Chapter 6a (Context free Language)

Example 2.4

Given L = Context free, Show that L^2 is also context free.

Answer: If S derives L then $S_1 \to SS$ derives L^2

Example 2.5

What language is generated by $S \to aSbb \mid a$? is it CF

keep in mind a in the "|a" therefore must be an extra a ... +1

~~$L = \{a^n b^{n*2} ; n \geq 1\}$~~

$L = \{a^{n+1} b^{2n} ; n \geq 0\}$

a
aSbb
aaSbbbb
aaaSbbbbbb

DFA→CFG

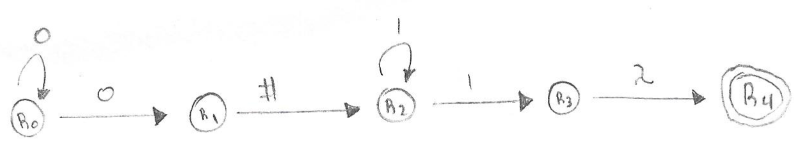
Not sure..

How to convert DFA to CFG:

1. Make a variable R_i for each state q_i Of the DFA.
2. Add the rule $R_i \to aR_j$ to the CFG if $\delta(q_i, a) = q_j$ is a transition in the DFA
3. Add the rule $R_i \to \lambda$ if q_i is an accept state of the DFA
4. Make R_0 the start variable of the grammer where q_0 is the start state of the machine

Lastly, verify on your own that the resulting GFG generates the same language

Ex: $L(G_1) = \{0^n \# 1^n \mid n > 0\}$



$\delta(R_0, 0, 00, 000, \ldots) \to \delta(R_1, \#) \to \delta(R_2, 1, 11, 111, \ldots) \to \delta(R_3, \lambda) \to \delta(R_4, accept)$

Is this correct?

What does step 2 mean?	My implementation of #3 is correct?

Chapter 6b (Push down Automata ie: PDA)

Example 2.9

The following is the formal description of the PDA that recognizes the language $\{0^n 1^n \mid n \geq 0\}$

Let M_1 be $(Q, \Sigma, \Gamma, \delta, q_1, \$, F)$ where

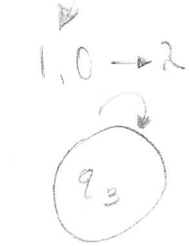
$Q = \{q_1, q_2, q_3, q_4\}$

$\Sigma = \{0, 1\}$

$\Gamma = \{0, \$\}$

$F = \{q_1, q_4\}$

input:	0			1			λ		
Stack:	0	\$	λ	0	\$	λ	0	\$	λ
q_1									$\{(q_2, \$)\}$
q_2			$\{(q_2, 0)\}$	$\{(q_3, \lambda)\}$					
q_3				$\{(q_3, \lambda)\}$				$\{(q_4, \lambda)\}$	
q_4									



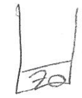
Variable	Description
Q	A finite set of States, like the states of a finite automaton
Σ	Is a finite input alphabet analoguos to the corresponding component of FA
Γ	Is the stack alphabet. The finite set of Symbols that we are allowed to push onto the stack
F	A set of accepting states or final states

PDA do not Start at 0 (q_0)

Chapter 6 b continued...

$i = 1$

abc

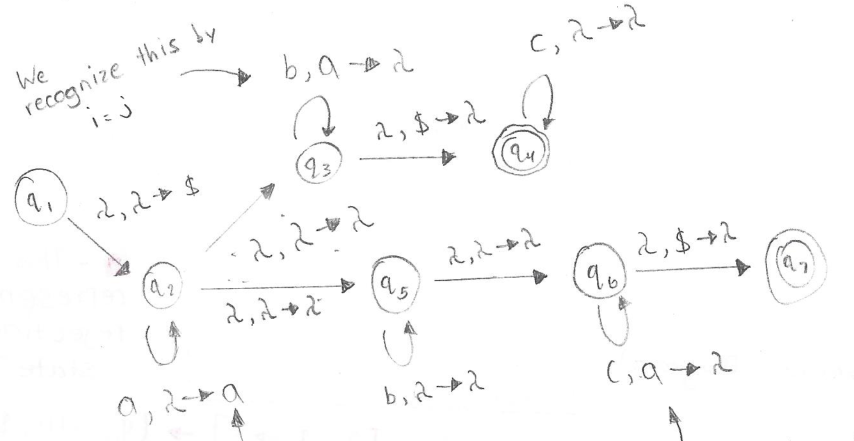


$\delta(q_2, abc, a)$

Example 2.10

$L : \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i = j \text{ or } i = k\}$

