









$$\begin{aligned} &1111111111 = \\ &512 + 256 + 128 + 64 + 32 + 16 + 8 + 4 + 2 + 1 = \\ &1023 \end{aligned}$$

Adding in Binary:

One of the Basic Operations of a CPU
&
A Useful Skill for a Computer Scientist

- Naming things in computer science is an important skill since the computer only understands on/off also known as 0/1 (**binary** digits or **bits**).

- Recall the “Naming Game” where we used bits to uniquely name items—after more than a handful of items, it becomes difficult to keep track of the names!

Item	Computer Name (using three bits)
	0 0 0
	0 0 1
	0 1 0
	0 1 1
	1 0 0
	1 0 1
	1 1 0
	1 1 1

- ***Binary addition*** is a strategy to help keep track of the uniqueness of computer names we assign to items.

The Rules of Binary Addition

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$$0 + 0 = 0$$

The Rules of Binary Addition

$$0 + 0 = 0$$

$$0 + 1 = 1$$

The Rules of Binary Addition

$$0 + 0 = 0$$

$$0 + 1 = 1$$

$$1 + 0 = 1$$

The Rules of Binary Addition

$$0 + 0 = 0$$

$$0 + 1 = 1$$

$$1 + 0 = 1$$

$$1 + 1 = 10$$

The Rules of Binary Addition

$$0 + 0 = 0$$

$$0 + 1 = 1$$

$$1 + 0 = 1$$

$$1 + 1 = 10$$

That last
rule is strange!

Let's recall how we did regular addition in elementary school...

e.g.

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Means "for
example"

e.g.

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e.g.

$$\begin{array}{r} 238 \\ + 5 \\ \hline \end{array}$$

Let's recall how we did regular
addition in elementary school...

Means "for
example"

e.g.

$$\begin{array}{r} 238 \\ + \quad 5 \\ \hline \end{array}$$



Let's recall how we did regular
addition in elementary school...

Means "for
example"

e.g.

$$\begin{array}{r} 238 \\ + \quad 5 \\ \hline \end{array}$$

3




Let's recall how we did regular
addition in elementary school...

Means "for
example"

e.g.

$$\begin{array}{r} 1 \\ 238 \\ + \quad 5 \\ \hline \end{array}$$

3



Let's recall how we did regular
addition

Means "for
example"

This digit
is "carried" to the next
column

e.g.

1

$$\begin{array}{r} 238 \\ + 5 \\ \hline 3 \end{array}$$

Let's recall how we did regular
addition

Means "for
example"

This digit
is "carried" to the next
column

e.g.

A vertical column addition problem is shown. The top number is 238, and the bottom number is 5. A plus sign is to the left of the 5. A horizontal line is drawn below the 5. Below the line, the number 3 is written. A pink arrow points from the 8 in the top number down to the 3 in the bottom number. Another pink arrow points from the 3 in the top number down to the 3 in the bottom number. A pink digit 1 is written above the first arrow, indicating a carry.

$$\begin{array}{r} 238 \\ + 5 \\ \hline 3 \end{array}$$

Let's recall how we did regular
addition

Means "for
example"

This digit
is "carried" to the next
column

e.g.

A vertical column addition problem is shown. The top number is 238, and the bottom number is 5. A plus sign is to the left of the numbers. A horizontal line is drawn below the 5. Below the line, the sum 43 is written. Two pink arrows point downwards: one from the 8 in 238 to the 3 in 43, and another from the 5 to the 4 in 43. A pink digit '1' is placed above the first arrow, indicating a carry of 1 from the ones column to the tens column.

$$\begin{array}{r} 238 \\ + 5 \\ \hline 43 \end{array}$$

Let's recall how we did regular
addition

Means "for
example"

This digit
is "carried" to the next
column

e.g.

$$\begin{array}{r} 238 \\ + 05 \\ \hline 43 \end{array}$$

Let's recall how we did regular
addition

Means "for
example"

This digit
is "carried" to the next
column

e.g.

$$\begin{array}{r} 238 \\ + 05 \\ \hline 43 \end{array}$$

We "imagine" this
digit

Let's recall how we did regular
addition

Means "for
example"

This digit
is "carried" to the next
column

e.g.

