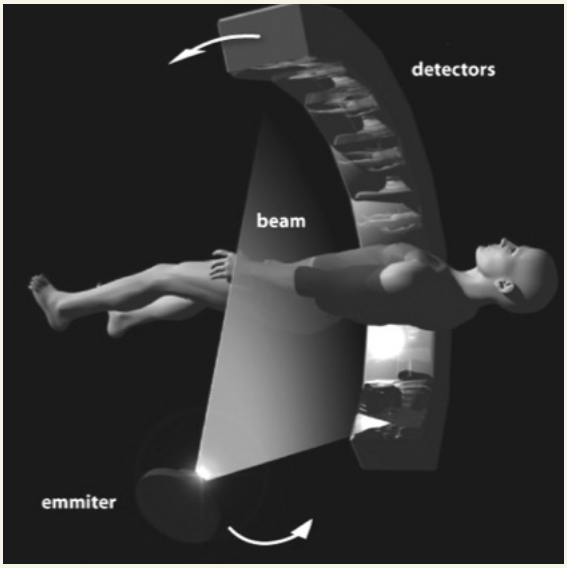
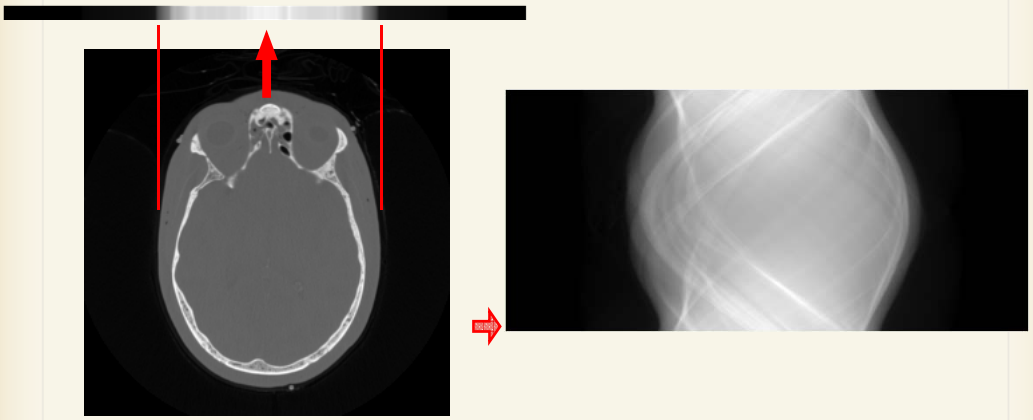
http://www.auh.dk/sks/afd/billeddi/dk/