We recognize this by $i = j$



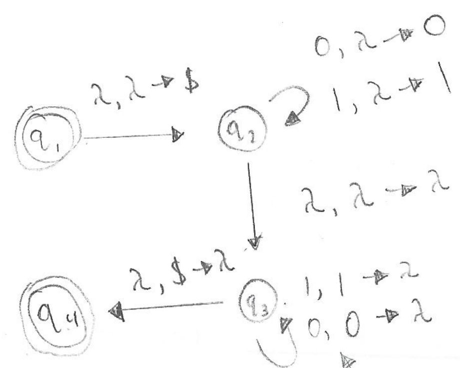
we recognize this (a), because i is determined by $i = j$ $i = k$, therefore values are placed on stack

We recognize this, because $i = k$

Example 2.11

$\{ww^R \mid w \in \{0,1\}^*\}$

$w = 0^i 1^i 1 0^i$



We recognize 0, 0 → λ, because it consist of the same amount of variables, thus $ww^R \mid w \in \{0,1\}^*$, ie the language

Deterministic Vs Non-deterministic

- Deterministic
  - Starts with q_1
  - Does NOT contain $ (stack not implemented)
- Non-deterministic
  - Starts with q_0

Example 2.11 A (A second solution to problem 2.11)

$L_{wwr} = \{ ww^R \mid w \in \{0,1\}^* \}$ ⟶ It is a CFL, generated by the grammar

$P \to 0P0 \mid 1P1 \mid \lambda$ ⟵ *

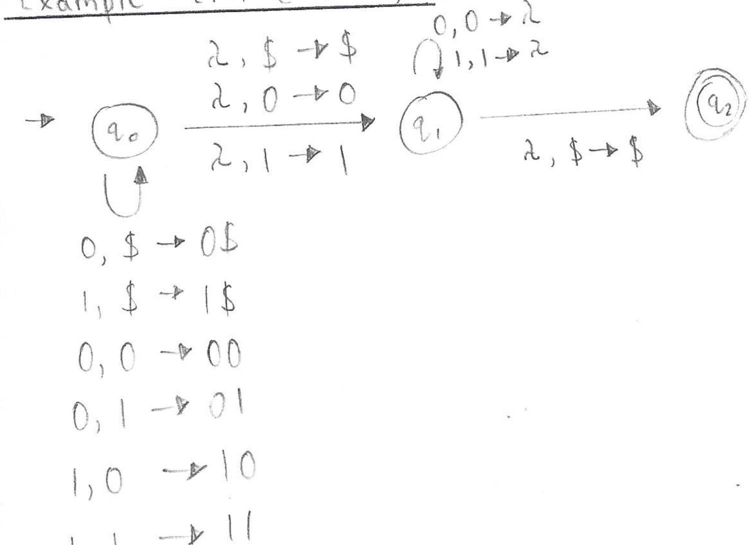
* Designing informal PDA

1. Start in a state q_0, we stack all values of w while on stack q_0.

2. This time, w will be on the stack, with the right end w at the top and the left end at the bottom

3. ...

