Lecture 1

- Book "Formal languages and automata" Peter Linz 4th edition

Grammar

extremely rigorous → Syntax - Rules

Semantic - Meaning

- Toream Machine proves that it is the powerful machine

Languages

- Alphabet (Σ "Sigma")
- $\Sigma = \{a, b\}$ is $\Sigma = \{0, 1\}$

Example (string)

Σ = abaaa

a = pq$rs#ab

Strings

- $\Sigma = \{a, b\}$, then abab

Strings : x, y, z

Symbol : a, b

Operation

- Addition & Multiplication

Sem.h

Concatination $w = a_1 a_2 \ldots a_n$ $v = b_1 b_2 \ldots b_m$

$wv = a_1 a_2 \ldots a_n b_1 b_2 \ldots b_m$

$vw = b_1 b_2 \ldots b_m a_1 a_2 \ldots a_n$

$(ab)^3$ is NOT ababab

Length of String

w = abaaa then $|w| = 5$

Empty String

λ symbol - Empty String with no symbols at all

↑

(ε - Epsilon , Λ - Capital Lamda)

Nuetral Element → $|\lambda| = 0$

$\lambda w = w\lambda = w$

Formal Definition Of Length

$|w^n| = n * |w|$

$|w^0| = \lambda$ for all w with $|w^0| = |\lambda| = 0$

The Operators * and +

*, +; Symbol are defined by the infinite sets

$a^* = \{a\}^* = \{\lambda, a, aa, aaa, \ldots\}$

$a^+ = \{a\}^+ = \{a, aa, aaa, \ldots\}$ - Computer cannot generate λ

$\emptyset$ Empty Set

$\{\lambda\}$ **NOT** Empty Set

- Languages are set therefor all sets are part of the properties

Example (New):

$\Sigma = \{a, b\}$ then Σ^* is given

$\Sigma^* = \{a, b\}^* = \{\lambda, a, b, aa, ab, ba, bb, \ldots\}$

Definition Language

- A **language** whose alphabet is Σ is defined very generally as a subset of Σ^*

simple - Regular Language

Example:

$L = \{a, aa, aab\}$ ← Finite Language (Boring)

Operations of language

$L^0 = \{\lambda\}$

$L^1 = L$

Lecture 2

$\boxed{E \cup O = N}$

- λ - does not add to the length
- length is always finite
- String is infinite
- We can count an infinite set (integers)
- We can count the rational numbers

$$\begin{array}{cccc} \frac{1}{1} & \frac{2}{1} & \frac{3}{1} & \frac{4}{1} \\ \frac{1}{2} & \frac{2}{2} & \frac{3}{2} & \frac{4}{2} \\ \frac{1}{3} & \frac{2}{3} & \frac{3}{3} & \frac{4}{3} \end{array}$$

Fundamental

Language:

Remember

* $L = \{a^n b^n : n > 0\} = \{\lambda, ab, a^2b^2 \ldots\}$

$L^* = L^0 \cup L^1 \cup L^2 \cup L^3$

$\boxed{\text{or} = \cup = +}$

<u>Example:</u>

$L = \{a^n b^n : n \geq 0\}$

$L = \{\lambda, ab, aabb, aaabbb, \ldots\}$

$LL = \{\lambda, ab, aabb \ldots\} \quad \{\lambda, ab, aabb \ldots\}$

$= \{\underbrace{\lambda, ab, aabb \ldots}_{\lambda L}, \underbrace{abab, abaabb \ldots}_{ab\,L} \; aabbab, aabbaabb \ldots$

- The empty set is a language
  $L = \{\} = \emptyset$ (Empty Set)

- $\emptyset^* = \{\lambda, \emptyset, \emptyset\emptyset, \emptyset\emptyset\emptyset\}$
  $= \{\lambda, \emptyset, \emptyset, \emptyset\}$
  $= \{\lambda, \emptyset\}$

- $\{\lambda\}^* = \{\lambda, \lambda\lambda, \lambda\lambda\lambda \ldots\}$
  $= \{\lambda\}$

- Two sets are identical then they are the same
  $\{a\} = \{a, a, a\}$ $\qquad$ $\{a\} \neq \{a, aa, aaa\}$
  Length = 1

Midterm #1 (Problem)

Prove / Disprove the following:

(a) True

(b) False

(c) False

(d)

$\lambda \in L_1$

$\lambda \in L_2$

$\Sigma^* \subseteq L_1 \Sigma^* L_2$ (Even if $L_1 = L_2 = \lambda$)

$L_1 \Sigma^* L_2 \subseteq \Sigma^*$ by def

II First set of a...

set is the empty ...

