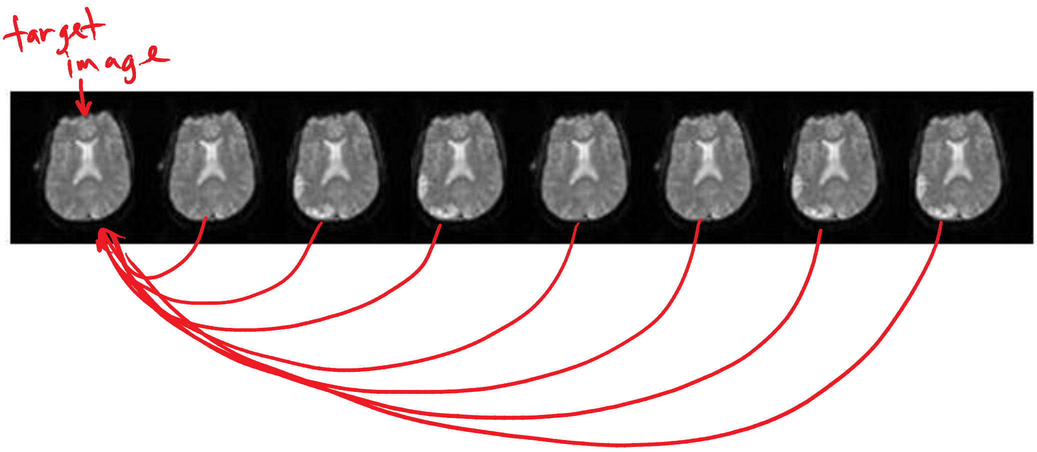
Correlation

L 26

Goal: Investigate correlation as a registration cost function, and how it can be computed.

Generally, this family of methods and cost functions is geared toward monomodal registration (aligning images of the same modality, eg. CT to CT).

A good example is motion correction for functional MRI (FMRI). A series of MRI snapshots are taken, yielding a whole time series for each pixel. These pixel time series are statistically analyzed. However, if the snapshots are not properly aligned, the pixel time series will be mixed and disrupted. This is a monomodal registration scenario. Typically, one snapshot is chosen as the "fixed" or "reference" or "target" image, and all other images are registered to it.



A typical FMRI experiment can easily have 100 to 200 images, and each snapshot can also be a volume.

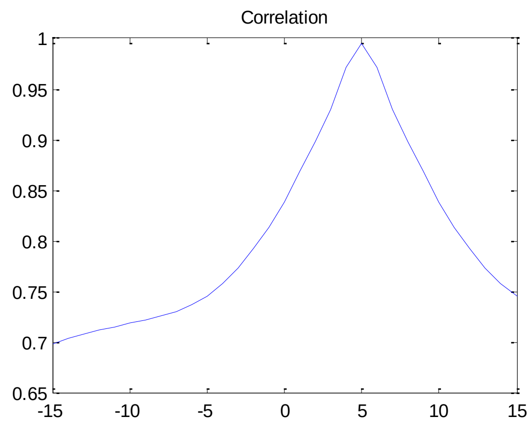
For such a monomodal registration scenario, one could use **cross-correlation** as a cost function.

$$CC(f,g) = \frac{\sum_{m,n} f_{mn} g_{mn}}{\sqrt{\sum_{mn} f_{mn}^2 \cdot \sum_{mn} g_{mn}^2}}$$

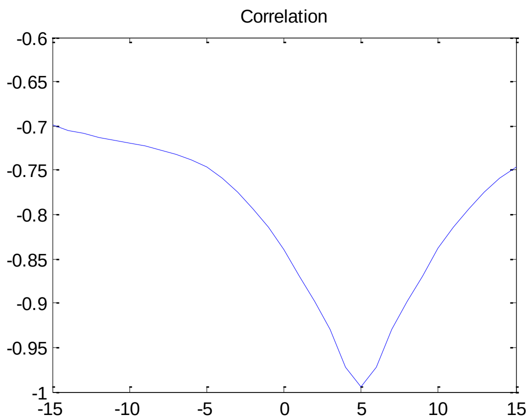
If $CC=1 \Rightarrow$ f & g are positively correlated
i.e. $f = \alpha g$ for some $\alpha > 0$

If $CC=0 \Rightarrow$ f & g are not correlated
i.e. f & g are independent (orthogonal)

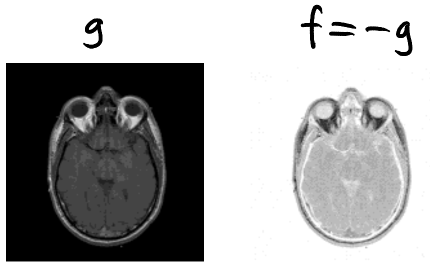
If $CC=-1 \Rightarrow$ f & g are negatively correlated
i.e. $f = \alpha g$ for some $\alpha < 0$



Same T1-weighted MRI used in the SAD example in L25. Max occurs at correct registration.



If the image intensity is negated, the correct registration gives a minimum.



One of the great things about cross-correlation is that it can be formulated as a **convolution** and evaluated efficiently for all integer translations using the FFT.

Consider a shifted version of f

$$g_{mn} = f_{m-a,n-b} = T(a,b) f_{mn}$$

To find the optimal shift, we can compute $CC(T(a,b)f, g)$ for all integer shifts (a,b) and choose the shift that gives the largest value (or absolute value).

$$CC(T(a,b)f, g) = \frac{\sum_{mn} f_{m-a,n-b}\, g_{mn}}{\sqrt{\sum_{mn} f_{mn}^2 \cdot \sum_{mn} g_{mn}^2}}$$

The numerator is almost a convolution.

Let $\bar{f}_{mn} = f_{-m,-n} \Rightarrow f_{m-a,n-b} = \bar{f}_{a-m,b-n}$

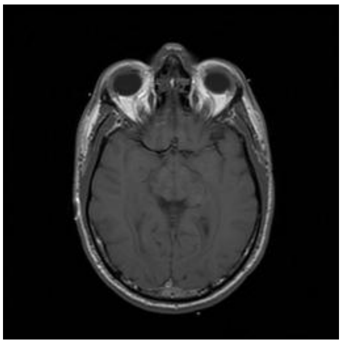
Then

$$CC(T(a,b)f, g) = \frac{\sum_{mn} \bar{f}_{a-m,b-n}\, g_{mn}}{\sqrt{\sum_{mn} \bar{f}_{mn}^2 \cdot \sum_{mn} g_{mn}^2}}$$

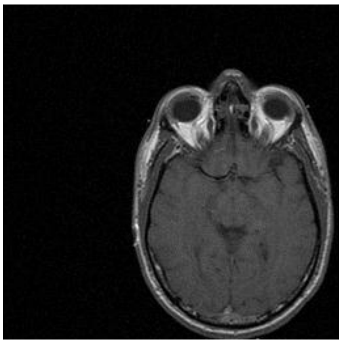
$$= \frac{(\bar{f} * g)_{ab}}{\sqrt{\sum_{mn} \bar{f}_{mn}^2 \cdot \sum_{mn} g_{mn}^2}}$$

$$= \frac{1}{\sqrt{\sum \bar{f}^2 \sum g^2}} \mathcal{F}^{-1}\left\{ \mathcal{F}\{\bar{f}\}_{kl}\, \mathcal{F}\{g\}_{kl} \right\}_{ab}$$

f



$g = f$ shifted $+$ noise



Maximum occurs at (a,b) that corresponds to correct shift.

$CC(T(a,b)f, g)$