Transform-Based Registration

L28

Goal: To see how we can take advantage of certain transformations for the purpose of registration.

Recall the Fourier Shift Theorem:

If $g_n = f_{n-d}$, then

$$G_k = e^{\frac{-2\pi i d k}{N}} F_k$$

Thus, to determine the shift, you can try to find a phase ramp that relates F_k and G_k.

$$\frac{G_k}{F_k} = e^{\frac{-2\pi i d k}{N}}$$

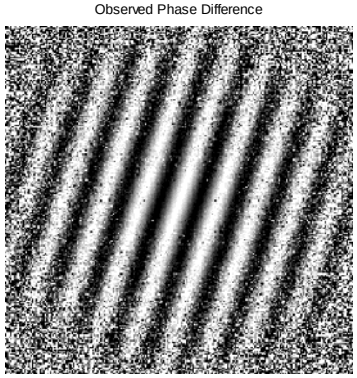
The slope of the ramp gives the relative shift → d

We can get a divide-by-zero if $F_k = 0$.
Instead, we can use the phase difference between G_k & F_k, which can be done using Matlab's **angle** function.

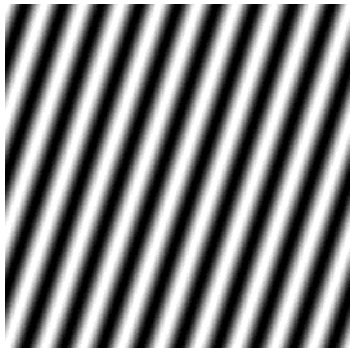
i.e. $\text{angle}(re^{i\theta}) = \theta$

(another awesome Matlab demo)

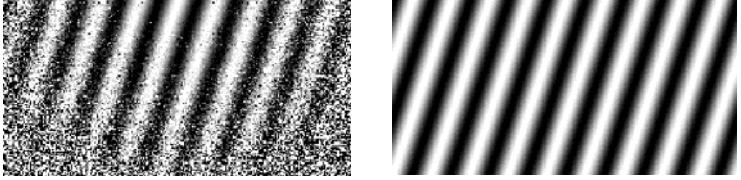
g is f shifted $\begin{bmatrix}3\\9\end{bmatrix}$ (+noise)



$\frac{G_{kl}}{F_{kl}} =$



$= e^{\frac{-2\pi i (3,9)\cdot(k,l)}{N}}$



However, as you can see, the phase difference gets progressively noisy in the high frequencies. But you can still rely on the low-frequency coefficients to try to estimate the wave front (frequency and direction).

Phase Correlation

Instead of looking in the frequency domain to estimate the phase ramp, let's look at the spatial-domain representation of the phase difference.

$$\mathcal{F}^{-1}\{\text{phase diff}\} = \sum_k e^{\frac{-2\pi i d k}{N}} e^{\frac{2\pi i n k}{N}}$$

$$= \sum_k e^{\frac{2\pi i k (n-d)}{N}}$$

$$= \delta_{n-d}$$

$\therefore$ it gives us a spike at d.

(cool Matlab demo continued)

Limitations of Phase Difference

These methods:

- Assume that the two images have the same **intensity mapping**
  (ie. corresponding objects in the two images have the same intensity, such as in monomodal images)
- Assume that the two images have the same **field of view.**
  The Fourier transform is a local↔ global transform

ie. changing 1 pixel changes all the Fourier coefficients, and changing 1 Fourier coef changes all pixels.
Hence, there seems to be no easy way to perform this type of registration on only a subimage.

- Only find the optimal **translation**
  There is a way to deal with rotations, though...