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Robert Martinez

cs3186: Midterm #3 (11/16/17)

Prob #	Pts
1	3
2	1
3	10
4	~~0~~ 1
5	11
Sum	

GRADE CALCULATION = round (
(pts#1/.4+ pts#2/.25+ pts#3/.25+ pts#4/.33+ pts#5/.41+ pts#6/.2)/6, 0)
NOTE: CFL = Context Free Language, RL = Regular Language, TM = Turing Machine

[10 pts] **Problem #1**
Use the **pumping lemma** (PL) to prove that the language L = $\{a^n b^{2n} c^{3n}, n \geq 0\}$ is **not** a CFL.
Do the cases when i = 0 and i = 2 (two cases only)

[10 pts] **Answer:**

- We assume pumping length for L is a CFL and obtain a contradiction. Let m be the pumping length for L that is guaranteed to exist by the pumping lemma.

Lemmas
① $uv^ixy^iz \in L$
② $|vy| > 0$
③ $|xvy| \leq m$

$m + 2m + 3m = 6m \geq m$ ✓

~~#~~3

- Both v & y contain same alphabet when we substitute i = 0 for uv^ixy^iz, therefore this does violate lemma condition 1

why? ✗

- When both v & y contain a value i = 2, the Language is recognize as uv^2xy^2z; which therefores offsets the value within the language of L, which is a contradiction and thus not CFL

✗

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[30pts] **Problem #2**
Given the language $L = \{a^n b^n c^m d^m \mid m, n \geq 1\}$. For each of the three cases below is L a CFL? if so, design (handwave) a PDA to prove it, otherwise say why it is not a CFL.
[10pts] (i) For arbitrary m's and n's.
[10pts] (ii) For m = n.
[10pts] (iii) For m $\neq$ n.
Answers:

(i) $L = \{a^n b^n c^m d^m \mid$ where m's & n's are arbitrary$\}$

aabbcd

① Push the a and b onto the stack

② Pop the a's with the c's, also Pop the b's with the d's

③ If there are a's and b's on the stack, accept

④ otherwise, reject

(ii) $L = \{a^n b^n c^n d^m \mid$ where $m = n\}$

① Push the a with b

② Pop the a's with c's and Pop the b's with d's

③ if there are no more a's and b's the the stack is empty, accept

④ Otherwise, reject!

(iii) $L = \{a^n b^n c^n d^m \mid$ where $m \neq n\}$

① Push the a's and the b's onto the stack

② Pop the a's with the c's Pop the b's with the d's

③ if there are more a's in the stack, accept

④ otherwise, reject!

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[30pts] **Problem #3**

Consider $F = \{w_n \in \Sigma^* \mid 1 \leq n \leq 100\}$. Then:

[10pts] (i) Prove that if L is a CFL then in general for all L, L – F is a CFL.

[10pts] (i) Prove that if L is not a CFL then in general for all L, L – F is not a CFL.

[10pts] (i) Prove that if L is not a CFL then in general for all L, $L \cup F$ is not a CFL.

NOTE: a correct "guess" gives you no points. You have to prove your answer.

Answer:

(i) Assuming that L is CF,

We know that L - F is Context Free Language, because by definition F is regular language (since its ~~CFL~~), therefore then in general for all L-F is CFL.

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(ii)

(iii) We assume that L is not regular (therefore it is not CFL), and we have F as a CFL therefore performing the union of both languages L ∪ F, this shows that the operation L ∪ F is not CFL.

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[30pts] **Problem #4**
Given the language $L = \{a^n b^n c^m d^m \mid m, n \geq 1\}$. Describe (handwave) a TM for each of the cases below.
[10pts] (i) For arbitrary m's and n's.
[10pts] (ii) For m = n.
[10pts] (iii) For m ≠ n.
Answers:

$L = \{a^n b^n c^m d^m \mid m, n \geq 1\}$

$n=1 \quad m=2$

$S \to AB$

$A \to aAb \mid ab$

$B \to cBd \mid cd$

(i) $\underline{\text{L}abccdd}$

1. We read an a on the left and write an w under the a
2. We read an c on the ~~right~~ and write an y under the y
3. If there are no more a and c that can't be read and both have the same number of w's and y's, accept
4. Otherwise, we restore & start over

