Robert Martinez

cs3186: Midterm #1

Probl #	Pts
1 9	55
2 20	25
3 13	25
4 5	40
5 10	35
6 0	35
Sum	100%

GRADE CALCULATION = round (
(pts#1/.55+ pts#2/.25+ pts#3/.25+ pts#4/.4+ pts#5/.35+ pts#6/.35, 0)

[55pts] **Problem #1**
Definition: The *Symmetric Difference* of two sets A, B, written SD(A, B), is their union without (minus) their intersection, that is:

$$SD(A, B) = (A \cup B) - (A \cap B) = (A - B) \cup (B - A) = (A \cap B^C) \cup (B \cap A^C)$$

11, 12, ~~1~~, 22, 3, 32

Given $\Sigma = \{1,2,3\}$, consider the following finite languages (sets) in Σ^*:
$L_1 = \{11, 12, 13\}$; $L_2 = \{1, 13, 22, 3, 32\}$; $L_3 = \{12, 13, 32\}$

[5pts] (i) Write the language $L_4 = L_2 - \Sigma\Sigma$
Answer:

1

$L_4 = \{1, 13, 22, 3, 32\} - \{1, 13, 22, 3, 32\}$ X

$L_4 = \{\} = \emptyset$

$(A \cup B) - (A \cap B)$

[5pts] (ii) Write the language $L_5 = SD(L_1, L_3)$
Answer:

5

$L_5 = SD(L_1, L_3) = \{11, 32\}$ ✓

11, ~~12~~, ~~13~~, 32 - ~~12~~, ~~13~~ = 11, 32

$(A - B) \cup (B - A) = 11, 32$

11 ∪ 32

= 11, 32

[5pts] (iii) Write the language $L_6 = SD(L_1, L_2)$
Answer:

3

$L_6 = SD(L_1, L_2) = \{11, 12, 1, 22, 3, 32\}$

[20pts] (iv) Write the sets $L_7 = SD(L_6, L_4)$ and $L_8 = SD(L_4, L_5)$. Briefly explain what happened.
Answers:

$L_7 = SD(L_6, L_4) = 11, 1, 12, 1, 22, 3, 32$ X

$L_8 = SD(L_4, L_5) = 11, 32$ X

Since L_4 is an $\emptyset$ then there is no affect to the language, because $(A \cup B) - (A \cap B)$ does not delete anything and the last two are union. Therefore answers are the same, since L_4 is $\emptyset$.

[20pts] (v) Define clearly *step by step* the set $((L_1 - L_3)^*)^C$. Then write 4 palindromes magnitude 4 from it. Each palindrome must include at least two 1's in it. Do not need to write the entire sets, just use ellipses, "…", when it is clear what comes next.
Answers:

9
20
13
5
10
―
113 → 57

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[25pts] **Problem #2**

Given $\Sigma = \{a\}$. Let $L_1 = \{w : |w| \bmod 5 = 3\}$ and $L_2 = \{w : |w| \bmod 7 = 2\}$.

(i) [5pts] Write 5 words in L_1

(ii) [5pts] Write 5 words in $(L_1)^c$

(iii) [5pts] Write 5 words in L_2

(iv) [5pts] Write 5 words in $L_3 = L_1 \cup L_2$

(v) [5pts] Write 4 words in $L_4 = L_1 \cap L_2$

Answer:

(i) $L_1 = \{w : |w| \bmod 5 = 3\}$

$|a^3| \bmod 5 = 3$

$|a^8| \bmod 5 = 3$

$|a^{13}| \bmod 5 = 3$

$|a^{18}| \bmod 5 = 3$

$|a^{23}| \bmod 5 = 3$

$L_1 = \{a^3, a^8, a^{13}, a^{18}, a^{23}\}$

5

a^{28}
a^{33}
a^{38}
a^{43}
a^{48}
a^{53}

(ii) $L_1 = \{a^1, a^2, a^4, a^5, a^6\}$

5

(iii)

$L_2 = \{w : |w| \bmod 7 = 2\}$

$|a^2| \bmod 7 = 2$

$|a^9| \bmod 7 = 2$

$|a^{16}| \bmod 7 = 2$

$|a^{23}| \bmod 7 = 2$

$|a^{30}| \bmod 7 = 2$

$L_2 = \{a^2, a^9, a^{16}, a^{23}, a^{30}\}$

5

a^{37}
a^{44}
a^{51}
a^{58}

(iv) $L_3 = L_1 \cup L_2$

$L_3 = \{a^2, a^3, a^8, a^9, a^{13}\}$

5

(v)

$L_4 = L_1 \cap L_2$

$L_4 = \{23, 0$

X

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[25pts] **Problem #3**
Given the productions B ➔ RR; R ➔ RRR | a | bR | Rb.
[5pts] (i) What are N (non-terminals), Σ (terminals), and S (starting symbol)?
[20pts] (ii) Derive at least 6 strings *before* you answer this question: What are the strings that this grammar generates?
Answer:

B → RR
R → RRR | a | bR | Rb

B = S ✓
N = BB, R, RRR
Σ = {a, b}

(i) R is your non-terminals

a & b are your terminals

B is your starting Symbol

(ii) ① B → RR → RRR RRR → a aa aaa ✓

② B → RR → RRR RRR → aaa aaRb → aaa aaab ✓

12

③ B → RR → aa. ✓

④ B → RR → RRR a → aaa a ✓

⑤ B → RR → bRa → baa ✓

⑥ B → RR → bR bR → baba. ✓

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[40pts] **Problem #4**
Write a grammar for the language $L = \{ a^{p-q}b^{p-s}: p \geq r\}$. Write necessary constraints to make your grammar consistent. **Assume q > s.**
[5pts] (i) Write at least 4 members of L
[5pts] (ii) Define a new "counting" variable considering that q > s
[5pts] (iii) Verify/prove that r - s is positive
[5pts] (iv) Re-write L using the new variable defined in step (ii)
[5pts] (v) Prepare the expression for L showing explicitly its terms before writing its grammar
[5pts] (vi) Write a concise expression for L
[10pts] (vii) Write a grammar for L
Answer:

(i) $L = \{ a^{p-q} b^{p-s} : p \geq r \}$

5

$L = \{ a^{p-q} b^{p-s}, a^{p-q+1} b^{p-s+1}, a^{p-q+2} b^{p-s+2}, a^{p-q+3} b^{p-s+3} \ldots \}$ ✓

(ii) $L = \{ a^{p-q} b^{p-s} : p \geq r \}$

replace p with r

$L = a^{r-q} b^{r-s}$

$p \geq r \rightarrow m = p - s$

$p \geq s$

$p = m + s$ ✗ ✓

(iii) $p = r \quad \therefore \quad m = r - s \geq 0$

$p \geq s$ constraints

(iv) $L = \{ a^{m+s-q} b^{m} : p \geq s \}$ ✗

(v) $L = \{ a^{m+s-q} b^{m}, a^{m+s-q+1} b^{m+1}, a^{m+s-q+2} b^{m+1}, a^{m+s-q+3} b^{m-s+1} \ldots \}$ ✗

(vi) $L = \{ a^{m+s-q} b^{m}, a^{m+s-q} a^{1} b^{m} b^{1}, a^{m+s-q} a^{2} b^{m} b^{2} \ldots \}$ ✗

$L = \{ a^{m+s-q} a^{L} b^{L} b^{m} : L \geq 0 \}$

(vii) $S \rightarrow a^{m+s-q} A b^{m}$

$A \rightarrow aAb \mid \lambda$ ✗

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[35pts] **Problem #5**
Given the alphabet $\Sigma = \{0, 1\}$, prove (true)or disprove (false) the following statements:

[5pts] The string (01*0)(01 + 10)*010 includes at least one string that begins with a double zero and ends with two zeroes separated by a one.

2 True ✓

[5pts] The string (01*0)(01 + 10)*010 includes many strings that begins with a double zero and end with a double zero separated by a one.

01^*0 Since all have 0 in between ✗

2 True ✓

[5pts] $\Sigma\Sigma$ - {00, 11, 10, 01} = {λ}

2 $\Sigma = \{0,1\}$ → ✗ $\{0, 1, 00, 01, 10, 11, 0000, 1111\} - \{00, 11, 10, 01\} \neq \{\lambda\}$

$\Sigma\Sigma$ and $\{00, 11, 10, 01\}$ cannot generate λ ✓

For any Σ prove or disprove the following: False

[5pts] $p^*q^* \cap (22)^* = \phi$

Both have λ

2 $\{\lambda, p, pp, ppp \ldots$ $\cap$ $\{\lambda, 22, 2222, 222222\}$ False ✓

$\lambda, q, qq, qqq\}$

[5pts] $\{w\} - \{s\} = \{w\} \cap (s)^C$

[5pts] $pqrs \in (p^*qs^*r^*)^*$

2 $pqrs \in (p^* q s^* r^*)^*$

$pqrs \in (p^* q^* s^* r^*)$

$\{\lambda, p, pp, ppp, \ldots \lambda, q, qq, qqq \ldots \lambda, s, ss, sss, \ldots \lambda, r, rr, rrr, \ldots\}$

cannot switch them around.. False ✓

[5pts] From the definition of L^+ and L^* it is clear that $L^+ \subseteq L^*$. Is there any case when they are equal?

$L^+ = \{L, LL, LLL, LLLL, \ldots\}$

$L^* = \{\lambda, L, LL, LLL, \ldots\}$

There is a case they are not equal... λ, because L^+ has no λ

False ✗

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[35pts] **Problem #6**

Given the alphabet $\Sigma = \{0, 1\}$, consider the alphabet $\Omega = \Sigma\Sigma$. Define a grammar for the language

$$L = \{w = x^n y^n:\ x \neq y \ \&\ x, y \in \Omega\}$$

Write 6 strings w representatives of the language L.

[20pts] (i) Write the general expression for all possible words in L

[5pts] (ii) Is u = 111111100000010 in L? Why?

[5pts] (iii) Is v = 00000000 in L? Why?

[5pts] (iv) Is z = 01010101011010101010 in L? Why?

Answers: