L44

Theory of MRI Reconstruction

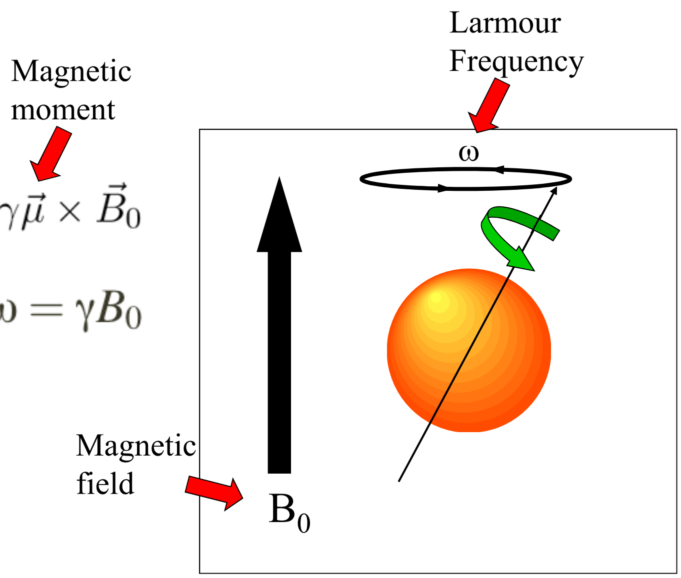
CS 473/673
Jeff Orchard

Goal: To find out how the MR signal can be turned into tomographic images.

Dipole Spin

$$\frac{d\vec{\mu}}{dt} = \gamma\vec{\mu} \times \vec{B}_0$$

$$\Rightarrow \omega = \gamma B_0$$



http://www.youtube.com/watch?v=IEwAry0GARw
http://www.youtube.com/watch?v=MOrk9ZQy1Dw&feature=share&list=ULMOrk9ZQy1Dw

Bloch's Equation

Net magnetization vector $\vec{M}$

$$\vec{M} = \vec{M}_x + \vec{M}_y + \vec{M}_z = M_x\vec{i} + M_y\vec{j} + M_z\vec{k}$$

Bloch's equation governs the behaviour of $\vec{M}$

$$\frac{d\vec{M}}{dt} = \boxed{\gamma\vec{M} \times \vec{B}_0} - \boxed{\frac{1}{T_2}\left(M_x\vec{i} + M_y\vec{j}\right)} - \boxed{\frac{1}{T_1}\left(M_z - M_0\right)\vec{k}}$$

Larmour precession | Transverse (x-y) decay | Longitudinal (z) decay

(see page 108 of Jeff's thesis for explanation of T_1 and T_2.)

Dynamics of M_{xy}

Recall:

$$\frac{d\vec{M}}{dt} = \gamma\vec{M} \times \vec{B}_0 - \frac{1}{T_2}\left(M_x\vec{i} + M_y\vec{j}\right) - \frac{1}{T_1}\left(M_z - M_0\right)\vec{k}$$

$$\frac{d\vec{M}}{dt} = \gamma\vec{M} \times \left(\vec{B}_0 + \vec{G}\cdot\vec{x}\right) - \frac{1}{T_2}\left(M_x\vec{i} + M_y\vec{j}\right) - \frac{1}{T_1}\left(M_z - M_0\right)\vec{k}$$

We introduce a gradient in the strength of the magnetic field.
The gradient is in the direction $\vec{x}$, with no z-component.

