverging from Adam (2007) is the information structure, which will be discussed later in detail.

2.1 Representative household

The representative household processes information perfectly and chooses consumption(Y) and labor supply(L) to maximize utility:

$$U(Y) - \nu V(L)$$

$$\text{s.t. } PY + T = WL + \Pi,$$

where W is the nominal wage and Π is the monopoly profits from firms. T is a nominal lump-sum tax collected by the government. The composite consumption good Y is produced with the Dixit-Stiglitz aggregator:

$$Y = [\int_0^1 (Y_i)^{(\theta-1)/\theta} di]^{\theta/(\theta-1)},$$

where θ measures the elasticity of substitution among goods and Y_i is the good produced by an individual firm i. P is the price index for the aggregate consumption Y that solves $PY = \int_0^1 P_i Y_i di$. The elasticity of substitution θ is a random variable with $E(\theta) = \bar{\theta}$ and induces variations in the desired markup of firms. The weight on disutility from labor $\nu > 0$ is a stochastic labor supply shifter with $E[\nu] = 1$. It causes fluctuations in the efficient level of output. The utility function satisfies normal properties: $U' > 0$, $U'' < 0$, $\lim_{Y\to\infty} U'(Y) = 0$, $V' > 0$, $V'' > 0$, and $V'(0) < U'(0)$. The representative household's optimality condition is then given by

$$W = \frac{\nu V'(L)}{U'(Y)} P. \tag{2.1}$$

2.2 Firms

The production sector consists of a continuum of monopolistically competitive firms indexed by $i \in (0, 1)$. Unlike the representative household, firms only have *finite* information processing capacity. Each firm i produces an intermediate good Y_i with labor input L_i according to a linear production function:

$$Y_i = L_i \tag{2.2}$$

and chooses price P_i to maximize the expected profit conditional on its own information set I_i:

$$\max_{P_i} E[(1+\tau)P_i Y_i(P_i) - W Y_i(P_i)|I_i]$$
$$\text{s.t. } Y_i(P_i) = \left(\frac{P_i}{P}\right)^{-\theta} Y,$$

where τ is an output subsidy provided by the government to correct distortion from monopolistic competition and is financed by the lump-sum tax T collected from the household. The government budget constraint is $T = \tau P Y$. Solving an individual firm's profit maximization problem yields the following FOC:

$$E\left[(1+\tau)(1-\theta)\left(\frac{P_i}{P}\right)^{-\theta} + \theta W \left(\frac{P_i}{P}\right)^{-\theta-1} |I_i\right] = 0. \tag{2.3}$$

Combining equation (2.3) with the household's optimality condition (2.1) as well as the production function (2.2) and linearizing it around the deterministic steady state delivers the firms' optimal pricing rule:[1]

$$p_i = E[p + \xi(y - y^*) + u|I_i], \tag{2.4}$$

where the lowercase variables denote percentage deviations from their deterministic steady states.

The pricing rule says given some information set I_i, each firm i sets price p_i optimally according to the aggregate price p, the output gap $y - y^*$, and the markup shock u, where the fluctuation in the efficient level of output y^* is determined by the stochastic labor supply shifter ν and the

[1]See Appendix A.1 in Adam (2007) for detailed derivation.

markup shock u is a function of the demand shock θ: $u \sim -(\theta - \bar{\theta})$. Firms charge a higher markup ($u > 0$) whenever the demand becomes less inelastic. Further, ξ measures how sensitive the optimal price is to the output gap.[1] Since the nominal aggregate demand q can be expressed as $q = p + y$, the firm i's price-setting rule can be rearranged to

$$p_i = E[(1-\xi)p + \xi q - \xi y^* + u|I_i]. \tag{2.5}$$

For simplicity, we assume that the labor supply shock and the markup shock are independent and follow normal distributions[2]: $y^* \sim \mathcal{N}(0, \sigma^2_{y^*})$ and $u \sim \mathcal{N}(0, \sigma^2_u)$. We only focus on these two shocks because they each represent a type of shock: the labor supply shock represents the type of shock that affects both the equilibrium output and the efficient level of output symmetrically,[3] whereas the markup shock represents the type of shock that only affects the equilibrium level of output. Conclusions drawn on each shock later in this paper can be extended to the corresponding type of shock. Also, we further assume that $\xi = 1$ [4] and the pricing rule (2.5) becomes $p_i = E[q - y^* + u|I_i]$.

We now focus on the firms' information sets. Since firms only have a finite information processing capacity, they optimally choose their information sets. Information processing capacity or attention is quantified using the concept of *Shannon mutual information*. Mutual information between a random vector Z and an information set I_i is defined as $\mathcal{I}(Z; I_i) = H(Z) - H(Z|I_i)$, where $H(\cdot)$ is the entropy of a random vector[5] and measures how uncertain a random variable is. The mutual information thus can be understood as the uncertainty about a random vector reduced by the observed information set. Paying more attention means a larger reduction in uncertainty and is equivalent to receiving more precise signals.[6] To maximize profits, firms want to process information more precisely, but their ability to do so is limited either because they only have a limited amount of processing capacity or because there is a cost to processing the information. Therefore, firms optimally decide what signals to pay attention to and the amount of attention to dedicate by

[1] $\xi = -U''(\bar{Y})\bar{Y}/U'(\bar{Y}) + V''(\bar{Y})\bar{Y}/V'(\bar{Y})$, where $\bar{Y}$ is the deterministic steady state of Y.

[2] In Adam (2007), y^* is referred to as a *supply shock*, and u is referred to as a *real demand shock*.

[3] The role of the labor supply shock is similar to the technology shock in Paciello and Wiederholt (2013).

[4] This assumption allows for analytical exposition when information is endogenous and there is trade-off between the central bank's dual mandates, but it assumes away the strategic actions among firms and higher-order effects. This paper focuses on the interaction between the central bank and the private sector. The main mechanism discussed in this paper still exists if this assumption is relaxed, and relaxing this assumption interacts with the current mechanism and is of great interest for future research.

[5] For instance, the entropy of a random vector $X \sim \mathcal{N}(0, \Omega)$ with length T is $H(X) = \frac{1}{2}\log_2((2\pi e)^T \det \Omega)$.

[6] See Mackowiak et al. (2018) for a review on rational inattention. Also see section 2 of Maćkowiak and Wiederholt (2009) for a detailed introduction on quantifying information flow.

solving the following optimization problem:

$$
\begin{aligned}
&\min_{g(s_i|z)} E[(p_i - p^*)^2] + f(k) \\
\text{subject to} \quad & p^* = q - y^* + u \\
& p_i = E(p^*|s_i) \\
& k = \mathcal{I}(Z; s_i) = H(Z) - H(Z|s_i) \\
& Z = \begin{pmatrix} y^* \\ u \end{pmatrix}
\end{aligned}, \tag{2.6}
$$

where $f(\cdot)$ is the cost function for attention with $f' > 0$ and $f'' > 0$[1] and the first term of the objective is the negative of the second-order approximation to the firms' profit maximization problem. p^* is the profit-maximizing price to set if firms had processed information perfectly. To maximize profit, the firms choose the signal(s) to observe, described by a conditional distribution $g(s_i|z)$ where z is the realization of Z. Intuitively, it includes both the form and the precision of the signal(s). Specifically, later in section 3, we compare the optimal monetary responses when firms process one signal as a linear combination of shocks and when firms process independent signals on both shocks.

2.3 CENTRAL BANK

The central bank also processes information perfectly like the representative household. It is benevolent and maximizes the representative household's utility by adjusting the nominal demand q as a linear combination of the fundamental shocks. Its objective is a second-order approximation to the representative household's utility function, and it takes the private sector's

[1] Examples of cost functions used in the rational inattention literature are $f(k) = \mu k$ and $f(k) = \mu 2^{2k}$, where μ is the parameter governing the magnitude of cost.

decisions as given. The central bank's problem is given by

$$
\begin{aligned}
&\min_{\phi_{y^*},\phi_u} E[(y-y^*)^2 + \gamma \int_0^1 (p_i - p)^2 di] \\
\text{subject to} \quad & q = y + p \\
& p = \int_0^1 p_i di \\
& p_i = E[q - y^* + u | I_i] \\
& q = \phi_{y^*} y^* + \phi_u u
\end{aligned} \qquad . \tag{2.7}
$$

The central bank minimizes the weighted loss from fluctuations in output gap and price dispersion. The weight on price dispersion γ is given by the mean of the price elasticity of demand $\bar{\theta}$.

2.4 Timing of events

The sequence of events taking place in the economy is as follows: (1) the central bank determines the monetary policy rule, (2) supply and demand shocks realize, (3) the central bank sets the policy according to its rule, (4) the firms process information and then set prices, and (5) the consumers demand products and productions take place.

2.5 Full information benchmark

Under full information, all firms process information perfectly and set prices in the same way: $p_i = p^* = q - y^* + u$. Since there is no nominal rigidity, output under full information is also the flexible price allocation. The corresponding full information equilibrium is as follows:

$$p^{FI} = q - y^* + u \tag{2.8}$$

$$y^{FI} = y^* - u. \tag{2.9}$$

We are back in the following two conventional results: (1) without nominal or information rigidity, there is no real effect from monetary policy, and (2) due to the markup shock, the flexible price allocation is not efficient. Welfare loss under flexible price is $E(y^{FI} - y^*)^2 = \sigma_u^2$, and it is purely from output gap fluctuations induced by the markup shock. For this reason, this type of shock is also referred to as the shock that causes inefficient fluctuations.

3 Endogenous Information and Optimal Monetary Policy

In this section, we derive the optimal monetary policy under different assumptions of the information structure. Studying which information structure is closer to reality is beyond the scope of this paper; instead the goal is to derive the optimal monetary policy and provide comparisons across different assumptions. We first present the optimal monetary policy when firms choose the form of the signal optimally as studied in Paciello and Wiederholt (2013).[1] The environment here in this paper is simpler, which helps to make the mechanism more transparent. The detailed discussion about the mechanism illustrates how the real effect of a monetary policy hinges on the marginal cost of attention and how the marginal cost of attention affects the trade-off between price stabilization and output gap stabilization faced by the central bank. We then derive the optimal monetary policy when firms process independent signals about the two fundamental shocks. Although processing independent signals is not optimal from the firms' perspective, the result shows that the welfare loss at the optimal monetary policy is smaller than the welfare loss at the optimal monetary policy when firms process the optimal signal. Last, we discuss the optimal monetary policy when firms do not understand the monetary policy rule and perceive the policy as an additional shock. Comparing the optimal policy under the last two scenarios adds new insight to the rule versus discretion debate. If firms know how monetary policy depends on shocks, it is *as if* the central bank announces a rule and commits to this rule. Similarly, when firms do not understand the rule and perceive the nominal demand as a separate shock, it is *as if* the central bank is discretionary. The comparison indicates welfare gain from commitment with rationally inattentive firms when the central bank places more and more weight on price dispersion.

3.1 Firms choose the optimal signal

In this subsection, we consider the case where firms optimally choose the form of the signal. Under a LQG framework with i.i.d. shocks, the optimal signal is a one-dimensional signal as a linear combination of the fundamentals.[2] With the current setup, it is a noisy signal on the optimal

[1] See section 6.2 in Paciello and Wiederholt (2013). The main body of the paper studies the case where firms process independent signals by turning only one shock on at one time, and the optimal signal case is presented later in section 6 as a robustness check.

[2] See Afrouzi (2017), Maćkowiak et al. (2018), Steiner et al. (2017), and Miao et al. (2019) for more detailed discussions.

price under full information: $s_i = p^* + e_i$,[1] where e_i is the processing error with variance σ_e^2. The optimization problem (2.6) then becomes

$$
\begin{aligned}
& \min_k E[(p_i - p^*)^2] + f(k) \\
\text{subject to} \quad & p^* = q - y^* + u \\
& p_i = (1 - 2^{-2k})(q - y^* + u + e_i) \\
& k = \frac{1}{2} \log 2 \frac{\sigma_{p^*}^2 + \sigma_e^2}{\sigma_e^2}
\end{aligned}
\quad . \tag{3.1}
$$

The solution to this problem is straightforward. When the full information optimal price p^* is more volatile, firms pay more attention to the signal. Since the optimal price p^* depends on monetary policy, the central bank's responses affect both the form and the precision of the signal chosen by the firms. This is the informational effect of the monetary policy.

Specifically, given some monetary policy, the equilibrium output is characterized below as

$$
y = (2^{-2k^*}(\phi_{y^*} - 1) + 1)y^* + (2^{-2k^*}(\phi_u + 1) - 1)u, \tag{3.2}
$$

where k^* is the optimal level of attention and also depends on the central bank's responses ϕ_{y^*} and ϕ_u. There are a few direct observations from equation (3.2). On one extreme, if firms could pay unlimited attention, equilibrium output becomes $y = y^* - u$. With unlimited attention, the firms adjust prices fully to absorb all the effects of the monetary policy, and there is no real effect from the monetary policy. On the other extreme, if firms don't pay attention at all, $y = \phi_{y^*} y^* + \phi_u u = q$. Price does not respond at all, and the monetary policy transmits into the real economy on a one-to-one scale. The real effect of the monetary policy is determined by surprises. When firms pay more attention, price adjusts more and absorbs a larger fraction of the nominal demand change. When the above results are combined with firms endogenously choosing information, the real effect of monetary policy is not monotonic in its response. Depending on how the firms dedicate their attention, larger responses from the central bank could end up with smaller real effects if those policies let firms pay more attention.