Example 2.11 ((6.3) (genaralized transition Diagram)



■ - This represents rejecting state

Initial ID: $(q_0, 1111, \$)$

(1) $(q_0, 1111, \$) \to [\lambda, \$ \to \$] \to (q_1, 1111, \$)$...
$(q_0, 1111, \$) \to [1, \$ \to 1\$] \to (q_0, 111, 1\$)$

(2) $(q_0, 111, 1\$) \to [\lambda, 1 \to] \to (q_1, 111, 1\$)$...
$(q_0, 111, 1\$) \to [1, 1 \to 11] \to (q_0, 11, 11\$)$

(3) $(q_0, 11, 11\$) \to [\lambda, 1 \to 1] \to (q_1, 11, 11\$)$

(4) $(q_1, 11, 11\$) \to [1, 1 \to \lambda] \to (q_1, 1, 1\$)$

(5) $(q_1, 1, 1\$) \to [1, 1 \to \lambda] \to (q_1, \lambda, \$)$

$\to (q_2, \lambda, \$)$

Accept

Value we are inputing & value we are taking that's on the stack already

The value we are outputing to the stack

Initial ID: $(q_0, 111, \$)$ (My Attempt)

(1) $(q_0, 111, \$) \to [1, \$ \to 1\$]$
(2) $(q_0, 11, 1\$) \to [1, 1 \to 11]$
(3) $(q_0, 1, 11\$) \to [\lambda, 1 \to 1]$
(4) $(q_1, 1, 11\$) \to [1, 1 \to \lambda]$
(5) $(q_1, \lambda, 1\$) \to$ Reject! Cannot continue iterating, the input has been exhausted

TM read write *

"We read an a on the left then

"We write an x under the a"

We restore then start over

□ Any <u>Regular language</u> is <u>context free</u>

Pumping Lema Theorem

① $uv^ixy^iz \in L$

② $|vy| > 0$

③ $|xvy| \leq m$

* Pumping Lema used to show that C is not CFL

Question:

Show that the language $L = \{a^n b^n : n > 0, n \neq 100\}$ is CF

Answer:

Let $L_1 = \{a^{100} b^{100}\}$, which is regular (finite). Also $(L_1)^c$ is regular

$$L = \{a^n b^n : n > 0\} \cap (L_1)^c$$

Therefore it is Context Free

Additional:

(a) $\{0^n 1^n 0^n 1^n ; n \geq 0\}$

Let $A = \{0^n 1^n 0^n 1^n ; n \geq 0\}$. Let m be the length of the pumping lemma. We show that $s = 0^m 1^m 0^m 1^m$ cannot be pumped. Let $s = uvxyz$ If either v or y contain more than one type of alphabet symbol, uv^2xy^2z does not contain the symbols in the correct order. Hence it cannot be member of A. Because s cannot be pumped without violating the pumping lemma conditions, A is not context Free

Extra Exercise

$(A \cup B)^c = (A^c) \cap (B^c)$

Compliment of context free is not context free.

Context Free Languages

- Union
- Con
- *
- Intersection
- Reverse (c^R)

8.7 Linz (Bedo)

Show that language $L = \{a^n b^n : n \neq 100, n \geq 0\}$ is CF.

$L_1 = a^{100} b^{100}$

Details $L = \{a^1b^1, a^2b^2, a^3b^3 \ldots\}$

$L = \{a^{98}b^{98}, a^{99}b^{99}, a^{101}b^{101} \ldots\}$

$(L \cup L_1)^c = L^c \cap L_1^c = L: \{a^n b^n ; n \geq 0\} \cap (L_1)^c$ $\boxed{L \text{ is CF}}$

Example 2.20

Use the pumping lemma to show that the language $B = \{a^n b^n c^n \mid n \geq 0\}$ is not CF

Answer:
- Suppose that B is a CFL and Thus $B = L(G)$ for some CFG Let m be the constant Specified by the pamping lemma (PL)
- Then $w = a^N b^N c^N$ with $N \geq m/3$ is in $L(G)$ and has the represenative $w = uvxyz$ Such that v ory is not the empty String λ (Note that $|w| \geq m$
- Since B is a CFL $uv^ixy^iz \in L(G)$ for each $i = 0, 1, 2 \ldots$
- Since $|vxy| \leq m$, vxy can contain at most 2 of the 3 symbols from $\{a, b, c\}$.
- Since $|vy| > 0$, v and y together contain at least one symbol
- Consider the string uv^2xy^2z
- This String