X-Ray

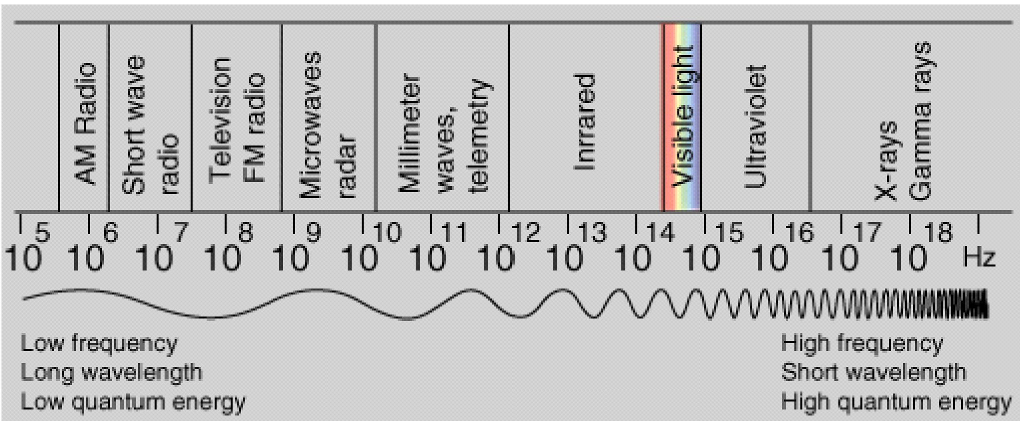
L12

Goal: To learn the physics behind x-rays, some of its uses and limitations.

Note: Much of the content here can also be found in section 2.1 of Alexei Ramotar's MMath thesis
http://hdl.handle.net/10012/2914 (PDF in UW Library, link also in D2L)

What Are X-Rays?

X-rays are a form of electromagnetic radiation, in the same spectrum as visible light & radio waves.



http://hyperphysics.phy-astr.gsu.edu/hbase/ems1.html

In 1895, Wilhelm Roentgen discovered these rays that could pass through wood & human tissues. He called them "x-rays", the "x" being a place-holder for the unknown.

http://en.wikipedia.org/wiki/Wilhelm_R%C3%B6ntgen

http://en.wikipedia.org/wiki/Xrays

X-Ray Physics

Like light, x-rays can be thought of as a particle or a wave.

Energy of an x-ray photon:

$$E = hf = h\frac{c}{\lambda}$$

h: Planck's constant
f: frequency
c: speed of light
λ: wavelength

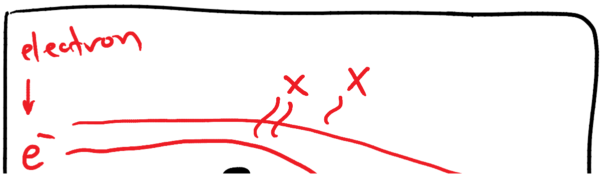
(thanks Einstein, 1905)

Producing X-Rays

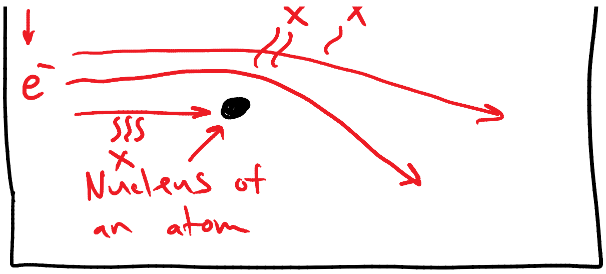
To make x-rays, you can shoot high-energy electrons into matter. We will discuss 2 such interactions:

1) Bremsstrahlung Radiation

- e^- is decelerated by charge of the nucleus.
- energy of photon

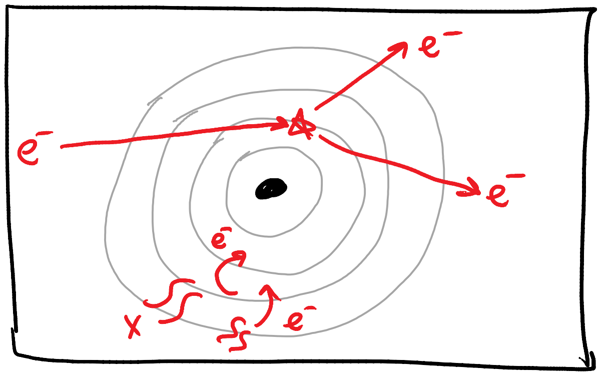


- energy of photon depends on charge of nucleus, & on trajectory of e^-.

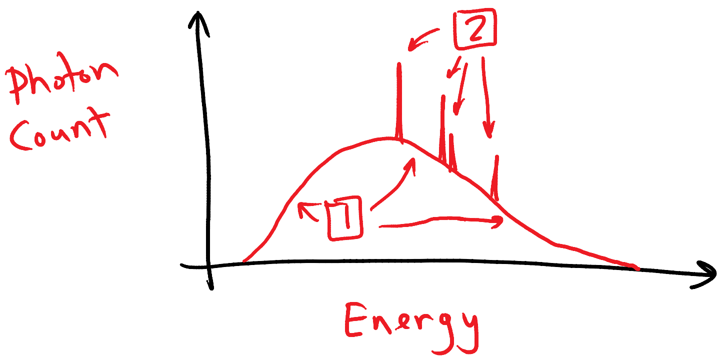


2) Characteristic Radiation

- knocks an e^- out of its shell, causing a cascade of e^-s dropping shells.
- energy of photon depends on the binding energies of the different electron shells.



Resulting X-Ray Spectrum:



In medical-imaging applications, x-rays are sent into tissue (the tissue is "irradiated").

X-Ray Interactions with Matter

We will discuss 3 common interactions:

1) Photoelectric Effect (earned Albert Einstein the Nobel Prize in Physics 1921)

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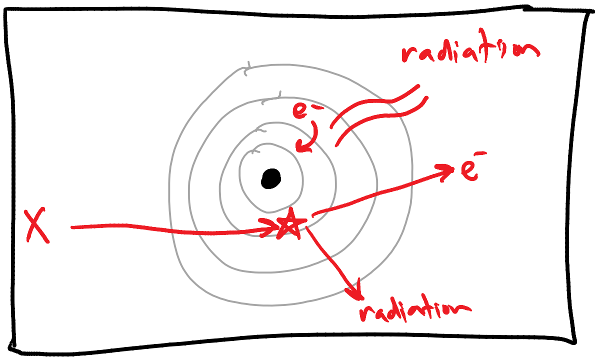


- an x-ray photon ejects an e^- from its shell ...
- probability of interaction

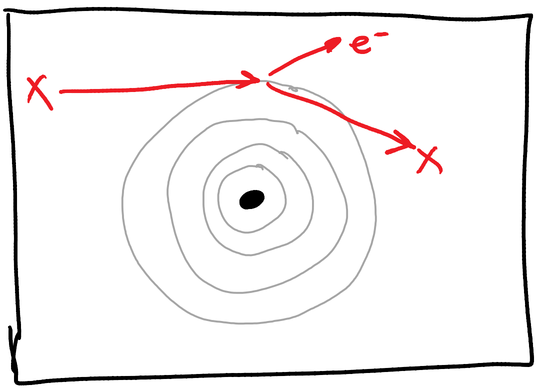
$$P \propto Z^3$$

↑ atomic # of atom

This effect is the principal effect that makes x-ray useful. Different tissues have different concentrations of atoms with high atomic #, so have different probabilities for this interaction. Thus, different tissues have different x-ray attenuation abilities.

X-ray images are essentially x-ray attenuation maps.