While designing its optimal policy, the central bank internalizes both its direct and informational effect and chooses the response coefficients to maximize the utility of the representative

[1] Miao et al. (2019) shows that when the levels of persistence of the underlying shocks are different, the optimal signal cannot be normalized to a noisy signal on the full information optimal price anymore. Here the model is static and $\rho_{y^*} = \rho_u = 0$.

household. We approach the optimal monetary policy by first focusing on the loss from price dispersion only and the loss from output stabilization only respectively.

Proposition 1. *When firms optimally choose the signal, more attention to the optimal one-dimensional signal always increases price dispersion.*

Proof. See Appendix A. □

Without nominal rigidity, price dispersion only comes from dispersed information. Since firms' information processing capacity is finite, they process information with individual processing errors. When firms pay more attention to the signal, their individual prices respond more to that signal, including their processing errors, which ends up with larger price dispersion.[1] Together with previous discussion about how the optimal level of attention is determined by the volatility of p^*, the monetary policy that fully stabilizes optimal price p^* ($\phi_{y^*} = 1$ and $\phi_u = -1$) could achieve zero attention and completely eliminate price dispersion. As monetary policy deviates further away from the policy that fully stabilizes p^*, p^* becomes more volatile. As a result, the optimal level of attention k^* stays at zero first and then increases, as shown in Figure 1. Where the zero attention threshold is depends on the marginal cost of paying attention as well as the volatility of p^*, which is jointly determined by the uncertainty from fundamentals and monetary policy. Higher levels of marginal cost of paying attention permit a larger range of monetary policy responses such that firms pay zero attention.

Note that the policy that fully stabilizes p^* and eliminates price dispersion ($\phi_{y^*} = 1$ and $\phi_u = -1$) also replicates the flexible price allocation, which is not efficient due to the existence of markup shock.[2] This is where the trade-off between stabilizing output and price dispersion stems from. If the markup shock is shut down, the central bank can simply replicate the efficient flexible price allocation by completely stabilizing the price level, which at the same time delivers zero attention and eliminates inefficient price dispersion.[3] Based on the intuition above, this result can be extended to all shocks that cause efficient fluctuations, and the conclusion that the response to shocks that cause efficient fluctuations does not face a trade-off is a standard one in the New Keynesian literature as long as the central bank is able to replicate the flexible price allocation by

[1] Mathematically, the loss from price dispersion is given by $Loss_p = \frac{(1-2^{-2k^*})f'(k^*)}{2\ln 2}$ and $\frac{\partial Loss_p}{\partial k^*} > 0$ for all $k^* > 0$. See Appendix A for more details.

[2] The welfare loss from the output gap at flexible price allocation is σ_u^2.

[3] If there is only labor supply shock, the optimal response is $\phi_{y^*} = 1$, and under this response, $p_i = p = 0$ and $y = y^*$

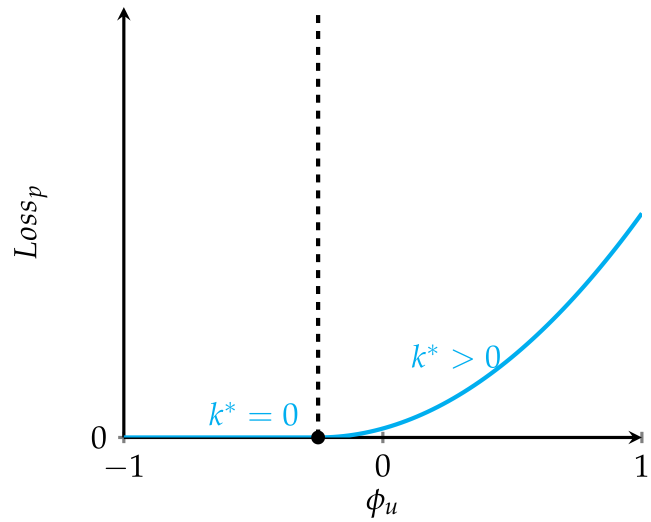


Figure 1: Loss from price dispersion as a function of ϕ_u

Note: The graph plots how loss from price dispersion changes as the response from markup shock changes, with a fixed ϕ_{y^*}. The plot of loss from price dispersion as a function of ϕ_{y^*} is similar. In that case, we consider how the loss increases as ϕ_{y^*} decreases from 1, the response that completely stabilizes the optimal price p^*.

completely stabilizing the price level.[1]

To understand the role of markup shock, we now fix the response to the labor supply shock at the optimal level ($\phi_{y^*} = 1$) and focus on the loss from output gap fluctuation caused by the markup shock.[2]

Proposition 2. *When firms optimally choose the signal, fixing the response to the labor supply shock at the optimal level ($\phi_{y^*} = 1$), the loss from output gap fluctuation is not monotonic in response to the markup shock. Specifically, if the corresponding optimal level of attention from firms k^* is positive, $\frac{\partial Loss_y}{\partial \phi_u} \leq 0$ when $\frac{f''(k^*)}{f'(k^*)} \geq 2\ln 2$ and $\frac{\partial Loss_y}{\partial \phi_u} \geq 0$ when $\frac{f''(k^*)}{f'(k^*)} \leq 2\ln 2$*

Proof. See Appendix A. □

This result is directly brought out by the endogenous information structure and is the main driving force for the conclusions in Paciello and Wiederholt (2013). While they focus on the conclusion, here we revisit the result to make the mechanism more transparent in a more simplified framework and focus on the mechanism itself.

[1]Section 3.2 presents a case where even the response to the shocks that cause efficient fluctuations does not face the trade-off itself; it is not optimal to set the response at the efficient level if paying attention to one shock changes the marginal cost of paying attention to another.

[2]We concentrate on the cases when $\phi_u > -1$, since when $\phi_u < -1$, it generates price dispersion, pushes the equilibrium output even further away from the efficient level, and cannot be optimal.

To understand the mechanism, we take a closer look at the real effect of ϕ_u:

$$E(y-y^*)^2 = \underbrace{2^{-4k^*}(\phi_{y^*}-1)^2\sigma_{y^*}^2}_{\text{fixed when } \phi_{y^*}=1} + [\underbrace{2^{-2k^*}}_{\text{info. eff.}}\underbrace{(\phi_u+1)}_{\text{direct eff.}}-1]^2\sigma_u^2. \tag{3.3}$$

As ϕ_u increases from -1, on the one hand, it helps to stabilize output as it directly counteracts a positive markup shock. On the other hand, it also attracts more attention from firms and makes individual firms' prices respond more. The effect of monetary policy is absorbed more by price adjustment instead, which works against the first direct channel. As plotted in Figure 2, when $\frac{f''(k^*)}{f'(k^*)} > 2\ln 2$, the first channel still dominates, and monetary policy that is less contractionary helps to stabilize output. Whereas when $\frac{f''(k^*)}{f'(k^*)} < 2\ln 2$, the second channel dominates. Although a less contractionary monetary policy reduces the fall in output in response to a positive markup shock, it attracts more attention from the firms, and price increases by more in response to the markup shock, which eventually causes output to decrease by more.[1]

The intuition for why condition $\frac{f''(k^*)}{f'(k^*)} < 2\ln 2$ is required to let the second effect dominate is that the marginal cost of attention must drop fast enough while the level of marginal cost itself is not too low so that the optimal level of attention k^* can increase by a large amount in response to a small change in policy response ϕ_u.

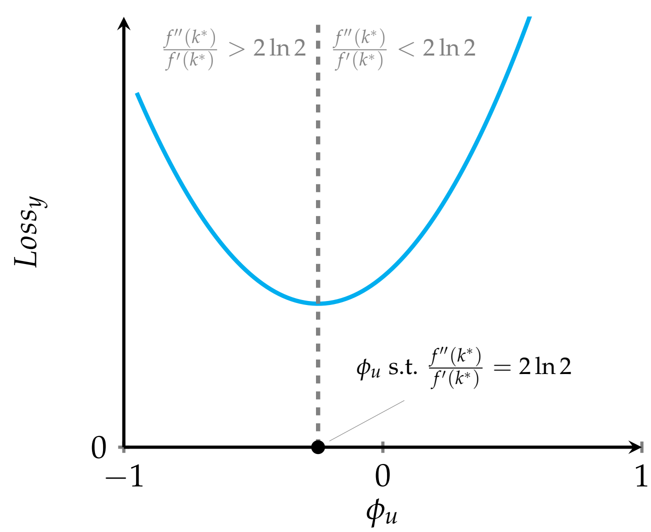


Figure 2: Loss from output gap as a function of ϕ_u ($k^* > 0$)

[1]Mathematically, as ϕ_u increases from -1, the direct eff. part in Equation (3.3) increases whereas the info. eff. part decreases. When $\frac{f''(k^*)}{f'(k^*)} \leq 2\ln 2$, the info. eff. part decreases more than the increase in the direct eff. part, driving the entire term away from 1 and vice versa.

To conclude, with firms endogenously deciding which pieces of information to pay attention to and how much attention to dedicate, the loss from output gap fluctuation is not monotonic in monetary policy response. The turning point depends on both the level and the slope of the marginal cost of paying attention. Combining Proposition 1 and Proposition 2, we can immediately get the following proposition.

Proposition 3. *When firms optimally choose the signal,*

1. *if $\sigma_u^2 \leq \frac{f'(0)}{2\ln(2)}$, the optimal responses from the central bank are given by*

$$\phi_{y^*} = 1, \quad \phi_u = 0;$$

2. *if $\sigma_u^2 > \frac{f'(0)}{2\ln(2)}$ and $\frac{f''(k)}{f'(k)} \leq 2\ln 2$ for all positive attention levels, monetary policy that completely eliminates price dispersion also minimizes output gap. The optimal responses from the central bank are given by*

$$\phi_{y^*} = 1, \quad \phi_u = -1 + \sqrt{\frac{f'(0)}{2\ln(2)\sigma_u^2}};$$

3. *if $\sigma_u^2 > \frac{f'(0)}{2\ln(2)}$ and $\frac{f''(k)}{f'(k)} \leq 2\ln 2$ is violated at low attention levels,[1] there is a trade-off between stabilizing the output gap and price dispersion. The optimal responses depend on the form of the cost function.[2]*

Proof. See Appendix A. □

When $\sigma_u^2 \leq \frac{f'(0)}{2\ln 2}$, by setting $\phi_{y^*} = 1$ and $\phi_u = 0$, the central bank can achieve the first best result, fully closing the output gap and eliminating price dispersion. Since fluctuation in markup shock is not efficient, if in equilibrium, neither the nominal demand nor individual firms' pricing responds to it, there is no inefficient fluctuation at all. This is only the case when the marginal cost of paying attention is significant relative to the uncertainty of the markup shock. In that case, firms do not pay attention at all, although the inaction policy leads to a relatively volatile p^*.[3] The main takeaway is that with the presence of the markup shock, increasing the overall cost of information could be welfare improving.

[1] A quadratic cost function $f(k) = k^2$ is an example.

[2] Proposition 3 is a special case of Paciello and Wiederholt (2013), with $\phi_c = \phi_\lambda = 1, \omega = 2$ in their environment. See section 6.2 in Paciello and Wiederholt (2013).

[3] This result itself does not hinge on whether the condition $\frac{f''(k)}{f'(k)} \leq 2\ln 2$ is satisfied, but $\sigma_u^2 \leq \frac{f'(0)}{2\ln 2}$ implies that $\frac{f''(k)}{f'(k)} \leq 2\ln 2$ is more likely to hold.

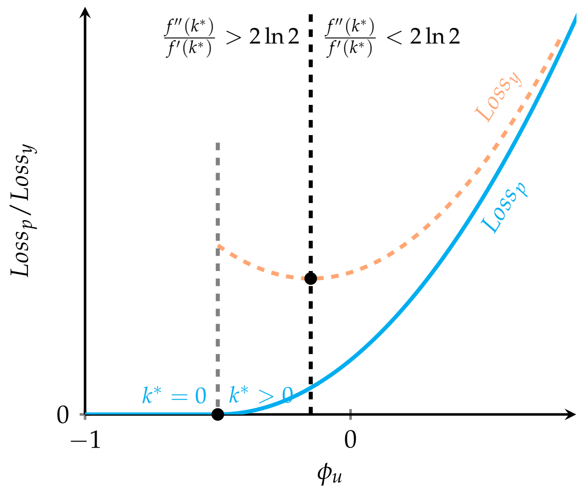
When $\sigma_u^2 > \frac{f'(0)}{2\ln 2}$, the first best result cannot be achieved anymore. If $\frac{f''(k)}{f'(k)} \leq 2\ln 2$ for all positive k, the optimal response is characterized in Point 2 of Proposition 3. Under the responses, firms do not pay attention to the optimal signal, and there is no price dispersion. At the same time, since $\frac{f''(k)}{f'(k)} \leq 2\ln 2$, from Proposition 2, ϕ_u needs to be more contractionary to stabilize the output gap. Therefore, the central bank should make its response more contractionary until the level of attention just becomes zero.[1] At this policy, the loss from the output gap fluctuation is minimized at $\phi_u^2\sigma_u^2$. If $\frac{f''(k)}{f'(k)} > 2\ln 2$, the trade-off presents, and we can no longer derive the optimal responses explicitly because they depend on the form of the cost function.

Figure 3 presents the loss from price dispersion and output gap as a function of ϕ_u by putting Figure 1 and Figure 2 together. Graphically, whether there is a trade-off depends on whether the turning point of the loss from output gap could appear at zero attention, which is further determined by whether $\frac{f''(k)}{f'(k)} \leq 2\ln 2$ is satisfied. When it is not satisfied, as shown in Panel (a), a less contractionary monetary policy stabilizes the output gap but attracts more attention from the private sector, creating inefficient price dispersion. When the constraint is satisfied, as shown in Panel (b), the less contractionary monetary policy not only attracts more attention but also induces an excessive increase in price in response to a positive markup shock, and it eventually decreases output by more. So there is no trade-off between these two objectives. The consistency comes from the fact that with endogenous information, the real effect of monetary policy is small when attention is large, which increases the losses from output fluctuation and price dispersion simultaneously. To achieve this consistency, the marginal cost of paying attention must be large enough.

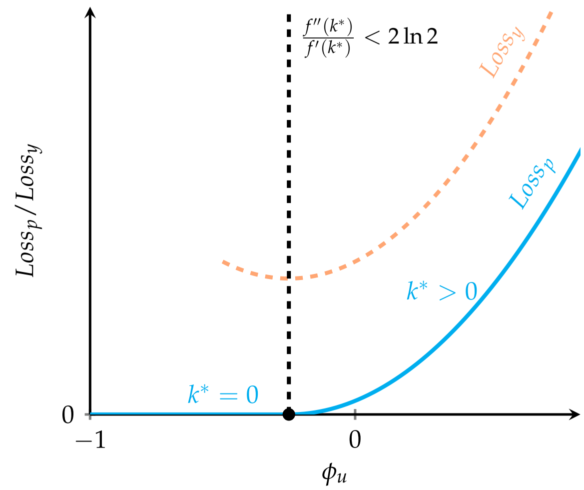
Due to the above discussed mechanism, Paciello and Wiederholt (2013) sharply conclude that complete price stabilization in response to the markup shock is optimal when $\frac{f''(k)}{f'(k)} \leq 2\ln 2$. They further show that the conclusion and the policy characterized in Proposition 3 are robust under several specifications including (1) when firms observe independent signals; (2) when firms observe a linear combination of the fundamentals (the optimal signal is just one of such combinations); (3) when the noises are correlated. The robustness of their result is not surprising since when condition $\frac{f''(k)}{f'(k)} \leq 2\ln 2$ is satisfied, the optimal monetary policy ends up with zero attention from the firms, making the signal structure irrelevant.

In the following subsection, we complement their results by showing that even when the con-

[1]When the attention level k^* becomes zero, a more contractionary policy creates more output fluctuation. The proposition holds only when $k^* > 0$.



(a). $\frac{f''(k^*)}{f'(k^*)} \geq 2\ln 2$



(b). $\frac{f''(k^*)}{f'(k^*)} \leq 2\ln 2$

Figure 3: Dual mandates and trade-off

dition $\frac{f''(k)}{f'(k)} \leq 2\ln 2$ is violated, the interaction between endogenous information and the real effect of monetary policy still plays a vital role and brings in new insights about optimal monetary policy and central bank communication. Specifically, in the following subsection, we assume that firms process independent signals on fundamentals. With firms processing independent signals, monetary policy can now direct attention to some specific shock by manipulating the marginal cost of paying attention to that specific shock. This feature interacts with the fact that attention to these two shocks has different implications for output gap stabilization[1] and provides fresh insights on optimal monetary policy.

3.2 Firms process independent signals

In this subsection, we first study the case where firms process independent signals and understand the monetary policy rule. We then explore the case where firms process independent signals without understanding the rule.

[1]Ceteris paribus, more attention to the shock that induces efficient fluctuation and less attention to the shock that induces inefficient fluctuation will help stabilize the output gap because the labor supply shock affects both the equilibrium output and efficient level of output, while the markup shock affects only the equilibrium output. This is consistent with Angeletos and Pavan (2007)'s result: providing more information improves welfare to the extent that the equilibrium and the efficient level of output are symmetrically affected by the shocks.

3.2.1 Firms understand the monetary policy rule

When firms process independent signals about the fundamental shocks with Gaussian noises $s_i = (y_i^*, u_i^*)$, where

$$\begin{aligned} y_i^* &= y^* + \epsilon_i^{y^*} \quad \text{with} \quad \epsilon_i^{y^*} \sim \mathcal{N}(0, \sigma_{ey^*}^2) \\ u_i &= u + \epsilon_i^u \quad \text{with} \quad \epsilon_i^u \sim \mathcal{N}(0, \sigma_{eu}^2) \end{aligned}. \tag{3.4}$$

Problem (3.1) becomes

$$\begin{aligned} &\min_{k_{y^*}, k_u} E(p_i - p^*)^2 + f(k) \\ \text{subject to} \quad &\underbrace{\frac{1}{2}\log_2 \frac{(\sigma_{y^*}^2 + \sigma_{ey^*}^2)}{\sigma_{ey^*}^2}}_{\equiv k_{y^*}} + \underbrace{\frac{1}{2}\log_2 \frac{(\sigma_u^2 + \sigma_{eu}^2)}{\sigma_{eu}^2}}_{\equiv k_u} \leq k \\ &p^* = (\phi_{y^*} - 1)y^* + (\phi_u + 1)u \\ &p_i = E(p^*|I_i) = (1 - 2^{-2k_y^*})(\phi_{y^*} - 1)y_i^* + (1 - 2^{-2k_u})(\phi_u + 1)u_i \end{aligned}. \tag{3.5}$$

Under the above attention allocation problem, the optimal monetary policy is the same as before: $\phi_{y^*} = 1, \phi_u = -1 + \sqrt{\frac{f'(0)}{2\ln(2)\sigma_u^2}}$, if $\left.\frac{f_{k_u k_u}}{f_{k_u}}\right|_{k_{y^*}=0} \leq 2\ln 2$ for all positive k_u where f_{k_u} and $f_{k_u k_u}$ are the first and second partial derivative of the cost function w.r.t. the attention to the markup shock respectively. The reason is as follows. We first consider the response to the markup shock. Given some ϕ_{y^*}, the solution is not different from the case when firms only process a one-dimensional signal, and the optimal response to the markup shock is the same as before with $k_u^* = 0$. The only difference is that the previous response to the markup shock needs to be modified to $\phi_u = -1 + \sqrt{\frac{f_{k_u}|_{k_{y^*}^*}}{2\ln(2)\sigma_u^2}}$ and the previous constraint on the cost function is modified to $\left.\frac{f_{k_u k_u}}{f_{k_u}}\right|_{k_{y^*}^*} \leq 2\ln 2$, where $k_{y^*}^*$ is the attention to the labor supply shock optimally chosen by firms under ϕ_{y^*}. We then consider the response to the labor supply shock. Since its fluctuation is efficient, replicating the flexible price allocation with $\phi_{y^*} = 1$ is optimal. Under such response, the attention to the labor supply shock k_{y^*} is zero, which delivers the optimal monetary policy described at the beginning of this paragraph.

What happens if k_{y^*} implied by the optimal ϕ_{y^*} is small enough such that $\left.\frac{f_{k_u k_u}}{f_{k_u}}\right|_{k_{y^*}^*} \leq 2\ln 2$ is not satisfied? Paciello and Wiederholt (2013) discuss this case and conclude that complete price stabilization in response to the efficient fluctuation may be suboptimal without explicitly showing it. In this subsection, we formalize their discussion by deriving the optimal responses. It turns out that the central bank's optimal monetary policy problem now becomes a question of whether the central bank should eliminate the trade-off induced by the markup shock at the cost of deviating its response to the labor supply shock from its previous efficient response.

Breaking down the condition $\left.\frac{f_{k_u k_u}}{f_{k_u}}\right|_{k_{y^*}} \leq 2\ln 2$ essentially imposes additional restrictions on the cross derivative of the cost function. To illustrate the mechanism transparently with analytical expressions, instead of choosing an endogenous k, we now assume that firms allocate a fixed amount of attention K across those two signals by removing the cost function from the objective and replacing the first constraint with $k_{y^*} + k_u \leq K$, where K is fixed and exogenous. This naturally breaks down the above condition, because the shadow marginal cost of paying attention becomes zero when the level of attention is zero. Yet the results in this subsection do not hinge on the assumption of a fixed K. When the total information processing capacity is endogenous, for the result in this subsection to hold, the cost function must satisfy the condition that $\frac{\partial^2 f}{\partial k_u \partial k^*_{y^*}}$ is positive and large enough so that the marginal cost of paying attention to the markup shock drops fast when attention to the labor supply shock approaches zero.[1]

The attention allocation problem (3.5) then becomes the original attention allocation problem in Maćkowiak and Wiederholt (2009). The more attention firms pay to one signal, the more precise that signal is, and individual prices respond more to that signal. The major principle of attention allocation is that firms allocate more attention to the more uncertain shock. Attention is allocated between those two signals based on the relative volatility of the optimal price p^*'s response to those two shocks. Therefore, unlike in the previous subsection where monetary policy that reduces the volatility of p^*'s response to one shock decreases the firms' attention to both shocks, under this case, monetary policy that reduces the volatility of p^*'s response to one shock increases the firms' attention to the other shock. Solving the central bank's problem yields the optimal monetary policy summarized in the following proposition.

Proposition 4. *If firms understand the monetary policy rule and process independent signals, the optimal monetary policy is characterized below.*

1. *If $\gamma(2^{2K} - 1)^2 \leq 1$, the optimal responses from the central bank are given by*

$$\phi_{y^*} = 1, \quad \phi_u = -1 + \frac{1}{\gamma(1 - 2^{-2K}) + 2^{-2K}} \tag{3.6}$$

[1]Since $k = k_u + k_{y^*}$, the restriction discussed here can be easily transformed into restrictions on $f(k)$ directly with the chain rule, which just says $f''(k)$ is large. Fixing K allows for analytical solutions for the case $f''(k)/f'(k) > 2\ln 2$ without specifying a functional form for the cost of attention.

At this policy, $k_{y^*} = 0$, $k_u = K$.

2. *If $\gamma(2^{2K}-1)^2 \geq 1$, the optimal responses from the central bank are given by*

$$\begin{aligned} \phi_{y^*} &= 1 - \frac{2^K}{1+\gamma(1-2^{-2K})+2^{-2K}} \cdot \frac{\sigma_u}{\sigma_{y^*}} \\ \phi_u &= -1 + \frac{1}{1+\gamma(1-2^{-2K})+2^{-2K}} \end{aligned} \tag{3.7}$$

At this policy, $k_{y^*} = K$, $k_u = 0$.

Proof. See Appendix B □

The limiting cases when $\gamma = 0$ and $\gamma = \infty$ correspond to the cases when the central bank cares about the output gap only or price dispersion only respectively. A quick conclusion from Proposition 4 is that under this scenario, there is always a trade-off between the central bank's dual mandates because the weight on loss from price dispersion γ enters the optimal responses.[1] Yet what drives trade-off is different, compared to an economy with exogenous information and markup shock only.

When $\gamma(2^{2K}-1)^2 \leq 1$, the optimal response to the labor supply shock eliminates inefficient price dispersion and completely closes the output gap induced by the labor supply shock, while the optimal response to the markup shock minimizes the weighted average loss from price dispersion and output gap fluctuation induced by the markup shock.[2] The optimal response to the labor supply shock is the response that replicates the flexible price allocation, which is efficient and makes firms pay zero attention to it. At the same time, as firms do not pay attention to the labor supply shock, the marginal cost of paying attention to the markup shock is low and the central bank's response to the markup faces a trade-off between reducing price dispersion and stabilizing output, as shown in Panel (a) of Figure 4. The result is consistent with the conclusion from the last section that when the marginal cost of paying attention to markup shock is low, there is a trade-off between the central bank's dual mandates, which maps into the case plotted in Panel (a) of Figure 3, with the condition modified to $\frac{f_{k_u k_u}}{f_{k_u}} > 2\ln 2$.

When $\gamma(2^{2K}-1)^2 \geq 1$, at the optimal monetary policy, firms do not pay attention to the

[1] Since Proposition 3 also holds when firms process independent signals, this case falls into Point 3 of Proposition 3 when $\frac{f''(k)}{f'(k)} > 2\ln 2$. But now we are able to characterize the optimal response analytically without specifying a specific form for the cost function.

[2] At the optimal responses, the loss is $\frac{\gamma(2^{2K}-1)}{\gamma(2^{2K}-1)+1}\sigma_u^2 < \sigma_u^2$.

markup shock and individual prices do not respond to the markup shock.[1] Therefore, there is no inefficient price dispersion from the markup shock. Meanwhile, the loss from output gap fluctuation brought by the markup shock is also minimized at the optimal monetary policy. The response to the markup shock thus does not face a trade-off between the central bank's two objectives as shown in Panel (b) of Figure 4. At the optimal monetary policy, firms pay all their attention to the labor supply shock. Even though there is no trade-off faced by the response to the labor supply shock itself, the previous optimal response $\phi_{y^*} = 1$ is not optimal any more. This is because at the original optimal response, the marginal cost of attention to the markup shock is too low. The response to the markup shock thus faces the trade-off, which could have been eliminated if the marginal cost of paying attention to the markup shock was high enough, just as shown in Panel (b) of Figure 3 where $\frac{f_{k_u k_u}}{f_{k_u}} \leq 2\ln 2$ is satisfied.

