## Convolution

L06

Goal: To define convolution, see what it does, and find out how it can be done using the Fourier transform.

Defin: The convolution between two functions f(n) and g(x) is an integral of the form  $(f*g)(x) = \int_{-\infty}^{\infty} f(t)g(x-t)dt$ 

For discrete signals  $f_n$  and  $g_n$ ,  $(f*g)_n = \sum_{k=0}^{N-1} f_k g_{n-k}$  n=0,...,N-1

Theorem: (continuous-domain version)

Let f(x) and g(x) be functions, and let  $F(w) = \mathcal{F}(f(x))(w)$  and  $G(w) = \mathcal{F}(g(x))(w)$ 

 $\cancel{\exists} f(f*g)(x) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(\tau) g(x-\tau) d\tau e^{-2\pi i \omega x} dx$ 

Change of variables: Let 
$$y = x - T = D x = y + T$$

$$dy = dx$$

$$= \iint_{\infty} f(t) g(y) dT e^{-2\pi i w} (y + T) dy$$

$$= \iint_{\infty} f(t) e^{-2\pi i w} dT \int_{-\infty}^{\infty} g(y) e^{-2\pi i w} dy$$

$$= F(w) \qquad G(w)$$

Note: One can just as easily prove the theorem  $\mathcal{F}'\{(F*G)(\omega)\}(x) = f(x)g(x)$ 

.: convolution in one domain is equivalent to element-wise multiplication in the other domain.

If f\*g(w) = F(w) G(w)

Conv. In spatial domain

D mult. In freq. domain

Fiff\*G](x) = f(x)g(x)

conv. in freq. domain

Denult. in spatial domain.

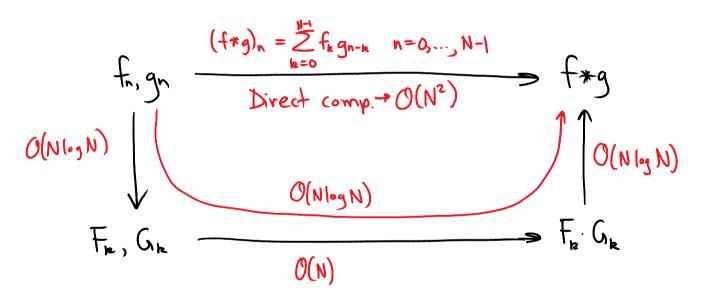
Theorem: (discrete-domain version)

Let  $f_n$  and  $g_n$ , n=0,...,N-1, be two discrete functions (signals).

TASK: Check out the Matlab script LOLe\_Convolution.m

Question: Why bother using the FT to compute convolution? Sure, it may be cool, but isn't it more work?

Consider the # of flops (floating-point operations) to compute (f\*g) using the 2 methods.



Thus, as N gets large, using the DFT is more efficient. What about convolving NxN images,  $f_{mn}$  and  $g_{mn}$ , where m=0,...,N-1 and n=0,...,N-1?

-> O(N2log N) flops