A vertical column addition problem is shown. The top number is 238, and the bottom number is 95. A plus sign is to the left of the 95. A horizontal line is drawn below the 95. Below the line, the sum 43 is written. A pink '1' is written above the 238, with a pink arrow pointing down to the 2. Another pink arrow points down from the 8 to the 3, and a third pink arrow points down from the 5 to the 8. The 43 is aligned under the 3 and 8 respectively.

$$\begin{array}{r} 238 \\ + 95 \\ \hline 43 \end{array}$$

We "imagine" this
digit

Let's recall how we did regular
addition

Means "for
example"

This digit
is "carried" to the next
column

e.g.

A vertical column addition problem. The top number is 238, the bottom number is 95, and the result is 243. A pink '1' is written above the 238, with a pink arrow pointing down to the 2 in the hundreds column. Another pink arrow points down from the 8 in the ones column to the 3 in the ones column of the result. A third pink arrow points down from the 5 in the tens column to the 4 in the tens column of the result. A fourth pink arrow points down from the 2 in the hundreds column to the 2 in the hundreds column of the result. A horizontal line is drawn between the numbers and the result.

$$\begin{array}{r} 1 \\ 238 \\ + 95 \\ \hline 243 \end{array}$$

We "imagine" this
digit

Let's see an example of binary
addition.

e.g.

Let's see an example of binary addition.

Means "for example"

e.g.

Let's see an example of binary addition.

Means "for example"

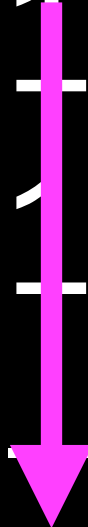
e.g.

$$\begin{array}{r} 101 \\ + 1 \\ \hline \end{array}$$

Let's see an example of binary addition.

Means "for example"

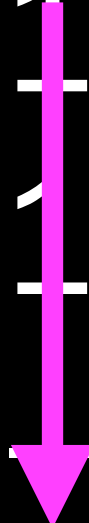
e.g.

$$\begin{array}{r} 101 \\ + 1 \\ \hline \end{array}$$


Let's see an example of binary addition.

Means "for example"

e.g.

$$\begin{array}{r} 101 \\ + 1 \\ \hline 110 \end{array}$$


The diagram illustrates the binary addition of 101 and 1. The numbers are aligned vertically, with a plus sign to the left. A horizontal dashed line separates the addends from the result. The result is 110. A pink arrow points from the 1 in the second row to the 0 in the result row.

Let's see an example of binary addition.

Means "for example"

e.g.

$$\begin{array}{r} 1 \\ 101 \\ + 1 \\ \hline 0 \end{array}$$

The diagram illustrates a binary addition problem. The first row shows the number 101. The second row shows a plus sign followed by the number 1. A horizontal dashed line separates the addends from the result. Below the line, the number 0 is shown. A pink arrow points from the 1 in the second row to the 0 in the result row, indicating a carry-over.

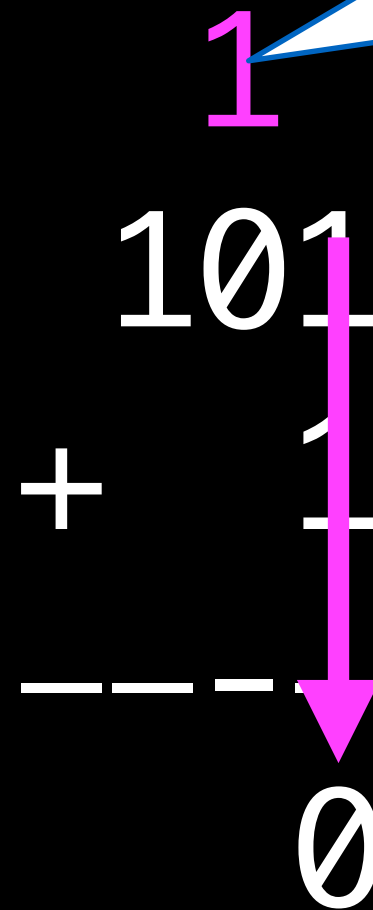
Let's see an example of binary

addition

Means "for example"

This digit is "carried" to the next column

e.g.

$$\begin{array}{r} 101 \\ + 1 \\ \hline 110 \end{array}$$


Let's see an example of binary

Means "for example"

This digit is "carried" to the next column

e.g.

$$\begin{array}{r} 101 \\ + 11 \\ \hline \end{array}$$

Diagram illustrating binary addition. The numbers 101 and 11 are added. A pink arrow points from the 1 in the 2's place of the second number to the 0 in the 2's place of the first number, indicating a carry. The result of the addition is 0, shown below the horizontal line.