2) Compton interaction

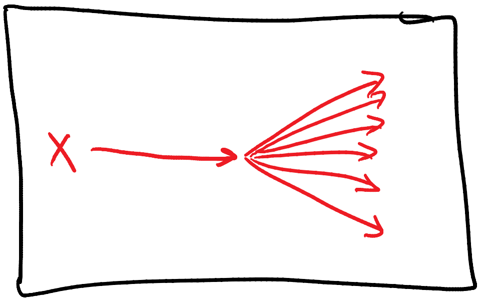


- ejects e^- in outer shell, so x-ray keeps most of its energy.
- but x-ray changes direction
- probability of this interaction

$$P \propto e^- \text{ density}$$

Since tissues tend to have little variation in e^--density, this interaction does not give us information about what is inside the body. → nuisance

3) Rayleigh (Coherent) Scattering



- no transfer of energy
- but a change in direction

This interaction also does not give us anatomical information. → nuisance

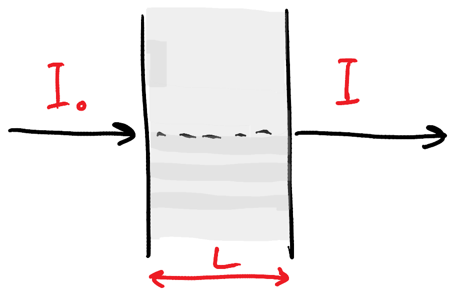
X-Ray Attenuation

Compton

$$I = I_0 e^{-(\tau + \sigma + \sigma_r)L}$$

Photoelectric

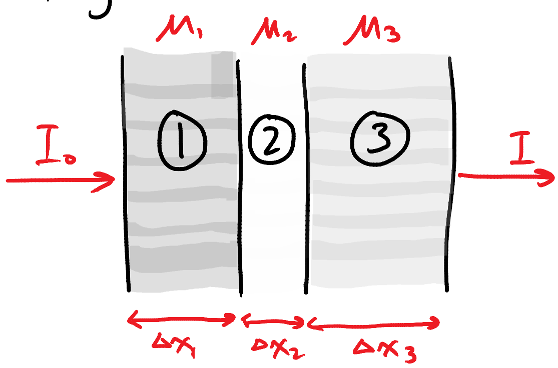
Rayleigh



Or, more simply,

$$I = I_0 e^{-\mu L}$$ "Beer-Lambert Law"

Now let's put a few layers together.



Exiting 1

$$I_1 = I_0 e^{-\mu_1 \Delta x_1}$$

This is the intensity that enters 2

$$I_2 = I_1 e^{-\mu_2 \Delta x_2}$$

$$I_3 = I_2 e^{-\mu_3 \Delta x_3}$$

$$I_3 = I_2 e^{-\mu_3 \Delta x_3}$$

Chain them all together...

$$I = I_0 e^{-\mu_1 \Delta x_1} e^{-\mu_2 \Delta x_2} e^{-\mu_3 \Delta x_3}$$

$$= I_0 e^{-(\mu_1 \Delta x_1 + \mu_2 \Delta x_2 + \mu_3 \Delta x_3)}$$

$$= I_0 e^{-\sum_n \mu_n \Delta x_n}$$

Take the limit as $\Delta x \to 0$

$$I = I_0 e^{-\int \mu(s)\, ds}$$

attenuation at s

Thus $\boxed{\ln \frac{I}{I_0} = -\int \mu(s)\, ds}$ ← Radon transform

The beam intensity (# of x-ray photons) that transmits through the tissue and emerges on the other side encodes attenuation information for its entire path.

Chest x-ray

The problem with x-ray images is that all structures are projected on top of each other, superimposed on the image superimposed on the image.

http://www.flickr.com/photos/evanskrig/2141439225/

Hounsfield Units

There is an absolute scale for x-ray attenuation coefficients.

$$H = \frac{\mu - \mu_{water}}{\mu_{water}} \times 1000$$

eg. $H_{air} = -1000$ HU

$H_{soft\ tissue} = -100$ to 60 HU

$H_{water} = 0$ HU

$H_{bone} = 250$ to 1000 HU