(ii) m = n

m = 2 n = 2

$\underline{\text{L}aabbccdd}$

x x x x y y y y

1. We read an ~~a or b~~ on the left side and we write an x under the a or b that was read
2. We read an c or d on the right side and we write an y under the c or d that was read
3. If there are no more a, b, c, d that can't be read and both have same number of x and y, accept
4. Otherwise, we restore & start over or reject

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[20pts] **Problem #5**

Given the language $L = \{a^n b^n \mid n \geq 1\}$. Someone told you that despite what you learned in class (CFL are not closed under complementation), the complement L^c of L is a CFL.

[10pts] (i) Describe (handwave) a PDA for L^c, the complement of L.

[10pts] (ii) Describe (handwave) a TM for L^c, the complement of L.

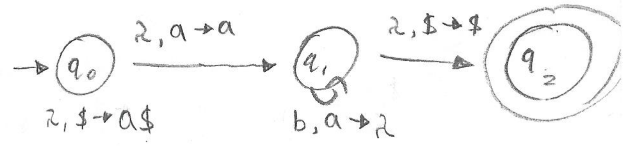
Answers:

(i) $L = \{a^n b^n \mid n \geq 1\}$

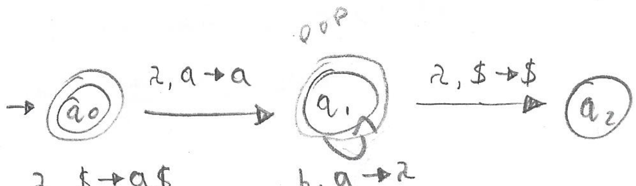
$L^c = \{a^n b^n \mid n \geq 1\}^c$

we first Start by doing PDA for L

PDA for L



Now that we have the PDA for L now we use this PDA to build PDA L^c



Design PDA for L^c

1. Push a into the stack
2. Pop the a's with the b's in state q_1 after transitioning
3. If there are a's in the stack accept
4. Otherwise, reject

There are few more cases!

3

(ii) ⌊aaabbb

x y x y x y

1. We read an a on the left then we write an x under the a
2. We read an b on the right then we write an y under the b
3. If both have the same number x's & y's and can't read anymore a's b's, reject!
4. otherwise if there are more x's then y's or more y's then x's then accept!

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esign a TM for L. (Handwork = describe)

aaa bbb cc
x y
x y
x y

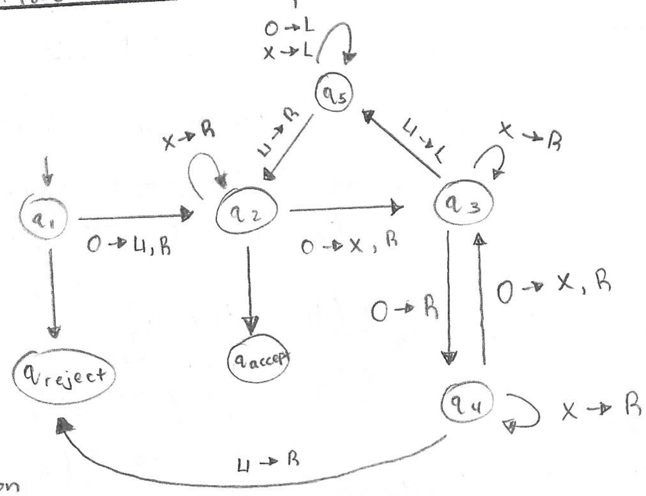
Accept!

Design:

1. We read an a on the left then we write an x under the a
2. We read an b on the right then we write an y under the b
3. If there are no more a's and b's that can't be read and both have the same number of x's and y's, accept.
4. Otherwise we restore & start over or reject...

