Introduction to Image Registration

L25

Goal: To define what registration is, and survey some methods.

Formulation

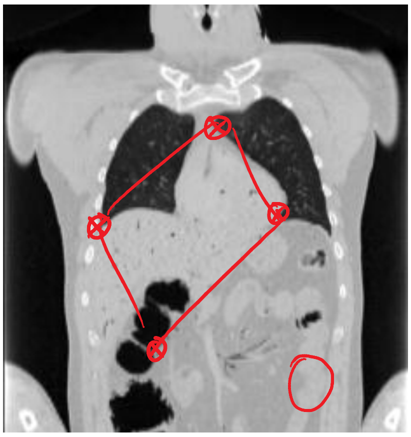
You have two images (or volumes) that you wish to register (align) so that they correspond on a pixel-by-pixel basis. Let the two images be *f* and *g*.
How does one go about registering *f* and *g*?

how to move f into alignment with g.

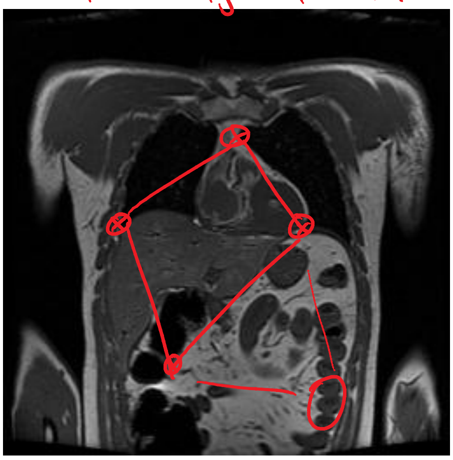
Point-Based

Someone who knows the anatomy can mark corresponding locations in both images and use those points to derive the transformation that moves *f* into alignment with *g*.

CT



T1-weighted MRI

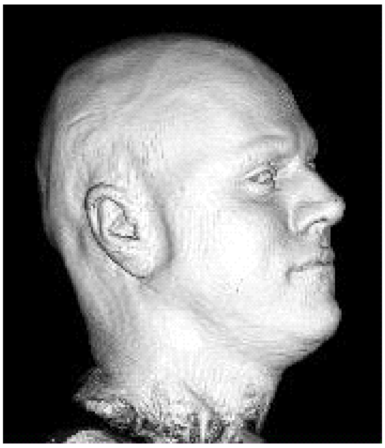


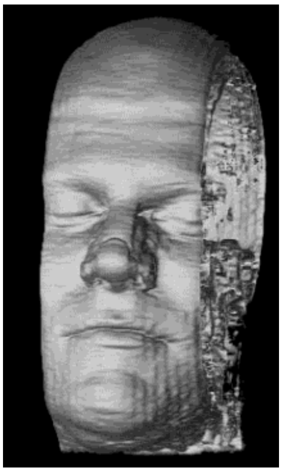
The problem is that an expert is needed, and it can be time-consuming. Also, content far from these matched points can have large registration errors.

Surface-Based

One can extract an iso-surface from each image (volume) and try to register the surfaces.

eg.





Each extracted from volumetric T1-weighted MRI.

The problem is that it can be difficut to automatically extract the same surface, from both volumes. Also, it doesn't guarantee a good fit away from the surface.

Intensity-Based

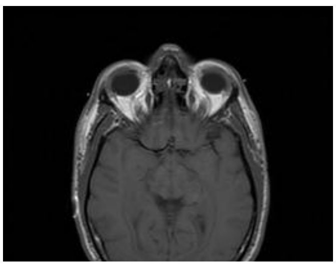
We can derive a "goodness-of-fit" value by comparing two images on a pixel-by-pixel basis. This measure is called a cost function or objective function, and is used to guage registration quality.

