Speed Functions

L42

Goal: Find out how to control the speed of the level set for image segmentation.

Recall that the embedding function ϕ evolves according to the PDE $\frac{\partial \phi}{\partial t} = -V_N \|\nabla \phi\|$.

The speed function, V_N , can be any smooth scalar function. To use level sets to segment images, we use image information (such as image gradients, etc.) to determine V_N.

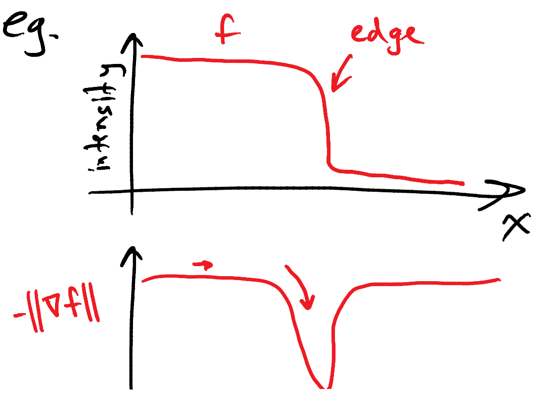
Edges:

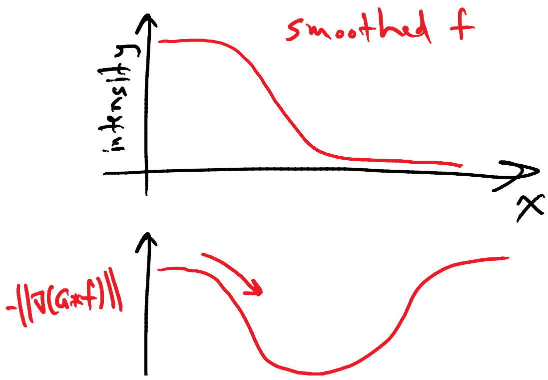
If we want the curve to stop at strong edges, then V_N has to be **close to zero** when the gradient of our image is **large** (where have we seen this before?).

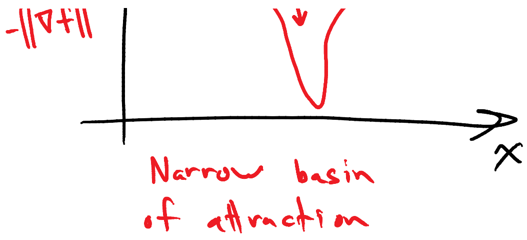
$$g(r) = e^{-\alpha r}$$

It is common to use the gradient of the **smoothed** image to widen the basin of attraction.

eg.







$$\therefore V_N = g\left(\|\nabla(G*f)\|\right) = e^{-\alpha\|\nabla(G*f)\|}$$

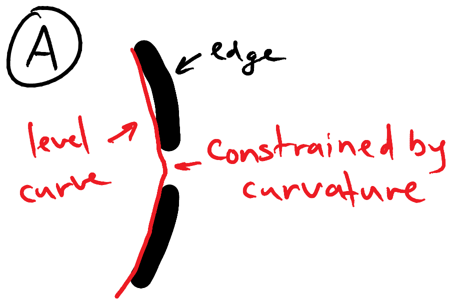
Another option:

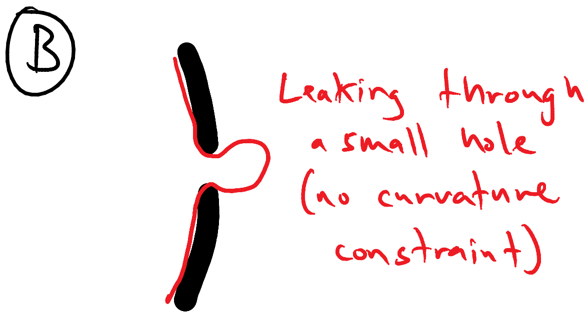
$$g(r) = \frac{1}{1+|r|}$$

Curvature:

We can also add a component to the speed function that keeps the curve **smooth.** This is similar to the rigidity term in the snakes formulation. One of its purposes is to prevent the set from **"leaking"** through tiny openings in the edge.

eg.



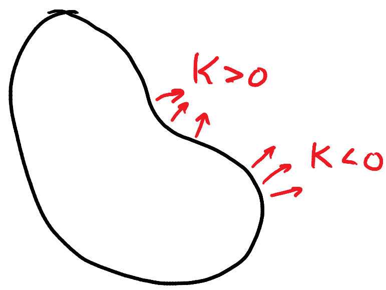


In **B**, the curve kept going where the edge was **absent** because the speed function only depended on the gradient magnitude. However, in **A** the curve stops because leaking through the hole would require the curve to take on a very high **curvature**, something that is discouraged if the speed function is chosen appropriately.

Curvature, κ, can be computed from the embedding function using

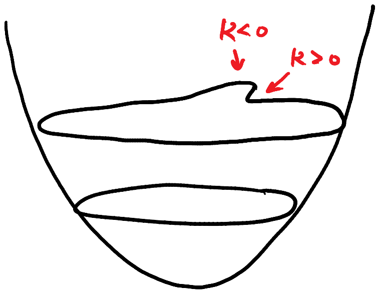
$$\kappa = -\nabla \cdot \frac{\nabla \phi}{\|\nabla \phi\|}$$

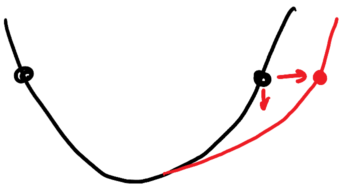
$$= -\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right) \cdot \underbrace{\frac{\left(\frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y}\right)}{\|\nabla \phi\|}}_{\text{unit normal vector}}$$



How can curvature influence ϕ? Let $V_N \propto \kappa$.

$$\frac{\partial \phi}{\partial t} = -\varepsilon \kappa \|\nabla \phi\|$$





If $\kappa > 0 \Rightarrow \frac{\partial \phi}{\partial t} < 0$

Hence, ϕ decreases which pushes the curve out.

If $\kappa < 0 \Rightarrow \frac{\partial \phi}{\partial t} > 0$

Hence, ϕ increases which puches the curve in.

Putting these speed factors together,

$$\frac{\partial \phi}{\partial t} = -g\left(\|\nabla(G * f)\|\right)\left(V_0 + \varepsilon \kappa\right)\|\nabla \phi\|$$

where

g is one of the edge-stopping functions

V_0 is a constant inflation (+) or deflation (-) speed

κ is curvature $(-\nabla \cdot \nabla \phi / \|\nabla \phi\|)$

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ε is a chosen smoothness constant