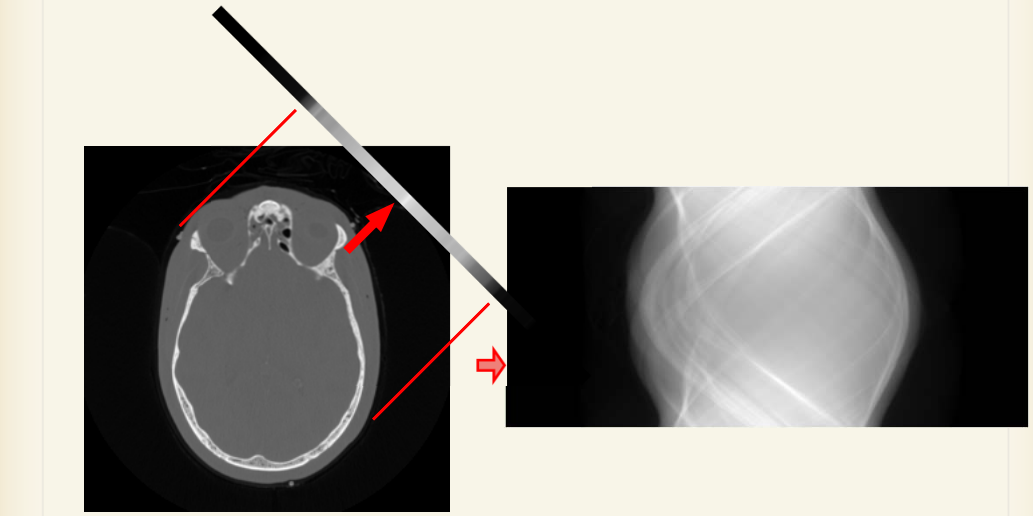
Algebraic CT Reconstruction



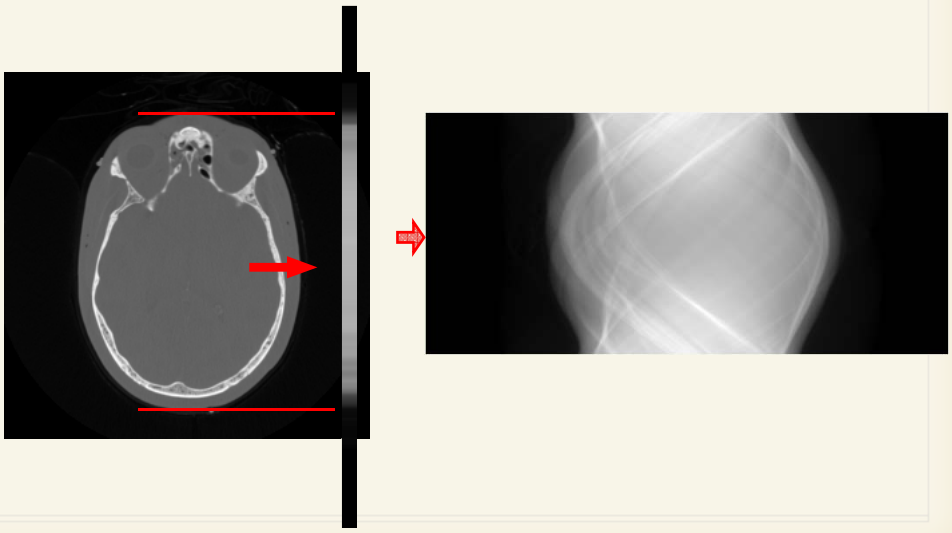
A slice from many small x-rays

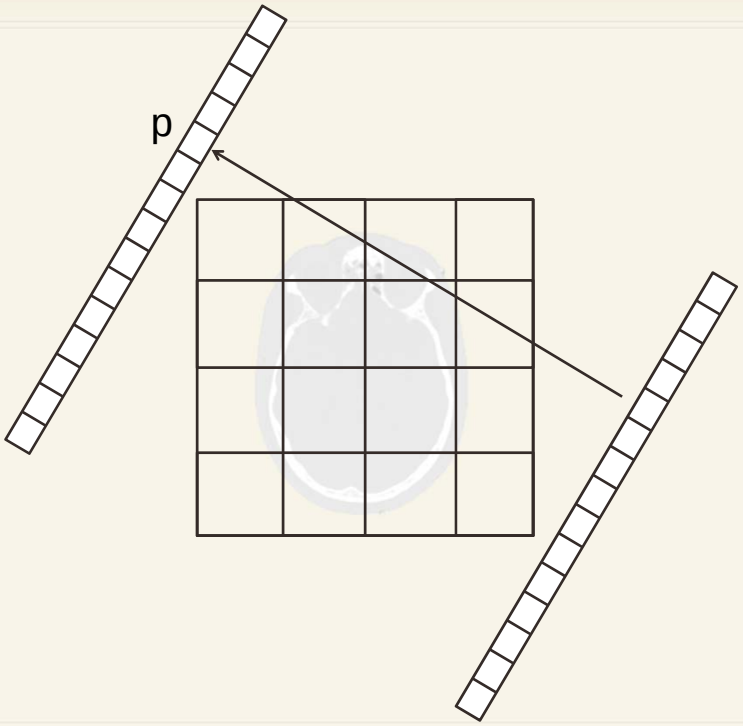


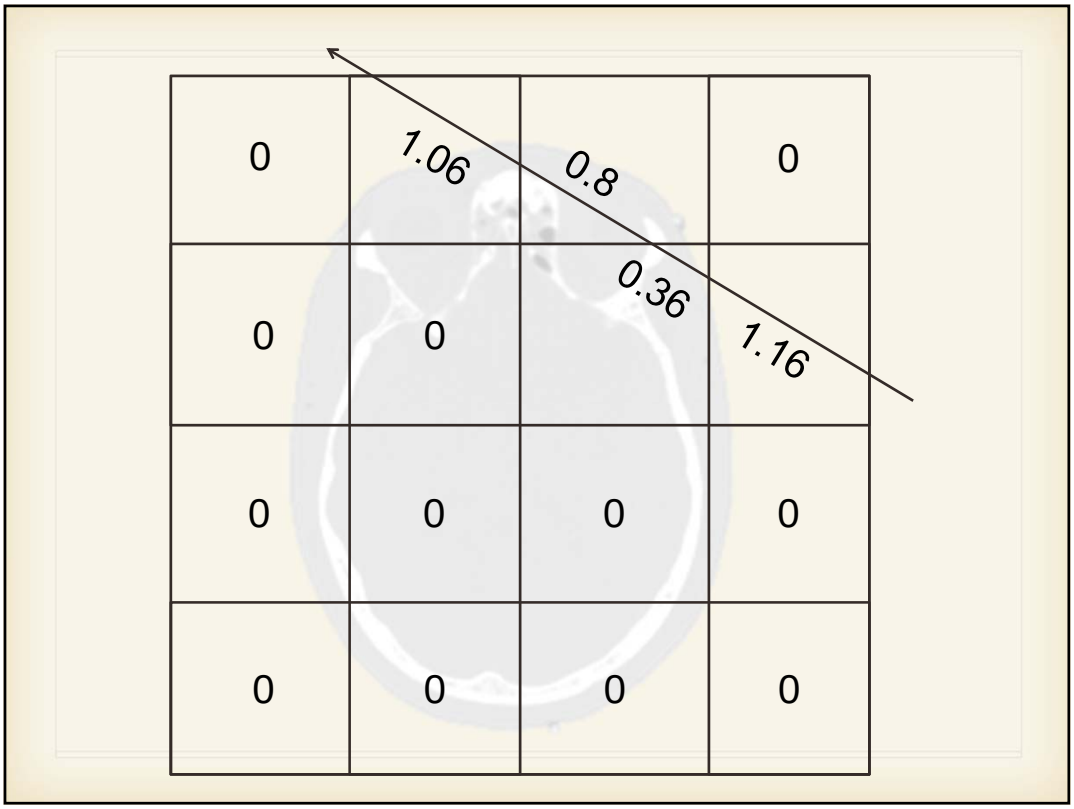
A slice from many small x-rays



A slice from many small x-rays