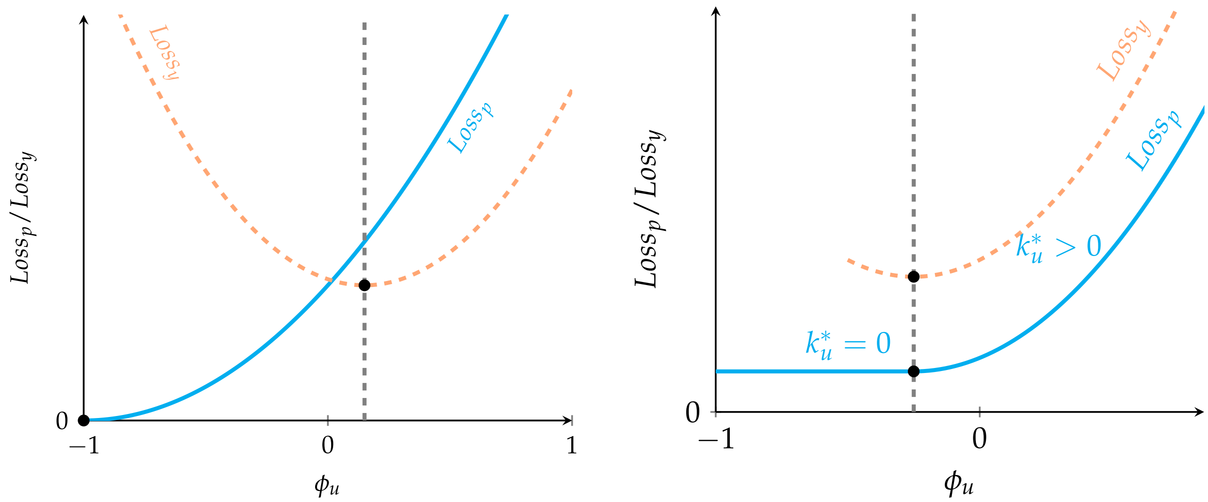
Proposition 4 says that it is welfare improving to eliminate that trade-off if the central bank puts a large weight on the loss from price dispersion ($\gamma(2^{2K}-1)^2 \geq 1$). Because when the weight on price dispersion increases, fixing $\phi_{y^*} = 1$ and letting the markup shock face the trade-off pushes the response to replicate the flexible price allocation, which decreases price dispersion but leads to a large loss from output gap fluctuation. Yet the response to the labor supply shock itself does not face such trade-off.[2] Although a small level of attention to it makes the response deviate from its original efficient level, it decreases the total welfare loss. Therefore, it is optimal for the central bank to respond in a way under which the firms still pay attention to the labor supply, the marginal cost of paying attention to the markup shock is high enough, and firms' attention is not distracted by the markup shock as the comparison shows in Figure 4.

It is also worth mentioning that if firms were to choose the signal optimally, then the optimal monetary policy would be the same as the first case when $\gamma(2^{2K}-1)^2 < 1$: $\phi_{y^*} = 1$ and $\phi_u = -1 + \frac{1}{\gamma(1-2^{-2K})+2^{-2K}}$. Now an individual firm's optimal price is $p_i = (1-2^{-2K})[(\phi_{y^*}-1)y^* + (\phi_u+1)u + e_i]$. Setting $\phi_{y^*} = 1$ mutes the firms' response to the labor supply and is also efficient in terms of output, so it is optimal. Then the equilibrium price and output become the same as that under $k_u = K$ and $k_{y^*} = 0$, which is exactly the first case implies.[3] Comparing this result with Proposition 4, although observing independent signals is not optimal for firms, welfare at

[1] At the optimal responses, the loss is $\frac{\gamma^2+(1-\gamma)\gamma\cdot 2^{-2K}}{\gamma^2-2^{-2K}(1-\gamma^2)} \cdot \sigma_u^2 < \sigma_u^2$.

[2] Recall that flexible price allocation in response to the labor supply shock is efficient.

[3] This is essentially the case when information is exogenous: K is fixed. But the result is not driven by the exogeneity of K. With an endogenous K, the optimal policy tends to decrease K. As long as $\frac{f''(K)}{f'(K)} > \ln 2$, the optimal K should be positive. Then a large γ can compensate the decrease in K to satisfy $\gamma(2^{2K}-1) > 1$.



(a) Loss as a function of ϕ_u
ϕ_{y^*} fixed at optimal level ($\phi_{y^*} = 1$)
when $\gamma(2^{2K} - 1)^2 \leq 1$

(b) Loss as a function of ϕ_u
ϕ_{y^*} fixed at optimal level ($\phi_{y^*} \neq 1$)
when $\gamma(2^{2K} - 1)^2 \geq 1$

Figure 4: Marginal cost of attention and trade-off

the optimal monetary policy is improved if the central bank values reducing price dispersion more heavily. The intuition is that assuming firms observe independent signals enables the central bank to increase the marginal cost of paying attention to the markup shock and eliminates the trade-off brought out by it.

To conclude, this section shows that when firms process independent signals, the central bank can manipulate the marginal cost of attention to an individual shock to mitigate the trade-off between its dual mandates induced by the markup shock. In this section, we assume that firms observe independent signals and it is not optimal. The mechanism is also present when firms observe multiple signals and could also apply to cases when agents have multiple choices and would optimally observe multiple orthogonal signals as shown in Kamdar (2019).

3.2.2 Firms do not understand the monetary policy rule

To understand the value of commitment under rational inattention, we study a case where firms do not understand the rule in this subsection. When firms understand the rule, they do not need to pay attention to the policy signal since they can back it out. But when firms do not understand the rule, they pay extra attention to the policy signal, which effectively increases the marginal cost of paying attention to other signals. As shown in the first part of Proposition 3, increasing the overall marginal cost of attention could be welfare improving. Meanwhile, the second part of Proposition 3 shows that increasing the marginal cost of paying attention to the markup shock helps mitigate the trade-off between the central bank's dual mandates. The result in this section confirms that not communicating the rule helps stabilize output and mitigate the trade-off. However it also generates unnecessary inefficient price dispersion.

Specifically, when firms do not understand the policy rule, they process one additional signal $q_i = q + \epsilon_i^q$, with $q \sim \mathcal{N}(0, \sigma_q^2)$ besides y_i^* and u_i, compared with the previous subsection.[1] Although de facto the policy is a linear combination of the fundamentals, private agents fail to build the connection and need to observe the additional policy signal. Problem (3.1) then becomes

$$\begin{aligned} &\min_{k_q, k_{y^*}, k_u} E(p_i - p^*)^2 \\ \text{subject to} \quad &\underbrace{\frac{1}{2}\log_2 \frac{(\sigma_q^2 + \sigma_{eq}^2)}{\sigma_{eq}^2}}_{\equiv k_q} + \underbrace{\frac{1}{2}\log_2 \frac{(\sigma_{y^*}^2 + \sigma_{ey^*}^2)}{\sigma_{ey^*}^2}}_{\equiv k_{y^*}} + \underbrace{\frac{1}{2}\log_2 \frac{(\sigma_u^2 + \sigma_{eu}^2)}{\sigma_{eu}^2}}_{\equiv k_u} \leq K \quad . \\ &p^* = (\phi_{y^*} - 1)y^* + (\phi_u + 1)u \\ &p_i = (1 - 2^{-2k_q})q_i - (1 - 2^{-2k_{y^*}})y_i^* + (1 - 2^{-2k_u})u_i \end{aligned} \tag{3.8}$$

The attention allocation mechanism is similar to the previous case. A more volatile shock attracts more attention. Different from the previous case where the central bank can freely allocate the firms' attention between the two fundamentals, now the central bank can only push the firms' attention to certain ranges as indicated by the solution to Problem (3.8). Figure 5 demonstrates the solution, which can be divided into seven cases.[2] Relative volatility between the two fundamental shocks $\sigma_{y^*}^2/\sigma_u^2$ determines all possible cases for attention allocation. For example, if $\sigma_{y^*}^2 = \sigma_u^2$, the attention allocation must be one of case 5, case 1, and case 4.[3] Monetary policy pins down which of them is the final allocation. If monetary policy is quite uncertain, the optimal allocation falls into case 5, where firms only pay attention to the policy signal. If it is quite certain, firms ignore it and only pay attention to the other signals as shown in case 4. For scenarios in between, the

[1] This subsection considers the case where the private sector perceives the policy as an independent shock. We could also set up an intermediate case where the policy can be decomposed into two parts. That is, $q = \phi_{y^*}y^* + \phi_u u = (1-\alpha)(\phi_{y^*}y^* + \phi_u u) + \alpha(\phi_{y^*}y^* + \phi_u u)$, of which the first term is the committed part and the second term is the discretionary part. Firms process an independent signal on the discretionary part $q_i = \alpha(\phi_{y^*}y^* + \phi_u u) + \epsilon_i^q$. When $\alpha = 0$, this setup corresponds to the full commitment case in section 3.2.1; when $\alpha = 1$, it corresponds to the case here in this subsection.

[2] A complete characterization of the solution to the attention allocation problem can be found in Appendix C.

[3] The slope of a line passing through the origin is $\sigma_{y^*}^2/\sigma_u^2$. When $\sigma_{y^*}^2/\sigma_u^2 = 1$, the firms' optimal allocation must fall on the line passing through the origin with a slope of 1.

optimal solution falls into case 1 and the firms pay attention to all signals. Nonetheless, monetary policy is unable to shift the firms' attention to any other case.

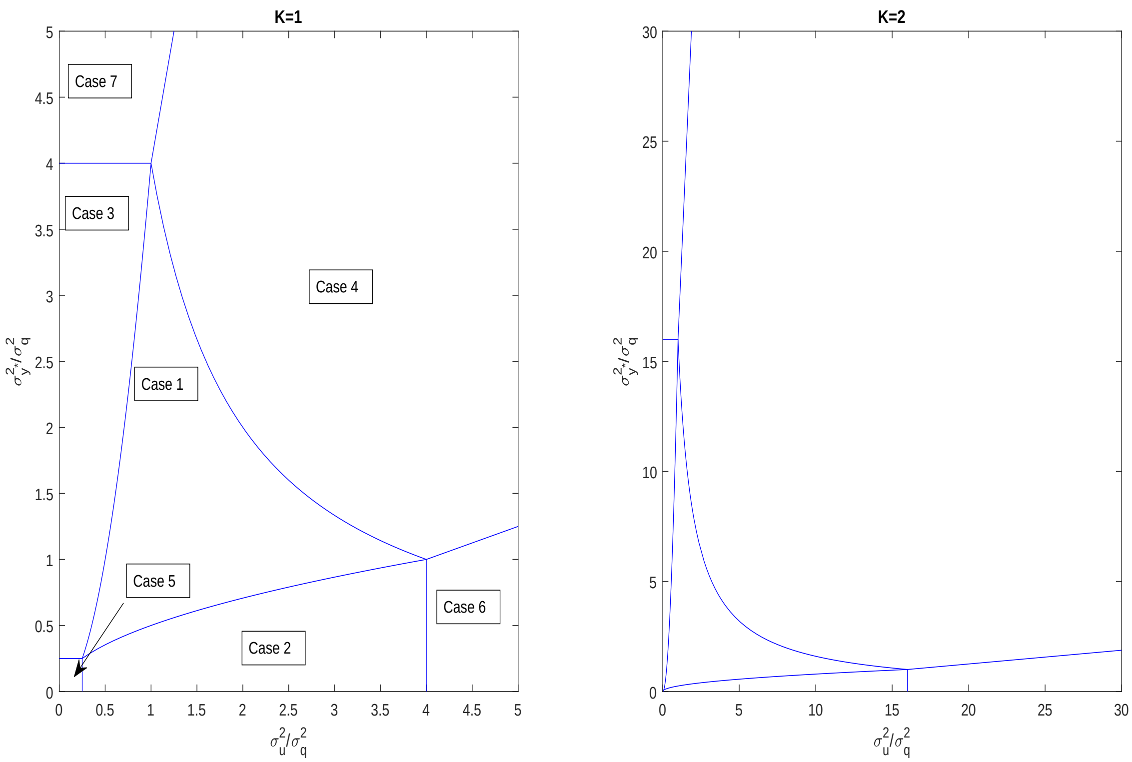


Figure 5: The role of monetary policy in determining the firms' attention allocation

Note: The graph shows how the firms' attention allocation is determined. Case 1 is the case where firms pay attention to all three signals. Under cases 2, 3, and 4, firms only pay attention to two signals. Under cases 5, 6, and 7, the firms allocate all their attention to one signal only. Complete characterization of the solution can be found in Appendix C. $\sigma^2_{y^*}/\sigma^2_u$ determines the candidate cases for firms to allocate attention. For example, if $\sigma^2_{y^*}/\sigma^2_u = 1$, the possible cases are case 5, case 1, and case 4; uncertainty from monetary policy σ^2_q pins down the final optimal attention allocation. If the policy is very uncertain, then the firms end up allocating all their attention to the monetary policy signal as in case 5. If it is quite certain, the firms ignore the monetary policy signal as in case 4. If it's comparable to other shocks, the firms' attention allocation falls in the interior solution as in case 1.

With the above attention allocation solution, solving the central bank's problem analytically becomes more involved. To explain the mechanism more clearly, we again start with one mandate at one time, and then combine them.