A sample run of M_2 0000

Turing Machines Chapter 8



TM Description

$\delta(q_1, 0) = (q_2, \sqcup, R)$
$\delta(q_2, 0) = (q_3, X, R)$
$\delta(q_3, 0) = (q_4, 0, R)$
$\delta(q_4, 0) = (q_3, X, R)$
$\delta(q_3, \sqcup) = (q_5, \sqcup, L)$
$\delta(q_5, X) = (q_5, X, L)$
$\delta(q_5, 0) = (q_5, 0, L)$
$\delta(q_5, X) = (q_5, X, L)$
$\delta(q_5, \sqcup) = (q_2, \sqcup, R)$

$\delta(q_2, X) = (q_2, X, R)$
$\delta(q_2, 0) = (q_3, X, R)$
$\delta(q_3, X) = (q_3, X, R)$
⋮
$\delta(q_2, \sqcup) = (q_{accept}, \sqcup, R)$

$q_1 0000$
$\sqcup q_2 000$
$\sqcup X q_3 00$
$\sqcup X 0 q_4 0$
$\sqcup X 0 X q_3 \sqcup$

$\sqcup X 0 q_5 X \sqcup$
$\sqcup X q_5 0 X \sqcup$
$\sqcup q_5 X 0 X \sqcup$
$q_5 \sqcup X 0 X \sqcup$

$\sqcup q_2 X 0 X \sqcup$
$\sqcup X q_2 0 X \sqcup$
$\sqcup X X q_3 X \sqcup$
$\sqcup X X X q_3 \sqcup$

$\sqcup X X q_5 X \sqcup$
$\sqcup X q_5 X X \sqcup$
$\sqcup q_5 X X X \sqcup$
$q_5 \sqcup X X X \sqcup$

$\sqcup q_2 X X X \sqcup$
$\sqcup X q_2 X X \sqcup$
$\sqcup X X q_2 X \sqcup$
$\sqcup X X X q_2 \sqcup$
$\sqcup X X X \sqcup q_{accept}$

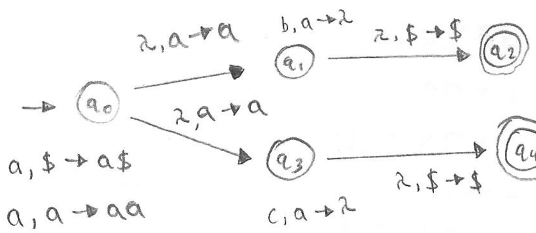
Jose Review

$L = \{a^i b^j c^k \ ; \ i = j \text{ or } i = k\}$

Design PDA for L: (Design = describe = handwork)

1. Push A into the stack
2. Pop the a's with the b's
3. If there are no more b's then the stack is empty, accept
4. Restore all the a's into the stack
5. Pop the a's with c's
6. If there are no more c's then the stack is empty, accept
7. otherwise reject

Build PDA for L: (Build = draw = Create)



Delta Transition: $\delta(aaabbbcc)$:

$\delta(q_0, aaabbbcc) \rightarrow \delta(q_0, a\$)$

→ $(q_0, aabbbcc) \rightarrow \delta(q_0, aa\$)$

→ $(q_0, abbbcc) \rightarrow \delta(q_0, aaa\$)$

→ $(q_1, \lambda bbbcc) \rightarrow \delta(q_0\ aaa)$

$\delta(q_1, bbbcc) \rightarrow \delta(q_1\ aaa)$

$\delta(q_1, bbcc) \rightarrow \delta(q_1, aa\$)$

$\delta(q_1, bcc) \rightarrow \delta(q_1, a\$)$

$\delta(q_1, \lambda cc) \rightarrow \delta(q_1, \$)$

$\delta(q_2, \$)$ ✓

accept

Trying to get to q_3 (Pop c)

$\delta(q_0, aa)$

$\delta(q_3, bbbcc) \rightarrow \delta(q_3, aa)$

$\delta(q_3, bbcc) \rightarrow \delta(q_3, aa)$

$\delta(q_3, bcc) \rightarrow \delta(q_3, aa)$

$\delta(q_3, cc) \rightarrow \delta(q_3, aa)$

$\delta(q_3, c) \rightarrow \delta(q_3, a)$

$\delta(q_3, \lambda) \rightarrow \delta(q_3, a\$)$ ✗ Reject