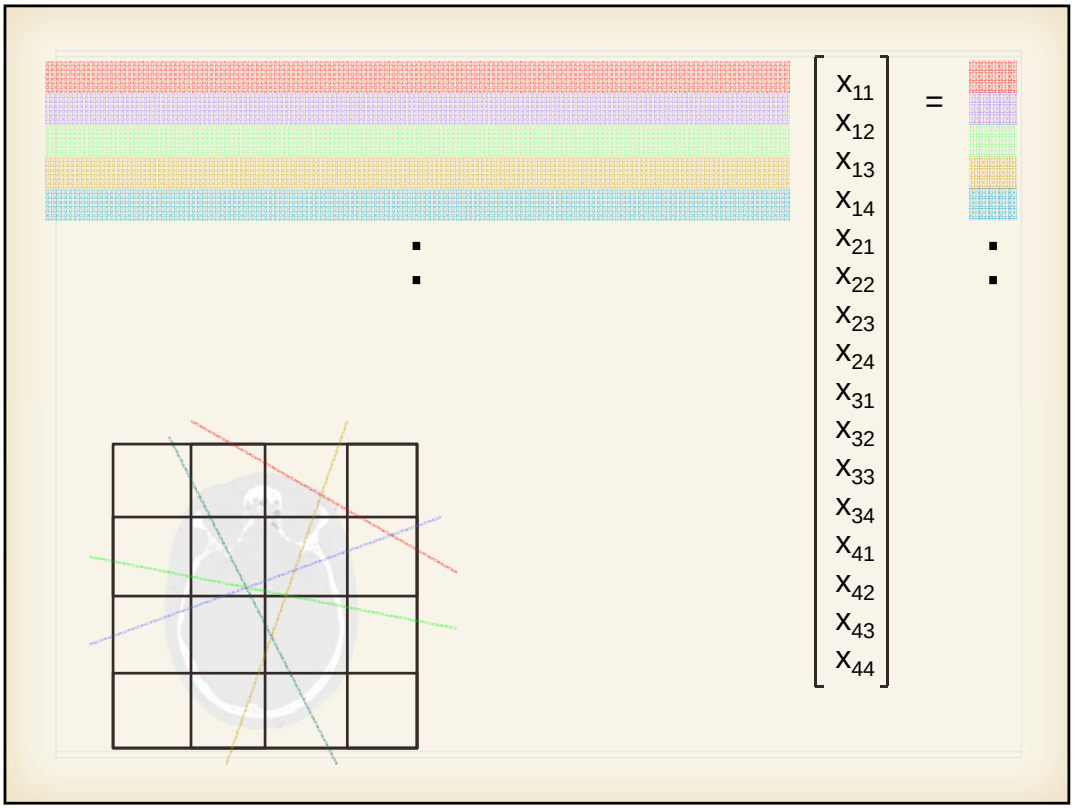






$$[\,0\ \ 1.06\ 0.8\ \ 0\ \ 0\ \ 0\ \ 0.36\ \ 1.16\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\,] \begin{bmatrix} x_{11} \\ x_{12} \\ x_{13} \\ x_{14} \\ x_{21} \\ x_{22} \\ x_{23} \\ x_{24} \\ x_{31} \\ x_{32} \\ x_{33} \\ x_{34} \\ x_{41} \\ x_{42} \\ x_{43} \\ x_{44} \end{bmatrix} = p$$

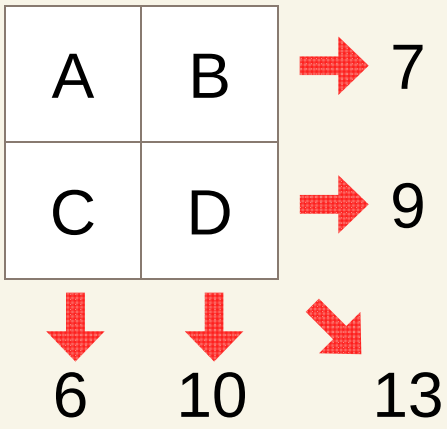
$0x_{11}$	$1.06x_{12}$	$0.8x_{13}$	$0x_{14}$
$0x_{21}$	$0x_{22}$	$0.36x_{23}$	$1.16x_{24}$
$0x_{31}$	$0x_{32}$	$0x_{33}$	$0x_{34}$
$0x_{41}$	$0x_{42}$	$0x_{43}$	$0x_{44}$