Proposition 5. *If firms do not understand the monetary policy and process independent signals, any monetary policy making the firms not pay attention to the policy signal minimizes price dispersion. Table 1 summarizes the results.*

Proof. See Appendix D □

Table 1: Optimal Response to Minimize Price Dispersion

State Space Partition	Attention at Optimal Responses	Loss from Price Dispersion
$\sigma_{y^*}^2/\sigma_u^2 \in [2^{2K}, \infty)$	$k_q^* = 0, k_{y^*}^* = K, k_u^* = 0$	$(2^{-2K} - 2^{-4K})\sigma_{y^*}^2$
$\sigma_{y^*}^2/\sigma_u^2 \in [2^{-2K}, 2^{2K}]$	$k_q^* = 0, k_{y^*}^* = \frac{1}{2}K + \frac{1}{4}\log 2(\sigma_{y^*}^2/\sigma_u^2), k_u^* = \frac{1}{2}K - \frac{1}{4}\log 2(\sigma_{y^*}^2/\sigma_u^2)$	$2 \cdot 2^{-K}\sigma_u\sigma_{y^*} - 2^{-2K}\sigma_u^2 - 2^{-2K}\sigma_{y^*}^2$
$\sigma_{y^*}^2/\sigma_u^2 \in (0, 2^{-2K}]$	$k_q^* = 0, k_{y^*}^* = 0, k_u^* = K$	$(2^{-2K} - 2^{-4K})\sigma_u^2$

Same as before, without nominal rigidity, price dispersion can only stem from dispersed information. Since firms only have limited information processing capacity, they observe signals with independent processing errors, leading to dispersion. However, different from the previous case, when the central bank reacts, the firms do not recognize that monetary policy depends on the fundamentals and allocate part of their attention to the policy signal. Failing to recognize the correlation purely increases the marginal cost of attention for the firms without any benefit in terms of receiving more precise signals about the fundamentals. Therefore, the optimal monetary policy to reduce price dispersion is the policy that does not attract any attention to the policy signal as described in Proposition 5.

To ensure zero attention to the policy signal, with the attention allocation mechanism discussed above, when $\sigma_{y^*}^2/\sigma_u^2 \geq 2^{-2K}$ ($\sigma_{y^*}^2/\sigma_u^2 \leq 2^{-2K}$) and the uncertainty is mostly from the labor supply (markup) shock, responses that make the firms pay full attention to the labor supply (markup) shock are optimal. When $\sigma_{y^*}^2/\sigma_u^2 \in [2^{-2K}, 2^{2K}]$ where the uncertainty of neither shock is dominating, any responses that make firms not pay attention to the policy signal are optimal, and the allocation between the two fundamentals is determined by the relative volatility $\sigma_{y^*}^2/\sigma_u^2$.

Comparing these findings with the results in section 3.2.1, if the firms fail to understand the correlation between the policy and the fundamentals, the last column of Table 1 says that price dispersion can never be completely eliminated and the central bank cannot replicate the flexible price allocation any more. Even the full information optimal price p^* is fully stabilized, the firms still pay unnecessary attention to the policy signal. At the same time, the optimal response is not unique under each case. Price dispersion is determined by monetary policy via the influence of σ_q^2, but σ_q^2 itself cannot pin down ϕ_{y^*} and ϕ_u uniquely. There exists a range for σ_q^2 (as long as it is small enough) such that the price dispersion is minimized.

In conclusion, when the firms do not understand the policy rule, they pay extra attention to the policy signal in addition to signals on the fundamentals, which creates unnecessary price dispersion. Compared with the case where the firms understand the rule, not only monetary policy is not effective in decreasing price dispersion induced by the fundamentals but it adds extra price dispersion. However, as will be introduced below, when firms need to pay separate attention to the policy, it lifts the overall marginal cost of attention in the economy, especially the marginal cost of attention to the markup shock, which helps to stabilize the inefficient output fluctuation brought by the markup shock.

Proposition 6. *If firms do not understand the monetary policy rule and process independent signals, the central bank can always close the output gap. The optimal responses are summarized in Table 2.*

Table 2: Optimal Responses to Fully Close the Output Gap

	State Space Partition	Optimal Responses
Case I	$\sigma_{y^*}^2/\sigma_u^2 \in [2^{2K}, \infty)$	$\phi_{y^*} = 2^{-2K}$, $\phi_u = 0$
		$\phi_{y^*} = 2^{2K}$, $\phi_u = 0$
		$\phi_{y^*} = 1$, $\phi_u = 0$
Case II	$\sigma_{y^*}^2/\sigma_u^2 \in (2^{-2K}, 2^{2K})$	$\phi_{y^*} = 2^{2K}$, $\phi_u = 0$
		$\phi_{y^*} = 2^{-K}\frac{\sigma_u}{\sigma_{y^*}}$, $\phi_u = 1 - 2^{-K}\frac{\sigma_{y^*}}{\sigma_u}$, if $\sigma_{y^*}/\sigma_u \in [\frac{3 \cdot 2^K - \sqrt{5 \cdot 2^{2K} - 4}}{2}, \frac{3 \cdot 2^K + \sqrt{5 \cdot 2^{2K} - 4}}{2}]$
Case III	$\sigma_{y^*}^2/\sigma_u^2 \in (0, 2^{-2K}]$	$\phi_{y^*} = 2^{2K}(1 - \sqrt{\Phi})$, $\phi_u = 2^{2K}(\sqrt{\Phi} - \Phi)$
		where $\Phi \equiv \min\{2^{-2K} - \sigma_{y^*}^2/\sigma_u^2, (1 - 2^{-2K})^2\}$

Proof. See Appendix E □

When $\sigma_u^2 \leq 2^{2K}\sigma_{y^*}^2$ (Case I and Case II), no response to the markup shock ($\phi_u = 0$) is always optimal. Under all those optimal responses, the firms do not pay attention to the markup shock, and price does not respond to it. The response of output to the markup shock is therefore efficient. As for the response to the labor supply shock, when the volatility is small (Case I), there are three responses that are efficient. A small response that does not attract much attention and has a large real effect is equivalent to a large response that attracts much attention and comes out less surprisingly. This is a result driven by the mechanism discussed in section 3.1 under endogenous information, and the analytics here provide a concrete example. However, the smaller responses then become suboptimal when the markup shock becomes more volatile (Case II). Under those responses, the increased volatility of the markup shock makes firms pay attention to the markup shock instead, causing an output gap.

When the markup shock is very volatile (Case III), the firms inevitably pay attention to it, and policy that tries to distract them is not optimal any more because that implies a policy driving all attention to the policy signal, which itself fluctuates with the markup shock and is not efficient.

Instead, it is optimal for the central bank to respond in a way that its response just counteracts the firms' response to the markup shock as described in the last row of Table 2.

Compared with the case in section 3.2.1, optimal responses that fully stabilize the output gap are not unique. This is not surprising since without knowing the rule, there is an extra dimension of firms' attention allocation problem. Attention to the markup shock and attention to the labor supply shock are not strictly substitutes any more, which adds more nonlinearity to the real effect of monetary policy. Meanwhile, from Case III to Case I as the volatility from markup shock decreases, there are more ways to fully stabilize output gap since it is easier to distract firms from less volatile markup shock, which helps to close the output gap.

To conclude, Proposition 6 says that the central bank can fully stabilize the output gap by carefully controlling its real effect. Compared with the previous case where firms understand the monetary policy rule, the central bank is now more flexible in stabilizing the output gap. Intuitively, with firms not knowing the rule, information is even more imperfect and the real effect of the monetary policy is strengthened.

Proposition 7. *When firms do not understand the monetary policy, there is no trade-off between price dispersion stabilization and output gap stabilization under the following three cases:*

1. $\sigma_u^2 \leq 2^{-2K}\sigma_{y^*}^2$;
2. $\sigma_u^2 \geq (2^{-2K} - (1-2^{-2K})^2)\sigma_{y^*}^2$;
3. $\sigma_{y^*}/\sigma_u \in [\frac{3\cdot 2^K - \sqrt{5\cdot 2^{2K}-4}}{2}, \frac{3\cdot 2^K + \sqrt{5\cdot 2^{2K}-4}}{2}]$.

Proof. See Appendix F □

Comparing Proposition 7 with Proposition 4 where there is always a trade-off between the central bank's dual mandates, when firms do not understand the rule and the marginal cost of information is increased, the trade-off between the central bank's two objectives can disappear as in the above three cases.

When $\sigma_{y^*}^2/\sigma_u^2 \geq 2^{2K}$ and uncertainty is mostly from the efficient fluctuation, the responses $\phi_{y^*} = 2^{-2K}$ and $\phi_u = 0$ fully close the output gap and minimize price dispersion. Under this policy, firms allocate all their attention to the labor supply shock. In this way, the equilibrium output does not respond to the markup shock and the output gap is closed. Although the response to the labor supply shock is muted compared with the efficient response, it is counteracted by the direct response of the monetary policy. At the same time, the firms do not pay attention to the policy signal, so price dispersion is minimized. This is similar to the first part of Proposition 3. No response to the markup shock now becomes optimal, and there is no trade-off. This optimal policy is missing in section 3.2.1 but appears here, because not communicating the rule increases the overall marginal cost of paying attention and firms are less inclined to pay attention to the markup shock, leaving the inaction response optimal.

When $\sigma_{y^*}^2/\sigma_u^2 \leq 2^{-2K} - (1-2^{-2K})^2$ and uncertainty is mostly from the markup shock, the policy $\phi_{y^*} = 1$ and $\phi_u = 1 - 2^{-2K}$ fully closes the output gap and minimizes price dispersion. Under this policy, the firms pay allocate all their attention to the markup shock. Thus the central bank needs to react to both shocks to stabilize the output gap. Also, the markup shock is quite uncertain. Firms pay much attention to it, which makes it easier for central banks not to draw attention to the policy signal and minimizes price dispersion. The non-negativity of the LHS also imposes that $K \in (0, -\frac{1}{2}\log_2 \frac{3-\sqrt{5}}{2})$.

When $\sigma_{y^*}/\sigma_u \in [\frac{3\cdot 2^K - \sqrt{5\cdot 2^{2K}-4}}{2}, \frac{3\cdot 2^K + \sqrt{5\cdot 2^{2K}-4}}{2}]$ and the volatilities of the two fundamentals are close to each other, the policy $\phi_{y^*} = 2^{-K}\frac{\sigma_u}{\sigma_{y^*}}$ and $\phi_u = 1 - 2^{-K}\frac{\sigma_{y^*}}{\sigma_u}$ fully closes the output gap and minimizes price dispersion. Under this optimal policy, the firms do not pay attention to the policy signal and the responses from the central bank adjust to stabilize the output gap.

When the firms do not understand the rule, the attention to the policy signal also causes a trade-off between stabilizing price dispersion and output gap aside from the existence of the markup shock. Paying attention to the policy signal mutes the firms' response and pushes responses (including the response to the labor supply shock) away from their efficient levels. By responding to the shocks directly, monetary policy can correct this deviation. However, it attracts attention from firms and produces more price dispersion. Under similar logic, to eliminate the trade-off, the marginal cost of paying attention to the policy signal must be high enough, and this is the case when one of the fundamental shocks is extremely uncertain or the volatility of the two fundamental shocks are similar as summarized by the above three cases.

3.2.3 Discussion on rule-based policy versus discretion

In this section, we compare the welfare losses under the optimal monetary policy under the setups in section 3.2.1 and section 3.2.2 respectively in Figure 6. We refer to the case in section 3.2.1 as a rule-based monetary policy and the case in section 3.2.2 as a discretionary monetary policy for the reasons discussed previously. Figure 6 shows that when the weight on price dispersion stabilization increases, the rule-based monetary policy generates a smaller total loss. Because

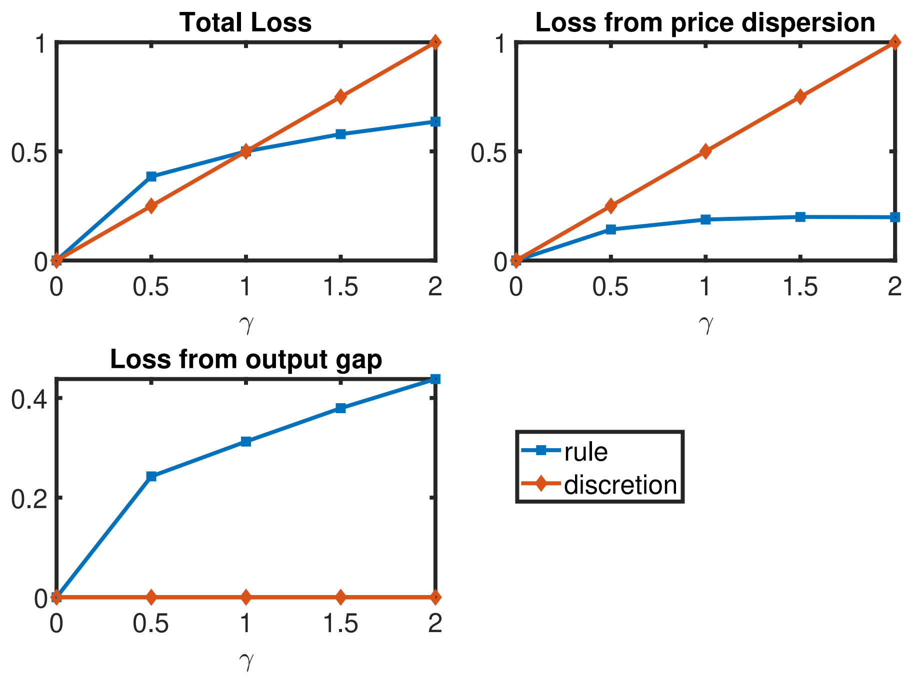


Figure 6: Comparison: discretion v.s. rule

Parameter values: $K = 1, \sigma_u^2 = \sigma_{y^*}^2 = 1, \gamma = (0, 0.5, 1, 1.5, 2)$. Given the parameter values here, the graph presents the case $\sigma_{y^*}/\sigma_u \in [\frac{3\cdot 2^K - \sqrt{5\cdot 2^{2K} - 4}}{2}, \frac{3\cdot 2^K + \sqrt{5\cdot 2^{2K} - 4}}{2}]$. Discretionary monetary policy strengthens the real effect of monetary policy and eliminates the trade-off brought out by the markup shock by increasing the marginal cost of attention. But it weakens the central bank's ability to reduce price dispersion. When the weight on the loss from price dispersion increases, there is welfare gain from a rule-based monetary policy.

discretionary policy increases the marginal cost of paying attention. With increased marginal cost of attention, monetary policy can better stabilize the output gap and eliminate the trade-off between the output gap fluctuation and price dispersion, but this occurs at the cost of inefficient price dispersion that cannot be reduced.