$A = B$

$A \subseteq B$

$B \subseteq B$

$A = B$

Midterm #1 (Problem)

$\Sigma\Sigma = \{a, b\}\{a, b\}$

$= \{aa, ab, ba, bb\}$

$u = u u^R u$

$w = ba\ ab\ ba$

All the words with $(\Sigma\Sigma)^*$ $\Rightarrow (\Sigma\Sigma)^* = \{\lambda, aa, ab, ba, bb, aaaa \ldots\}$

Personal Practice (8/26/17)

Excercise 1:

a. $\lambda^* = \{\lambda\}^* = \{\lambda, \lambda\lambda, \lambda\lambda\lambda \ldots\}$

$= \{\lambda, \lambda, \lambda \ldots \lambda\}$

$\{\lambda\}^* = \{\lambda\}$

b. $L, \emptyset L = L\emptyset = L$ $\qquad L\emptyset = \emptyset$

$L, \emptyset L = L\emptyset \neq L$ $\qquad L\emptyset \neq L$

c. $L, \emptyset \cap L = L \cap \emptyset = L$

$L, \emptyset = \emptyset = L$ They are not the same

D. $\lambda \in L_1 \subseteq \Sigma^*$; $\lambda \in L_2 \subseteq \Sigma^*$ then $(L_1 \Sigma^* L_2)^* = \Sigma^*$

$L_1 = A$ $\qquad A = B$

$L_2 = B$ $\qquad A \subseteq B$

$\underline{B \subseteq B}$

$A = B$

Cartesian Product:

$L_1 = \{in, out\}$

$L_2 = \{come, law, door\}$

$L_1 \circ L_2 = \{income, inlaw, indoor, outcome, outlaw, outdoor\}$

$L_2 \circ L_1 = \{comein, comeout, lawin, lawout, door in, doorout\}$

Example:

$L = \{01, 1, 100\}$, then

$110001110011 \in L^*$,

concatenation...

$1 \circ 100 \circ 01 \circ 1 \circ 100 \circ 1 \circ 1$

Personal Practice (8/26/17)

Excercise 1:

a. $\lambda^* = \{\lambda\}^* = \{\lambda, \lambda\lambda, \lambda\lambda\lambda \ldots\} \ldots$

$= \{\lambda, \lambda, \lambda \ldots \lambda\}$

$\{\lambda\}^* = \{\lambda\}$

b. $L, \emptyset L = L\emptyset = L$ $\quad$ $L\emptyset = \emptyset$

$L, \emptyset L = L\emptyset \neq L$ $\quad$ $L\emptyset \neq L$

C. $L, \emptyset \cap L = L \cap \emptyset = L$

$L, \emptyset = \emptyset = L$ They are not the same

D. $\lambda \in L_1 \subseteq \Sigma^*$ & $\lambda \in L_2 \subseteq \Sigma^*$ then $(L_1 \Sigma^* L_2)^* = \Sigma^*$

$L_1 = A$ $\quad$ $A = B$

$L_2 = B$ $\quad$ $A \subseteq B$

$\underline{B \subseteq B}$

$A = B$

Cartesian Product:

$L_1 = \{in, out\}$

$L_2 = \{come, law, door\}$

$L_1 \circ L_2 = \{income, inlaw, indoor, outcome, outlaw, outdoor\}$

$L_2 \circ L_1 = \{comein, comeout, lawin, lawout, door in, doorout\}$

Example:

$L = \{01, 1, 100\}$, then

$110001110011 \in L^*$,

concatenation...

$1 \circ 100 \circ 01 \circ 1 \circ 100 \circ 1 \circ 1$

Problem

Given $\Sigma = \{a,b\}$, give some examples of strings in and not in, these sets where:

A) $\{W \mid \text{for some } u \in \Sigma\,\Sigma,\ w = u\,u^R\,u\}$

Σ $w = a,\ aa,\ ab,\ ba,\ aaa$

Σ $w^R = b,\ bb,\ ba,\ ab,\ bbb$

$\Sigma = \{a,b\}$ $\Sigma = \{a,b\}$

$\Sigma\,\Sigma =$

Automata Theory (Lecture 3)

$\langle \rangle$ - Variable

...g ...ool

Example - Grammer

productions

$S \to NP \ Pred$

$NP \to Art \ N$

$Pred \to Verb$

$Art \to a \mid the$

$N \to boy / dog$

$Verb \to runs / walks / sing$

Nonterminal variables

Terminals

$S \to \underbrace{NP}$ $Pred$

$\to Art \ N \ Pred$

$\to \underbrace{a \quad dog \quad sings}$

↑ terminals

This is a sentence / string / word

Definition

Terminals are the alphabet

Example 1.11

Consider the grammer

$G = (\{S\}, \{a, b\}, S, P)$

with P given by

$P = \{S \to aSb$

$S \to \lambda\}$

$S \to aSb / \lambda$

OR

$G = \{N, T, S, P\}$

Memorize

$G = (\{S\}, \{a,b\}, S, P)$

$P = \{S \to aSb$

$S \to \lambda\}$

λ never part of alphabet

Ex of strings

① $S \to \lambda$

② $S \to aSb \to a\lambda b = ab$

③ $S \to aSb \to aaSbb = a^2b^2$

$\searrow aSb \qquad = a^3b^3$

④ $S \to aSb \to aaSbb \to aaaSbbb$

$\searrow aSb \qquad \searrow aaaSbbb$

The language of this grammar is:

$\to L = \{a^n b^n : n \geq 0\}$

Generates

$S \to Ab$

$\downarrow$

$a^n A b^n$

$a^n \quad b^n$

$L = \{a^n b^{n+1} : n \geq 0\} \to L = \{a^n b^n b$

$S \to Ab$

$A \to aAb \mid \lambda$

Grammer

① $S \to Ab \to b$

$\quad \hookrightarrow \lambda$

② $S \to Ab \to aAbb \to abb \to ab^2$

$\quad \hookrightarrow aAb \qquad \hookrightarrow \lambda$

$G = (\{S, A\}, \{a, b\}, S, P)$

Example 2: number of a = number of b

$L = \{w : n_a(w) = n_b(w)\}$

$= \{aabb\,ab,\ aabb\,aabb,\ aabb,\ ab\ ab,\ ababab,\ bbaa\}$

$aSb \qquad SSS \qquad bSa$

$S \to SS$

$S \to SSS$

$S \to SS \to aSbaSb$

$\hookrightarrow \lambda \quad \hookrightarrow abSab$

$\hookrightarrow \lambda$

* P12 Midterm (chapter 2)

Personal Practice (8/31/17)

Example: 1-12 $L = \{a^n b^{n+1} : n \geq 0\}$ $G = \{\{S, Q\}, \{a, b\}, S, P\}$

$L_0 = \{a^0 b\} \Rightarrow L = \{Sb\}$

Let

$P = \{S = Qb, Q = aQb, Q = \lambda\}$

→ ① $S \to Qb \to \lambda b \to b$

② $S \to Qb \to aQbb\ (aQb^2) \to a\lambda b^2 \to ab^2$

③ $S \to Qb \to aQbb(aQb^2) \to a^2Qb^3 \to a^2\lambda b^3 \to a^2b^3$

Practice Example: 1-12.2

$L = \{a^n b^{n+2} : n \geq 0\}$ $G = \{\{S, Q\}, \{a, b\}, S, P\}$

Let $P = \{S =$

Automata Theory (lecture 4)

P2 (c) $S \to aXb$

$X \to \begin{cases} \lambda & a\lambda b = ab \\ aX & aaXb = aa\lambda b = \end{cases}$

redundant $\to Xa$

$\begin{cases} bX \\ Xb \end{cases}$

P3) $S \to aaA \mid \lambda$

$A \to bS$

① $S \to \lambda$

② $S \to aaA \to aab\lambda \to aab$

③ $S \to aaA \to aabS \to aabaaA \to aabaabS$

aabaab

④ $S \to aaA \to aabS \to aabaaA \to aabaabS \to aabaabaaA$

$\to aabaabaabS$

$\to aabaabaab\lambda$

$\to$ aabaabaab

$(aab)^3$

P4) What language does the language

P5b) $L_2 = \{a^{3n}b^{2n} : n \geq 2\}$

$L_2 = \{a^6b^4, b^9b^6, \ldots\}$

$n \geq 2 \to m = n - 2$ $\qquad n = m + 2$

if $n = 2 \Rightarrow m = 0$

* important

$L_2 = \{a^{3(m+2)}b^{2(m+2)} : m \geq 0\}$

$L_2 = \{a^6a^{3m}b^4b^{2m} : m \geq 0\}$

$L_2 = \{a^6(aaa)^m b^4(bb)^m : m \geq 0$

$L_2 = \{a^6(aaa)^m(bb)^m b^4 : m \geq 0\}$

* Problem (new)

$L_3 = \{ a^{n+3} b^n \quad n \geq 2 \}$ $\qquad m = n - 2 \qquad n = m + 2$

$n = 2 \Rightarrow m = 0$

$L_3 = \{ a^3 a^n b^n \quad n \geq 2 \}$

$L_3 = \{ a^3 a^{m+2} b^{m+2} : m \geq 0 \}$

$L_3 = \{ a^5 a^m b^m b^2 : m \geq 0 \}$

$b^{-1} = \lambda$

* Problem $\quad L_4 = \{ a^n b^{n-3} : n \geq 3$

$m = n - 3 \qquad n = m + 3$

if $n = 3 \Rightarrow m = 0$

$L_4 = \{ a^{m+3} b^m : m \geq 0 \}$

$L_4 : \{ a^3 \underbrace{a^m b^m}_{B} : m \geq 0 \}$

$B \to aBb \mid \lambda$

$S \to aaaB$

Personal Practice Problem

Example 1-12: $L = \{a^n b^{n+1} : n \geq 0$

$G = \{ \{S, A\}, \{a, b\}, S, P \}$ $P = \{ S = Ab, \ A = aAb, \ A = \lambda$

① $S = Ab \rightarrow S = \lambda b \rightarrow S = b$

② $S = Ab \rightarrow S = aAbb \rightarrow S = a\lambda bb \rightarrow S = ab^2$

③ $S = Ab \rightarrow S = aAbb \rightarrow S = aaAbbb \rightarrow S = aa\lambda bbb \rightarrow S = a^2 b^3$

