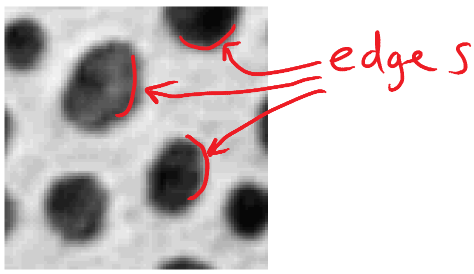
Edge Detection

L22

Goal: To lay the foundation for detecting and using the edges in image content.

Edges in images are boundaries between regions with different intensities.

eg.

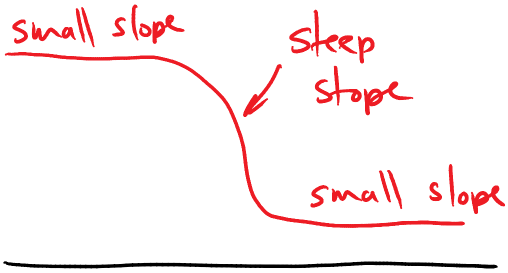


There are many reasons to want to locate the edges.

- delineate organ boundaries
- align images

Edges are also used in other image-processing operations (anisotropic diffusion, level-set segmentation, etc.)

Anatomy of an Edge



An edge is indicated by a rapid change in intensity

⇒ Gradient vector with a large magnitude

So if we compute the gradient, we can get the edges.

Approximating Image Derivatives

Recall, for $f(x, y, z)$, $\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{bmatrix}$

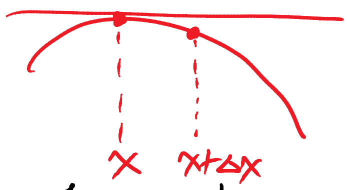
By definition,

$$\frac{\partial f}{\partial x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

If we approximate $f(x+\Delta x)$ using Taylor's formula,

$$f(x+\Delta x) = f(x) + \frac{\partial f}{\partial x}\Delta x + O(\Delta x^2)$$

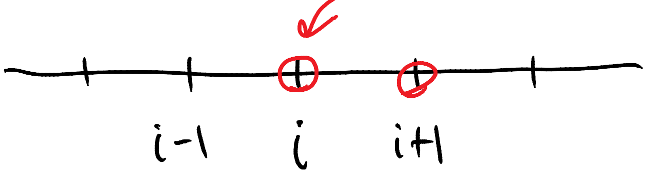
$$\Rightarrow \frac{\partial f}{\partial x} \approx \frac{f(x+\Delta x) - f(x)}{\Delta x}$$



For images, we can treate Δx as 1 pixel

$$\frac{\partial f_i}{\partial x} \approx f_{i+1} - f_i$$

This is known as **forward differencing**. It involves the pixel where the derivative is desired, as well as the pixel in the positive direction.



The opposite is called **backward differencing.**

$$\frac{\partial f_i}{\partial x} \approx f_i - f_{i-1}$$

$$\frac{\partial f_i}{\partial x} \approx f_i - f_{i-1}$$

Central differencing: Start with Taylor's for both...

$$f(x+\Delta x) = f(x) + \frac{\partial f}{\partial x}\Delta x + O(\Delta x^2)$$

$$-f(x-\Delta x) = f(x) - \frac{\partial f}{\partial x}\Delta x + O(\Delta x^2)$$

$$f(x+\Delta x) - f(x-\Delta x) = 2\frac{\partial f}{\partial x}\Delta x$$

$$\Rightarrow \frac{\partial f}{\partial x} \approx \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x}$$

$$\text{or} \quad \frac{\partial f_i}{\partial x} \approx \frac{f_{i+1} - f_{i-1}}{2}$$

Central differencing is $O(\Delta x^2)$ accurate, while forward and backward differencing are both $O(\Delta x)$.

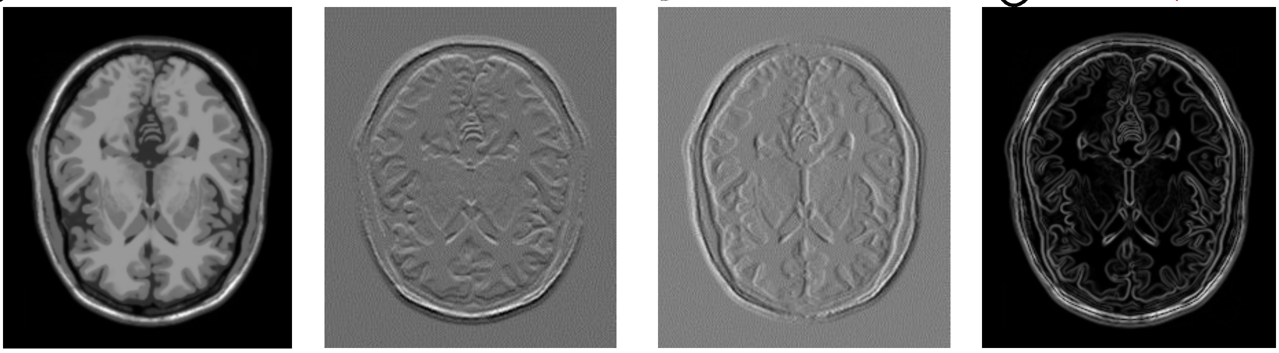
Image Gradient

For each pixel, compute its gradient $\left(\frac{\partial f_{ij}}{\partial r}, \frac{\partial f_{ij}}{\partial c}\right)$.

```
f = imread('t1.jpg');
f = double( f(:,:,1) );
imshow(f,[])

dfdr = ( circshift(f,[-1 0]) - circshift(f,[1 0]) ) / 2;
imshow(dfdr,[])

dfdc = ( circshift(f,[0 -1]) - circshift(f,[0 1]) ) / 2;
imshow(dfdc,[])

grad_mag = sqrt(dfdr.^2 + dfdc.^2);
imshow(grad_mag, [])
```

① f ② $\frac{\partial f}{\partial r}$ ③ $\frac{\partial f}{\partial c}$ ④ $\|\nabla f\|$

1 f 2 $\frac{\partial f}{\partial r}$ 3 $\frac{\partial f}{\partial c}$ 4 $\|\nabla f\|$



BrainWeb simulated MRI data http://www.bic.mni.mcgill.ca/brainweb