4 Conclusion

This paper uses a simple static model to analytically explore the informational role of monetary policy in forming private sectors' expectation, its interaction with the traditional direct effect of monetary policy, and the corresponding implication for optimal monetary policy. The effectiveness of monetary policy is not only determined by the central bank's response but also the marginal cost of paying attention. When the marginal cost of paying attention is high, the central bank does not face the trade-off between its dual mandate. The monetary policy trying to not attract the private sector's attention stabilizes both price and output gap fluctuation. When the marginal cost of attention is low, the monetary policy aimed at stabilizing output gap attracts attention from the private sector and generates inefficient price dispersion.

One direction for future research is to extend the model into a dynamics setting and study how the intertemporal trade-off in paying attention affects optimal monetary policy. Dynamic rational inattention problems are generally much more difficult to solve. But very recently, there has been a growing body of literature tackling the problem such as Maćkowiak et al. (2018), Miao et al. (2019), and so on, which could be utilized for future research.

References

Adam, K. (2007). Optimal monetary policy with imperfect common knowledge. *Journal of monetary Economics 54*(2), 267–301.

Afrouzi, H. (2017). Strategic inattention, inflation dynamics and the non-neutrality of money. *Working Paper*.

Angeletos, G.-M. and J. La'O (2011). Optimal monetary policy with informational frictions. Technical report, National Bureau of Economic Research.

Angeletos, G.-M. and A. Pavan (2007). Efficient use of information and social value of information. *Econometrica 75*(4), 1103–1142.

Aoki, K. (2003). On the optimal monetary policy response to noisy indicators. *Journal of monetary economics 50*(3), 501–523.

Baeriswyl, R. and C. Cornand (2010). The signaling role of policy actions. *Journal of Monetary Economics 57*(6), 682–695.

Ball, L., N. G. Mankiw, and R. Reis (2005). Monetary policy for inattentive economies. *Journal of monetary economics 52*(4), 703–725.

Berkelmans, L. (2011). Imperfect information, multiple shocks, and policy's signaling role. *Journal of Monetary Economics 58*(4), 373–386.

Galí, J. (2015). *Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications*. Princeton University Press.

Kamdar, R. (2019). The inattentive consumer: Sentiment and expectations. *Manuscript*.

Luo, Y. (2008). Consumption dynamics under information processing constraints. *Review of Economic dynamics 11*(2), 366–385.

Maćkowiak, B., F. Matějka, and M. Wiederholt (2018). Dynamic rational inattention: Analytical results. *Journal of Economic Theory 176*, 650–692.

Mackowiak, B., F. Matejka, and M. Wiederholt (2018). Rational inattention: A disciplined behavioral model.

Maćkowiak, B. and M. Wiederholt (2009). Optimal sticky prices under rational inattention. *The American Economic Review 99*(3), 769–803.

Mankiw, N. G. and R. Reis (2002). Sticky information versus sticky prices: a proposal to replace the new keynesian phillips curve. *The Quarterly Journal of Economics 117*(4), 1295–1328.

Miao, J., J. Wu, and E. Young (2019). Multivariate rational inattention. *Manuscript*.

Mondria, J. (2010). Portfolio choice, attention allocation, and price comovement. *Journal of Economic Theory 145*(5), 1837–1864.

Orphanides, A. (2003). Monetary policy evaluation with noisy information. *Journal of monetary economics 50*(3), 605–631.

Paciello, L. (2012). Monetary policy and price responsiveness to aggregate shocks under rational inattention. *Journal of Money, Credit and Banking 44*(7), 1375–1399.

Paciello, L. and M. Wiederholt (2013). Exogenous information, endogenous information, and optimal monetary policy. *Review of Economic Studies 81*(1), 356–388.

Shannon, C. E. (1948). A mathematical theory of communication. *ACM SIGMOBILE Mobile Computing and Communications Review 5*(1), 3–55.

Sims, C. A. (2003). Implications of rational inattention. *Journal of monetary Economics 50*(3), 665–690.

Sims, C. A. (2010). Rational inattention and monetary economics. In *Handbook of monetary economics*, Volume 3, pp. 155–181. Elsevier.

Steiner, J., C. Stewart, and F. Matějka (2017). Rational inattention dynamics: Inertia and delay in decision-making. *Econometrica 85*(2), 521–553.

Svensson, L. E. and M. Woodford (2003). Indicator variables for optimal policy. *Journal of monetary economics 50*(3), 691–720.

Tutino, A. (2013). Rationally inattentive consumption choices. *Review of Economic Dynamics 16*(3), 421–439.

Appendices

A Proof for Propositions 1-3

1. Given some monetary policy (ϕ_{y^*}, ϕ_u) and the corresponding optimal attention allocation (k^*), the equilibrium price and output are given by

$$p = (1 - 2^{-2k^*})(q - y^* + u) \tag{A.1}$$

$$y = [2^{-2k^*}(\phi_{y^*} - 1) + 1]y^* + [2^{-2k^*}(\phi_u + 1) - 1]u. \tag{A.2}$$

2. Solving the firms' optimal attention problem

$$\begin{aligned} &\min_k E[(p_i - p^*)^2] + f(k) \\ \text{subject to} \quad & p^* = q - y^* + u \\ & p_i = (1 - 2^{-2k})(q - y^* + u + e_i) \end{aligned}, \tag{A.3}$$

yields the following solution

$$k^* \begin{cases} = 0, & \text{if } 2 \cdot \ln(2) Var(p^*) \leq f'(0) \\ \text{solves } 2 \cdot \ln(2) Var(p^*) = f'(k)2^{2k}, & \text{otherwise} \end{cases}. \tag{A.4}$$

3. We first focus on the zero attention case. If $k^* = 0$, individual prices don't respond to shocks at all. The price level is completely stabilized. Output gap is characterized by $y - y^* = (\phi_{y^*} - 1)y^* + \phi_u u$. Setting $\phi_{y^*} = 1$ and $\phi_u = 0$ achieves the first-best result, as long as k^* is indeed zero under the responses, which requires $\sigma_u^2 \leq \frac{f'(0)}{2\ln(2)}$.

4. We now turn to the case when $\sigma_u^2 > \frac{f'(0)}{2\ln(2)}$ and the first best result is not feasible any more. To study the effect of monetary policy on price dispersion and output gap fluctuation, we first show how monetary policy affects the optimal level of attention and then show how the optimal level of attention affects welfare. If $k^* > 0$ and satisfies the above optimality

condition A.4, from the implicit function theorem, we know that:

$$\frac{\partial k^*}{\partial \phi_{y^*}} = \frac{4\ln(2)(\phi_{y^*}-1)\sigma_{y^*}^2}{g'(k)} < 0, \text{if } \phi_{y^*} < 1 \tag{A.5}$$

$$\frac{\partial k^*}{\partial \phi_u} = \frac{4\ln(2)(\phi_u+1)\sigma_u^2}{g'(k)} > 0, \text{if } \phi_u > -1 \tag{A.6}$$

where $g(k) = f'(k)2^{2k}$.

As for the effect of attention on price dispersion, we first derive the loss from price dispersion

$$Loss_p = E\int_0^1 (p - p_i)d_i = 2^{-2k^*}(1-2^{-2k^*})Var(p^*) = \frac{(1-2^{-2k^*})f'(k^*)}{2\ln(2)}, \tag{A.7}$$

and then take partial derivative of it w.r.t. the optimal attention, and get the following result

$$\frac{\partial Loss_p}{\partial k^*} = \frac{(1-2^{-2k^*})f''(k^*) + 2\ln 2\cdot 2^{-2k^*}f'(k^*)}{2\ln 2} > 0, \tag{A.8}$$

which finishes the proof for Proposition 1.

Then we consider the effect of monetary policy on output gap stabilization. The loss from output gap is

$$Loss_y = E(y-y^*)^2 = 2^{-4k^*}(\phi_{y^*}-1)^2\sigma_{y^*}^2 + [2^{-2k^*}(\phi_u+1)-1]^2\sigma_u^2. \tag{A.9}$$

Fixing ϕ_u and taking partial derivative w.r.t. response to the labor supply shock if $\phi_{y^*} < 1$ and $k^* > 0$ yields

$$\begin{aligned}\frac{\partial Loss_y}{\phi_{y^*}} &= 2\cdot 2^{-4k^*}(\phi_{y^*}-1)\sigma_{y^*}^2 \\ &+ \frac{\partial k^*}{\partial \phi_{y^*}}[-4\cdot\ln 2\cdot 2^{-4k^*}(\phi_{y^*}-1)^2\sigma_{y^*}^2 - 4\cdot\ln 2\cdot 2^{-2k^*}(2^{-2k^*}(\phi_u+1)-1)]\sigma_u^2 \\ &= 2\cdot 2^{-4k^*}(\phi_{y^*}-1)\sigma_{y^*}^2 + \frac{\partial k^*}{\partial \phi_{y^*}}(-2\cdot 2^{-2k^*})f'(k^*)2\cdot\ln 2\cdot\sigma_u^2[-\phi_u(\phi_u+1)2^{-2k^*}-1] < 0.\end{aligned} \tag{A.10}$$

Thus $\phi_{y^*} = 1$ is optimal, regardless of the value of ϕ_u and it doesn't face a trade-off between price dispersion and output gap stabilization. Fixing ϕ_{y^*} at its optimal value $\phi_{y^*} = 1$, we can evaluate the effect of ϕ_u on output gap fluctuation,

$$\begin{aligned}\frac{\partial Loss_y}{\partial \phi_u} &= 2 \cdot [2^{-2k^*}(\phi_u + 1) - 1][2^{-2k^*} - 2 \cdot \ln 2 \cdot (\phi_u + 1)2^{-2k^*}\frac{\partial k^*}{\phi_u}] \\ &= 2 \cdot [2^{-2k^*}(\phi_u + 1) - 1][2^{-2k^*} - \frac{4 \cdot \ln 2 \cdot f'(k^*)}{g'(k^*)}] \\ &= 2 \cdot [2^{-2k^*}(\phi_u + 1) - 1]\frac{(f''(k^*) - 2 \cdot \ln 2 f'(k^*))}{g'(k^*)} \geq 0, \text{ if } \frac{f''(k^*)}{f'(k^*)} \leq 2 \cdot \ln 2\end{aligned} \tag{A.11}$$

where $g(k^*) = 2^{2k^*}f'(k^*)$, for all $k^* > 0$. Therefore, response to the markup shock doesn't face trade-off and the policy that minimizes price dispersion also minimizes output gap fluctuation. As ϕ_u decreases towards -1, the optimal response to markup shock should decrease until the optimal attention k^* is just zero. At this point, $\phi_u = -1 + \sqrt{\frac{f'(0)}{2 \cdot \ln 2 \cdot \sigma_u^2}}$. This finishes the proof for Propositions 2 and 3.

B Proof for Proposition 4

1. Given a monetary policy that makes firms pay attention according to (k_{y^*}, k_u), the equilibrium is characterized by the following three parts.

   Individual pricing for firm i:

   $$p_i = (1 - 2^{-2k_{y^*}})(\phi_{y^*} - 1)y_i^* + (\phi_u + 1)(1 - 2^{-2k_u})u_i \tag{B.1}$$

   Aggregate price level:

   $$p = (1 - 2^{-2k_{y^*}})(\phi_{y^*} - 1)y^* + (\phi_u + 1)(1 - 2^{-2k_u^*})u \tag{B.2}$$

   Aggregate output:

   $$\begin{aligned} y &= q - p \\ &= [2^{-2k_{y^*}}(\phi_{y^*} - 1) + 1]y^* + [2^{-2k_u}(\phi_u + 1) - 1]u \end{aligned} \tag{B.3}$$

   Loss from output gap fluctuation is given by:

   $$E(y - y^*)^2 = [2^{-2k_{y^*}}(\phi_{y^*} - 1)]^2\sigma_{y^*}^2 + [2^{-2k_u}(\phi_u + 1) - 1]^2\sigma_u^2 \tag{B.4}$$

Loss from price dispersion is given by:

$$\int_0^1 (p-p_i)^2 d_i = (\phi_{y^*}-1)^2[2^{-2k_{y^*}}(1-2^{-2k_{y^*}})]\sigma_{y^*}^2 + (\phi_u+1)^2[2^{-2k_u}(1-2^{-2k_u})]\sigma_u^2 \quad \text{(B.5)}$$

2. Given some monetary policy, the pair of optimal allocation of attention (k_{y^*}, k_u) is characterized by

$$k_u = K - k_{y^*}, k_{y^*} = \begin{cases} 0, & if \frac{(\phi_{y^*}-1)^2\sigma_{y^*}}{(\phi_u+1)^2\sigma_u^2} \in [0, 2^{-2K}] \\ \frac{1}{2}K + \frac{1}{4}\log_2 \frac{(\phi_{y^*}-1)^2\sigma_{y^*}}{(\phi_u+1)^2\sigma_u^2}, & if \frac{(\phi_{y^*}-1)^2\sigma_{y^*}}{(\phi_u+1)^2\sigma_u^2} \in (2^{-2K}, 2^{2K}) \\ K, & if \frac{(\phi_{y^*}-1)^2\sigma_{y^*}}{(\phi_u+1)^2\sigma_u^2} \in [2^{2K}, \infty) \end{cases} \quad \text{(B.6)}$$

3. Suppose the monetary policy responses are in a range such that $\frac{(\phi_{y^*}-1)^2\sigma_{y^*}}{(\phi_u+1)^2\sigma_u^2} \in (0, 2^{-2K}]$, then the firms' optimal attention allocation is $k_{y^*} = 0$ and $k_u = K$. The loss from output gap fluctuation is $E(y-y^*)^2|_{k_u=K, k_{y^*}=0} = (\phi_{y^*}-1)^2\sigma_{y^*}^2 + [2^{-2K}(\phi_u+1)-1]^2\sigma_u^2$, and the loss from price dispersion is $\int_0^1 (p-p_i)^2 d_i|_{k_u=K,k_{y^*}=0} = (\phi_u+1)^2[2^{-2K}(1-2^{-2K})]\sigma_u^2$. Since when $\phi_{y^*} = 1$, the relevant loss from output gap fluctuation is minimized and it doesn't put a restriction on ϕ_u, $\phi_{y^*} = 1$ is optimal. Then solving the following optimization problem $\min_{\phi_u} E(y - y^*)^2 + \gamma \int_0^1 (p-p_i)^2 d_i$ yields the optimal response $\phi_u = -1 + \frac{1}{\gamma(1-2^{-2K})+2^{-2K}}$. Finally, we can check that with the monetary policy responses, the restriction that $\frac{(\phi_{y^*}-1)^2\sigma_{y^*}}{(\phi_u+1)^2\sigma_u^2} \in (0, 2^{-2K}]$ is satisfied. The loss at this policy is given by $\frac{\gamma(2^{2K}-1)}{\gamma(2^{2K-1})+1} \cdot \sigma_u^2 < \sigma_u^2$ which is the loss under flexible price.