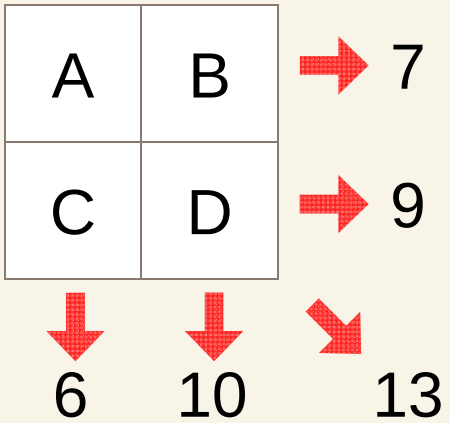
Arithmetic Reconstruction

5	2	→	7
1	8	→	9
↓ 6	↓ 10	↘	13

Arithmetic Reconstruction

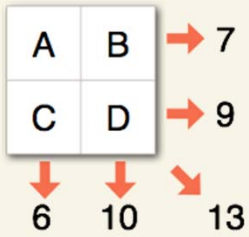


Arithmetic Reconstruction



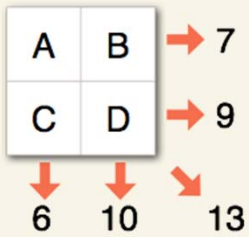
A+B=7
C+D=9
A+D=13
A+C=6
B+D=10

Arithmetic Reconstruction



$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} 7 \\ 9 \\ 13 \\ 6 \end{bmatrix}$$

Arithmetic Reconstruction



$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} \approx \begin{bmatrix} 7 \\ 9 \\ 13 \\ 6 \\ 10 \end{bmatrix} ?$$

$$\mathbf{MX} \approx \mathbf{P}$$

Least-Squares Solution

$$\mathbf{MX} = \mathbf{P}$$

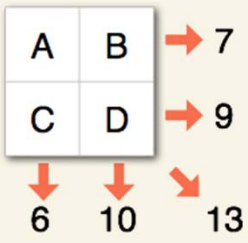
$$\mathbf{M}^{\mathrm{T}}\mathbf{MX} = \mathbf{M}^{\mathrm{T}}\mathbf{P}$$

$$\mathbf{X} = \left(\mathbf{M}^{\mathrm{T}}\mathbf{M}\right)^{-1}\mathbf{M}^{\mathrm{T}}\mathbf{P}$$

$$\text{Minimizes} \quad \|\mathbf{MX} - \mathbf{P}\|_2^2$$

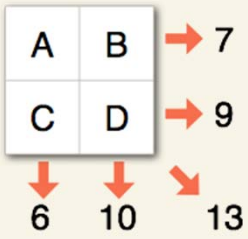
Iterative Reconstruction

A	B
C	D



Iterative Reconstruction

3.5	3.5	→ 7
4.5	4.5	→ 9

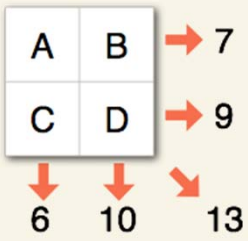


Distribute row-sums across each row.

Iterative Reconstruction

3.5	3.5
4.5	4.5

↓ 8 ↓ 8

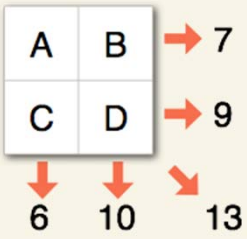


Split new residuals evenly over columns.

Iterative Reconstruction

2.5	4.5
3.5	5.5

6 10

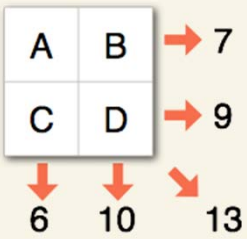


Split new residuals evenly over columns.

Iterative Reconstruction

2.5	4.5
3.5	5.5

8

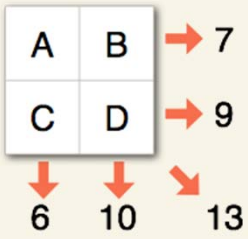


And now the diagonal vs. off-diagonals.

Iterative Reconstruction

5	2
1	8

13



And now the diagonal vs. off-diagonals.