In matrix form…

$$\frac{d\vec{M}}{dt} = -\begin{bmatrix} \frac{1}{T_2} & -\gamma\left(B_0 + \vec{G}\cdot\vec{x}\right) & 0 \\ \gamma\left(B_0 + \vec{G}\cdot\vec{x}\right) & \frac{1}{T_2} & 0 \\ 0 & 0 & \frac{1}{T_1} \end{bmatrix}\vec{M} + \frac{1}{T_1}\vec{M}_0$$

Solution for M_{xy}

$$M_{xy}(x,y,t) = c\underbrace{e^{-i\gamma B_0 t}}_{\substack{\text{Normal precession}\\ \text{at Larmour}\\ \text{frequency}}}\overbrace{e^{\frac{-t}{T_2}}}^{\text{T2 Relaxation}}\underbrace{e^{-i(k_x x + k_y y)}}_{\substack{\text{Frequency/Phase}\\ \text{state (the } k\text{'s depend}\\ \text{on the gradients)}}}$$

$$k_x(t) = \int_0^t G_x(\tau)d\tau \qquad k_y(t) = \int_0^t G_y(\tau)d\tau$$

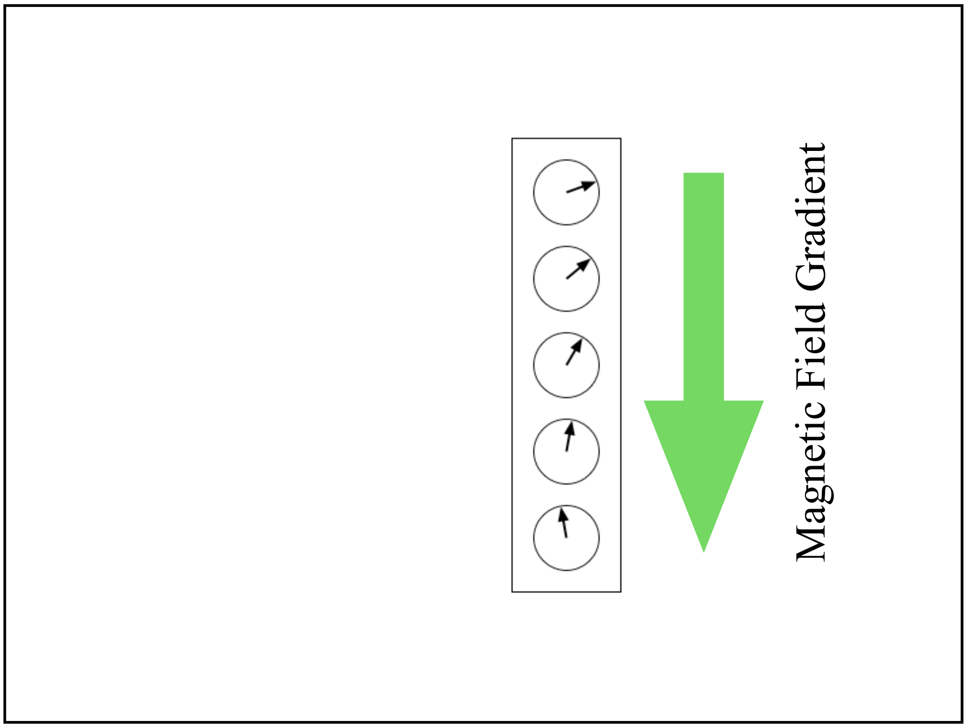
MR Signal

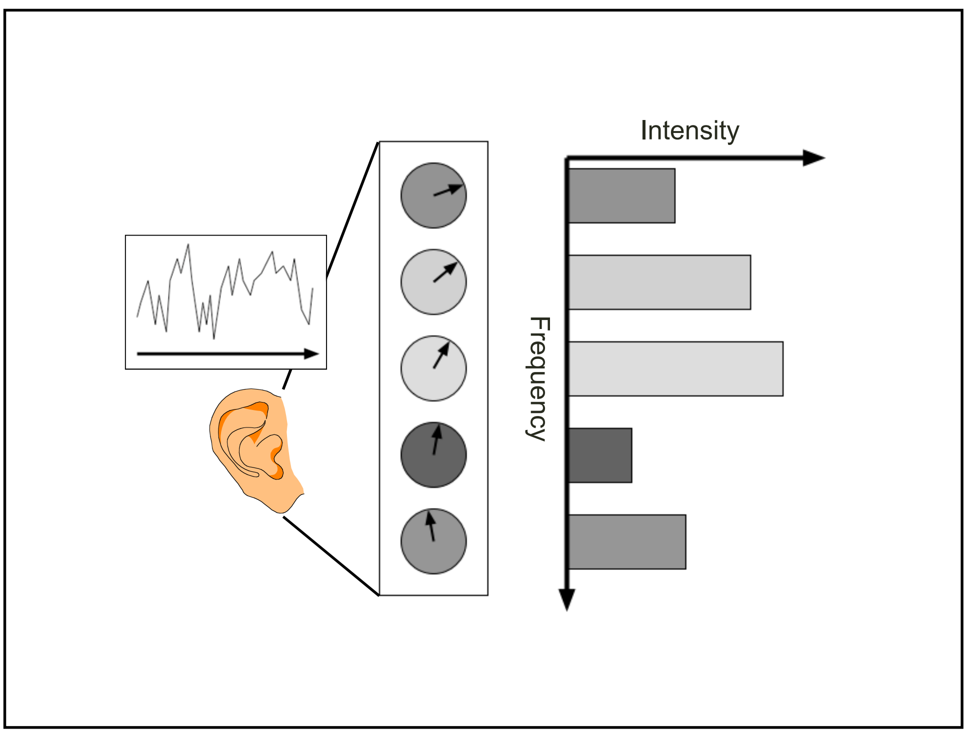
The MR signal is the sum of all the excited $\boldsymbol{M_{xy}}$'s (ignoring t now).

$$S(k_x, k_y) = \iint M_{xy}(x,y)e^{-i(k_x x + k_y y)}dxdy$$

This looks just like a Fourier Transform, where (k_x, k_y) are the frequency variables. Thus, its inverse is just like the inverse FT,