4. Suppose that the monetary policy responses are in a range such that $\frac{(\phi_{y^*}-1)^2\sigma_{y^*}}{(\phi_u+1)^2\sigma_u^2} \in [2^{2K}, \infty)$, then under this policy, firms pay attention according to $k_u = 0$ and $k_{y^*} = K$. The loss from output gap fluctuation is $E(y-y^*)^2|_{k_{y^*}=K,k_u=0} = [2^{-2K}(\phi_{y^*}-1)]^2\sigma_{y^*}^2 + \phi_u^2\sigma_u^2$, and the loss from price dispersion is $\int_0^1 (p-p_i)^2 d_i|_{k_{y^*}=K,k_u=0} = (\phi_{y^*}-1)^2[2^{-2K}(1-2^{-2K})]\sigma_{y^*}^2$. The optimal responses are obtained by solving the following constrained optimization problem:

$$\begin{aligned} &\min_{\phi_{y^*},\phi_u} E(y-y^*)^2 + \gamma \int_0^1 (p-p_i)^2 d_i \\ &\text{subject to} \quad (\phi_{y^*}-1)^2\sigma_{y^*} \geq 2^{2K} \cdot (\phi_u+1)^2\sigma_u^2 \end{aligned} \quad \text{(B.7)}$$

The loss at the optimal monetary policy is given by $\frac{\gamma(1-2^{-2K})+2^{-2K}}{1+\gamma(1-2^{-2K})+2^{-2K}} \cdot \sigma_u^2 < \sigma_u^2$, which is the loss under flexible price.

5. Suppose that the monetary policy responses are in a range such that $\frac{(\phi_{y^*}-1)^2\sigma_{y^*}}{(\phi_u+1)^2\sigma_u^2} \in (2^{-2K}, 2^{2K})$, solving the optimization problem gives $\phi_u = -1 + \frac{\gamma}{\gamma^2 - 2^{-2K}(1-\gamma)^2}$ and $\phi_{y^*} = 1 - \frac{(1-\gamma)2^{-2K}}{\gamma^2-2^{-2K}(1-\gamma)^2} \cdot \frac{\sigma_u}{\sigma_{y^*}}$. At this policy, total loss is given by $\frac{\gamma^2+(1-\gamma)\gamma 2^{-2K}}{\gamma^2-2^{-2K}(1-\gamma^2)} \cdot \sigma_u^2 > \sigma_u^2$, which is the loss under flexible price.

6. Comparing the welfare losses across the cases shown from step 3-5, it's clear that case in step 5 can be eliminated immediately. Comparing the results in the other two cases, it can be shown that if and only if $\gamma(2^{2K}-1)^2 \le 1$, loss under the case in step 3 is smaller than that in step 4. This finishes the proof for Proposition 4.

C Solution to Firms' Attention Allocation Problem (3.8)

Case 1: $2^{-4K} < \frac{\sigma_u^2}{\sigma_q^2}\frac{\sigma_{y^*}^2}{\sigma_q^2} < 2^{2K}$, $2^{-2K} < \frac{\sigma_u^2}{\sigma_q^2}\frac{\sigma_u^2}{\sigma_{y^*}^2} < 2^{4K}$, and $2^{-2K} < \frac{\sigma_{y^*}^2}{\sigma_q}\frac{\sigma_{y^*}^2}{\sigma_u^2} < 2^{4K}$

$$k_{y^*}^* = \frac{1}{3}K + \frac{1}{3}\log 2\frac{\sigma_{y^*}^2}{\sigma_q^2} - \frac{1}{6}\log 2\frac{\sigma_u^2}{\sigma_q^2}$$
$$k_u^* = \frac{1}{3}K + \frac{1}{3}\log 2\frac{\sigma_u^2}{\sigma_q^2} - \frac{1}{6}\log 2\frac{\sigma_{y^*}^2}{\sigma_q^2}$$
$$k_q^* = \frac{1}{3}K - \frac{1}{6}\log 2\frac{\sigma_{u^*}^2}{\sigma_q^2} - \frac{1}{6}\log 2\frac{\sigma_{y^*}^2}{\sigma_q^2}$$

Case 2: $\frac{\sigma_{y^*}^2}{\sigma_q}\frac{\sigma_{y^*}^2}{\sigma_u^2} \le 2^{2K}$ and $2^{-2K} < \frac{\sigma_q^2}{\sigma_u^2} < 2^{2K}$

$$k_{y^*}^* = 0, \qquad k_u^* = \frac{1}{2}K - \frac{1}{4}\log 2\frac{\sigma_q^2}{\sigma_u^2}, \qquad k_q^* = \frac{1}{2}K + \frac{1}{4}\log 2\frac{\sigma_q^2}{\sigma_u^2}$$

Case 3: $\frac{\sigma_u^2}{\sigma_q^2}\frac{\sigma_u^2}{\sigma_{y^*}^2} \le 2^{-2K}$ and $2^{-2K} < \frac{\sigma_q^2}{\sigma_{y^*}^2} < 2^{2K}$

$$k_{y^*}^* = \frac{1}{2}K - \frac{1}{4}\log 2\frac{\sigma_q^2}{\sigma_{y^*}^2}, \qquad k_u^* = 0, \qquad k_q^* = \frac{1}{2}K + \frac{1}{4}\log 2\frac{\sigma_q^2}{\sigma_{y^*}^2}$$

Case 4: $\frac{\sigma_u^2}{\sigma_q^2}\frac{\sigma_{y^*}^2}{\sigma_q^2} \ge 2^{2K}$ and $2^{-2K} < \frac{\sigma_{y^*}^2}{\sigma_u^2} < 2^{2K}$

$$k_{y^*}^* = \frac{1}{2}K + \frac{1}{4}\log 2\frac{\sigma_{y^*}^2}{\sigma_u^2}, \qquad k_u^* = \frac{1}{2}K - \frac{1}{4}\log 2\frac{\sigma_{y^*}^2}{\sigma_u^2}, \qquad k_q^* = 0$$

Case 5: $\frac{\sigma_q^2}{\sigma_u^2} \geq 2^{2K}$ and $\frac{\sigma_q^2}{\sigma_{y^*}^2} \geq 2^{2K}$

$$k_{y^*}^* = 0, \qquad k_u^* = 0, \qquad k_q^* = K$$

Case 6: $\frac{\sigma_u^2}{\sigma_{y^*}^2} \geq 2^{2K}$ and $\frac{\sigma_u^2}{\sigma_q^2} \geq 2^{2K}$

$$k_{y^*}^* = 0, \qquad k_u^* = K, \qquad k_q^* = 0$$

Case 7: $\frac{\sigma_{y^*}^2}{\sigma_u^2} \geq 2^{2K}$ and $\frac{\sigma_{y^*}^2}{\sigma_q^2} \geq 2^{2K}$

$$k_{y^*}^* = K, \qquad k_u^* = 0, \qquad k_q^* = 0$$

D Proof for Proposition 5

Case I: $\frac{\sigma_{y^*}^2}{\sigma_u^2} \geq 2^{2K}$

Under the parameter restrictions, the firms allocate their attention according to case 7, case 5, and case 3 depending on the policy. To minimize price dispersion, central bank's optimal policy is to drive firms' attention to case 7.

If the policy drives firms' optimal attention to case 7, the loss from price dispersion becomes:

$$E[(p_i - p)^2]_7 = (2^{-2K} - 2^{-4K})\sigma_{y^*}^2 \tag{D.1}$$

If the policy drives firms' optimal attention to Case 5, the loss from price dispersion becomes:

$$E[(p_i - p)^2]_5 = (2^{-2K} - 2^{-4K})\sigma_q^2 \geq (2^{-2K} - 2^{-4K})\sigma_{y^*}^2 \tag{D.2}$$

where the inequality comes from the restriction of firms' optimal attention allocation falling into case 5.

If the policy drives firms' optimal attention to case 3, the loss from price dispersion becomes:

$$E[(p_i - p)^2]_3 = 2 \cdot 2^{-K}\sigma_{y^*}\sigma_q - 2^{-2K}\sigma_{y^*}^2 - 2^{-2K}\sigma_q^2 \tag{D.3}$$

We then show that $E[(p_i-p)^2]_7 \leq E[(p_i-p)^2]_3$.

$$\begin{aligned} E[(p_i-p)^2]_3 - E[(p_i-p)^2]_7 &= 2\cdot 2^{-K}\sigma_{y^*}\sigma_q - 2^{-2K}\sigma_{y^*}^2 - 2^{-2K}\sigma_q^2 - (2^{-2K}-2^{-4K})\sigma_{y^*}^2 \\ &= -(\sigma_{y^*} - 2^{-K}\sigma_q)^2 + (1-2^{-2K})^2\sigma_{y^*}^2 \\ &= \sigma_{y^*}^2 2^{-K}(\frac{\sigma_q}{\sigma_{y^*}} - 2^{-K})(1-2^{-2K}+1-2^{-K}\frac{\sigma_q}{\sigma_{y^*}}) \geq 0 \end{aligned} \tag{D.4}$$

Both the second and the third terms are nonnegative due to the restriction for case 3 attention solution: $2^{-K} \leq \frac{\sigma_q}{\sigma_{y^*}} \leq 2^K$.

Case II: $2^{-2K} \leq \frac{\sigma_{y^*}^2}{\sigma_u^2} \leq 2^{2K}$

Under the parameter restrictions, the firms allocate their attention according to case 5, case 4, and case 1 depending on the policy. To minimize price dispersion, central bank's optimal policy is to drive firms' attention to case 4.

If the policy drives firms' optimal attention to case 4, the loss from price dispersion becomes:

$$E[(p_i-p)^2]_4 = 2\cdot 2^{-K}\sigma_u\sigma_{y^*} - 2^{-2K}\sigma_u^2 - 2^{-2K}\sigma_{y^*}^2 \tag{D.5}$$

If the policy drives firms' optimal attention to case 1, the loss from price dispersion in as follows:

$$\begin{aligned} E[(p_i-p)^2]_1 =& 3\cdot(2^{-K}\sigma_u\sigma_{y^*}\sigma_q)^{\frac{2}{3}} \\ &- 2^{-\frac{4}{3}K}(\sigma_u^{\frac{4}{3}}\sigma_{y^*}^{\frac{4}{3}}\sigma_q^{-\frac{2}{3}} + \sigma_q^{\frac{4}{3}}\sigma_{y^*}^{\frac{4}{3}}\sigma_u^{-\frac{2}{3}} + \sigma_u^{\frac{4}{3}}\sigma_q^{\frac{4}{3}}\sigma_{y^*}^{-\frac{2}{3}}) \\ &\geq 3\cdot 2^{-K}\sigma_u\sigma_{y^*} - 2^{-K}\sigma_u\sigma_{y^*} - 2^{-2K}\sigma_{y^*}^2 - 2^{-2K}\sigma_u^2 = E[(p_i-p)^2]_4 \end{aligned} \tag{D.6}$$

The last inequality uses the restriction $2^{-K}\sigma_u\sigma_{y^*} \leq \sigma_q^2 \leq 2^{-2K}\sigma_u\sigma_{y^*}$ so that the firms' attention is according to case 1.

If the policy drives firms' optimal attention to case 5, the loss from price dispersion becomes:

$$E[(p_i-p)^2]_5 = (2^{-2K}-2^{-4K})\sigma_q^2 \tag{D.7}$$

We then show that $E[(p_i - p)^2]_4 \leq E[(p_i - p)^2]_5$.

$$\begin{aligned} E[(p_i - p)^2]_4 &= -(2^{-K}\sigma_u - \sigma_{y^*})^2 + (1 - 2^{-2K})\sigma_{y^*}^2 \\ &\leq -(2^{-K}\sigma_u - \sigma_{y^*})^2 + (1 - 2^{-2K})2^{-2K}\sigma_q^2 \\ &\leq E[(p_i - p)^2]_5 \end{aligned} \tag{D.8}$$

The second line is from the restriction for firms attention allocation being case 5.