For example: Sum of Absolute Differences (SAD)

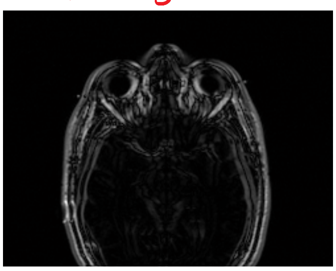
Given images $f, g \in \mathbb{R}^{M \times N}$

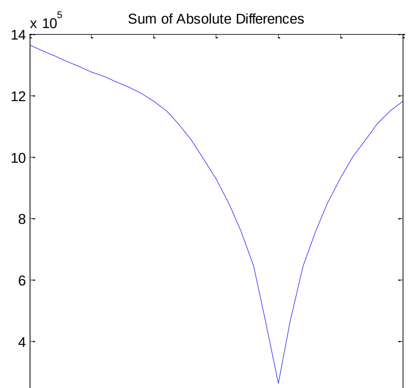
$$SAD(f,g) = \sum_{m=1}^{M} \sum_{n=1}^{N} |f_{mn} - g_{mn}| = \sum_{mn} |f_{mn} - g_{mn}|$$

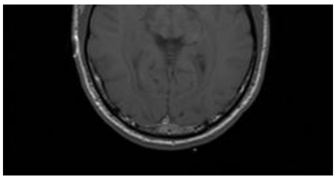
f

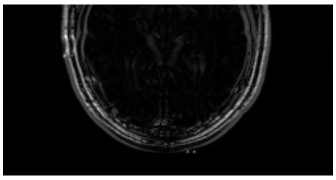


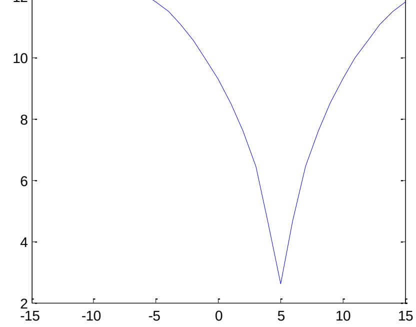
$|f-g|$









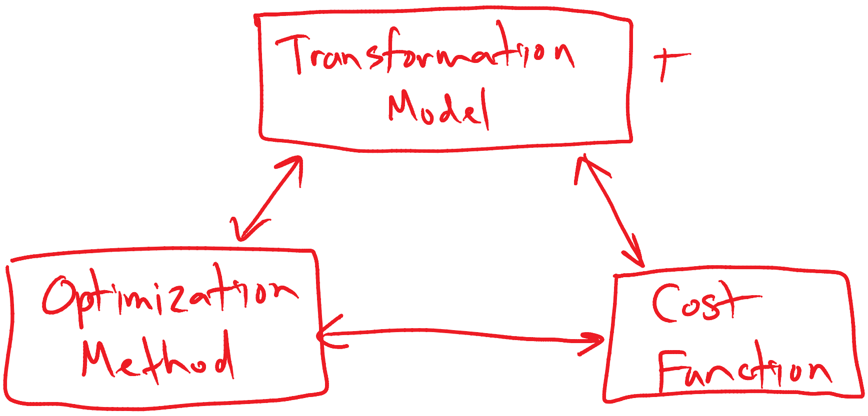


g is f, but offset by 5 pixels

Min of cost function corresponds to correct shift

Registration Process

Each registration process involves an interplay between three main components:



Each choice can potentially influence the others. We will look at a number of combinations.

In general:

- Let T represent a motion transformation.
  - Eg. Translation only, rotation only, rigid-body, affine, non-affine, etc.
  - We represent a transformed image as Tf .
    In this context, I call T an image operator because it applies the intended transform to the image itself. Alternatively, we could use the notation $f(Mx)$ where M is a coordinate operator, applied to the pixel locations

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$$(Tf)(x) = f(M^{-1}x)$$

Note that $T^{-1} \Leftrightarrow M$

Sometimes it is helpful to be explicit about the dependence of the transform on motion parameters,

Eg. $m = [\Delta row, \Delta col, \theta]^T$

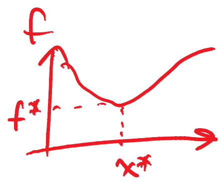
$$T(m)f$$

Ie. T is a function of m, but operates on f

- Let $d(f,g)$ be a cost function that quantifies the registration quality.

eg. $$d(f,g) = \sum_m \sum_n |f_{mn} - g_{mn}| \quad \text{for SAD}$$

- Then the registration problem boils down to the optimization problem



$$m = \arg\min_m d(T(m)f, g)$$

$$f^* = \min_x f(x)$$

$$x^* = \arg\min_x f(x)$$

or

$$m = \arg\max_m d(T(m)f, g)$$

depending on the cost function. For example, we minimize SAD, but maximize correlation (later). But our goal is to find the motion parameters m that achieve that optimal value.