Example (My own) $L = \{ a^{n-1} b^{n+1} ; n > 2 \}$

$G = \{ \{S, A\}, \{a, b\}, S, P \}$ $P = \{ S = aaAbbbb \quad A = aAb \quad , A = \lambda$

① $S = aaAbbbb \rightarrow aa\lambda bbbb \rightarrow a^2 b^4$

② $S = aaAbbbb \rightarrow aaaAbbbbb \rightarrow a^3 \lambda b^5 = a^3 b^5$

③ $S = aaAbbbb \rightarrow aaaAbbbbb \rightarrow aaaaAbbbbbb = a^4 \lambda b^6 = a^4 b^6$

Example: $L = \{ a^{n-2} b^{n+2} : n \geq 3 \}$

$G = \{\{S, A\}, \{a, b\}, S, P\}$ $P = \{ S = aAbbbbb, \ A = aAb, \ A = \lambda \}$

① $S = aAbbbbb \rightarrow a\lambda bbbbb \rightarrow ab^5$

② $S = aAbbbbb \rightarrow aaAbbbbbb \rightarrow a^2 \lambda b^6 \rightarrow a^2 b^6$

Examples Rearding languages and their grammers:

P1. Given the alphabet $\Sigma = \{a\}$, find a grammar for the language $L = \{a^n$, where n is even and $n > 3\}$.

$G = \{ \{S\}, \{a\}, S, P\}$ $P = \{S = aaaaA \ ; \ A = aaA \ ; \ A = \lambda$

① $S = aaaaA \rightarrow aaaa\lambda = a^4$

② $S = aaaaA \rightarrow aaaaaaA = a^6$

$= a^8$

P2(a) All strings with exacty two a's

$G = \{ \{S, A\}, \{a, b\}, S, P \}$ $P: \{ S: aaA, A = bA, A = \lambda$

① $S: aaA \to aa\lambda \to aa$
② $S: aaA \to aabA \to aab\lambda \to aabA$
③ $S: aaA \to aabA \to aabbA \to aabb\lambda \to aabb$

P2(c): All strings with no more than 3A's

$G: \{ \{S, A\}, \{a, b\}, S, P \}$ $P = \{ S: AA, AaA, AaAaA, AaAaAa$

P2(d) All string with atleast three a's

$G = \{ \{S, A\}, \{a, b\}, S, P \}$ $P = \{ S: AaAaAaA$

$A: aA$

$A = Ab$

$A = \lambda$

P2(e) All string that start with a end with b

$G = \{ \{S, A\}, \{a, b\}, S, P \}$ $P = \{ S: aAb, A = aA, A: Ab, A = \lambda \}$