Let's see an example of binary

Means "for example"

This digit is "carried" to the next column

e.g.

$$\begin{array}{r} 101 \\ + 11 \\ \hline 100 \end{array}$$

The diagram illustrates the binary addition of 101 and 11. The numbers are aligned by their least significant bits. A horizontal line separates the addends from the sum. The sum 100 is written below the line. Two pink arrows highlight the carry process: one arrow points from the '1' in the units place of the sum (0) up to the '1' in the tens place of the sum (0), and another arrow points from the '1' in the tens place of the sum (0) up to the '1' in the hundreds place of the sum (1). This shows how the carry from the units place is added to the tens place, and the carry from the tens place is added to the hundreds place.

Let's see an example of binary

Means "for example"

This digit is "carried" to the next column

e.g.

$$\begin{array}{r} 101 \\ + 01 \\ \hline 110 \end{array}$$

The diagram illustrates the binary addition of 101 and 01. The numbers are aligned by their least significant bits. A horizontal dashed line separates the addends from the result. The result 110 is shown below the line. Two magenta arrows indicate the carry process: one arrow points from the '1' in the units place of the addends down to the '0' in the units place of the result, and a second arrow points from the '1' in the tens place of the addends down to the '1' in the tens place of the result, representing the carry of 1 to the next column.

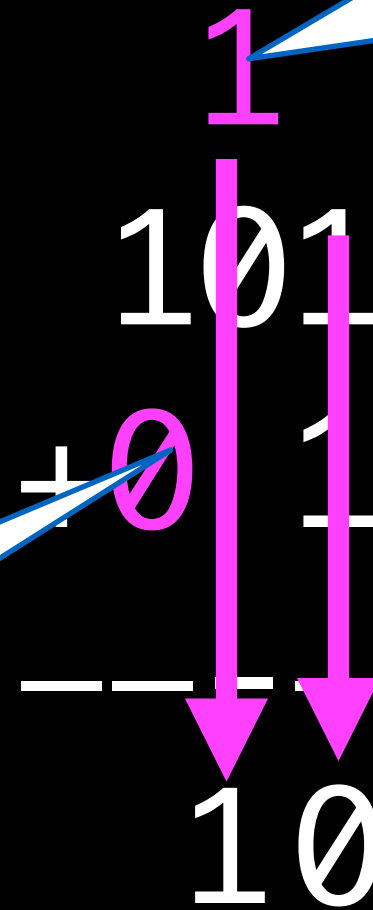
Let's see an example of binary

addition

Means "for example"

This digit is "carried" to the next column

e.g.

$$\begin{array}{r} 101 \\ + 01 \\ \hline 110 \end{array}$$


We "imagine" this digit

Let's see an example of binary

Means "for example"

e.g.

This digit is "carried" to the next column

$$\begin{array}{r} 101 \\ + 011 \\ \hline 100 \end{array}$$

We "imagine" this digit

Let's see an example of binary

addition

e.g.

Means "for example"

This digit is "carried" to the next column

$$\begin{array}{r} 101 \\ + 011 \\ \hline 110 \end{array}$$

We "imagine" this digit

You try one:

You try one:

$$\begin{array}{r} 1101 \\ + 101 \\ \hline \end{array}$$

You try one:

$$\begin{array}{r} 1101 \\ + 101 \\ \hline \end{array}$$

0

You try one:

$$\begin{array}{r} 1 \\ 1101 \\ + 101 \\ \hline \end{array}$$

0

You try one:

$$\begin{array}{r} 1 \\ 1101 \\ + 101 \\ \hline 10 \end{array}$$

You try one:

$$\begin{array}{r} 1 \\ 1101 \\ + 101 \\ \hline 010 \end{array}$$

You try one:

$$\begin{array}{r} 11 \\ 1101 \\ + 101 \\ \hline 010 \end{array}$$

You try one:

$$\begin{array}{r} 11 \\ 1101 \\ + 101 \\ \hline 0010 \end{array}$$

You try one:

$$\begin{array}{r} 111 \\ 1101 \\ + 101 \\ \hline 0010 \end{array}$$

You try one:

$$\begin{array}{r} 111 \\ 01101 \\ + 101 \\ \hline 0010 \end{array}$$

You try one:

$$\begin{array}{r} 111 \\ 01101 \\ + 101 \\ \hline 10010 \end{array}$$