$$M_{xy}(x,y) = \iint S(k_x, k_y)e^{i(k_x x + k_y y)}dk_x dk_y$$

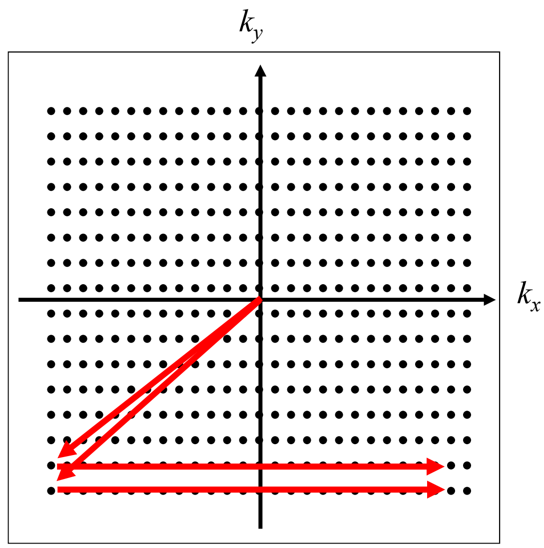

Magnetic Field Gradient


Intensity
Frequency

k-Space Traversal

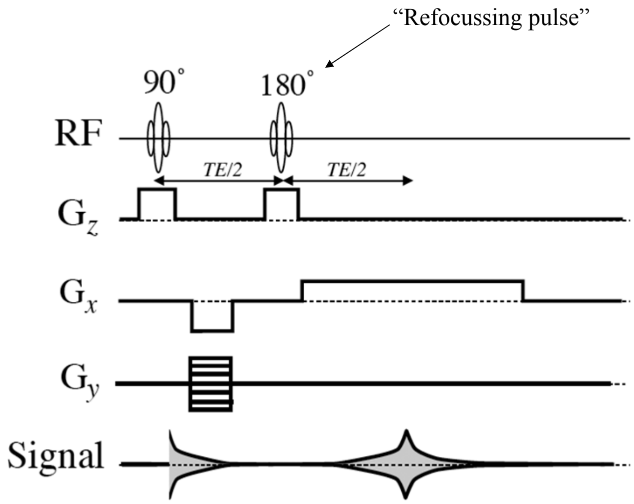
$$k_x(t) = \int_0^t G_x(\tau)d\tau$$

$$k_y(t) = \int_0^t G_y(\tau)d\tau$$



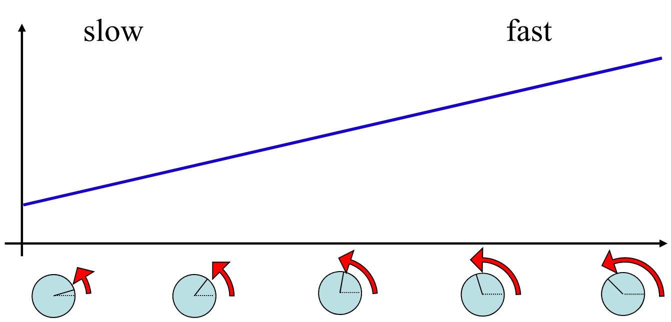
How to control the gradients

Spin-Echo Imaging

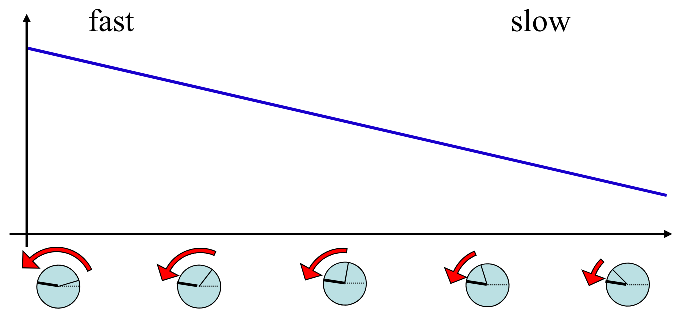


Gradient-Echo Imaging

Instead of flipping all the dipoles by 180°, reverse the phase difference to refocus the dipoles.



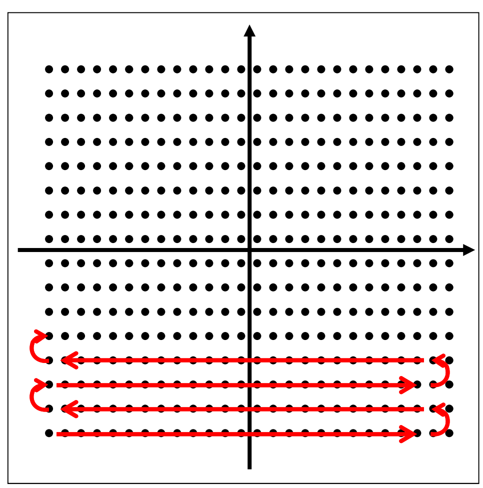
Gradient-Echo Imaging



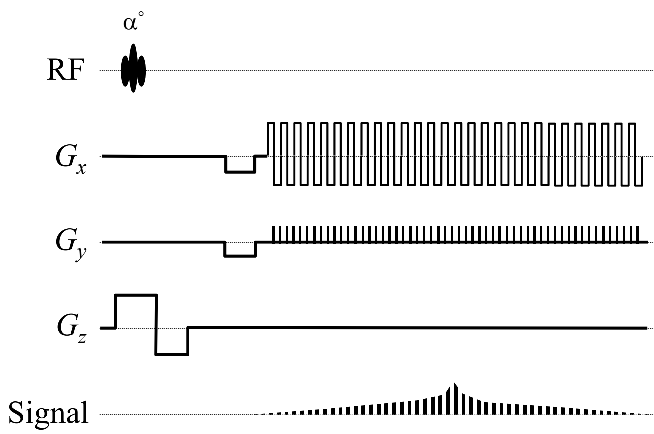
Echo-Planar Imaging (EPI)

A gradient-echo technique that allows one to collect a whole slice in one excitation.

Exhibits $T2^*$ contrast, not T2 contrast.



EPI Scan Sequence



EPI Ghost Artifact