① $S: aAb \to a\lambda b \to ab$
② $S: aAb \to aaAb \to a^2\lambda b \to a^2 b$

P5(a) $L_1 = \{a^n b^m : n \geq 1 ; m < n\}$

$m = n - 1$ $m = 0$

$n = m + 1$

$L_1 = a$

$n = 1$

① $S_1 = aA_1 \to a\lambda \to a$

② $S_1 = aA_1 \to aaA_1 \to aa\lambda \to a^2$

$S_1 = aA_1 \mid AA \mid \lambda$

③ $S_1 = aA_1 \to aA_1 \to aaA_1b \to a^2\lambda b \to a^2b$

$A_1 = aA_1b \mid \lambda A_1 \mid \lambda$

P5(b) $L_2 = \{a^{3n} b^{2n} : n \geq 2\}$

$n - 2 = m$ $m = 0$

$n = 2 + m$

$L_2 = \{a^{3(m+2)} b^{2(m+2)}\} = a^6 a^{3m} b^4 b^{2m}$

$\cancel{a^6 \, aaa \, b^4 \, bb}$

$S_2 \to aaaaaaAbbbb$

$A \to aaaAbb \mid \lambda$

P5(c) $L_3 = \{a^{n+3} b^n : n \geq 2\}$

$n - 2 = m$

$n = 2 + m$ $m = 0$

$L_3 = \{a^{(m+2)+3} b^{(m+2)}\} = a^{m+5} b^{m+2} = a^5 a^m b^2 b^m$

$S_3 \to aaaaaA_3bb$

$A_3 \to aA_3b \mid \lambda$

P5(d) $L_4 = \{a^n b^{n-3} : n \geq 3\}$

$= \{a^{3+m} b^{(3+m)-3} : n \geq 3\}$

$= \{a^3 a^m b^0 b^m\}$

Expand: $a^3b^0 = a^3$

$a^4b^1 =$

a^5b^2

$S_4 = aaaA_4$

$A_4 = aAb \mid \lambda$

$n - 3 = m$

$n = 3 + m$

$m = 0$

P6(a) Find the grammar that generates $L = L_1 L_2$

$S = S_1 S_2$

$S_1 = aaaA_1$

$A_1 = A_1 b \mid \lambda$

$S_2 = bbbbA_2$

$A_2 = A_2 aa \mid \lambda$

P5(f): Find the grammar that generates $L = L_1 \cup L_2$

$L_1 = S_1$

$S_1 = aaA_1$

$A_1 = A_1 b \mid \lambda$

$L_2 = S_2$

$S_2 = aA_2$

$A_2 = bA_2 \mid \lambda$

P5(g): Find the grammars that generates $L = (L_1)^3$

$L_1 = S_1 S_1 S_1$

$L_1 = A_1 B_1 A_1 B_1 A_1 B_1$

$A_1 = aB_1 a \mid bA_1 b \mid \lambda$

$B_1 = bA_1 b \mid aB_1 a \mid \lambda$

Automata Theory (lecture 5)

Practice Problems:

$L_1 = a^n b^m : n \geq 1 ; m < n\}$ $\quad S_1 \to A_1 B_1$ $\quad L_1 = a, a^2, a^2 b, a^3, a^3 b$

$A_1 \to a A_1 \mid a$

$B_1 \to a B_1$

$L_2 = \{a^{3n} b^{2n} : n \geq 2\}$ $\quad S_2 = aaaaaa A_2 bbbb$

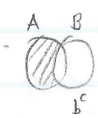
$A_2 \to aaa A_2 bb \quad \lambda$

$S_3 \to aa$

$A_3 \to aaaaa \ldots\ldots$

Practice Problem

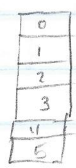
$l = L_1^*$ $\quad A - B = A \cap B^c$



$L = \{\lambda, L_1, L_1 L_1, L_1 L_1 L_1, \ldots\}$

$L_1 - (L_4)^c = L_1 \cap ((L_4)^c)^c = L_1 \cap L_4$

mod $(17, 3) = 2$



$17 \bmod 6 = 5$

P7 (a):

$L = \{w : |w| \bmod \overset{3k}{3} > 0 = \{0, 1, 2\}$

$\{|w| \bmod 3 > 0\}$ = Word of size $K \cdot 1$ or $K \cdot 2$ $= \begin{cases} a^{3k+1} \\ a^{3k+2} \end{cases}$ $k = 0, 1, 2, 3$

$= a^1, a^2, a^4, a^5, a^7, a^8 \ldots$

4 mod 3 $\quad$ 5 mod 3

$S \to S_1 \mid S_2$

$S_1 \to aaa S_1 \mid a$

$S_2 \to aaa S_2 \mid aa$

P8. Show that the grammars

$S \to aSb \mid bSa \mid SS \mid a$ Produces aa

$S \to$

P9. Equivalent

★ P10) $L = \{ww^R : w \in \{a,b\}^*\}$

P12) $L = \{a^{n+p} b^{n-q} : n \geq s\} = \{a^{s+p} b^{s-q}, a^{s+p+1} b^{s+1-q}$

$s, s+1, s+1+1$ $s-q \geq 0$

$s \geq q$

$\begin{cases} m = s-q \geq 0 \to \underline{s = m+q} \\ \cancel{n \geq q \to n \geq m+q} \end{cases}$

$L = a^{s+p} b^{s-q}, a^{s+p+1} b^{s+1-q}, a^{s+p+2} b^{s+2-q}$

$= a^{m+q+p} b^{m}, a^{m+p+q+1} b^{m+1}, a^{m+p+q+2} b^{m+2}$

$L = a^{p+q} a^m b^m, a^{p+q} a^{m+1} b^{m+1}, \ldots$

$L = a^{p+q} a^m b^m : m \geq 0$

$S \to a^{p+q} A_{jr}$

$A_{jr} \to a A_{jr} b \mid \lambda$

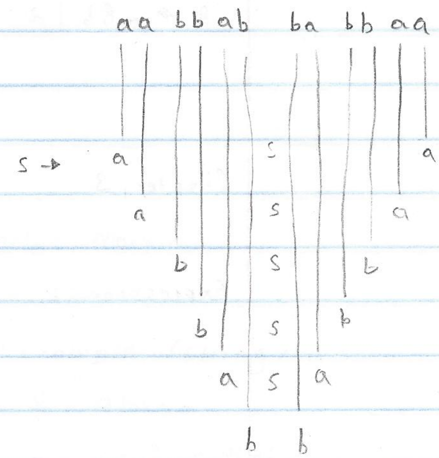
Automata Theory Class

Example Problem:

$L = \{ a^{n+b} b^{n-a} \; ; \; n \geq s \}$

$L = \{ w = uu^R \; ; \; u^R \in \Sigma^* = (a,b)^* \}$

Let $u = aabbab$; $w = \underbrace{aabbab}_{u} \underbrace{ba\ bb\ aa}_{u^R}$



$S \to aSa \mid bSb \mid aa \mid bb \mid \lambda$

$S \to aSa \to aaSaa \to aabSbaa$

$\to aabbSbbaa \to aabbaSabbaa$

$\to aabbabbabbaa$

Example

$L = \{ a^{n+3} b^{n-2} \; ; \; n \geq 2 \}$

$(n = 2, 3, 4, \ldots)$ $\quad n - 2 = m$ if $n = 2 \to m = 0$

$m = 0, 1, 2, 3 \ldots$

$n = m + 2$

$L = a^{m+2+3} b^{m+2-2}$

$L = \{ a^m a^5 \; b^m \; ; \; m \geq 0 \}$

$aaaaa \; a^m \quad b^m$

$S \to a^5 B$

$B \to aBb \mid \lambda$

Example Problem

$L = \{ a^{n-p} b^{n-q} \; ; \; n \geq s \}$ In this case $n = s, s+1, s+2, \ldots$

$L = \{ a^{s+p} b^{s-q}, \; a^{s+1+p} b^{s+1-q}, \; a^{s+2+p} b^{s+2-q}, \ldots \}$

$L = \{ a^{s+p} b^{s-q}, \; a^{s+p+1} b^{s-q+1}, \; a^{s+p+2} b^{s-q+2}, \ldots \}$

Let $m = s - q \to s = m + q$

$L = \{ a^{m+q+p} b^m, \; a^{m+q+p+1} b^{m+1}, \; a^{m+p+q+2} b^{m+2}, \ldots \}$

$L = \{ a^{p+q} a^m b^m, \; a^{p+q} a^{m+1} b^{m+1}, \; a^{p+q} a^{m+2} b^{m+2}, \ldots \}$

$L = \{ a^{p+q} a^m b^m, \; m \}$

$\to m = s - q \geq 0$ $\quad \boxed{\text{Constraint: } s \geq q}$

Continued...

$L = \{ a^{p+q} a^m b^m : m \geq 0 \}$

$S \rightarrow a^{p+q} B$

$B \rightarrow aBb \mid \lambda$

Chapter 3

Expressions: λ, $\emptyset$, $a \in \Sigma$; if a and b $(a \circ \beta)$, $(a \cup B)$; a^* — Nothing is a regular expression unless it follows (1) through (5)

- Regular expessions represent a language called Regular Language

Reg Exp	Reg Lang
λ	$\{\lambda\}$
$\emptyset$	$\emptyset$ is a set
$a \in \Sigma$	$\{a\}$
$\alpha \in \Sigma$	(α) is a set
$\beta \in \Sigma$	$(\alpha\beta)$ is a set
	$(\alpha \cup \beta)$ union set

Notation

$(\alpha \cup \beta) \leftrightarrow (\alpha + \beta)$

$\emptyset^* = \{ \}^* = \{\lambda, \dots\} = \{\lambda\}$

$(110)^* (0+1) = ((110)^* 0 + (110)^* 1)$

$(110)^* = \{\lambda, 110, 110110, 110110110, \dots\}$

$\Sigma = \{0,1\} = \{0\} \cup \{1\} = (0+1)$

$\Sigma\Sigma = \{00, 01, 10, 11\} = (0+1)(0+1) = (00 + 01 + 10 + 11) = (00, 01, 10, 11)$

Oct 31, 2016

$L = \{a^i b^j c^k : i < j < k$

1. $uv^ixy^iz \in L$
2. $|vy| > 0$
3. $|vxy| \leq m$

$S = a^m b^{m+1} c^{m+2} \Rightarrow |S| = 3m+3 > m$

$S = uvxyz$ ← Comes from Theorem (Pumping Lema)

i is tyically 0 or 2

$u = \lambda$
$v = a^{m/2}$
$x = \lambda$
$y = b^{m/2}$
$z = b^{\frac{m}{2}+1} c^{m+2}$

$u = \lambda$
$v = \lambda$
$a^m \begin{cases} x = a \\ y = a^{m-1} \end{cases}$
$z = b^{m+1} c^{m+2}$

$S = uvxyz$
$= a^{m/2} a^{m/2} \cdot \lambda \cdot b^{m/2+1} c^{m+2}$
$= a^m b^{m+1} c^{m+2}$

$S = uvxyz$
$= \lambda \cdot \lambda \cdot a \cdot a^{m-1} \cdot b^{m+1} \cdot c^{m+2}$
$= a^m b^{m+1} \cdot c^{m+2} = S$