Case III: $\frac{\sigma_{y^*}^2}{\sigma_u^2} \leq 2^{-2K}$

This case is symmetric to Case I. Under the restriction, the firms allocate their attention according to case 6, case 5, and case 2. To minimize price dispersion, central bank's optimal policy is to drive firms' attention to case 6.

If the policy drives firms' optimal attention to case 6, the loss from price dispersion becomes:

$$E[(p_i - p)^2]_6 = (2^{-2K} - 2^{-4K})\sigma_u^2 \tag{D.9}$$

If the policy drives firms' optimal attention to case 5, the loss from price dispersion becomes:

$$E[(p_i - p)^2]_5 = (2^{-2K} - 2^{-4K})\sigma_q^2 \geq (2^{-2K} - 2^{-4K})\sigma_u^2 \tag{D.10}$$

since $\frac{\sigma_q^2}{\sigma_u^2} \geq 2^{2K}$ must be satisfied for case 5.

If the policy drives firms' optimal attention to case 2, the loss from price dispersion becomes:

$$E[(p_i - p)^2]_2 = 2 \cdot 2^{-K}\sigma_u\sigma_q - 2^{-2K}\sigma_u^2 - 2^{-2K}\sigma_q^2 \tag{D.11}$$

We then show that $E[(p_i - p)^2]_2 - E[(p_i - p)^2]_6 \geq 0$.

$$\begin{aligned} E[(p_i - p)^2]_2 - E[(p_i - p)^2]_6 &= 2 \cdot 2^{-K}\sigma_u\sigma_q - 2^{-2K}\sigma_u^2 - 2^{-2K}\sigma_q^2 - (2^{-2K} - 2^{-4K})\sigma_u^2 \\ &= -(\sigma_u - 2^{-K}\sigma_q)^2 + (1 - 2^{-2K})^2\sigma_u^2 \\ &= \sigma_u^2 2^{-K}(\frac{\sigma_q}{\sigma_u} - 2^{-K})(1 - 2^{-2K} + 1 - 2^{-K}\frac{\sigma_q}{\sigma_u}) \geq 0 \end{aligned} \tag{D.12}$$

The second and third terms are nonnegative due to restriction for Case 3 attention solution: $2^{-K} \leq \frac{\sigma_q}{\sigma_u} \leq 2^K$.

E Proof for Proposition 6

The output gap is given by

$$\begin{aligned} y - y^* &= 2^{-2k_q^*} q - 2^{-2k_{y^*}^*} y^* - (1 - 2^{-2k_u^*}) u \\ &= (2^{-2k_q^*} \phi_{y^*} - 2^{-2k_{y^*}^*}) y^* + (2^{-2k_q^*} \phi_u - (1 - 2^{-2k_u^*})) u \end{aligned} \tag{E.1}$$

Case I: $\frac{\sigma_{y^*}^2}{\sigma_u^2} \geq 2^{2K}$

Under this parameter restriction, depending on central bank's policy, the possible attention allocation by firms are case 7, case 5, and case 3 as shown by figure 5. Under each attention allocation case, there is a solution to stabilize output gap completely.

If firms' attention allocation ends up with Case 5 where $k_q^* = K$ and $k_{y^*}^* = k_u^* = 0$, we must have $\phi_{y^*} = 2^{2K}$ and $\phi_u = 0$. We then need to check that under this policy, the firms will pay attention in the form of case 5. $\frac{\sigma_q^2}{\sigma_u^2} = 2^{4K} \frac{\sigma_{y^*}^2}{\sigma_u^2} \geq 2^{6K}$ is indeed greater than 2^{2K}. And $\frac{\sigma_q^2}{\sigma_{y^*}^2} = 2^{4K}$ is greater than 2^{2K}.

If firms' attention allocation ends up with case 7 where $k_{y^*}^* = K$ and $k_u^* = k_q^* = 0$, to fully stabilize the output gap, we must have:

$$\phi_{y^*} - 2^{-2K} = 0$$

$$1 \cdot \phi_u - 0 = 0$$

The only possible response is $\phi_{y^*} = 2^{-2K}$ and $\phi_u = 0$. We again need to check that under this policy response, the firms actually end up with paying attention in the form of case 7. $\frac{\sigma_q^2}{\sigma_{y^*}^2} = 2^{-4K}$ is indeed smaller than 2^{-2K}.

If firms' attention allocation ends up with case 3 where $k_{y^*}^* = \frac{1}{2}K - \frac{1}{4}\log 2\frac{\sigma_q^2}{\sigma_{y^*}^2}$,$k_u^* = 0$, and $k_q^* = \frac{1}{2}K + \frac{1}{4}\log 2\frac{\sigma_q^2}{\sigma_{y^*}^2}$,to fully stabilize output, we must have:

$$\phi_{y^*} = 2^{2k_q^* - 2k_{y^*}^*} = \frac{\sigma_q^2}{\sigma_{y^*}^2} = \phi_{y^*}^2$$

$$\phi_u = 0$$

Then we have $\phi_{y^*} = 1, \phi_u = 0$ or $\phi_{y^*} = 0, \phi_u = 0$. Again, we need to check under the policy, the firms will end with case 3 attention allocation, i.e., $\frac{\sigma_u^2}{\sigma_q^2}\frac{\sigma_u^2}{\sigma_{y^*}^2} \leq 2^{-2K}$ and $2^{-2K} < \frac{\sigma_q^2}{\sigma_{y^*}^2} < 2^{2K}$ should be satisfied and that excludes the case where $\phi_{y^*} = 0$, $\phi_u = 0$. And under the optimal response, $k^*_{y^*} = k^*_q = \frac{K}{2}$ and $k^*_u = 0$.

Case II: $2^{-2K} \leq \frac{\sigma^2_{y^*}}{\sigma^2_u} \leq 2^{2K}$

Under this parameter restriction, the possible attention allocation by firms are case 5, case 4, and case 1.

If the firms' attention allocation ends up with case 5 where $k^*_q = K$, $k^*_u = k^*_y = 0$, setting $\phi_{y^*} = 2^{2K}$ and $\phi_u = 0$ again can fully stabilize output gap. We then check under this policy, the firms pay attention according to Case 5: $\frac{\sigma^2_q}{\sigma^2_{y^*}} = \phi^2_{y^*} = 2^{4K} \geq 2^{2K}$ and $\frac{\sigma^2_q}{\sigma^2_u} = \phi^2_{y^*}\frac{\sigma^2_{y^*}}{\sigma^2_u} \geq 2^{4K} \cdot 2^{-2K} = 2^{2K}$.

If the firms' attention allocation ends up with case 4 where $k^*_{y^*} = \frac{1}{2}K + \frac{1}{4}\log 2\frac{\sigma^2_{y^*}}{\sigma^2_u}, k^*_u = \frac{1}{2}K - \frac{1}{4}\log 2\frac{\sigma^2_{y^*}}{\sigma^2_u}$, and $k^*_q = 0$, to fully stabilize output gap, we must have

$$\phi_{y^*} = 2^{-2k^*_{y^*}} = 2^{-K}\frac{\sigma_u}{\sigma_{y^*}}$$
$$\phi_u = 1 - 2^{-K}\frac{\sigma_{y^*}}{\sigma_u}$$

To make sure that under this policy, the firms' attention allocation falls into case 4, $\frac{\sigma^2_u}{\sigma^2_q}\frac{\sigma^2_{y^*}}{\sigma^2_q} \geq 2^{2K}$ should be satisfied. Plug the policy into the inequality, we have the following restriction on the parameter space:

$$2^{-2K} - 2^{-K}\frac{\sigma_{y^*}}{\sigma_u} + (1 - 2^{-K}\frac{\sigma_{y^*}}{\sigma_u})^2 \leq 0 \tag{E.2}$$

Take partial derivative of the LHS w.r.t. $\frac{\sigma_{y^*}}{\sigma_u}$, we have $\frac{\partial LHS}{\partial \sigma_{y^*}/\sigma_u} = 2 \cdot 2^{-K}\frac{\sigma_{y^*}}{\sigma_u} - 3 < 0$, because $2^{-K} < \frac{\sigma_{y^*}}{\sigma_u} < 2^K$. As σ_{y^*}/σ_u increases, the LHS decreases. Solve the constraint as a function of $\frac{\sigma_{y^*}}{\sigma_u}$, the above constraint becomes $\sigma_{y^*}/\sigma_u \in [\frac{3 \cdot 2^K - \sqrt{5 \cdot 2^{2K} - 4}}{2}, \frac{3 \cdot 2^K + \sqrt{5 \cdot 2^{2K} - 4}}{2}]$.

If the firms' attention allocation ends up with case 1, existence of the solution to fully output gap stabilization depends on the parameter values. The analytical solution is complex, but it doesn't affect the conclusions in Proposition 6 and Proposition 7. So it's omitted here.

Case III: $\frac{\sigma^2_{y^*}}{\sigma^2_u} \leq 2^{-2K}$

Under this parameter restriction, the possible firms attention allocation cases are case 6, case 5, and case 2. The solution depends on the parameter values.

If the firms' attention allocation is in case 5, then the only possible full output stabilization policy should be $\phi_{y^*} = 2^{2K}$ and $\phi_u = 0$. However, given this policy the firms won't allocate attention according to case 5. $\frac{\sigma^2_q}{\sigma^2_u} = \phi^2_{y^*}\frac{\sigma^2_{y^*}}{\sigma^2_u} \leq 2^{2K}$ cannot be greater 2^{2K}. The equality case can be included in case II.

If the firms' attention allocation is in case 6 where $k_{y^*}^* = k_q^* = 0$ and $k_u^* = K$, to fully stabilize output gap $\phi_{y^*} = 1$ and $\phi_u = 1 - 2^{-2K}$. To make sure that under this policy, the firms end up with case 6, the restriction that $\frac{\sigma_q^2}{\sigma_u^2} = \phi_{y^*}^2 \frac{\sigma_{y^*}^2}{\sigma_u^2} + \phi_u^2 \leq 2^{-2K}$ needs to be satisfied, which can be rearranged as:

$$\frac{\sigma_{y^*}^2}{\sigma_u^2} \leq 2^{-2K} - (1 - 2^{-2K})^2 \tag{E.3}$$

Due to the non-negativity of LHS, we have a further restriction to information capacity K:

$$K \in (0, -\frac{1}{2} \log 2 \frac{3 - \sqrt{5}}{2}) \tag{E.4}$$

If the firms' attention allocation ends up with Case 2 where $k_{y^*}^* = 0, k_u^* = \frac{1}{2}K - \frac{1}{4}\log 2 \frac{\sigma_q^2}{\sigma_u^2}$, and $k_q^* = \frac{1}{2}K + \frac{1}{4}\log 2 \frac{\sigma_q^2}{\sigma_u^2}$, monetary policy can fully stabilize output by setting

$$\phi_{y^*} = 2^{2k_q^*} = 2^K \frac{\sigma_q}{\sigma_u} \quad \Longrightarrow \quad \phi_{y^*}^2 (1 - 2^{2K} \frac{\sigma_{y^*}^2}{\sigma_u^2}) = 2^{2K} \phi_u^2$$

$$\phi_u = 2^{2k_q^*} - 2^{2(k_q^* - k_u^*)} = \phi_{y^*} - \phi_{y^*}^2 \frac{\sigma_{y^*}^2}{\sigma_u^2} - \phi_u^2$$

Solve the above system, we get $\phi_{y^*} = 2^{2K}(1 - \sqrt{2^{-2K} - \frac{\sigma_{y^*}^2}{\sigma_u^2}})$ and $\phi_u = 2^{2K}(\sqrt{2^{-2K} - \frac{\sigma_{y^*}^2}{\sigma_u^2}} - 2^{-2K} + \frac{\sigma_{y^*}^2}{\sigma_u^2})$. We then check that under the policy firms pay attention as Case 2. The following condition needs to be satisfied:

$$2^{-2K} \leq \frac{\sigma_q^2}{\sigma_u^2} = \phi_{y^*}^2 2^{-2K} \leq 2^{2K} \tag{E.5}$$

$$\frac{\sigma_{y^*}^2}{\sigma_q^2} \frac{\sigma_{y^*}^2}{\sigma_u^2} \leq 2^{2K} \tag{E.6}$$

If (E.5) is satisfied, (E.6) is satisfied automatically due to $\frac{\sigma_{y^*}^2}{\sigma_u^2} \leq 2^{-2K}$, thus the restriction boils down to:

$$\frac{\sigma_{y^*}^2}{\sigma_u^2} \geq 2^{-2K} - (1 - 2^{-2K})^2 \tag{E.7}$$

Combining the above two situations, we can see under Case III when $\sigma_{y^*}^2 / \sigma_u^2 \leq 2^{-2K}$, there always exists a solution for the central bank to stabilize output gap completely. And under any given parameter, the solution is unique. The central bank cannot stabilize output gap as flexibly as in Case I and Case II.

F Proof for Proposition 7

Compare the table in from proposition 5 and 6, Proposition 7 can be derived quickly. Under the same parameter space, if the policy achieving fully output stabilization and optimal price dispersion, then there is no trade-off between output gap and price dispersion. Since output gap can always be completely stabilized, whenever the two objectives are not consistent, there is a trade-off.