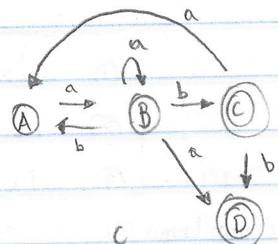
$S = a^m b^{m+1} c^{m+2}$
$S = uvxyz$

$u = a^n b^{m+1}$
$v = c^{m/2}$
$x = \lambda$] $|xyz| = m$
$y = c^{m/2}$
$z = c^2$

$uv^ixy^iz \in L$
$a^m b^{m+1} (c^{\frac{m}{2}})^i \cdot \lambda \, (c^{m/2})^i c^2$
$i = 0 \quad a^m b^{m+1} \quad c^2 \notin L$

NFA

$L_1 =$



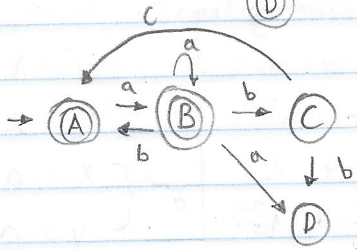
L_1^c

L_1^R

$(L_1^c)^R$

$L_1 - L_2 = L_1 \cap L_2^c$

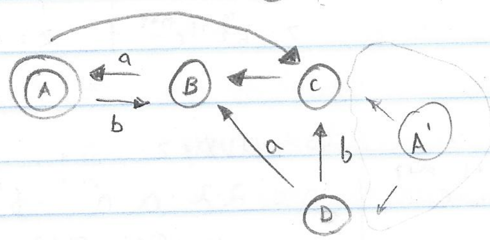
$L_1 = L^c$



$s = abb$ you have to

$s = bba$ begin their

L^R



$L_1 - L_2^R = L_1 \cap (L_2^R)^R$

$L_1 \cap L_2 = (L_1^c \cup L_2^c)^c$

Chapter 8 November 2, 2017

3.1 Turing Machines

$B = \{w = u \# u \in \Sigma^*\}$ ← A finite language $\Sigma = \{0, 1, \#\}$

$w_1 = 0101 \# 0101 \in B$

$w_2 = 111 \# 111 \in B$

$w_3 = 001 \# 001 \in B$

$$\boxed{\begin{array}{l} \forall i \geq 0 \quad uv^ixy^iz \in B \\ vy > 0 \checkmark \\ |vxy| \leq m \checkmark \end{array}}$$

(0) Cannot have # when $= v^iy^i$

$v^2 \neq \#\#$

(1) Let $s = \underbrace{0^m 1^m}_{uv} \underset{x}{\#} \underbrace{0^m 1^m}_{yz}$ $|s| = 4m + 1 > m$

$u = 0^m 1^{m/2}$ $y = \lambda$

$v = 1^{m-1}$ $z = 0^{\frac{m+1}{2}} 1^m$

$i = 0$

$uv^ixy^iz = 0^m 1 \# 0^m 1^m \notin B$

$i = 2$

$uv^ixy^iz = 0^m 1 (1^{m-1})^i \# 0^m 1^m$

$= 0^m 1^{mi-i+1} \# 0^m 1^m$

$= 0^m 1^{2m-1} \# 0^m 1^m \notin B$

$|vy| = m - 1 > 0$

$|vxy| = |1^{m-1} \cdot \# \cdot \lambda| = m \leq m$

∎ proof ✓

(2) $i = 0$ $s = uvxyz =$

$u \; v \; x \; y \; z = 0^m 1 (1^{m-1})^i \# 0^m 1^m = 0^m 1 (1^{m-1})^i \# 0^m 1^m$

$0^m 1 \;\; 1^{m-1} \;\; \# \;\; \lambda \;\; 0^m 1^m$

$$\overbrace{\underbrace{\cancel{0}\cancel{1}\cancel{1}\cancel{0}\cancel{0}\cancel{1}\cancel{1}\cancel{0}}_{M} \# \underbrace{\cancel{0}\cancel{1}\cancel{1}\cancel{0}\cancel{0}\cancel{1}\cancel{1}\cancel{0}}_{M}}^{w}$$

If no pound # is found then reject

M = "On input w:

1. Check (from L → R) that w contains one # symbol, if not rejected

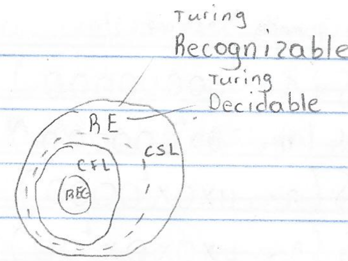
$L = \{w = u \# u^R$

↑ Context Free!

If anything is left then reject

Determines if string is the language

Turing recognizable



☆ Turing on Midterm (write the algorithem that operate's the machine)

Exercise 1:

1. Visualize language

$A = \{0^n \mid m = 2^n \text{ and } n \geq 0\}$

$A = \{0, 00, 0000, \underbrace{00000000}_{W} \ldots\}$

W = Any String in A

M_2 = "On input w:

1.

1000000001

1 0 0 0 0 1

1 0 0

1 0 accept

Exercise 2:

$\Sigma = \{0\}$ $A = \{0, 00, 0000$

M_2 = "On input w"

1000000

10 0 0

10 0

Reject by step 3.

Figure 3.4 Turing Machine

$\delta(q_1, \overset{\downarrow}{0}0000000) \to \delta(q_2, \sqcup\overset{\downarrow}{0}000000)$

$\delta(q_3, \sqcup x\overset{\downarrow}{0}00000) \to \delta(q_4, \sqcup x0\overset{\downarrow}{0}0000)$

$\delta(q_3, \sqcup x0x\overset{\downarrow}{0}000) \to \delta(q_4, \sqcup x0x0\overset{\downarrow}{0}00)$

$\delta(q_3, \sqcup x0x0x\overset{\downarrow}{0}0) \to \delta(q_4, \sqcup x0x0x0\overset{\downarrow}{0})$

$\delta(q_3, \overset{\downarrow}{\sqcup}x0x0x0x)$

$\downarrow$

$\delta(q_5, \sqcup x0x0x\overset{\downarrow}{0}x) \to \delta(q_5, \sqcup x0x0\overset{\downarrow}{x}0x)$

$\delta(q_5, \sqcup x0x\overset{\downarrow}{0}x0x) \to \delta(q_5, \sqcup x0\overset{\downarrow}{x}0x0x$

Do table → Picture

Picture → Do table

Thursday Midterm

Nov 9, 2017 Chapter 6a Context Free Languages

☆ DFA → CFG (Page 7) Midterm Question

Skip 2.3

Important 2.4, 2.5

Skip Chomsky

Chapter 6b Push Down Automaton (One, Not Plural)

(PDA) ☆ Midterm - Given the following gramme Prove that it is a PDA

☆ Midterm - Formal lanaguag

☆ Before 2.10

☆ 2.10 - Midterm - Build PDA, Turing machine

☆ 2.11A

☆ 2.11B

☆ 2.11C - Midterm forsure (non-deterministic)

23 ~ Ignore (From Summary) - diamond

Chapter 7 Closure properties Context Free languages

☆ Closed Under
- Union
- Concatenation
- Star
- Reversal
- Intersection with Reg Lang

☆ - Any Reg Lang is Context Free

Not Closed
- Intersection
- Complementation
- SD $(A,B) = A \cup B - A \cap B$
- Difference $A \cap B^c$

☆ Ex 8.7 (in linz)

☆ Pumping Lemma for context free language

- Using PDA recognize this is context free
- then build turing machine

Final - Delta Description

November 9, 2017

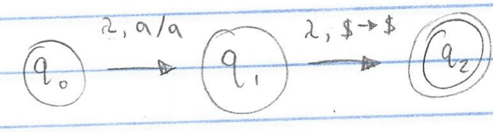
$L = \{a^n b^n; n \geq 0\}$

Design / Handwrite:

PDA

1. Here come the a's. I push them into the stack
2. Here come b's. I pop the stack such that for each a in the stack, there a b poping it
3. If at the end there are no were b's and the stack is empty accept
4. If there are still a's in the stack and there are no more b's, reject.
5. If the stack is empty and there are still b's, reject

Build a PDA:



ab

aabb

a, $ → a$

a, a → aa

b, a → λ

Poping of a

Design a Turing Machine for it (call it M_1)

M_1 = "On input w:

1. Assume the head is all the way to the left
2. Cross of the a's (write an x ontop of it)
3. Continue to the right until "a, b" is formal. (cone off the b (write a y on top of it)
4. Go all the way back to the left until the 1st "x" is found. Go to step 2
5. (Accept / Reject statement)

If no more a's check if no more b's if not reject...

aaabbb
xaabbb
xaaybb
xaaybb
xxaybb
xxayyb
xxxyyb

If no b is found then go to step 5

If there are

Reject is there are nomore "a" but

aabb
aaabbb } Reject

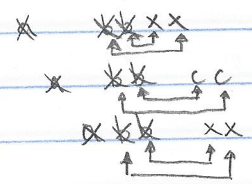
Chapter 8 (Turing Machine) November 21, 2017

* 1 or 2 question about decider
* CFL & Recursive Language

<u>TM</u>

$a^3 b^2 c^6$

$aaabbcccccc \in a^+b^+c^+$



You cross out this b, you cross out this c

3 times 2

2 times 3

$a^i b^j c^k \quad i \times j \neq c$

Flip accept & reject in statement if asked to do the compliment

---

$L = \{a^n b^n : n \geq 0\}^c$

Let M be the TM that accepts L

M = "On input w"

1. Scan the input and check if it is ~~from~~ of the form $a^+|b^+$ if it is, go to sleep — otherwise, accept.
2. Cross of a "b" for each "a" (zigzag, shodow)
3. If there are no more "a's" and there is still "b's" or if we run out of "b's" and there are still "b's", accept
3. Otherwise, reject